Sensor & Simulation Notes
Note 369

An Exponentially Tapered Transmission Line Antenna

Nicholas H. Younan, Bobby L. Cox,
Clayborne D. Taylor, and William D. Prather

Abstract—**In this paper, the analysis and design of an exponentially tapered transmission line antenna is presented. The exponentially tapered transmission line is designed to operate such that it has radiator characteristics at high frequency and serves as a matching section at low frequency. The NEC-2 is used to model the antenna at frequencies ranging from 500 MHz to 1 GHz to obtain the input impedance and the desired radiation pattern.**

I. Introduction

In general, the illumination of large system structures can be accomplished via the use of continuous-wave (CW) testing, where a test object, such as an aircraft, is placed under the structure driven by either a common-mode or a differential-mode excitation, for electromagnetic pulse (EMP) effects. CW testing can be used to identify EMP coupling and perform hardness surveillance testing to evaluate hardening techniques.

For EMP applications, uniform illumination is required. However, nonuniform illumination may occur at frequencies where the test object characteristic dimension exceeds several wavelengths. Accordingly, an exponentially tapered transmission line is designed to operate, over the frequency range of 100 kHz to 1 GHz, such that it has radiator characteristics at the high frequencies and serves as a matching section at low frequencies. Since the characteristic impedance of this section of transmission line will be different at one end than at the other, this exponentially tapered transmission line has sometimes been referred to as a transition section.

In this paper, the analysis and design of an exponentially tapered transmission line is presented. Due to the wide operating frequency range, a numerical rather than an analytical analysis is performed. Accordingly, a procedure that incorporates the use of the NEC code is developed [1]. As a result, the desired radiation pattern and the corresponding input impedance are determined to ascertain the performance of the antenna at high frequencies.

II. Analysis

A. Exponentially Tapered Transmission Line

A transmission line whose impedance per unit length, Z, and admittance per unit length, Y, vary with distance down the line is known as a nonuniform transmission line. This transmission line is illustrated in Fig. 1.

This configuration can be analyzed as a transmission line with varying characteristic impedance. One such nonuniform transmission line is the case in which Z and Y vary exponentially with distance down the line z as follows [2]–[4]:

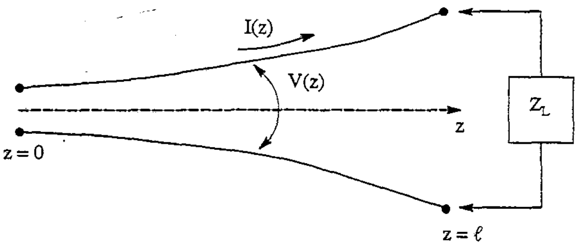

Fig. 1. The exponentially tapered transmission line.

$$Z(z) = j\omega L_0 e^{qz} \tag{1}$$
$$Y(z) = j\omega C_0 e^{-qz}. \tag{2}$$

Here, L_0 and C_0 are the inductance and capacitance per unit length at the input and q is the taper factor. The characteristic impedance at any point z on the transmission line can be expressed as

$$Z_c(z) = \sqrt{\frac{Z(z)}{Y(z)}} = e^{qz}\sqrt{\frac{L_0}{C_0}} = Z_{c0}e^{qz} \tag{3}$$

where Z_{c0} is the input characteristic impedance (i.e., at $z = 0$). Note that $Z_c(z)$ does not represent the ratio of V/I for an infinite line.

For an exponentially tapered transmission line (ETTL), the voltage and current vary with distance down the line z as follows:

$$V(z) = V_1 e^{-\gamma_1' z} + V_2 e^{-\gamma_1'' z} \tag{4}$$
$$I(z) = I_1 e^{-\gamma_2' z} + I_2 e^{-\gamma_2'' z} \tag{5}$$

where the subscripts 1 and 2 denote $+z$ and $-z$ traveling waves, respectively, and the exponent terms are defined as

$$\gamma_{1,2}' = \mp\frac{q}{2} + j\beta \tag{6}$$

and

$$\gamma_{1,2}'' = \mp\frac{q}{2} - j\beta \tag{7}$$

where

$$\beta = \sqrt{\omega^2 L_0 C_0 - \left(\frac{q}{2}\right)^2}. \tag{8}$$

The variation of the voltage and current along the transmission line, in differential forms, are as follows:

$$\frac{dV(z)}{dz} = -Z(z)I(z) \tag{9}$$

and

$$\frac{dI(z)}{dz} = -Y(z)V(z). \tag{10}$$

Accordingly, the $+z$ and $-z$ traveling current can be expressed as

$$I_1 = \frac{V_1\gamma_1'}{j\omega L_0} \tag{11}$$

and

$$I_2 = \frac{V_2\gamma_1''}{j\omega L_0}. \tag{12}$$

Manuscript received March 23, 1992; revised October 12, 1993.
N. H. Younan, B. L. Cox, and C. D. Taylor are with the Department of Electrical and Computer Engineering, Mississippi State University, Mississippi State, MS 39762.
W. D. Prather is with Phillips Laboratory, Kirtland AFB, NM 87117.
IEEE Log Number 9216504.

B. Input Impedance Derivation

The input impedance to the transmission line can be expressed as

$$Z_{\text{in}} = \frac{V(0)}{I(0)} = \frac{V_1 + V_2}{I_1 + I_2}. \tag{13}$$

Substituting I_1 and I_2 from (11) and (12) into (13) yields

$$Z_{\text{in}} = \frac{1 + \frac{V_1}{V_2}}{\frac{V_1}{V_2} + \frac{\gamma_1''}{\gamma_1'}} \left(\frac{j\omega L_0}{\gamma_1'}\right). \tag{14}$$

If the line has no tapering, i.e., $q = 0$, (6) becomes

$$\gamma_{1,2}' = j\omega\sqrt{L_0 C_0} = j\omega\sqrt{\mu_0 \epsilon_0} = j\beta_0. \tag{15}$$

Evaluating (3) at $z = 0$, L_0 can then be solved as

$$L_0 = Z_c(0)\sqrt{\mu_0 \epsilon_0} = \frac{\beta_0}{\omega} Z_c(0). \tag{16}$$

Consequently, (14) for Z_{in} can be re-written as

$$Z_{\text{in}} = \frac{1 + \frac{V_1}{V_2}}{\frac{V_1}{V_2} + \frac{\gamma_1''}{\gamma_1'}} \left(\frac{j\beta_0}{\gamma_1'}\right) Z_c(0). \tag{17}$$

If an arbitrary load is placed on the transmission line at $z = \ell$ (see Fig. 1), the voltage and current on the line at this termination point will obey the following relationship:

$$Z_L = \frac{V(\ell)}{I(\ell)} = \frac{V_1 e^{-\gamma_1'\ell} + V_2 e^{-\gamma_1''\ell}}{V_1 e^{-\gamma_2'\ell} + \frac{\gamma_1''}{\gamma_1'} V_2 e^{-\gamma_2''\ell}} \left(\frac{j\omega L_0}{\gamma_1'}\right). \tag{18}$$

Solving for the ratio V_1/V_2 yields

$$\frac{V_1}{V_2} = e^{j2\beta\ell} \frac{j\beta_0 Z_c(0) e^{q\ell} - Z_L \gamma_1''}{Z_L \gamma_1' - j\beta_0 Z_c(0) e^{q\ell}}. \tag{19}$$

Thus the input impedance to an exponentially tapered transmission line with input characteristic impedance $Z_c(0)$ and loaded with an arbitrary complex load Z_L can be expressed as

$$Z_{\text{in}} = j\beta_0 Z_c(0) \cdot \frac{Z_L(\gamma_1' - \gamma_1'' e^{j2\beta\ell}) + j\beta_0 Z_c(0) e^{q\ell}(e^{j2\beta\ell} - 1)}{Z_L \gamma_1' \gamma_1''(1 - e^{j2\beta\ell}) + j\beta_0 Z_c(0) e^{q\ell}(\gamma_1' e^{j2\beta\ell} - \gamma_1'')}. \tag{20}$$

The above expression for the input impedance can be simplified further to yield

$$Z_{\text{in}} = Z_c(0) \cdot \frac{Z_L[q \tan(\beta\ell) + 2\beta] + j2\beta_0 Z_c(0) e^{q\ell} \tan(\beta\ell)}{Z_c(0) e^{q\ell}[2\beta - q \tan(\beta\ell)] + j2 Z_L \beta_0 \tan(\beta\ell)}. \tag{21}$$

For an open-circuit line, i.e., $Z_L = \infty$, (21) reduces to

$$Z_{\text{in}} = Z_c(0) \frac{q \tan(\beta\ell) + 2\beta}{j2\beta_0 \tan(\beta\ell)} \tag{22}$$

which is purely capacitive when the line is electrically short and $q\ell \ll 1$. For a short-circuit line, i.e., $Z_L = 0$, (21) becomes

$$Z_{\text{in}} = Z_c(0) \frac{j2\beta_0 \tan(\beta\ell)}{2\beta - q \tan(\beta\ell)} \tag{23}$$

which is purely inductive when the line is electrically short and $q\ell \ll 1$. Note that if $q = 0$, the line has no tapering. Accordingly, (21)–(23) for the input impedance reduce to the lossless uniform transmission line [5].

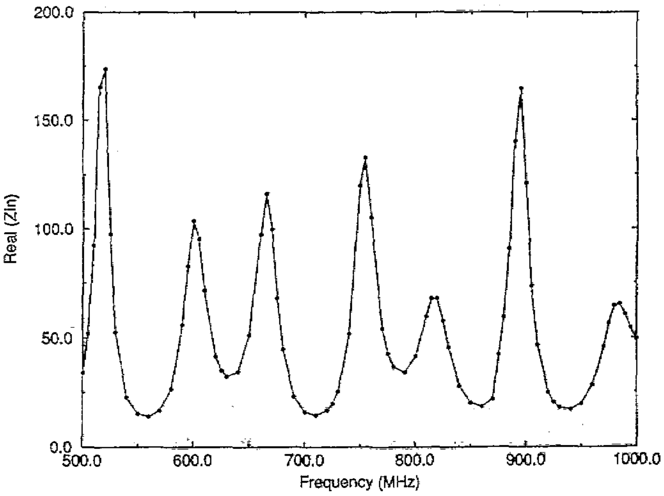

Fig. 2. Resistive part of the input impedance of the ETTL.

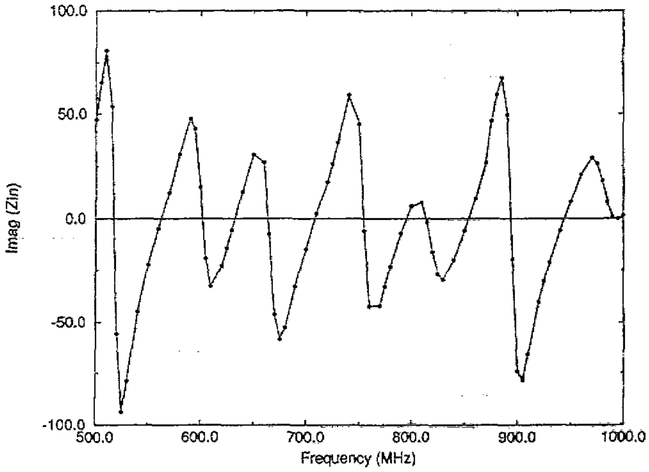

Fig. 3. Reactive part of the input impedance of the ETTL.

C. Bandwidth Considerations

The exponentially tapered transmission line antenna has a lower cutoff frequency that occurs at the frequency where $\beta = 0$, i.e.,

$$\omega_c = \frac{1}{\sqrt{L_0 C_0}} \left(\frac{q}{2}\right) \tag{24}$$

where q can be obtained from (3). To illustrate the performance of such an antenna, design parameters of $\ell = 1.935$ m, $Z_{c0} = 50\ \Omega$, and $Z_c(\ell) \approx 414\ \Omega$ are used. Details of this particular design are given in [6]. These design parameters used in (3) yield a q value of 1.0924. Accordingly, a cutoff frequency of 26 MHz is obtained. Note that for frequencies below cutoff, exponential attenuation occurs along the length of the tapered line.

III. NEC Modeling

The Numerical Electromagnetics Code, NEC-2, is used to model only a portion of the exponentially tapered structure. Electrically short straight-line segments are used with the NEC-2 code to approximate the curvature of the structure shown in Fig. 1. However, due to limitations on wire spacing in the NEC-2 program [1], that portion of the structure where the wire separation becomes a small fraction of a wavelength is not modeled. A Thevenin equivalent source is used to replace that section of the structure not included in the NEC-2 code model. The source impedance and voltage are obtained via

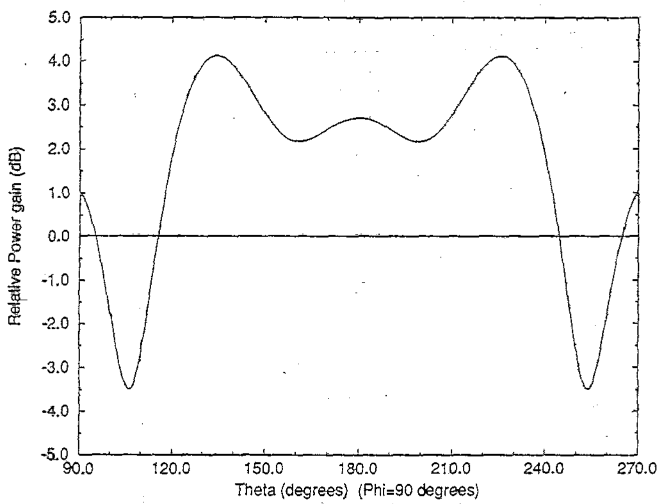

Fig. 4. Power pattern of the ETTL at 500 MHz, horizontal polarization.

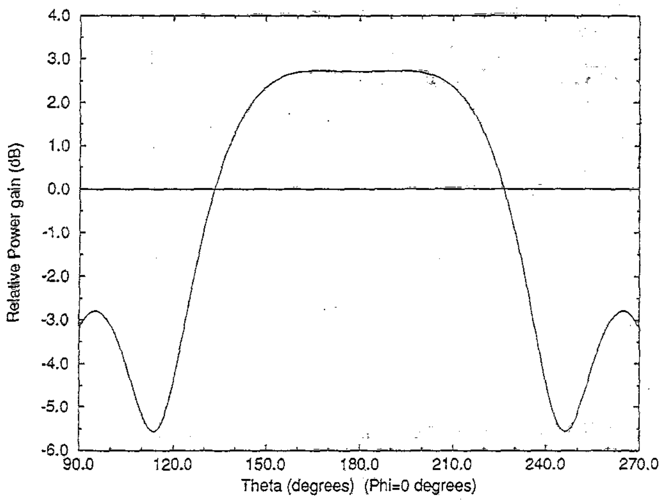

Fig. 5. Power pattern of the ETTL at 500 MHz, vertical polarization.

transmission line theory. Similarly, the input impedance for the total structure is obtained from the impedance seen looking into the section modeled by the NEC-2 code as transformed by the remaining portion of the tapered transmission line. Details on the Thevenin model and the NEC-2 code implementation are given in [6]. Further information on the NEC-2 code in general are given in [1].

IV. Results

Design considerations for an input section to drive an electrically large wire structure have been developed from nonuniform transmission line theory. Since transmission line theory does not account for radiation, the NEC-2 computer code is used to verify the analysis. A comparison of the input impedance from transmission line analysis is made with the corresponding results. For an open-end configuration, i.e., an open-circuit termination, transmission line analysis indicates that the input impedance is purely imaginary above cutoff, see (22). Since the numerical solution includes radiation, the input impedance to the exponentially tapered transmission line will have a resistive component that is directly proportional to the radiated power for a constant input current.

The resistive component of the input impedance, for the frequency range 500 MHz to 1 GHz, is shown in Fig. 2. A series of resonant peaks are exhibited with the impedance varying between 25 and 75 Ω over most of the frequency range. An ideal performance of the antenna would yield a constant 50-Ω input resistance with no reactance. If transmission line analysis were applied over the entire structure, a zero input resistance would be obtained. The reactive component of the input impedance is shown in Fig. 3. An ideal performance would yield a zero reactance. Note that the magnitude of the reactance is less than 50 Ω over most of the frequency range.

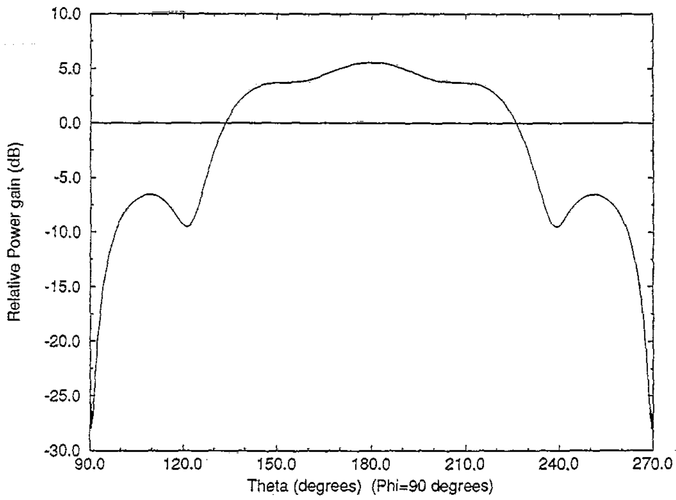

Fig. 6. Power pattern of the ETTL at 800 MHz, horizontal polarization.

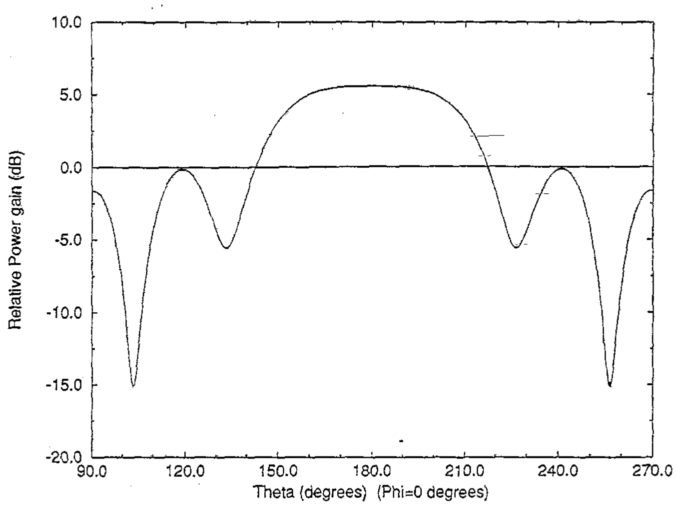

Fig. 7. Power pattern of the ETTL at 800 MHz, vertical polarization.

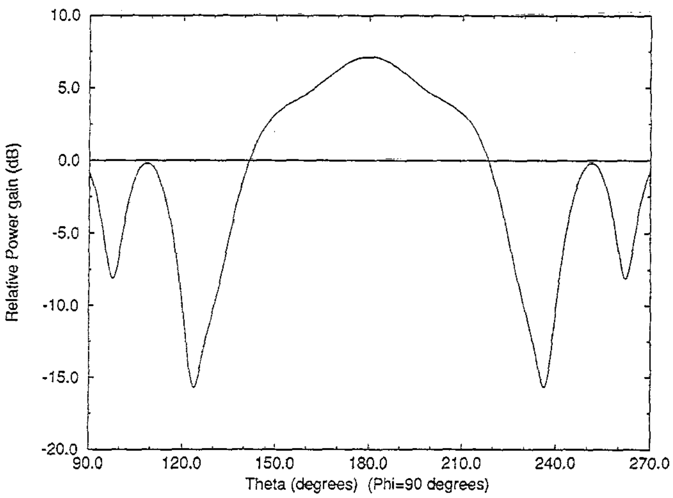

Fig. 8. Power pattern of the ETTL at 1 GHz, horizontal polarization.

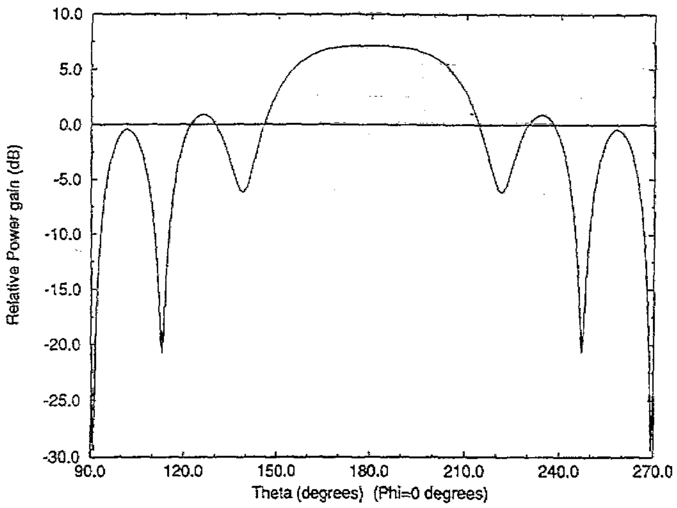

Fig. 9. Power pattern of the ETTL at 1 GHz, vertical polarization.

Clearly, the results indicate that the tapered transmission line is smoothing the input impedance variation with frequency.

In the high-frequency regime, the tapered transmission line should radiate effectively. This is seen from the pattern calculations shown in Figs. 4–9. Here, the patterns have been plotted for a series of frequencies over the range shown in Figs. 2 and 3. As the frequency increases, there is a general narrowing of the pattern as would be expected.

V. Conclusion

Design considerations have been presented for an exponentially tapered transmission line to match a low-impedance source to an electrically large wire structure. As the frequency increases, the exponentially tapered transmission line becomes an effective radiator. The design is accomplished through the use of transmission line theory. However, the evaluation of the design is accomplished by using the NEC-2 computer code.

The exponentially tapered transmission line did not perform as well as was hoped. Although, the impedance did exhibit resonant behavior, the variation with frequency was appreciably smoothed. With the use of a source that is tolerant of some impedance variation, the exponentially tapered transmission line should provide an acceptable solution to driving an electrically large antenna structure from a low-impedance source.

References

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