Sensor and Simulation Notes

Note 281

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EFFECT OF PULLING A WIRE OUT OF A CABLE BUNDLE ON THE DISTRIBUTION OF CURRENT IN THE BUNDLE

Robert L. Gardner
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ABSTRACT

In this note, we apply a methodology of transmission line junction theory which considers the variation across a cable bundle in the continuous variable approximation. The application is the calculation of the current on a wire which has been pulled out of a bundle to apply a current probe relative to the bundle current. For the 45-wire bundle we consider there are about 20 surface wires, resulting 5 percent of the bundle current on each wire for uniform excitation. The method predicts around 17 percent of the bundle current on the wire selected for measurement rather than 5 percent. This conclusion agrees with distributions of measured single wire currents in bundles which have minimums of 20 percent of the bundle current on each wire.

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transmission lines, theory, cables, wire, excitation

CONTENTS

ILLUSTRATIONS

I. DISCUSSION

One of the results of various cable drive tests that have been run on aircraft, missiles, and satellites is the distribution of currents induced on wires in a bundle. In an example cable drive test treated here a bundle of about 45 shield wires was studied in detail and individual shield currents were measured on each of the 45 shields. As is usual for these kinds of measurements the shield currents were measured by pulling the wire out of the bundle and enclosing the wire inside a current probe. A typical distribution of individual shield peak currents measured was from 0.3 to 1.2 times the peak bundle current. Under uniform excitation and uniform termination conditions each shield on the surface of the bundle should carry 5 percent of the bundle current. There are, of course, phasing differences for the individual incident currents in the real case, without sensors. The same sort of distribution of peak currents also appears in other systems with other excitation sources. In some aircraft measurements a distribution of peak current $0.2 \leq I_{shield}/I_{peak} \leq 2$ was observed with some individual wire currents observed as high as 3.5 times the bundle current. The existance of a minimum peak current that is much larger than the uniform excitation value indicates that there may be a perturbation of the measurement due to the installation of the sensor. The cables were all alike and terminated identically in a short. The individual cables were coaxial cables with braided shields and an exterior insulation layer.

As part of a program to develop methods of treating cable bundles with large numbers of component wires, a methodology of treating the bundles using a continuous variable approximation has been developed. The following is a subset of that treatment which incorporates the theory of junctions (Ref. 1) in the continuous variable context. The current distribution for a wire pulled out of a bundle is then treated as an example. Several particular geometries are treated to maximize the understanding gleaned from the example problems.

II. JUNCTION CONDITIONS

The above conditions determine the current flow in the continuous representation of the cable bundle until a discontinuity is encountered. Discontinuities, in this case, include branches, terminations, and, in some sense, redistribution of wires within the bundle. The effects of each of these discontinuities may be calculated using a continuous variable approximation to the junction condition (Refs. 1 and 2) by computing the scattering matrix of the junction.

Consider a three tube network in which a single bundle branches into two smaller bundles according to a mapping operator, M. M operates only on the coordinates and maps position r_1 in tube 1 to r_2 in tube 2 or r_3 in tube 3, depending on r_1. There are some restrictions on the mapping. First the area of tube 2 plus the area of tube 3 must equal that of tube 1 to conserve the total number of wires. In conserving area we do not conserve perimeter so that sufficient wires (unit areas) must be mapped from the interior of tube 1 to fill out the perimeter of tubes 2 and 3. For example, the splitting of circular tube 1 into two semi-circular tubes 2, 3 is performed by

$$\vec{r}_2 = \vec{r}_1 \qquad 0 \le \phi_1 < \pi$$

$$\vec{r}_3 = \vec{r}_1 \qquad \pi \le \phi_1 < 2\pi \tag{1}$$

The center wire goes to the edge of tube 2. From our solution to the propagation equations above we have $J(r,t)$ and $\rho(r,t)$ incident on the junction from tube 1. We need the reflected waves into the three tubes, which may be derived from Kirchhoff's Laws for the junction. The charge density is related to the potential in a wire through $\nabla^2 V = \rho/\varepsilon_0$. The formal inverse of the capacitance matrix C (similar to $-6b^2C_{od}\nabla_{\perp}^2$ in operator form, i.e., CV = Q, where C_{od} is the capacitance between the wire of interest and one of its nearest neighbors, and b is one-half the wire spacing; C and the above differential operator are the same in that either operating on V yield Q. In one case V is a vector; in the other V is a continuous variable, the infinite dimensional extension of the vector V.) is the inductance matrix v^2L. A formal inverse to the transverse Laplacian may be found by considering a formal operator L_{op} which must have the property

$$L_{op}\ \nabla_{\perp}^2 V = V \tag{2}$$

The solution is

$$L_{op} = \int dS'\ G(r,r') \tag{3}$$

where G is the solution of

$$\nabla_{\perp}^2\ G(r,r') = \delta(r-r') \tag{4}$$

and where S' is the cross section of the appropriate tube. The operator approach is not necessary for those geometries that need numerical treatment since the scattering matrix may be found knowing the self-capacitance of each cell in the tube, without going through the operator step. Kirchhoff's Laws then require that the voltages for the simple connections are the same at both ends of the connection and the current into the connection must be balanced by a current flow out of the connection. Then

$$V(\vec{r}_1) = MV(\vec{r}_1) = V(M\vec{r}_1) \tag{5}$$

$$J(\vec{r}_1) = -MJ(\vec{r}_1) = -J(M\vec{r}_1) \tag{6}$$

In this symbology V and I are functions of position and are continuous. Let Y be an operator which converts V into J, i.e., J = YV. Note this operator must act on the local voltage. That is, the voltage on tubes 2 and 3, formed from MV, uses the characteristic admittance operator for those tubes. Since the characteristic admittance operator is proportional to $\int dS\ G$, the Green's function and integration surface for the correct tube must be used. The Green's function must obey the boundary conditions of the respective tube, requiring numerical solution of most problems. This formal problem may now be solved.

In matrix-operator form, Kirchhoff's Laws, for continuity of current and voltage, are

$$\begin{pmatrix} 1 \\ YZ \end{pmatrix} \begin{pmatrix} -1 \\ YZ \end{pmatrix} \begin{pmatrix} V \\ MV \end{pmatrix} = 0 \tag{7}$$

where Z is some useful characteristic impedance and is included to provide for a unitless matrix operator. The incident and reflected waves are related by

$$V^{tot} = V^{inc} + V^{ref} \tag{8}$$

$$J^{tot} = J^{inc} - J^{ref} \tag{9}$$

for all tubes. Then

$$\begin{pmatrix} 1 & -1 \\ YZ & YZ \end{pmatrix} \begin{pmatrix} V^{inc} \\ MV^{inc} \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -YZ & -YZ \end{pmatrix} \begin{pmatrix} V^{ref} \\ MV^{ref} \end{pmatrix} = 0 \tag{10}$$

or in the scattering matrix-operator form

$$\begin{pmatrix} V^{ref} \\ MV^{ref} \end{pmatrix} = - \begin{pmatrix} 1 & -1 \\ -YZ & -YZ \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 \\ YZ & YZ \end{pmatrix} \begin{pmatrix} V^{inc} \\ MV^{inc} \end{pmatrix} \tag{11}$$

Note that with proper selection of mapping function and characteristic admittance matrix both redistribution of wires within a bundle and terminations may be treated.

III. APPLICATION TO SENSOR INSERTION

An immediate application of the above procedure is to determine the effect of pulling a wire out of the bundle and placing a sensor around the shield. Measurements have indicated that the phasing of the current on the wires is such that each individual shield carries a fifth to a third of the bundle current, minimum. For the "phasing" to be such that all 45 wires in the bundle to have large currents when measured, indicates there is some perturbation by the sensor.

A single example of how current may be increased is to consider a large bundle from which a single wire is split off, and all are terminated is a short to ground. The appropriate geometry is shown in Figure 1. The simple circuit modeling this arrangement, for the case that the shields (except that of the wire being pulled out) are assumed well connected within a bundle, is a large inductor branching into small and large inductors in parallel. For the example case there is an insulation layer, which is treated in an approximate sense in a later section. The current will flow predominantly into the large inductor, i.e., the large wire. However, since the inductance of the wire scales as a log and the division of the current for the equal excitation case scales as the diameter of the bundle more current will flow on the small wire than would have flowed if the

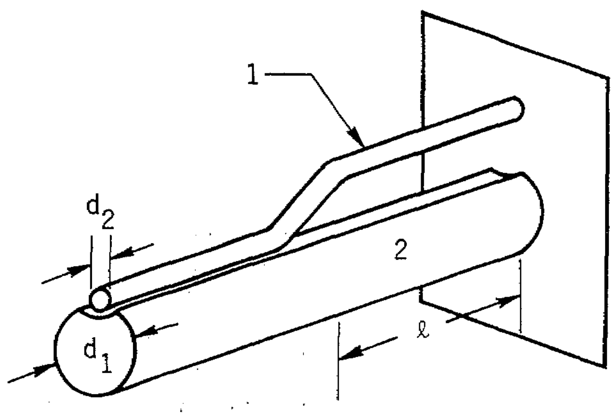

Figure 1. Geometry of a single wire pulled out of a bundle of wires.

separation had not occurred. The applicability of this example is yet to be established, since the electrical connectivity between wires is not as good as one might like. This connectivity problem will be considered later in a 6-tube example.

Consider the breakout near the end of a cable bundle where one exterior wire is removed from the bundle, and consider the first reflection of the peak signal. If the wire is conductor 1 and the bundle is conductor 2, the capacitance matrix is, approximately, before the breakout

$$C_1 = \begin{pmatrix} \frac{C_b}{n} + C_m & -C_m \\ -C_m & C_b + C_m \end{pmatrix} \tag{12}$$

and, after the breakout

$$C_2 = \begin{pmatrix} C_s & 0 \\ 0 & C_b \end{pmatrix} \tag{13}$$

where C_b is the bundle over the ground plane, C_m is the mutual term between the wires, C_s is the capacitance of a single wire above the ground plane, and n is the number of perimeter wires. As a first approximation the mutual capacitance between the pulled out wire and the bundle are assumed zero. A better calculation of those mutuals is given in the next section. Consider frequencies low compared to the length over which the section is removed.

Specific example values for the coefficients, for a 49 wire bundle, are

$\ell = 5$ cm

$\omega = 10^8\ s^{-1}$

$d_1 = 1.4$ cm

$d_2 = 0.2$ cm

$n = 22$

$$C_b = 4 \times 10^{-11} \text{ F/m}$$

$$C_s = 1.6 \times 10^{-11} \text{ F/m}$$

$$C_m = 4 \times 10^{-10} \text{ F/m} \tag{14}$$

and a typical bundle height is $h = 2.4$ cm. A useful figure of merit is $v/\omega\ell = 60$, where v is the propagation velocity of a signal along the transmission line. The velocity is assumed constant through the bundle.

The characteristic admittance of the wire before the breakout is

$$Y_1 = vC_1 \tag{16}$$

where v is the propagation velocity. The impedance of the breakout section is

$$Z_L = j\omega\ell L_2 \tag{17}$$

so the admittance is

$$Y_2 = \frac{v^2}{j\omega\ell} \; C_2 = \frac{v^2}{j\omega\ell} \begin{pmatrix} C_s & 0 \\ 0 & C_b \end{pmatrix} \tag{18}$$

Using these sign conventions

$$\xrightarrow{\quad I^{ref} \quad}$$

$$\qquad\qquad \xrightarrow{\quad I^{trans} \quad}$$

$$\xrightarrow{\quad I^{inc} \quad}$$

then the voltage and current must obey

$$V^i + V^r = V^t$$

$$I^i + I^r = I^t \tag{19}$$

The voltage and currents are related by

$$I^i = Y_1 V^i$$

$$I^r = -Y_1 V^r$$

$$I^t = Y_2 V^t \tag{20}$$

Solving these equations leads to

$$V^t = 2(Y_1 + Y_2)^{-1} Y_1 V^i$$

$$I^t = 2Y_2(Y_1 + Y_2)^{-1} Y_1 V^i \tag{21}$$

The useful V^i is

$$V^i = \begin{pmatrix} V_0 \\ V_0 \end{pmatrix} \tag{22}$$

representing a uniform excitation.

Substituting the appropriate values of the admittance matrices into the above equations

$$I^i = Y_1 V^i = v V_0 \begin{pmatrix} C_b/n \\ C_b \end{pmatrix} \tag{23}$$

and

$$(Y_1 + Y_2)^{-1} = \frac{1}{v\Delta} \begin{pmatrix} C_b + C_m + \dfrac{vC_b}{j\omega\ell} & C_m \\ C_m & \dfrac{C_b}{n} + C_m + \dfrac{vC_s}{j\omega\ell} \end{pmatrix} \tag{24}$$

where

$$\Delta = \frac{C_b^2}{n} + C_bC_m + \frac{C_bC_m}{n} + \frac{v}{j\omega\ell}\left\{C_sC_b + C_sC_m + \frac{C_b^2}{n} + C_bC_m\right\} + \frac{v^2C_sC_b}{(j\omega\ell)^2}$$

Finally, the transmitted current vector is

$$I^t = \frac{2vV_o}{\Delta}\left(\frac{v}{j\omega\ell}\right)\begin{pmatrix} \dfrac{C_sC_b^2}{n} + \dfrac{C_mC_bC_s}{n} + \dfrac{vC_b^2C_s}{j\omega\ell n} + C_bC_mC_s \\ \\ \dfrac{C_b^2C_m}{n} + \dfrac{C_b^3}{n} + C_mC_b^2 + \dfrac{vC_sC_b^2}{j\omega\ell} \end{pmatrix} \tag{25}$$

A couple of cases are of interest. If $C_m \to 0$, (i.e, there is no mutual coupling) and we keep the terms ordered

$$C_b >> C_s >> \frac{C_b}{n} \tag{26}$$

then the transmitted current is

$$I^t = 2vV_0\begin{pmatrix} C_b/n \\ C_b \end{pmatrix} = 2I^i \tag{27}$$

or the transmitted current is twice the incident current, which is pleasing.

In the ordering $C_m >> C_b >> C_s >> C_b/n$, the transmitted wave is (if $C_m >> vC_s/j\omega\ell$) for the tightly coupled bundle case

$$I^t = 2vV_o\begin{pmatrix} C_s \\ C_b \end{pmatrix} \tag{28}$$

The measurements from which we take our numerical example is

$$\frac{v}{\omega\ell}C_s > C_m > C_b > C_s > \frac{C_b}{n} \tag{29}$$

with

$$C_m > \frac{v}{\omega\ell}\left(\frac{C_b}{n}\right) \tag{30}$$

and the dominant terms turn out to be

$$I^t = 2vV_o\left(\frac{\dfrac{j\omega\ell\ C_m}{v}}{C_b}\right) \tag{31}$$

The fraction of current on the single wire is

$$\frac{\omega\ell\ C_m}{v\ C_b} \approx 0.17 \tag{32}$$

For a uniformly excited bundle each perimeter wire carries a current of about 0.05 the bundle current so even this approximation suggests a factor of 3.7 increase in the current on the wire.

Confirming this limiting estimate, a calculation of the current split using Equation 21 was performed for frequencies between 1 MHz and 100 MHz. The result is shown in Figure 2.

<u>Effect of Mutual Terms on Current Redistribution</u>

The expression for C_2 in Equation 13 ignores the contribution of the capacitance between the bundle and the wire pulled out. An estimate of the mutual capacitance may be found by computing the elastance matrix for the following geometry by image theory and inverting the matrix to find the capacitance matrix. The geometry used for the image theory calculation is shown in Figure 3. As shown, only six image charges are used in the calculation.

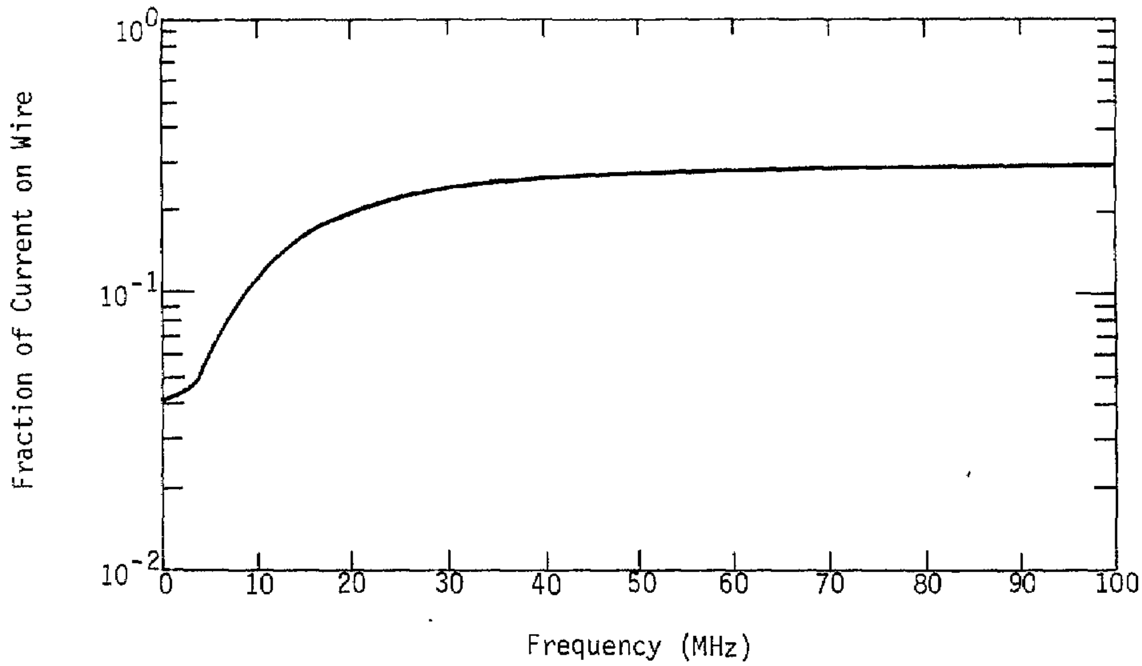

Figure 2. Current division as a function of frequency for the simple 4-tube case

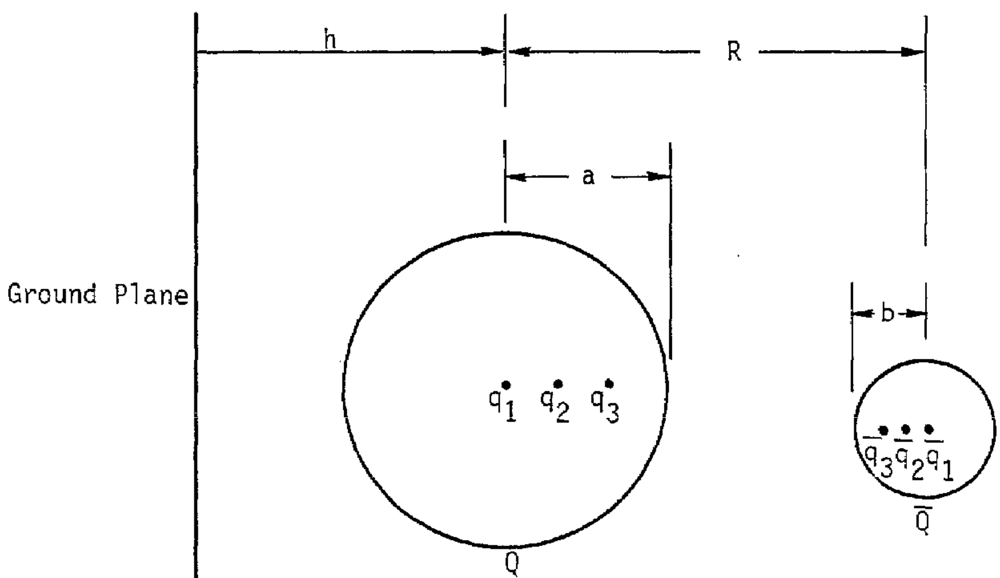

Figure 3. Image theory solution geometry.

If charges Q and $\overline{Q}$ are placed on the large and small cylinders above, then the resulting potentials can be used to calculate the elastance matrix elements. The charges q_1 and $\overline{q}_1$ are located at the center of each of the cylinders and are used to control the total charge on each of the cylinders. q_2 is the image of $\overline{q}_1$ in the surface of the bundle. q_3 is the image of $\overline{q}_2 + \overline{q}_3$ in the surface of the bundle. Similarly, $\overline{q}_2$ and $\overline{q}_3$ are chosen to maintain a uniform potential on the surface of the wire. Note, the image theory series is truncated by assuming $q_2 + q_3$ generates only one image. A complete solution is an infinite series of image charges, so this solution is best when the separation between bundle and wire is more than a wire radius. A numerical solution is required for better calculation of the capacitance matrix. Numerical treatment is required since the wire actually enters the bundle when it gets close. Further, the bundle is not a conductor but is a collection of capacitively coupled conductors.

The equations that the charges must obey are found by demanding that the image plus impressed charge contributions result in a uniform potential around the cylinder surface (Ref. 3). The equations are:

$$q_1 + q_2 + q_3 = Q \qquad \overline{q}_1 + \overline{q}_2 + \overline{q}_3 = \overline{Q}$$

$$q_2 = -\frac{a}{R}\overline{q}_1 \qquad \overline{q}_2 = -\frac{bq_1}{R}$$

$$q_3 = -\frac{a(\overline{q}_2 + \overline{q}_3)}{R - b^2/R} \qquad \overline{q}_3 = -\frac{b(q_2 + q_3)}{R - a^2/R} \tag{33}$$

If we let

$$\overline{K}_1 = ab\left(\frac{1}{R^2} - \frac{1}{R(R - a^2/R)}\right) \tag{34}$$

and

$$\bar{K}_2 = \left(1 + ab\left(\frac{1}{R(R - b^2/R)} - \frac{1}{(R - a^2/R)(R - b^2/R)}\right)\right) \tag{35}$$

then

$$\bar{q}_1 = -\frac{b/R\ Q - \bar{K}_2\bar{Q}}{R_1 - R_2} \tag{36}$$

and

$$K_1 = ab\left(\frac{1}{R^2} - \frac{1}{R(R - b^2/R)}\right) \tag{37}$$

$$K_2 = \left(1 + ab\left(\frac{1}{R(R - a^2/R)} - \frac{1}{(R - a^2/R)(R - b^2/R)}\right)\right) \tag{38}$$

$$q_1 = -\frac{a/R\ \bar{Q} - K_2 Q}{K_1 - K_2} \tag{39}$$

The other charges are found from

$$q_2 = -\frac{a}{R}\bar{q}_1 \tag{40}$$

$$q_2 = -\frac{\bar{b}\ q_1}{R} \tag{41}$$

$$\bar{q}_3 = \frac{\left(\dfrac{-bq_2}{R - a^2/R} + \dfrac{ab}{(R - a^2/R)(R - b^2/R)}\,\bar{q}_2\right)}{1 - \dfrac{ab}{(R^2 - a\ /R)(R^2 - b\ /R)}} \tag{42}$$

and

$$q_3 = -\frac{a}{R - b^2/R}(\overline{q}_2 + \overline{q}_3) \tag{43}$$

Finally, the locations of the charges are also determined by the boundary conditions (Ref. 3). For a coordinate system centered at the center of the bundle and the x-axis through the center of the wire, the locations are:

$$x_1 = 0$$

$$x_2 = a^2/R$$

$$x_3 = a^2/(R - b^2/R)$$

$$x_4 = R - b/(R - a^2/R)$$

$$x_5 = R - b^2/R$$

$$x_6 = R \tag{44}$$

where we have reindexed the charges to $q_{1,2,3} = \overline{q}_{1,2,3}$ and $q_4 = \overline{q}_3$, $q_5 = \overline{q}_2$, and $q_6 = \overline{q}_1$.

The elastance matrix elements are then found by adding up the contributions of the various line charges to the potential at the surface of the bundle and wire.

The distances from the bundle surface for each of the charges are

$$d_1 = a$$

$$d_2 = a - a^2/R$$

$$d_3 = a - a^2/(R - b^2/R)$$

$$d_4 = R - a - b^2/(R - a^2/R)$$

$$d_5 = R - a - b^2/R$$

$$d_6 = R - a \tag{45}$$

The distances from the wire surface

$$e_1 = R - b$$

$$e_2 = R - b - a^2/R$$

$$e_3 = R - b - a^2/(R - b^2/R)$$

$$e_4 = b - b^2/(R - a^2/R)$$

$$e_5 = b - b^2/R$$

$$e_6 = b \tag{46}$$

The elastance matrix elements are then:

$$Q = Q \qquad \overline{Q} = 1$$

$$P_{11} = - \frac{1}{2\pi\varepsilon_0} \left[\sum_{i=1}^{6} q_i \ln d_i - \ln[2(h+R)] \right]$$

$$P_{21} = - \frac{1}{2\pi\varepsilon_0} \left[\sum_{i=1}^{6} q_i \ln e_i - \ln[2h+R] \right]$$

$$Q = 1 \qquad \overline{Q} = 0$$

$$P_{12} = -\frac{1}{2\pi\varepsilon_0}\left[\sum_{i=1}^{6} q_i \ln d_i - \ln[2h+R]\right]$$

$$P_{22} = -\frac{1}{2\pi\varepsilon_0}\left[\sum_{i=1}^{6} q_i \ln e_i - \ln[2h]\right] \tag{47}$$

where the additional logarithmic terms account for reflection in the ground plane.

The elastance matrix above was used in the calculation of C_2 above as a function of separation distance. Otherwise the problem was the same as that described in the previous section. The current division as a function of separation distance of the bundle and wire (surface to surface distance) is shown in Figure 4. Near the bundle the pulled out wire arries about 5 percent of the current, as it should. Farther from the bundle, for 100 MHz in particular, the fraction of current transferred to the wire becomes larger, reaching 21 percent of the total current. Both self and mutual capacitances are calculated with the image theory technique.

To confirm the conclusion of the effect of pulling out the wire two more geometries were treated using the same junction theory technique. The first of these geometries addresses the assumption, used above, that the bundle is a solid conductor and is shown in Figure 5.

This problem is now a junction of three conductors and the capacitance matrix corresponding to those in Equation 12 is

$$C_1 = \begin{pmatrix} C_b/n + C_{ms} & -C_{ms} & 0 \\ -C_{ms} & 2C_b/n + C_{mb} + C_{ms} & -C_{mb} \\ 0 & -C_{mb} & (n-3)/n\ C_b + C_{mb} \end{pmatrix} \tag{48}$$

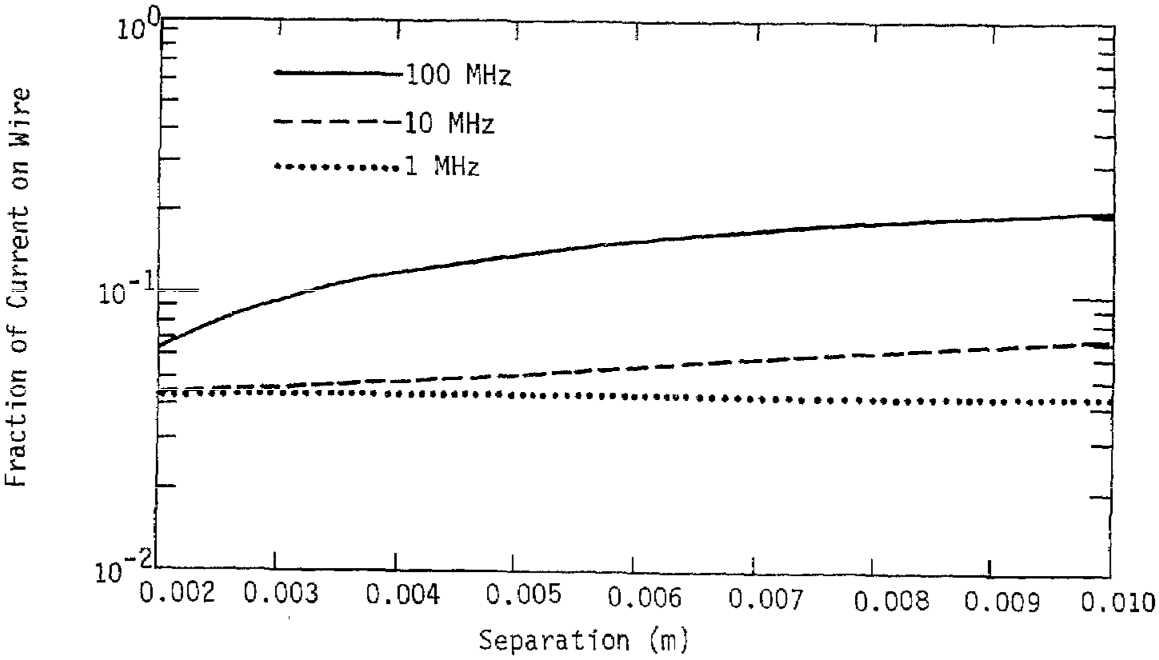

Figure 4. Current division for various frequencies as a function of the separation of the surfaces of the bundle and wire.

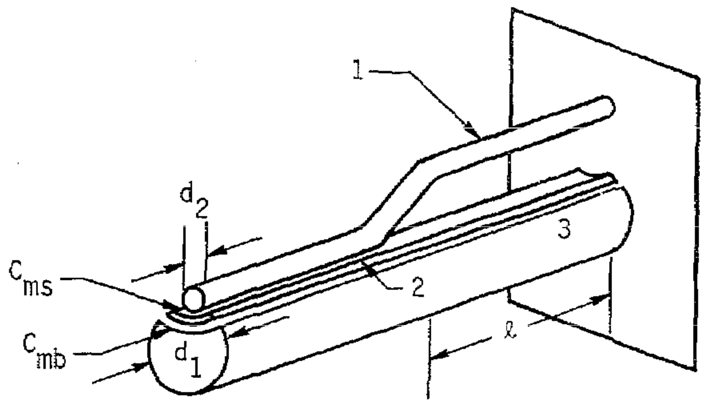

Figure 5. Modification of geometry in Figure 1 showing additional 3-wire section which allows first order treatment of the capacitive (rather than conductive) coupling between the wires in the bundle.

where C_1 represents the capacitance matrix before the junction, and as shown in Figure 4, C_{ms} and C_{mb} are the mutual capacitances between the intermediate layer and the wire and the bundle and intermediate layer respectively. After the junction the capacitance between the wire and intermediate layer is assumed negligible but the mutual capacitance between the layer and the bundle is unchanged. The resulting capacitance matrix after the junction is:

$$C_2 = \begin{pmatrix} C_s & 0 & 0 \\ 0 & 2C_b/n + C_{mb} & -C_{mb} \\ 0 & -C_{mb} & (n-3)/n\ C_b + C_{mb} \end{pmatrix} \qquad (49)$$

Figure 6 shows the variation with frequency of the fraction of the total current that is shifted from the wire. At frequencies associated with the

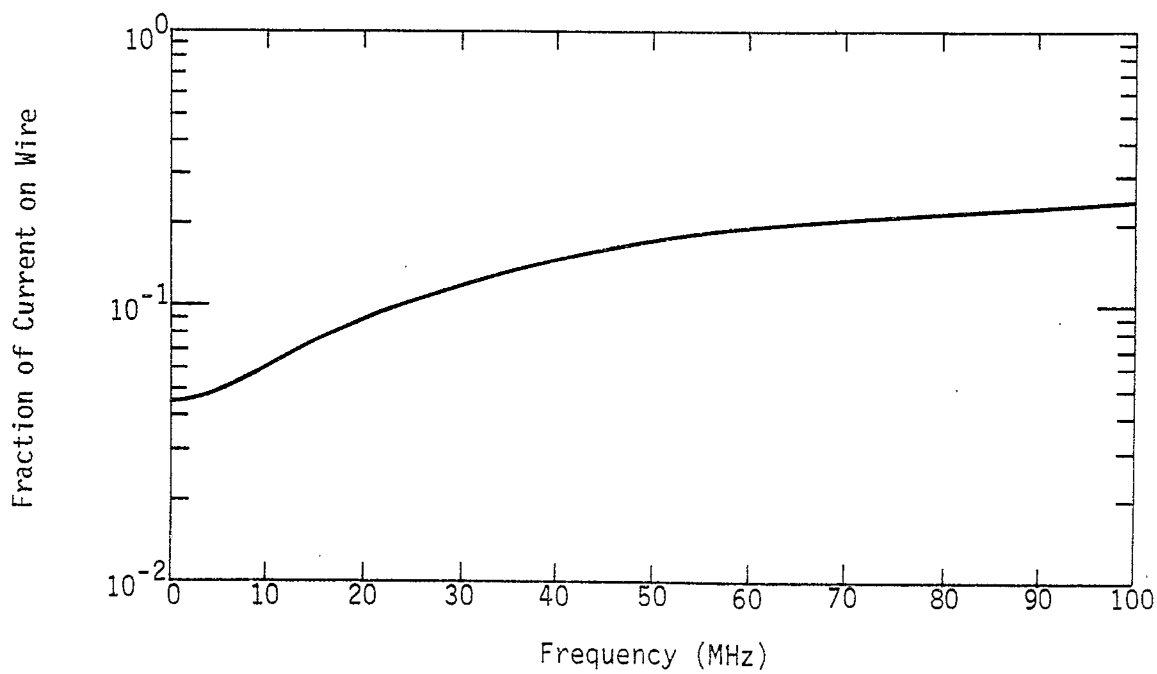

Figure 6. Fraction of bundle current flowing on single wire pulled out from the bundle for the case of the wire and nearest neighbors treated as tubes separate from the main bundle.

rise time of the electric drive pulses used here a substantial fraction of the total current is shifted to the wire with the sensor. The effect is similar for this geometry to that for the 2 x 2 (or 4 tube) problem.

A final geometry is shown in Figure 7 which has two sensor wires pulled out in the same way from a bundle of 45 wires (22 surface wires). In this case the initial capacitance matrices where the bundle wire mutual capacitance is C_m.

$$C_1 = \begin{pmatrix} C_b/n + C_m & 0 & -C_m \\ 0 & C_b/n + C_m & -C_m \\ -C_m & -C_m & C_b + 2C_m \end{pmatrix} \tag{50}$$

and

$$C_2 = \begin{pmatrix} C_s & 0 & 0 \\ 0 & C_s & 0 \\ 0 & 0 & C_b \end{pmatrix} \tag{51}$$

The fraction of the total current on each of the two wires is shown in Figure 8.

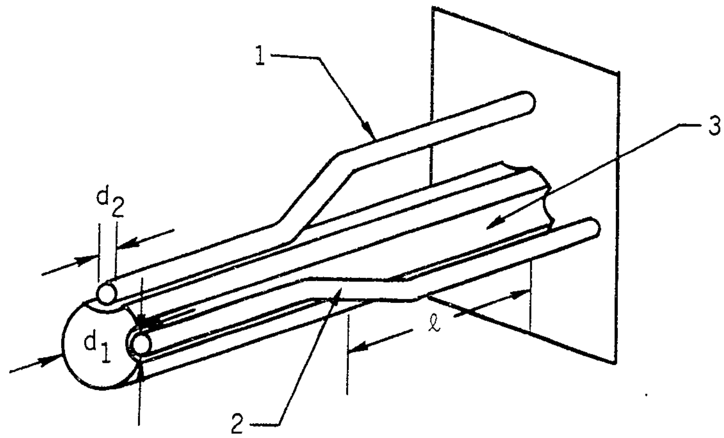

Figure 7. Geometry showing two wires pulled out of the bundle.

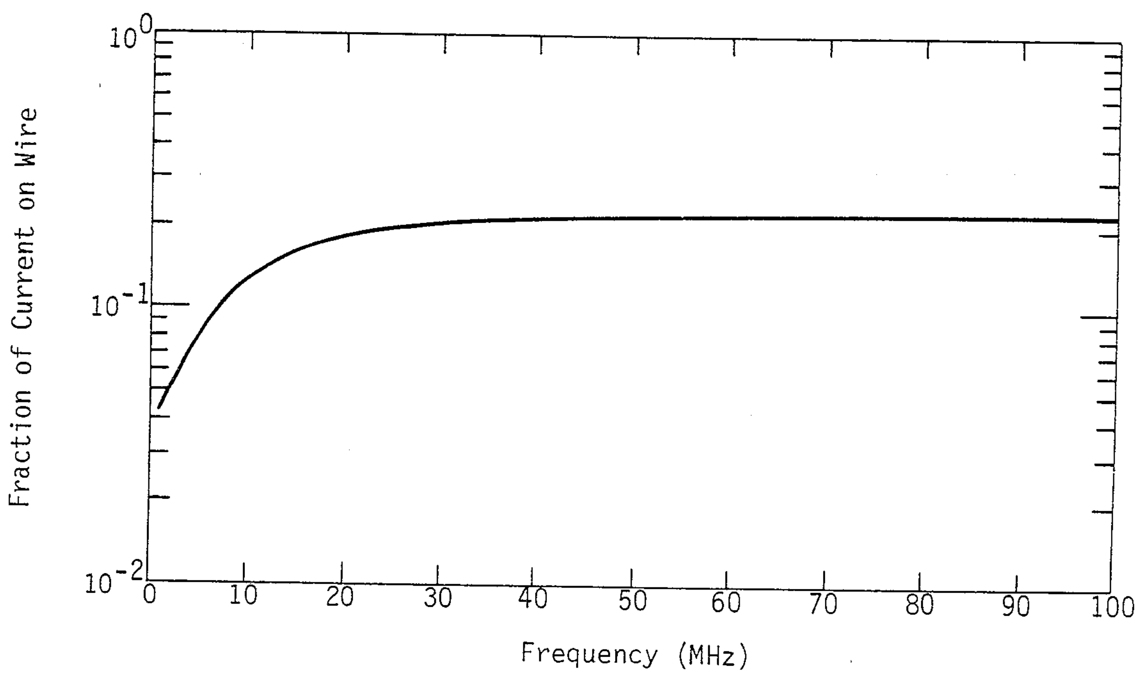

Figure 8. Fraction of bundle current flowing on each of two wires pulled out from the main bundle.

IV. CONCLUSIONS

It is evident from the previous calculations that the change in geometry caused by installing a sensor on a wire in a bundle perturbs the distribution of current in the bundle, at least for the first peak. Our calculations do not consider reflections. Certainly there are other reasons for the distribution of peak currents being a larger ratio of the bundle currents than one might expect intuitively. The particular phasing of the components of the bundle is one of those reasons. However, these calculations cast doubt on many of the minimum values for current peaks observed on individual wires in bundles.

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APPENDIX A
POWER FRACTION SPLIT OFF ONTO WIRE

To confirm that the splitting of the current from the bundle to the single wire occurs for the excitation used in the sample measurements a power spectrum weighted average was carried out. Consider the current on the single wire as the product of a transfer function $T(\omega)$ and a bulk current $I(\omega)$. Then a power spectrum weighted average of the fraction F of the bulk current on the single wire is given by:

$$F^2 = \frac{\frac{R}{2\pi} \int_{-\infty}^{\infty} T^*(\omega)\ T(\omega)\ I^*(\omega)\ I(\omega)\ d\omega}{\frac{R}{2\pi} \int_{-\infty}^{\infty} I^*(\omega)\ I(\omega)\ d\omega} \tag{A1}$$

where R is a characteristic, constant resistance.

For the example cable drive test, a cable bundle was excited by driving a meter long and a few centimeter wide plate parallel to the satellite ground plane with a voltage across the plate and a termination impedance at the center of the plate. The plate's long dimension ran along the cable.

For this calculation the current was assumed proportional to dV/dt, where V is the voltage applied to the parallel plate for the cable drive. The waveform used for the applied electric field was developed from the following curve fit:

$$E_\rho = 2.25 \times 10^5 \left(\frac{e^{-\alpha t}}{1 + Ge^{-\beta t}} - \frac{e^{-\gamma t}}{1 + Ge^{-\delta t}} \right) \tag{A2}$$

where

$$\alpha = 2 \times 10^9$$

$$\beta = \gamma = 2 \times 10^8$$

$$\delta = 2 \times 10^9$$

$$G = 1/(\beta/\alpha - 1)$$

The resulting waveform is shown in Figure A1. This waveform was chosen to eliminate the usual difficulty with the time derivates at $t = 0$ with the double exponential. The result of this averaging process was that F = 16 percent of the bulk current was shifted to the wire pulled out of the bundle, a factor of 3 more than one would expect for the uniform excitation case.

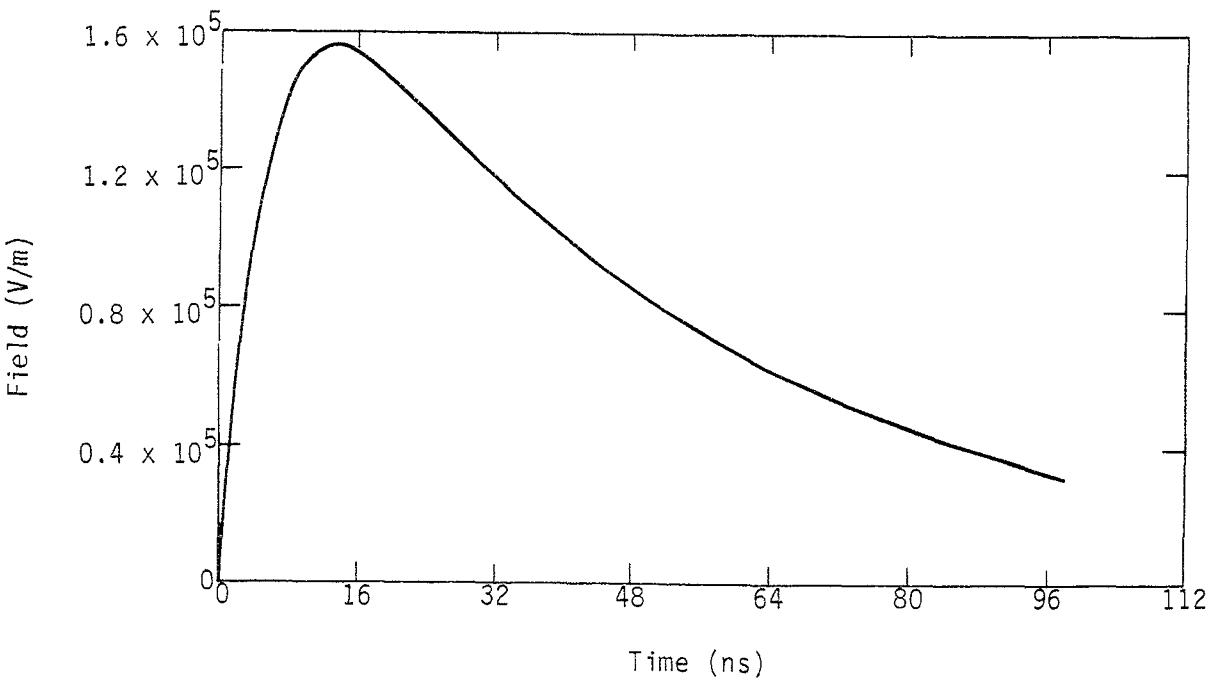

Figure A1. Waveform used for the weighted average of the current split off onto the small wire.