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Coupled Transmission-Line Model of Periodic Array of Wave Launchers

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Abstract

This paper considers the situation in which the individual wave launchers in an array are long compared to their cross-section dimensions. A two-wire (plus reference) transmission-line model is formulated applicable to symmetrical in-line wave launchers. The flat plates of the launchers are chosen such that a particular form of the characteristic-impedance matrix is realized. This leads to an analytic solution for the early-time propagation on the wavefront. From this one finds the transmission and reflection of the wavefront of the aperture plane.

electromagnetic coupling, transmission lines, plates

I. Introduction

In synthesizing an electromagnetic wave one can establish the required tangential electric field for the desired wave (satisfying the Maxwell equations) on boundary surfaces [1,3]. In practice the source surface is composed of some set of sources, each source involving a set of conductors connected to switches and related hardware. Each source occupies a portion of the surface. For wavelengths large compared to this portion (low frequencies) the discrete nature or granularity of the source surface is not significant.

At high frequencies, for which wavelengths are small compared to the portion of the source surface assigned to an individual source, the picture is somewhat different. As discussed in [2,4] one can construct each of the sources as a conical wave launcher. Summing the early-time contributions from the spherical TEM waves in each launcher one can find the behavior of the early-time or high-frequency field far from the source array. For simultaneous step-function excitation of the individual launchers the wave propagating normal to an infinite planar array rises as a ramp function. Comparing to the late-time constant field a characteristic time t_1 can be defined for the rise of the distant wave.

It is observed in [4] that as the length ℓ of the conical wave launcher is increased (for fixed dimensions 2a x 2b in the aperture plane) $t_1 \to 0$. However, as ℓ becomes large compared to a and b there is an interaction between the adjacent wave launchers before the wave can reach the aperture plane. This interaction can produce signals from one launcher coupling into a second launcher and arriving at the aperture plane very soon behind the initial wave (in a time short compared to a/c and b/c for sufficiently large ℓ).

This paper considers a transmission-line model of the coupled wave launchers. This is valid for the wavelength $\lambda >>$ a,b. If in turn $\ell >> \lambda$, then a transmission-line model should yield useful information.

In [5] the high-frequency behavior of nonuniform multiconductor transmission lines in uniform media (such as free space) is considered. This leads to formulae for the propagation of a wavefront on such coupled transmission lines. This has to be interpreted as being limited in that the launcher cross-section dimensions 2a and 2b are assumed electrically small for present calculations.

II. Problem Definition

As indicated in fig. 2.1, we have a symmetric conical wave launcher of height 2b, width 2a, and length ℓ. There is an aperture plane on z = 0 and the wave is launched from a theoretical apex at $\vec{r} = (0,0,\ell)$ where the coordinates are

$$\vec{r} = (x\ ,\ y\ ,\ z) \tag{2.1}$$

Now the wave launcher on some intermediate cross-section plane $-\ell < z < 0$ has edges at $x = \pm a'$, $y = \pm b'$ expanding to $x = \pm a$, $y = \pm b$ at the aperture plane. In the illustration the edges expand in proportion from apex to aperture plane so as to form straight lines. However, let us allow for our transmission-line calculations that a' and b' can be chosen in a more general manner so that the edges may have more general curved shapes. In general the plates need not be flat either, but for present calculations they will be chosen to be flat so that b' is proportional to the distance from the apex.

On each intermediate cross-section plane the array of wave launchers looks as in fig. 2.2. Here the in-line configuration is chosen. Conveniently, the symmetry of the problem has electric boundaries on the constant-y planes midway between launchers and in the center of the launchers as

$$\begin{aligned} &y = mb \equiv \text{electric boundaries} \\ &m = 0, \pm 1, \pm 2, \ldots \end{aligned} \tag{2.2}$$

These electric boundaries have electric field normal to them, and are used to define voltages in the transmission-line approximation. Similarly, we have magnetic boundaries on constant x planes as

$$\begin{aligned} &x = na \equiv \text{magnetic boundaries} \\ &n = 0, \pm 1, \pm 2, \ldots \end{aligned} \tag{2.3}$$

These magnetic boundaries have magnetic field normal to them, and are used to define currents in the transmission-line approximation.

Further note that a wave launcher, say the central one, occupies the space

$$\begin{aligned} &-a \le x \le a \\ &-b \le y \le b \end{aligned} \tag{2.4}$$

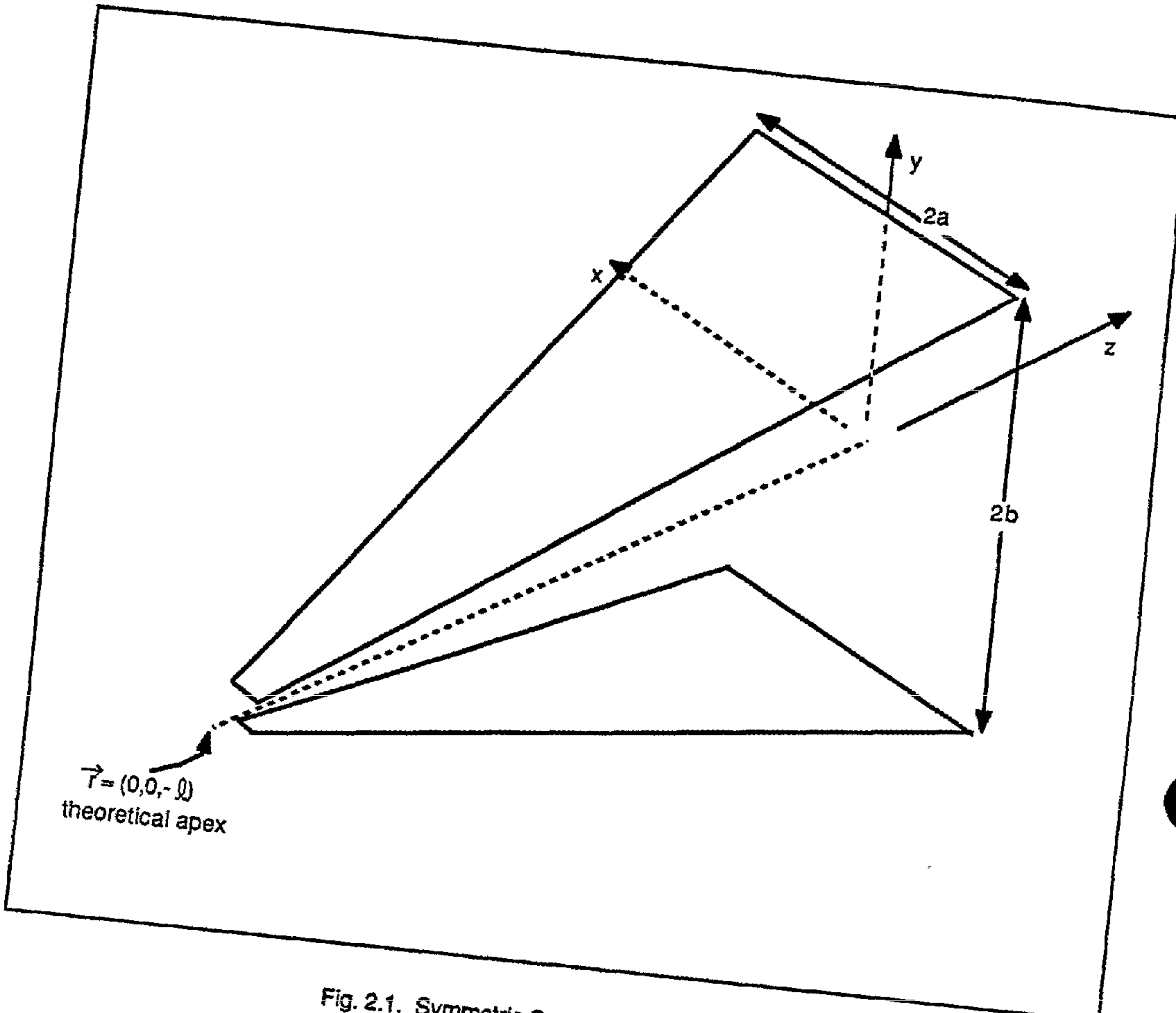

Fig. 2.1. Symmetric Conical Wave Launcher

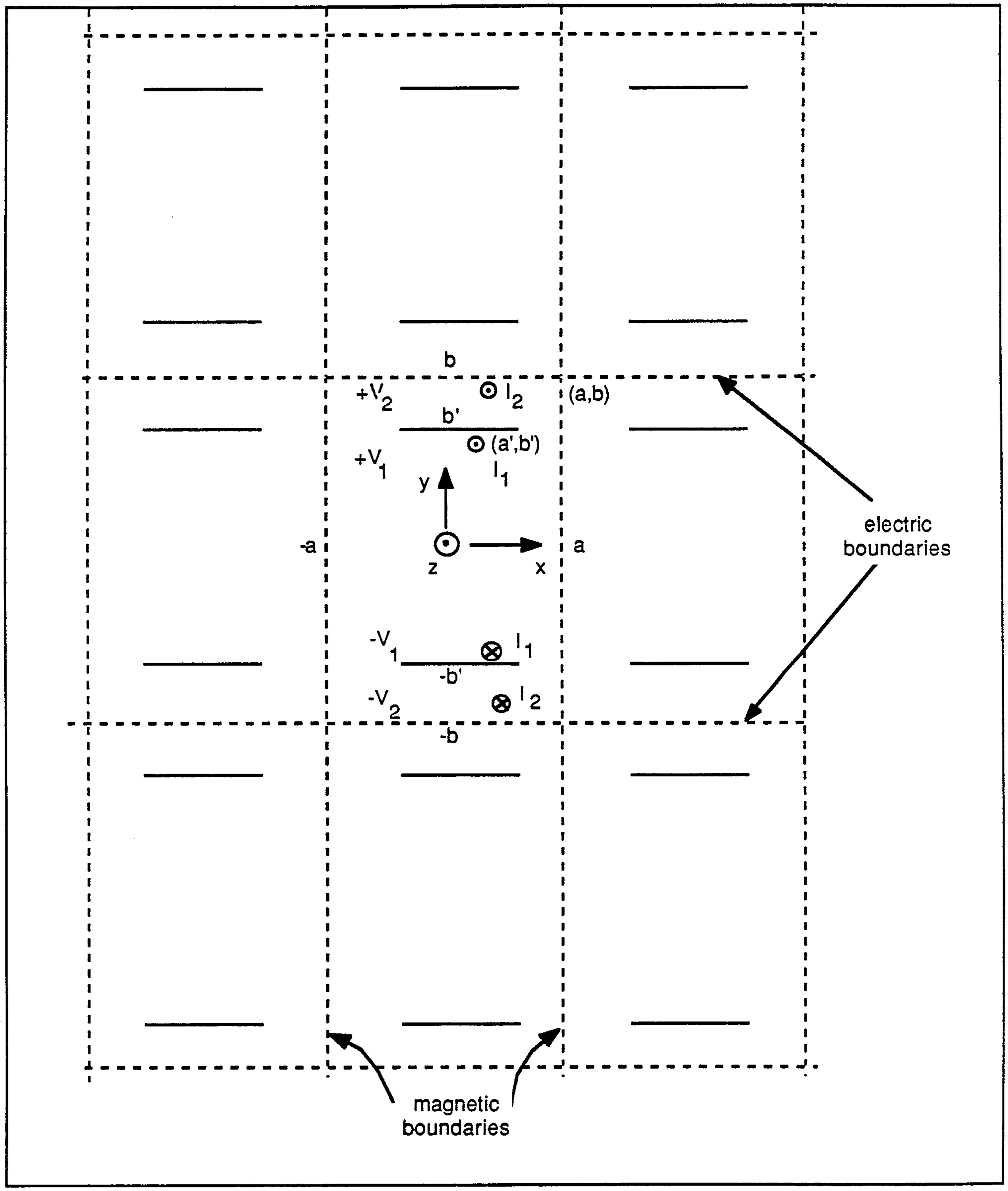

Fig. 2.2. Cross-Section Plane and Unit Cell for In-Line Configuration ($-\ell < z < 0$)

bounded by these electric and magnetic boundaries across which no energy flows. The rest of the array is replaced by these boundaries as in (2.4). In this unit cell (on some cross section) by symmetry we have voltages $\pm V_1$ on the two planes and $\pm V_2$ on the two electric boundaries. The potential difference between the two plates is $2V_1$. On the top plate current I_1 comes out of the page and on the bottom the current goes in. On the top electric boundary (underside) current I_2 comes out and goes in on the bottom magnetic boundary (topside). Noting that $2V_1$ and $2V_2$ are the appropriate potential differences for use in the transmission-line equations we have for a wave propagating in the +z direction

$$\begin{aligned}(2V_1, 2V_2) &= \left(Z_{c_{n,m}}\right) \cdot (I_1, I_2) \\ \left(Z_{c_{n,m}}\right) &= Z_0\left(f_{g_{n,m}}\right) \\ Z_0 &= \sqrt{\frac{\mu_0}{\varepsilon_0}} = \text{characteristic impedance of free space}\end{aligned} \tag{2.5}$$

Note that for a nonuniform transmission line we can think of this wave propagating in one direction in the high-frequency limit as in [5].

The geometric-impedance-factor matrix $\left(f_{g_{n,m}}\right)$ is dimensionless, has non-negative eigenvalues, and is also used to calculate

$$\begin{aligned}\left(L'_{n,m}\right) &= \mu_0\left(f_{g_{n,m}}\right) \\ &\equiv \text{inductance-per-unit-length matrix} \\ \left(C'_{n,m}\right) &= \varepsilon_0\left(f_{g_{n,m}}\right)^{-1} \\ &\equiv \text{capacitance-per-unit-length matrix}\end{aligned} \tag{2.6}$$

This can be calculated by considering one-quarter of the unit cell in fig. 2.2 by use of the electric boundary on y = 0 and the magnetic boundary on x = 0. This cuts the voltages and currents in half for the quarter cell so that (2.5) is replaced by

$$(V_1, V_2) = Z_0\left(f_{g_{n,m}}\right) \cdot \left(\frac{I_1}{2}, \frac{I_2}{2}\right) \tag{2.7}$$

with the geometric-impedance-factor matrix unchanged.

As in fig. 2.3 the quarter cell is exhibited with the voltages and currents as in (2.7) except of a reversal in the currents as the wave is propagating in the -z direction. Setting $I_1/2 = 0$ and driving the outer electric boundaries gives a uniform field in the quarter cell (noting the presence of the two magnetic boundaries). This allows us to find two components of the geometric-impedance-factor matrix as

$$f_{g_{1,2}} = \frac{b'}{a}$$

$$f_{g_{2,2}} = \frac{b}{a} \tag{2.8}$$

Applying reciprocity gives

$$f_{g_{2,1}} = f_{g_{1,2}} = \frac{b'}{a} \tag{2.9}$$

This leaves $f_{g_{1,1}}$ which is determined by setting $I_2/2 = 0$ and finding the ratio of V_1 to the current $I_1/2$ as in fig. 2.3. One of the dimensions a,b,a',b' can be used to normalize the results (as a in (2.8) and (2.9)) giving $f_{g_{1,1}}$ as a function of three parameters.

Collecting we have

$$\left(f_{g_{n,m}}\right) = \begin{pmatrix} f_{g_{1,1}} & \frac{b'}{a} \\ \frac{b'}{a} & \frac{b}{a} \end{pmatrix} \tag{2.10}$$

Define a normalized form of this as

$$\left(F_{n,m}\right) \equiv \frac{a}{b}\left(f_{g_{n,m}}\right) = \begin{pmatrix} F_{1,1} & \frac{b'}{b} \\ \frac{b'}{b} & 1 \end{pmatrix}$$

$$F_{1,1} = \frac{a}{b} f_{g_{1,1}} \tag{2.11}$$

This is in general a function of position z along the transmission-line (2 conductors with reference). Let us define a normalized coordinate as

$$\zeta \equiv \frac{z}{\ell} + 1 \tag{2.12}$$

so the wave propagates from $\zeta = 0$ (the apex) to $\zeta = 1$ (the aperture plane).

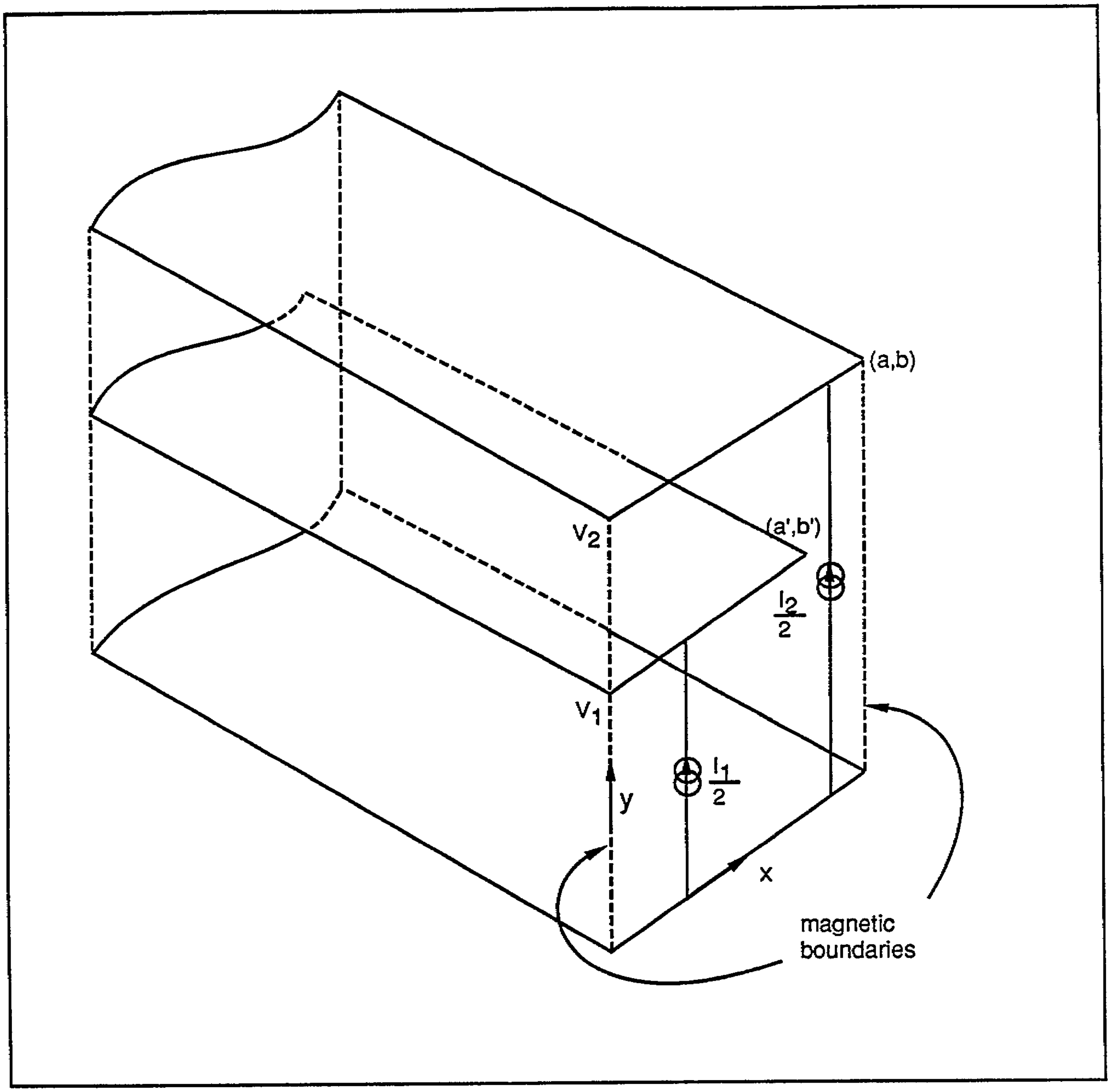

Fig. 2.3. Quarter Cell Driven at Some Cross Section with No z-Variation of Dimensions

III. Special Case

As a special case let us choose

$$\left(F_{n,m}(\zeta)\right) \equiv \begin{pmatrix} 1 & \zeta \\ \zeta & 1 \end{pmatrix} \tag{3.1}$$

This corresponds to

$$\frac{b'}{b} = \zeta \tag{3.2}$$

in which the wave-launcher plates are flat, starting with zero height at $\zeta = 0$ and filling the full height of the unit cell at $\zeta = 1$. The choice of

$$F_{1,1}(\zeta) \equiv \frac{a}{b} f_{g_{1,1}} \equiv 1 \tag{3.3}$$

is the simplest choice as a constant and makes $a' = a$ at the aperture plane so that the wave launcher occupies the full width at the aperture plane.

Note that

$$\frac{d}{d\zeta}\left(F_{n,m}(\zeta)\right) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{3.4}$$

which is a constant matrix, simplifying matters somewhat. In (3.1) the off-diagonal elements ζ could be replaced by some function of ζ which would introduce the derivative of this function in (3.4) as a scalar multiplier; such a case is not much more difficult than the present one. If, however, one makes $F_{1,1}$ a function of ζ (instead of (3.3)) there are significant complications.

IV. Matrix Differential Equation

As discussed in [5] the high-frequency propagation on a nonuniform multiconductor transmission line is given by

$$\left(V_n(\zeta)\right) = \left(\phi_{n,m}(\zeta)\right) \cdot \left(V_n(0)\right) \tag{4.1}$$

where the voltage vector is taken here in retarded time and interpreted as the discontinuity across a step-rising wavefront. The boundary condition at $\zeta = 0$ is

$$\left(V_n(0)\right) = V_0(1, 0) \tag{4.2}$$

which indicates a step rising voltage of amount V_0 on the wave launcher. The second element (i.e., 0) indicates that initially V_2 is zero at $\zeta = 0$ since the height 2b' of the wave launcher is zero at $\zeta = 0$ which does not influence V_2 across the corresponding height 2b between electric boundaries. As the wave propagates along the wave launcher V_2 increases from zero.

Now $(\phi_{n,m}(\zeta))$ satisfies a differential equation [5]

$$\begin{aligned}
&\frac{d}{d\zeta}\left(\phi_{n,m}(\zeta)\right) = \left(A_{n,m}(\zeta)\right) \cdot \left(\phi_{n,m}(\zeta)\right) \\
&\left(A_{n,m}(\zeta)\right) = \frac{1}{2}\left[\frac{d}{d\zeta}\left(Z_{c_{n,m}}(\zeta)\right)\right] \cdot \left(Y_{c_{n,m}}(\zeta)\right) \\
&\left(Z_{c_{n,m}}(\zeta)\right) = \left(Y_{c_{n,m}}(\zeta)\right)^{-1} = Z_0\left(f_{g_{n,m}}\right) \\
&\qquad = \frac{b}{a} Z_o\left(F_{n,m}(\zeta)\right)
\end{aligned} \tag{4.3}$$

$\equiv$ characteristic impedance matrix

Note that z has been replaced by the normalized distance ζ. In integrating the above it does not matter if we use $d\zeta$ or dz as it merely corresponds to a change of variable. Similarly, $d\zeta$ could be replaced by $df(\zeta)$ if $f(\zeta)$ is monotonic and continuous in ζ. This corresponds in (3.1) and (3.2) to a more general shape of the wave launcher. In addition, if $f(0) = 0$ and $f(1) = 1$ the same solution at $\zeta = 1$ will result.

Noting that

$$\det\big(\big(F_{n,m}(\zeta)\big)\big) = 1 - \zeta^2 \tag{4.4}$$

which is nonnegative, we have

$$\big(F_{n,m}(\zeta)\big)^{-1} = \left[1 - \zeta^2\right]^{-1}\begin{pmatrix} 1 & -\zeta \\ -\zeta & 1 \end{pmatrix} \tag{4.5}$$

This gives

$$\begin{aligned}
\big(A_{n,m}(\zeta)\big) &= \frac{1}{2}\left[\frac{d}{d\zeta}\big(F_{n,m}(\zeta)\big)\right]\cdot\big(F_{n,m}(\zeta)\big)^{-1} \\
&= \frac{1}{2}\left[1 - \zeta^2\right]^{-1}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cdot\begin{pmatrix} 1 & -\zeta \\ -\zeta & 1 \end{pmatrix} \\
&= \frac{1}{2}\left[1 - \zeta^2\right]^{-1}\begin{pmatrix} -\zeta & 1 \\ 1 & -\zeta \end{pmatrix} \qquad \begin{pmatrix} -\zeta & 1 \\ 1 & -\zeta \end{pmatrix} \\
&= \frac{-\zeta}{2}\left[1 - \zeta^2\right]^{-1}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\left[1 - \zeta^2\right]^{-1}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\end{aligned} \tag{4.6}$$

Since one of the two matrices in the last result is the identity, the two matrices commute.

This allows us to write the matrizant solution [5] as

$$\begin{aligned}
\big(\phi_{n,m}(\zeta)\big) &= e^{\int_0^{\zeta}\left(A_{n,m}(\zeta')\right)d\zeta'} \\
&= e^{g_1(\zeta)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}\cdot e^{g_2(\zeta)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}
\end{aligned}$$

$$\begin{aligned}
g_1(\zeta) &\equiv \int_0^{\zeta}\frac{-\zeta'}{2\left[1 - \zeta'^2\right]}d\zeta' = \frac{1}{4}\ell n\left(1 - \zeta^2\right) \\
g_2(\zeta) &\equiv \int_0^{\zeta}\frac{d\zeta'}{2\left[1 - \zeta'^2\right]} = \frac{1}{4}\ell n\left(\frac{1+\zeta}{1-\zeta}\right)
\end{aligned} \tag{4.7}$$

where the integrals are found in standard tables [6]. The exponential matrices are evaluated using the series representation of the exponential. First we have

$$e^{g_1(\zeta)\begin{pmatrix}1&0\\0&1\end{pmatrix}} = e^{g_1(z)}\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$= \left[1-\zeta^2\right]^{\frac{1}{4}}\begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{4.8}$$

Second, noting

$$\begin{pmatrix}0&1\\1&0\end{pmatrix}^2 = \begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{4.9}$$

then in the series expansion of the exponential the even terms are all scalars times the identity, and the odd terms are all scalars times this other simple matrix giving

$$e^{g_2(\zeta)\begin{pmatrix}0&1\\1&0\end{pmatrix}} = \cosh\left(g_2(\zeta)\right)\begin{pmatrix}1&0\\0&1\end{pmatrix} + \sinh\left(g_2(\zeta)\right)\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$= \frac{e^{g_2(\zeta)} + e^{-g_2(\zeta)}}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix} + \frac{e^{g_2(\zeta)} - e^{-g_2(\zeta)}}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$= \frac{e^{g_2(\zeta)}}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix} + \frac{e^{-g_2(\zeta)}}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}$$

$$= \frac{1}{2}\left[\frac{1+\zeta}{1-\zeta}\right]^{\frac{1}{4}}\begin{pmatrix}1&1\\1&1\end{pmatrix} + \frac{1}{2}\left[\frac{1-\zeta}{1+\zeta}\right]^{\frac{1}{4}}\begin{pmatrix}1&-1\\-1&1\end{pmatrix} \tag{4.10}$$

Multiplying out the terms the matrizant solution is

$$
\left(\phi_{n,m}(\zeta)\right) = \frac{1}{2}\left[1-\zeta^2\right]^{\frac{1}{4}}\left\{\left[\frac{1+\zeta}{1-\zeta}\right]^{\frac{1}{4}}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + \left[\frac{1-\zeta}{1+\zeta}\right]^{\frac{1}{4}}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\right\}
$$

$$
= \frac{1}{2}\left\{\left[1+\zeta\right]^{\frac{1}{2}}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + [1-\zeta]^{\frac{1}{2}}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\right\} \tag{4.11}
$$

Applying the boundary condition at $\zeta = 0$ gives

$$
\left(V_n(\zeta)\right) = \left(\phi_{n,m}(\zeta)\right)\cdot\left(V_n(0)\right) = V_0\left(\phi_{n,m}(\zeta)\right)\cdot(1,0)
$$

$$
= \frac{V_0}{2}\left\{[1+\zeta]^{\frac{1}{2}}(1,1) + [1-\zeta]^{\frac{1}{2}}(1,-1)\right\} \tag{4.12}
$$

At $\zeta = 1$ this gives

$$
\left(V_n(1)\right) = \frac{V_0}{\sqrt{2}}(1,1) \tag{4.13}
$$

so that V_1 and V_2 are both $V_0/\sqrt{2}$. This is consistent with the fact that the wave launcher plates meet the electric boundaries on $y = \pm b$ at $\zeta = 1$ (i.e., $z = 0$).

V. Power Considerations on the Wavefront

The power just behind the wavefront on the quarter cell is

$$P = (V_n(\zeta)) \cdot \left(\frac{1}{2} I_n(\zeta)\right) = (V_n(\zeta)) \cdot \left(Y_{c_{n,m}}(\zeta)\right) \cdot (V_n(\zeta))$$

$$= (V_n(\zeta)) \cdot \left\{ \frac{a}{b} Z_0^{-1} \left[1 - \zeta^2\right]^{-1} \begin{pmatrix} 1 & -\zeta \\ -\zeta & 1 \end{pmatrix} \right\} \cdot (V_n(\zeta))$$

$$= \frac{a}{b} Z_0^{-1} \left[1 - \zeta^2\right]^{-1} \left[V_1^2 - 2\zeta V_1 V_2 + V_2^2\right]$$

$$= \frac{a}{b} \frac{V_0^2}{4Z_0} \left[1 - \zeta^2\right]^{-1} \left\{ \left[[1+\zeta]^{\frac{1}{2}} + [1-\zeta]^{\frac{1}{2}} \right]^2 \right.$$

$$\left. - 2\zeta \left[[1+\zeta]^{\frac{1}{2}} + [1-\zeta]^{\frac{1}{2}} \right] \left[[1+\zeta]^{\frac{1}{2}} - [1-\zeta]^{\frac{1}{2}} \right] + \left[[1+\zeta]^{\frac{1}{2}} - [1-\zeta]^{\frac{1}{2}} \right]^2 \right\}$$

$$= \frac{a}{b} \frac{V_0^2}{4Z_0} \left[1 - \zeta^2\right]^{-1} \left\{4 - 4\zeta^2\right\}$$

$$= \frac{a}{b} \frac{V_0^2}{Z_0} \neq \text{function of } \zeta \tag{5.1}$$

So power is conserved along the wavefront as it should be, since in the high-frequency limit there are no reflections along a smoothly varying multiconductor transmission line, and the line is lossless.

At $\zeta = 0$ the power is all associated with the wave launcher since V_2 and $I_2/2$ are both zero. At $\zeta = 1$ the situation is more complicated. As illustrated in fig. 5.1 the boundary condition there is visualized. Noting that the incident wave has (from (4.13))

$$V_1 = V_2 = \frac{V_0}{\sqrt{2}} \tag{5.2}$$

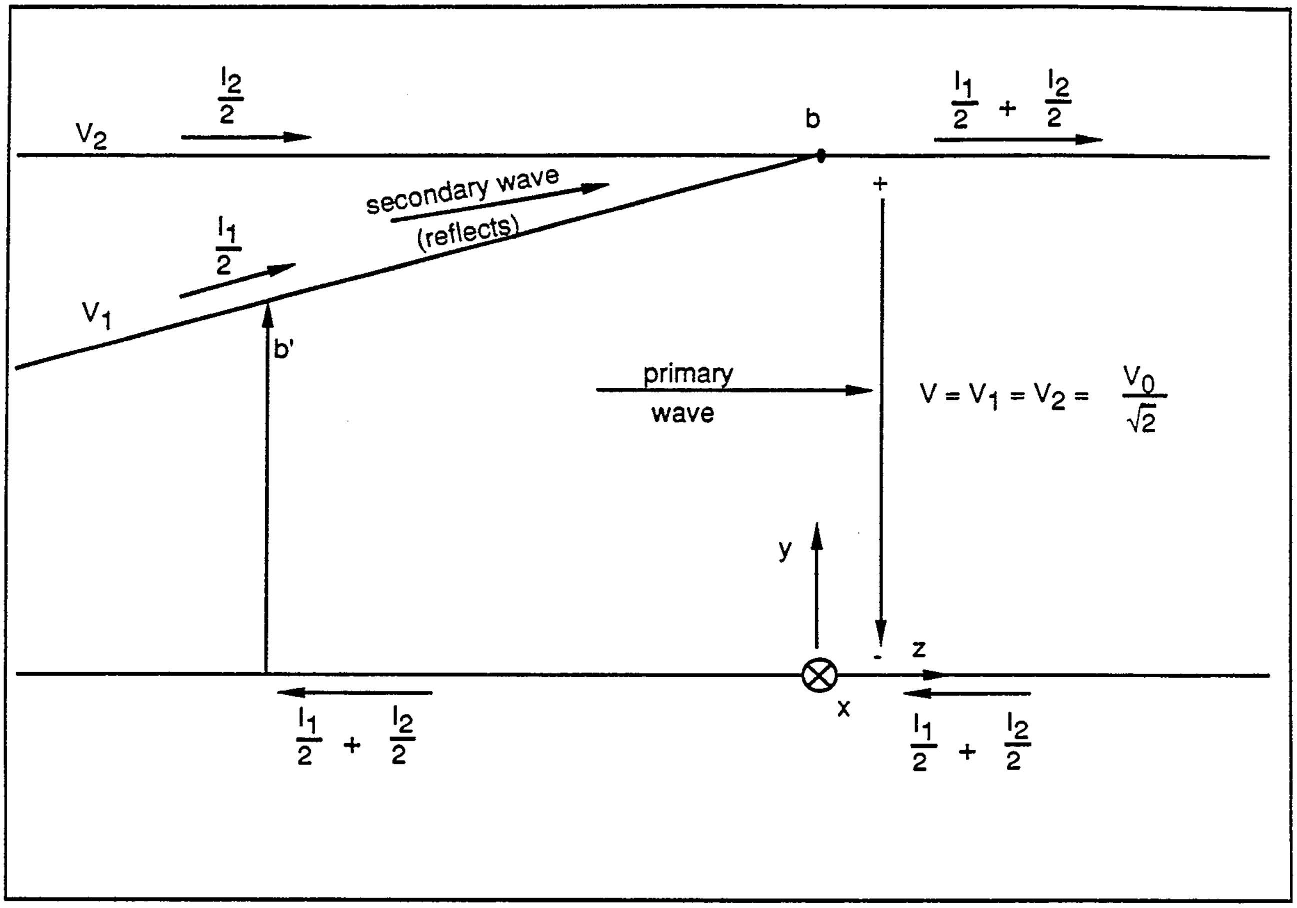

Fig. 5.1. Transmission of Wave Through Aperture Plane

The associated incident currents are

$$\left(\frac{1}{2}I_n(\zeta)\right) = \left(Y_{c_{n,m}}(\zeta)\right)^{-1} \cdot (V_n(\zeta))$$
$$= \frac{a}{b}Z_0^{-1}\left[1-\zeta^2\right]^{-1}\begin{pmatrix}1 & -\zeta \\ -\zeta & 1\end{pmatrix}\cdot (V_n(\zeta))$$
$$= \frac{a}{b}Z_0^{-1}\left[1-\zeta^2\right]^{-1}\left(V_1(\zeta) - \zeta V_2(\zeta), -\zeta V_1(\zeta) + V_2(\zeta)\right) \tag{5.3}$$

which gives the total current

$$\frac{I_1(\zeta)}{2} + \frac{I_2(\zeta)}{2} = \frac{a}{b}Z_0^{-1}[1+\zeta]^{-1}\left(V_1(\zeta) + V_2(\zeta)\right) \tag{5.4}$$

Taking the limit as $\zeta \to 1$

$$\frac{I_1}{2} + \frac{I_2}{2} = \frac{a}{b}\frac{1}{2Z_0}\left(V_1 + V_2\right) = \frac{a}{b}\frac{1}{\sqrt{2}}\frac{V_0}{Z_0} \tag{5.5}$$

Note that this matches through the interface since

$$\frac{V_1}{\frac{I_1}{2} + \frac{I_2}{2}} = \frac{b}{a}Z_0 \tag{5.6}$$

which is the impedance of the unit cell for $z > 0$ where the wave launcher has merged with the electric boundaries (top and bottom) to form an effective single conductor (plus reference) transmission line with only one mode of propagation. So Kirchoff's laws are satisfied at the aperture plane.

As the wave launcher reaches the aperture plane the wave for y between 0 and b' is effectively shielded from that between b' and b. The power in this "primary" wave is just

$$P_1 = \frac{V_1^2}{Z_0} = \frac{V_0^2}{2Z_0} = \frac{1}{2}P \tag{5.7}$$

i.e., half the total incident power. This matches through the aperture plane consistent with (5.2) and continuity of voltage. The remaining power in the "secondary" wave is

$$P_2 = P - P_1 = \frac{1}{2}P = P_1 \qquad (5.8)$$

This additional power is trapped between the wave launcher and the electric boundaries (i.e., between b' and b). Note that at the aperture plane a' → a allowing no "leakage" into the region of the primary wave. This secondary wave is reflected in the -z direction.

VI. Concluding Remarks

Considering the character of the solution in this special case some observations are in order. First, this forms a canonical case for which an analytic solution (for the early-time response in the transmission-line approximation) is available. It is not necessarily the best choice for the wave launcher shape, but one to which others can be compared.

A more optimal shape might be one with

$$0 < f_{g_{1,1}}\Big|_{z=-\ell} < \frac{b}{a} \tag{6.1}$$

so that for a given V_0 more power is launched to compensate for the reflection at $z = 0$ and make the voltage of the wave transmitted through the aperture plane closer to V_0 (as compared to the present case of $V_0 / \sqrt{2}$). As position is varied from $z = -\ell$ to $z = 0$ one can have

$$f_{g_{1,1}} \to \frac{b}{a} \quad \text{as} \quad z \to 0- \tag{6.2}$$

with smooth variation in between.

It would also be useful if one could suppress the reflection from the aperture plane since this can partially reflect from the source at $z = -\ell$. Noting in fig. 5.1 that near the aperture plane the secondary wave (between b' and b as well as -b' and -b) is effectively isolated from the primary wave. Then one may place some absorber material in the space occupied by the secondary wave with minimal impact on the primary wave. This, however, will likely only partially remove the secondary wave.

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