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A Lens Technique for Transitioning Waves Between Conical and Cylindrical Transmission Lines

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Abstract

Conical transmission lines can be used to launch and terminate plane waves on cylindrical transmission lines. However, some mismatch is introduced in joining the two transmission lines. This mismatch may be significantly reduced by the insertion of a synthetic space lens at the juncture of the two transmission lines. The lens is made of a material which has the same wave impedance as free space, but which also has a slower propagation velocity. Such a material may also have other uses in other kinds of electromagnetic devices.

I. Introduction

In a previous note we discussed the use of conical transmission lines in launching plane waves on cylindrical transmission lines.[1] While the field distribution on the conical line can be made to closely match that on the cylindrical line, the wave on the conical line is spherical, not planar, introducing a time dispersion across the cross section of the cylindrical line. This time dispersion can be reduced by lengthening the conical line which also makes the transition more gradual.

Another way to reduce this time dispersion is to slow down the spherical wavefront in different degrees over the cross section of the transmission lines at their juncture so that the spherical wave is made nearly planar. This is just the concept of a lens. In the more familiar case of an optical lens, if a light source is placed at one of the foci, the light passing through the lens is focused into a parallel (non-diverging) beam. For the present application consider the apex of the conical transmission line as the wave source which is at one focal length, the length of the conical line, from the lens. Of course the typical lens works for wavelengths much less than the diameter of the lens, but it is for such high frequencies that the lens is needed. For longer wavelengths the conductors of the transmission line guide the wave. Combining a lens with the matched conical and cylindrical transmission lines may then improve the rise time characteristics of the plane wave launched on a cylindrical transmission line.

The typical lens achieves its effect because of its dielectric properties. That is, its permittivity, ε, (or dielectric constant) is greater than ε_o and thus electromagnetic waves propagate through the lens at a speed less than c, the speed of light in free space (and very nearly in normal air also). This type of lens, however, has an undesirable property in that electromagnetic waves are partially reflected (except in the case of the Brewster angle) in passing into and out of the lens material. But suppose that we have a special material from which to make a lens. This special material (which we choose to call synthetic space) has an increased permeability, μ, as well as permittivity, ε, in such a proportion that their ratio is the same as that for free space, i.e., μ_o/ε_o. In other words, this material has the same wave impedance as free space. The propagation speed in this material, however, is less, so that it can be used for a lens. An electromagnetic wave in a free space medium which is normally incident on a semi-infinite medium (or a slab) of this synthetic space undergoes no reflection. For angles near normal incidence the reflection is small compared to a dielectric lens. A synthetic space lens used with a gradual conical transition might be built with angles of incidence close to normal incidence and thus avoid this reflection problem.

In order to make this synthetic space we require some material with μ/μ_o greater than one and greater than or equal to $\varepsilon/\varepsilon_o$ for the same material. Also the conductivity, σ, should be sufficiently low such that the relaxation time, ε/σ, of the material be much larger than appropriate times of interest (such as transit times on the structure). The permeability and permittivity should be approximately independent of frequency for frequencies of interest.

1. Capt Carl E. Baum, Sensor and Simulation Note XXXI, The Conical Transmission Line as a Wave Launcher and Terminator for a Cylindrical Transmission Line, January 1967.

One group of materials which might be considered for this application is the ferrites. If the ratio, μ/ε , of the material is too large it might be lowered by mixing the material with an insulating dielectric in appropriate proportions. If the propagation speed in the material is slower than desired it might be increased by lowering both μ and ε by decreasing the mass density of the material, possibly by foaming or mixing with a foam.

Besides lenses for launching plane waves on cylindrical transmission lines, such a synthetic space material might have other uses. As suggested (private communication) by Dr. R. Partridge of LASL such a material might also be advantageously employed in electromagnetic sensors. If one is willing to vary the propagation speed in the synthetic space as a function of position perhaps one could further improve devices such as lenses by smoothly bending electromagnetic waves in particular desirable manners.

Particular interest lies in the case in which the wave impedance is the same as free space because of the typical application (say for electromagnetic plane wave simulators) in which the electromagnetic wave is in air. There might be other cases, however, in which one is concerned with media (such as dielectrics) with wave impedances other than that of free space. In such cases materials with different electromagnetic propagation speeds but with the same wave impedance may still be used to achieve the same kind of effects. Even though a particular wave impedance is considered in this note, the results apply for a general wave impedance.

II. Reflection and Refraction at an Interface With a Synthetic Space Medium

Consider, then, what happens at the interface of two semi-infinite media as illustrated in figure 1. Medium 1 is defined by negative z and medium 2 by positive z with scalar electromagnetic parameters (ε, μ, and σ) for each medium appropriately subscripted. The incident wave impinges on the interface from medium 1 at an angle, ξ_1, with the negative z axis. The direction of propagation of the incident wave (and of the other two waves as well) is parallel to the (x, z) plane, determining the direction of the x and y axes. The incident, transmitted, and reflected waves are designated by subscripts 1, 2, and 3, respectively, and similarly have angles defining their propagation directions as illustrated in figure 1.

First, consider the relations among the angles defining the propagation directions for the three waves. The electric field, E, in the three waves is normal to the propagation directions and has the forms for the incident wave

$$E = E_1 e^{-jk_1[x\sin(\xi_1)+z\cos(\xi_1)]} \tag{1}$$

for the transmitted wave

$$E = E_2 e^{-jk_2[x\sin(\xi_2)+z\cos(\xi_2)]} \tag{2}$$

and for the reflected wave

$$E = E_3 e^{-jk_1[x\sin(\xi_3)-z\cos(\xi_3)]} \tag{3}$$

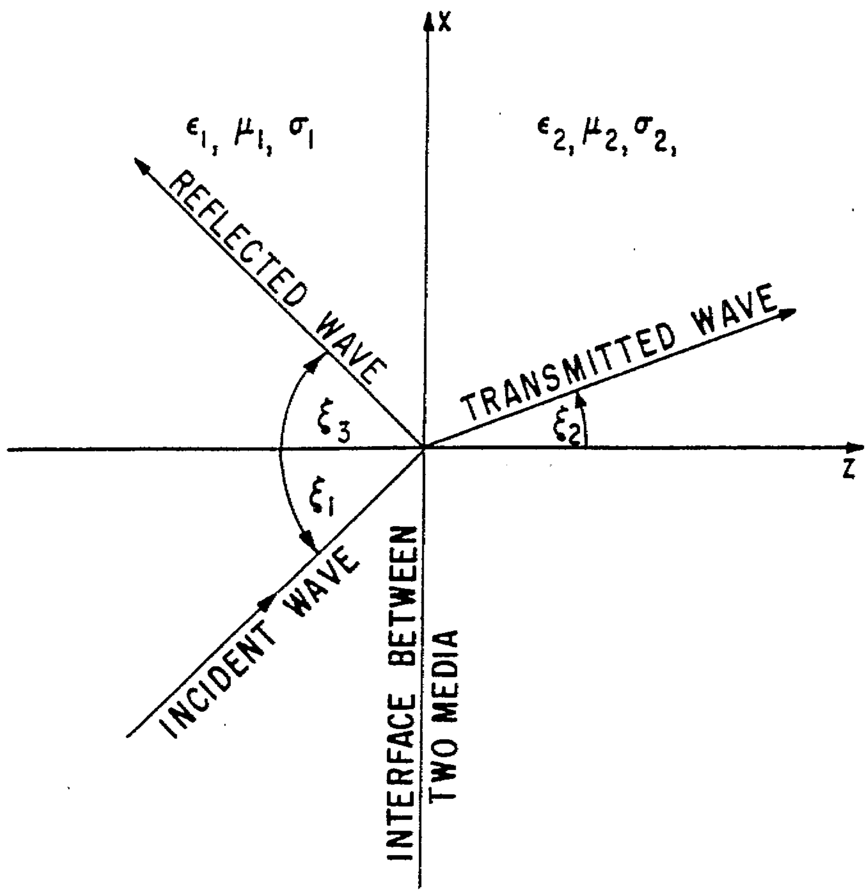

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FIGURE I. REFLECTION AND REFRACTION GEOMETRY where in the formulation (and in others to follow) a time dependence of the form $e^{j\omega t}$ is assumed, but suppressed from the expressions. Similar expressions apply for the magnetic field in the three waves. Either the electric or magnetic field is assumed parallel to the y axis and a more general polarization of the incident wave can be decomposed into these two cases, which are considered later. The propagation constants and wave impedances are of the forms

$$k = \sqrt{-j\omega\mu(\sigma+j\omega\varepsilon)} \tag{4}$$

and

$$Z = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}} \tag{5}$$

respectively, and these parameters are subscripted to apply to each specific medium. For a polarization of the incident wave with one field component parallel to the y axis, the electric and magnetic fields are related by the wave impedance in each medium and the results apply to many cases if we let the angles defining the propagation directions be complex numbers.[2] For this note the interesting cases have these angles as real numbers.

If the three waves as in equations (1) through (3) are to match along the interface (z = 0), then it is required that

$$k_1\sin(\xi_1) = k_2\sin(\xi_2) = k_1\sin(\xi_3) \tag{6}$$

This reduces to the familiar relationships

$$\xi_1 = \xi_3 \tag{7}$$

and

$$k_1\sin(\xi_1) = k_2\sin(\xi_2) \tag{8}$$

which are known as Snell's laws. From equation (7) then ξ_3 is dropped from further consideration. Since we are interested in the case for which the incident wave is a plane wave with ξ_1 a real number we can see from equation (8) that for ξ_2 to also be real it is necessary that the ratio k_1/k_2 be real. From equations (4) and (5)

$$\frac{k_1}{k_2} = \frac{\mu_1}{\mu_2}\,\frac{Z_2}{Z_1} \tag{9}$$

For real μ's and equal Z's (which is required later) this ratio is a real number, and then ξ_2 is also a real number. Note that this relationship does not require that the σ's be zero, although this is also required later.

2. J. A. Stratton, Electromagnetic Theory, Chap. IX, 1941.

Now consider the transmission and reflection of the waves at the interface between the two media. The transmission and reflection coefficients are given by Fresnel's equations. First, let the electric field in the incident wave (and also the electric field in the other two waves) be parallel to the y axis as illustrated in figure 2A. This polarization is designated by a subscript, e. The direction of the field components indicated in the figure is taken as positive for each field component. Applying the boundary conditions at the interface gives for tangential E

$$E_1 + E_3 = E_2 \tag{10}$$

and for tangential H

$$H_1\cos(\xi_1) - H_3\cos(\xi_1) = H_2\cos(\xi_2) \tag{11}$$

Replacing H by E/Z gives

$$\left(E_1 - E_3\right)\frac{\cos(\xi_1)}{Z_1} = E_2\,\frac{\cos(\xi_2)}{Z_2} \tag{12}$$

Eliminating one of the E's in equations (10) and (12), the ratio of the other two can be found. This gives a transmission coefficient

$$T_e = \frac{E_2}{E_1} = \frac{Z_2}{Z_1}\frac{H_2}{H_1} = 2\left[1 + \frac{Z_1}{Z_2}\frac{\cos(\xi_2)}{\cos(\xi_1)}\right]^{-1} \tag{13}$$

and a reflection coefficient

$$R_e = \frac{E_3}{E_1} = \frac{H_3}{H_1} = \frac{1 - \dfrac{Z_1}{Z_2}\dfrac{\cos(\xi_2)}{\cos(\xi_1)}}{1 + \dfrac{Z_1}{Z_2}\dfrac{\cos(\xi_2)}{\cos(\xi_1)}} \tag{14}$$

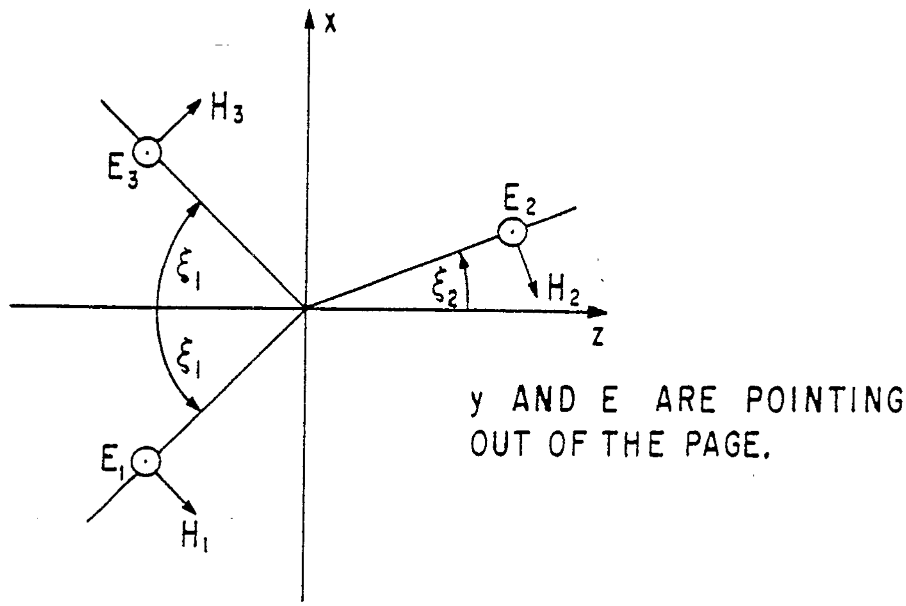
where these are related as

$$T_e = 1 + R_e \tag{15}$$

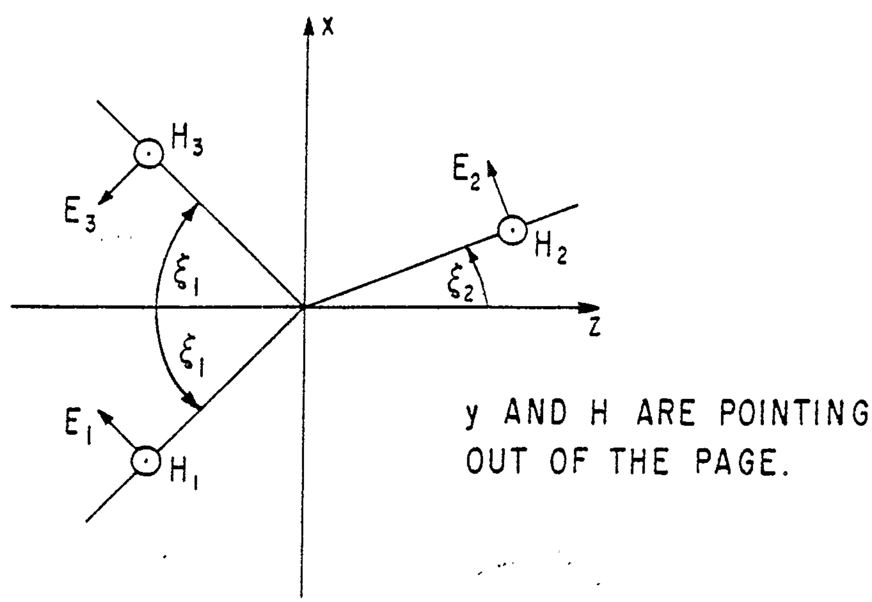
With equation (8) T_e and R_e can be expressed in terms of either ξ_1 or ξ_2, the other being eliminated.

Similarly consider the transmission and reflection coefficients for the case that the magnetic field in the incident wave (and also the magnetic field in the other two waves) is parallel to the y axis. This case is illustrated in figure 2B and is designated by a subscript, h. Applying the boundary conditions at the interface gives for tangential H

$$H_1 + H_3 = H_2 \tag{16}$$



A. E PARALLEL TO INTERFACE



B. H PARALLEL TO INTERFACE

FIGURE 2. REFLECTION AND REFRACTION CASES

and for tangential E

$$E_1\cos(\xi_1) - E_3\cos(\xi_1) = E_2\cos(\xi_2) \tag{17}$$

Replacing E by HZ gives

$$(H_1 - H_3)\, Z_1\cos(\xi_1) = H_2Z_2\cos(\xi_2) \tag{18}$$

Eliminating one of the H's in equations (16) and (18), the ratio of the other two can be found. This gives a transmission coefficient

$$T_h = \frac{H_2}{H_1} = \frac{Z_1}{Z_2}\frac{E_2}{E_1} = 2\left[1 + \frac{Z_2}{Z_1}\frac{\cos(\xi_2)}{\cos(\xi_1)}\right]^{-1} \tag{19}$$

and a reflection coefficient

$$R_h = \frac{H_3}{H_1} = \frac{E_3}{E_1} = \frac{1 - \dfrac{Z_2}{Z_1}\dfrac{\cos(\xi_2)}{\cos(\xi_1)}}{1 + \dfrac{Z_2}{Z_1}\dfrac{\cos(\xi_2)}{\cos(\xi_1)}} \tag{20}$$

where these are related as

$$T_h = 1 + R_h \tag{21}$$

Again using equation (8) we can eliminate ξ_1 or ξ_2 from T_h and R_h.

Suppose now that we restrict the electromagnetic parameters of the two media by the relationship

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{\mu_2}{\mu_1} = \frac{\sigma_2}{\sigma_1} \tag{22}$$

Also let these parameters be independent of frequency (over a range of interest). Then the ratio of the propagation constants, k_2/k_1, is a real number, independent of frequency, so that from equation (8) for a fixed real ξ_1, ξ_2 is a real number and independent of frequency. The two wave impedances are also equal. Referring to equations (13), (14), (19), and (20), note that the transmission and reflection coefficients now have the same form, independent of polarization. These now have the form for the transmission coefficient

$$T = 2\left[1 + \frac{\cos(\xi_2)}{\cos(\xi_1)}\right]^{-1} \tag{23}$$

and for the reflection coefficient

$$R = \frac{1 - \frac{\cos(\xi_2)}{\cos(\xi_1)}}{1 + \frac{\cos(\xi_2)}{\cos(\xi_1)}} \tag{24}$$

Substituting for ξ_2 from equation (8) gives

$$T = \frac{2\cos(\xi_1)}{\cos(\xi_1) + \sqrt{1 - \left(\frac{k_1}{k_2}\right)^2 \left(\sin(\xi_1)\right)^2}} \tag{25}$$

and

$$R = \frac{\cos(\xi_1) - \sqrt{1 - \left(\frac{k_1}{k_2}\right)^2 \left(\sin(\xi_1)\right)^2}}{\cos(\xi_1) + \sqrt{1 - \left(\frac{k_1}{k_2}\right)^2 \left(\sin(\xi_1)\right)^2}} \tag{26}$$

Note that for normal incidence ($\xi_1 = 0$) the reflection coefficient is zero and the transmission coefficient is unity.

Consider now the case that one of the media has parameters equivalent to free space, i.e., it has permittivity, ε_o, permeability, μ_o, and conductivity, zero, over some frequency range of interest. From equation (22) the other medium has zero conductivity and has the same wave impedance as, but a slower propagation speed than, the former medium. We call this other medium a synthetic space medium. Define for this kind of medium a relative permittivity

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o} \tag{27}$$

a relative permeability

$$\mu_r = \frac{\mu}{\mu_o} \tag{28}$$

and a relative propagation speed

$$\beta = \frac{\sqrt{\mu_o \varepsilon_o}}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \tag{29}$$

For a synthetic space medium these three parameters are related as

$$\varepsilon_r = \mu_r = \frac{1}{\beta} \tag{30}$$

Then by specifying β the other parameters of the synthetic space medium are determined. Actually, if one regards these parameters as applying to the relative values of the permittivity, permeability, and propagation speeds between two media (without requiring that one medium have the free space parameters), the equations for the transmission and reflection coefficients still apply.

For an electromagnetic wave travelling between a medium with free space parameters and a synthetic space medium there are two cases to consider. First, let the incident wave be in a medium with free space parameters. Using a subscript, 1, for this case, there is a transmission coefficient

$$T_1 = \frac{2\cos(\xi_1)}{\cos(\xi_1) + \sqrt{1 - \beta^2(\sin(\xi_1))^2}} \tag{31}$$

and a reflection coefficient

$$R_1 = \frac{\cos(\xi_1) - \sqrt{1 - \beta^2(\sin(\xi_1))^2}}{\cos(\xi_1) + \sqrt{1 - \beta^2(\sin(\xi_1))^2}} \tag{32}$$

Second, let the incident wave be in a synthetic space medium. Using a subscript, 2, for this case, there is a transmission coefficient

$$T_2 = \frac{2\cos(\xi_1)}{\cos(\xi_1) + \sqrt{1 - \frac{1}{\beta^2}(\sin(\xi_1))^2}} \tag{33}$$

and a reflection coefficient

$$R_2 = \frac{\cos(\xi_1) - \sqrt{1 - \frac{1}{\beta^2}(\sin(\xi_1))^2}}{\cos(\xi_1) + \sqrt{1 - \frac{1}{\beta^2}(\sin(\xi_1))^2}} \tag{34}$$

There is a certain antisymmetry in these two cases, as can be seen by rewriting R_2 in terms of ξ_2 as

$$R_2 = -\frac{\cos(\xi_2) - \sqrt{1 - \beta^2(\sin(\xi_2))^2}}{\cos(\xi_2) + \sqrt{1 - \beta^2(\sin(\xi_2))^2}} \tag{35}$$

Then, for a synthetic space lens, if it can be arranged to interchange the values of ξ_1 and ξ_2 for the electromagnetic wave on entering and on leaving the lens, R_2 will be the negative of R_1. The possibility of interchanging the values of ξ_1 and ξ_2 is consistent with equation (8) since the values of k_1 and k_2 are interchanged between entering and leaving the synthetic space medium.

For small angles (ξ_1 and ξ_2) the reflection coefficients are small and can be expanded to give similar expressions. Equation (32) reduces to

$$R_1 \approx \frac{\left(1 - \frac{\xi_1^2}{2}\right) - \left(1 - \beta^2 \frac{\xi_1^2}{2}\right)}{\left(1 - \frac{\xi_1^2}{2}\right) + \left(1 - \beta^2 \frac{\xi_1^2}{2}\right)} \approx -\frac{\xi_1^2}{4}(1-\beta^2) \tag{36}$$

Similarly equation (35) reduces to

$$R_2 \approx \frac{\xi_2^2}{4} (1-\beta^2) \tag{37}$$

For small angles then the reflections are quite small, and may produce negligible effect in many cases.

III. Approximate Synthetic Space Lens

With such a synthetic space medium, then construct a lens as illustrated in figure 3. As illustrated, let this example of a lens be a plano-convex lens. This lens is placed at the juncture of a conical and a cylindrical transmission line. The parameters for these transmission lines are the same as those used in a previous note.[3] For convenience, we list some of the pertinent equations which relate these parameters from this previous note and expand the expressions for small angles (θ's). There is the transition paraboloid along which the potential distributions for the TEM modes on the two transmission lines match given by

$$\left(\frac{r_1}{2z_o}\right)^2 = 1 - \frac{z_1}{z_o} \tag{38}$$

in coordinates appropriate to the cylindrical line or

$$\frac{\rho_1}{z_o} = \frac{2}{1+\cos(\theta_1)} \approx 1 + \frac{\theta_1^2}{4} \tag{39}$$

in coordinates appropriate to the conical line. Also on the transition paraboloid the two coordinate systems relate as

$$\frac{r_1}{2z_o} = \tan\left(\frac{\theta_1}{2}\right) \approx \frac{\theta_1}{2} \tag{40}$$

The two transmission lines are considered as equivalents of each other as discussed in the previously referenced note in that the conductors of each transmission line, if extended in their particular geometry to the transition paraboloid, match each other on that surface. Inside the lens, however, the conductors will not necessarily have exactly the same geometry in this case. Note that the relations for the transition paraboloid also give an optimum relation for mapping potential distributions from the conical line to the cylindrical line and vice versa. Ideally a lens preserves this mapping relation.

Considering θ_3 as the maximum angle for significant fields on the conical transmission line, make the intersection of this line with the transition paraboloid the maximum angular extent of the lens. This

3. See reference 1.

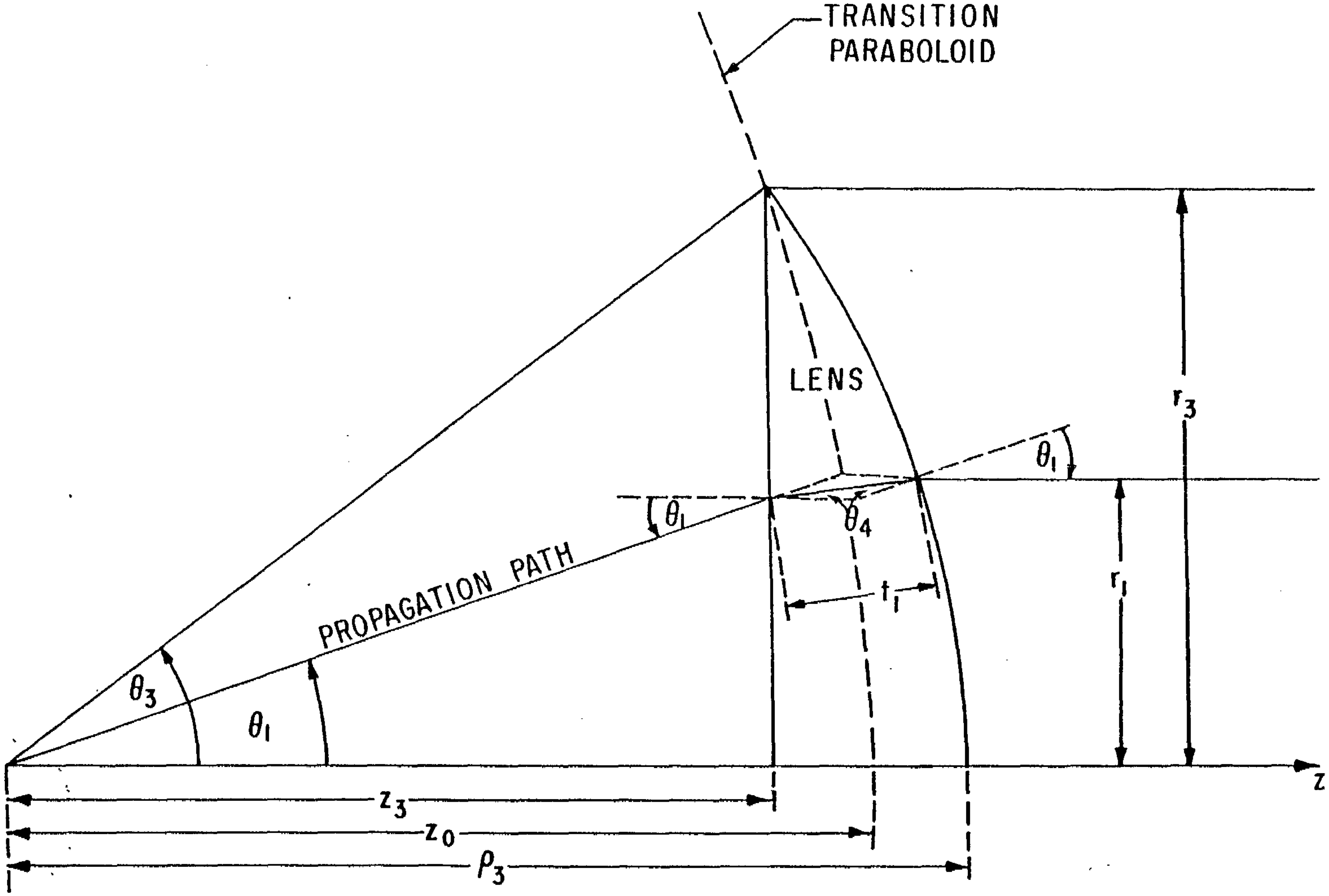


FIGURE 3. PLANO-CONVEX SYNTHETIC SPACE LENS

intersection with the transition paraboloid is a circle with coordinates (ρ_3, θ_3) and (r_3, z_3) as appropriate to the two coordinate systems. The lens being considered has boundaries given by a plane, $z = z_3$, which is perpendicular to the z axis, and a sphere, $\rho = \rho_3$, which is centered on the apex of the conical transmission line. This lens has cylindrical symmetry about the z axis. Since r_3 is the maximum r for significant fields, one would normally expect all the conductors on the cylindrical line to have r's less than or equal to r_3. The transmission line conductors then go right through the lens to join the two transmission lines. Significant fields may not extend to r_3 for all angles (ϕ) aroung the z axis, in which case part of the lens near its edge (for such angles) may be removed without significant effect on the lens performance.

From the previously referenced note we also have a dispersion distance given by

$$d_1 = \frac{r_1^2}{2z_o} = 2z_o \tan^2\left(\frac{\theta_1}{2}\right) \simeq z_o \frac{\theta_1^2}{2} \tag{41}$$

This represents the approximate extra distance that a wave off the z axis travels, over that travelled by the wave on the z axis, in going between a spherical wavefront on the conical line and a planar wavefront on the cylindrical line (in either direction). This dispersion distance is largest for the largest r of significance, r_3, giving

$$d_3 = \frac{r_3^2}{2z_o} = 2z_o \tan^2\left(\frac{\theta_3}{2}\right) \simeq z_o \frac{\theta_3^2}{2} \tag{42}$$

The effect of the lens is to slow down the wave for distances which decrease, the farther the propagation path is from the z axis, in such a manner as to compensate for the above dispersion distance. Consider a typical propagation path designated by θ_1 on the conical line and r_1 on the cylindrical line as illustrated in figure 3. Ideally these two coordinates are related as in equation (40). Since we are considering the case of small θ_1 the distance, t_1, that the wave travels through the lens is approximately the same as if the lens were not there. The propagation velocity is, however, less so that the transit time over this distance is longer. In order to approximately match the transit time between a spherical surface (constant ρ) on the conical line and a plane (constant z) on the cylindrical line set

$$\left(\frac{1}{\beta} - 1\right) t_1 \simeq d_3 - d_1 \tag{43}$$

The left side of the equation represents the additional transit time introduced by the lens, but converted to an equivalent distance. This equivalent extra distance is used to increase the dispersion distance, d_1, for the propagation path of interest to the maximum dispersion distance, d_3. If we can choose β such that the equation holds for $0 \leq \theta_1 \leq \theta_3$ then the transit times will be matched for all propagation paths of interest in going between the two transmission lines.

For small θ_1 equations (41) and (42) give

$$d_3 - d_1 \approx \frac{z_o}{2} \left(\theta_3^2 - \theta_1^2\right) \tag{44}$$

Also the lens thickness for small θ_1 is approximately

$$t_1 = \rho_3 \,[\cos(\theta_1)-\cos(\theta_3)] \approx \frac{\rho_3}{2} (\theta_3^2 - \theta_1^2) \approx \frac{z_o}{2} (\theta_3^2 - \theta_1^2) \tag{45}$$

Equation (43) is then satisfied by

$$\beta = \frac{1}{2} \tag{46}$$

independent of θ_1 (for small θ_3). This choice is then used for further considerations. Note that equation (43) is exactly satisfied for $\theta_1 = \theta_3$ where $t_1 = 0$. Also for $\theta_1 = 0$, using equation (39) for ρ_3, t_1 becomes

$$\begin{aligned} t_1\Big|_{\theta_1=0} &= \rho_3 - z_3 = \rho_3[1-\cos(\theta_3)] \\ &= 2z_o \frac{1-\cos(\theta_3)}{1+\cos(\theta_3)} = 2z_o\tan^2\left(\frac{\theta_3}{2}\right) \end{aligned} \tag{47}$$

which is precisely the same as d_3. Thus, with this choice of β, equation (43) is exactly satisfied for $\theta_1 = 0$. This plano-convex lens then does closely match the transit times between the two transmission lines.

Another consideration is the degradation of a wave in passing through the lens due to reflections at the surfaces of lens. Let the wave be travelling from the conical to the cylindrical line. The angle of incidence at the planar surface of the lens is θ_1 as illustrated in figure 3. From equation (8) we have for the angle, θ_4, of the transmitted wave

$$\beta\sin(\theta_1) = \sin(\theta_4) \tag{48}$$

which for small θ_1 and the chosen β reduces to

$$\theta_4 \approx \beta\theta_1 = \frac{\theta_1}{2} \tag{49}$$

In going between the two transmission lines the propagation path should bend through a total angle, θ_1. Conveniently, for small angles, half of the bending is accomplished at the first lens surface, leaving the other half to the second surface to minimize the reflections. The reflection at the first interface is now, from equation (36),

$$R_1 \approx -\frac{\theta_1^2}{4}(1-\beta^2) = -\frac{3}{16}\theta_1^2 \tag{50}$$

which for small θ_1 is rather small. At the second surface of the lens (the spherical surface), for small θ_3 so that $t_1 << z_o$, the angle of incidence is $\theta_1 - \theta_4$ or approximately θ_4 since the surface is normal to a line of constant θ_1 from the conical apex and θ_4 is the deviation, in the lens, of the propagation path from that line. Then from equation (8) the angle of transmitted wave is approximately θ_1 so that the propagation path ends up parallel to the z axis as desired. The reflection at the second interface is, from equation (37),

$$R_2 \approx \frac{\theta_1^2}{4} (1-\beta^2) = \frac{3}{16} \theta_1^2 \tag{51}$$

There is then a net transmission coefficient, T_3, counting only first reflections, due to the reflections at the two surfaces of

$$T_3 = (1+R_1)(1+R_2) \approx \left(1- \frac{3}{16} \theta_1^2\right)\left(1+ \frac{3}{16} \theta_1^2\right) = 1- \frac{9}{256} \theta_1^4 \tag{52}$$

For small θ_1, T_3 is very close to one.

From equation (52) one can see another advantage in bending the propagation path through half the total angle at each surface. The terms involving the square of the angles cancel. Since the angles for the incident and transmitted waves at the two surfaces of the lens interchange at each surface the net transmission coefficient of equation (52) applies to waves going in either direction. This combination of a synthetic space lens with equivalent conical and cylindrical transmission lines can then be used to launch and/or terminate plane waves on a cylindrical line. If the lens properties of matching the transit times and of small reflections are frequency independent over a sufficiently large frequency band, then the combination of the lens with the two transmission lines can be used with pulsed waves.

In combining the two equivalent transmission lines with the plano-convex lens, there are various ways one might join the transmission lines and the lens. The conductors of the two transmission lines might extend directly into the lens, joining at the transition paraboloid. Alternatively, and a little better, the conductors can join by following a path which is also a propagation path through the lens. The transition of the conductors then follows the transition of the wave.

This plano-convex synthetic space lens is, of course, but one possible lens design, albeit one with some desirable characteristics. It may be advantageous to use a synthetic space material with a β other than 1/2, in which case the lens geometry would change somewhat. One might even make a synthetic space lens with β as a function of position in the lens, in an attempt to make an even better match and/or allow a conical transmission line with larger θ_3. Perhaps more than one lens could be used in a transition assembly, such as one in which one conical line matches into a more gradually tapered conical line which in turn matches into a cylindrical line. This lens technique might even be combined with the multiple conical transition assembly discussed in the previously referenced note. In that case a separate lens might be used to improve the match of each conical line in the array of conical lines into the one cylindrical transmission line. There are many possibilities for employment of synthetic space lens devices and the above list probably does not exhaust such possibilities.

IV. Summary

A conical transmission line, matched into a cylindrical transmission line, is used to launch and/or terminate a plane wave on the cylindrical line. The match of the two transmission lines can be improved by placing an appropriate lens at the juncture of the two lines. This lens transforms a spherical wave on the conical line into a plane wave on the cylindrical line and/or vice versa. An example of such a lens is the plano-convex lens considered in this note. One can note for the lenses that their focal length should be the same as the length of the conical transmission line. Thus, a plane wave on the cylindrical line, propagating toward the conical line is focused at the apex of the conical line. Certain concepts from geometrical optics can then be applied to such a lens system. This plano-convex lens is not perfect in that it only matches the wavefronts on both transmission lines for small divergence angles for the conical line. In other words, it has spherical aberration, a common feature found in optical lenses.

There are important features required of a lens for matching a conical and cylindrical line which are not necessarily required of an optical lens. The reflection coefficients must be small and the propagation velocity nearly constant over a frequency band of interest, particularly for use with pulsed waves. One way to achieve these low reflections is to make the lens from a synthetic space material which is, by definition, a material with the same wave impedance as free space but with a slower propagation velocity than free space. In addition, the angles of incidence of a wave on the surfaces of a synthetic space lens should be kept close to normal incidence to minimize the reflections. Another feature of this kind of lens matching system is that the conductors which form the transmission lines may pass right through the lens, or in some cases form part of the boundary for the lens.

A limitation on the design of such synthetic space lenses is, of course, the degree to which synthetic space materials with various propagation velocities can be realized under the present and future state-of-the-art. Perhaps a mixture of some insulating, high permeability material, such as a ferrite, with an insulating dielectric would be appropriate. For pulsed waves the permittivity and permeability of this synthetic space material should be approximately constant over an appropriate broad frequency band. With such synthetic space materials, various new electromagnetic devices may be possible, including lenses for transitioning waves between conical and cylindrical transmission lines.