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Diffraction-Free Microwave Propagation

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Abstract

The properties of three mathematically diffraction-free microwave propagation modes are reviewed. It is shown that each of these modes either is of limited practical value or exhibits the usual diffraction rate for physically realizable antennas.

I. Introduction

Conventional optics theory predicts that a well collimated microwave beam expands in cross-sectional area as $(L_r/z)^2$ after propagating beyond a Rayleigh range. Here, z is the propagation distance,

$$L_r = A/\lambda \qquad (1)$$

is the Rayleigh range, A is the antenna area, and λ is the radiation wavelength. As a numerical example, a 3 GHz microwave beam (λ = 0.1 m) launched from a 10 m radius antenna has a 3 km Rayleigh range. Achieving high microwave power densities at many tens of km requires fluxes a few hundred times greater at the antenna according to this model. Air breakdown at the antenna becomes the limiting factor in some applications.

However, several theoretical and experimental studies during the past six years purport to show free-space wave propagation without transverse spreading, i. e., diffraction-free propagation. This paper reviews these potentially exciting results. Unfortunately, we find that the three diffraction-free propagation schemes appearing in the scientific literature either are not substantiated by the published research or have limited practical value. Basically, the mathematically correct diffraction-free wave fields derived by the various authors are not physically realizable with finite aperture and risetime antennas.

The following Section employs Huygen's principal to obtain an upper bound on microwave energy density at large distances from an antenna. This upper bound proves that a microwave beam cannot propagation indefinitely without diffraction but does not preclude diffraction-free propagation over limited distances. T. T. Wu's "electromagnetic missile" concept is analyzed in Section III. Although electromagnetic missiles do not diffract in the usual sense, they are subnanosecond in duration, carrying little energy and requiring exceedingly fast rise times. Section IV treats J. Durnin's suggestion that the well known wave fields with Bessel-function radial profiles allow microwave energy transport at better than the diffraction limit. In fact, neither his calculations nor his experiment support this conclusion. The intriguing "focused-wave modes" of J. N. Brittingham and R. W. Ziolkowski are treated in Section V. While a focused-wave mode has a central wave packet that does not diffract, most of its energy is contained in wings that extend over great distances. Moreover, finite antennas proposed for launching these waves lead to energy transport no better than for conventional wave patterns. The concluding bibliography lists all relevant literature known to us; references in the text are by first author and date of publication.

This paper deals throughout with scalar rather than vector fields. The scalar solutions can be thought of as Hertz potentials, from which free-space electromagnetic fields can be derived [A. Sezginer, 1985]. Additionally, the scalar field pattern is itself a good approximation to that of the vector fields, if characteristic wavelengths are short compared to the antenna size. Scalar waves also are of interest in their own right, e. g., as acoustic waves in water.

II. Estimates of Microwave Diffraction

Fields generated by an antenna can be expressed in terms of Huygen's principle [Ziolkowski, 1989a], an approximation to the scalar wave equation Green's theorem.

$$f(x,y,z,t) = \int \frac{dA}{4\pi R} \left[\frac{\partial f}{\partial z} - \frac{z}{cR}\frac{\partial f}{\partial t} - \frac{z}{R^2} f \right] \tag{2}$$

$$R^2 = (x - x')^2 + (y - y')^2 + (z - z')^2$$

The wave function $\underline{f}$ in the right side of Eq. (2) is evaluated at positions (x',y',z') on the antenna surface and at retarded time t - R/c.

Let the antenna be oriented along the z-axis and located near z' = 0. Then, near the z-axis and far from the antenna, the field can be approximated as

$$f(x,y,z,t) \approx \int \frac{dA}{4\pi R} \left[\frac{\partial f}{\partial z} - \frac{1}{c}\frac{\partial f}{\partial t} \right] \tag{3}$$

From this follows an upper bound on the magnitude of f:

$$|f(x,y,z,t)| \leq \frac{A}{4\pi R} \left| \frac{\partial f}{\partial z} - \frac{1}{c} \frac{\partial f}{\partial t} \right|_{max} \tag{4}$$

It is evident from Eq. (4) that f must fall off as z^{-1} (and power as z^{-2}) or faster at sufficiently large distances from the antenna. Diffraction-free propagation to infinity is impossible [Wu, 1985b]. However, Eq. (4), being an upper bound, does to rule out diffraction-free propagation over finite distances. The field pattern from a focusing antenna can increase with z over a limited range, for instance.

The Rayleigh range also can be obtained from Eq. (3). Specify the field at the antenna to be a plane wave, $f = f_0 \exp[i(kz - \omega t)]$ with $\omega/c = k$. Then, in the far field region

$$f(x,y,z,t) \approx f_0\, \omega A/2\pi cz \tag{5}$$

With the substitution $\omega = 2\pi c/\lambda$, the scale length $\omega A/2\pi c$ becomes A/λ, as desired. Alternative, more rigorous derivations exist.

III. The Electromagnetic Missile Concept

About five years ago T. T. Wu showed that transient wave packets with frequency dependence $\omega^{-\frac{1}{2}-\epsilon}$ for large ω decay in energy as $z^{-2\epsilon}$ far from the antenna [Wu, 1985a]. The wave amplitude varies as $z^{\frac{1}{2}-\epsilon}$, and the pulse length as z^{-1} [Shen, 1989]; $0 < \epsilon < 1$. Wu dubbed his wave packet the "electromagnetic missile." Diffraction can be made arbitrarily slow by the appropriate choice of ϵ. This result is reconciled easily with Eq. (4). A $\omega^{-\frac{1}{2}-\epsilon}$ high frequency variation implies a $t^{\epsilon-\frac{1}{2}}$ temporal variation at the head of the microwave pulse [Lee, 1987]. Thus, the integrand in Eq. (3) is singular at early times, and the simple arguments leading to Eq. (4) fail. Truncating the distribution at a finite frequency destroys the slow spatial fall-off rate at distances exceeding one-half of the Rayleigh range, defined in terms of the cutoff frequency [Wu, 1989b].

The essence of the electromagnetic missile concept is well illustrated by a simple example, considered in the time domain. A disk antenna driven by a current pulse I(t) produces the on-axis field,

$$f(0,0,z,t) = I(t - z/c) - I(t - R/c) \tag{6}$$

where now $R^2 = z^2 + a^2$, a being the antenna radius [Shen, 1988]. For a step function pulse (analagous to $\epsilon = ½$), f is a square wave of width $\Delta z = R - z$. Expanding R for large z shows the width to be decreasing steadily, $\Delta z \approx a^2/2z$. Thus, although the very front of the pulse does not diffract, the tail does and in such a way that the total energy in the packet decreases inversely with propagation distance. A similar picture for more general pulses, valid out to a distance a from the axis, has been derived [Lee, 1987; Shen, 1989]. Qualitatively, losses occur only in the tail of the wave packet until Δz becomes less than the pulse rise time, whereupon diffraction eats into the head as well.

This simple analysis is consistent with an experiment [Shen, 1988] in which a very fast rising electromagnetic pulse was launched from a 2 ft. radius parabolic antenna and measured at distances of up to 49 ft. Beyond about 4 ft. the transmitted energy decreased very nearly as z^{-1}. The pulse width at 49 ft. was found to be about 50 psec, also in approximate agreement with theoretical predictions.

Even though the energy decay of an electromagnetic missile is of order z^{-1} instead of the z^{-2} decay in ordinary diffracting pulses, electromagnetic missiles do not seem practical for transmitting large amounts of energy over long distances. For instance, a wave packet launched from a 100 m radius antenna would be only 16 psec long at 1000 km. The same pulse length results for a 10 m radius antenna at 10 km or for a 1 m radius antenna at 0.1 km. Moreover, producing such a pulse requires phasing the antenna to an accuracy of 16 psec or better.

IV. The Bessel-Function Profile Beam Concept

One exact solution to the scalar wave equation is

$$f = f_0 J_0(\kappa r) \exp[i(kz - \omega t)] \quad (7)$$

with $\omega^2/c^2 = k^2 + \kappa^2$. J_0 is the Bessel function of order zero. The radial pattern of Eq. (7) consists of a central peak of radius $2.4/\kappa$ surrounded by concentric rings spaced by approximately π/κ. Although the ring amplitudes decrease with radius as $(\kappa r)^{-1/2}$, each ring contains about the same amount of energy as the central peak. Nonaxisymmetric solutions corresponding to Eq. (7) also exist but are of less interest.

J. Durnin and coworkers have demonstrated computationally [Durnin, 1987a] and experimentally [Durnin, 1987b] that bounded wave packets similar to Eq. (7) can be generated from finite antennas provided $\kappa a/\pi >> 1$; i. e., many rings across the antenna face. These waves propagate a distance $\pi a a_0/1.2\lambda$ before diffracting significantly. Here, a_0 is the radius of the central peak, and $\lambda = 2\pi c/\omega$ is the wavelength. Durnin also generated Gaussian profile waves of half-radius a_0 and found them to propagate only $\pi a_0{}^2/\lambda$ before beginning to diffract. On this basis he concluded that Bessel-function profile beams have remarkably good diffraction properties.

This comparison is, however, misleading. The diffraction length of Durnin's waves should, instead, be compared to the Rayleigh range of the antenna, Eq. (1). Doing so shows that these waves propagate less than the Rayleigh range by a factor of $2/\kappa a$ for a specified antenna size and wave frequency. Of course, the waves require less energy than the plane waves used to obtain Eq. (1), the ratio being $2/\pi\kappa a$. The Bessel-function profile beam is seen to be a little more efficient than the plane wave beam in terms of diffraction-free range per energy invested but is much less efficient in terms of antenna area.

It has been pointed out the Bessel-function profile beam, as produced by a finite-sized antenna, can be viewed as a line focus [DeBeer, 1987].

V. The Focused-Wave Mode Concept

Research on diffraction-free propagation was triggered by Brittingham's discovery of the focused-wave mode, [Brittingham, 1983]

$$f = f_0 \exp[-ks]/[z_0 + i(z - ct)] \tag{8}$$

$$s = r^2/[z_0 + i(z - ct)] - i(z + ct)$$

It propagates at speed $\underline{c}$ in the positive $\underline{z}$ direction and falls off in all directions from its central peak at $z = ct$. The width in $\underline{r}$ of the central peak is $z_0/k^{1/2}$, and the length in $\underline{z}$ is approximately z_0. That $\underline{f}$ is complex is not a difficulty: The real part is taken as the physical wave. Nonaxisymmetric waves corresponding to Eq. (8) also are known [Belanger, 1984; Sezginer, 1985].

Eq. (8) has a serious shortcoming, infinite energy [Wu, 1984]. However, as with ordinary plane waves, a superposition of focused-wave modes can yield a finite energy wave packet [Ziolkowski, 1985]. One such solution is [Ziolkowski, 1989a]

$$f = f_0 \exp[-ks]/[z_0 + i(z - ct)][ks + b] \tag{9}$$

The constant $\underline{b}$ sets the spherical radius at which the tail of the focused-wave mode is largely cut off. It also sets the maximum propagation distance for which there is no dispersion, $z \approx b/2k$.

Another difficulty with focused-wave modes is that they are acausal, with energy components flowing in both the positive and negative axial directions [Heyman, 1987a]. R. W. Ziolkowski argues, however, that the negative component in Eq. (9) can be made exponentially small by choosing k/b large. (The validity of this claim is unclear to the author.) In any case, the causality issue is subsidiary to whether the focused-wave mode can be launched from a realizable antenna.

Ziolkowski numerically investigated focused-wave mode generation from disk antennas. Parameters $z_0 = 1.67\,10^{-5}$ m, $k = 1.67\,10^{-4}$ m^{-1}, and $b = 10^{10}$ were used. The time scale corresponding to z_0 is 0.1 psec. With an antenna radius of $a = 0.5$ m, the diffraction-free propagation distance of the central pulse was found from Eq. (2) to be 10^3 m, less than the Rayleigh range by a factor of five. Increasing the antenna radius in his calculations increased the diffraction-free distance only linearly [Ziolkowski, 1989a]. Note also that essentially all the energy in the numerical examples was in the pulse wings.

Acoustic focused-wave mode propagation experiments in water were performed as well [Ziolkowski, 1989c]. In this case the rectangular antenna used had an effective area of 36 cm^2, and z_0 was about 0.2 cm. Hence, the Rayleigh range was 180 cm. The focused-wave mode propagated roughly 100 cm before showing evidence of diffraction. A Gaussian wave launched from the antenna did less well, but its 1.5 cm radius was not necessarily optimal. Ziolkowski is preparing experiments with a shorter Rayleigh range in order to put his models to a more incisive test.

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