Sensor and Simulation Notes

Note 331

12 September 1991

EFFECTS OF WAVEGUIDE DISPERSION ON HIGH-POWER MICROWAVE SIGNALS

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Abstract

The objective of this note is to investigate the effects of waveguide dispersion on high power microwave (HPM) signals carried from the extraction port of the HPM source to the radiating system [1]. Although HPM sources are designed to be monochromatic, they do in practice possess a narrow band of frequencies, typically a few percent of of the center frequency. The waveguide runs from the extraction port of the source to the feed horn can be several meters long. We have investigated the effects of waveguide dispersion on the HPM signals, assuming a dominant mode of propagation. It is found that for the parameters (frequency, pulse durations) considered, the pulse broadening is negligible if the waveguide lengths are kept under 100 m. This is practically feasible and hence group dispersion effects are seen to be negligible.

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high power microwaves (HPM), waveguides

Preface

This work was performed jointly by Dr. D.V.Giri (Pro-Tech) and Prof.Y.Rahmat-Samii of the Department of Electrical Engineering, University of California (UCLA), and was sponsored by the U.S.Army. The authors are thankful to Dr.Carl Baum of Phillips Laboratory for valuable discussions.

Contents

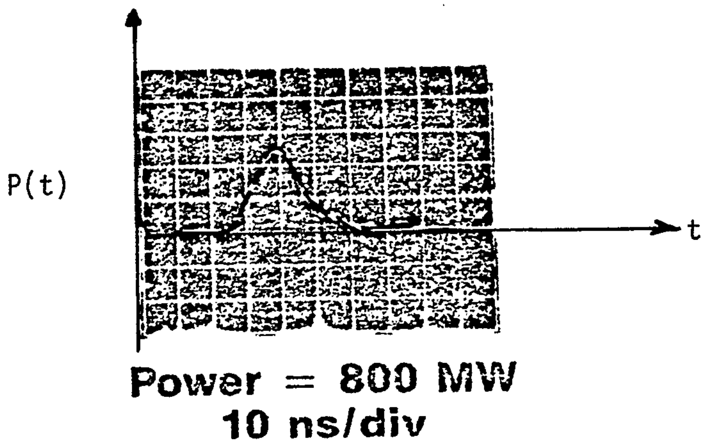
1. Introduction

HPM sources are designed to be monochromatic e.g., by precision tuning of the microwave cavities in a relativistic magnetron or controlling the anode-cathode gap in a VIRCATOR. However, devices in practice do have a narrow bandwidth, typically a few percent of the center frequency. Examples of this are:

a) Benford et al., [2] demonstrated a bandwidth of 2% or (70 MHz at 3.5 GHz) in a VIRCATOR source in 1987.

b) Fazio et al., [3] measured a bandwidth of 0.3% or (2.48 MHz at 825 MHz) in a VIRCATOR source in 1990.

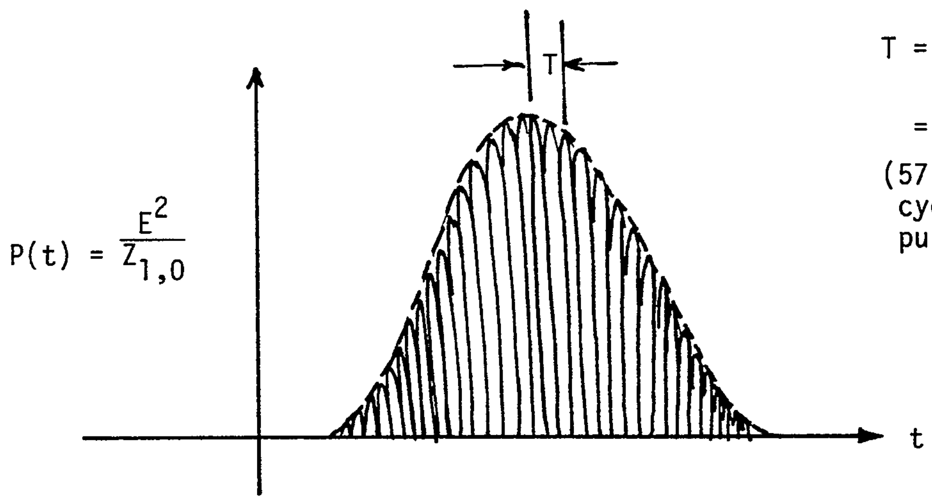
The envelope of the output power is sometimes a near Gaussian shaped curve as indicated by an S-Band magnetron example in figure 1a. Quite often, the power envelope is not as "clean" as shown in this figure, due to inefficient ways of power extraction. If the waveguides get overmoded, the output power as a function of time can become somewhat of arbitrary shapes with many spikes. In the interest of maximizing the field at a distance [4,5,6] it is important to extract the microwave power from the source into an evacuated waveguide, wherein the power flow is in the dominant mode only. This power will be referred to as the "useful power" from the HPM source. Several features of the waveguide and horn design were considered in [7] including the optimization of the waveguide for its power handling capabilities. However, an underlying question is that of the effect of waveguide dispersion on the propagation of such a pulse. We consider gaussian envelopes for both field and power waveforms and estimate the pulse broadening due to waveguide dispersion in the following sections.



Example [8]

S-Band Magnetron
Peak Power = 0.8 GW
freq. = 2.85 GHz
width ~ 20 ns

(a) An example of output power as a function of time.

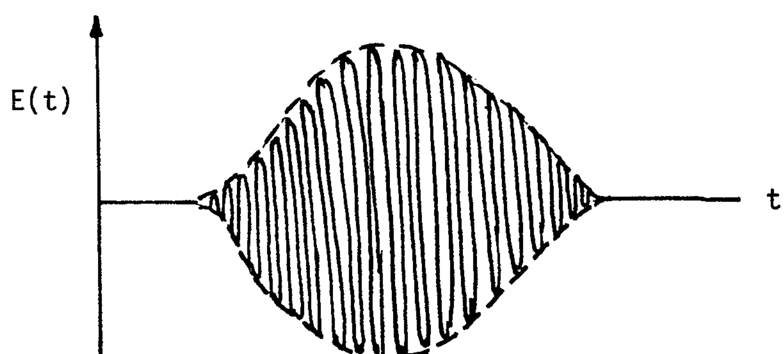


$T = (2.85)^{-1}$ ns for (a) above

$= 0.35$ ns

(57 cycles or 114 half cycles in a 20 ns wide pulse)

(b) A more detailed look at (a)



(c) Electric field as a function of time in the waveguide.

Figure 1. HPM power and electric field waveforms.

2. Waveguide Dispersion

In the context of an HPM radiating system, it is possible that there are waveguide runs over lengths of the order of several meters between the power extraction ports on the source and the feed horns. It would then be useful to investigate the effects of wavegude dispersion on the HPM signal propagating in the waveguide. It is well known [9] that the waveguide modes including the dominant mode of propagation, suffer dispersion even when the waveguides are evacuated.

It is assumed that there is only the dominant mode propagating in the waveguide and that the center frequency of operation f is near and below the cut off of the next higher order mode. HPM signal as a function of time, contains a band of frequencies, however narrow. If it happens that the phase velocity v_p is the same for all frequencies and that there is no attenuation, the spectral components will add in proper phase at all locations in the waveguide, to reproduce the original waveshape exactly. The only difference in the signal is that it is delayed by the propagation delay time (z/v_p). However, if v_p changes with frequency, we have dispersion and the HPM signal will change its shape as it travels in the waveguide. For an evacuated rectangular waveguide with cross sectional dimensions of a (longer side) and b (shorter side), the frequency dependent propagation "constant" is given by

$$\begin{aligned}\beta_{m,n}(\omega) &= \omega\sqrt{\mu_0\varepsilon_0}\left[1-\frac{\omega^2_{c_{m,n}}}{\omega^2}\right]^{1/2} \\ &= \sqrt{\mu_0\varepsilon_0}\sqrt{\omega^2-\omega^2_{c_{m,n}}}\end{aligned} \tag{1}$$

The phase velocity v_p is then given by

$$v_p = \frac{\omega}{\beta_{m,n}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\left[\frac{\omega^2}{\omega^2-\omega^2_{c_{m,n}}}\right]^{1/2} \tag{2}$$

and the group velocity v_g is given by

$$v_g = \left[\frac{d\beta_{m,n}}{d\omega}\right]^{-1} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \left[\frac{\omega^2 - \omega_{c_{m,n}}^2}{\omega^2}\right]^{1/2} \tag{3}$$

It is observed that

$$v_p v_g = \frac{1}{\mu_0 \varepsilon_0} = c^2 \tag{4}$$

where $c \equiv$ speed of light in vacuum and the two velocities v_p and v_g are schematically illustrated in figure 2. We could specialize the velocity expressions for the $H_{1,0}$ mode as follows

$$\lambda_{c_{1,0}} = 2a \;;\; f_{c_{1,0}} = \frac{c}{2a} \;;\; \omega_{c_{1,0}} = \frac{c\pi}{a} \tag{5}$$

$$v_p = c\left[\frac{\omega^2}{\omega^2 - \left[\frac{c\pi}{a}\right]^2}\right]^{1/2} = c\left[1 - \left[\frac{c\pi}{a\omega}\right]^2\right]^{-1/2} \tag{6}$$

$$v_g = c\left[\frac{\omega^2 - \left[\frac{c\pi}{a}\right]^2}{\omega^2}\right]^{1/2} = c\left[1 - \left[\frac{c\pi}{a\omega}\right]\right]^{+1/2} \tag{7}$$

The above expressions for $\beta_{m,n}(\omega)$, v_p and v_g as functions of ω are graphically shown in figure 3. We can now use the above expressions for the phase and group velocities for the particular signal of $E(t)$ in the waveguide shown in figure 1c.

In general the time dependence of the electric field designated by $E(t)$ may be considered as an amplitude modulated carrier as

$$E(t) = \mathrm{Re}\left\{E_0 e^{j\omega_c t}[1 + mf(t)]\right\} \tag{8}$$

where $\omega_c \equiv$ angular HPM frequency

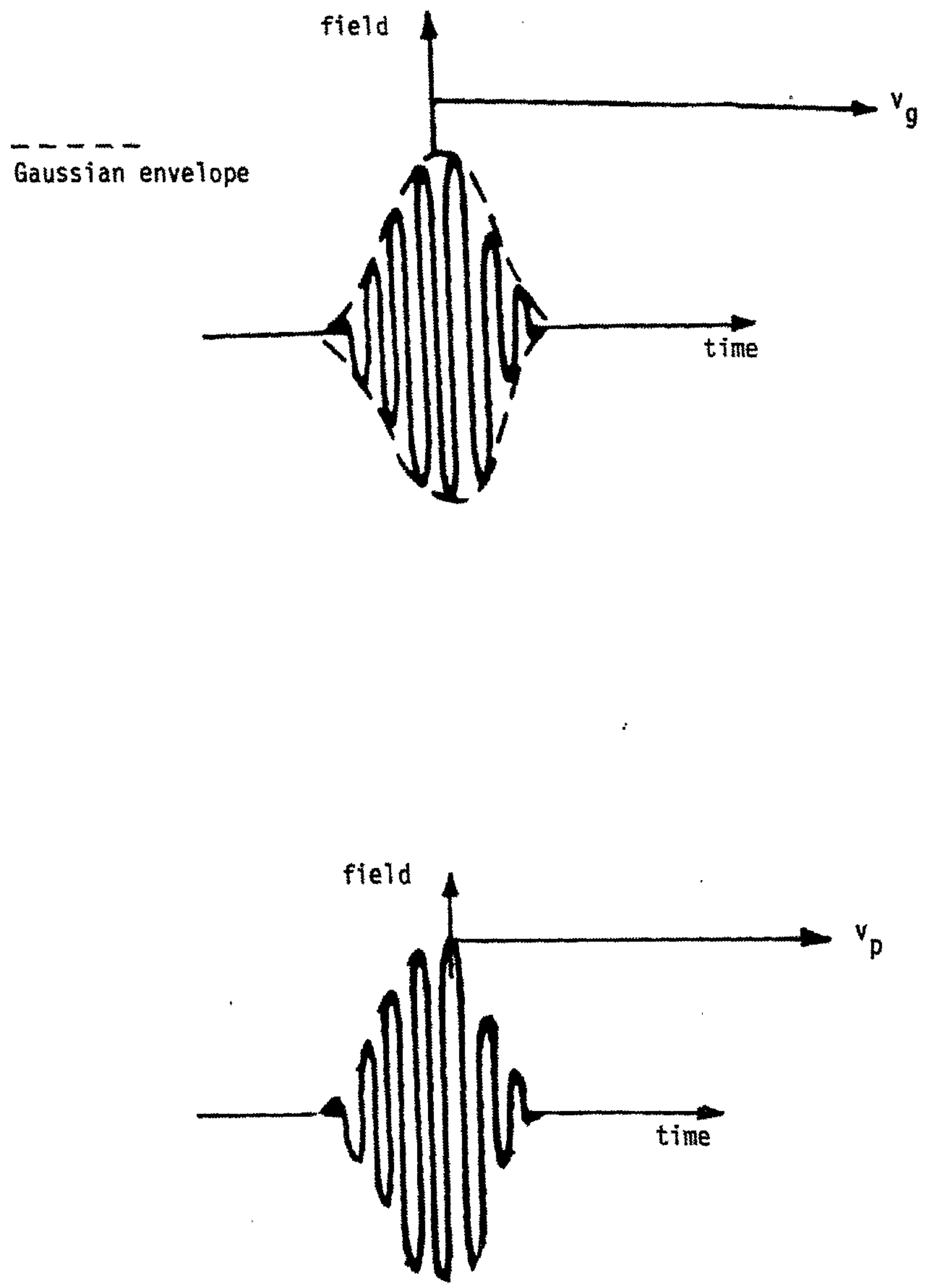
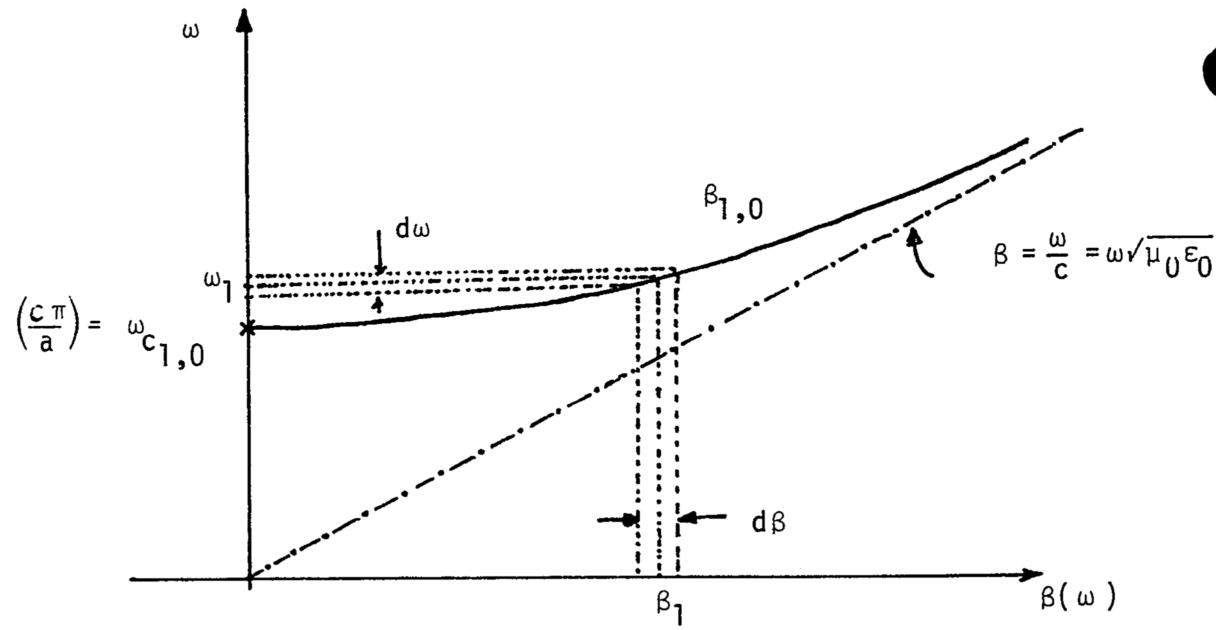
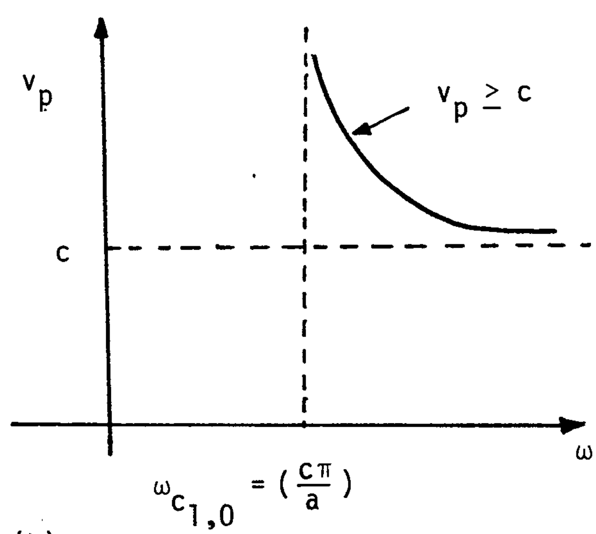


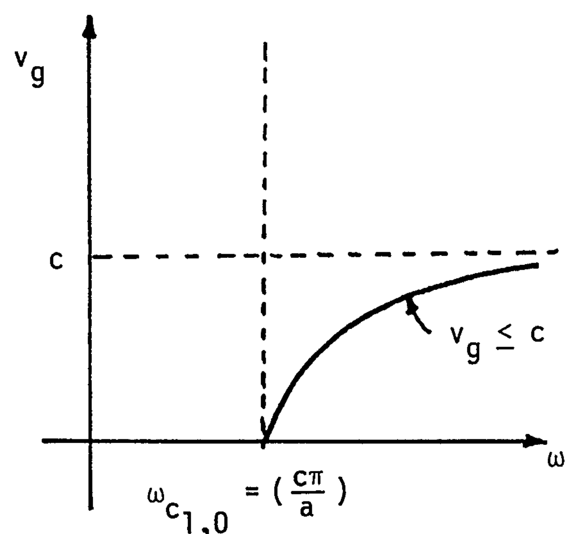
Figure 2. Group and phase velocities of a signal.



(a) ω-β diagram for the dominant $H_{1,0}$ mode.



(b) phase velocity



(c) group velocity

Figure 3. Frequency characteristics of the dominant $H_{1,0}$ mode in a rectangular waveguide.

$m \equiv$ modulation index
$f(t) \equiv$ modulating signal

If $f(t)$ is a band limited signal, it may then be written as

$$f(t) = \frac{1}{2\pi} \int_{-\omega_b}^{\omega_b} F(\omega_m)\, e^{j\omega_m t}\, d\omega_m \tag{9}$$

Now, the electric field $E(t,z)$ at $z = 0$ is given by

$$E(t,0) = \mathrm{Re}\left\{ E_0\, e^{j\omega_c t} \left[1 + \frac{m}{2\pi} \int_{-\omega_b}^{\omega_b} F(\omega_m)\, e^{j\omega_m t}\, d\omega_m \right] \right\} \tag{10}$$

With a change of variable $\omega = \omega_m + \omega_c$, we have

$$E(t,0) = \mathrm{Re}\left\{ E_0 \left[e^{j\omega_c t} + \frac{m}{2\pi} \int_{\omega_c-\omega_b}^{\omega_c+\omega_b} F(\omega-\omega_c)\, e^{j\omega t}\, d\omega \right] \right\} \tag{11}$$

If such an electric field propagates in the waveguide where the propagation "constant" is $\beta(\omega)$ and for a distance of z meters, the output is

$$E(t,z) = \mathrm{Re}\left\{ E_0 \left[e^{j(\omega_c t - \beta(\omega) z)} + \frac{m}{2\pi} \int_{\omega_c-\omega_b}^{\omega_c+\omega_b} F(\omega-\omega_c)\, e^{j[\omega t - \beta(\omega) z]}\, d\omega \right] \right\} \tag{12}$$

Now, expanding $\beta(\omega)$ in its Taylor series about ω_c

$$\beta(\omega) = \beta(\omega_c) + (\omega-\omega_c) \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c} + \frac{(\omega-\omega_c)^2}{2} \left.\frac{d^2\beta}{d\omega^2}\right|_{\omega=\omega_c} + \cdots \tag{13}$$

Since the operating frequency of the HPM signal is above and near the cut off frequency we may initially keep only the first two terms. Defining

$$\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c} = \frac{1}{v_g} \quad \text{and} \quad \beta(\omega_c) \equiv \beta_c \tag{14}$$

$$E(t,z) = \mathrm{Re}\left\{E_0\left[e^{j(\omega_c t-\beta_c z)} + \frac{m}{2\pi}\int_{\omega_c-\omega_b}^{\omega_c+\omega_b} F(\omega-\omega_c)\, e^{j\left[\omega t-\beta_c z-\frac{\omega-\omega_c}{v_g}z\right]}\, d\omega\right]\right\} \tag{15}$$

reversing the change of variable $\omega_m = \omega - \omega_c$

$$E(t,z) = \mathrm{Re}\left\{E_0 e^{j(\omega_c t-\beta_c z)}\left[1 + \frac{m}{2\pi}\int_{-\omega_b}^{\omega_b} F(\omega_m)\, e^{j\omega_m\left[t-\frac{z}{v_g}\right]}\, d\omega_m\right]\right\} \tag{16}$$

Comparing (16) and (9) we find

$$E(t,z) = \mathrm{Re}\left\{E_0 e^{j(\omega_c t-\beta_c z)}\left[1 + m f\left[t-\frac{z}{v_g}\right]\right]\right\} \tag{17}$$

resulting back in

$$E(t,0) = \mathrm{Re}\left\{E_0\, e^{j\omega_c t}[1 + m f(t)]\right\} \tag{18}$$

which is consistent with (8).

However, we can go a step further[10] and consider the modulating signal to be a gaussian and also include the next term in the Taylor series expansion of $\beta(\omega)$, as follows

$$f(t) \equiv \text{modulating signal} = e^{-(2t/\tau)^2} \tag{19}$$

$$F(\omega_m) = \int_{-\infty}^{\infty} e^{(-2t/\tau)^2} e^{j\omega_m t}\, dt = \frac{\sqrt{\pi}\,\tau}{2}\, e^{-\left[\frac{\omega_m \tau}{4}\right]^2} \tag{20}$$

$$D \equiv \left.\frac{d^2\beta}{d\omega^2}\right|_{\omega = \omega_c} \tag{21}$$

So, we now have

$$E(t,z) = \mathrm{Re}\left\{E_0 e^{j(\omega_c t - \beta_c z)} \left[1 + \frac{m\tau}{4\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\left[\frac{\omega_m \tau}{4}\right]^2} e^{j\left[\omega_m \left[t - \frac{z}{v_g}\right] - \frac{\omega_m^2}{2} Dz\right]} d\omega_m\right]\right\} \tag{22}$$

Using

$$\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)} dx = \sqrt{\frac{\pi}{a}}\, e^{(b^2-4ac)/4a} \tag{23}$$

the integral in (22) becomes

$$\int_{-\infty}^{\infty} [\quad] \, d\omega_m = \sqrt{\frac{\pi}{\frac{\tau^2}{16} + j\,\frac{Dz}{2}}}$$

$$= \frac{\sqrt{\pi}}{\left[\left[\frac{\tau^2}{16}\right]^2 + \left[\frac{Dz}{2}\right]^2\right]^{1/4}}\, e^{-\frac{j}{2}\tan^{-1}\left[\frac{8Dz}{\tau^2}\right]} e^{-\left[\frac{\left[t - \frac{z}{v_g}\right]^2 \left[\frac{\tau^2}{4} - 2jDz\right]}{(\tau^2/4)^2 + 4D^2 z^2}\right]}$$

leading to

$$E(t,z) = \mathrm{Re}\left\{ E_0 e^{j(\omega_c \tau - \beta_c z)} \left[1 + \frac{m\tau}{4} \frac{e^{-\frac{j}{2}\tan^{-1}\left[\frac{8Dz}{\tau^2}\right]}}{\left[\left[\frac{\tau^2}{16}\right]^2 + \left[\frac{Dz}{2}\right]^2\right]^{1/4}} e^{-\left[\frac{\left[t - \frac{z}{v_g}\right]^2 \left[\frac{\tau^2}{4} - 2jDz\right]}{\left[\frac{\tau^2}{4}\right] + (2Dz)^2}\right]} \right] \right\}$$

(24)

From the above expression, we observe that after propagating through a distance of z in the waveguide the HPM signal of frequency ω_c with a gaussian envelop, changes in amplitude and phase. The peak of the gaussian is delayed by (z/v_g) and the width is modified from τ to τ' given by

$$\frac{\tau'^2}{4} = \frac{\left[\frac{\tau^2}{4}\right]^2 + (2Dz)^2}{(\tau^2/4)}$$

or

$$\tau' = \tau\sqrt{1 + \left[\frac{8Dz}{\tau^2}\right]^2} \qquad (25)$$

This result is consistent with the expected result in Problem 8.16d of [9]. It is also observed that the amplitude of the gaussian field envelope diminishes by an amount $(\sqrt{\tau'/\tau})$ and the amplitude of the gaussian power envelop diminishes by an amount (τ/τ'). These results may be derived from (24). In a waveguide with rectangular cross section ($a \times b$), and a dominant propagating mode, we can deduce D as

$$
\begin{aligned}
D &= \frac{d^2\beta}{d\omega^2} = \frac{d^2\beta_{1,0}}{d\omega^2} \\
&= \frac{d^2}{d\omega^2}\left[\frac{1}{c}\sqrt{\omega^2 - \omega_{c_{1,0}}^2}\right] \\
&= \frac{1}{c}\frac{d}{d\omega}\left[\frac{\omega}{\sqrt{\omega^2 - \omega_{c_{1,0}}^2}}\right] \\
&= -\frac{1}{c}\frac{\omega_{c_{1,0}}^2}{(\omega^2 - \omega_{c_{1,0}}^2)^{3/2}}
\end{aligned}
\tag{26}
$$

Substituting for the cut off frequency, we have

$$
D = -\frac{1}{c}\left[\frac{c\pi}{a}\right]^2\left[\omega^2 - \left[\frac{c\pi}{a}\right]^2\right]^{-3/2}\left[\frac{\sec^2}{m}\right] \tag{27}
$$

Substituting the above expression for D into (25), we have

$$
\tau' = \tau\left[1 + \left[\frac{8\pi^2 zc\tau}{a^2}\right]^2\left\{(\omega\tau)^2 - \left[\frac{c\tau\pi}{a}\right]^2\right\}^{-3}\right]^{1/2} \tag{28}
$$

We observe that since the second term in the parenthesis is positive, the Gaussian pulse of width τ is broadened to a new width τ' ($> \tau$) after propagating through the waveguide for a distance z. In the expression for τ' above in (28), ω is the angular frequency $= 2\pi f$ of the HPM signal, τ is the initial width, c is the speed of light in vacuum and a is the longer side of the waveguide cross section. It is now a simple matter to estimate this pulse broadening due to dispersion by substituting some numerical parameters.

3. Numerical Examples

We reproduce (28) and consider some numerical examples

$$\tau' = \tau\left[1 + \left[\frac{8\pi^2 z c\,\tau}{a^2}\right]^2\left\{(\omega\tau)^2 - \left[\frac{c\,\tau\pi}{a}\right]^2\right\}^{-3}\right]^{1/2} \tag{29}$$

We consider two frequencies (f_1 = 1 GHz and f_2 = 3 GHz) and the corresponding waveguides are [5] WR 975 and WR 340 with the relevant dimensions of a_1 = 247.65 mm and a_2 = 86.36 mm. Let us consider initial pulse widths τ_i = 25 ns 50 ns, 100 ns, 500 ns and 1 μs. The waveguide run lengths are varied from z = 0 to 100 m. The output pulse widths are computed by rewriting the above expression as follows

$$\tau' = \tau\left[1 + A\,\frac{z^2}{\tau^4}\right]^{1/2} \quad s \tag{30}$$

where

$$A = \left[\frac{8\pi^2 c}{a^2}\right]^2\left\{\omega^2 - \left[\frac{c\,\pi}{a}\right]^2\right\}^{-3} \quad (s^4/m^2) \tag{31}$$

Substituting the numerical constants for π, c, a, we have the following

$$\tau' = \tau\left[1 + 9.49\,\frac{z^2}{\tau_{ns}^4}\right]^{1/2} \quad \text{at } f = 1\ GHz \tag{32}$$

$$\tau' = \tau\left[1 + 0.7654\,\frac{z^2}{\tau_{ns}^4}\right]^{1/2} \quad \text{at } f = 3\ GHz \tag{33}$$

noting that τ_{ns} in above is τ expressed in ns.

The above expressions are used in estimating the output pulse widths listed in Tables 1 and 2 for frequencies 1 GHz and 3 GHz respectively.

f = 1 GHz

z (m) \ τ (ns)	25	50	100	500	1000
0	25	50	100	500	1000
25	25.189 051	50.023 744	100.002 966	500.000 024	1000.000 002
50	25.747 876	50.094 910	100.011 862	500.000 095	1000.000 011
100	27.871 356	50.378 567	100.047 439	500.000 380	1000.000 047

TABLE 1. Output pulse width τ' as a function of input pulse width τ and distance of propagation z in WR 975 waveguide at 1 GHz.

NOTE:

1) In all of the cases above the pulse is broadened by less than 12 % of its initial value.

2) Consequently, the amplitude of the Gaussian power envelope will be reduced by less than 12% also. The corresponding reduction in the amplitude of the Gaussian field envelope is about 6 %.

f = 3 GHz

z (m) \ τ(ns)	25	50	100	500	1000
0	25	50	100	500	1000
25	25.015 303	50.001 914	100.000 239	500.000 002	1000.000 000
50	25.061 156	50.007 656	100.000 957	500.000 008	1000.000 001
100	25.243 737	50.030 616	100.003 827	500.000 031	1000.000 004

TABLE 2. Output pulse width τ' as a function of input pulse width τ and distance of propagation z in WR 340 waveguide at 3 GHz.

NOTE:

1) In all of the cases above the pulse is broadened by less than 1 % of its initial value.

2) Consequently, the amplitude of the Gaussian power envelope will be reduced by less than 1 %. The corresponding reduction in the amplitude of the field envelope (also Gaussian) is about 0.5 %.

Some comments about the results in the two tables are in order. They are listed below

1) The pulse is broadened by propagating in the waveguide.
2) The pulse amplitude is reduced due to both dispersion and also attenuation in the waveguide.
3) The pulse is phase shifted (frequency domain) or delayed in time domain.
4) The pulse broadening due to dispersion, which is the focus of this paper appears negligible for practical situations, where waveguide run lengths from the source to feed horns are expected to be under 100 m, for all pulse widths ranging from 25 ns to 1 μs.
5) The pulse broadening decreases as the pulse width increases.
6) It is recalled that we have considered two frequencies 1 GHz and 3 GHz here and the corresponding waveguides are WR 975 and WR 340 with dominant mode propagation.

4. Summary

In this report, we have investigated the effect of group dispersion on the pulse broadening of HPM signal passing through a section of rectangular waveguide in its dominant mode of propagation. Two frequencies are considered (1 GHz and 3 GHz) along with a range of waveguide lengths (0 to .1 km) and a range of pulse widths (25 ns to 1 μs). Following the analysis required by Problem 8.16 d of [9], an expression for the output pulse width in terms of the input pulse width, frequency, waveguide dimensions is available and has been used in estimating the output pulse width.

It is generally found that for the parameters (frequency, pulse widths) considered, the pulse broadening is negligible if the waveguide lengths are kept under 100 m. This is practically feasible and hence group dispersion effects should not be a limiting factor in HPM radiation systems.

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