Sensor and Simulation Notes

Note 316

1 April 1989

Impedance Matrix Characterization of an Incremental Length of a Periodic Array of Wave Launchers

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Abstract

In this paper, we consider the problem of impedance characterization of an incremental length of a periodic array of wave launchers in a symmetric in-line configuration. Baum [1] has formulated a two-wire (plus reference) transmission-line model and chose the flat plates of the launchers such that a particular form of the characteristic-impedance matrix was realized. Three of the four impedance matrix elements can be determined analytically. In this paper, a numerical solution of the Laplace equation is presented, which permits one to find all four elements of the characteristic impedance matrix for a given set of geometrical parameters in an arbitrary cross section of the unit cell. The numerical method presented here yield results that agree well with the three known elements, thus enhancing our confidence in the computed values for the single unknown element of the characteristic-impedance matrix.

I. Introduction

The concept of distributed sources invokes the uniqueness theorem in electromagnetic theory whereby the fields are known everywhere if they are specified on a closed surface. This is a classical electromagnetic analysis problem. On the other hand, in synthesizing an electromagnetic wave, one can attempt to establish the required tangential electric field on boundary surfaces [2,3] for the desired wave satisfying the Maxwell's equation [4,5]. One of many possible ways of configuring the conical wave launchers is the symmetric in-line type, for which Baum [1] formulated a two-wire (plus reference) transmission-line model for the coupled wave launchers. It is emphasized that such a transmission-line model is valid for wavelengths large compared to cross sectional dimensions. In addition, if the individual launcher lengths are large compared to the wavelengths, the transmission-line model will yield useful information, e.g., about the power transferred from the source to the aperture plane.

The two-wire (plus reference) transmission-line model of·the coupled wave launcher array may be characterized by a (2×2) characteristic-impedance matrix if one considers an incremental length of the array. It was observed in [1], that there is a particular form of this matrix that leads to an analytical expression for the output voltage in the aperture plane. This particular form of the characteristic impedance (not necessarily the most optimal choice) leads to exactly half the input power going into the primary radiated wave and the other half in a secondary wave surrounding the plates and basically uncoupled to the primary wave. If $V_o\, u(t)$ is the input voltage at each launcher, the above condition leads to $(V_o/\sqrt{2}) = 0.707\, V_o$ as the peak output voltage in the aperture plane.

In this paper, we have considered the problem of determining the 4 elements of the characteristic-impedance matrix for an arbitrary cross sectional geometry by solving the Laplace equation numerically. As it was observed in [1], three of the four elements can be written down by inspection. The numerical method presented here gives results that agree with the three known elements enhancing our confidence in the computed values for the one element that cannot be written down analytically. A precise knowledge of the characteristic-impedance matrix elements will be useful in determining optimum shapes of wave launchers in future studies.

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2. Transmission-Line Model

Consider a periodic array of conical wave launchers in a symmetric in-line configuration as shown in figure 1. We use a rectangular coordinate system (x,y,z) with its origin at the center of the central launcher (say) in the aperture plane $z = 0$. $z = -l$ is the plane on which the theoretical apexes lie and $z = -l_1$ is the plane in which physical switches (sources of transient energy) lie. Note that 2a and 2b are the dimensions of the launcher (width and separation)in the aperture plane. l is the length of the launcher. One can define a normalized coordinate ζ according as

$$\zeta = \frac{z}{l} + 1 \tag{1}$$

so that ζ extends from 0 to 1 as z varies from $-l$ to 0 i.e., from the apex plane to the aperture plane.

Now consider a general cross section between the apex and aperture planes for which $z = z_1$ and $\zeta = \zeta_1$ such that $-l < z_1 < 0$ or $0 < \zeta_1 < 1$. Such a cross-sectional view is shown in figure 2. It is observed that by virtue of the periodicity of the wave launchers, there are periodic horizontal lines separated by $2b$, which are electric boundaries, and periodic vertical lines separated by $2a$, which are magnetic boundaries. Electric field lines are normal to an electric boundary and magnetic field lines are normal to a magnetic boundary. PQRS is observed to be an "unit cell" of this particular array of wave launchers (figure 3). It is further observed that in this cross section $y = 0$ is once again an electric wall (Voltage = 0 or "reference" potential) and $x = 0$, likewise, is a magnetic wall and consequently, it is adequate to consider one-quarter or the first quadrant of the unit cell for impedance calculations. Thus we end up with the basic problem illustrated in figure 4 for which we seek to determine the characteristic-impedance matrix elements.

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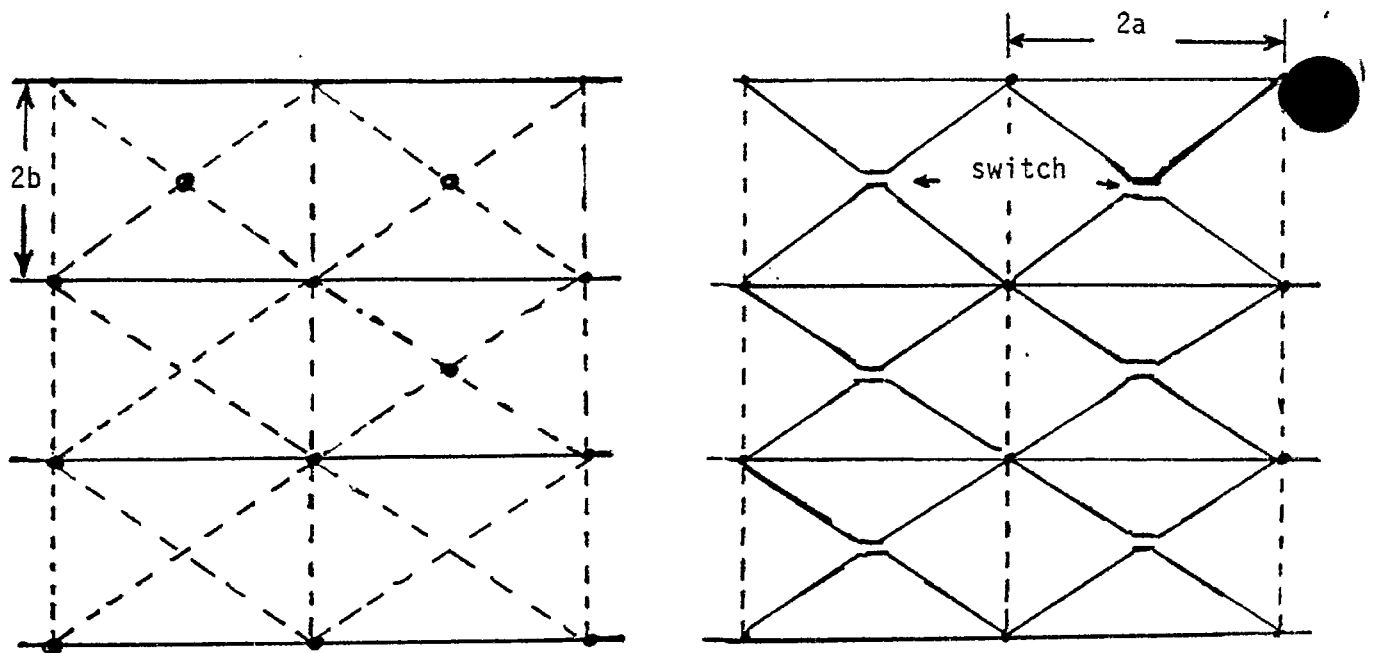
a) Front view

b) Back view
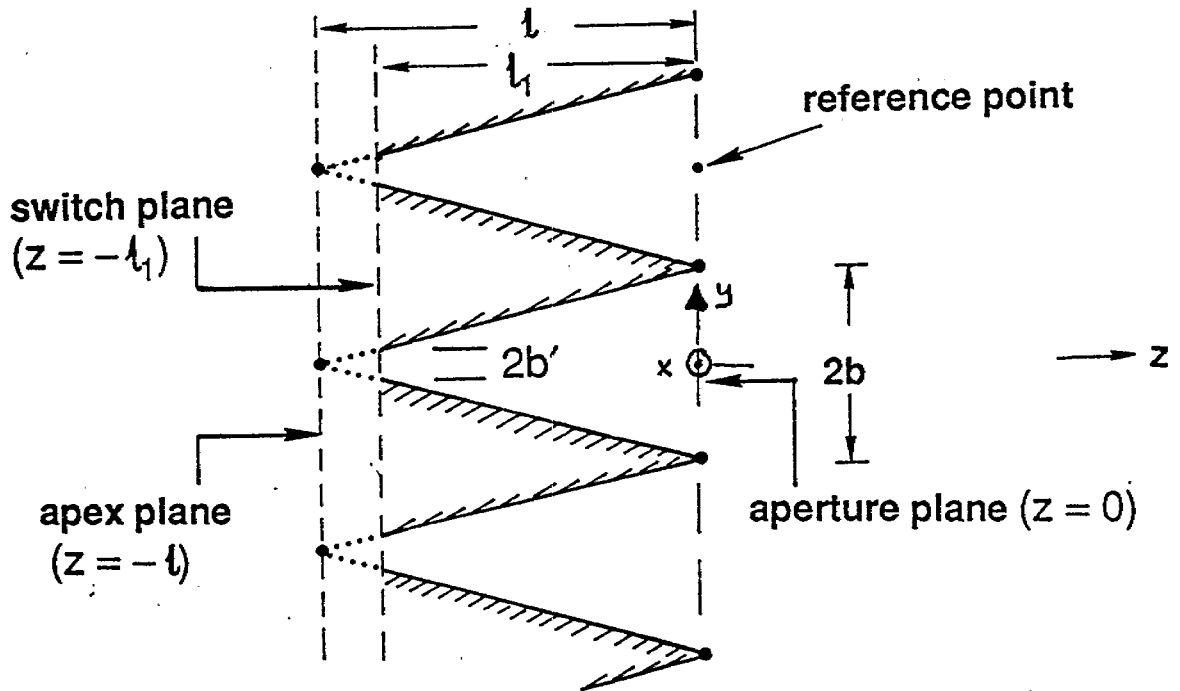
c) Side view

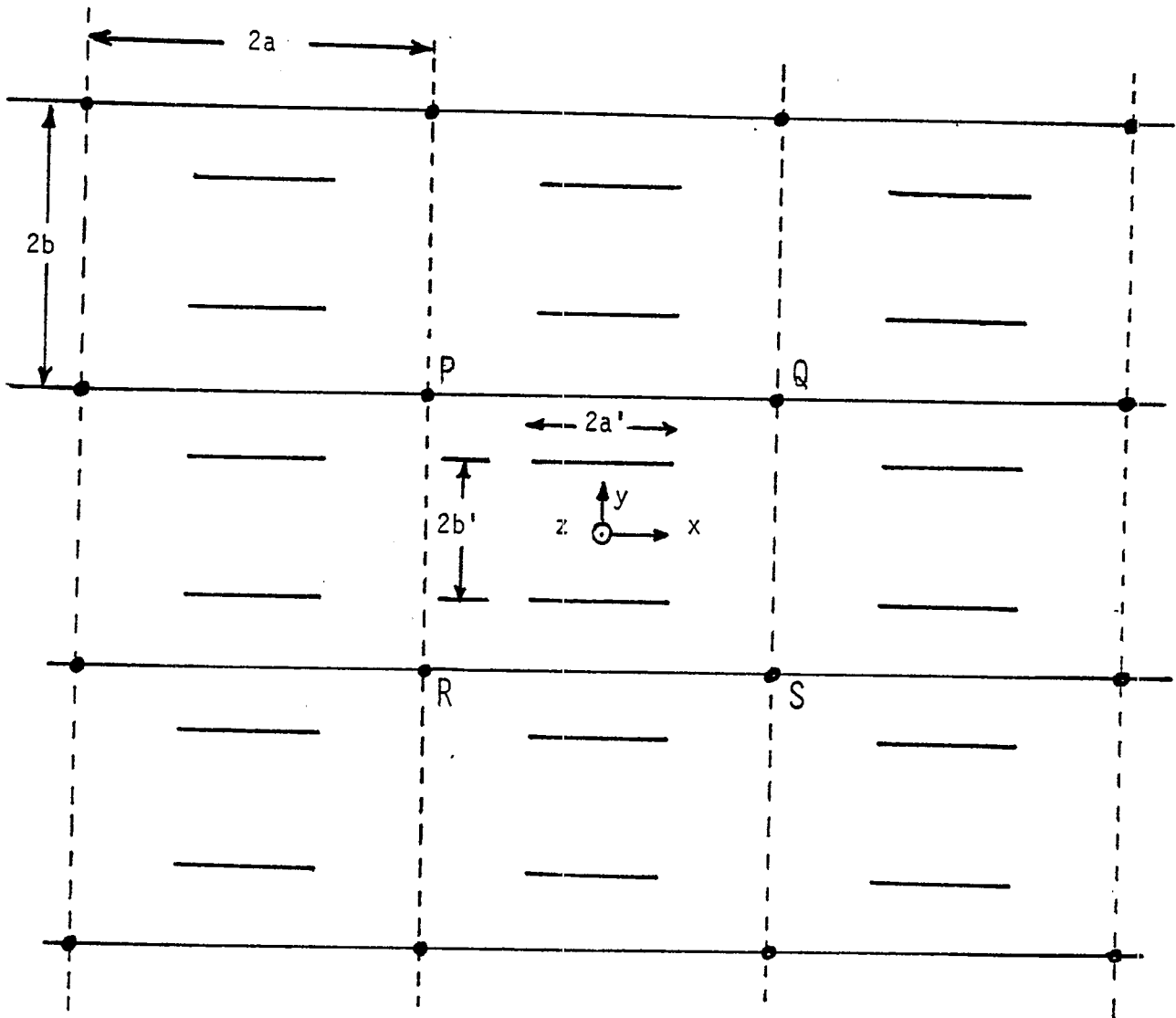Figure 1. Symmetric in-line configuration of a periodic array of conical wave launchers

Figure 2. Cross-sectional view (somewhere between z=0 and z= -l planes)
of the symmetric in-line configuration of figure 1, identifying
PQRS as an unit cell
NOTE: a'(ζ) and b'(ζ) attain their maximum values in the
aperture plane (ζ = 1 or z = 0); a'(1)=a and b'(1)=b

—————————— primary electric boundaries

– – – – – – –   primary magnetic boundaries
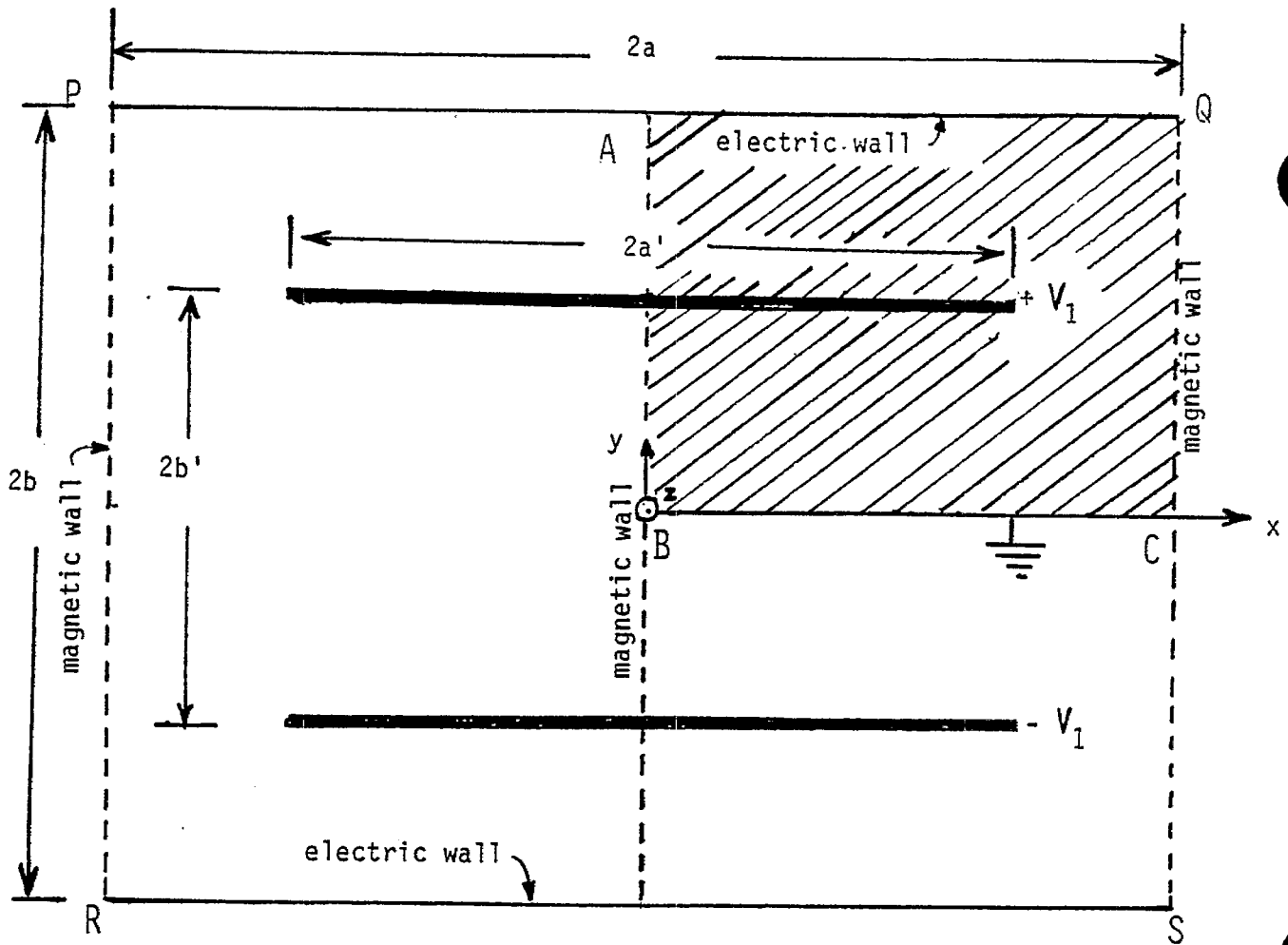(additional boundaries within the unit cell are identified later)
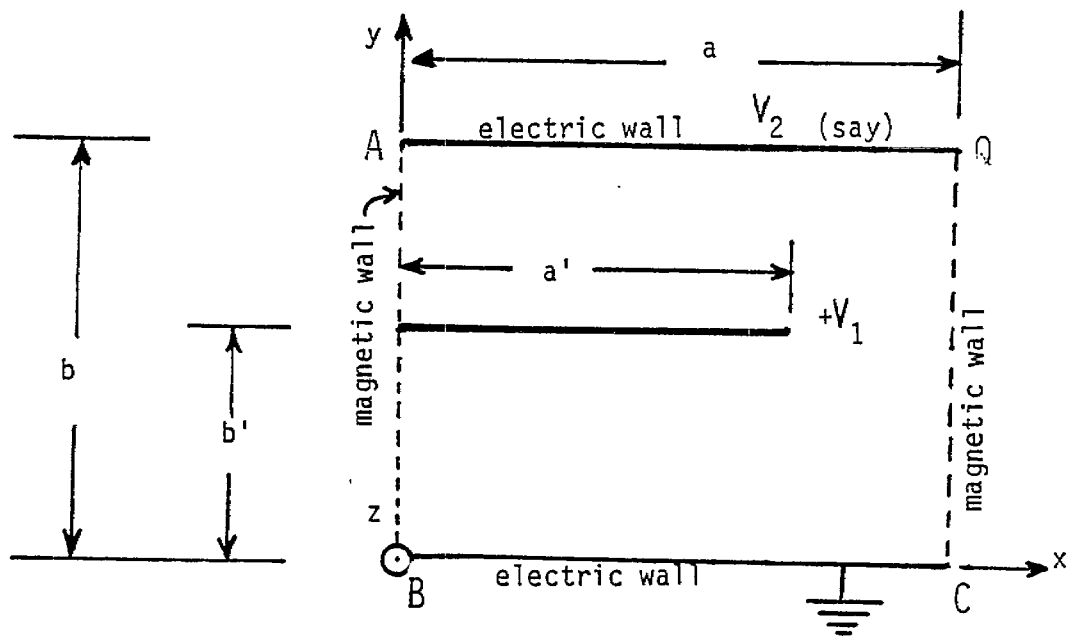
Figure 3. Enlarged view of a unit cell PQRS

Figure 4. The basic problem (first quadrant of the unit cell above)

3. Characteristic-Impedance Matrix

The basic problem shown again in figure 5 has two conductors at potentials V_1 and V_2 with respect to the reference conductor which is an electric wall on the x axis ranging from $x = 0$ to $x = a$. The two conductors carry currents I_1 and I_2 and their charge per unit length are Q_1 and Q_2. Let there be a current of I_o and per-unit-length-charge of Q_o on the reference conductor and since the entire system is charge neutral

$$Q_o + Q_1 + Q_2 = 0 \tag{2}$$

which is useful later on. This system of conductors is representable by

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{3}$$

and

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{1,1} & Z_{1,2} \\ Z_{2,1} & Z_{2,2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{4}$$

where the $(C_{i,j})$ is the capacitance-per-unit length matrix and $(Z_{i,j})$ is the characteristic-impedance matrix. They are interrelated for the present case according as

$$\begin{bmatrix} Z_{i,j} \end{bmatrix} = \frac{1}{c} \begin{bmatrix} C_{i,j} \end{bmatrix}^{-1} \tag{5}$$

with $c = speed\ of\ light$ in the free space medium in which the conical wave launchers are assumed to be immersed. We can now proceed to determine the various matrix elements.

1) Determination of $Z_{2,2}, Z_{1,2}$ and $Z_{2,1}$

By definition,

$$Z_{2,2} = \left. \frac{V_2}{I_2} \right|_{I_1 = 0} \tag{6}$$

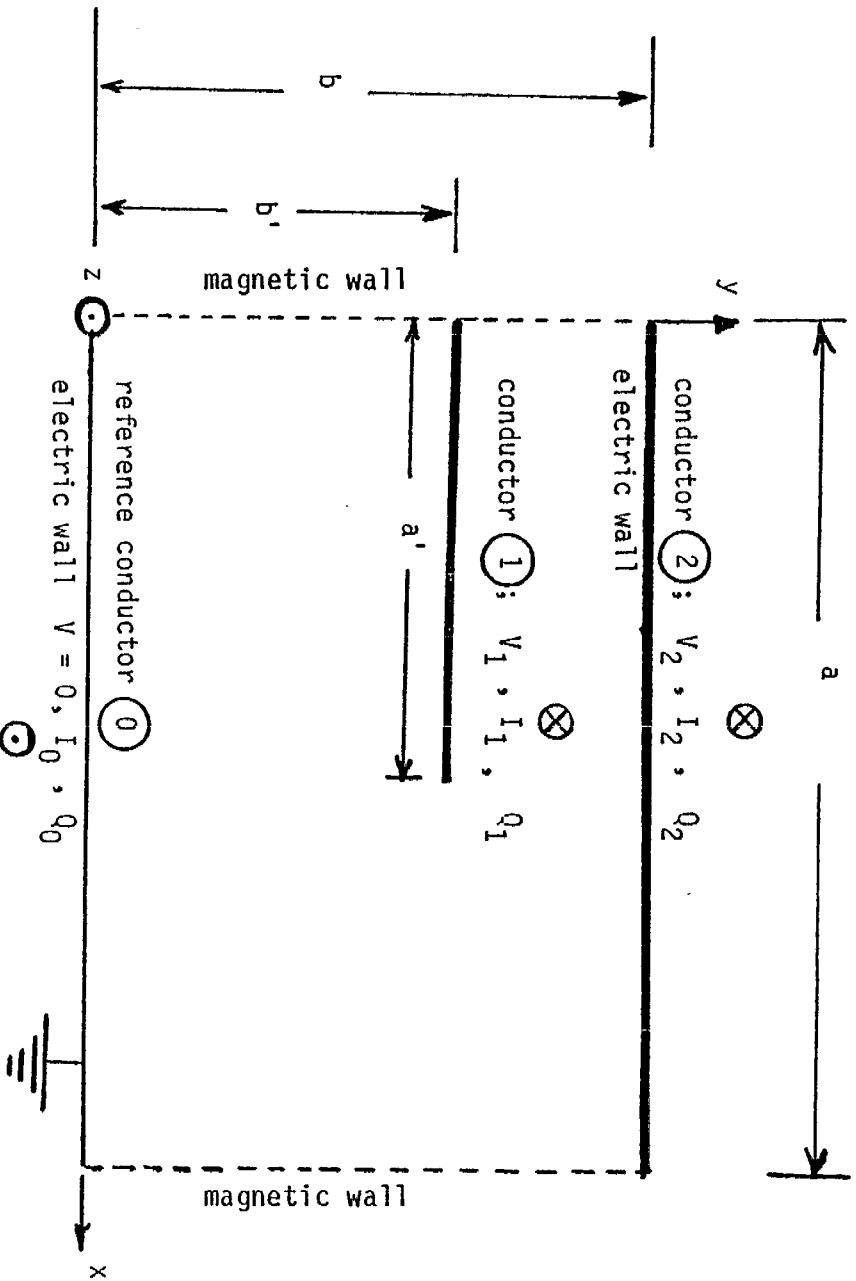The condition $I_1 = 0$ can be achieved by setting

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Figure 5. The basic problem for determining the elements of the (2 X 2) characteristic impedance matrix.

NOTE: $Q_0 + Q_1 + Q_2 = 0$ (charge neutral system)

$$I_0 = I_1 + I_2$$

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$$V_2 = V_o \quad \text{and} \quad V_1 = (b'/b) \, V_o \qquad\qquad (7)$$

resulting in a uniform field situation where $I_1 = Q_1 = 0$. The capacitance per unit length of the system between conductor ② and reference, when conductor ① has no effect is simply

$$C_2 = \varepsilon_o \, (a/b) \qquad \text{Farads/m}$$

and

$$Z_{2,2} = \frac{1}{cC_2} = \frac{1}{c\varepsilon_o} \left[\frac{b}{a}\right] = Z_o \left[\frac{b}{a}\right] \qquad\qquad (8)$$

Furthermore,

$$Z_{1,2} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{(b'/b)\,V_o}{I_2}\bigg|_{I_1=0} = \frac{b'}{b} \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$$\qquad\qquad (9)$$

$$= \frac{b'}{b} Z_{2,2} = \frac{b'}{b} \frac{b}{a} Z_o = Z_o \left[\frac{b'}{a}\right]$$

and because of reciprocity or symmetry $Z_{1,2} = Z_{2,1}$

Gathering the above results, we have

$$(Z_{i,j}) \equiv Z_o \begin{bmatrix} f_{g_{1,1}} & f_{g_{1,2}} \\ f_{g_{2,1}} & f_{g_{2,2}} \end{bmatrix} \qquad\qquad (10)$$

where $f_{g_{1,1}}$ is as yet unknown and the remaining three elements are known and listed below.

$$f_{g_{1,2}} = f_{g_{2,1}} = (b'/a) $$

$$\qquad\qquad (11)$$

$$f_{g_{2,2}} = (b/a)$$

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leading to

$$\frac{1}{Z_o}(Z_{i,j}) = (f_{g_{i,j}}) = \begin{bmatrix} f_{g_{1,1}} & \dfrac{b'}{a} \\[2mm] \dfrac{b'}{a} & \dfrac{b}{a} \end{bmatrix}$$

$$= \frac{b}{a} \begin{bmatrix} F_{1,1} & \dfrac{b'}{b} \\[2mm] \dfrac{b'}{b} & 1 \end{bmatrix} \tag{12}$$

It is noted that three of the four elements are known and the only unknown is

$$F_{1,1} = \frac{a}{b} f_{g_{1,1}} = (\frac{a}{b}) \frac{1}{Z_o} Z_{1,1} \tag{13}$$

or

$$Z_{1,1} = Z_o (\frac{b}{a}) F_{1,1} \tag{14}$$

$F_{1,1}$ is a function of $(a, a', b$ and $b')$. However, one of the four variables, e.g., a can be normalized out and $F_{1,1}$ is then a function of 3 independent variables, e.g., (a'/a), (b/a) and (b'/a). Certain analytical considerations in determining $F_{1,1}$ are discussed in the following section, before proceeding with the numerical computations.

4. Analytical Considerations in Determining $F_{1,1}$

With reference to figure 5, it is noted that there is only a single unknown element $Z_{1,1}$, which may be written as

$$Z_{1,1} = Z_o \left[\frac{b}{a} \right] F_{1,1} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0} \tag{15}$$

It is recalled that $F_{1,1}$ is a function of three independent variables e.g., (b/a), (a'/a) and (b'/a). One could decompose the problem represented by (15) into two separate problems. This decomposition is illustrated in figure 6. The original problem of figure 6a may be thought of as a combination of the uniform field part (figure 6b) and the remainder problem of figure 6c. In the uniform field problem, conductor 1 has no current and charge and in the remainder problem conductor 2 is grounded, i.e., voltage is constrained to be zero. It is important to observe that the voltages, currents and charges per unit length on all conductors in problems illustrated by figures 6b and 6c add up to the values in the original problem of figure 6a.

Next, we consider the problem illustrated in figure 6c. Let the impedance element of interest for this remainder problem be denoted by $Z_{1,1}^{(aux)}$. It is given by

$$Z_{1,1}^{(aux)} = \frac{V_{1c}}{I_{1c}} = \frac{V_1 - V_2(b'/b)}{I_1}$$

$$= \left\{ \frac{V_1}{I_1} - \left[\frac{b'}{b} \right] \frac{V_2}{I_1} \right\}$$

$$= Z_{1,1} - \left[\frac{b'}{b} \right] Z_{1,2} \tag{16}$$

or

$$Z_{1,1} = \left[\frac{b'}{b} \right] Z_{1,2} + Z_{1,1}^{(aux)} \tag{17}$$

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(a) Problem for finding $F_{1,1}$

(b) Uniform field part
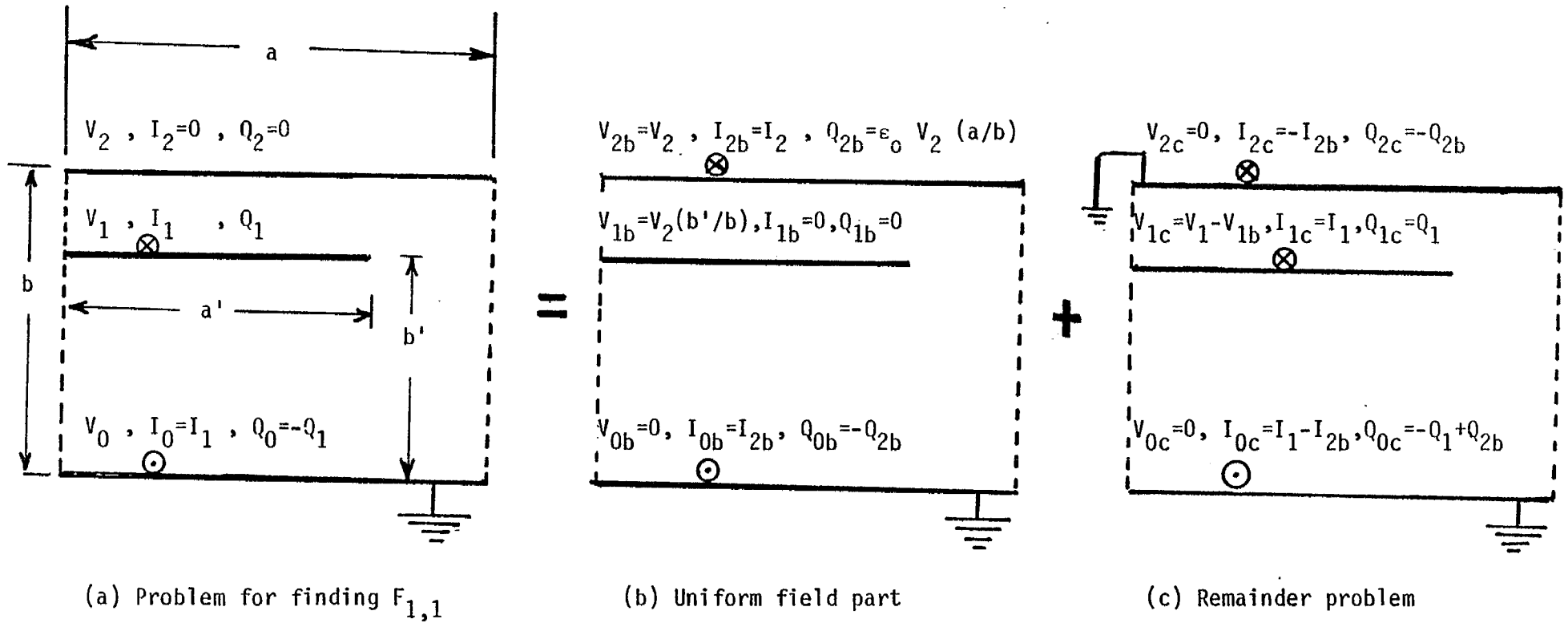
(c) Remainder problem

Figure 6. Decomposition of the problem for finding $F_{1,1}$

NOTE: Voltages, currents and charges per unit length on all conductors in problems (b) and (c) add up to those in problem (a)

If the above equation is divided through out first by Z_o and then by $Z_o(b/a)$, we get

$$f_{g_{1,1}} = \left\{ \left[\frac{b'}{b} \right] f_{g_{1,2}} + f_{g_{1,1}}^{(aux)} \right\} = \left\{ \left[\frac{b'}{b} \right] \left[\frac{b'}{a} \right] + f_{g_{1,1}}^{(aux)} \right\} \qquad (18)$$

$$F_{1,1} = \left\{ \left[\frac{b'}{b} \right] F_{1,2} + F_{1,1}^{(aux)} \right\} = \left\{ \left[\frac{b'}{b} \right]^2 + F_{1,1}^{(aux)} \right\} \qquad (19)$$

Some observations can be made with regards to $F_{1,1}^{(aux)}$. They are listed below.

a) $\quad F_{1,1}^{(aux)} (\frac{b}{a}, \frac{a'}{a}, \frac{b'}{b}) = F_{1,1}^{(aux)} (\frac{b}{a}, \frac{a'}{a}, \frac{b-b'}{b})$ $\qquad$ (*symmetry*) $\qquad$ (20)

b) In view of the above it is adequate to consider a parametric range of $0 \le (b'/b) \le (1/2)$

c) Once $F_{1,1}^{(aux)}$ is known, (19) may be used in obtaining $F_{1,1}$.

Condition (a) merely reflects the symmetry in the solution as conductor ① is moved above or below the line $y = b/2$ in figure 6C. In other words the problem of figure 6C is symmetric with respect to a reflection of the inner plate about the line $y = b/2$. Although $F_{1,1}^{(aux)}$ is also not analytically determinable, certain special cases may be considered as follows.

A. Special case 1; $a' = a$

In this case, launchers are made of rectangular plates instead of triangular plates. With reference to figure 6c, under the condition $a' = a$, we observe

$$f_{g_{1,1}}^{(aux)} = \frac{1}{\left[\frac{a}{b'} \right] + \left[\frac{a}{b-b'} \right]} = \frac{b'}{a} \left[1 - \frac{b'}{b} \right] \qquad (21)$$

Using (21) in (18) we have

$$f_{g_{1,1}} = \left[\frac{b'}{b} \right] \left[\frac{b'}{a} \right] + \left[\frac{b'}{a} \right] \left[1 - \frac{b'}{b} \right] = \left[\frac{b'}{a} \right] \quad \text{when } a' = a \qquad (22)$$

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or

$$F_{1,1} = \frac{a}{b} f_{g_{1,1}} = \left[\frac{b'}{b} \right] \quad \text{when } a' = a \tag{23}$$

B. Special case 2; $(a'/a) \ll 1$ and $(a'/b') \ll 1$

Under this special case, it is possible to approximate conductor 1 of half width a' by a round conductor of diameter $= a'$ centered at the intersection of conductor 1 with the magnetic boundary at $x = 0$. If desired, one could get $F_{1,1}^{(aux)}$ under the above approximations and finally obtain $F_{1,1}$ valid for this special case.

Special case 3; $(a'/a) \ll 1$ and $(b'/b) \ll 1$

Under this special case, $f_{g_{1,1}}^{(aux)}$ may be approximated by that of a parallel plate of half width a' located at a height b' above a ground plane. This problem has been solved in detail and results are tabulated in [6]. Once again, analytical approximations are seen to be possible under the stated approximations.

In concluding this section, we note that closed form analytical expressions are available for three elements $(F_{2,2}, F_{1,2}, F_{2,1})$, but not for $F_{1,1}$. Furthermore for the single unknown element $F_{1,1}$, it is possible to write it as a sum of two parts, the first of which is known analytically and the second part has to be evaluated numerically. It is also observed that certain approximate analytical expressions may be found for the second part and hence for $F_{1,1}$ under special limiting cases.

We now proceed to determine $F_{1,1}$ by a numerical procedure in the following section.

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5. Numerical Method for Finding $F_{1,1}$

The numerical method consists of solving Laplace equation for the potential distribution for a given set of conductors and boundary conditions in a two-dimensional geometry. On an electric boundary, the potential is constant and on a magnetic boundary, the normal derivative of the electric potential is zero. A finite difference scheme that uses a relaxation method is employed in writing the required computer programs. The input variables to the program are a, a', b and b' and the output variables from the program are $F_{1,1}, F_{1,2}, F_{2,1}$ and $F_{2,2}$. It is noted that the symmetry conditions ($F_{1,2} = F_{2,1}$) and the fact that three of the four elements are known is _not_ used in the numerical calculations. Verifying the numerically evaluated values of the three analytically known elements enhances our confidence in the value of $F_{1,1}$ computed numerically. The numerical procedure is summarized below.

Input a, a', b, b'

Step 1 set $V_1 = 1\ Volt$ $V_2 = 0$ in figure 6a

Find Q_1 and Q_2
Then $C_{1,1} = Q_1$ and $C_{2,1} = Q_2$

Step 2 set $V_1 = 0$ and $V_2 = 1$ volt   in figure 6a

Find Q_1 and Q_2 (different values from step 1 above)
Then $C_{2,2} = Q_2$ and $C_{1,2} = Q_1$

Step 3 Check to see if $C_{1,2} = C_{2,1}$

Step 4 Find normalized Impedance matrix elements

$$F_{1,1} = \left[\frac{a}{b}\right]\frac{1}{Z_o}Z_{1,1} = \left[\frac{a}{b}\right]\frac{1}{Z_o}\frac{1}{c\Delta}C_{2,2} = \left[\frac{a}{b}\right]\left[\frac{\varepsilon_o}{\Delta}\right]C_{2,2}$$

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$$F_{1,2} = \left[\frac{a}{b}\right] \frac{1}{Z_o} Z_{1,2} = \left[\frac{a}{b}\right] \left[\frac{-\varepsilon_o}{\Delta}\right] C_{2,1}$$

$$F_{2,1} = \left[\frac{a}{b}\right] \frac{1}{Z_o} Z_{2,1} = \left[\frac{a}{b}\right] \left[\frac{-\varepsilon_o}{\Delta}\right] C_{1,2}$$

$$F_{2,2} = \left[\frac{a}{b}\right] \frac{1}{Z_o} Z_{2,2} = \left[\frac{a}{b}\right] \left[\frac{\varepsilon_o}{\Delta}\right] C_{2,2}$$

where $\Delta \equiv \det(C_{i,j}) = C_{1,1} C_{2,2} - C_{1,2} C_{2,1}$

Output   Tabulate (b/a), (b'/a), (b'/b), $F_{1,1}$, $F_{1,2}$, $F_{2,1}$ & $F_{2,2}$

NOTE   1) Q_1 is more easily determined by using $Q_1 = -(Q_o + Q_2)$ to avoid the numerical problems at plate edge arising from the charge build up at the plate edge.

2) The way the output is tabulated, it is easy to verify $F_{1,2} = F_{2,1} = (b'/b)$ and $F_{2,2} = 1$ within acceptable numerical inaccuracies.

A. Illustrative Example

Consider a sample case where

$a = 100$, $b = 30$, $a' = 20$, and $b' = 12$ in any units of length.

This set results in

$(b/a) = 0.3$; $(b'/a) = 0.12$; $(a'/a) = 0.2$ and $(b'/b) = 0.4$

The potential distribution for step 1 $(V_1 = 1 \text{ Volt } V_2 = 0)$ and step 2 $(V_1 = 0 \text{ and } V_2 = 1 \text{ Volt})$ are shown in figures 7 and 8. These distributions appear as one would expect. The computer routine evaluates electric fields and charges on conductors from these distributions of the potential. The normalized impedance matrix elements computed and expected (from analytical results) are listed below:
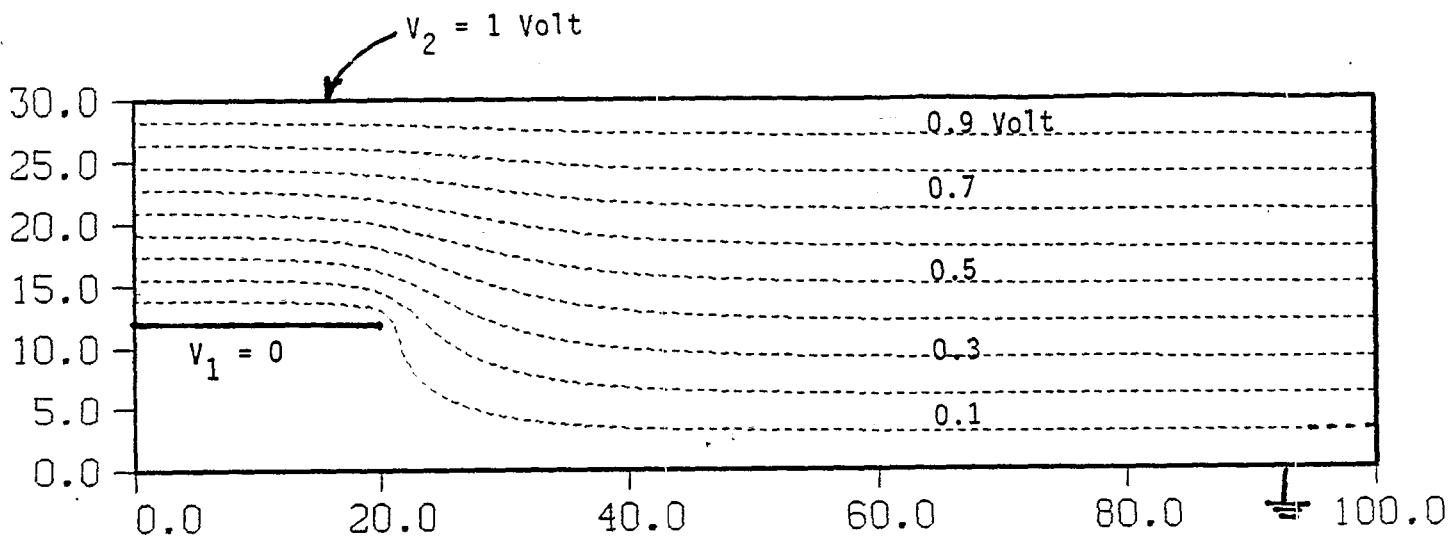
Figure 7. Potential distribution when $V_1 = 0$ and $V_2 = 1$ Volt

NOTE : (b/a)= 0.3 ; (b'/a)= 0.12 ; (a'/a)= 0.2 ; (b'/b) = 0.4
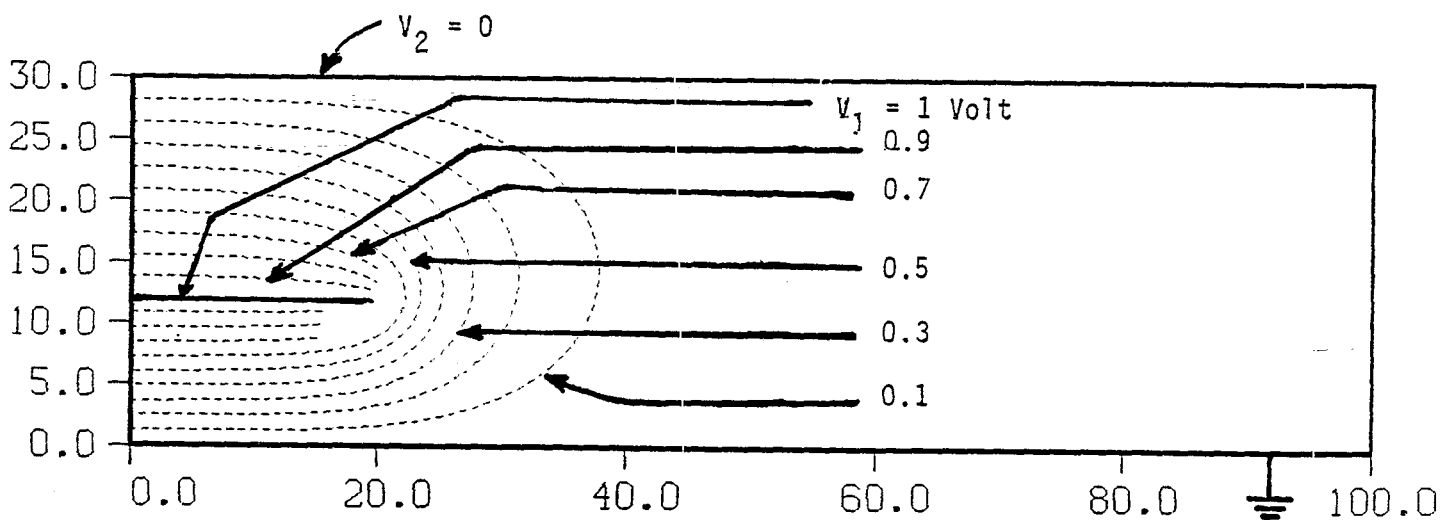
Figure 8. Potential distribution when $V_1 = 1$ Volt and $V_2 = 0$

Quantity	Numerical	Analytical
$F_{1,1}$	1.0627	unavailable
$F_{1,2}$	0.3999	$0.400 = (b'/b)$
$F_{2,1}$	0.4010	$0.400 = (b'/b)$
$F_{2,2}$	0.9999	1.0000

The accuracy of numerical values is considered adequate and can be improved if needed by more stringent convergence criterion in the calculations.

6. Nomograms for Cross-sectional Dimensions to Achieve a Given Impedance.

The numerical procedure described in the previous section yielded results for many test cases that established the accuracy of results for the unknown $F_{1,1}$ element for the characteristic-impedance matrix. That is, when the other three elements ($F_{1,2}$, $F_{2,1}$ and $F_{2,2}$) computed numerically agree within 3 digit accuracy with analytically predicted values, our confidence in the accuracy of the computed $F_{1,1}$ element is improved.

In this section, we present the numerically computed results of $F_{1,1}$ in such a manner that it is useful in determining the cross-sectional dimensions to achieve a given $F_{1,1}$ profile. Several cases have been considered and $F_{1,1}$ is computed in each case. The parameter ranges are listed below.

$(b/a) = 0.1, 0.3, 0.5, 0.88, 1.00, 2.00, 3.00$ and 10.00

$(b'/a) = 0.2(b/a), 0.4(b/a), 0.6(b/a), 0.8(b/a)$ and (b/a)

$(a'/a) = 0.1, 0.2, 0.4, 0.6, 0.8$ and 1.0

The above choice of (b'/a) results in (b'/b) of 0.2, 0.4, 0.6, 0.8 and 1.0. The computed values of $F_{1,1}$ are listed for the above cases in Table 1. The results are plotted in figure 9. In this figure, for each fixed (b/a) value, $F_{1,1}$ is plotted as a function of (b'/b) for five different cases of (a'/a). The special case of $(b/a) \approx 0.88$ corresponds to the case of equal early and late-time impedances [4].

The usefulness of these results may be illustrated with an example. Let us say $F_{1,1} = 1$ is the desired value for the case of $b'(\zeta)$ varying as $b\zeta$ for $0 \le \zeta \le 1$. This means $\{b'(\zeta)/b\}$ has a constant slope. This special case of $F_{1,1} = 1$ was considered analytically in [1] yielding a result of half the power in the aperture plane. If one draws $F_{1,1} = 1$ line say for the case of $(b/a) = 0.88$, the intersections of this line with the plotted curves give the value of (a'/a) for each (b'/b). Thus a profile of the launcher plate can be generated for a desired impedance value of $F_{1,1}$. It is also noted that (b'/b) need not necessarily vary linearly for $0 \le \zeta \le 1$. For some specified $\{b'(\zeta)/b\}$, the plotted and similar results are still useful in obtaining $a'(\zeta)$ to achieve a given impedance. If one has some $F_{1,1}(\zeta)$ and some specified $b'(\zeta)/b$, then one has specified $F_{1,1}$ as a function of b'/b. Then, such a curve can be superimposed on the plotted figure, with the intersections yielding

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TABLE 1. Numerically computed values of $F_{1,1}$

b/a = 0.1

b'/a	a'/a	b'/b	$F_{1,1}$
0.020	0.100	0.200	1.3988
0.020	0.200	0.200	0.7748
0.020	0.400	0.200	0.4230
0.020	0.600	0.200	0.2990
0.020	0.800	0.200	0.2357
0.020	1.000	0.200	0.2000
0.040	0.100	0.400	2.1077
0.040	0.200	0.400	1.2352
0.040	0.400	0.400	0.7271
0.040	0.600	0.400	0.5451
0.040	0.800	0.400	0.4515
0.040	1.000	0.400	0.4000
0.060	0.100	0.600	2.3077
0.060	0.200	0.600	1.4351
0.060	0.400	0.600	0.9171
0.060	0.600	0.600	0.7402
0.060	0.800	0.600	0.6486
0.060	1.000	0.600	0.6000
0.080	0.100	0.800	1.9988
0.080	0.200	0.800	1.3670
0.080	0.400	0.800	1.0214
0.080	0.600	0.800	0.8986
0.080	0.800	0.800	0.8357
0.080	1.000	0.800	0.8000

b/a = 0.3

b'/a	a'/a	b'/b	$F_{1,1}$
0.060	0.100	0.200	1.1002
0.060	0.200	0.200	0.6814
0.060	0.400	0.200	0.3960
0.060	0.600	0.200	0.2864
0.060	0.800	0.200	0.2285
0.060	1.000	0.200	0.2000
0.120	0.100	0.400	1.5918
0.120	0.200	0.400	1.0627
0.120	0.400	0.400	0.6753
0.120	0.600	0.400	0.5206
0.120	0.800	0.400	0.4377
0.120	1.000	0.400	0.4000
0.180	0.100	0.600	1.7918
0.180	0.200	0.600	1.2562
0.180	0.400	0.600	0.8731
0.180	0.600	0.600	0.7194
0.180	0.800	0.600	0.6370
0.180	1.000	0.600	0.6000
0.240	0.100	0.800	1.7004
0.240	0.200	0.800	1.2767
0.240	0.400	0.800	0.9946
0.240	0.600	0.800	0.8857
0.240	0.800	0.800	0.8281
0.240	1.000	0.800	0.8000

b/a = 0.5

b'/a	a'/a	b'/b	$F_{1,1}$
0.100	0.100	0.200	0.9067
0.100	0.200	0.200	0.5986
0.100	0.400	0.200	0.3687
0.100	0.600	0.200	0.2731
0.100	0.800	0.200	0.2217
0.100	1.000	0.200	0.2000
0.200	0.100	0.400	1.2881
0.200	0.200	0.400	0.9241
0.200	0.400	0.400	0.6268
0.200	0.600	0.400	0.4965
0.200	0.800	0.400	0.4262
0.200	1.000	0.400	0.4000
0.300	0.100	0.600	1.4881
0.300	0.200	0.600	1.1241
0.300	0.400	0.600	0.8268
0.300	0.600	0.600	0.6965
0.300	0.800	0.600	0.6262
0.300	1.000	0.600	0.6000
0.400	0.100	0.800	1.5067
0.400	0.200	0.800	1.1987
0.400	0.400	0.800	0.9687
0.400	0.600	0.800	0.8732
0.400	0.800	0.800	0.8218
0.400	1.000	0.800	0.8000

b/a = 0.88

b'/a	a'/a	b'/b	$F_{1,1}$
0.180	0.100	0.205	0.7209
0.180	0.200	0.205	0.5130
0.180	0.400	0.205	0.3395
0.180	0.600	0.205	0.2613
0.180	0.800	0.205	0.2196
0.180	1.000	0.205	0.2045
0.360	0.100	0.409	1.0078
0.360	0.200	0.409	0.7816
0.360	0.400	0.409	0.5753
0.360	0.600	0.409	0.4769
0.360	0.800	0.409	0.4255
0.360	1.000	0.409	0.4091
0.520	0.100	0.591	1.1897
0.520	0.200	0.591	0.9635
0.520	0.400	0.591	0.7572
0.520	0.600	0.591	0.6588
0.520	0.800	0.591	0.6074
0.520	1.000	0.591	0.5909
0.700	0.100	0.795	1.3121
0.700	0.200	0.795	1.1043
0.700	0.400	0.795	0.9306
0.700	0.600	0.795	0.8525
0.700	0.800	0.795	0.8107
0.700	1.000	0.795	0.7955

TABLE 1. Continued and concluded

b/a = 1.0

b'/a	a'/a	b'/b	$F_{1,1}$
0.200	0.100	0.200	0.6652
0.200	0.200	0.200	0.4836
0.200	0.400	0.200	0.3246
0.200	0.600	0.200	0.2521
0.200	0.800	0.200	0.2135
0.200	1.000	0.200	0.2000
0.400	0.100	0.400	0.9326
0.400	0.200	0.400	0.7370
0.400	0.400	0.400	0.5510
0.400	0.600	0.400	0.4611
0.400	0.800	0.400	0.4145
0.400	1.000	0.400	0.4000
0.600	0.100	0.600	1.1327
0.600	0.200	0.600	0.9345
0.600	0.400	0.600	0.7499
0.600	0.600	0.600	0.6606
0.600	0.800	0.600	0.6143
0.600	1.000	0.600	0.6000
0.800	0.100	0.800	1.2655
0.800	0.200	0.800	1.0815
0.800	0.400	0.800	0.9239
0.800	0.600	0.800	0.8519
0.800	0.800	0.800	0.8135
0.800	1.000	0.800	0.8000

b/a = 2.0

b'/a	a'/a	b'/b	$F_{1,1}$
0.400	0.100	0.200	0.4650
0.400	0.200	0.200	0.3676
0.400	0.400	0.200	0.2753
0.400	0.600	0.200	0.2301
0.400	0.800	0.200	0.2069
0.400	1.000	0.200	0.2000
0.800	0.100	0.400	0.6767
0.800	0.200	0.400	0.5773
0.800	0.400	0.400	0.4802
0.800	0.600	0.400	0.4315
0.800	0.800	0.400	0.4070
0.800	1.000	0.400	0.4000
1.200	0.100	0.600	0.8769
1.200	0.200	0.600	0.7775
1.200	0.400	0.600	0.6804
1.200	0.600	0.600	0.6317
1.200	0.800	0.600	0.6071
1.200	1.000	0.600	0.6000
1.600	0.100	0.800	1.0656
1.600	0.200	0.800	0.9681
1.600	0.400	0.800	0.8758
1.600	0.600	0.800	0.8306
1.600	0.800	0.800	0.8074
1.600	1.000	0.800	0.8000

b/a = 3.0

b'/a	a'/a	b'/b	$F_{1,1}$
0.600	0.100	0.200	0.3829
0.600	0.200	0.200	0.3168
0.600	0.400	0.200	0.2528
0.600	0.600	0.200	0.2208
0.600	0.800	0.200	0.2047
0.600	1.000	0.200	0.2000
1.200	0.100	0.400	0.5854
1.200	0.200	0.400	0.5189
1.200	0.400	0.400	0.4538
1.200	0.600	0.400	0.4211
1.200	0.800	0.400	0.4047
1.200	1.000	0.400	0.4000
1.800	0.100	0.600	0.7858
1.800	0.200	0.600	0.7194
1.800	0.400	0.600	0.6543
1.800	0.600	0.600	0.6217
1.800	0.800	0.600	0.6052
1.800	1.000	0.600	0.6000
2.400	0.100	0.800	0.9840
2.400	0.200	0.800	0.9180
2.400	0.400	0.800	0.8538
2.400	0.600	0.800	0.8219
2.400	0.800	0.800	0.8057
2.400	1.000	0.800	0.8000

b/a = 10.0

| b'/a	a'/a	b'/b	$F_{1,1}$
2.000	0.100	0.200	0.2493
2.000	0.200	0.200	0.2324
2.000	0.400	0.200	0.2146
2.000	0.600	0.200	0.2055
2.000	0.800	0.200	0.2011
2.000	1.000	0.200	0.2000
4.000	0.100	0.400	0.4494
4.000	0.200	0.400	0.4324
4.000	0.400	0.400	0.4146
4.000	0.600	0.400	0.4055
4.000	0.800	0.400	0.4011
4.000	1.000	0.400	0.4000
6.000	0.100	0.600	0.6501
6.000	0.200	0.600	0.6331
6.000	0.400	0.600	0.6154
6.000	0.600	0.600	0.6063
6.000	0.800	0.600	0.6018
6.000	1.000	0.600	0.6000
8.000	0.100	0.800	0.8506
8.000	0.200	0.800	0.8336
8.000	0.400	0.800	0.8159
8.000	0.600	0.800	0.8068
8.000	0.800	0.800	0.8024
8.000	1.000	0.800	0.8000
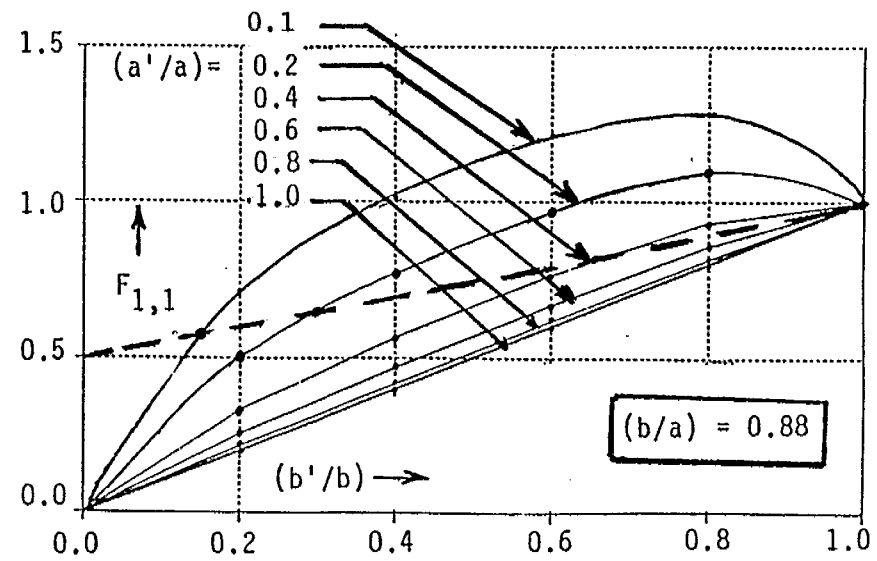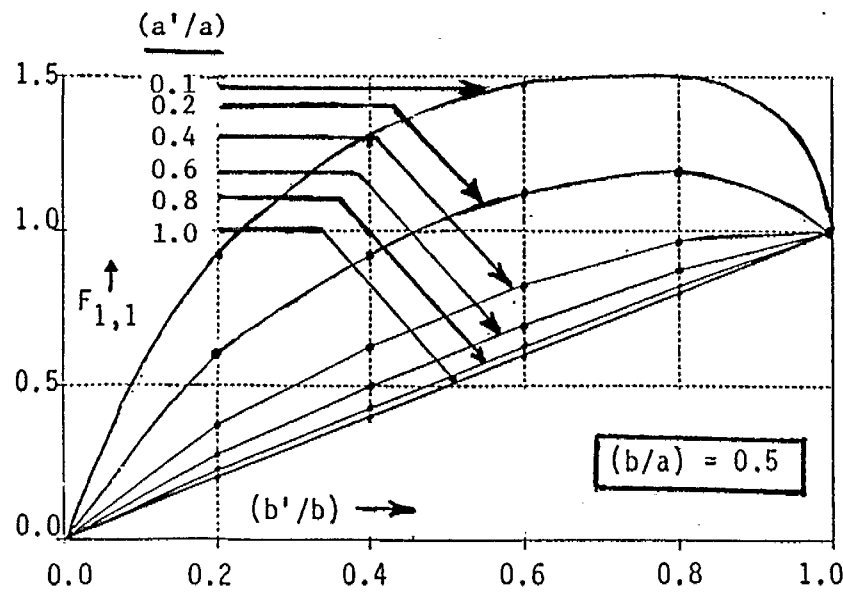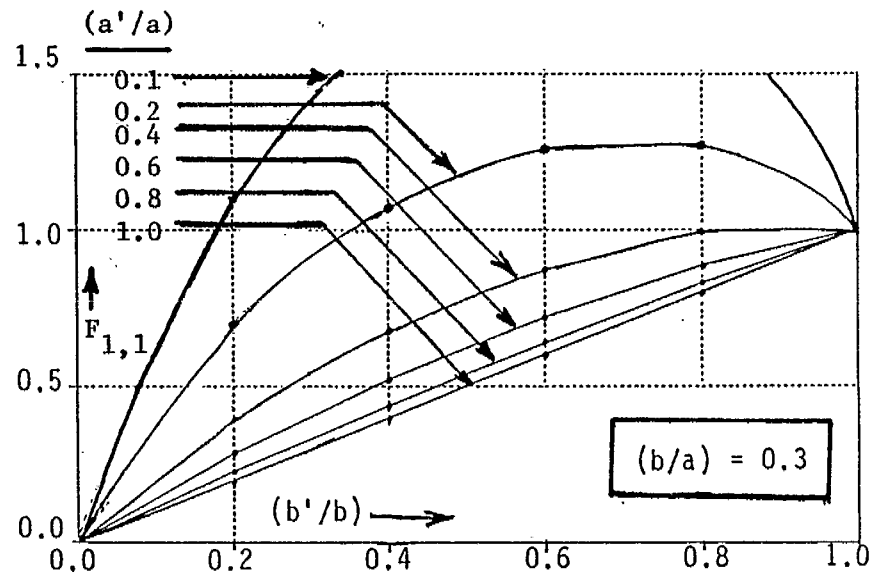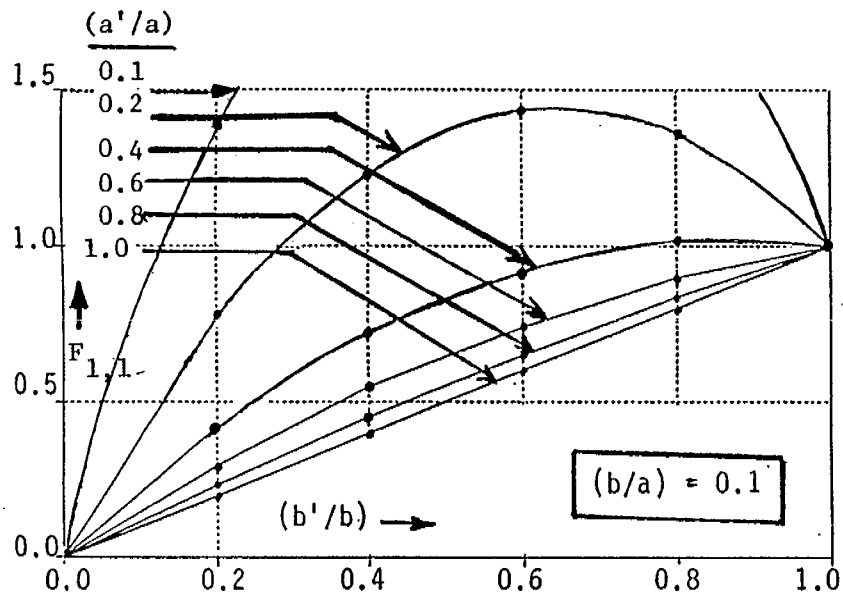
Figure 9. Normalized characteristic- impedance element $F_{1,1}$ for different launcher geometries
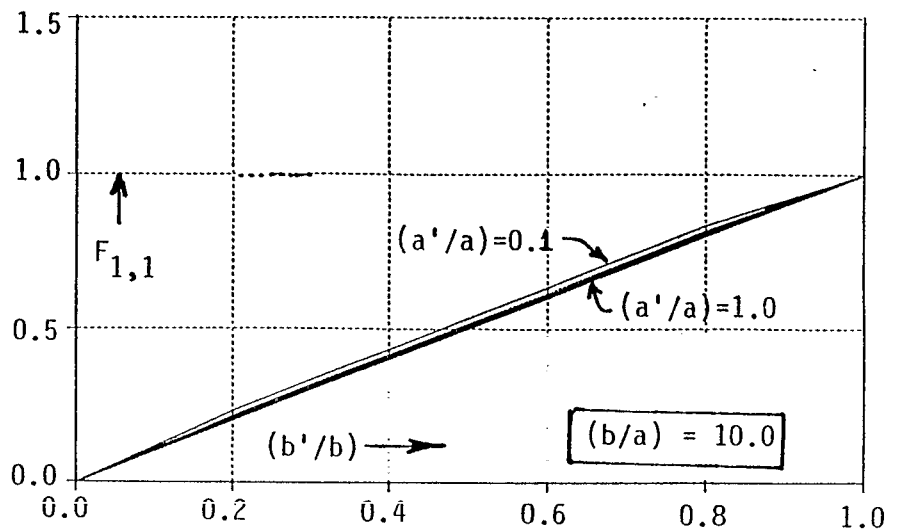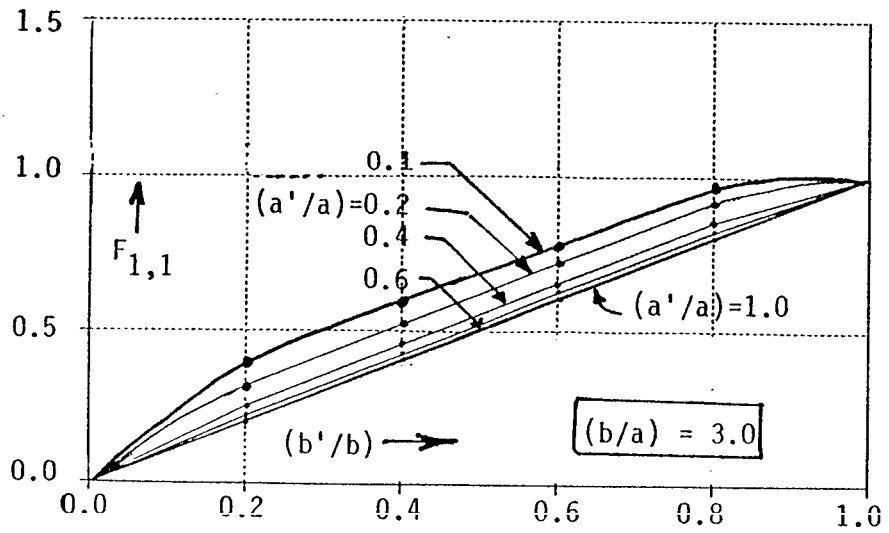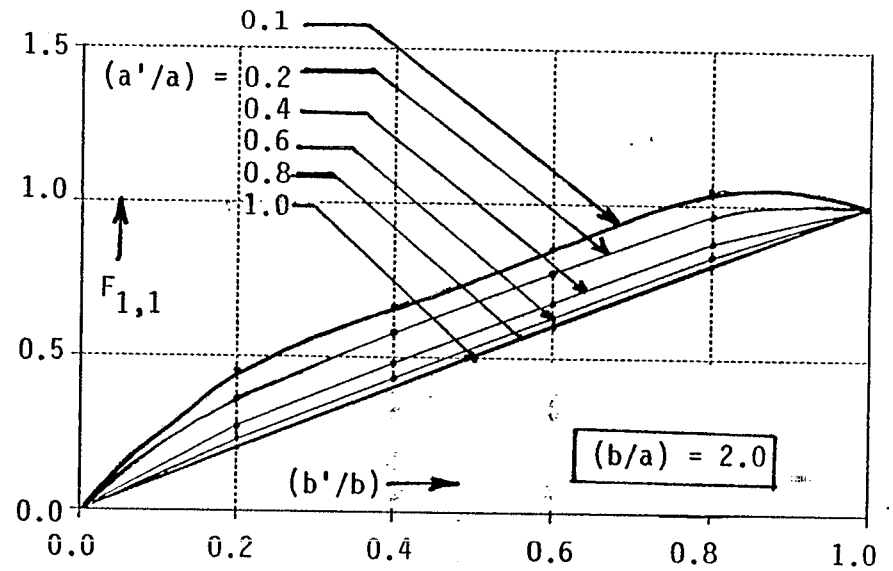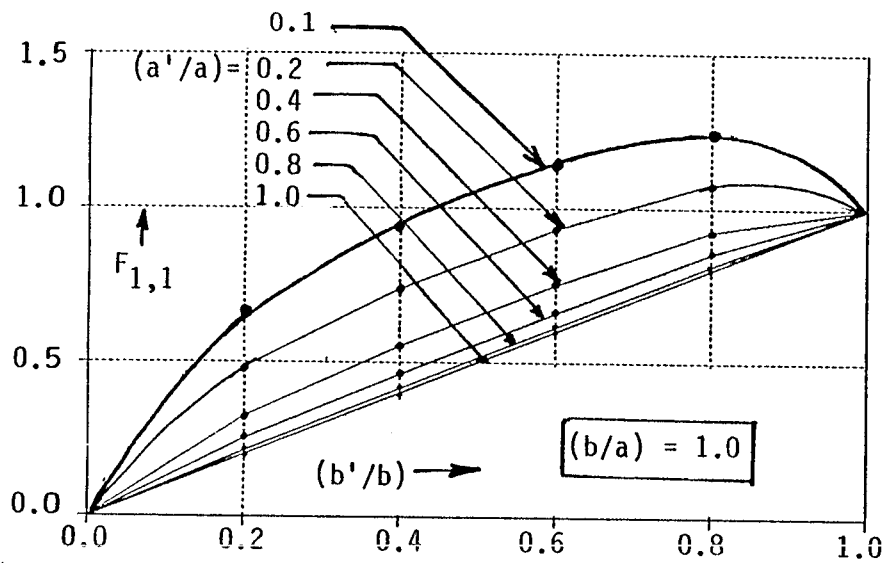Note that $Z_{1,1}$ = Z_0 (b... $F_{1,1}$

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Figure 9. Continued and concluded

the value of (a'/a) for each (b'/b). As an example suppose $F_{1,1}$ was to go linearly from 0.5 to 1 as (b'/b) goes from 0 to 1, illustrated by a dashed line in figure 9 (for the special case of $b/a = 0.88$), the intersections of this dashed line with the plotted curves lead to the construction of the launcher plate geometry. Although $F_{1,1} = 1$ leads to analytical results for the output voltage at the aperture plane, it may not necessarily be the optimal choice. It is not clear at this time, if other choices of $F_{1,1}$ exist which improve the power transfer from launch to the aperture plane, recalling $F_{1,1} = 1$ leads to an output voltage of $(V_o/\sqrt{2})$ and power transferred to the aperture plane is one-half the input power. Future studies could address this issue of optimizing the impedance.

7. Summary

A numerical method is presented in evaluating the characteristic-impedance matrix elements for an incremental length of an array of periodic conical wave launchers in a symmetric in-line configuration. This method solves for the potential distribution for the prescribed set of conductors, electric walls, magnetic walls and assigned potentials on conductors. Once the potential distribution is known, electric fields are numerically determined and charges on conductors are evaluated using the normal electric field values. From a knowledge of charges and the capacitance per-unit length matrix the characteristic-impedance matrix elements are easily computed.

It is expected results such as reported here will be useful in arriving at optimal shapes of launcher plates in other analytical efforts.

References

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