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Equivalent Displacement for a High-Voltage Rollup on the Edge of a Conducting Sheet

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Abstract

It is commonplace to rollup the edges of high-voltage electrodes in the shape of flat or conical plates, in order to avoid electrical breakdown to nearby electrodes. Such a rollup has the effect of reducing the field enhancement at the edges of the plates. In this note we address the problem of estimating the equivalent displacement of a high-voltage rollup at the plate edge, for the case of a commonly employed cylindrical rollup.

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It is commonplace to rollup the edges of high-voltage electrodes in the shape of flat or conical plates, in order to avoid electrical breakdown to nearby electrodes. Such a rollup has the effect of reducing the field enhancement at the edges of the plates. In this note we address the problem of estimating the equivalent displacement of a high-voltage rollup at the plate edge, for the case of a commonly employed cylindrical rollup.

electrical breakdown, electrodes

Contents

I. Introduction

In the practice of high-voltage engineering, often times, the edges of plates or wire meshes that serve as high voltage electrodes are rolled up to prevent electrical breakdown to nearby electrodes. The electric field has a singularity at the edges of a plate, as in the case of a two-parallel-plate transmission line [1], for instance. The total electric field in the TEM mode propagating in such a transmission line is indicated in figure 1 and the field enhancement at the edges of both plates is evident. The behavior of the fields at the edges is similar, for the case of a conical transmission line [2,3] made up of two triangular shaped conductors as shown in figure 2. In the context of a conical wave launcher, wherein the plates are triangular in shape, we may now consider a rollup at the edges of the plates. Two possible rollups are schematically shown in figure 3. The conical roll has a smaller radius of rollup towards the apex, whereas the cylindrical roll has a constant rollup radius and consequently, the plates do not go all the way back to their apexes. In either case, it would be useful to determine an equivalent displacement of the plate with the rollup. This displacement is indicated by the dashed lines in figure 3 and is useful in characterizing the plate, as for example in the determination of the effective impedance of the wave launcher. The problem at hand may then be stated as that of estimating the equivalent displacement of a plate with its edges rolled up. This is accomplished by investigating the current or charge distribution near an edge (Section II), and also near an edge with a rollup (Section III). By equating the distant fields to a first order, an equivalent displacement can be derived for the rollup (Section IV). This note is concluded with a summary in Section V followed by a list of references.

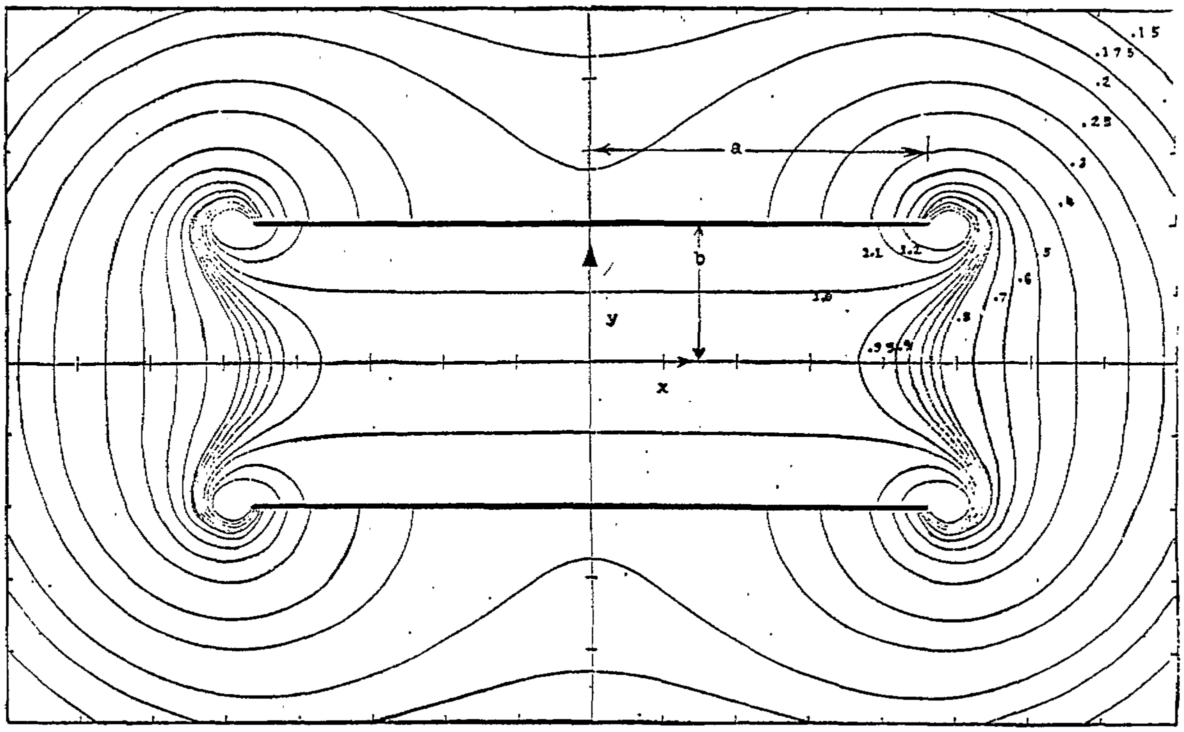


Figure 1. Contours of constant $|E_{rel}|$ in a two-parallel plate transmission line with b/a = 0.40679.

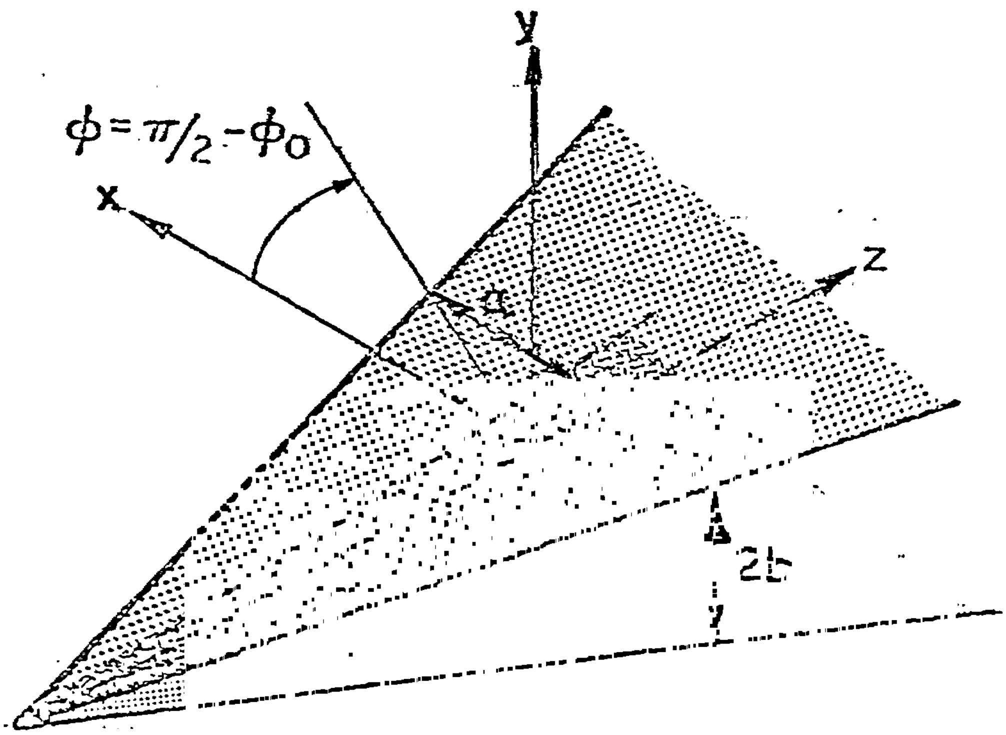
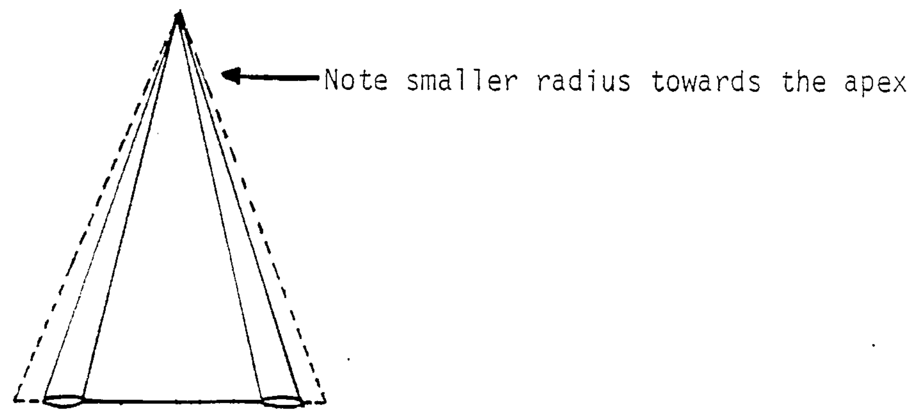
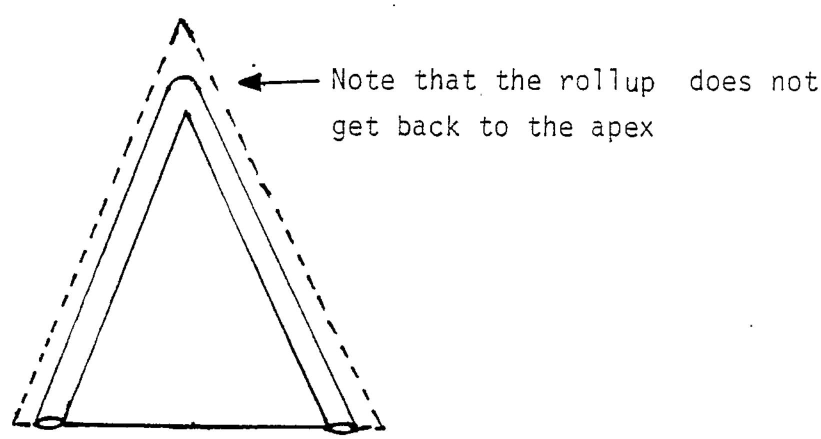


Figure 2. Geometry of a conical transmission line made up of two triangular shaped conductors.



(A) Conical roll



(B) Cylindrical roll

Figure 3. Two possible rollup configurations in a conical sheet with equivalent displacements indicated by dashed lines.

II. Current or Charge Distribution Near an Edge

In this section, we consider the current or charge distribution near a conducting edge. Such a conducting edge is shown in figure 4 wherein the complex coordinate $z' = (x' + j\,y')$ uniquely identifies the location of an observation point. The conducting edge is at a fixed electric potential with respect to infinity, and it extends from $x' = -\infty$ to $x' = b$ on the real axis of the z' plane. The equipotentials u_i' and streamlines v_i' are also sketched in figure 4. Next, one may consider a mapping function given by

$$z_1 = \sqrt{z' - b} \tag{1}$$

which maps the entire z' plane into the right half of the complex z_1 plane as indicated in figure 5. The equipotentials and streamlines are now uniformly distributed in the z_1 plane, leading to the conformal transformation,

$$w'(x', y') \equiv u'(x', y') + j\,v'(x',y') \tag{2}$$

or

$$w'(z') \equiv u'(z') + j\,v'(z')$$

$$= \sqrt{z' - b} \tag{3}$$

In summary, the conformal transformation may be thought of as one that takes the uniform field in the w' plane and transforms it so that it fits the given boundary conditions in the z' plane, while keeping the required properties of an electrostatic field.

In the following section, we find a similar transformation for the case of a conducting edge with a cylindrical roll. Once the conformal mapping solutions (potential and consequently the complex field) are separately known for these two problems, we can address the question of equivalent displacement.

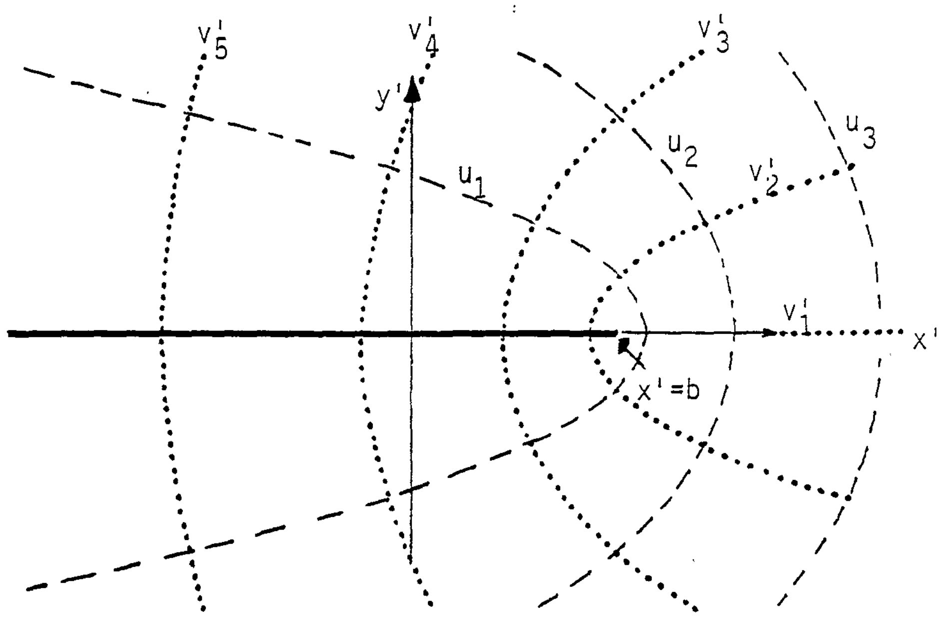


Figure 4. Potential and stream functions near and around a conducting edge (z' plane).

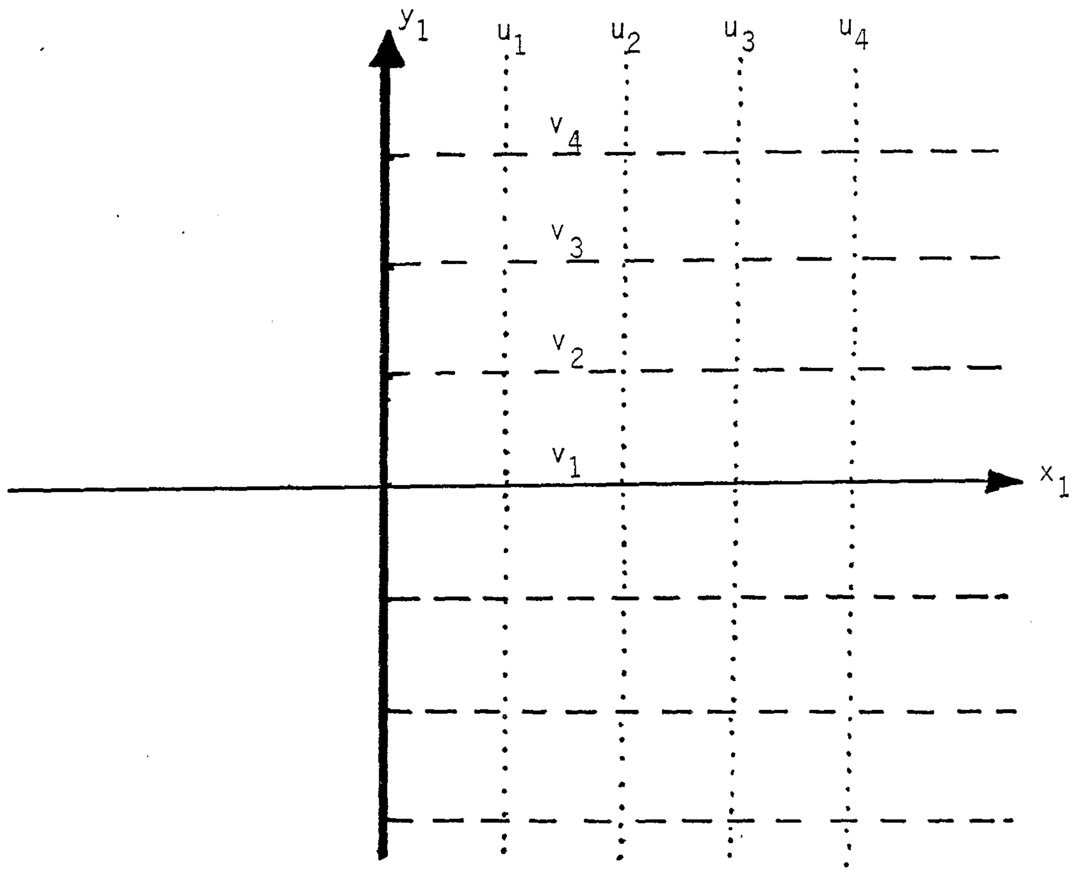


Figure 5. Conformal transformation $z_1 = \sqrt{z' - b}$ of above resulting in a uniform potential-stream function distribution.

III. Current or Charge Distribution Near a Cylindrical Roll on an Edge

We now consider the current or charge distribution, or equivalently the potential and stream function distribution near a conducting edge with a cylindrical roll. The geometry of the problem is shown in figure 6 wherein the complex coordinate $z = x + j\, y$ uniquely identifies the location of an observation point. As before, the conducting edge with the cylindrical roll is held at a fixed potential with respect to a reference conductor at infinity. The edge with the roll extends from $x = -\infty$ and the origin of the complex z plane coincides with the center of the cylindrical roll of radius a.

Next, one may consider transforming the z plane to a plane in which the potential and stream function distribution is uniform for comparison with the case of the edge with no rollup. This transformation is accomplished in two steps as follows. Initially,

$$z_2(z) = \sqrt{z} \tag{4}$$

and this mapping which takes the $z = (x + j\, y)$ plane into $z_2 = (x_2 + j\, y_2)$ plane is illustrated in figure 7 with the associated equipotentials and stream lines. A second mapping intended to map the z_2 plane into z_3 plane with uniform potential and field distribution is as follows

$$z_3(z) = z_2 - \frac{a}{z_2} \tag{5}$$

Inserting (4) into (5), the two mappings can be combined to yield

$$z_3(z) = \left(\sqrt{z} - \frac{a}{\sqrt{z}}\right) \tag{6}$$

The $z_3 = (x_3 + j\, y_3)$ plane is shown in figure 8 and it is observed that the potential and field distribution is uniform and similar to the z_1 plane we had obtained for the case of the conducting edge with no rollup.

In effect, the conformal transformation of (6) takes the uniform field in the z_3 plane and transforms it so that it fits the given

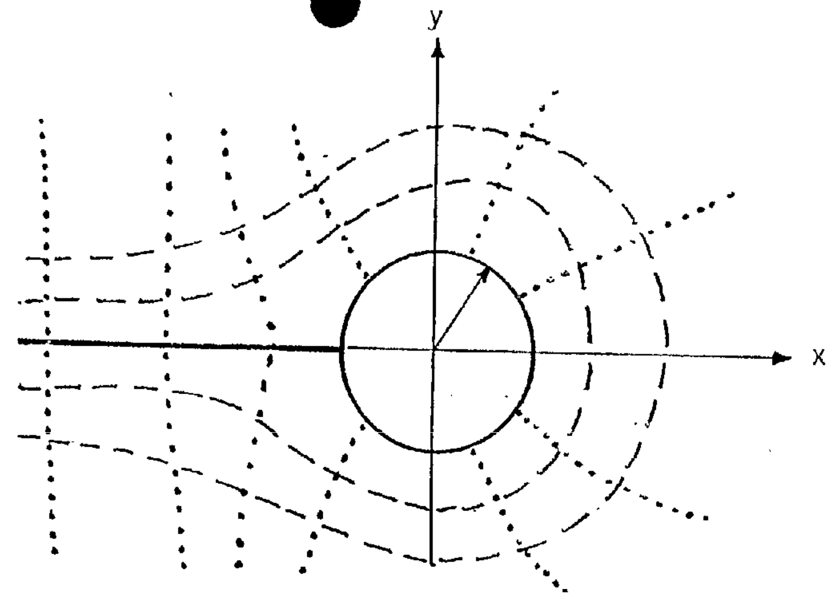


Figure 6. Conducting edge with a cylindrical roll (z-plane)

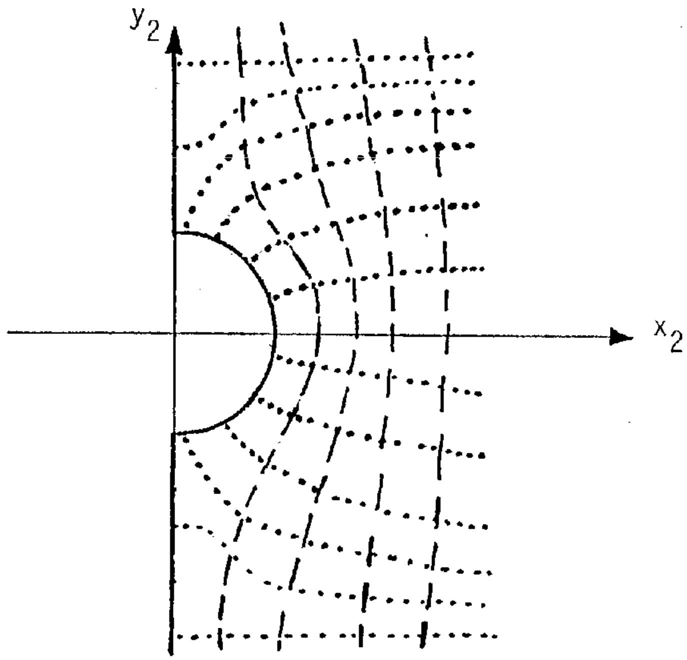


Figure 7. An intermediate transformation ($z_2 = \sqrt{z}$)

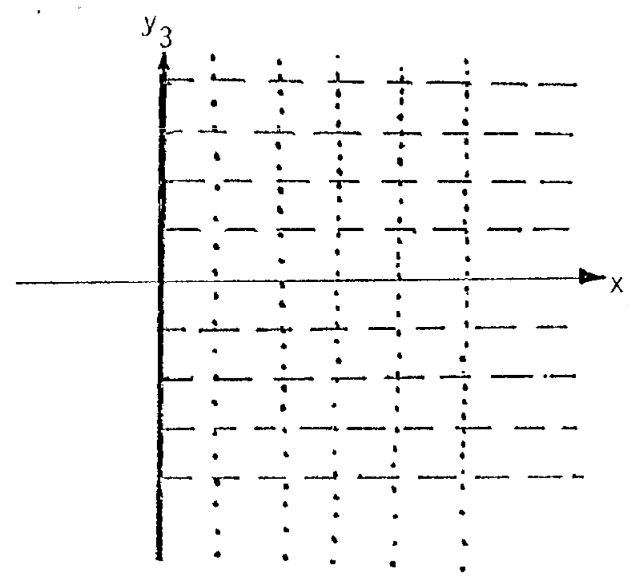


Figure 8. Final transformation $z_3 = [z_2 - (a/z_2)]$ resulting in a uniform potential and stream function distribution.

boundary conditions in the z plane, while keeping the required properties of an electrostatic field. Since we now have such a transformation for both cases of an edge with and without a cylindrical roll, we are now in a position to establish an equivalence between the two. This forms the subject of the following section.

IV. Equivalent Displacement

Recall that in the preceeding sections, we obtained conformal transformations for a conducting edge without and with a cylindrical roll. In both cases the transformations lead to a plane in which the equipotentials and streamlines are uniformly distributed. These are the z_1 plane in figure 5 and the z_3 plane in figure 8 respectively for the cases of conducting edges without and with a cylindrical roll. Now, the complex potential in the two cases are given by

$$w' = z_1 = \sqrt{z' - b} \qquad \text{(without a roll)}$$

$$w = z_3 = \left(\sqrt{z} - \frac{a}{\sqrt{z}}\right) \qquad \text{(with a cylindrical roll)} \tag{7}$$

An equivalence between the two cases may be established by equating the complex potentials as follows

$$w' = w \tag{8}$$

implying

$$\left(\sqrt{z} - \frac{a}{\sqrt{z}}\right) = \sqrt{z' - b} \tag{9}$$

Taking square roots in the upper half plane, and setting $z = x$ and $z' = x'$, we have

$$\left(\sqrt{x} - \frac{a}{\sqrt{x}}\right) = \sqrt{x' - b} \tag{10}$$

For x and x' tending to large negative values, the above may be rewritten as

$$j\sqrt{-x} - \frac{a}{j\sqrt{-x}} = j\sqrt{-x' + b}$$

or

$$\sqrt{-x} + \frac{a}{\sqrt{-x}} = \sqrt{-x' + b}$$

$$= \sqrt{-x'} \left[1 + \frac{b}{(-x')}\right]^{1/2}$$

$$= \sqrt{-x'} \left[1 + \frac{1}{2} \frac{b}{(-x')} + 0\left(\left(\frac{b}{x'}\right)^2\right)\right] \tag{11}$$

or

$$\left[1 + \frac{a}{(-x)}\right] = \left[\sqrt{\frac{x'}{x}}\right]\left[1 + \frac{1}{2} \frac{b}{(-x')} + 0\left(\left(\frac{b}{x'}\right)^2\right)\right] \tag{12}$$

For large negative values of x and x', we find that we have established an equivalence to an order of (b^2/x'^2) by picking b = 2a. Referring to figures 6 (an edge with a cylindrical roll of radius a) and 4 (an edge without a roll), the equivalent edge displacement Δ is then given by

$$\Delta = b - a = 2a - a = a \tag{13}$$

or a normalized edge displacement Δ_n is

$$\Delta_n = \frac{\Delta}{a} = \left(\frac{b - a}{a}\right) = 1 \tag{14}$$

This equivalent displacement is explicitly displayed in figure 9. It implies that at large negative x and x', the stream function v gives the same total charge and current to that large -x and -x'. Furthermore, one has the same potential u as one leaves the boundary in both cases, i.e.,

$$\frac{\partial u}{\partial y} = \frac{\partial u'}{\partial y'} \quad \text{as } (x, x') \to -\infty \tag{15}$$

which means, one has the same local charge density in both cases at large distance away from the edges.

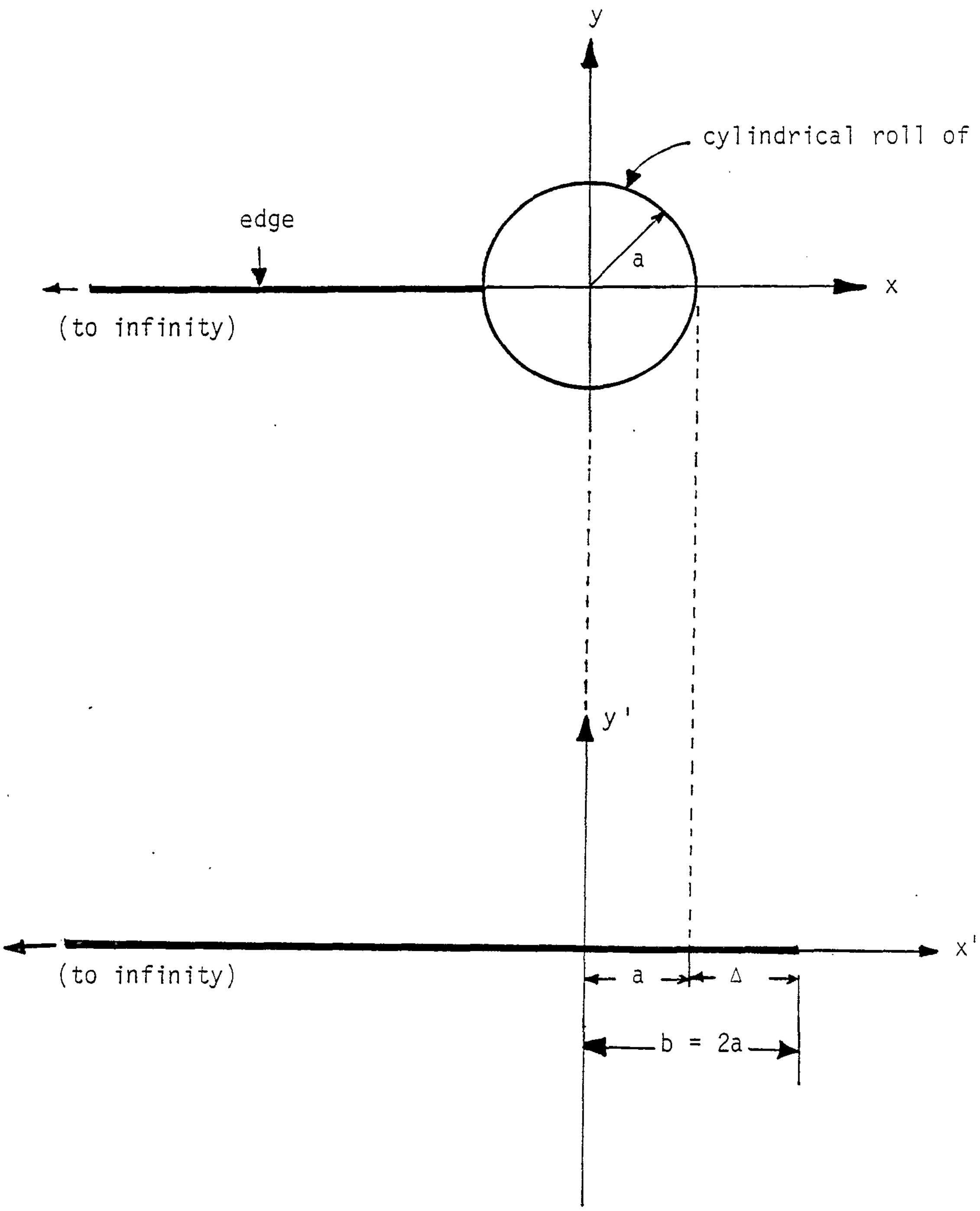


Figure 9. Equivalent displacement of an edge with a cylindrical roll of radius a.

The equivalent displacement established above is reminscent of the equivalence between a circular cylinder and cylinders of other cross sections. A general proof of such an equivalence is available in the literature [4] and for a flat strip of width d, the equivalent circular cylinder has a radius a given by $a = d/4$. Of course, this equivalence is valid for frequencies where $d \ll$ wavelength and also requires that the length of the flat strip be very long compared to its width. The equivalence implies that the total axial current in the flat strip is the same as that in the equivalent circular-cylindrical conductor. In the context of the problem at hand, referring to the z' plane in the bottom of figure 9, a portion of the edge of width $2b = 4a$ centered at the origin, may be replaced by a cylindrical conductor with a circular cross section of radius a.

V. Summary

It is customary to roll the edges of flat conductors when they are used in high-voltage applications for launching or guiding an electromagnetic wave. The roll near the edge effectively smooths or reduces the field enhancement, but could affect the impedance characteristics for example, between the rolled up conductor and another reference conductor.

In this note, the problem of obtaining the equivalent displacement of an edge with a cylindrical roll has been addressed. In other words, a conducting edge with a cylindrical roll at the edge is equivalent to a conducting edge displaced by an amount that can be quantified. We have made the assumption that the conducting edge is semi-infinite in a direction normal to the edge. In a practical situation, this requires that the distance of the edge from any change from flat-plate geometry be large compared to the rollup radius. The equivalence derived here implies that the total current or charge out to a large distance away from the edge as well as the local surface current and charge densities at that large distance are the same in both cases.

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4. R.W.P. King, The Theory of Linear Antennas, Harvard University Press, Cambridge, MA, p. 18, 1956