Sensor and Simulation Notes

Note 336

A TIME-DOMAIN INCIDENT-FIELD EXTRAPOLATION TECHNIQUE BASED ON THE SINGULARITY EXPANSION METHOD

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ABSTRACT

In this report, a method is presented to extrapolate measurements from Nuclear Electromagnetic Pulse (NEMP) assessments directly in the time domain. This method is based on a time-domain extrapolation function which is obtained from the Singularity Expansion Method representation of the measured incident field of the NEMP simulator.

Once the time-domain extrapolation function is determined, the responses recorded during an assessment can be extrapolated simply by convolving them with the time-domain extrapolation function.

It is found that to obtain useful extrapolated responses, the incident-field measurement needs to be made minimum phase; otherwise unbounded results can be obtained.

Results obtained with this technique are presented, using data from actual assessments.

[1] The research of which the results are presented in this note was carried out during a stay at the Defence Research Establishment Ottawa, Ottawa, Ontario, Canada

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1 INTRODUCTION

Most Nuclear Electromagnetic Pulse (NEMP) simulators do not reproduce the expected NEMP threat as layed down by AEP 4, 1984 [1]. They fail to reproduce both the waveform and the peak field strength of the perceived threat level (also known as the criterion environment), which includes reflections from the earth for ground-based facilities, but not so for airborne systems. To compensate for these shortcomings, the measured responses in NEMP assessments have to be corrected (extrapolated) to calculate the response that would be expected from a NEMP.

This is a particular problem for radiating and hybrid simulators, which produce a waveform that is significantly different from the waveform of the perceived threat. The measurements obtained from NEMP assessments using such simulators have therefore always to be extrapolated.

In this report, what is known as *incident-field extrapolation* will be addressed (see Baum [2], type 3A). This type of extrapolation not only corrects for the difference in waveform, but also tries to correct the different spatial behaviour of the incident field of the simulator compared with the criterion environment. An extrapolation function which is an average over the space of interest, i.e., the test volume of the simulator, is therefore constructed.

An overview of the basic incident-field extrapolation method is given in Chapter 2, and some properties of the extrapolation function based on signal theory considerations are derived. How the incident-field extrapolation method has been implemented in the past is also addressed in Chapter 2. Traditional implementations are without exception based on frequency-domain techniques.

An extrapolation technique which uses time-domain techniques is presented in Chapter 3. This technique constructs the extrapolation function entirely in the time domain. Results are presented in Chapter 4.

2 INCIDENT-FIELD EXTRAPOLATION

With incident-field extrapolation, an extrapolation function is constructed which is an average over the space of interest, i.e., the test volume of the simulator. Furthermore, the Equipment Under Test (EUT) is assumed to be configured in the normal operating-mode for the system, and the interaction between the simulator structure and the object is neglected.

Extrapolation to correct differences in polarization, angle of incidence, or direction of propagation of the incident field between the criterion environment and the simulation will not be addressed in this report. This simplifies the analysis and notation. Also geometrical differences between the test environment and the normal operating environment, most importantly the presence or absence of the influence of the earth, will not be considered. Therefore, the type of extrapolation addressed in this report is limited to airborne systems in bounded wave simulators and ground-based facilities for radiating simulators.

2.1 The Basic Formulation

Let the response of a linear and time-invariant system in its normal operating-mode and environment to an incident NEMP be denoted by $g(t)$. The response $g(t)$ can be, for example, an electric or a magnetic field, a current or a voltage. Then $g(t)$ is the response of the system in the criterion environment, and is given by[3]

$$g(t) = \int_0^t h(t-\tau)\ e_{EMP}(\tau)\ d\tau = h(t) * e_{EMP}(t), \tag{1}$$

where the asterisk denotes the convolution operator, and $h(t)$ is the impulse response of the system. When necessary, the latter takes into account reflections from the earth. Furthermore, $e_{EMP}(t)$ is the waveform of the perceived threat of the NEMP, and can be the incident electric or the incident magnetic field. For a high-altitude NEMP environment, the incident electric field is usually given by (Bell Laboratory waveform)

$$e_{EMP}(t) = A(e^{-\alpha t} - e^{-\beta t}), \tag{2}$$

with

$$\begin{aligned} A &= 5.278\times10^4\ [V/m], \\ \alpha &= 3.705\times10^6\ [s^{-1}], \\ \beta &= 3.908\times10^8\ [s^{-1}]. \end{aligned} \tag{3}$$

The impulse response $h(t)$ of the system during the simulation will be the same as the impulse response during its normal operating-mode, only if the following three conditions are satisfied:

[3] For simplicity a scalar notation has been employed throughout the text.

- the interaction between the EUT and the simulator structure can be neglected,
- the system is configured the same as during its normal operating-mode,
- the test environment is the same as the normal operating-mode environment (i.e., an airborne system must be tested without the influence of the earth, and vice versa for a ground-based system).

Assuming that during the NEMP assessment the above mentioned conditions are satisfied, the response of the system in the NEMP simulator is given by (assuming a linear system)

$$g_{sim}(t) = h(t) * e_{sim}(t), \tag{4}$$

where $e_{sim}(t)$ is the incident electric or incident magnetic field of the simulator. In this context, the incident field is the field in the working volume of the simulator in absence of the EUT, and without any reflections from the earth's surface. See Farr *et al.* [3] and Farr [4] for details on how to measure the incident field of a NEMP simulator. See also Section 3.3 of this report. Furthermore, the system response $g_{sim}(t)$ is the same physical quantity as $g(t)$ in Eq.(1).

It is well known that an approximation to the response $g(t)$ can be reconstructed in the following way (see Baum [2], type 3A)

$$g_x(t) = \mathcal{L}^{-1}\{X(s)G_{sim}(s)\}, \tag{5}$$

where $g_x(t)$ denotes the extrapolated response, which is, unfortunately, not necessarily equal to $g(t)$. The difference between $g_x(t)$ and $g(t)$ is the (unknown) error in the extrapolation. Furthermore in Eq.(5), a quantity indicated with a capital letter denotes a complex frequency-domain quantity, $\mathcal{L}^{-1}\{.\}$ denotes the inverse Laplace transform operator, and s denotes the complex-frequency variable $s = \sigma + j\omega$. $X(s)$ denotes the extrapolation transfer function given by,

$$X(s) = E_{EMP}(s)/E_{sim}(s). \tag{6}$$

Instead of using Eq.(5), another representation for the extrapolated response is

$$g_x(t) = x(t) * g_{sim}(t). \tag{7}$$

Eqs.(5) and (7) clearly show that $X(s)$ plays the role of a transfer function, and $x(t)$ that of the impulse response pertaining to the transfer function $X(s)$.

The extrapolation impulse response is determined by

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\{E_{EMP}(s)/E_{sim}(s)\}, \tag{8}$$

or directly in the time domain

$$x(t) = e_{EMP}(t) * e_{sim}^{-1}(t), \tag{9}$$

in which $e_{sim}^{-1}(t)$ is the inverse signal of $e_{sim}(t)$, defined by

$$e_{sim}^{-1}(t) \doteq \mathcal{L}^{-1}\{1/E_{sim}(s)\}. \tag{10}$$

Note that $e_{sim}(t)$ and $e_{sim}^{-1}(t)$ are related by

$$e_{sim}(t) * e_{sim}^{-1}(t) = \delta(t), \tag{11}$$

where $\delta(t)$ denotes the Dirac delta function.

When the incident field of the simulator not only differs in waveform and peak field strength from the criterion environment, but also exhibits a different spatial behaviour, the extrapolation transfer function $X(s)$ depends on the point of observation also. This is usually the case with NEMP assessments of large systems with radiating simulators such as TNO-FEL's EMIS-3. Such simulators radiate a spherical wave instead of a plane wave. It is advantageous, however, to define an "average" extrapolation function which will be used for all positions in the test volume. In that case, $E_{sim}(s)$ may be taken as a geometrical average, i.e., the average of several field-mapping measurements at different positions in the test volume.

2.2 Some Properties of the Extrapolation Function

Although the formulation of the incident-field extrapolation is quite straightforward, some difficulties arise which we will address in this chapter. But before we do so, we first introduce some definitions (Zadeh *et al.* [5]).

Definition 1: A signal $f(t)$ is said to be bounded if and only if there exists a finite positive constant M, such that

$$|f(t)| < M, \quad \forall t.$$

Definition 2: A transfer function $H(s)$ is said to be stable if and only if its impulse response $h(t)$ is bounded.

The latter definition does not give any information on the response of the system. Therefore, the following definition is introduced:

Definition 3: A transfer function $H(s)$ is said to be *strictly* stable if and only if its response to a bounded input is bounded.

Definition 3 leads to the following theorem:

Theorem 1: A transfer function $H(s)$ is strictly stable if and only if its impulse response $h(t)$ satisfies the inequality

$$\int_{-\infty}^{+\infty} |h(t)| \, dt < \infty.$$

The extrapolation process can be called successful and of practical use, only if the extrapolated response is causal and bounded. From Eq.(7) and Definition 3, we conclude that the extrapolation impulse response $x(t)$ must then be causal, and the extrapolation transfer function $X(s)$ strictly stable.

Since the extrapolation impulse response is a convolution of two causal signals, causality is always guaranteed. Whether or not the extrapolation transfer function is strictly stable, however, depends on $e_{sim}^{-1}(t)$. In fact, it is easy to show that $X(s)$ is strictly stable if and only if $e_{sim}^{-1}(t)$ is bounded. This puts some restrictions on $e_{sim}^{-1}(t)$[4].

To analyze the restrictions we have to impose on $e_{sim}^{-1}(t)$, consider a bounded signal $f(t)$. In principle, the Laplace transform $F(s)$ of $f(t)$ has a number of poles, a number of zeros, and some branch points in the complex-frequency plane[5]. It can be proven easily that a necessary (but not sufficient) requirement for $f(t)$ to be bounded is, that its poles must be located in the left half-plane or on the $j\omega$-axis of the complex-frequency plane. Since $F^{-1}(s) = 1/F(s)$, the poles of $F(s)$ are the zeros of $F^{-1}(s)$. But what is more important, the zeros of $F(s)$ are the poles of $F^{-1}(s)$. Hence, for $f^{-1}(t)$ to be unbounded the zeros of $F(s)$ must also be located in the left half-plane of the complex-frequency plane. This yields the following theorem:

Theorem 2: For a bounded signal $f(t)$ to have a bounded inverse signal $f^{-1}(t)$, where $f(t) * f^{-1}(t) = \delta(t)$, it is necessary but not sufficient that all poles and zeros of its Laplace transform $F(s)$ lie in the left half-plane or on the $j\omega$-axis of the complex-frequency plane.

A signal whose Laplace transform $F(s)$ has the above mentioned properties is called a minimum-phase signal. See Zadeh *et al.* [5] for a more elaborate treatment of minimum-phase signals.

From the above discussion, we conclude that for the extrapolation transfer function $X(s)$ to be strictly stable, $E_{sim}(s)$ needs to be a minimum-phase signal, or needs to be made minimum phase if it is not. With respect to the latter remark, it is important to observe that the magnitude of the spectrum of a signal whose Laplace transform has some zeros located in the right half-plane, is the same as that of a

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[4] Note that $e_{sim}^{-1}(t) \neq 1/e_{sim}(t)$.

[5] Observe that the number of poles and zeros can also be infinite.

minimum-phase signal whose Laplace transform has those zeros reflected with respect to the $j\omega$-axis into the left half-plane.

Note that if $F(s)$ in Theorem 2 is a rational function, it is sufficient for $f^{-1}(t)$ to be bounded that its poles and zeros are located in the left half-plane, as a rational function does not have branch points.

2.3 Traditional Implementations of Incident-Field Extrapolation

Traditional implementations of incident-field extrapolation compute the extrapolated response $g_x(t)$ by applying the inverse Fast Fourier Transform (FFT) to $X(j\omega)G_{sim}(j\omega)$. The difference is the way in which $X(j\omega)$ is computed. We mention the following three methods for determining $X(j\omega) = E_{EMP}(j\omega)/E_{sim}(j\omega)$ (see also Table I):

1. compute $E_{sim}(j\omega)$ with a FFT;
2. compute $E_{sim}(j\omega)$ with a FFT, but use a minimum-phase fit of the magnitude of $E_{sim}(j\omega)$ (see Fisher *et al.* [6]);
3. approximate $e_{sim}(t)$ by a Singularity Expansion Method (SEM) representation, then $E_{sim}(j\omega)$ is also known (see Van de Sande [7]).

For an elaborate treatment of the SEM the reader is referred to Baum [8].

Method 1 has the disadvantage that it can yield an unstable extrapolation transfer function, as has been pointed out in Section 2.2. To circumvent this, Method 2 has been employed. In this method, the magnitude of the spectrum of $E_{sim}(j\omega)$ is computed from the FFT of $e_{sim}(t)$. Subsequently, the phase is obtained from the Hilbert transform applied to $\log|E_{sim}(j\omega)|$. The resulting signal is then a minimum-phase signal. This assures a strictly stable extrapolation transfer function. For more details on how to construct the phase of a signal from the magnitude of its spectrum see Oppenheim *et al.* [9].

The advantages of Method 3 are that no aliasing error occurs, that no high-frequency quantization noise and no high-frequency noise as a result of a truncated time window are introduced. However, Method 3 does not guarantee a stable extrapolation transfer function. With care these difficulties can be overcome and an extrapolation procedure based on Method 3 and the considerations given in Section 2.2 will be developed in the next chapter.

Table I Summary of traditional extrapolation techniques.

Time-domain	FFT	Frequency-domain
Measure incident field $e_{sim}(t)$ in the simulator with EUT removed		
Method 1:	⟹	*Spectrum of incident field $E_{sim}(j\omega)$*
Method 2:	⟹	*Spectrum of incident field $E_{sim}(j\omega)$* *Construct minimum-phase spectrum of $E_{sim}(j\omega)$*
Method 3: *Approximate incident field with SEM (Prony's method)*		*Construct spectrum of incident field $E_{sim}(j\omega)$ from poles and residues of SEM representation*
		Determine extrapolation transfer function $X(j\omega) = E_{EMP}(j\omega) / E_{sim}(j\omega)$
Repeat for all measurements		
System response in simulator $g_{sim}(t)$	⟹	*Spectrum of system response in simulator $G_{sim}(j\omega)$*
Extrapolated system response $g_x(t)$	⟸	$G_x(j\omega) = X(j\omega)\, G_{sim}(j\omega)$
Until done		

3 TIME-DOMAIN INCIDENT-FIELD EXTRAPOLATION

Method 3 of Section 2.3 has some useful properties. We mention:

- the extrapolation transfer function contains no aliasing errors;
- the extrapolation transfer function contains no quantization noise, and no high-frequency noise due to a truncated time window;
- the extrapolation transfer function is known for all frequencies, which enhances the low-frequency resolution.

When the SEM representation is used to represent the incident field of the simulator, the extrapolation transfer function is known analytically. Then it is possible to perform the extrapolation entirely in the time domain. This has the added advantage that time truncated signals can be extrapolated as well. Once the extrapolation impulse response is computed, the extrapolated responses can be found by convolving the measured signals with the extrapolation impulse response. This idea will be pursued in the subsequent sections.

3.1 The SEM Representation of a Transient Signal

The SEM postulates that a transient signal can be written as a series of exponentials with complex-valued arguments. So, according to the SEM a causal transient signal $f(t)$ can be represented as

$$f(t) = \sum_{i=1}^{N} A_i e^{s_i t} U(t), \tag{12}$$

with

- s_i : a simple pole,
- A_i : the residue pertaining to the pole s_i,
- $U(t)$: the Heaviside step-function,
- N : number of poles.

In general, the poles and residues are complex valued, but since the signal $f(t)$ is real valued they occur in complex-conjugate pairs. For $f(t)$ to be bounded, all the poles s_i have to lie in the left half-plane or on the $j\omega$-axis of the complex-frequency plane, i.e., $\{s_i \in \mathbb{C}: \Re(s_i) \leq 0\}$.

To extract the poles and residues of a transient signal, several methods are known. We mention Prony's method (see Kay [10]), and the Pencil-Of-Functions (POF) technique (see MacKay [11]). Treatment of these methods is beyond the scope of this report.

Once the poles and residues of $f(t)$ are computed with either Prony's method or the POF technique, the Laplace transform of the signal is also known. It is given by (partial-fraction expansion)

$$F(s) = \sum_{i=1}^{N} A_i \frac{1}{s-s_i}. \tag{13}$$

This representation was used in Van de Sande [7] to approximate $E_{sim}(s)$. The extrapolation transfer function was then constructed using the Laplace transform of Eq.(2), and the extrapolated response was computed by applying the inverse FFT to $X(j\omega)G_{sim}(j\omega)$ (see Table I, Method 3).

The inverse FFT, however, can be circumvented entirely by determining the extrapolation impulse response analytically. For that purpose, the partial-fraction expansion of Eq.(13) will be casted in a rational form, i.e.,

$$F(s) = \sum_{i=1}^{N} A_i \prod_{\substack{j=1 \\ j\neq i}}^{N} (s-s_j) / \prod_{j=1}^{N} (s-s_j) = \frac{p(s)}{q(s)}, \tag{14}$$

where $p(s)$ is the polynomial of the numerator which is of degree $N-1$, and $q(s)$ is the polynomial of the denominator and is of degree N. The polynomial $p(s)$ is given by

$$p(s) = \sum_{i=1}^{N} A_i p_i(s), \tag{15}$$

and

$$q(s) = \prod_{j=1}^{N} (s-s_j), \tag{16}$$

in which

$$p_i(s) = \prod_{\substack{j=1 \\ j\neq i}}^{N} (s-s_j) = \frac{q(s)}{(s-s_i)}. \tag{17}$$

As the complex poles and residues occur in complex-conjugate pairs, it can be proven that the coefficients of both $p(s)$ and $q(s)$ are real valued.

Generally, the polynomial $p(s)$ is of degree $N-1$ [6], so it has $N-1$ zeros. This allows the following representation for $p(s)$

$$p(s) = c\prod_{j=1}^{N-1} (s-z_j), \tag{18}$$

where the z_j's are the zeros of $p(s)$ (and of $F(s)$), i.e., $p(z_j) = 0$ and c is a proportionality constant to be determined later. It can be shown that $c \in \mathbf{R}$, which also follows from the fact that the complex poles

[6] It can be proven that if $f(0) = 0$, $p(s)$ is of degree $N-2$

and residues occur in complex-conjugate pairs.

The zeros z_j can be found from

$$p(s) = \sum_{i=1}^{N} A_i p_i(s) = 0, \tag{19}$$

and have to be determined numerically with a root find algorithm, such as the IMSL subroutine ZPLRC (see [12]). Observe that the zeros depend on the poles and residues, but a direct relation cannot be established.

Once the zeros z_j are known, the constant c can be found from the value of $F(s)$ at $s = 0$. After substituting Eq.(18) in Eq.(14), we find

$$F(s) = \frac{p(s)}{q(s)} = \frac{c\prod_{j=1}^{N-1}(s-z_j)}{q(s)}, \tag{20}$$

and after equating this result with Eq.(13), this yields for c at $s = 0$

$$c = \frac{(\prod_{j=1}^{N} s_j) \sum_{i=1}^{N} A_i s_i^{-1}}{\prod_{j=1}^{N-1} z_j}. \tag{21}$$

3.2 The Extrapolation Impulse Response

To determine the extrapolation impulse response, the incident field of the simulator $e_{sim}(t)$ is approximated with a SEM representation. To be able to do so, first the poles and residues of $e_{sim}(t)$ have to be determined with a pole extraction method, e.g. with Prony's method or the POF-technique. Subsequently, the zeros of $E_{sim}(s)$ are determined from its poles and residues in the way that has been described in Section 3.1. This yields the following representation for $E_{sim}(s)$ (cf. Eq.(14))

$$E_{sim}(s) = \frac{p(s)}{q(s)}, \tag{22}$$

where $q(s)$ and $p(s)$ are given by Eqs.(16) and (18), respectively. The roots of $q(s)$ are the poles of $E_{sim}(s)$, while the roots of $p(s)$ are the zeros of $E_{sim}(s)$. Since $e_{sim}(t)$ is a real-valued signal, any complex-valued zeros occur in complex-conjugate pairs.

It was proven in Section 2.2 that, for the extrapolation transfer function to be strictly stable, every zero of $E_{sim}(s)$ is required to lie in the left half-plane. In general, this is not the case, so that $E_{sim}(s)$ has to be made minimum phase simply by negating the real part of any offending zeros.

Using the representation of Eq.(22) for $E_{sim}(s)$, the extrapolation transfer function is given by

$$X(s) = \frac{q(s)}{p(s)} E_{EMP}(s). \tag{23}$$

If the double-exponential waveform of Eq.(2) is used as the waveform to which the response is required, we find for $E_{EMP}(s)$

$$E_{EMP}(s) = A\left(\frac{1}{s+\alpha} - \frac{1}{s+\beta}\right) = A(\beta - \alpha)\frac{1}{(s+\alpha)(s+\beta)}. \tag{24}$$

After substituting Eq.(24) in Eq.(23), and after applying a partial-fraction expansion of $X(s)$, we finally get

$$X(s) = \sum_{i=1}^{N+1} B_i \frac{1}{s - z_i}, \tag{25}$$

in which we have ordered the zeros so that $\{z_i \in \mathbf{C}: i=1, ..., N-1\}$ are the zeros of the minimum-phase signal of $E_{sim}(s)$, $z_N = -\alpha$, and $z_{N+1} = -\beta$. $B_i \in \mathbf{C}$ denotes the residue pertaining to the zero z_i given by

$$B_i = \lim_{s \to z_i} (s - z_i)\, X(s) = \frac{A}{C}(\beta - \alpha)\frac{\prod_{j=1}^{N} (z_i - s_j)}{\prod_{\substack{j=1 \\ j \neq i}}^{N+1} (z_i - z_j)}. \tag{26}$$

The corresponding extrapolation impulse response is easily found from Eq.(25). It is given by

$$x(t) = \sum_{i=1}^{N+1} B_i e^{z_i t} U(t). \tag{27}$$

Since $x(t)$ is real valued, any complex-valued zeros z_i and residues B_i occur in complex-conjugate pairs.

Obviously, since $\{z_i \in \mathbf{C}: \Re(z_i) \leq 0\}$, $X(s)$ is strictly stable, which follows from Theorem 1.

If the criterion waveform does not have a simple Laplace transform as in Eq.(24) (which is the case with the reciprocal double-exponential waveform), $x(t)$ can be determined, either analytically or numerically, from Eq.(9).

The extrapolated responses can now be found (see Eq.(7)) by convolving the measured signals with the extrapolation impulse response of Eq.(27).

3.3 The Extrapolation Procedure

To summarize the results of this chapter, we have depicted the complete time-domain extrapolation procedure in Table II. The first step is to measure the incident field of the NEMP simulator, which usually involves measuring both the electric and the magnetic fields in the working volume of the simulator with the EUT removed (see Farr [4]). The incident electric field can then be found by adding the normalized measured magnetic field (normalized by Z_0 = 377 Ω) to the measured electric field, and after dividing this result by two. The incident magnetic field can be found in a similar way. This technique is based on the property that the reflected electric field adds to the incident electric field, while the reflected magnetic field subtracts from the incident magnetic field.

A novel approach to determine the incident field is to use an "*incident-field sensor*" (see Farr *et al.* [3]). Such a sensor has a directional dependent sensitivity, so that it ignores ground reflections.

After the incident field has been determined, the second step is to compensate for sensor droop (see Klaasen [13]) which is due to the low-cutoff frequency of the field sensor. The steps 3-6 in Table II have already been discussed in this chapter. After the extrapolation impulse response has been determined, the system responses measured during the simulation have to be extrapolated by convolution with the extrapolation impulse response.

Table II Summary of the complete time-domain incident-field extrapolation procedure.

1. *Measure incident field of simulator (EUT removed)*
2. *Compensate measurement for sensor droop*
3. *Approximate incident field with SEM representation (Prony's method)*
4. *Compute the zeros from the poles and the residues of the SEM representation*
5. *Make the SEM representation of the incident field minimum phase*
6. *Compute the extrapolation impulse response*
7. *Repeat for all measurements*

    If necessary, compensate measurement for sensor droop

    Convolve measurement with extrapolation impulse response

   Until done

4 NUMERICAL RESULTS

A good test for extrapolation methods is to extrapolate the incident-field measurement itself. When this signal is extrapolated, the result should be approximately equal to the waveform of the criterion environment. The procedure outlined in the previous chapter has been employed to a field mapping of the Vertically Polarized Dipole (VPD) version of TNO-FEL's EMIS-3 simulator (a transportable radiating simulator). We will use the double-exponential waveform given by Eq.(2) as the waveform to which the response is required (the criterion environment). This waveform is depicted in Figure 1.

Figure 2 shows an incident magnetic field measurement of the above mentioned simulator. The telemetry system that was used has a bandwidth of 500 MHz, and a signal-to-noise ratio of 35-40 dB. The digitizer has a record length of 512 samples, and an eight-bit resolution. Observe that since the rise time of the incident field of the simulator is larger than that of the criterion environment waveform, we are extrapolating a signal with a smaller bandwidth to a signal with a higher bandwidth.

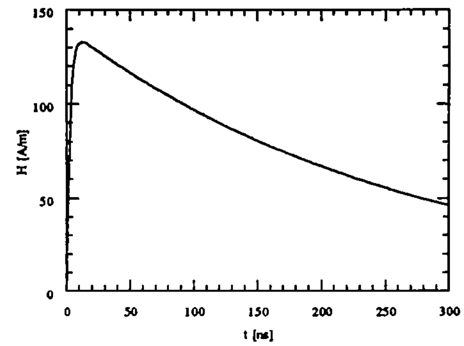


Figure 1 The prevailing waveform in the criterion environment.

Firstly, the field-mapping measurement has been approximated with a SEM representation using Prony's method. The number of poles (and residues) to approximate the original signal is 17. The signal that has been reconstructed using the 17 poles and residues is shown in Figure 3.

Secondly, using the poles and residues generated by the Prony program, the zeros and the proportionality constant of the rational representation of the SEM approximated signal have been determined. It is found that some zeros are located in the right half-plane of the complex-frequency plane, so that a minimum-phase signal has been constructed simply by negating the real part of the offending zeros. The resulting minimum-phase signal is shown in Figure 4. The magnitude of the spectrum of the minimum-phase signal is not shown, because it is the same as that of Figure 3b. To make the comparison easier, the SEM approximated signal is repeated in Figure 4a. Figure 4a shows that the only noticeable difference between the two signals is around the peak value.

Subsequently, the extrapolation impulse response has been constructed using the double-exponential waveform and the minimum-phase signal of Figure 4a. Figure 5a shows the results. The magnitude and the phase of the spectrum of the corresponding extrapolation transfer function are depicted in Figures 5b and 5c, respectively. Figure 5c shows the phase of the non-stable extrapolation transfer function also. Note that the extrapolation impulse response is completely noise free.

Finally, to show the effects of each step in the process of obtaining the extrapolation impulse response, the extrapolation impulse response has been convolved with the following three signals:

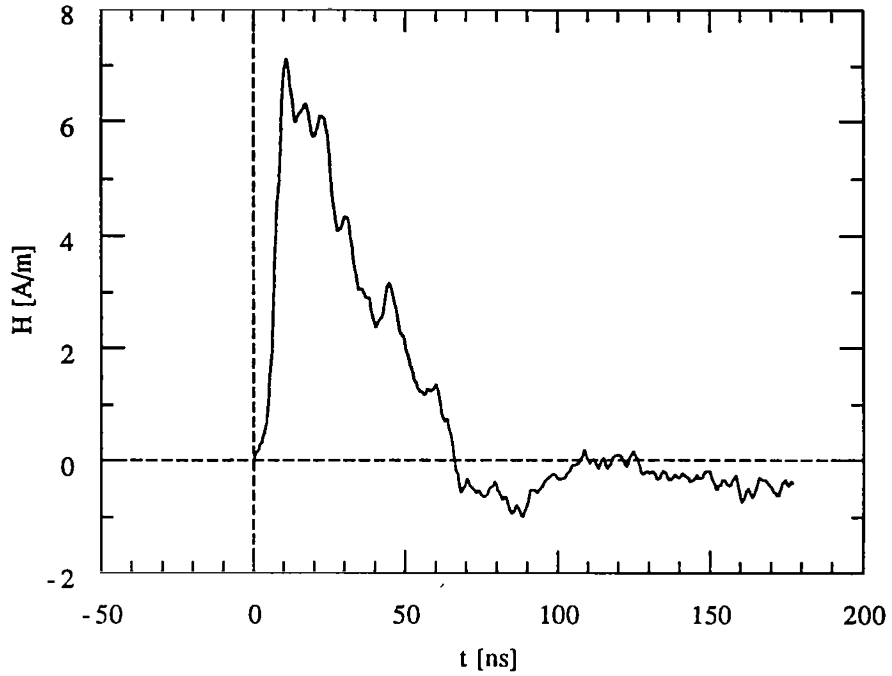
1. the minimum-phase signal of Figure 4a,
2. the SEM approximated signal of Figure 3a,
3. the incident magnetic field measurement of Figure 2a.

Figures 6, 7 and 8, respectively, depict the results. The convolution has been determined with the procedure described in Appendix A. Each of the first two data sets contains the same number of samples as the original signal, i.e., 512 samples.

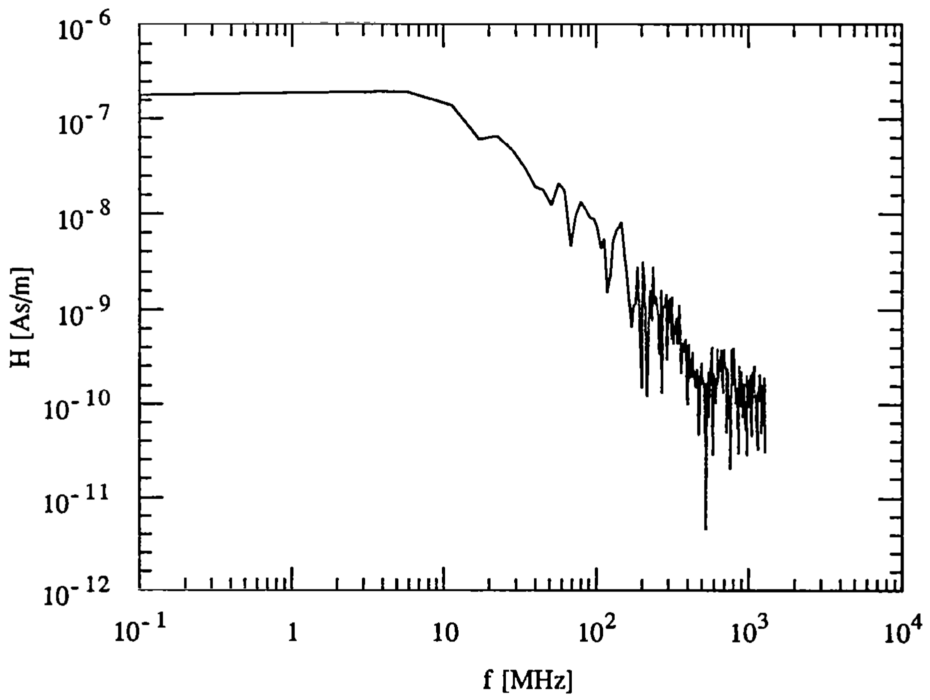
Obviously, convolving the minimum-phase signal with the extrapolation impulse response, which has been constructed from the minimum-phase signal, yields exactly the same waveform as the criterion environment. This is shown in Figure 6 (compare this figure with Figure 1).

The influence on the extrapolated signal of making the SEM approximated signal minimum phase can be seen from Figure 7. This shows that (in this case) the effect is small.

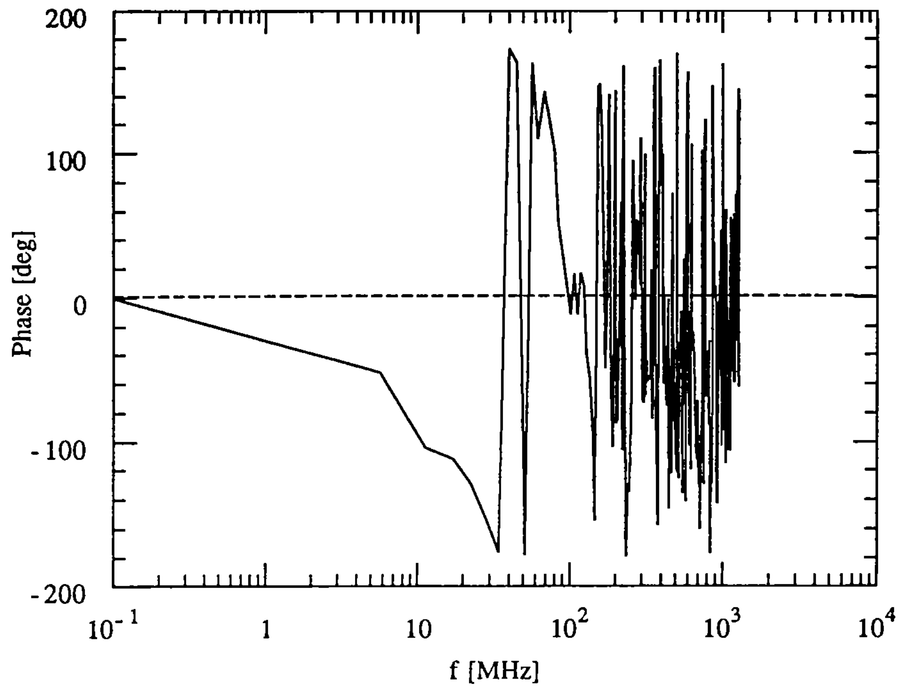
The total influence of approximating the original signal with a SEM representation, and making the resulting signal minimum phase is depicted in Figure 8. When judging this last plot, one has to keep in mind that the extrapolation transfer function enhances the high frequencies by a factor of approximately 100 (the incident field has a smaller bandwidth than the criterion waveform), so that noise and quantization errors in the original signal are amplified. Obviously, any extrapolation technique fails if the high-frequency content of the incident field of the NEMP-simulator is below the noise level of the measuring equipment. An exact criterion is difficult to give.


H [A/m]
t [ns]
8
6
4
2
0
-2
-50
0
50
100
150
200

(a)

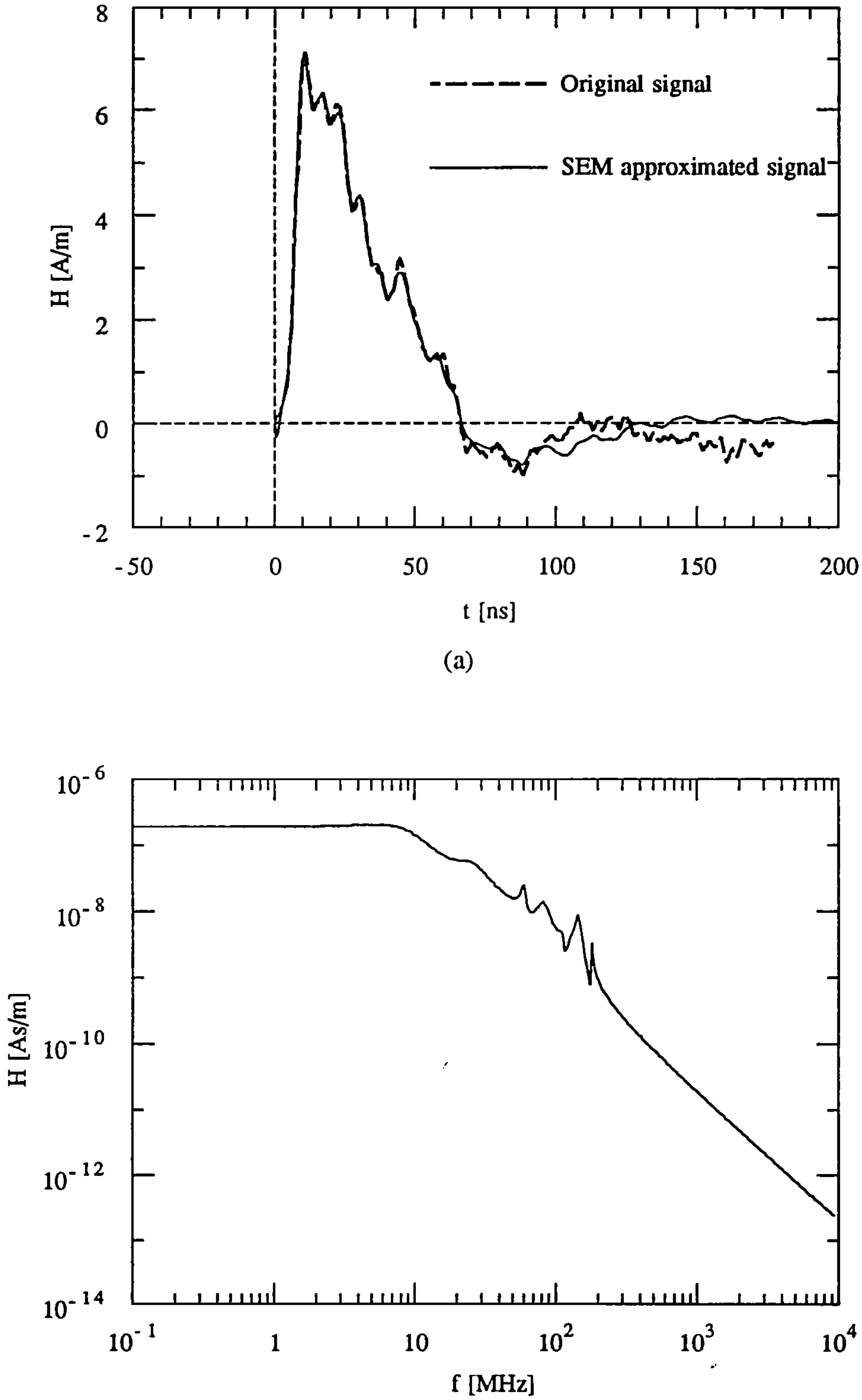

H [As/m]
f [MHz]
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-1
1
10
10^2
10^3
10^4

(b)



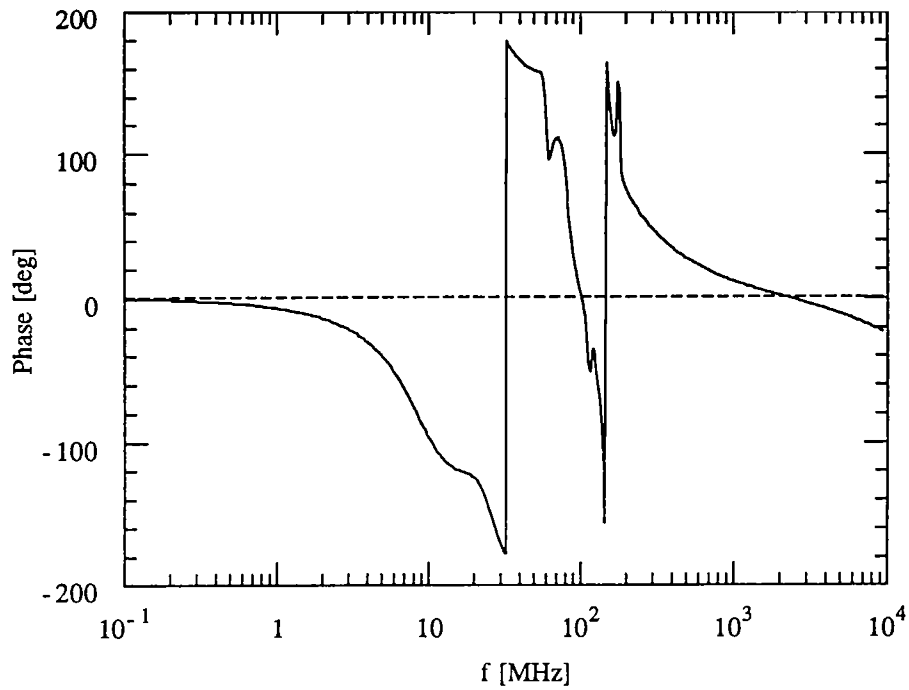
(c)

Figure 2 Incident magnetic field of the EMIS-3 simulator.
a) time domain,
b) magnitude of the spectrum,
c) phase of the spectrum.



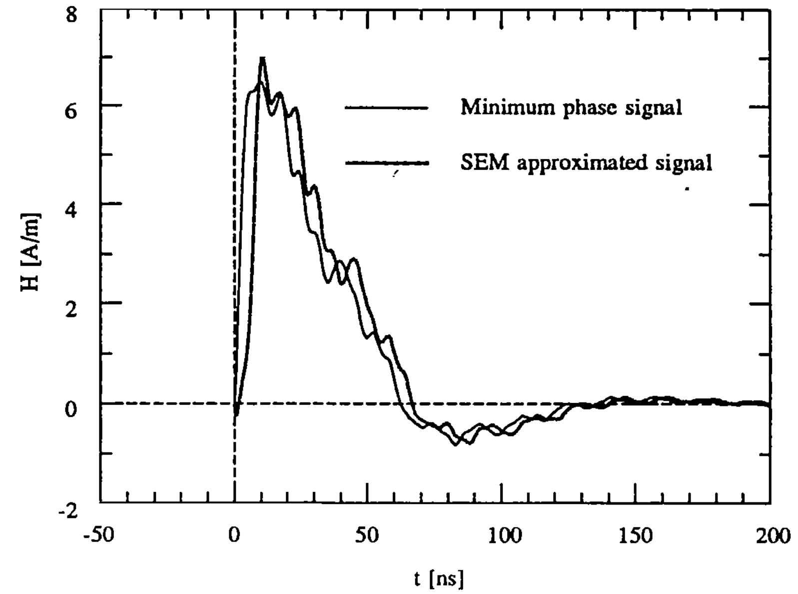
(a)

(b)

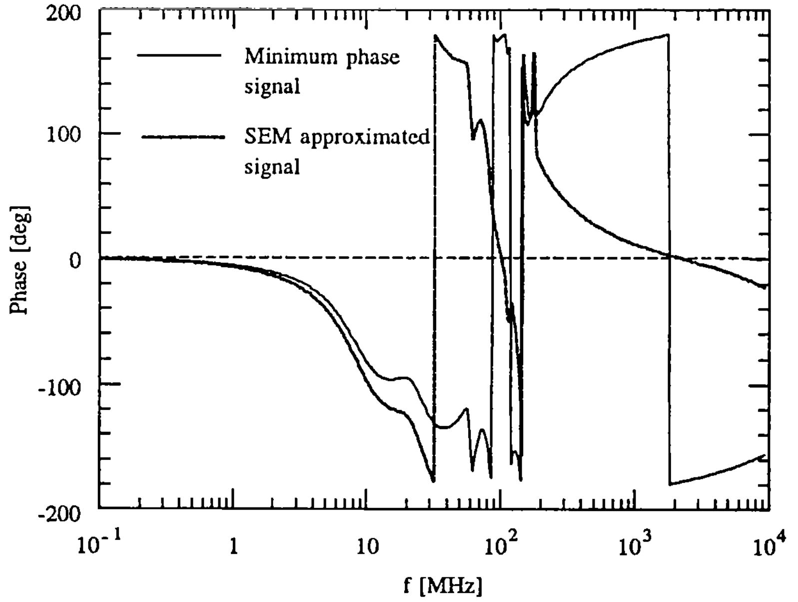


(c)

Figure 3 SEM approximated incident magnetic field (17 poles).
a) time domain,
b) magnitude of the spectrum,
c) phase of the spectrum.

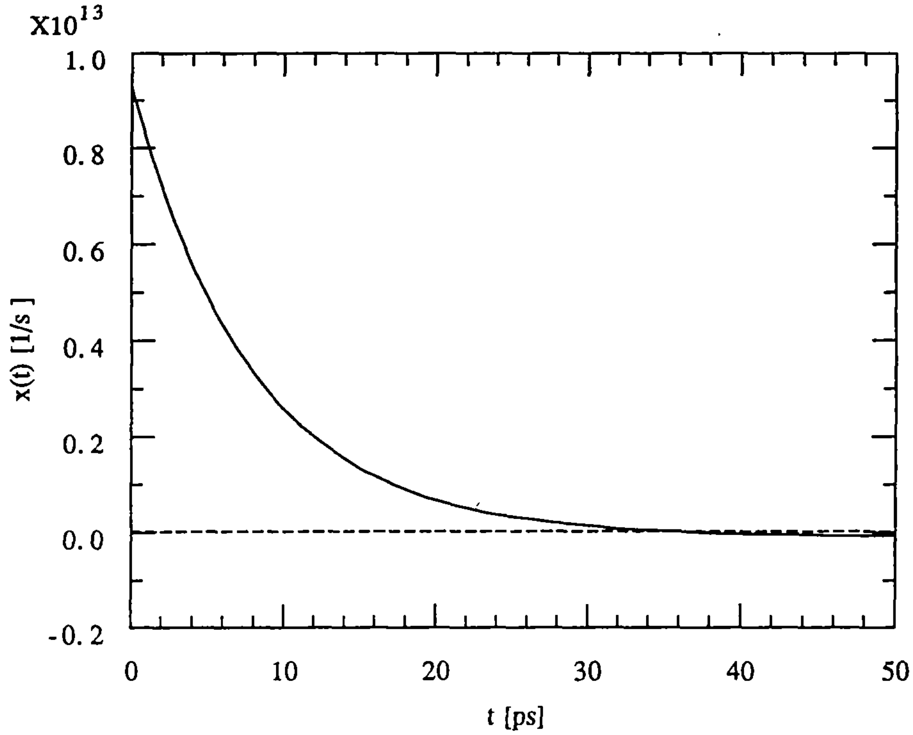


(a)

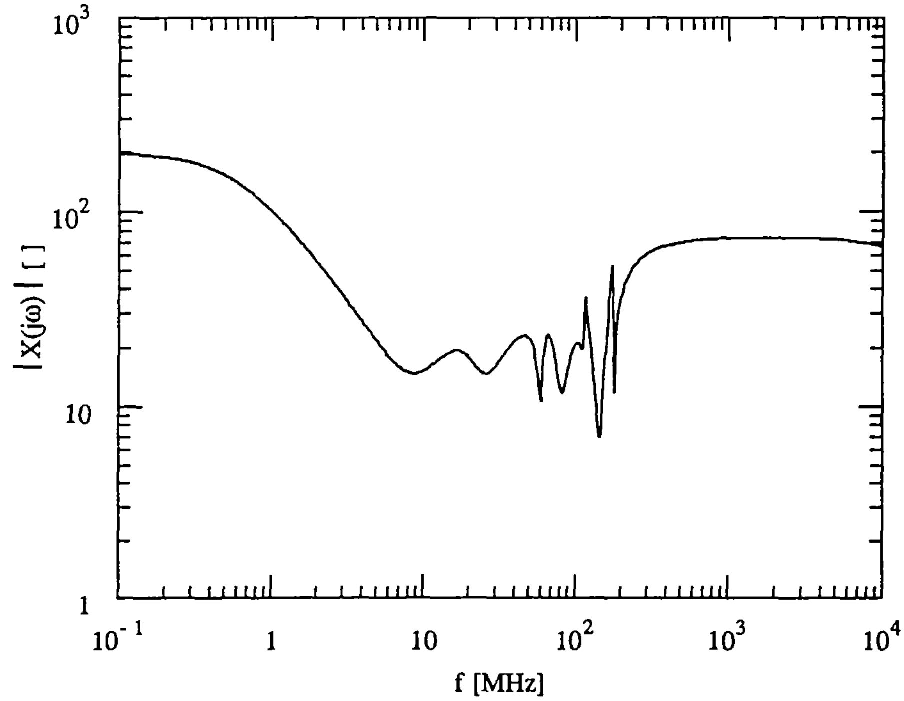


(b)

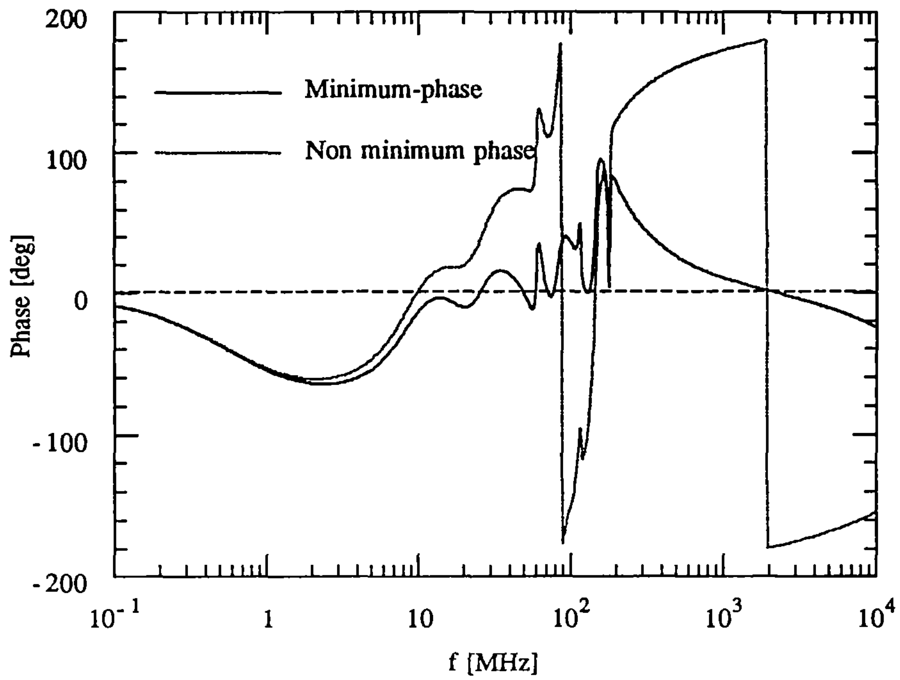
Figure 4 Minimum-phase signal of SEM approximated incident magnetic field.
a) time domain,
b) phase of the spectrum.


X10^13
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
x(t) [1/s]
0
10
20
30
40
50
t [ps]

(a)


10^3
10^2
10
1
|X(jω)| [ ]
10^-1
1
10
10^2
10^3
10^4
f [MHz]

(b)



(c)

Figure 5 The extrapolation function pertaining to the signal of Figure 2a:
a) extrapolation impulse response,
b) magnitude of the extrapolation transfer function,
c) phase of the stable and the non-stable extrapolation transfer function.

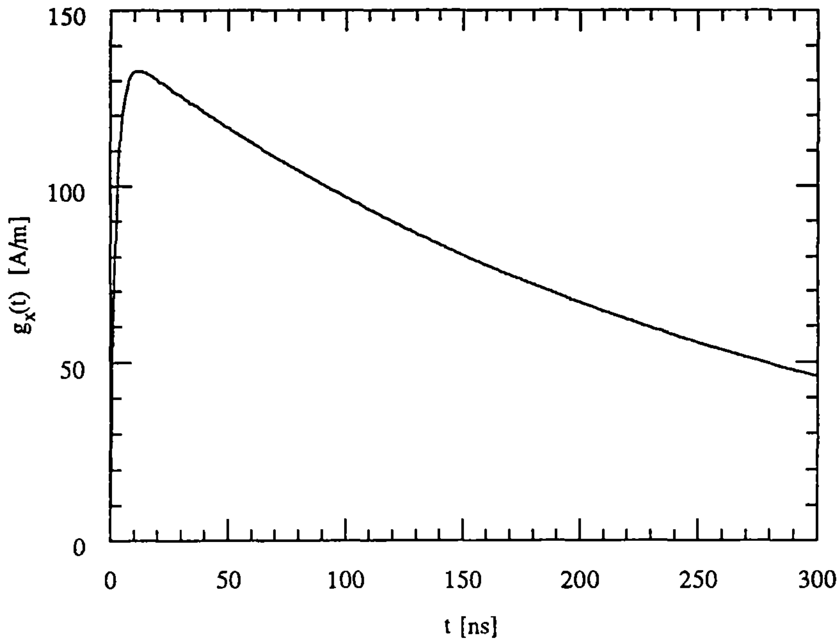


Figure 6 Result of the convolution of the extrapolation impulse response with the minimum-phase signal of Figure 4a.

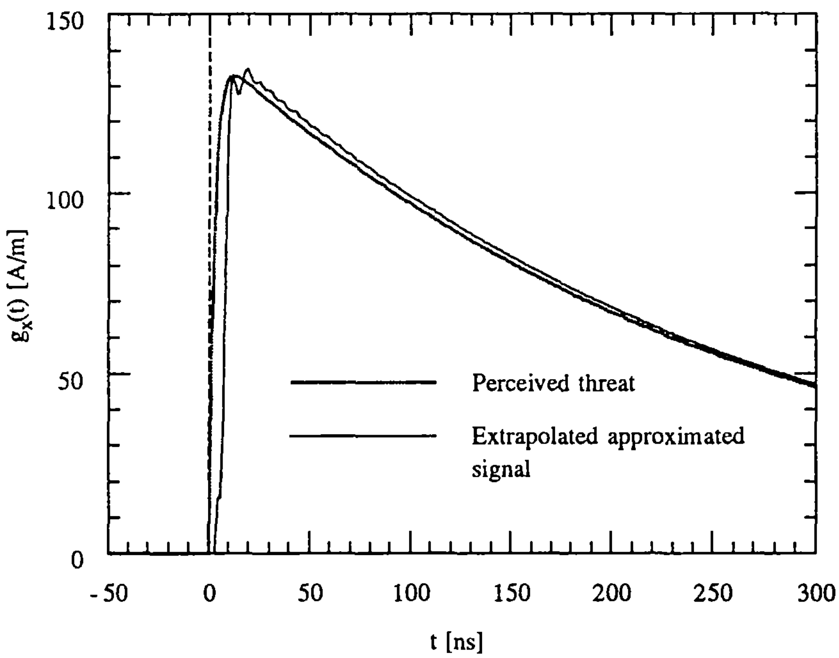


Figure 7 Result of the convolution of the extrapolation impulse response with the approximated signal of Figure 3a.

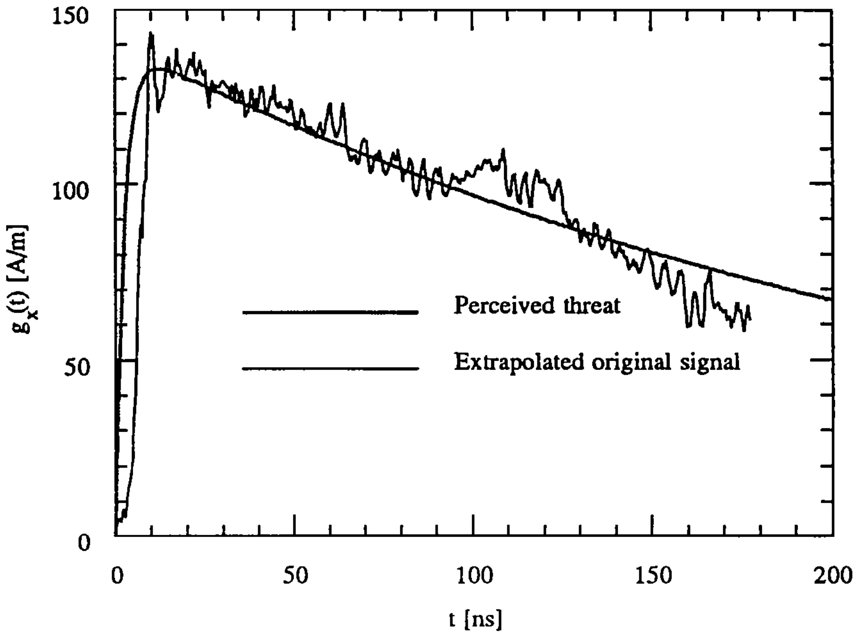


Figure 8 Result of the convolution of the extrapolation impulse response with the original signal of Figure 2.

5 CONCLUSIONS

A method has been developed and implemented to perform the incident-field extrapolation.

A necessary requirement for the extrapolation transfer function to be strictly stable, is that it is a minimum-phase signal. Making the extrapolation transfer function minimum phase can be accomplished very easily with this method.

Because the method which has been presented does not use a Fast Fourier Transform, the extrapolation transfer function is free of aliasing errors, high-frequency quantization noise and high-frequency noise due to a truncated time window. With this method, time truncated signals can be extrapolated.

Further research is needed to investigate the relation between the zeros of a signal, and its poles and residues; and how these relate for a minimum-phase signal. It is also desirable to investigate how pole extraction methods can approximate a signal in a minimum-phase sense.

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A EVALUATION OF THE CONVOLUTION INTEGRAL

The time scales of the extrapolation impulse response $x(t)$ and the signal to be extrapolated $g_{sim}(t)$ can differ significantly, so that special care has to be taken to compute the convolution integral given by (cf. Eq.(7))

$$g_x(t) = x(t) * g_{sim}(t) = \int_0^t g_{sim}(t-\tau)\, x(\tau)\, d\tau. \tag{A.1}$$

Using the fact that the extrapolation impulse response is known in analytical form, however, the convolution integral can be computed very accurately.

Let $t = n\Delta t$, where Δt is the time step of the sampled data $g_{sim}(t)$, then $g_x(n) \doteq g_x(n\Delta t)$ is given by

$$g_x(n) = \sum_{i=1}^{n} \int_{(i-1)\Delta t}^{i\Delta t} g_{sim}(n\Delta t - \tau)\, x(\tau)\, d\tau. \tag{A.2}$$

The time step Δt is assumed to be small over the interval of integration $[(i-1)\Delta t,\ i\Delta t]$, so that $g_{sim}(n\Delta t - \tau)$ may be approximated by

$$g_{sim}(n\Delta t - \tau) \approx \frac{g_{sim}(n-i+1) - g_{sim}(n-i)}{\Delta t}(i\Delta t - \tau) + g_{sim}(n-i). \tag{A.3}$$

After substituting Eq.(A.3) in Eq.(A.2), this yields

$$\begin{aligned} g_x(n) = \sum_{i=1}^{n} & g_{sim}(n-i)x_1(i) - g_{sim}(n-i+1)\, x_1(i-1) \\ & + \frac{g_{sim}(n-i+1) - g_{sim}(n-i)}{\Delta t} \int_{(i-1)\Delta t}^{i\Delta t} x_1(\tau)\, d\tau, \end{aligned} \tag{A.4}$$

where $x_1(i)$ denotes the integrated extrapolation impulse response given by

$$x_1(i) \doteq x_1(i\Delta t) = \int_0^{i\Delta t} x(\tau)\, d\tau = \sum_{i=1}^{N+1} \frac{B_j}{s_j}(e^{s_j i\Delta t} - 1). \tag{A.5}$$

Using the notation $x_2(i)$ for the twice integrated extrapolation impulse response, i.e.,

$$x_2(i) \doteq x_2(i\Delta t) = \int_0^{i\Delta t} x_1(\tau)\, d\tau = \sum_{j=1}^{N+1} \frac{B_j}{s_j}\left(\frac{e^{s_j i\Delta t}-1}{s_j} - i\Delta t\right), \tag{A.6}$$

we obtain

$$\begin{aligned} g_x(n) = \sum_{i=1}^{n} & g_{sim}(n-i)\, x_1(i) - g_{sim}(n-i+1)\, x_1(i-1) \\ & + \frac{g_{sim}(n-i+1) - g_{sim}(n-i)}{\Delta t}(x_2(i) - x_2(i-1)). \end{aligned} \tag{A.7}$$

After collecting terms, Eq.(A.7) is finally rewritten as

$$\begin{aligned} g_x(n) = & -[x_1(0) - \frac{x_2(1)-x_2(0)}{\Delta t}]\, g_{sim}(n) + [x_1(n) - \frac{x_2(n)-x_2(n-1)}{\Delta t}]\, g_{sim}(0) \\ & + \sum_{i=1}^{n-1} \frac{x_2(i-1) - 2x_2(i) + x_2(i+1)}{\Delta t} g_{sim}(n-i). \end{aligned} \qquad n \geq 1 \tag{A.8}$$

Note that $x_1(0) = x_2(0) = 0$, and that the only value of $x_1(i)$ in Eq.(A.8) is $x_1(n)$.