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Unified View of EMP Interaction, Threat Extrapolation, and Alternate Simulation/ Transfer Coefficients

by

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Abstract

This report presents two complementary theoretical developments leading to the conclusion that for a large class of important systems, transfer coefficients represents the unique and total relationship between external interaction quantities and the voltage and current drives of subsystems. If external interaction is given a role in any coupling effort, e.g., internal cable or pin prediction, threat extrapolation, or alternate simulation, then transfer coefficients must play a role. It is also argued that transfer coefficients must play a role in assigning or verifying certain hardness specifications. For internal prediction, transfer coefficients must be explicitly determined. For certain threat extrapolation that utilizes external interaction quantities as well as alternate simulation, the existence of transfer coefficients have always been explicitly or implicitly assumed to be provided by nature. For these activities it is noted that the nature provided transfer coefficients depend on the physical environment external to the system and this dependence bears on the accuracy of the procedures. Data is presented quantifying the extent of this dependence for a controlled laboratory situation. Despite these laboratory results, it is recommended that studies be initiated to minimize this dependence for real system applications. The potential benefits of transfer coefficients on system testing are discussed.

It is argued that present computational technology cannot be relied upon to calculate transfer coefficients for a complex coupling configuration. As compensation, an experimental procedure for measuring transfer coefficients is presented. This procedure has the desirable features that it is self-testing and that the excitation sources have considerably fewer constraints placed on them compared to the constraints on full scale simulators. Data is presented showing that these transfer coefficients can be accurately measured.

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I. INTRODUCTION AND SUMMARY

This report introduces the concept of transfer coefficients, describes their role in determining a system's response to an impinging electromagnetic field, and presents a procedure for measuring them. An important aspect of this work is to clarify how transfer coefficients have been implicitly assumed in existing electromagnetic pulse coupling analyses. Despite the fact that transfer coefficients have been implicitly assumed in past efforts, this work suggests that they should receive a different treatment in future work. It is noted that this work, as well as the large amount of related work that implicitly assumed the existence of transfer coefficients, are linear theories.

The basic argument that transfer coefficients have been implicitly assumed in past EMP coupling analyses is summarized as follows. Transfer coefficients represent the unique and total relationship between the interaction current and charge densities induced on the exterior of a system, and voltage and currents driving subsystems that are contained within the system. A more detailed description of the systems to which this concept pertains are discussed in Section II. For this class of systems, if an electromagnetic coupling situation is treated in a manner where one is interested in subsystem excitation and a role is assigned to external interaction, then one must deal with transfer coefficients.

This statement clearly bears directly on the ability to make internal coupling predictions, and it is closely related to threat extrapolation and alternate simulation. The relationship between transfer coefficients to threat extrapolation and alternate simulation will subsequently be further discussed. Transfer coefficients must also play a role in assigning or verifying shielding hardness specifications of a system if the shielding specification, in any manner, references any external coupling quantity.

Furthermore, as part of the transfer coefficient concept, it follows that external coupling quantities have no inherent meaning unless they are determined on seals placed over the POE's of the system. If external coupling quantities are determined elsewhere on the surface of the system, for convenience sake, then their significance to any coupling consideration must be addressed by relating that quantity to the external coupling quantity that would be excited on a POE seal. The arbitrary nature of where to choose the points to determine the external coupling quantities is resolved by the transfer coefficient concept.

The concepts underlying transfer coefficients have previously been recognized to a large extent by workers in the EMP field and in parallel communities (e.g., EMC). The relationships between this work and other EMP efforts will be presented in the text as concepts are developed. The relationship between this work and the work of Schuman (Refs. 1, 2) in support of parallel communities will be described in this introduction.

The major new contributions of this study are as follows:

- A procedure for measuring transfer coefficients is presented and it is argued that there is no analytic or computational alternative to this measurement procedure for all but the simplest of systems. This procedure allows transfer coefficients to be determined for systems with electromagnetic sources that are required to satisfy much less stringent conditions than existing simulator requirements. The procedure has a self-testing aspect that allows the accuracy of the transfer coefficients to be determined.

- It is theoretically shown that transfer coefficients depend on the physical environment external to the system of interest. Experimental data is presented to quantify the extent of this dependence. This aspect of the theory has not appeared in the literature with which the author is familiar. This fact is relevant to present EMP threat extrapolation procedures as well as to alternate simulation.
- The transfer coefficient concept and procedure is not restricted to small isolated points of entry.

Regarding the first contribution, the measurement procedure, it is now argued that present computational technology cannot be relied upon to calculate transfer coefficients for even a moderately complex coupling configuration so that they must be measured. Consider two different types of computations of transfer coefficients. One type of computation would rely on an analytic formulation. This analytic formulation is presented in detail in the text since it serves as part of the basis for defining transfer coefficients. This analytic formulation utilizes the fact that certain Green's dyadics exist in principle but can never be theoretically explicitly obtained for calculation purposes for any three dimensional shape other than the sphere. Despite the fact that explicit representations of Green's dyadics limit computations, this fact does not, in any manner, limit the utility of the procedure for measuring transfer coefficients.

The other type of transfer coefficient computation is the direct brute force numerical solution of Maxwell's equations, e.g., finite difference computation. This direct numerical solution applied to a system of some complexity will yield a result having a certain confidence level ascribed to it. By the same token, measurements performed on a system of some complexity yield results of less than total confidence. It is the author's view that the confidence level resulting from the measurement of transfer coefficients is much greater than the confidence level resulting from the direct numerical computation. All of the concepts of underlying the assignment of a quantitative or qualitative measure to confidence are based on a number of experiences related to the issue being assessed the confidence level. The author has seen direct numerical computation yield qualitatively poor results for situations less complex than those encountered to perform a transfer coefficient computation on even a moderately complex structure. Along these lines, data for an extremely simple coupling configuration is presented along with the detailed presentation of a prominent coupling analysis. It is shown that the analysis is not in good agreement with the data. Along these lines, other related disciplines measure electromagnetic responses rather than compute, for situations of less complexity than the complexity encountered for a transfer coefficient calculation, e.g., antenna pattern prediction.

A more direct argument that the confidence level in direct numerical computation suffers in comparison to measurement of transfer coefficients is based on the amount of knowledge one must have to measure as compared to compute. At this stage of our coupling technology, we cannot justify the elimination of any of the internal complexity of a real system for the numerical prediction of transfer coefficients. It is not clear that we can even physically survey a complex system in sufficient detail to provide the required information for the numerical computation. In contrast, the procedure for measuring transfer coefficients as presented in this report, is totally insensitive to the detailed knowledge of the system's internal complexity. In this procedure, one need only identify the location where one wishes to measure a quantity of interest, e.g., a voltage or current, and nature accounts for the internal complexity. The self-testing aspect of the procedure indicates the adequacy of the measurements.

We now return to the second contribution of this report, the external physical environment dependence of the transfer coefficients and, in particular, the effect of this dependence on threat extrapolation as well as on alternate simulation (direct drive). A prominent method of threat extrapolation (Ref. 3) utilizes external coupling quantities measured on an aircraft during full scale simulator exposure as well as external coupling quantities measured on a scale model of that aircraft when it is illuminated in an anachoic chamber. It is known that this procedure has some problems which effect its accuracy, e.g., the external coupling quantities are not measured on seals placed over the POE's of the aircraft and not all of the POE's can be located. An experiment conducted at the University of Michigan and described in the text, was designed to eliminate all known deficiencies with the extrapolation procedure with the exception of the external environment dependence of transfer coefficients, e.g., there was only one POE, it was sealed for the external coupling measurements, and only one scalar external coupling quantity played a role for all aspects of the experiment. For this case, the variation in the transfer coefficient with the external environment is a direct measure of the error that the external environment dependence can introduce to the extrapolation procedure. This experiment was designed to emphasize the existence of the external environment dependence and these results should not be taken to mean that external coupling quantities should not play a role in threat extrapolation. A way that they could be used in such a procedure is as follows. Experimentally test whether existing or potential ground based external environments used in aircraft testing have an acceptably small effect on transfer coefficients. The procedure described in the text should be employed to determine the transfer coefficients in these environments. This procedure requires both external and internal measurements on a test system in the appropriate ground environment as well as in the free space in-flight environment. The in-flight requirement forces us to consider laboratory experiments which should then treat a sufficient number of generic test objects in order to come to conclusion about real aircraft. If such a study yields positive results, i.e., transfer coefficients measured on the ground are acceptably close to transfer coefficients corresponding to those for the in-flight environment, then we could have an extrapolation procedure which utilizes external coupling quantities. This procedure might be one suggested by Baum or some extension of it. Such a procedure would allow the internal system response to be determined for EMP threats having polarization and incident angles variations, by only determining the external interaction quantities induced on the system for these angle variations. This is the case because it would be explicitly or implicitly based on transfer coefficients and transfer coefficients do not depend on these incident angles. The author is not aware of any other "transfer function" characterization of a system that is independent of these incident angles.

The impact of the external environment dependence of transfer coefficients on alternate simulation is similar to the impact on threat extrapolation. If a system can be direct driven in the same environment, it will actually be utilized, e.g., a shielded building, then this external dependence plays no role. For such situations, the transfer coefficient concept allows the alternate simulation goals to totally focus on the exterior of the facility. In particular, the local direct drive sources need only excite the same external interaction quantities at sealed POE's as would an EMP, in order to excite the interior of the unsealed system in the same manner as would the EMP when no seals are in place. When the system cannot conveniently be direct driven in the same environment in which it will operate, e.g., an aircraft in-flight, then this environment must be shown to have an acceptably small effect on transfer coefficients.

The data presented in this report was taken from two sources. The first was from an experimental effort intentionally coupled to this work and managed by Val Liepa at the University of Michigan. The second was from the work of Schuman (Refs. 1, 2). Liepa's data was used for two purposes. One was to demonstrate the limitations of prominent coupling theory, to calculate transfer coefficients for a relatively simple coupling situation. The other Michigan data displays the dependence of transfer coefficients on the exterior environment. It should be noted that Val Liepa has some reservations about the quality of this data, especially above 1 GHz (a frequency that would be of interest for EMP when scaled to real system dimensions). His data was also used to determine the transfer coefficients in a self-testing manner consistent with the procedure described in this work, however, the data processing was not complete at the time of this writing. Because of the significance of Liepa's data, the author has recommended that the experiment be repeated.

Schuman's data (Refs. 1, 2) is reproduced in this report with a minor change in notation. His data demonstrates the accuracy with which it is possible to measure transfer coefficients. It should be noted that the case treated by Schuman was less complex than the case by Liepa in that no phase measurements were required.

Finally, it is appropriate to describe the relationship between Schuman's work and that of the author. The transfer coefficient concept was first presented by the author in July 1978 at the Lightning Analysis for Aircraft Design Workshop held at the Naval Ocean Systems Center. Shortly thereafter, a joint effort sponsored by the Defense Nuclear Agency was begun with Val Liepa to determine the accuracy with which transfer coefficients could be measured. In January 1979, before any definitive results were obtained by Val Liepa, Schuman published some work (Ref. 1) that interpreted an experiment in a manner equivalent to transfer coefficients. The author then established an on-going communication with Schuman in which the general experimental procedure described in this work was sent to him in a draft report. The procedure in this draft report was then described in a recent invited paper by Schuman (Ref. 2).

II. OPERATOR APPROACH TO TRANSFER COEFFICIENTS

Transfer coefficients are relevant to those systems which are predominantly metallic but have breaks in their skin which allow EMP penetration. The basis of this approach is a set of equations that recognizes the essential features of classical aperture coupling analysis and also has relevance to complex systems. Since this approach is based on aperture coupling equations, one might be concerned with its relevance to other types of penetrators, i.e., deliberate antennas. Such penetrators must have associated apertures or else no energy could penetrate the sealed skin of the system corresponding to that penetrator. The equation that forms the basis for this unified approach is as follows:

$$L\underline{J}_m(\underline{r}') = \underline{J}_{E.I.}(\underline{r}) \qquad (1)$$

where the meaning and significance of each term requires considerable attention. This equation was derived in Reference 4 and is also presented in Appendix A. The theoretical implications of this equation have not previously been utilized as fully as possible; however, when specialized to very simple geometries, this equation has formed the basis of the classical aperture coupling results (Refs. 5,6). The viewpoint expressed here is that this equation as applied to complex systems should not be viewed as requiring solution but instead should be used to define self-testing experimental procedures.

First, we must explain that Eq. (1) describes the relationship between electrical quantities on two different physical systems. One system is the actual system of interest, and the other system is the original system modified by metallic shorting surfaces covering all apertures (including those associated with antennas). The notation $\underline{J}_m$ was chosen to denote "magnetic current," but it is simply $\hat{n}(\underline{r}') \times \underline{E}_t(\underline{r}')$ where $\underline{r}'$ varies over all of the mathematical surfaces corresponding to the open apertures in the original system, $\hat{n}(\underline{r}')$ is the outward normal at $\underline{r}'$, and $\underline{E}_t(\underline{r}')$ is the tangential component of the electric field induced in the open aperture. The quantity $\underline{J}_{E.I.}(\underline{r})$ is the "external interaction" current density induced on the shorted system with $\underline{r}$ ranging only over the shorting surfaces. It is important to note that even though $\underline{r}$ and $\underline{r}'$ refer to different physical systems, they mathematically refer to the same set of points. This distinction allows a discussion of the mathematical nature of Eq. (1) that is not confused by the dual system aspect of the problem. It remains to discuss the meaning of L in Eq. (1) in order to proceed. This operator (as can be seen in Appendix A) depends on the Green's dyadic corresponding to the environment external to the shorted system, $\underline{\underline{G}}_E$, as well as the Green's dyadic $\underline{\underline{G}}_I$ internal to the shorted system. For a very limited number of geometries, we have explicit representations of $\underline{\underline{G}}_E$ and $\underline{\underline{G}}_I$ (Refs. 7,8); however, the recognition of this fact does not limit our ability to define the experimental program and instead suggests the need for such a program. The recognition that L depends on the external environment, through $\underline{\underline{G}}_E$, implies that simulator/test object interaction must be viewed differently than in the past. Further discussion of this point will be deferred until the development has reached the point where this topic can be viewed as a special case of the general discussion.

Having explained the meaning of Eq. (1), we must now introduce several additional but more well-known concepts. First, the fact that a $\underline{J}_m(\underline{r}')$ is sufficient to determine a variety of internal electrical quantities Q_β (i.e., β can correspond to a voltage, a current, or a field component) is expressed as

$$Q_\beta = L_\beta \underline{J}_m(\underline{r}') \tag{2}$$

and L_β depends on the $\underline{\underline{G}}_I$ for the shorted system no matter what the quantity is that is indexed by β. Next we introduce a step, the legitimacy of which is best studied from a field equivalence point of view. Specifically, it is assumed that the L appearing in Eq. (1) has a unique inverse, L^{-1}, so that from Eq. (1) we can obtain

$$\underline{J}_m = L^{-1}\underline{J}_{E.I.} \tag{3}$$

The field equivalence argument will be presented later. Combining Eqs. (2) and (3) we obtain

$$Q_\beta^\alpha = T_\beta^\alpha \underline{J}_{E.I.} \tag{4}$$

where

$$T_\beta^\alpha = L_\beta L^{-1} \tag{5}$$

and the superscript is explicitly introduced to show that T_β^α depends on the external environment as a result of the dependence of L, and consequently L^{-1}, on $\underline{\underline{G}}_E$. It is possible to present the desired unified approach on Eq. (4); however, that equation will be modified to conform to the prevalent notion that both the external interaction current density $\underline{J}_{E.I.}$ and the external interaction charge density $\sigma_{E.I.}$ are required for the ultimate determination of the internal quantities Q_β^α. For nonzero frequency, it follows from the fact that $\nabla_s \cdot \underline{J}_{E.I.} = i\omega\sigma_{E.I.}$, the determination of $\underline{J}_{E.I.}$

automatically determines $\sigma_{E.I.}$, so the requirement that $\sigma_{E.I.}$ be separately determined must be superfluous. When the experimental procedure for obtaining the transfer coefficients is presented, the relative merits of not introducing the charge density as a separate entity will be discussed. Viewing the charge density as a separate entity leads to

$$Q^{\alpha}_{\beta} = T^{\alpha}_{J\beta}\underline{J}_{E.I.} + T^{\alpha}_{\sigma\beta}\sigma_{E.I.} \qquad (6)$$

as the basic equation.

The theoretical aspects of Eq. (6) will now be highlighted with respect to EMP interaction, simulator/test object interaction, and alternate simulation.

EMP Interaction

- A very limited number of the interaction analyses based on topological factoring are capable of being self-quantifying in view of the limited number of explicit representations of $\underline{\underline{G}}_E$ and $\underline{\underline{G}}_I$.

Simulator/Test Object Interaction

- The assessment and compensation for simulator/test object interaction based only on determining $\underline{J}_{E.I.}$ and $\sigma_{E.I.}$ for the threat and simulation environments are theoretically unjustified because the transfer operators $T^{\alpha}_{J\beta}$ and $T^{\alpha}_{\sigma\beta}$ also depend on these two environments. Assessment and compensation based on $\underline{J}_{E.I.}$ and $\sigma_{E.I.}$ could be justified if the $T^{\alpha}_{J\beta}$ and $T^{\alpha}_{\sigma\beta}$ could be demonstrated to be approximately independent of α (i.e., the external environments).

Alternate Simulation

- The distribution of proximate sources used for alternate simulation need only have the limited objective of duplicating the external interaction quantities caused by a threat source provided the environment external to the system is the same for the local sources as it is in the threat situation. If these conditions are satisfied, all quantities excited within the system when no shorting surfaces are present will be the same for the threat and the local sources.

- An assessment of the failure to duplicate the external interaction quantities by the two different sources requires the determination of the $T^{\alpha}_{J\beta}$ and $T^{\alpha}_{\sigma\beta}$ for the system and its external environment.

III. EXPERIMENTAL PROCEDURE FOR OBTAINING TRANSFER COEFFICIENTS

EMP interaction information has been and is currently being obtained by both system level tests and laboratory experiments. The determination of an appropriate balance between these two types of experiments is a subject of current interest. For the purpose of obtaining generic transfer coefficient information, laboratory experiments eliminate the system level test problem of POE tracing. This problem is eliminated because in a laboratory experiment, all physical structure complexity is introduced in a controlled and known manner. For either real systems or laboratory models having more than one aperture, a problem of POE definition is encountered and its solution is addressed as part of the described experimental procedure.

We will now give Eq. (6) the proper interpretation to define an experimental program. An essential part of the transition of Eq. (6) to the set of equations directly related to the experiment is the conversion of the transfer operators $T^{\alpha}_{J\beta}$ and $T^{\alpha}_{\sigma\beta}$ to transfer coefficients. This is accomplished by noting that Eq. (6) requires as inputs the external interaction quantities at the shorting surfaces. This implies that the external coupling quantities are only required to be known on portions of closed surfaces. The abrupt discontinuities associated with apertures are of no consequence in causing a rapid variation on the true required input external interaction quantities. This in turn implies that the external interaction quantities will not vary in an abrupt manner, and we can expect that a limited number of sample values at the shorting surfaces will provide an adequate description of the external interaction quantities. We now introduce a local coordinate system at each shorting surface, having $\hat{s}$ and $\hat{t}$ as the unit tangent vectors. These coordinate systems are required in order to be able to deal with scalar quantities. We are now in a position to relate Eq. (6) to a discrete set of equations. First, the following definitions are introduced:

$J_{si,j}$ = the $\hat{s}$ component of $\underline{J}_{E.I.}$ at the $j^{\underline{th}}$ location on the $i^{\underline{th}}$ shorting surface.

$J_{ti,j}$ = the $\hat{t}$ component of $\underline{J}_{E.I.}$ at the $j^{\underline{th}}$ location on the $i^{\underline{th}}$ shorting surface.

$\sigma_{i,j}$ = value of $\sigma_{E.I.}$ at the $j^{\underline{th}}$ location on the $i^{\underline{th}}$ shorting surface.

$N_p(i)$ = number of sample points on the i^{th} shorting surface (depends on geometrical and electrical size of the penetrator).

$Q_\beta^{\alpha,n}$ = an internal scalar electrical quantity at the $n^{\underline{th}}$ internal location (β, as before, denotes type of quantity, i.e., voltage, current, field component; and α denotes external environment of the system, i.e., free space or simulator).

N_T = the number of penetrators.

$T_\beta^{\alpha,n}(s,t,\sigma)i,j$ = a transfer coefficient.

Utilizing these quantities, an approximation to Eq. (6) becomes

$$Q_\beta^{\alpha,n} \approx \sum_{i=1}^{N_t} \sum_{j=1}^{N_p(i)} \left(T_{\beta si,j}^{\alpha,n} J_{si,j} + T_{\beta ti,j}^{\alpha,n} J_{ti,j} + T_{\beta\sigma i,j}^{\alpha,n} \sigma_{i,j} \right) \tag{7}$$

In describing the experimental procedure to obtain the various quantities in Eq. (7), it is essential to recognize that this equation relates quantities on two different physical systems.

The quantities that we are truly interested in causing to be excited by the simulator in the same manner that they would be excited by a threat are the $Q_{\beta}^{\alpha,n}$ (i.e., we would like $Q_{\beta}^{S,n} = Q_{\beta}^{T,n}$) where the external environment to the system index α is labeled S for the simulator environment and T for the threat environment. Past simulator/test object interaction studied implicitly assumed it was sufficient to cause the same $\underline{J}_{E.I.}$ and $\sigma_{E.I.}$ to be excited by the simulator and the threat in order to assess and compensate for simulator/test object interaction. This information would be sufficient only if

$$T^{S,n}_{\beta(s,t,\sigma)i,j} \approx T^{T,n}_{\beta(s,t,\sigma)i,j} \tag{8}$$

and we will now discuss how to test Eq. (8) experimentally.

In order to investigate Eq. (8), we will use a test system that has a limited number of penetrators, N_T, and we will have a limited number of interior sample points corresponding to the range of n. Two different external environments to the system will be used in the sequence of experiments. One external environment corresponding to a particular common threat will be free space, and the experiment will take place either in an anechoic chamber or over a large metallic symmetry plane. For the purposes of this discussion, the second external environment corresponding to a simulator environment is a scale model bounded wave simulator containing the test object or a model of a radiating simulator in the proximity of the test object which is resting on a lossy half space. The transfer coefficients are determined from appropriate measured (or calculated) sets of $Q_\beta^{\alpha,n}$, $J_{si,j}$, $J_{ti,j}$, and $\sigma_{i,j}$ where α is labeled F for the free-field environment and S for either of the simulator environments. Later we will describe a simple experiment that has been performed to assess the dependence of the external environment on the transfer coefficients.

A procedure for obtaining the $T_{\beta(s,t,\sigma)i,j}^{\alpha,n}$ will now be described. At this point we again emphasize that all $J_{E.I.}$ and $\sigma_{E.I.}$ are determined (measured or calculated) with all penetrators sealed; however, the sealed system is in the appropriate α environment. Rigid sources are introduced to excite the sealed system and they are not necessarily the sources associated with the model simulator. For the purpose of illustration consider that we first choose to characterize the fifth aperture alone and begin by examining whether $N_p(5) = 1$ is sufficient. For this case we introduce three rigid sources for each α environment all at the same frequency. In practice the three sources might be the same source at three different orientations or locations and the sets of three sources might be different for each α environment. For each source in each α environment we measure (determine) $Q_\beta^{\alpha,n,(k)}$, $J_{s5,j}^{(k)}$, $J_{t5,j}^{(k)}$, and $\sigma_{5,j}^{(k)}$ and form the equations

$$Q_\beta^{\alpha,n,(k)} = \sum_{j=1}^{N_p(5)} \left(T_{\beta s5,j}^{\alpha,n} J_{s5,j}^{(k)} + T_{\beta t5,j}^{\alpha,n} J_{t5,j}^{(k)} + T_{\beta\sigma 5,j} \sigma_{5,j}^{(k)} \right) \tag{9}$$

$$k = 1,\cdots,3N_p(5)$$

The quantities on the left-hand side of Eq. (9) are determined with all but the fifth seal in place, and the quantities on the right-hand side of Eq. (9) are on the fifth seal with all seals in place. The range of k, i.e., $3N_p(5)$, is the number of sources that must be used in each α environment. For $N_p(5) = 1$, Eq. (9) represents two sets (one for each α) of three equations for the two sets of three unknown (one for each α) $T^{\alpha,n}_{\beta s5,1}$, $T^{\alpha,n}_{\beta t5,1}$, and $T^{\alpha,n}_{\beta\sigma 5,1}$. These transfer coefficients can be solved for by inverting each " α " 3 x 3 matrix ($3N_p(5)$ x $3N_p(5)$ in general) and then by introducing a fourth ($3N_p(5)+1$ in general) testing source. Then these steps must be followed. Determine the external interaction quantities for this testing source and use the calculated transfer coefficients together with Eq. (9) (for $k = 3N_p(5) + 1$) to predict the $Q^{\alpha,n,(3N_p(5)+1)}_{\beta}$ that will be excited when the fifth seal is removed. Remove the seal and test the adequacy of the transfer coefficient determination. If not adequate, repeat the procedure for a larger $N_p(5)$.

Transfer coefficients for a particular aperture in a particular α environment can always be determined one aperture at a time according to the procedure just described. The question arises whether transfer coefficients determined in such a manner are the ones that should be used in Eq. (7). The need to be concerned with a one aperture at a time characterization is eliminated because of the concept that $N_p(i)$ is allowed to be greater than one. For large apertures we would expect $N_p(i) > 1$ and for a cluster of not necessarily small apertures in close proximity we would expend no more effort determining the transfer coefficients for the cluster than for a single large aperture. By measuring the external interaction quantities on the sealed cluster and then simultaneously removing the seals on all of the apertures comprising the cluster and measuring the internal quantities, the same procedure would be employed as would be necessary to determine the transfer coefficients for a single aperture requiring the same $N_p(i)$ as the total for the cluster. The concept of a cluster can now be generalized to any combination of apertures that require its transfer coefficients to be simultaneously determined. This combination may or may not have the physical appearance of a cluster (i.e., one aperture may be quite long, coupling a number of remote apertures).

Even though there is no difference between determining the transfer coefficients for a combination of apertures (combined aperture) than for a single large aperture having the same $N_p(i)$, it is still worthwhile to determine the minimum number of apertures that must be characterized in combination. This can be done by comparing transfer coefficients determined first individually and then in progressively larger subsets of a suspected combination. The minimum set is determined after no significant change in the transfer coefficients is found by including more apertures in the subset. We now see that the minimum subset of apertures and a single large aperture behave in exactly the same manner with regard to their effects on transferring external interaction quantities to internal electrical quantities. This leads to the generalized concept of a POE as being the minimum combination of apertures requiring that their transfer coefficients be determined in combination.

After a particular POE has been demonstrated to be adequately characterized, through the addition of a testing source, one might be tempted to focus only on the largest $T^{\alpha,n}_{\beta(s,t,\sigma)i,j}$'s that comprise the characterization. Even though we might focus on those i,j locations, we should not dismiss the possibility that an i,j location having relatively small transfer coefficients could significantly contribute to the excitation of internal quantities. This is possible because larger external interaction quantities might be capable of being excited at the i,j locations having smaller transfer coefficients.

The ability to test the transfer coefficients allows us to experimentally resolve a sticky theoretical point. This point, as described earlier, is related to the fact that σ is determined from a linear operation on J_s and J_t. This leads to the questions of whether the $T^{\alpha,n}_{\beta\sigma i,j}$'s are necessary. First we point out that part of the linear operation involved a derivative which implies that for $N_p(i) = 1$ we do not have sufficient information to even approximately determine σ from J_s and J_t. This implies that for the particular (but very important) case of $N_p(i) = 1$ we should expect to require that $T^{\alpha,n}_{\beta\sigma i,1}$ be determined. Whether $N_p(i) = 1$ or $N_p(i) > 1$ we can test whether predictions made that do not include $T^{\alpha,n}_{\beta\sigma i,j}$ are adequate by introducing additional testing sources after transfer coefficients are determined without the inclusion of these quantities. For larger apertures, it is possible that $N_p(i) > 1$ and the charge density transfer coefficient may not be required.

The special case ($N_p(i) = 1$) includes all previous work (Refs. 3 ,11) and it includes this work under less restrictive conditions. The work of Latham (Ref. 9) and Lee and Baum (Ref. 10) is restricted to the case where the $Q^{\alpha,n}_{\beta}$ corresponds to a voltage or current associated only with a TEM mode excited within a simple enclosure having apertures. The external environment factor corresponds only to free space. These works depend on small aperture assumptions as well as a knowledge of electric and magnetic polarizabilities associated with the apertures. The work of Tesche (Ref. 11) also depends on small aperture assumptions and a knowledge of aperture polarizabilities in order to relate external interaction quantities to internal quantities. His work is more general than the previously cited work in that he allows both the external and internal geometries to be complex; however, his work contains no explicit procedures or results. At this point we should emphasize that our procedure for obtaining transfer coefficients automatically includes the polarizabilities for $N_p(i) = 1$ and automatically eliminates the need to generalize the concept of polarizabilities when $N_p(i) > 1$. The least restrictive work related to transfer coefficients is contained in the work of Baum (Ref. 3). In that work, Baum allows the more general concept of aperture to include apertures associated with antennas and he also eliminates the requirement that aperture polarizabilities be known. Baum does have some assumptions that are eliminated by the described combined aperture theory. This is possible because N_p (i) is allowed to be greater than one. Baum's work was directed toward developing extrapolation techniques for interpreting the results of tests in EMP simulators in terms of EMP criteria. For that application, it is essential that the dependence of the transfer coefficiencies on the complete exterior system environment be understood and accounted for. The viewpoint taken here is that Eq. (8) must be tested according to our described procedure and the results of the test will impact future simulator/test object interaction studies. The potential benefits of an extrapolation that utilizes external coupling quantities as suggested by Baum has been discussed in the introduction.

More recently another effort (Ref. 12) was initiated to assess the accuracy of the extrapolation techniques suggested by Baum (Ref. 3). In that work no attempt was made to quantify the effects of the external environment on the transfer coefficients. It should be noted that none of the cited works contain the self-testing experimental procedures presented here.

IV. COMPARISON OF ANALYTIC APPROXIMATION TRANSFER COEFFICIENTS WITH EXPERIMENTAL DATA

The University of Michigan experimental program discussed in the introduction dealt with the following coupling situation. A cylinder of finite length having hemispherical end caps was illuminated by a plane wave. The interior of this structure is a cylindrical cavity having flat end plates. Within the cavity is a single wire running through its center and metallically connected to one of the interior end plates. The other end of the wire extends through the other end plate to become the center conductor of a 50-Ω coaxial cable. A circular aperture is cut in the structure allowing the incident plane wave to excite a voltage across the load impedance presented by the coaxial line. A more detailed description of the structure is presented in Figure 2 (page 27).

The prominent transmission line coupling model for the structure described in Figure 2 is presented in References 8 and 9 and is depicted in Figure 1. The parameters of this transmission line are as follows:

d = Distance from the center of the aperture to the interior flat plate to which the interior wire is shorted.

p = Distance from the center of the aperture to the other interior flat plate.

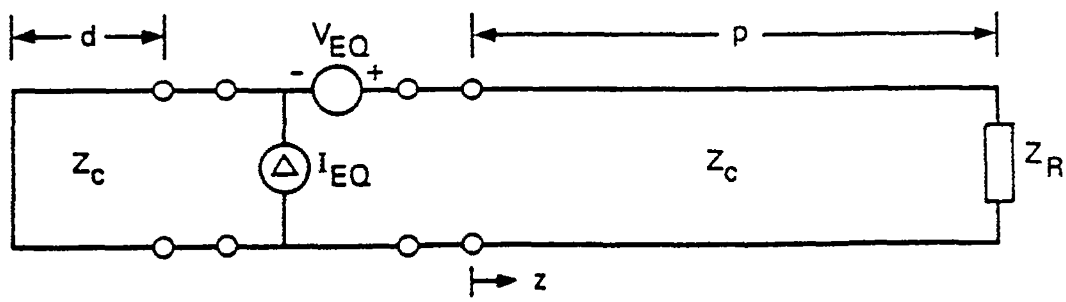

Figure 1. Transmission line model.

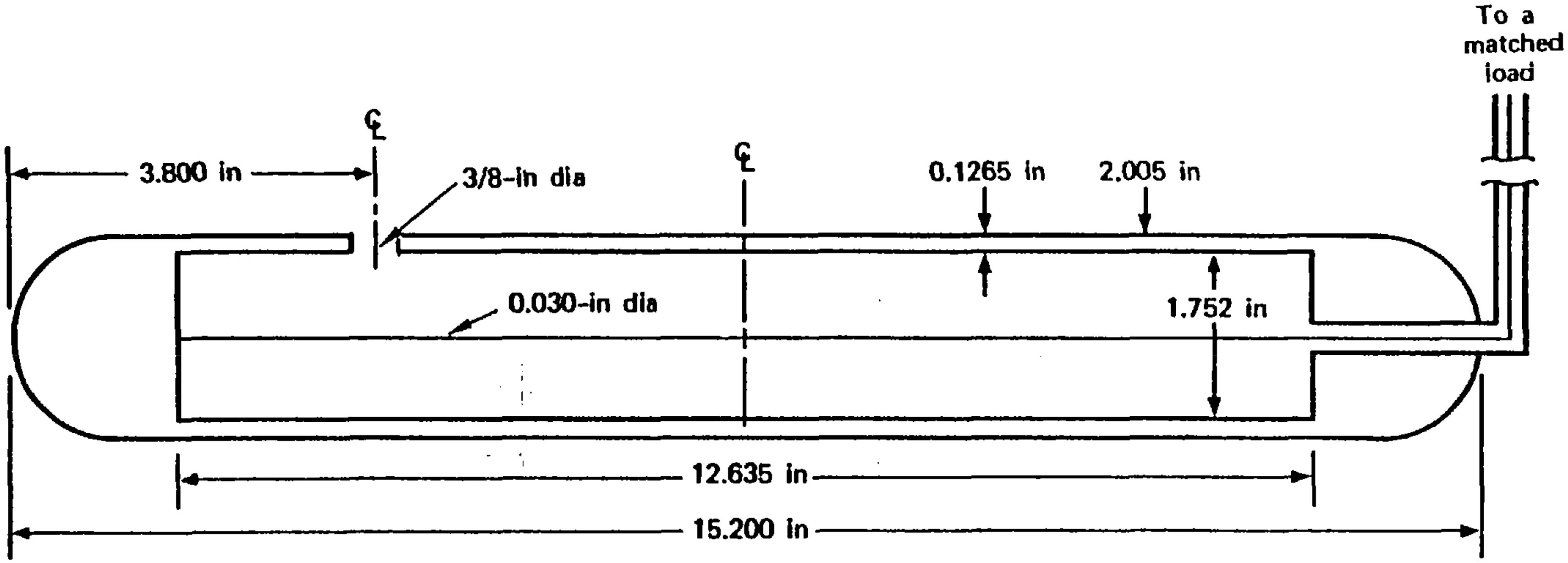

Figure 2. Geometry of the first experimental structure.

Z_R = Effective load impedance presented to the interior wire by the 50-Ω coaxial line.

V_{EQ} = Voltage source which depends on the external interaction current density induced on the metallic seal placed over the aperture.

I_{EQ} = Current source which depends on the external interaction charge density induced on the metallic seal placed over the aperture.

Z_c = Characteristic impedance of the transmission line which is the same as that of the interior coaxial cavity corresponding to the actual structure.

It is a straightforward transmission-line problem to determine the voltage across the load in terms of the quantities just defined. The more complex electromagnetic aspects of the coupling problem were treated in References 9 and 10 in arriving at the transmission-line model and in representing the sources and characteristic impedance as follows:

$$V_{EQ} = KJ_Z \tag{10}$$

$$I_{EQ} = K \frac{\alpha_e}{\alpha_m} \frac{1}{Z_0 Z_c} E_n \tag{11}$$

$$K = -ik_0\alpha_m Z_0/2\pi a_b \tag{12}$$

$$\alpha_e = D^3/12 \tag{13}$$

$$\alpha_m = 2\alpha_e \tag{14}$$

$$Z_c = (Z_0/2\pi) \ \ln(a_b/a) \tag{15}$$

These quantities are defined as follows:

- J_z = Longitudinal component of current density induced by the exterior source on a metallic seal placed over the aperture.
- E_n = Normal component of electric field induced by the exterior source on a metallic seal placed over the aperture.
- D = Diameter of the circular aperture.
- a = Radius of the wire within the cavity.
- a_b = Interior radius of the cavity.
- k_0 = Free space wave number.
- Z_0 = Intrinsic impedance of free space.

Using standard transmission line theory presented in Appendix B, the voltage across the load is determined in terms of the described parameters including V_{EQ} and I_{EQ}. The dependence of these equivalent sources on J_z and E_n allows the transmission line result to be written as the transfer coefficient equation for the voltage V_R across the load as follows:

$$V_R = T_z J_z + T_\phi J_\phi + T_n E_n \tag{16}$$

where

$$T_\phi \doteq 0 \tag{17}$$

$$T_z = KF\left(1 + e^{2ik_o d}\right) \tag{18}$$

$$T_n = (KF/2Z_0)\left(1 - e^{2ik_o d}\right) \tag{19}$$

$$F = \frac{(1-\rho_R)\ e^{ik_o p}}{2\left(1+\rho_R\ e^{2ik_o(p+d)}\right)} \qquad (20)$$

$$\rho_R = \frac{Z_c - Z_R}{Z_c + Z_R} \qquad (21)$$

All quantities in these transfer coefficients have been defined and can be given numerical values by referring to Figures 1 and 2 with the exception of Z_R. In the simplest approximation to Z_R, it is taken as 50 Ω; however, the following remarks are applicable for any correction to the 50-Ω description that was considered. The analytic transfer coefficient approximations given by Eqs. (17), (18), and (19) were not in good agreement with the data taken. This can be seen by viewing Figures 3 to 5. No value for Z_R could explain the peaking of the analytic transfer coefficients at the higher frequencies to account for the internal voltage pickup displayed in Figure 5.

In contrast to the failure of Eqs. (17), (18), and (19), they were adequate to explain the design of an experiment where transfer coefficients were measured with considerable success. In particular, when $k_o d = \pi$, Eq. (19) implies $T_n = 0$, and when $k_o d = \pi/2$, $T_z = 0$. For these choices, the experiment hypothesis is that

$$|V_R|^2 = |T_z|^2\ |J_z|^2 \qquad k_o d = \pi \qquad (22)$$

and

$$|V_R|^2 = |T_n|^2\ |E_n|^2 \qquad k_o d = \pi/2 \qquad (23)$$

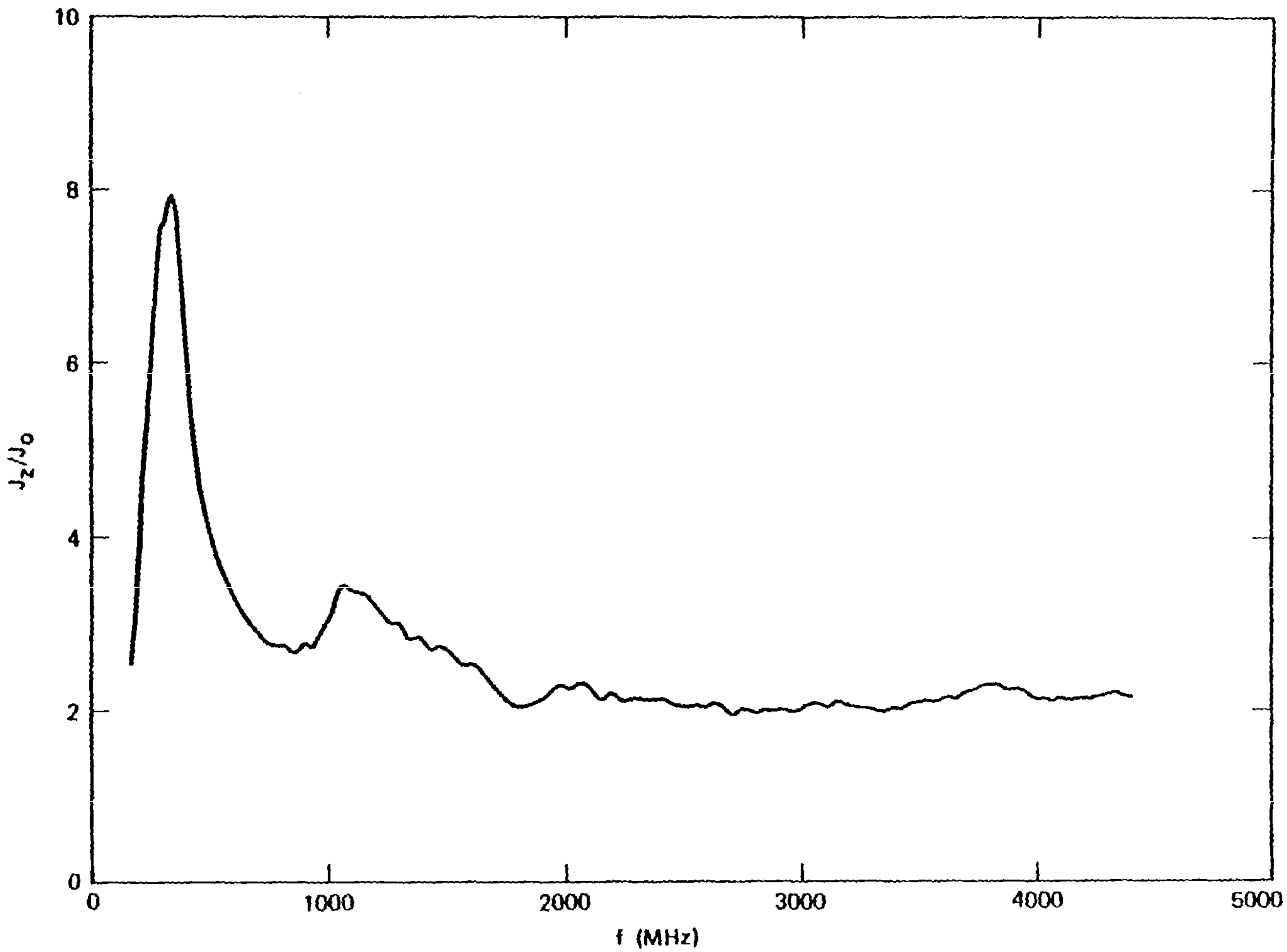

Figure 3. Induced longitudinal current density on shorting surface.

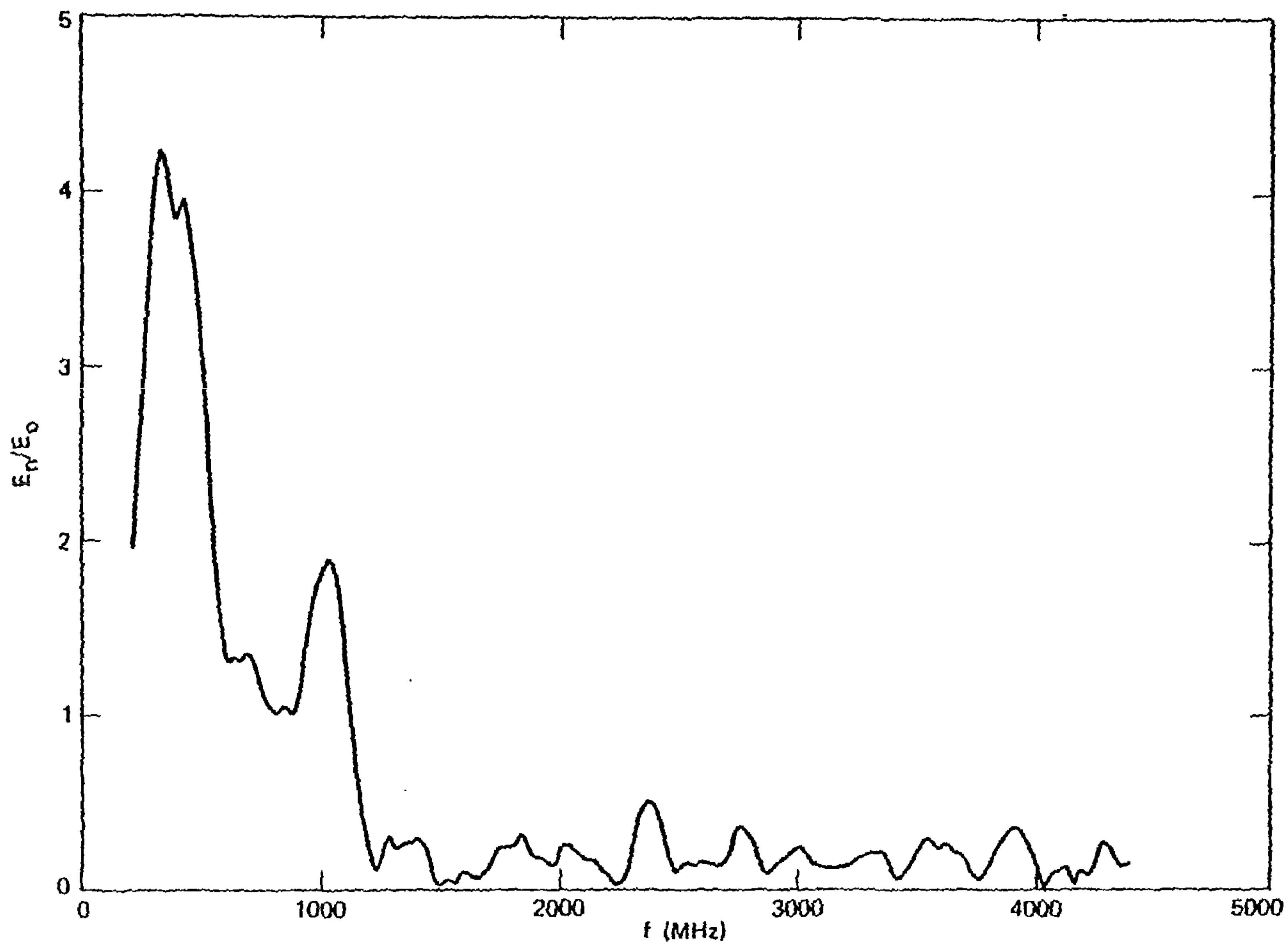

Figure 4. Induced charge density on shorting surface.

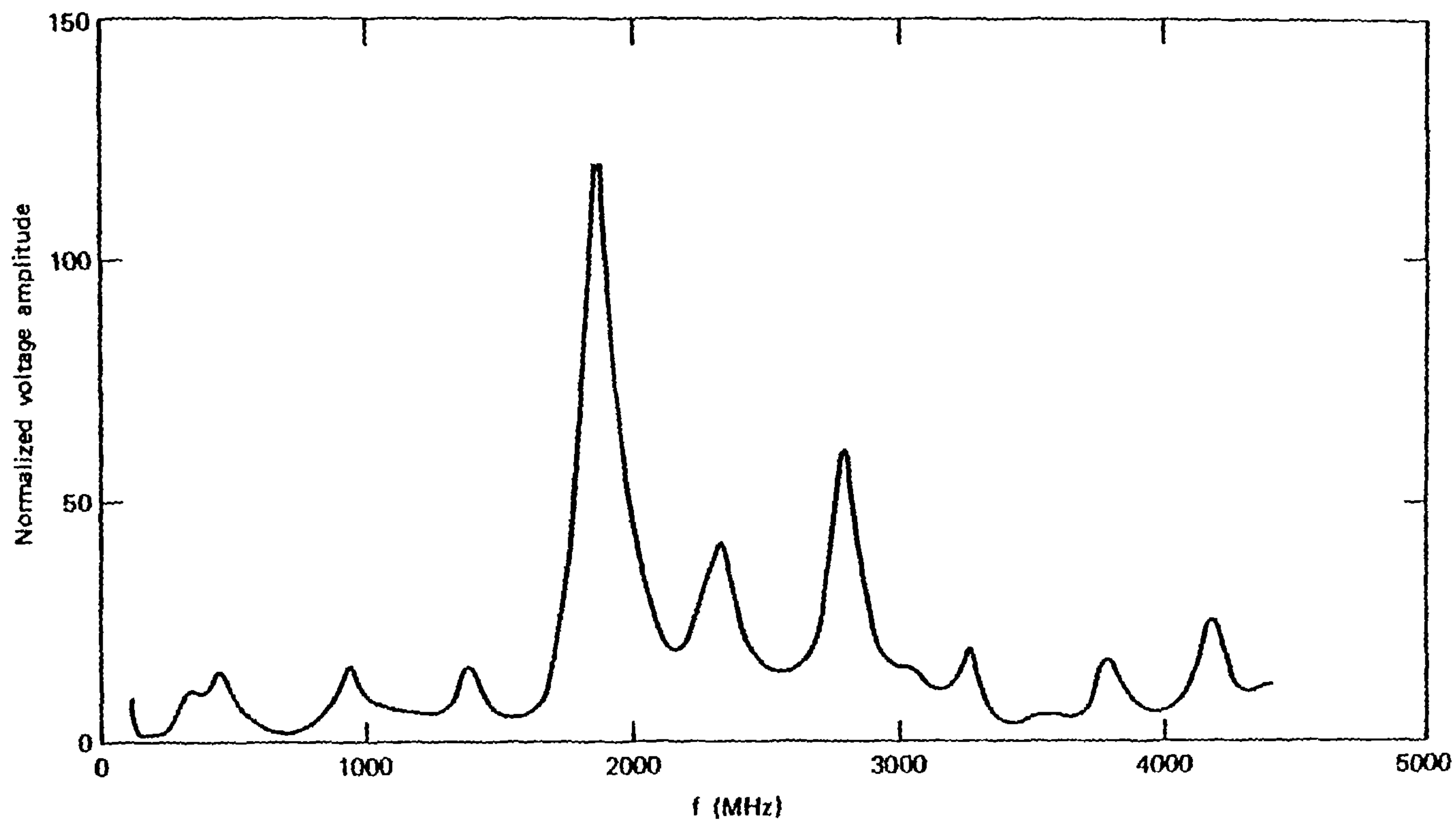

Figure 5. Voltage amplitude induced across the internal load when shorting surface is removed.

The experimental advantage of this hypothesis is that it can be tested without measuring phase angles. The experimental setup described and treated by Schuman (Ref. 1) is illustrated in Figure 6. The experiments were performed at the Naval Surface Weapons Center (Dahlgren, Virginia) and at the University of Colorado (Boulder, Colorado).

Figure 7 demonstrates how well a single constant multiplier, the value of which is not important to this discussion, relates $|J_z|^2$ induced on the short placed over the aperture to $|V|^2$ induced across the load impedance when the seal is removed. For the situation in Figure 7, $k_o d = \pi$; Figure 8 corresponds to the case $k_o d = \pi/2$ and shows how well a single constant multiplier relates $|E_n|^2$ to $|V|^2$. For these two cases, i.e., $k_o d$ values, any single incident angle could be used to determine the constant multiplier of the appropriate external interaction quantity. After that single measurement, the internally excited quantity $|V|^2$ could be determined for all incident angles by only determining external interaction quantities either through computer codes which are becoming increasingly more reliable for this type of calculation or through external interaction experiments. This is an example of the transfer coefficient procedure described in the previous section.

A final note is that the data presented in Figures 7 and 8 bear on extrapolation to threat used in the EMP community. It shows that the external geometry of a system can have no bearing on the choice of whether to emphasize current density or charge density in the extrapolation. For the data taken in these two figures, the exterior geometry was identical; however, the dependence of the internal pickup depended solely on the external current density in one case and on the external charge density in the other.

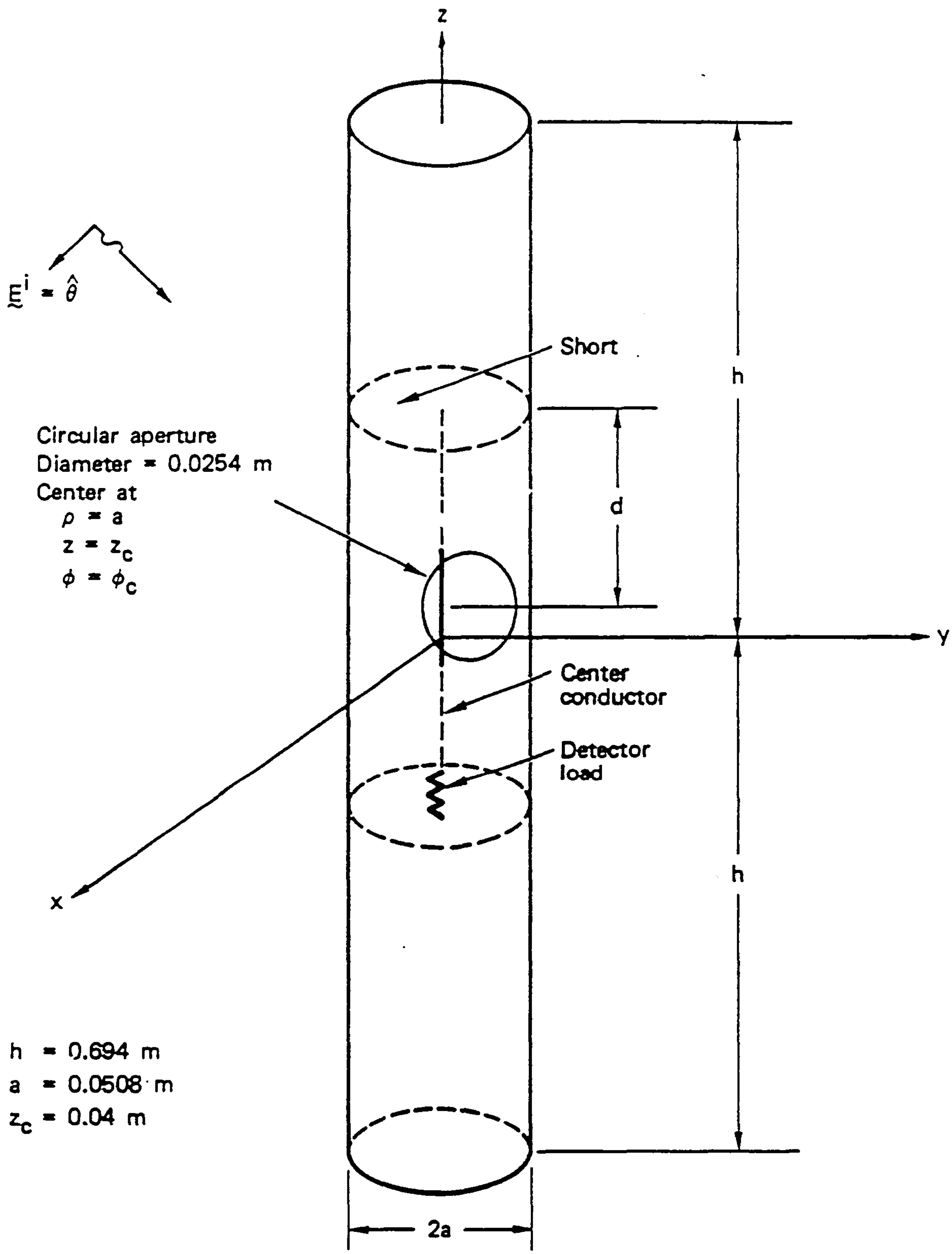

Figure 6. Test object.

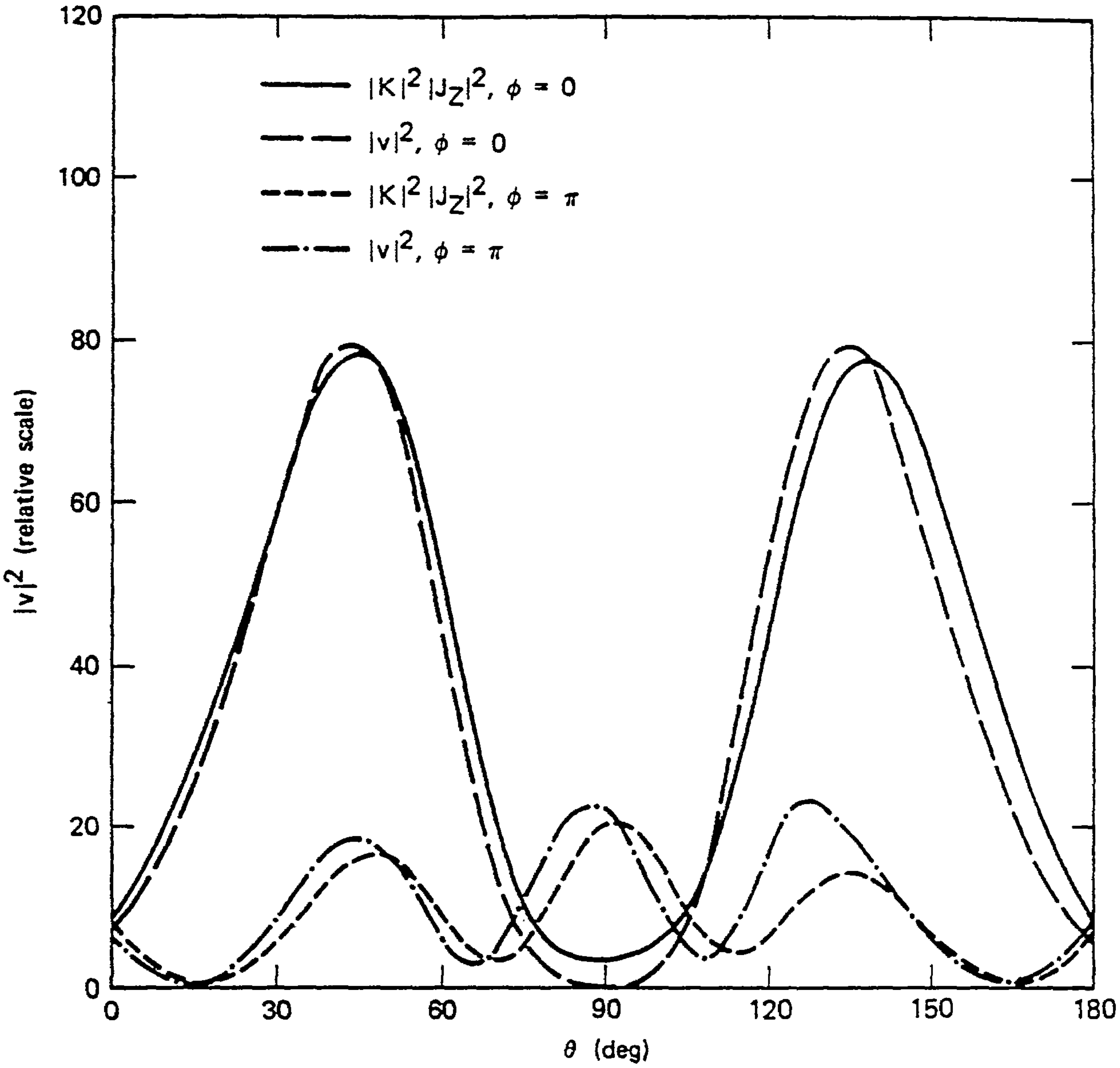

Figure 7. Comparison of transfer coefficient calculation with experimental data.

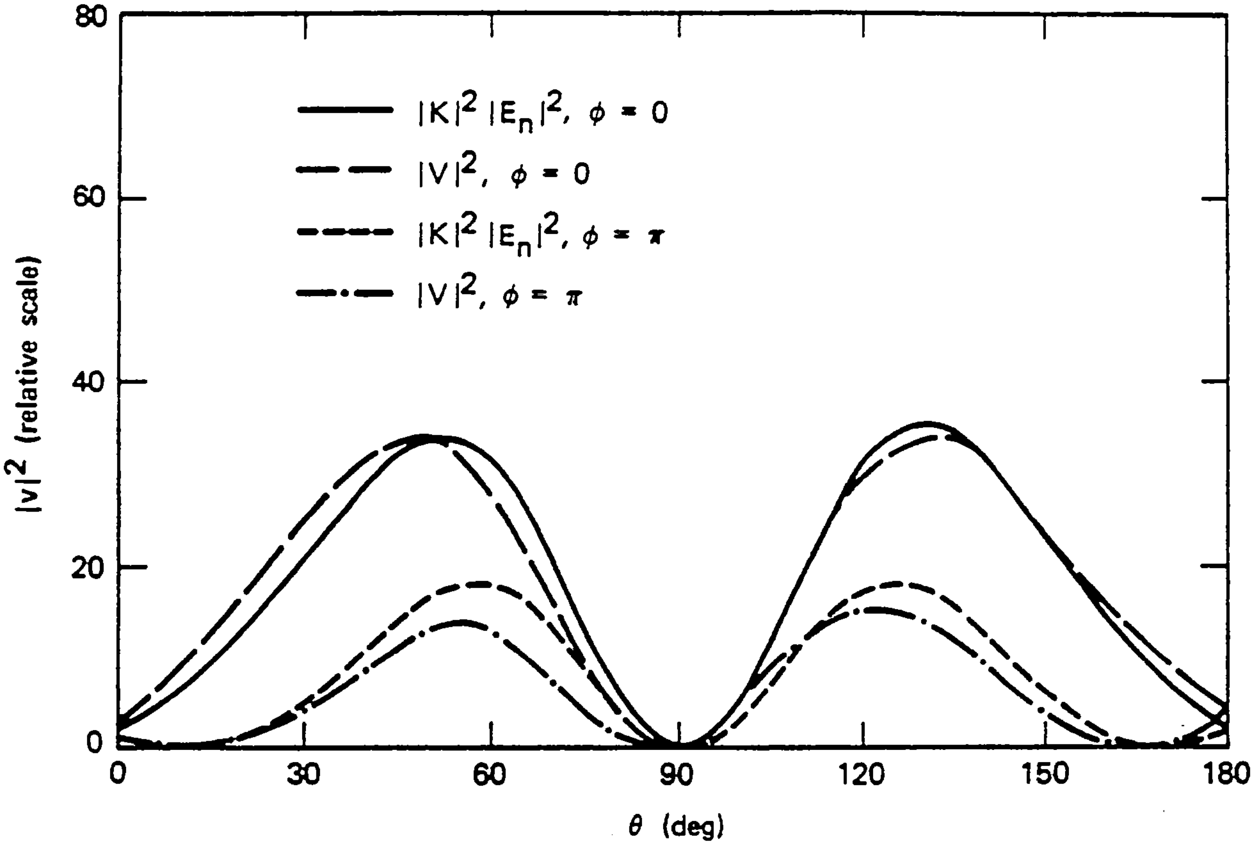

Figure 8. Comparison of transfer coefficient calculation with experimental data.

V. DEMONSTRATION OF TRANSFER COEFFICIENT DEPENDENCE ON THE EXTERIOR ENVIRONMENT

It is shown in this report that transfer coefficients do depend on the external environment. This fact is very significant if one were to measure transfer coefficients on a system in one environment, e.g., on an aircraft on the ground or in a bounded wave simulator, and the real interest was the survivability of the aircraft in the in-flight mode. For other situations, e.g., a shielded building, the transfer coefficients would be measured in the actual environment of interest and the fact that they depended on the external environment would be of no consequence.

In this section an experiment is described which demonstrates the dependence of transfer coefficients on the environment. This experiment was designed to enhance the external environment effect, and the results will be viewed in this light. The experiment consisted of the cylindrical structure depicted in Figure 9 placed in front of a metallic ground screen as depicted in Figure 10. The aperture is cut in the longitudinal center of the cylinder and it is symmetrically excited as depicted in Figure 10. For this symmetric situation, neither J_ϕ nor E_n can be excited on the seal placed over the aperture. The transfer coefficient equation becomes

$$V = TJ_z \tag{24}$$

independent of the internal geometry, the separation of the cylinder from the ground plane, D, or the orientation of the aperture, ϕ. Figures 11, 12, and 13 are plots of $|T|$ corresponding to the indicated three choices of the aperture orientation. The curves on each of these figures correspond to the three indicated separation values of D. On each figure, the deviation of one curve from the other as a function of D

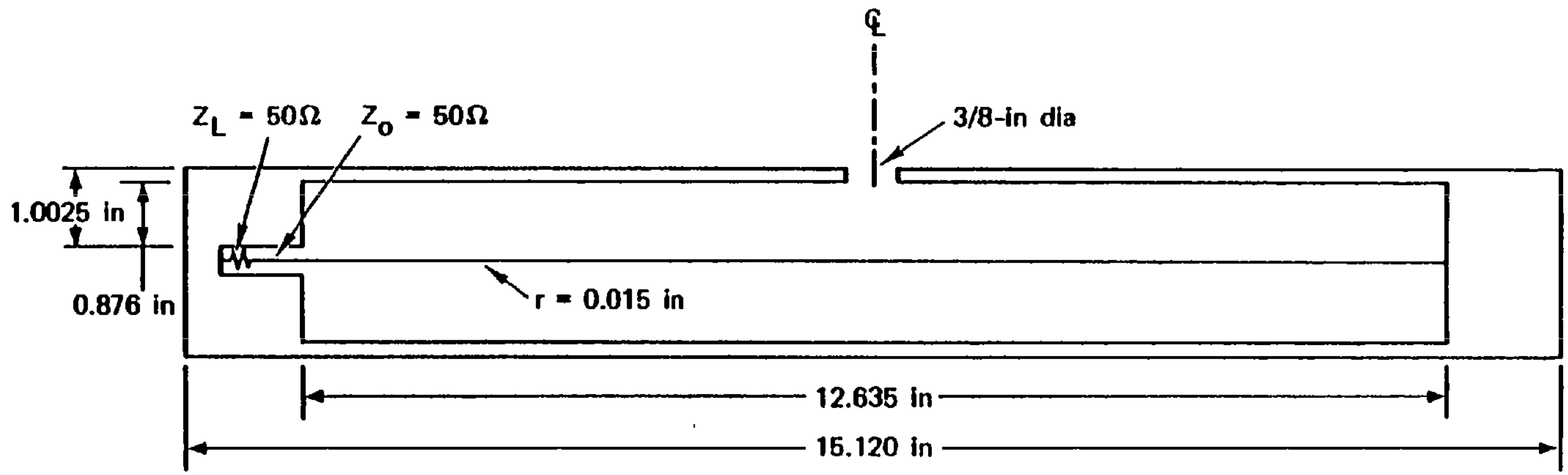

Figure 9. Test object for ground plane experiment.

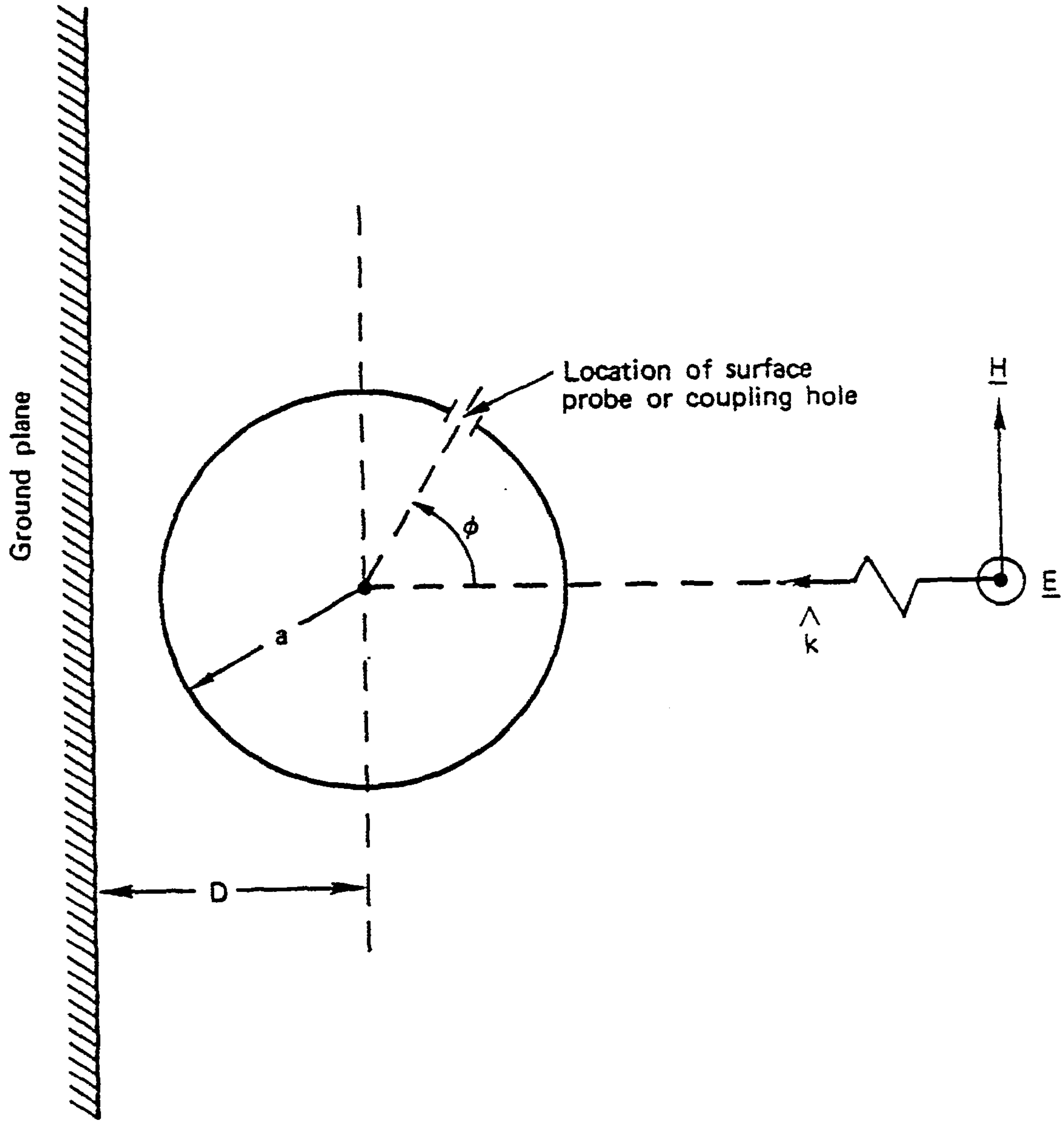

Figure 10. Geometry for ground plane experiment.

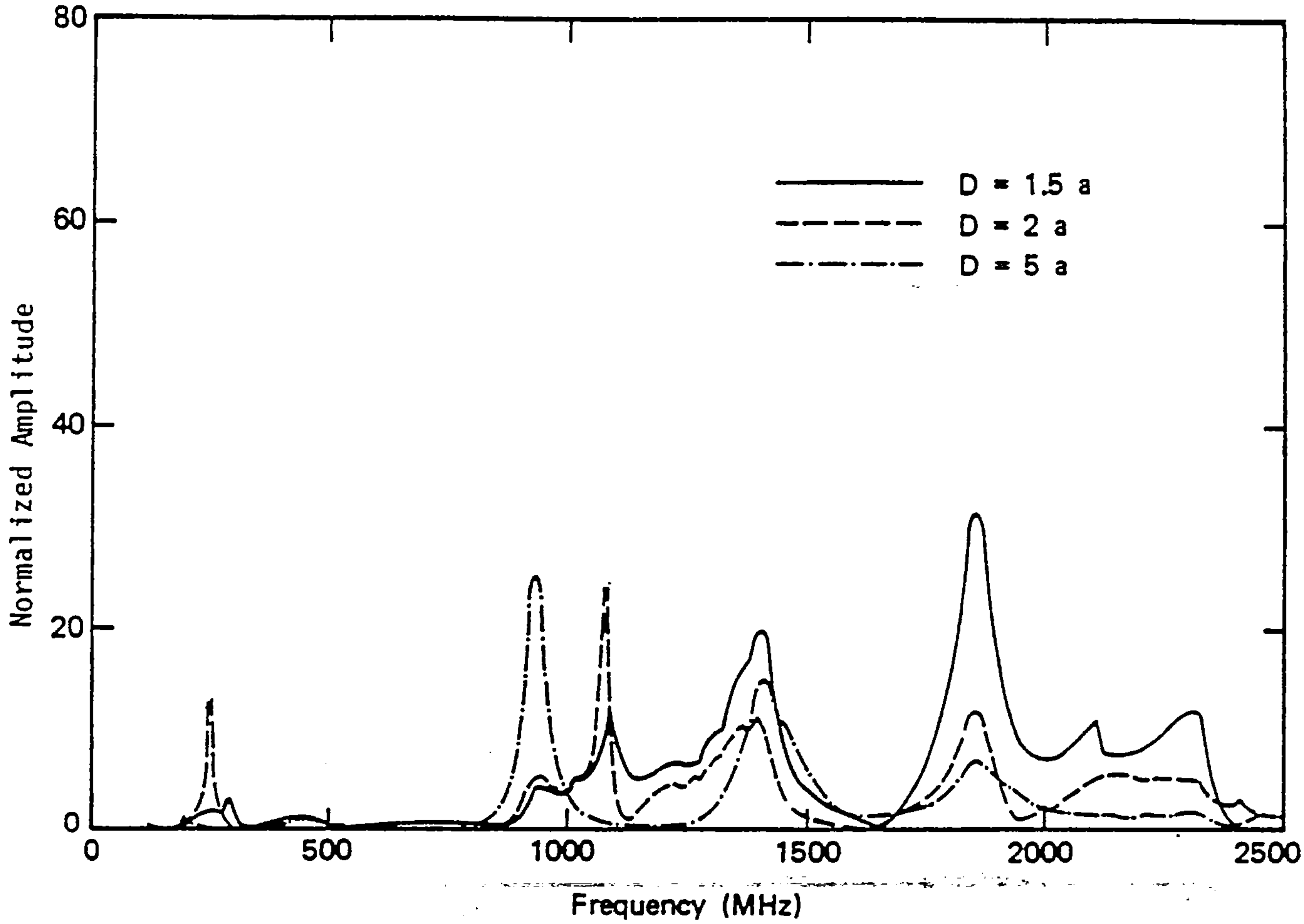

Figure 11. Transfer coefficients as a function of D for $\phi = 0^{\circ}$

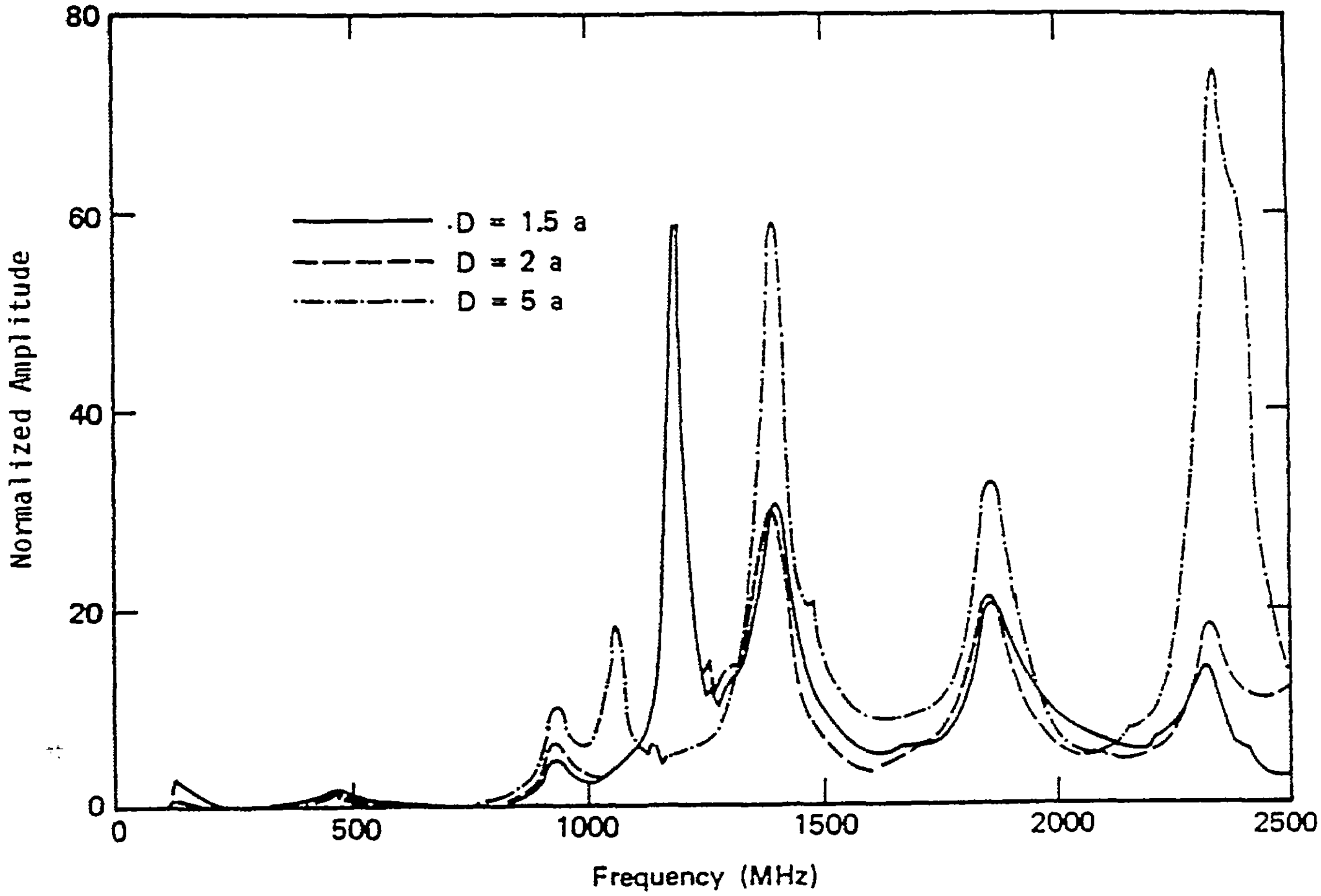

Figure 12. Transfer coefficients as a function of D for $\phi = 90^{o}$

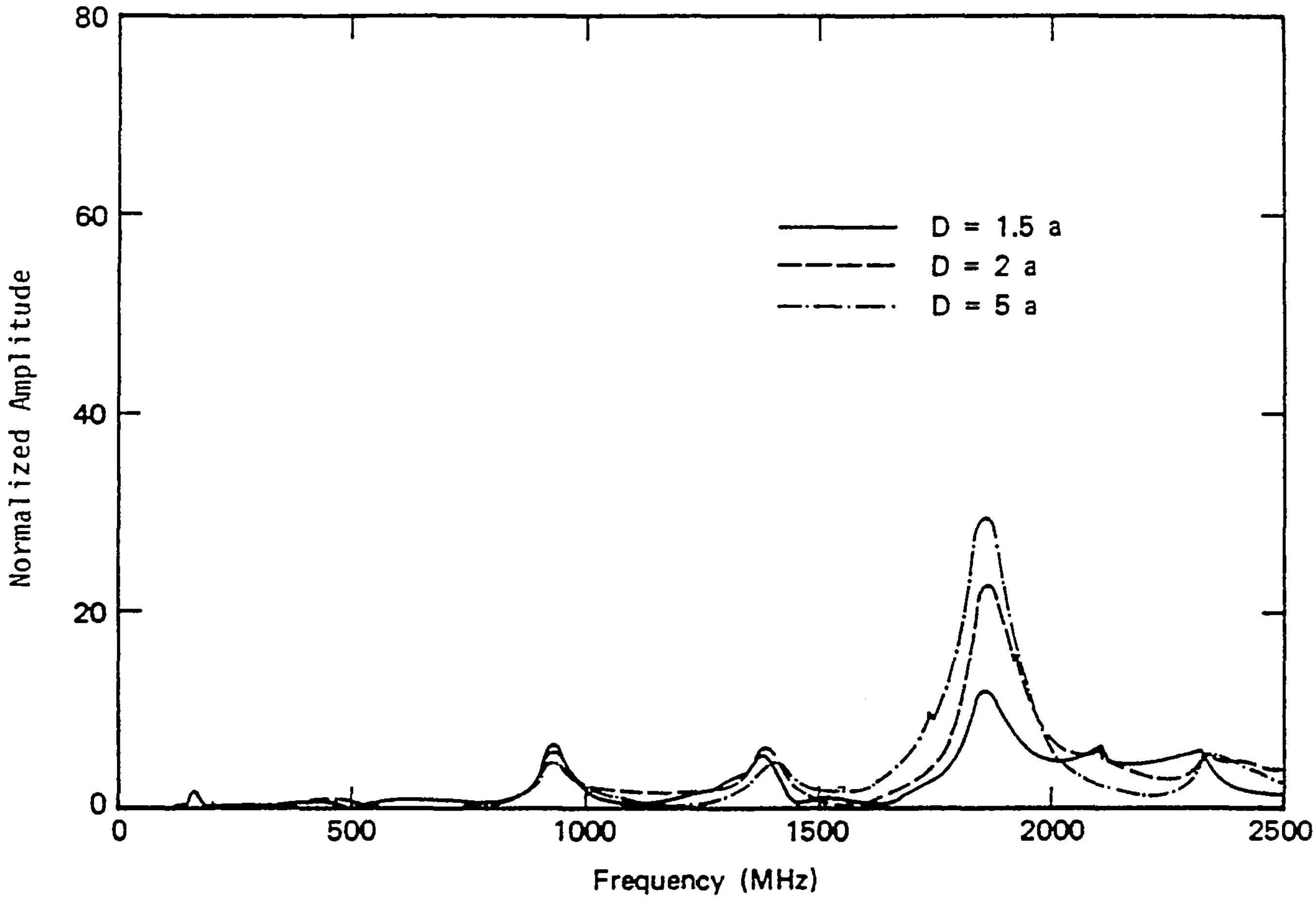

Figure 13. Transfer coefficients as a function of D for $\phi = 180^{\circ}$.

demonstrates the extent to which the transfer coefficients depend on the external environment. Figures 14, 15, and 16 are another representation of the same data to further illustrate the external environment dependence. For a fixed separation, D, they exhibit the variation with the angle ϕ. The extent to which these curves deviate from each other further illustrates the external environment dependence of transfer coefficients. A final note is that many external environments are not as severe as a proximate metallic ground plane, e.g., lossy earth, and the transfer coefficient dependence on such an environment should be less than indicated by the data just presented.

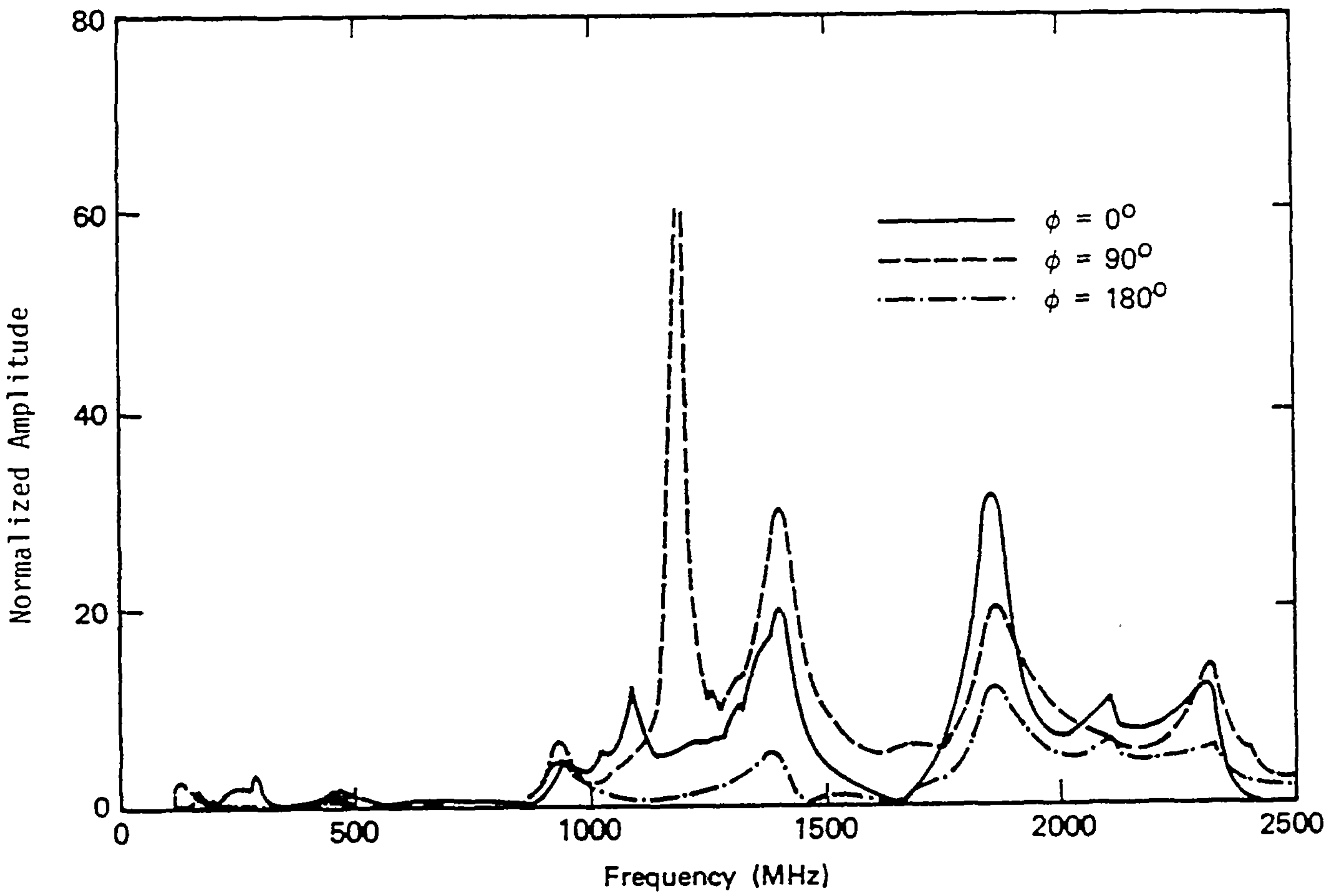

Figure 14. Transfer coefficients as a function of ϕ for D = 1.5a.

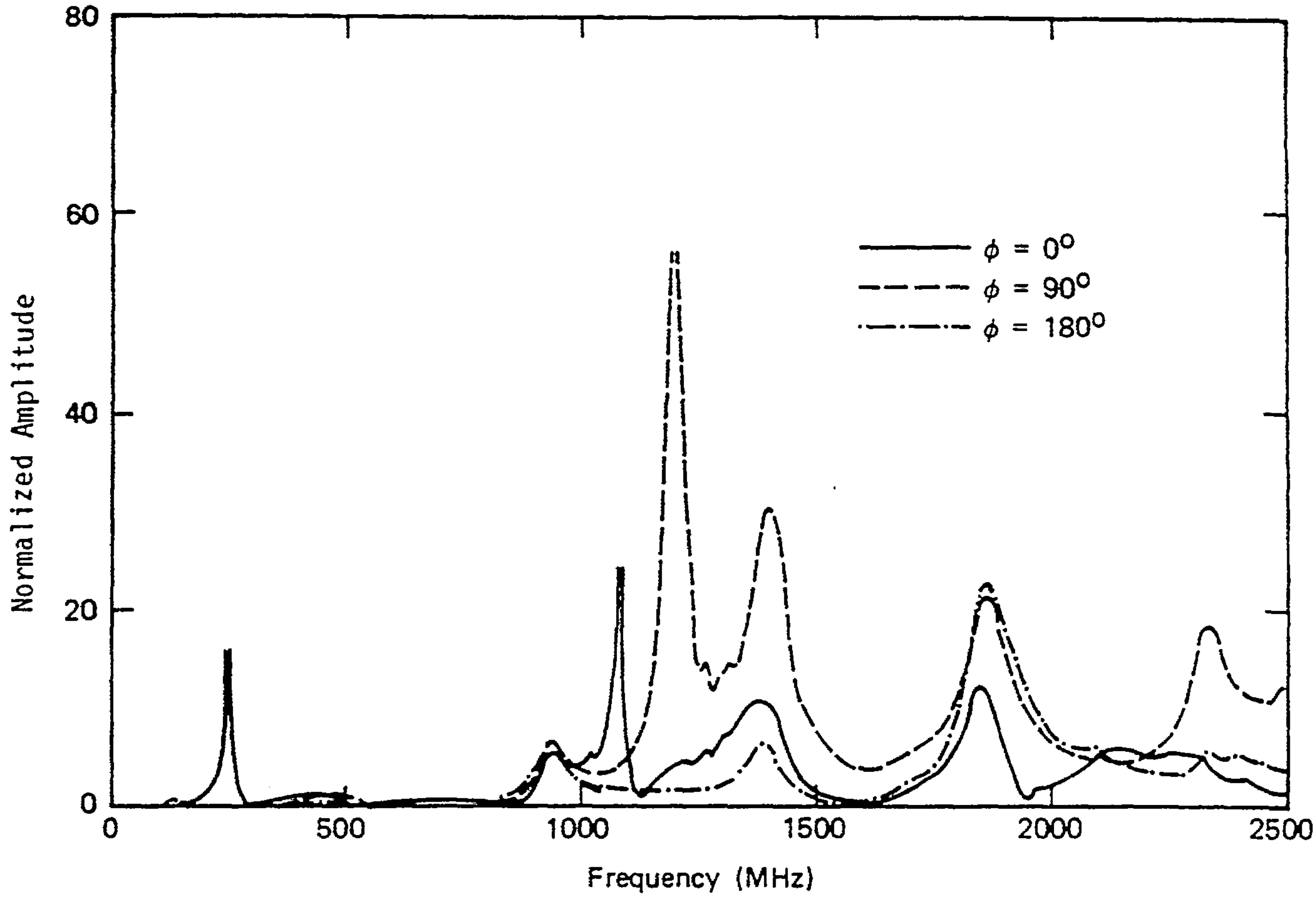

Figure 15. Transfer coefficients as a function of ϕ for D = 2a.

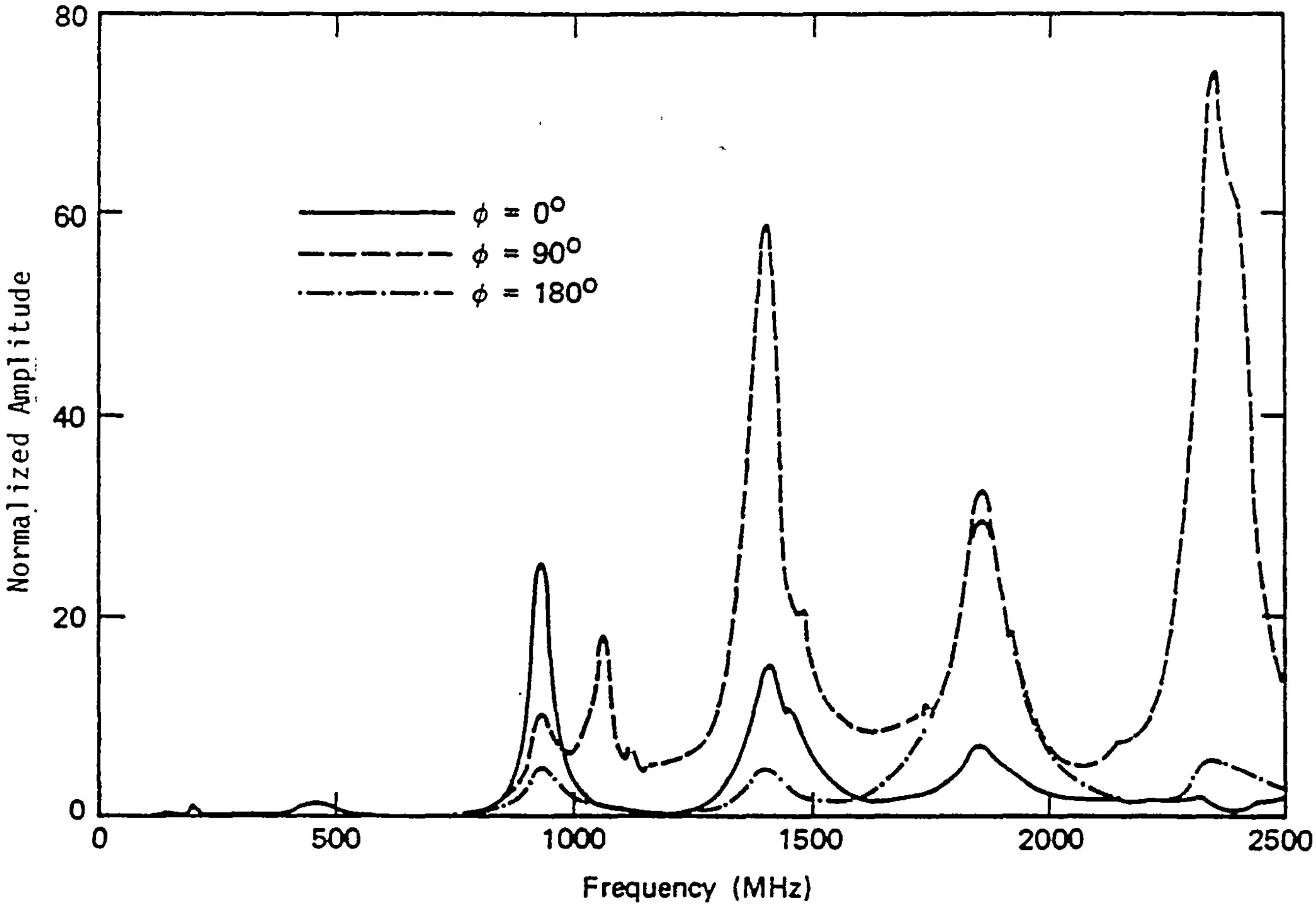

Figure 16. Transfer coefficients as a function of φ for D = 5a.

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APPENDIX A. DERIVATION OF OPERATOR EQUATIONS

This appendix will present detailed derivations of Eqs. (1) and (2) for the situation depicted in Figure A1. First we introduce the following definitions:

S_m = The surface of the metallic enclosure (aircraft) augmented by the mathematical surfaces S_1 and S_2.

V_L = The volume of a lossy medium in the proximity of the enclosure (earth, water).

S_L = The surface bounding V_L.

V_p = The volume of an object in the proximity of the enclosure (i.e., an aircraft carrier).

S_p = The surface bounding V_p.

V_s = The volume of a subsystem contained within the enclosure.

S_s = The surface bounding V_s.

V_0 = The volume exterior to S_m bounded by S_m, S_p, S_L, S_r, and the hemisphere at infinity.

V_I = The volume interior to S_m bounded by S_m and S_s.

V_J = The volume of a rigid source of an electromagnetic wave, $\underline{J}$, and it is contained in V_0.

V_r = The volume of the portable radiator.

S_r = The surface of the portable radiator.

S_g = The portion of S_r over which the surface tangential electric field is rigidly specified.

The essential equation that this approach is based on is the dyadic identity

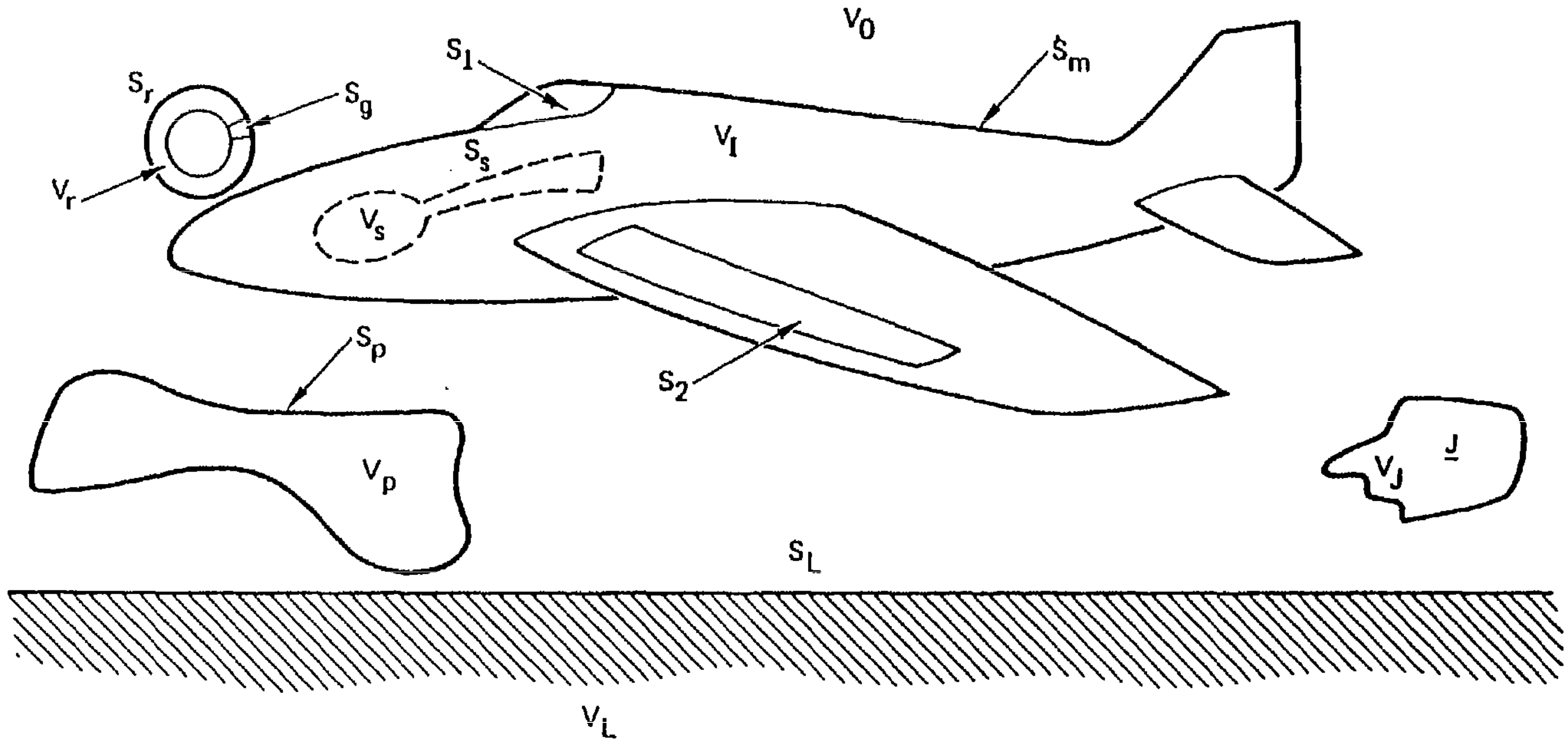

Figure A1. Aircraft and its environment with rigid and nonrigid sources.

$$\int_V \left\{ \underline{A}(\underline{r}')\cdot\left[\nabla'\times\nabla'\times\underline{\underline{D}}(\underline{r}';\underline{r})\right] - \left[\nabla'\times\nabla'\times\underline{A}(\underline{r}')\right]\cdot\underline{\underline{D}}(\underline{r}';\underline{r}) \right\} dV'$$

$$= \int_S \left\{ \underline{A}(\underline{r}')\cdot\left[\hat{n}(\underline{r}')\times\left\{\nabla'\times\underline{\underline{D}}(\underline{r}';\underline{r})\right\}\right] + \left[\left(\nabla'\times\underline{A}(\underline{r}')\right)\times\hat{n}(\underline{r}')\right]\cdot\underline{\underline{D}}(\underline{r}';\underline{r}) \right\} dS' \tag{A1}$$

where $\underline{A}(\underline{r}')$ and $\underline{\underline{D}}(\underline{r}';\underline{r})$ are, at this point, a general vector and a general dyadic that must satisfy certain behavior requirements (e.g., differentiability) but not necessarily any equations. In Eq. (A1), S is the surface bounding V, and $\hat{n}(\underline{r}')$ is the outward normal to V. Next the volume, bounding surface, $\underline{A}(\underline{r}')$, and $\underline{\underline{D}}(\underline{r}';\underline{r})$ are specialized. V is chosen, in turn, as V_0 and V_I, and $\underline{A}(\underline{r}')$ is chosen as $\underline{H}_0(\underline{r}')$ and $\underline{H}_i(\underline{r}')$. We also choose $\underline{\underline{D}}(\underline{r}';\underline{r})$ as appropriate Green's dyadics $(\underline{\underline{G}}_0(\underline{r}';\underline{r}),\ \underline{\underline{G}}_I(\underline{r}';\underline{r}))$ that satisfy the vector wave equation

$$\left(\nabla'\times\nabla'\times-k_0^2\right)\underline{\underline{G}}_\alpha(\underline{r}'_\alpha,\underline{r}_\alpha) = \underline{\underline{I}}\,\delta(\underline{r}'_\alpha-\underline{r}_\alpha) \quad \alpha=0,I \quad \underline{r}'_\alpha,\underline{r}_\alpha\in V_\alpha \tag{A2}$$

and subsequently the α subscript of $\underline{r}$ and $\underline{r}'$ will automatically be implied by the subscript on $\underline{\underline{G}}_\alpha$ when it is not explicitly indicated. Boundary conditions to be satisfied are

$$\hat{n}(\underline{r}')\times\left(\nabla'\times\underline{\underline{G}}_I(\underline{r}',\underline{r})\right) = 0 \quad \underline{r}'\in S_m \tag{A3}$$

$$\hat{n}(\underline{r}')\times\left(\nabla'\times\underline{\underline{G}}_0(\underline{r}',\underline{r})\right) = 0 \quad \underline{r}'\in S_r \cup S_p \cup S_m \tag{A4}$$

$$\hat{n}(\underline{r}')\times\left(\nabla'\times\underline{\underline{G}}_0(\underline{r}',\underline{r})\right) = \hat{n}(\underline{r}')\times\left(\nabla'\times\underline{\underline{G}}_L(\underline{r}',\underline{r})\right) \quad \underline{r}'\in S_L \tag{A5}$$

$$\hat{n}(\underline{r}')\times\varepsilon_0\underline{\underline{G}}_0(\underline{r}',\underline{r}) = \hat{n}(\underline{r}')\times\varepsilon\underline{\underline{G}}_L(\underline{r}',\underline{r}) \quad \underline{r}'\in S_L \tag{A6}$$

The equation satisfied by $\underline{\underline{G}}_L(\underline{r}',\underline{r})$ is

$$\left(\nabla' x \nabla' x - \omega^2 \mu_0 \varepsilon\right) \underline{\underline{G}}_L(\underline{r}',\underline{r}) = 0 \quad \underline{r}' \epsilon V_L, \underline{r} \epsilon V_0 \tag{A7}$$

The equations satisifed by the $H_\alpha(\underline{r}')$ are

$$\left(\nabla' x \nabla' x - k_0^2\right) \underline{H}_\alpha(\underline{r}') = \begin{cases} 0 & \alpha = I \\ \nabla' x \underline{J}(\underline{r}') & \alpha = 0 \end{cases} \tag{A8}$$

It also follows from Maxwell's equations

$$\nabla' x \underline{H}_\alpha(\underline{r}') = -i\omega\varepsilon_0 \underline{E}_\alpha(\underline{r}') \quad \underline{r}' \epsilon S_m \tag{A9}$$

Substituting Eqs. (A2), (A8), and (A9) into (A1) for $V = V_0$ or V_I and using the property of the δ function, we obtain

$$\underline{H}_0(\underline{r}_0) = \underline{I}(\underline{r}_0) + \sum_{q=m,p,r,L} \int_{S_q} \left\{ \hat{n}_q, \underline{E}_0, \underline{H}_0, \underline{\underline{G}}_0 \right\} dS' + \int_{S_\infty} \left\{ \hat{a}_{r'}, \underline{E}_0, \underline{H}_0, \underline{\underline{G}}_0 \right\} dS' \tag{A10}$$

$$\underline{H}_I(\underline{r}_I) = \sum_{q=m,s} \int_{S_q} \left\{ \hat{n}_q, \underline{E}_I, \underline{H}_I, \underline{\underline{G}}_I \right\} dS' \tag{A11}$$

where $\hat{a}_{r'}$ is the unit outward normal to the sphere at infinity S_∞,

$$\int_S \left\{ \hat{n}, \underline{E}, \underline{H}, \underline{\underline{G}} \right\} dS'$$

$$= \int_S \left\{ \underline{H}(\underline{r}') \cdot \left[\hat{n}(\underline{r}') \times \left\{ \nabla' \times \underline{\underline{G}}(\underline{r}';\underline{r}) \right\} \right] + i\omega\varepsilon \left[\hat{n}(\underline{r}') \times \underline{E}(\underline{r}') \right] \cdot \underline{\underline{G}}(\underline{r}';\underline{r}) \right\} dS' \tag{A12}$$

where ε is the appropriate dielectric permittivity and

$$\underline{I}(\underline{r}_0) = \int_{V_J} \nabla' \times \underline{J}(\underline{r}') \cdot \underline{\underline{G}}_0(\underline{r}';\underline{r}_0)\, dV' \tag{A13}$$

Using Eqs. (A3) and (A4) as well as the fact that

$$\hat{n}(\underline{r}') \times \underline{E}(\underline{r}') = 0 \qquad \underline{r}' \in (S_m - S_1 - S_2) \cup S_p \cup (S_r - S_g) \tag{A14}$$

we find that

$$\int_{S_p} \left\{ \hat{n}, \underline{E}_0, \underline{H}_0, \underline{\underline{G}}_0 \right\} dS' = 0 \tag{A15}$$

$$\int_{S_r} \left\{ \hat{n}, \underline{E}_0, \underline{H}_0, \underline{\underline{G}}_0 \right\} dS' = \int_{S_g} i\omega\varepsilon_0 \left[\hat{n}(\underline{r}') \times \underline{E}(\underline{r}') \right] \cdot \underline{\underline{G}}_0(\underline{r}';\underline{r}_0)\, dS'$$

$$\equiv \underline{S}(\underline{r}_0) \tag{A16}$$

$$\int_{S_m} \left\{ \hat{n}_\alpha, \underline{E}_\alpha, \underline{H}_\alpha, \underline{\underline{G}}_\alpha \right\} dS' = i\omega\varepsilon_0 \left(\int_{S_1} \left[\hat{n}_\alpha(\underline{r}') \times \underline{E}_\alpha(\underline{r}') \right] \cdot \underline{\underline{G}}_\alpha(\underline{r}';\underline{r}_\alpha)\, dS' + \int_{S_2} \left[\hat{n}_\alpha(\underline{r}') \times \underline{E}_\alpha(\underline{r}') \right] \cdot \underline{\underline{G}}_\alpha(\underline{r}';\underline{r}_\alpha)\, dS' \right)$$

$$\alpha = 0, I \tag{A17}$$

and because $\underline{E}_0$, $\underline{H}_0$, and $\underline{\underline{G}}_0$ satisfy the radiation condition

$$\int_{S_\infty} \left\{ \hat{a}_{r'}, \underline{E}_0, \underline{H}_0, \underline{\underline{G}}_0 \right\} dS' = 0 \qquad \text{(A18)}$$

The remaining quantities to evaluate in Eqs. (A10) and (A11) are the surface integrals over S_L and S_s. Substituting the equations appropriate for the lossy half space, that is

$$(\nabla' \times \nabla' \times -\omega^2 \mu_0 \varepsilon) \underline{H}_L(\underline{r}') = 0 \quad \underline{r}' \epsilon V_L \qquad \text{(A19)}$$

and

$$\nabla' \times \underline{H}_L(\underline{r}') = -i\omega\varepsilon \underline{E}_L(\underline{r}') \quad \underline{r}' \epsilon V_L \qquad \text{(A20)}$$

as well as Eqs. (A7) into (A1), we obtain

$$\int_{S_L} \left\{ \underline{H}_L(\underline{r}') \cdot \left[\hat{n}(\underline{r}') \times \left\{ \nabla' \times \underline{\underline{G}}_L(\underline{r}';\underline{r}) \right\} \right] + i\omega\varepsilon \left[\hat{n}(\underline{r}') \times \underline{E}_L(\underline{r}') \right] \cdot \underline{\underline{G}}_L(\underline{r}';\underline{r}) \right\} dS'$$

$$+ \int_{S_\infty} \left\{ \underline{H}_L(\underline{r}') \cdot \left[\hat{n}(\underline{r}') \times \left\{ \nabla' \times \underline{\underline{G}}_L(\underline{r}';\underline{r}) \right\} \right] + i\omega\varepsilon \left[\hat{n}(r') \times \underline{E}_L(r') \right] \cdot \underline{\underline{G}}_L(\underline{r}';\underline{r}) \right\} dS' = 0$$

(A21)

The second integral in Eq. (A20) is zero due to the losses in V_L (or the radiation condition if V_L is lossless). Using the fact that the tangential components of $\underline{E}$ and $\underline{H}$ are continuous across S_L as well the boundary conditions in Eqs. (A5) and (A6), we see that the integral over S_L in Eq. (A10) is equal to the integral over S_L in Eq. (A21) which in turn we have just shown to equal zero. The integral over S_s will also equal zero, and the manner in which this can be seen depends on the physical properties of the subsystem occupying V_s. If it were totally metallic, the boundary conditions on $\underline{E}_I$ and $\underline{\underline{G}}_I$ would make the surface integral vanish in the same manner they did for the integral over S_p. If it were a homogeneous dielectric, then the boundary conditions would cause the surface integral over S_s to vanish in the same manner the surface integral over S_L was caused to vanish. If it were some hybrid of dielectric and metal, a combination of the arguments would be used to cause the surface integral to vanish.

We can now write Eqs. (A10) and (A11) as

$$\underline{H}_0(\underline{r}_0) = \underline{F}(\underline{r}_0) - K_0\underline{J}_m(\underline{r}') \tag{A22}$$

and

$$\underline{H}_I(\underline{r}_I) = K_I\underline{J}_m(\underline{r}') \tag{A23}$$

where

$$\underline{F}(\underline{r}_0) = \underline{I}(\underline{r}_0) + \underline{S}(\underline{r}_0) \tag{A24}$$

with $\underline{I}(\underline{r}_0)$ and $\underline{S}(\underline{r}_0)$ defined by Eqs. (A13) and (A16) and the operators K_α are defined by

$$K_\alpha\underline{J}_m(\underline{r}') = i\omega\varepsilon_0\left(\int_{S_1}\underline{J}_m(\underline{r}')\cdot\underline{\underline{G}}_\alpha(\underline{r}',r_\alpha)\,dS' + \int_{S_2}\underline{J}_m(r')\cdot\underline{\underline{G}}_\alpha(\underline{r}',\underline{r}_\alpha)\,dS'\right)$$

$$\alpha = 0, I \tag{A25}$$

and we have made use of the fact that the tangential components of the electric field are continuous through the apertures so that

$$-\hat{n}_0(\underline{r}')\times\underline{E}_0(\underline{r}') = \hat{n}_I(\underline{r}')\times\underline{E}_I(\underline{r}') = \underline{J}_m(\underline{r}') \qquad \underline{r}'\in S_1 \cup S_2 \tag{A26}$$

Now we focus our attention on $\underline{F}(\underline{r}_0)$ appearing in Eq. (A22). It would be a very difficult task to evaluate Eqs. (A13), (A16), and (A24) in order to determine the full significance of $\underline{F}(\underline{r}_0)$. Instead, we will simply utilize certain key features of those equations as well as Eqs. (A22) and (A25) to determine what $\underline{F}(\underline{r}_0)$ must be if all the required equations were evaluated. First, we note according to Eq. (A13) that $\underline{I}(\underline{r}_0)$ is excited by the rigid (interaction independent) source $\underline{J}(\underline{r}')$ and that according to Eq. (A16), $\underline{S}(\underline{r}_0)$ is excited by the rigidly specified $\hat{n}(\underline{r}')\times\underline{E}(\underline{r}')$ for $\underline{r}'\in S_g$. Next, we note that according to these equations, both $\underline{I}(\underline{r}_0)$ and $\underline{S}(\underline{r}_0)$ are insensitive to the size of the apertures S_1 and S_2 and in fact they are insensitive to whether or not these apertures are even present. Using these observations in conjunction with Eq. (A25) as the aperture size becomes zero and using the result in Eq. (A22), we see that $\underline{F}(\underline{r}_0)$ equals $\underline{H}_0(\underline{r})$ for the special case where all apertures are sealed (short circuited). Mathematically, we express this evaluation of $\underline{F}(\underline{r}_0)$ as

$$\underline{F}(\underline{r}_0) = \underline{H}_0^{s.c.}(\underline{r}_0) \tag{A27}$$

where the superscript is introduced to indicate "short circuit." We note that $\underline{F}(\underline{r}_0)$ is the short circuit magnetic field at some point $\underline{r}_0$ with apertures sealed, but all other aspects of the external environment including the proximity and structure of the radiator, S_r, are unchanged.

Substituting Eq. (A27) into Eq. (A22) we obtain

$$\underline{H}_0(\underline{r}_0) = \underline{H}_0^{s.c}(\underline{r}_0) - K_0\underline{J}_m(\underline{r}') \qquad (A28)$$

Next, we define $\hat{n}(\underline{r}) = \hat{n}_I(\underline{r}) = -\hat{n}_0(\underline{r})$ for $\underline{r}\epsilon S_1 U S_2$, use the fact that

$$\hat{n}(\underline{r}) \times \underline{H}_0(\underline{r}) = \hat{n}(\underline{r}) \times \underline{H}_I(\underline{r}) \qquad (A29)$$

and employ Eqs. (A22) and (A28) to obtain

$$\lim_{\substack{\underline{r}_0 \to \underline{r} \\ \underline{r}_I \to \underline{r}}} \hat{n}(r) \times (K_0+K_I)\ \underline{J}_m(\underline{r}') = \underline{J}_{E.I.}(\underline{r}) \qquad (A30)$$

where we have used the definition

$$\hat{n}(\underline{r}) \times \underline{H}_0^{s.c.}(\underline{r}) \equiv \underline{J}_{E.I.}(\underline{r}) \qquad (A31)$$

and we have the desired result, in that Eq. (A30) is the more detailed representation of Eq. (1).

Before we can present our theoretical conclusions, we must present our more detailed representation of Eq. (2). In fact, we already have a representation of Eq. (2) for the case where the desired internal electrical quantity is the magnetic field. For that use we might choose the symbol β as H so that $Q_H = \underline{H}$ and $L_\beta = L_H = K_I$. Another example where the structure of L_β changes depending on the choice of Q_β is readily demonstrated by considering the case where the desired internal electrical quantity is the electric field $\underline{E}$ and we denote β as E so that $Q_E = \underline{E}$. For this case Eq. (2) becomes

$$Q_E = L_E \underline{J}_m(\underline{r}) \tag{A32}$$

where

$$L_E = -\frac{1}{i\omega\varepsilon_0} \nabla \times K_I \tag{A33}$$

Finally, we will discuss the more important case where the desired internal electrical quantity is a current. For this discussion consider that part of the internal subsystem occupying volume V_s in Figure A1 contains a wire and we choose a local cylindrical coordinate system having its axis along the wire and having the local azimuthal vector denoted $\hat{\phi}_w(\ell')$ at the point on the wire where we wish to determine the current. The argument of this unit vector, ℓ', denotes the circumferential position on the wire. With these definitions, the current on the wire is

$$I = \oint d\ell' \hat{\phi}_w(\ell') \cdot \underline{H}_I(\underline{r}_I) \tag{A34}$$

We see from Eqs. (A29) and (A34) that

$$Q_c = L_c \underline{J}_m(\underline{r}') \tag{A35}$$

where we have denoted $I = Q_c$ and

$$L_c \underline{J}_m(\underline{r}') \equiv \oint d\ell' \hat{\phi}_w(\ell') \cdot K_I \underline{J}_m(\underline{r}') \tag{A36}$$

We have now presented Eqs. (1) and (2) in sufficient detail to draw our desired conclusions. The specific points we wish to make are:

1. The external interaction current density, $\underline{J}_{E.I.}$, can be excited by either a rigid source, a non-rigid source, or a combination of the two types.
2. The transfer operator, T^{α}_{β}, depends on the external environment to the system.
3. T^{α}_{β} depends on the internal environment.
4. T^{α}_{β} depends on the internal electrical quantity, Q_{β}, being determined.
5. T^{α}_{β} depends on the rigidity of the source.

Equations that specifically illustrate each of these points are identified with the numbered points as follows:

1. Eqs. (A13), (A16), (A24), (A27), and (A31);
2. Eqs. (A4), (A5), and (A6).
3. Eq. (A3) as well as the argument that eliminated the integral over S_s;
4. Eqs. (A33) and (A36);
5. Eq. (A4).

APPENDIX B. TRANSMISSION LINE THEORY

Referring to Figure 1, we replace the sources and transmission line to the left of z = 0 by a Thevenin equivalent circuit as depicted below (Fig. B1).

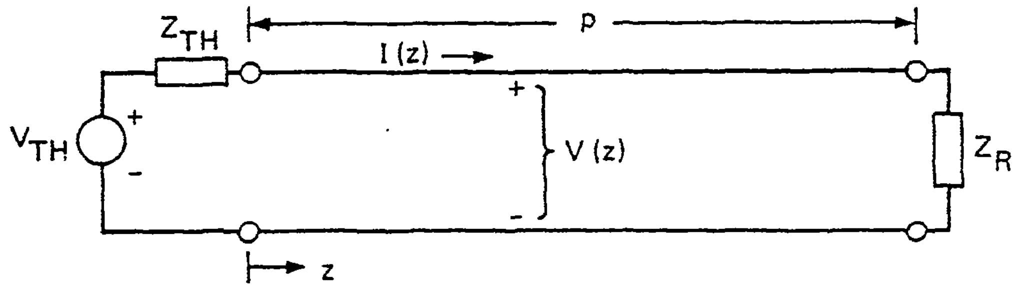

Figure B1. Transmission line model with Thevenin equivalent representing shorted end.

The source for this transmission line problem is given the subscript notation "TH" (V_{TH}, Z_{TH}) to indicate that we have applied Thevenin's theorem for the region to the left of the aperture. Details of the Thevenin equivalent will be presented later. The equations satisfied by V(z) and I(z) are:

$$\frac{d^2V(z)}{dz^2} + k_0^2 V(z) = 0 \tag{B1}$$

$$\frac{d^2I(z)}{dz^2} + k_0^2 I(z) = 0 \tag{B2}$$

The general solutions to these equations have the form

$$V = Ae^{ik_0 z} + Be^{-ik_0 z} \tag{B3}$$

$$I = Ce^{ik_0 z} + De^{-ik_0 z} \qquad (B4)$$

and it remains to solve for the four constants A,B,C, and D. From the required relationship

$$\frac{\partial V}{\partial z} = ik_0 Z_c I$$

and the linear independence of the functions $e^{ik_0 z}$ and $e^{-ik_0 z}$, it follows that

$$C = A/Z_c \qquad (B5)$$

$$D = -B/Z_c \qquad (B6)$$

From conditions imposed at the source, we have

$$V_{TH} = I(0)\ Z_{TH} + V(0)$$

or

$$V_{TH} = (C+D)\ Z_{TH} + A + B \qquad (B7)$$

From conditions imposed at the load, we have

$$V(p) = I(p)\ Z_R$$

or

$$0 = (CZ_R-A)\ e^{ik_0 p} + (DZ_R-B)\ e^{-ik_0 p} \qquad (B8)$$

It can be verified by substitution that a regrouping of the terms according to whether they multiply $e^{ik_0 z}$ or $e^{-ik_0 z}$ in the following expressions:

$$V(z) = \frac{Z_c}{Z_c + Z_{TH}} \frac{e^{ik_0 z} - \rho_R e^{ik_0(2p-z)}}{1 - \rho_{TH}\rho_R e^{2ik_0 p}} V_{TH} \tag{B9}$$

$$I(z) = \frac{1}{Z_c + Z_{TH}} \frac{e^{ik_0 z} + \rho_R e^{ik_0(2p-z)}}{1 - \rho_{TH}\rho_R e^{2ik_0 p}} V_{TH} \tag{B10}$$

gives A,B,C, and D which satisfy Eqs. (B5) through (B8). The remaining definitions of the terms appearing in Eqs. (B9) and (B10) are:

$$\rho_{TH} = \frac{Z_c - Z_{TH}}{Z_c + Z_{TH}} \tag{B11}$$

$$\rho_R = \frac{Z_c - Z_R}{Z_c + Z_R} \tag{B12}$$

A successive use of Eqs. (B9) and (B10) is required to obtain the expressions pertaining to the model depicted in Figure 1. That is, what we need are explicit expressions for V_{TH} and Z_{TH} in terms of V_{EQ}, I_{EQ}, and transmission line parameters. Consider Figure B2.

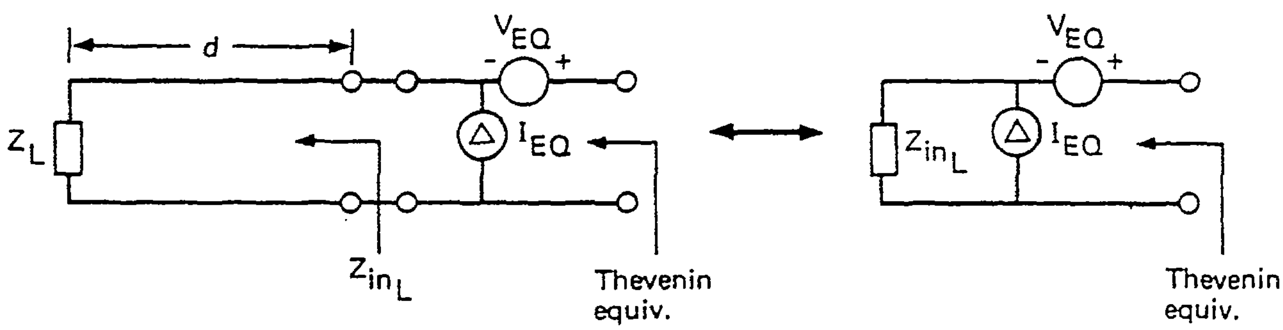

Figure B2. Model for the Thevenin equivalent construction.

The Thevenin equivalent parameters are easily obtained from the right-hand portion of Figure B2. The short-circuit current that would flow through a zero-impedance wire connecting the two terminals in that figure is

$$I_{sc} = I_{EQ} + V_{EQ}/Z_{inL} \tag{B13}$$

The open-circuit voltage that would appear across those terminals is

$$V_{0.c.} = V_{EQ} + I_{Eq} Z_{inL} \equiv V_{TH} \tag{B14}$$

The Thevenin impedance is

$$Z_{TH} = \frac{V_{0.c.}}{I_{s.c.}} = Z_{inL} \tag{B15}$$

From Eqs. (B14) and (B15) we see that we would have the desired description if we had Z_{inL} in terms of the parameters of interest. To obtain this representation, consider the left-hand side of Figure B2 reoriented (Fig. B3)

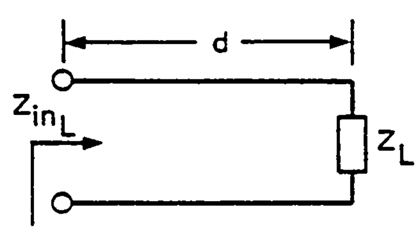

Figure B3. Portion of the transmission line reoriented.

and consider the situation (Fig. B4)

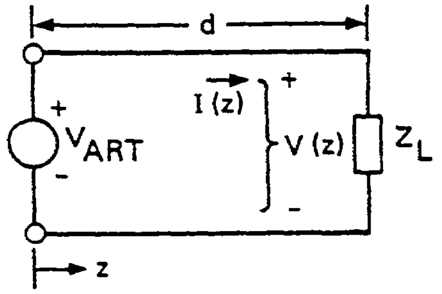

Figure B4. Model using V_{ART}.

where we have introduced an artificial voltage source V_{ART} to the transmission line as well as a coordination system description V(z) and I(z) analogous to the one introduced in Figure B1. The reason is that

$$Z_{inL} = \frac{V(0)}{I(0)} \tag{B16}$$

and this ratio is independent of V_{ART}. Specifically, using the analysis developed for the situation depicted in Figure B1, we can immediately find

$$V(0) = V_{ART} \tag{B17}$$

$$I(0) = \frac{1}{Z_c} \frac{1 + \rho_R' \, e^{2ik_0 d}}{1 - \rho_R' \, e^{2ik_0 d}} V_{ART} \tag{B18}$$

$$\rho_R' = \frac{Z_c - Z_L}{Z_c + Z_L} \tag{B19}$$

so that

$$Z_{inL} = Z_c \frac{1 - \rho_R' e^{2ik_0 d}}{1 + \rho_R' e^{2ik_0 d}}$$

and for our case $Z_L = 0$, so $\rho_R' = 1$, and

$$Z_{inL} = Z_c \frac{1 - e^{2ik_0 d}}{1 + e^{2ik_0 d}} \equiv Z_{TH} \tag{B20}$$

and

$$V_{TH} = V_{EQ} + I_{Eq} Z_{TH} \tag{B21}$$

Substituting Eqs. (B20) and (B21) into Eqs. (B9) and (B10) gives us our desired result. Performing this substitution and evaluating the expression for V(z) at z = p, we have

$$V(p) = T_{J_z}^{(c)} J_Z + T_{E_n}^{(c)} E_n \tag{B22}$$

where

$$T_{J_z}^{(c)} = KF\left(1 + e^{2ik_0 d}\right) \tag{B23}$$

and

$$T_{E_n}^{(c)} = (KF/2Z_0)\left(1 - e^{2ik_0 d}\right) \tag{B24}$$

with K defined in Eq. (12) and

$$F = \frac{(1-\rho_R)\ e^{ik_0 p}}{2\left(1 - \rho_R\ e^{2ik_0 L}\right)} \tag{B25}$$

and ρ_R is given in Eq. (B12).

This appendix presents a field equivalence principle that could be used as an alternative to the operator approach for serving as the basis for the transfer coefficient measurement procedure described in the text. The major difference of the work presented in this appendix and field equivalence principles that appear in the literature is the emphasis placed on external environment dependence. It is shown that fields penetrating a metallic enclosure retain a dependence on the environment exterior to the enclosure that is not completely accounted for by the effect of the exterior environment on the short circuit current density.

The field equivalent derivation is performed relative to the situation depicted in Figure C1.

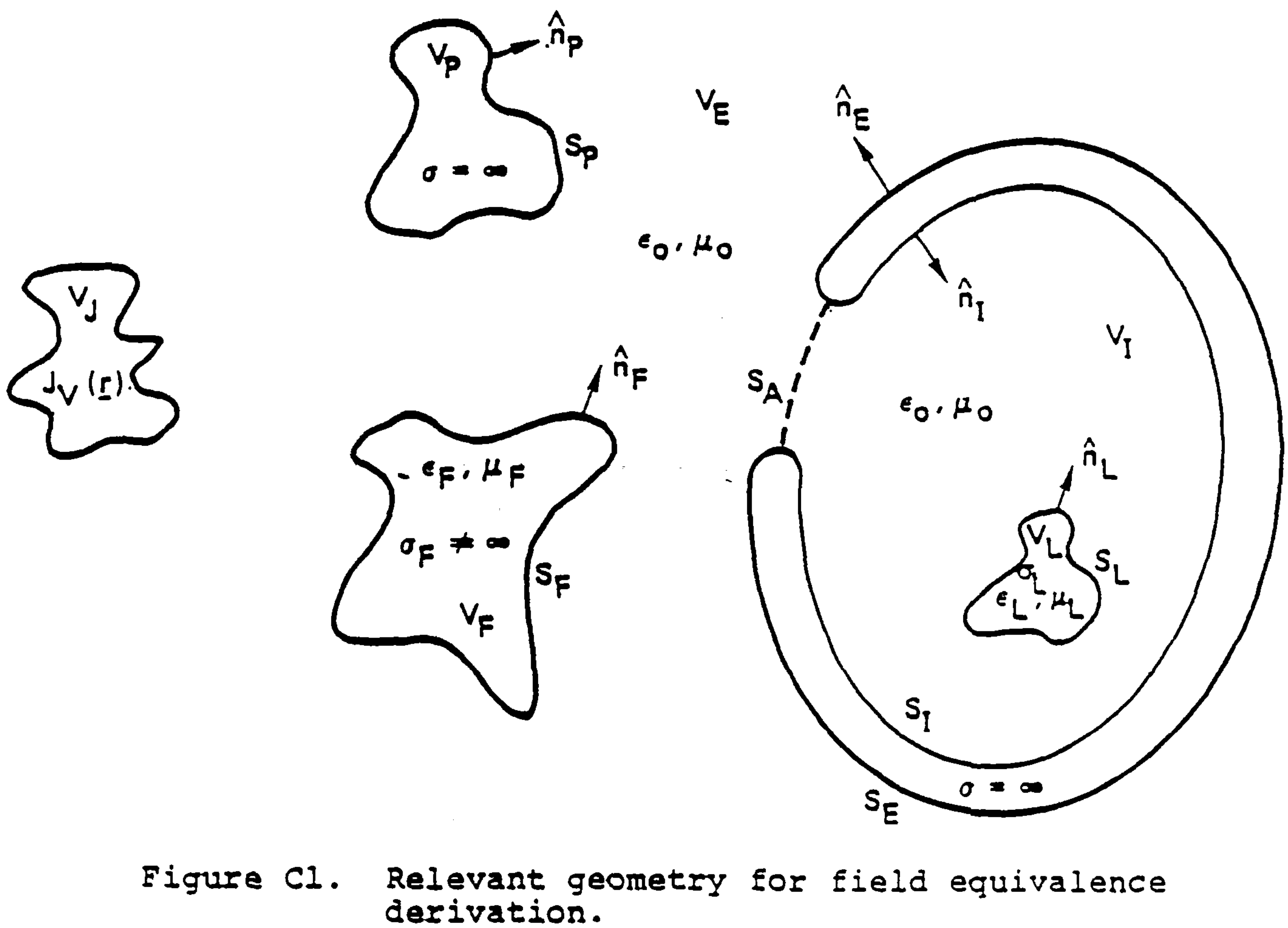

Figure C1. Relevant geometry for field equivalence derivation.

The metallic enclosure is bounded by the surfaces S_I and S_E, and the enclosure is not required to have zero thickness. The definition of a volume interior and exterior to the enclosure requires that we choose a mathematical surface S_A that becomes the definition of the surface of the aperture. The volume interior to the enclosure, V_I, is the one bounded by the surface S_I together with S_A. The volume exterior to the enclosure, V_E, is the one bounded by the surfaces S_E together with S_A as well as a surface at infinity. The exterior volume contains the volume V_J corresponding to the source volume current density $\underline{J}_V(\underline{r})$ which is the original source for all fields. In addition, V_E contains the volumes which comprise the "exterior environment" to the enclosure. These volumes are denoted V_p corresponding to a perfect conductor of general shape and V_F to finitely conducting volume of general shape. For generality, V_I contains lossy material occupying the volume V_L. We now write Maxwell's equations for the entire volume $V_E - V_p$ as well as for V_I:

$$\nabla \times \underline{E}_{\alpha\beta}(\underline{r}) = i\omega\mu_\beta \underline{H}_{\alpha\beta}(\underline{r}) \tag{C1}$$

$$\nabla \times \underline{H}_{\alpha\beta}(\underline{r}) = -i\omega\epsilon_\beta \underline{E}_{\alpha\beta}(\underline{r}) + \sigma_\beta \underline{E}_{\alpha\beta}(\underline{r}) + \underline{J}_V(\underline{r}) \tag{C2}$$

and the double subscripts are introduced to make both boundary conditions as well as subsequent definitions explicit. The subscript α represents either E or I according to whether $\underline{r}\epsilon\underline{V}_E$ or $\underline{r}\epsilon\underline{V}_I$ and β represents either 0, F, or L according to subvolumes in which $\underline{r}$ is located. It is also noted that $\underline{J}_V(\underline{r})$ is nonzero only if $\underline{r}\epsilon V_J$. In addition to satisfying Eqs. (C1) and (C2), the fields satisfy the following boundary conditions:

$$\hat{n}_p(\underline{r}) \times \underline{E}_{E0}(\underline{r}) = 0 \qquad \underline{r}\epsilon S_p \tag{C3}$$

$$\hat{n}_E(\underline{r}) \times \underline{E}_{E0}(\underline{r}) = 0 \qquad \underline{r} \epsilon S_E \tag{C4}$$

$$\hat{n}_I(\underline{r}) \times \underline{E}_{I0}(\underline{r}) = 0 \qquad \underline{r} \epsilon S_I \tag{C5}$$

$$\hat{n}_F(\underline{r}) \times \underline{E}_{EF}(\underline{r}) = \hat{n}_F(\underline{r}) \times \underline{E}_{E0}(\underline{r}) \qquad \underline{r} \epsilon S_F \tag{C6}$$

$$\hat{n}_F(\underline{r}) \times \underline{H}_{EF}(r) = \hat{n}_F(\underline{r}) \times \underline{H}_{E0}(\underline{r}) \qquad \underline{r} \epsilon S_F \tag{C7}$$

$$\hat{n}_L(\underline{r}) \times \underline{E}_{IL}(\underline{r}) = \hat{n}_L(\underline{r}) \times \underline{E}_{I0}(\underline{r}) \qquad \underline{r} \epsilon S_L \tag{C8}$$

$$\hat{n}_L(\underline{r}) \times \underline{H}_{IL}(\underline{r}) = \hat{n}_L(\underline{r}) \times \underline{H}_{I0}(\underline{r}) \qquad \underline{r} \epsilon S_L \tag{C9}$$

In addition, $\underline{E}_{E0}$ and $\underline{H}_{E0}$ satisfy the radiation condition at infinity. These boundary conditions are sufficient to guarantee a unique solution to Eqs. (C1) and (C2). In order to proceed, it is necessary to define the external interaction fields. These are fields $\underline{E}_{E\beta}^{E.I.}$ and $\underline{H}_{E\beta}^{E.I.}$ that satisfy Eqs. (C1) and (C2) as well as the boundary conditions

$$\hat{n}_p(\underline{r}) \times \underline{E}_{E0}^{E.I.}(\underline{r}) = 0 \qquad \underline{r} \epsilon S_p \tag{C10}$$

$$\hat{n}_E(\underline{r}) \times \underline{E}_{E0}^{E.I.}(\underline{r}) = 0 \qquad \underline{r} \epsilon S_E U S_A \tag{C11}$$

$$\hat{n}_F(\underline{r}) \times \underline{E}_{EF}^{E.I.}(\underline{r}) = \hat{n}_F(\underline{r}) \times \underline{E}_{E0}^{E.I.}(\underline{r}) \qquad \underline{r} \epsilon S_F \tag{C12}$$

$$\hat{n}_F(\underline{r}) \times \underline{H}_{EF}^{E.I.}(\underline{r}) = \hat{n}_F(\underline{r}) \times \underline{H}_{E0}^{E.I.}(\underline{r}) \qquad \underline{r} \epsilon S_F \tag{C13}$$

as well as the radiation condition at infinity. These boundary conditions guarantee a unique solution. Next we define the fields

$$\underline{E}^{S}_{E\beta} = \underline{E}_{E\beta} - \underline{E}^{E.I.}_{E\beta} \quad \text{(C14)}$$

$$\underline{H}^{S}_{E\beta} = \underline{H}_{E\beta} - \underline{H}^{E.I.}_{E\beta} \quad \text{(C15)}$$

The fields on the right-hand side of Eqs. (C14) and (C15) are unique, so it follows that the left-hand sides are unique.

Next we introduce the fields $\underline{\tilde{E}}_{\alpha\beta}$ and $\underline{\tilde{H}}_{\alpha\beta}$ which satisfy Eqs. (C1) and (C2) with $\underline{J}_V(\underline{r}) = 0$ even when $\underline{r} \epsilon V_J$. These fields satisfy the boundary conditions given in Eqs. (C3) through (C9), the radiation condition, and the following equation defines their source of excitation

$$\hat{n}_E(\underline{r}) \times \left(\underline{\tilde{H}}_{I0}(\underline{r}) - \underline{\tilde{H}}_{E0}(\underline{r})\right) = \hat{n}_E(\underline{r}) \times \underline{H}^{E.I.}_{E0}(\underline{r})$$

$$\equiv \underline{J}_{E.I.}(\underline{r}) \qquad \underline{r} \epsilon S_A \quad \text{(C16)}$$

Shortly it will be shown that

$$\underline{\tilde{E}}_{E\beta} = \underline{E}^{S}_{E\beta} \quad \text{(C17)}$$

$$\underline{\tilde{H}}_{E\beta} = \underline{H}^{S}_{E\beta} \quad \text{(C18)}$$

$$\underline{\tilde{E}}_{I\beta} = \underline{E}_{I\beta} \quad \text{(C19)}$$

$$\underline{\tilde{H}}_{I\beta} = \underline{H}_{I\beta} \quad \text{(C20)}$$

Eqs. (C16), (C19) and (C20) are the key equations in describing the field equivalence principle. Eq. (C16) exhibits that the external interaction current density is the only source for the $\tilde{E}_{\alpha\beta}$, $\tilde{H}_{\alpha\beta}$ fields. Eqs. (C19) and (C20) state that those fields excited within the enclosure are the same as those excited by the actual volume source $\underline{J}_V(\underline{r})$. In addition, it is important to note that $\underline{J}_{E.I.}(\underline{r})$ excites the fields $\tilde{\underline{E}}_{I\beta}$ and $\tilde{\underline{H}}_{I\beta}$ in a manner that simultaneously requires $\tilde{\underline{E}}_{E\beta}$ and $\tilde{\underline{H}}_{E\beta}$ to satisfy boundary conditions in the environment external to the cavity. This fact will be required to prove Eqs. (C19) and (C20). This fact is also a statement that the external environment dependence of the cavity fields is not totally accounted for by its effect on $\underline{J}_{E.I.}$.

To prove Eqs. (C17) through (C20), we introduce

$$\underline{E}^D_{E\beta} = \tilde{\underline{E}}_{E\beta} - \underline{E}^S_{E\beta} \tag{C21}$$

$$\underline{H}^D_{E\beta} = \tilde{\underline{H}}_{E\beta} - \underline{H}^S_{E\beta} \tag{C22}$$

$$\underline{E}^D_{I\beta} = \tilde{\underline{E}}_{I\beta} - \underline{E}_{I\beta} \tag{C23}$$

$$\underline{H}^D_{I\beta} = \tilde{\underline{H}}_{I\beta} - \underline{H}_{I\beta} \tag{C24}$$

and form the relationship utilizing a vector identity and Maxwell's equations without a volume source

$$\nabla \cdot \left[\underline{E}^D_{\alpha\beta} \times \underline{H}^{D*}_{\alpha\beta}\right] = \underline{H}^{D*}_{\alpha\beta} \cdot \nabla \times \underline{E}^D_{\alpha\beta} - \underline{E}^D_{\alpha\beta} \cdot \nabla \times \underline{H}^{D*}_{\alpha\beta}$$

$$= i\omega\mu_\beta \left|\underline{H}^D_{\alpha\beta}\right|^2 - i\omega\epsilon_\beta \left|\underline{E}^D_{\alpha\beta}\right|^2 + \sigma_\beta \left|\underline{E}^D_{\alpha\beta}\right|^2 \tag{C25}$$

Next, we use the divergence theorem and integrate this equation over the volumes $V_E - V_p - V_F$, V_F, $V_I - V_L$, and V_L to obtain

$$\int_{V_E - V_p - V_F} \left\{ i\omega\left(\mu_0 \left|\underline{H}^D_{E0}\right|^2 - \epsilon_0 \left|\underline{E}^D_{E0}\right|^2\right) + \sigma_0 \left|\underline{E}^D_{E0}\right|^2 \right\} dV$$

$$= \int_{S\infty} \hat{n}_\infty \cdot \left[\underline{E}^D_{E0} \times \underline{H}^{D*}_{E0}\right] dS + \int_{S_p} - \hat{n}_p \cdot \left[\underline{E}^D_{E0} \times \underline{H}^{D*}_{E0}\right] dS$$

$$+ \int_{S_F} - \hat{n}_F \cdot \left[\underline{E}^D_{E0} \times \underline{H}^{D*}_{E0}\right] dS + \int_{S_E} - \hat{n}_E \cdot \left[\underline{E}^D_{E0} \times \underline{H}^{D*}_{E0}\right] dS$$

$$+ \int_{S_A} - \hat{n}_E \cdot \left[\underline{E}^D_{E0} \times \underline{H}^{D*}_{E0}\right] dS \qquad \text{(C26)}$$

$$\int_{V_F} \left\{ i\omega\left(\mu_F \left|\underline{H}^D_{EF}\right|^2 - \epsilon_F \left|\underline{E}^D_{EF}\right|^2\right) + \sigma_F \left|\underline{E}^D_{EF}\right|^2 \right\} dV$$

$$= \int_{S_F} \hat{n}_F \cdot \left[\underline{E}^D_{EF} \times \underline{H}^{D*}_{EF}\right] dS \qquad \text{(C27)}$$

$$\int_{V_I - V_L} \left\{ i\omega\left(\mu_0 \left|\underline{H}^D_{I0}\right|^2 - \epsilon_0 \left|\underline{E}^D_{I0}\right|^2\right) + \sigma_0 \left|\underline{E}^D_{I0}\right|^2 \right\} dV$$

$$= \int_{S_L} - \hat{n}_L \cdot \left[\underline{E}_{I0}^D \times \underline{H}_{I0}^{D*} \right] dS + \int_{S_I} - \hat{n}_I \cdot \left[\underline{E}_{I0}^D \times \underline{H}_{I0}^{D*} \right] dS$$

$$+ \int_{S_A} - \hat{n}_I \cdot \left[\underline{E}_{I0}^D \times \underline{H}_{I0}^{D*} \right] dS \qquad \text{(C28)}$$

$$\int_{V_L} \left\{ i\omega \left(\mu_L \left| \underline{H}_{IL}^D \right|^2 - \varepsilon_L \left| \underline{E}_{IL}^D \right|^2 \right) + \sigma_L \left| \underline{E}_{IL}^D \right|^2 \right\} dV$$

$$= \int_{S_L} \hat{n}_L \cdot \left[\underline{E}_{IL}^D \times \underline{H}_{IL}^{D*} \right] dS \qquad \text{(C29)}$$

Eqs. (C26) through (C29) are now summed, and the boundary conditions given in Eqs. (C3) through (C9) as well as the radiation condition are used to eliminate all the surface integrals with the exception of the integrals over S_A. The resulting equation, using the fact that $\hat{n}_E = -\hat{n}_I$, is written as

$$\int_{V_E - V_P - V_F} \left\{ i\omega \left(\mu_0 \left| \underline{H}_{E0}^D \right|^2 - \varepsilon_0 \left| \underline{E}_{E0}^D \right|^2 \right) + \sigma_0 \left| \underline{E}_{E0}^D \right|^2 \right\} dV$$

$$+ \int_{V_F} \left\{ i\omega \left(\mu_F \left| \underline{H}_{EF}^D \right|^2 - \varepsilon_F \left| \underline{E}_{EF}^D \right|^2 \right) + \sigma_F \left| \underline{E}_{EF}^D \right|^2 \right\} dV$$

$$+\int_{V_I-V_L}\left\{i\omega\left(\mu_0\left|\underline{H}_{I0}^D\right|^2-\varepsilon_0\left|\underline{E}_{I0}^D\right|^2\right)+\sigma_0\left|\underline{E}_{I0}^D\right|^2\right\}dV$$

$$+\int_{V_L}\left\{i\omega\left(\mu_L\left|\underline{H}_{IL}^D\right|^2-\varepsilon_L\left|\underline{E}_{IL}^D\right|^2\right)+\sigma_L\left|\underline{E}_{IL}^D\right|^2\right\}dV$$

$$=\int_{S_A}\left\{\underline{E}_{E0}^D\cdot\left[\hat{n}_E\times\underline{H}_{E0}^{D*}\right]-\underline{E}_{I0}^D\cdot\left[\hat{n}_E\times\underline{H}_{I0}^{D*}\right]\right\}dS$$

$$=\int_{S_A}\left\{\left[-\hat{n}_E\times\left(\hat{n}_E\times\underline{E}_{E0}^D\right)\right]\cdot\left(\hat{n}_E\times\underline{H}_{E0}^{D*}\right)\right.$$

$$\left.-\left[-\hat{n}_E\times\left(\hat{n}_E\times\underline{E}_{I0}^D\right)\right]\cdot\left(\hat{n}_E\times\underline{H}_{I0}^{D*}\right)\right\}dS \qquad \text{(C30)}$$

Next, it will be argued that the right-hand side of of Eq. (C30) is zero. The terms appearing in this are expanded using the previously presented definitions

$$\hat{n}_E\times\underline{E}_{E0}^D=\hat{n}_E\times\underline{E}_{E0}^{E.I.}+\hat{n}_E\times\underline{\tilde{E}}_{E0}-\hat{n}_E\times\underline{E}_{E0} \qquad \text{(C31)}$$

$$\hat{n}_E\times\underline{E}_{I0}^D=\hat{n}_E\times\underline{\tilde{E}}_{I0}-\hat{n}_E\times\underline{E}_{I0} \qquad \text{(C32)}$$

$$\hat{n}_E\times\underline{H}_{E0}^D=\underline{J}^{E.I.}+\hat{n}_E\times\underline{\tilde{H}}_{E0}-\hat{n}_E\times\underline{H}_{E0} \qquad \text{(C33)}$$

$$\hat{n}_E\times\underline{H}_{I0}^D=\hat{n}_E\times\underline{\tilde{H}}_{I0}-\hat{n}_E\times\underline{H}_{I0} \qquad \text{(C34)}$$

It is necessary to argue that

$$\hat{n}_E \times \underline{E}_{E0} = \hat{n}_E \times \underline{E}_{I0} \qquad \underline{r} \in S_A \tag{C35}$$

and

$$\hat{n}_E \times \underline{H}_{E0} = \hat{n}_E \times \underline{H}_{I0} \qquad \underline{r} \in S_A \tag{C36}$$

These two equations follow from the fact that $\underline{E}_{\alpha 0}$ and $\underline{H}_{\alpha 0}$ are determined under the conditions that S_A is just a mathematical surface having no physical significance. In contrast, $\tilde{\underline{E}}_{\alpha 0}$ and $\tilde{\underline{H}}_{\alpha 0}$ are excited by a physical source at the location S_A. In order to draw our desired conclusions, it is necessary to specify boundary conditions in addition to the previously presented ones for these fields. These boundary conditions are

$$\hat{n}_E \times \tilde{\underline{E}}_{E0} = \hat{n}_E \times \tilde{\underline{E}}_{I0} \qquad \underline{r} \in S_A \tag{C37}$$

Now it is noted that Eq. (C11) implies

$$\hat{n}_E \times \underline{E}_{E0}^{E.I.} = 0 \qquad \underline{r} \in S_A \tag{C38}$$

Substituting Eqs. (C16) and (C31) through (C38) into the right-hand side of Eq. (C30) leads to the conclusion that this right-hand side is zero. This in turn implies that the left-hand side of Eq. (C30) is zero. Equating the real and imaginary parts of the left-hand side of Eq. (C30) leads to the conclusion

$$|\underline{H}_{\alpha\beta}^{D}|^2 = 0 \tag{C39}$$

$$|\underline{E}_{\alpha\beta}^{D}|^2 = 0 \tag{C40}$$

Eqs. (C39) and (C40) imply our desired conclusion given by Eqs. (C19) and (C20). A final note is that the proof made use of the standard procedure of considering $\sigma_0 \neq 0$ even though the case of interest is $\sigma_0 = 0$. The usual argument of stating that all media have some amount of conductivity could suffice to justify this procedure or one could argue that the solution is continuous as σ_0 approaches zero.

An analysis which makes the dependence of $\underline{\tilde{E}}_{I\beta}$, $\underline{\tilde{H}}_{I\beta}$ on the external environment more explicit will now be presented. The fields $\underline{E}^{Xi}_{\alpha\beta}$, $\underline{H}^{Xi}_{\alpha\beta}$ are introduced which satisfy the equations

$$\nabla \times \underline{E}^{Xi}_{\alpha\beta}(\underline{r},\underline{r}_D) = i\omega\mu_\beta \underline{H}^{Xi}_{\alpha\beta}(\underline{r},\underline{r}_D) \tag{C41}$$

$$\begin{aligned} \nabla \times \underline{H}^{Xi}_{\alpha\beta}(\underline{r},\underline{r}_D) &= -i\omega\epsilon_\beta \underline{E}^{Xi}_{\alpha\beta}(\underline{r},\underline{r}_D) \\ &+ \sigma_\beta \underline{E}^{Xi}_{\alpha\beta}(\underline{r},\underline{r}_D) + \hat{X}_i \delta(\underline{r}-\underline{r}_D) \qquad \underline{r}_D \epsilon V_I - V_L \end{aligned} \tag{C42}$$

and they satisfy the same boundary conditions as do $\underline{E}_{\alpha\beta}$, $\underline{H}_{\alpha\beta}$. Using a standard vector identity and Eqs. (C1), (C2), (C41) and (C42), we obtain

$$\nabla \cdot \left[\underline{E}^{Xi}_{\alpha\beta} \times \underline{\tilde{H}}_{\alpha\beta} - \underline{\tilde{E}}_{\alpha\beta} \times \underline{H}^{Xi}_{\alpha\beta}\right] = \underline{\tilde{E}}_{\alpha\beta}(\underline{r}) \cdot \hat{X}_i \delta(\underline{r}-\underline{r}_D) \tag{C43}$$

Integrating Eq. (C43) over $V_E - V_P$, V_F, $V_I - V_L$, and V_L and using the boundary conditions, we obtain

$$\int_{S_A} -\hat{n}_E \cdot \left[\underline{E}^{Xi}_{E0} \times \underline{\tilde{H}}_{E0} - \underline{\tilde{E}}_{E0} \times \underline{H}^{Xi}_{E0}\right] dS = 0 \tag{C44}$$

$$\int_{S_A} -\hat{n}_I \cdot \left[\underline{E}^{Xi}_{I0} \times \underline{\tilde{H}}_{I0} - \underline{\tilde{E}}_{I0} \times \underline{H}^{Xi}_{I0}\right] dS = \hat{X}_i \cdot \underline{E}_{I0}(\underline{r}_D) \tag{C45}$$

From the described boundary conditions it follows that

$$\hat{n}_E \cdot \left(\hat{\underline{E}}_{E0} \times \underline{H}_{E0}^{Xi}\right) = -\hat{n}_I \cdot \left(\tilde{\underline{E}}_{I0} \times \underline{H}_{I0}^{Xi}\right) \qquad \text{(C46)}$$

Combining Eqs. (C44), (C45), and (C46) we have

$$\int_{S_A} \left\{ -\hat{n}_I \cdot \left[\underline{E}_{I0}^{Xi} \times \tilde{\underline{H}}_{I0}\right] - \hat{n}_E \cdot \left[\underline{E}_{E0}^{Xi} \times \tilde{\underline{H}}_{E0}\right]\right\} dS = \hat{x}_i \cdot \underline{E}_{I0}(\underline{r}_D) \qquad \text{(C47)}$$

Using the continuity of $\hat{n}_E \times \underline{E}_{\alpha 0}^{Xi}$ on S_α as well as Eq. (C16), we have

$$\int_{S_A} \underline{E}_{I0}^{Xi}(\underline{r},\underline{r}_D) \cdot \underline{J}_{E.I.}(\underline{r})\, dS = \hat{x}_i \cdot \underline{E}_{I0}(\underline{r}_D) \qquad \text{(C48)}$$

Multiply both sides of Eq. (C48) by $\hat{x}_i$ and summing over the three orthogonal directions $\hat{x}_1$, $\hat{x}_2$, and $\hat{x}_3$ we obtain

$$\int_{S_A} \underline{\underline{G}}(\underline{r},\underline{r}_D) \cdot \underline{J}_{E.I.}(\underline{r})\, dS = \underline{E}_{I0}(\underline{r}_D) \qquad \text{(C49)}$$

where

$$\underline{\underline{G}}(\underline{r},\underline{r}_D) = \sum_{i=1}^{3} \hat{x}_i \underline{E}_{I0}^{Xi}(\underline{r},\underline{r}_D) \qquad \text{(C50)}$$

Eq. (C49) demonstrates that the external environment dependence of $\underline{E}_{I0}(\underline{r}_D)$ is contained in $\underline{\underline{G}}(\underline{r},\underline{r}_D)$ as well as through $\underline{J}_{E.I.}(\underline{r})$. This is the case because $\underline{E}_{I0}^{Xi}(\underline{r},\underline{r}_D)$ must be determined simultaneously with $E_{E\beta}^{Xi}(\underline{r},\underline{r}_D)$ which satisfies boundary conditions in the external environment. The magnetic field at $\underline{r}_D$ is given by

$$\underline{H}_{I0}(\underline{r}_D) = (i\omega\mu_0)^{-1} \nabla_D \times \int_{S_A} \underline{\underline{G}}(\underline{r},\underline{r}_D) \cdot \underline{J}_{E.I.}(\underline{r})dS \qquad \text{(C51)}$$

A large variety of internal electrical quantities are linear operators on $\underline{E}_{I0}(\underline{r}_D)$ and $\underline{H}_{I0}(\underline{r}_D)$, and Eqs. (C49) and (C51) imply Eq. (4). The assumption of an inverse operator regarding that equation has also been proved.

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