Sensor and Simulation Notes

Note XXVII

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Impedances and Field Distributions for Symmetrical Two Wire and Four Wire Transmission Line Simulators

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Abstract

The impedances and field distributions of the symmetrical two wire and four wire transmission lines are calculated. For a particular ratio of the cross section dimensions, the four wire case has a very uniform field distribution near the center.

I. Introduction.

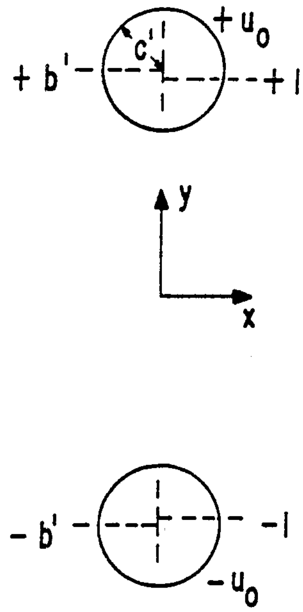
In the simulation of nuclear EMP fields, it is desirable to produce comparable fields in the form of a TEM wave on a cylindrical transmission line. Furthermore, it is often desired that there be a uniform electric and magnetic field distribution over some part of the cross section of the transmission line. Thus, part of a uniform free-space plane wave is approximated.

By convention, the z axis of a Cartesian or cylindrical coordinate system is taken as the direction of propagation of the TEM wave. The (x, y) or (r, ϕ) plane at constant z (the geometry being independent of z) is the cross section of the transmission line. In a previous note we have discussed two types of parallel plate transmission lines.[1] While these are useful for producing reasonably uniform fields over a significant fraction of the cross section enclosed by the conductors, there may be other types of transmission lines with such a property. Although the parallel plate structures are convenient to use in some cases, in others they might be unnecessarily cumbersome, in particular for very large structures. In this note we consider some structures which are somewhat sparser than the parallel plate type.

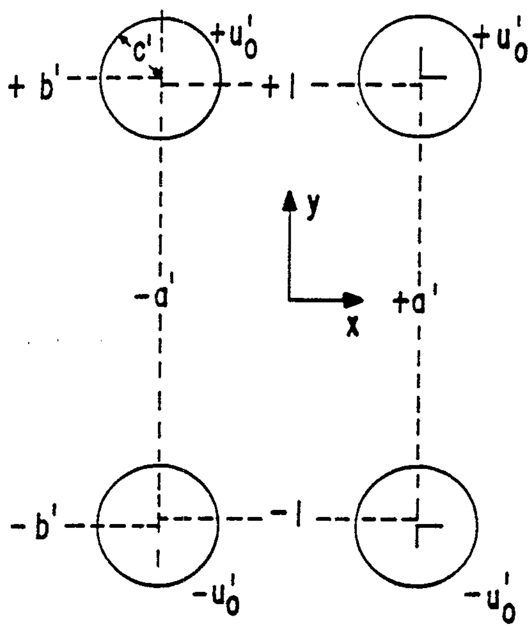
The transmission lines to be considered are the symmetrical two wire and four wire structures. The cross sections of these transmission lines are illustrated in figure 1. In each case there are two planes of symmetry, given by $x = 0$ and $y = 0$, respectively. It is assumed that the structures are far enough away from perturbing objects, such as the ground, to be insignificantly affected by their presence. The circularly cylindrical wires, of radius, c, are separated by a distance, 2b, between centers in the y direction. In the four wire case (figure 1B), the wires are located at the corners of a rectangle with sides of length, 2a and 2b. For convenience in the analysis, the positions have been normalized by dividing by b_o. The separation of infinitesimal line charges and/or currents, from which the field distribution for the two wire transmission line is developed, is $2b_o$. The normalized positions are denoted by the addition of a prime over the letter.

Using a conformal transformation we first investigate some of the characteristics of a symmetrical two wire transmission line. The symmetrical four wire transmission line is then developed by superimposing the field distributions of two, two wire lines. For a certain ratio of dimensions (a particular a') the four wire structure can sustain a very uniform field near the center of its cross section. For both types of transmission line the impedance, field efficiencies (or effectiveness in converting voltage and current to uniform electric and magnetic fields), and field distributions are discussed. The solutions for both the two wire and four wire transmission lines also apply (when appropriate factors of two are included) to transmission lines formed by replacing the wires at negative y by a sufficiently large conducting plane at $y = 0$ (a plane of symmetry).

1. Lt Carl E. Baum, Sensor and Simulation Note XXI, Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.



A. SYMMETRICAL TWO WIRE TRANSMISSION LINE



B. SYMMETRICAL FOUR WIRE TRANSMISSION LINE

FIGURE 1. NORMALIZED TRANSMISSION LINE GEOMETRIES

II. Symmetrical Two Wire Transmission Line.

Consider first the symmetrical two wire transmission line as in figure 1A. In normalized form we have the conformal transformation[2]

$$w = \ln\left(\frac{z+j}{z-j}\right) = 2j\,\mathrm{arccot}\ (z) \tag{1}$$

where

$$z = x+jy \tag{2}$$

and

$$w = u+jv \tag{3}$$

This corresponds to equal and opposite line charges and/or currents at $(x,y) = (0,\pm 1)$. The equipotentials and magnetic field lines are given by constant u and the electric field lines by constant v. These are illustrated for positive x and y in figure 2. This field plot can be extended to all four quadrants by symmetry.

Expanding equation (1) gives for the potential functions

$$u = \frac{1}{2}\ \ln\left[\frac{x^2+(1+y)^2}{x^2+(1-y)^2}\right] \tag{4}$$

and

$$v = \arctan\left[\frac{2x}{x^2+y^2-1}\right] \tag{5}$$

Rewriting equation (1) as

$$z = \cot\left(-\frac{jw}{2}\right) = j\coth\ \left(\frac{w}{2}\right) \tag{6}$$

it can also be expanded as

$$x = \frac{\sin(v)}{\cosh(u)-\cos(v)} \tag{7}$$

and

$$y = \frac{\sinh(u)}{\cosh(u)-\cos(v)} \tag{8}$$

The contours of constant u and v (figure 2) are made from these last two equations.

Considering curves of constant u, equation (4) gives

$$e^{u}\ [x^2+y^2-2y+1] = e^{-u}\ [x^2+y^2+2y+1] \tag{9}$$

2. W. R. Smythe, Static and Dynamic Electricity, 2nd ed., 1950, p. 76.

FOR $u<.5\pi$, u AND v ARE IN INCREMENTS OF $.05\pi$.
FOR $u>.5\pi$, u AND v ARE IN INCREMENTS OF $.1\pi$.

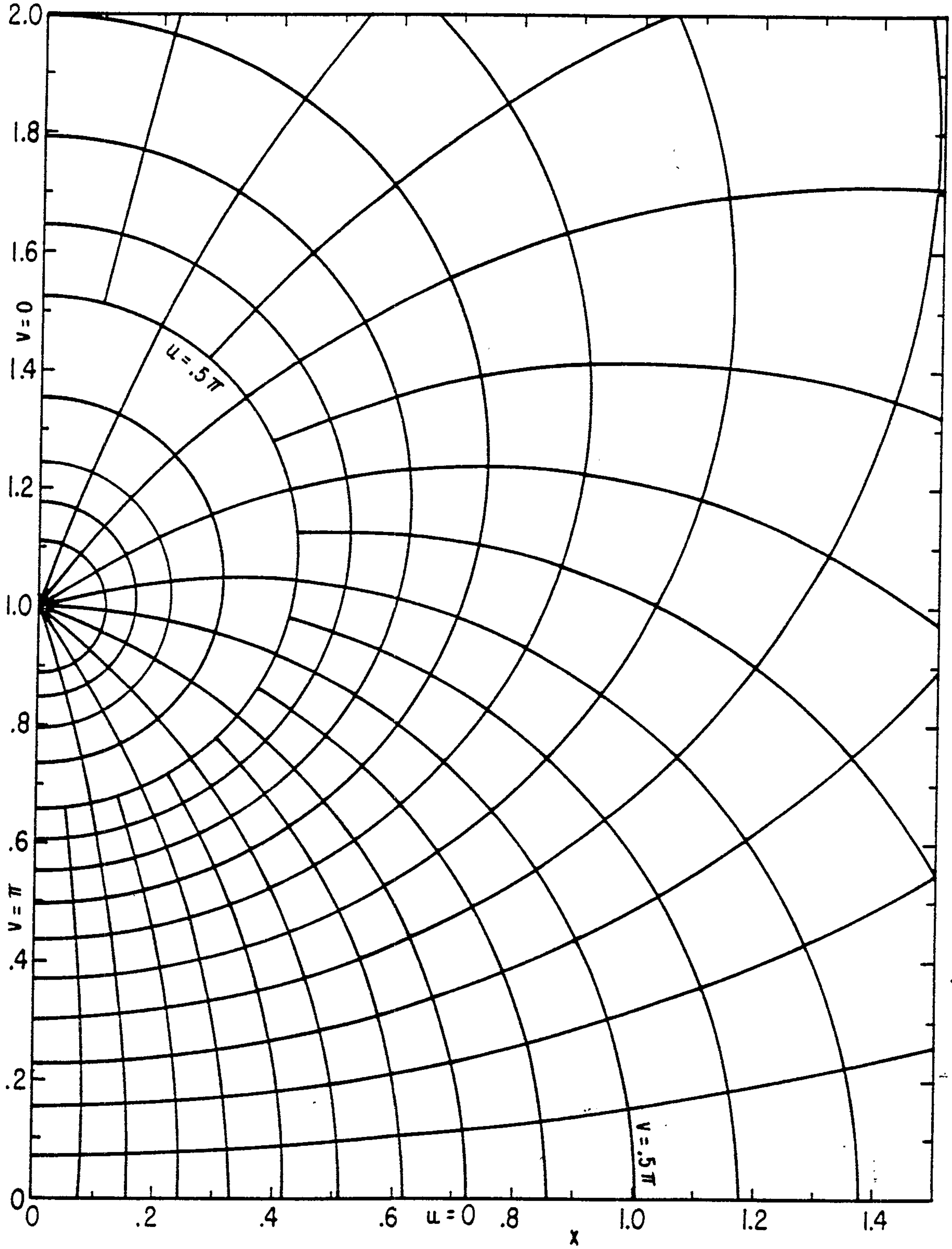


FIGURE 2. FIELD AND POTENTIAL DISTRIBUTION FOR SYMMETRICAL TWO WIRE TRANSMISSION LINE

which can be rearranged as

$$x^2+y^2+1 - 2y \coth(u) = 0 \tag{10}$$

or

$$x^2+ (y - \coth(u))^2 = \operatorname{csch}^2(u) \tag{11}$$

This last equation shows that the lines of constant u, equipotentials, are circles. As such, conducting circular cylinders can be placed with their surfaces at constant u without disturbing the field pattern. These conductors are extended in the z direction. Consider two such wires, each of normalized radius, c', and centered at the symmetrical positions given by $(x,y) = (0,\underline{+}1)$. These two conductors are at potentials, $\underline{+}\, u_o$. The upper conductor intersects the y axis (from equation (11)) at

$$y = \coth(u_o) \pm \operatorname{csch}(u_o) \tag{12}$$

Thus, the normalized center of the conductor is

$$b' = \coth(u_o) \tag{13}$$

and the normalized radius is given by

$$c' = \operatorname{csch}(u_o) \tag{14}$$

The ratio of dimensions is then

$$\frac{b}{c} = \frac{b'}{c'} = \cosh(u_o) \tag{15}$$

We can now relate b and c to the position of an equivalent line charge and/or current, b_o. From equation (13)

$$b' = \frac{b}{b_o} = \frac{\cosh(u_o)}{[\cosh^2(u_o)-1]^{1/2}} = \frac{\frac{b}{c}}{[(\frac{b}{c})^2-1]^{1/2}} \tag{16}$$

or

$$b^2 = b_o^{\,2} + c^2 \tag{17}$$

In normalized form

$$b'^2 = 1+c'^2 \tag{18}$$

Given b and c we can calculate b_o to relate the symmetrical two wire transmission line to the normalized coordinates.

The pulse impedance, Z_L, of the transmission line is related to the wave impedance, Z, by

$$Z_L = f_g Z \tag{19}$$

where f_g is a dimensionless geometric factor. This is given by

$$f_g = \frac{2u_o}{2\pi} \tag{20}$$

where $2u_o$ is the difference in the potential function, u, between the two wiresand 2π is the change in v for a path circling one wire. Thus, using equation (15)

$$f_g = \frac{1}{\pi} \text{ arccosh } \left(\frac{b}{c}\right) = \frac{1}{\pi} \ln\left[\frac{b}{c} + \left(\left(\frac{b}{c}\right)^2 - 1\right)^{1/2}\right] \tag{21}$$

This and other parameters for the two wire case are plotted later along with the corresponding parameters for the four wire case.

Next, let us relate the electric and magnetic fields to the voltages and currents on the transmission line. Specifically, consider the electric and magnetic fields at the origin of the coordinate system ((x,y) = (0,0)) around which the field distribution is symmetric. Considering first the electric field, define an effective half-spacing, b_1, relating the potential of the wires to the electric field between the two wires. Then in normalized form

$$\frac{1}{b_1'} = \frac{1}{u_o} \left.\frac{\partial u}{\partial y}\right|_{\substack{x=0 \\ y=0}} \tag{22}$$

Since, for $|y| < 1$,

$$u\Big|_{x=0} = \ln\left[\frac{1+y}{1-y}\right] \tag{23}$$

and

$$\left.\frac{\partial u}{\partial y}\right|_{x=0} = \frac{1}{1+y} + \frac{1}{1-y} \tag{24}$$

then

$$b_1' = \frac{u_o}{2} = \frac{1}{2} \text{ arccosh } \left(\frac{b'}{c'}\right) \tag{25}$$

Or,

$$b_1 = \frac{b_o}{2} \text{ arccosh } \left(\frac{b}{c}\right) = \frac{\sqrt{b^2-c^2}}{2} \text{ arccosh } \left(\frac{b}{c}\right) \tag{26}$$

For the two wires at potentials, $\pm V$, the electric field, E_y, is in the y direction and is just given by V/b_1. This is just the result for the electric field near the center of a two conductor, parallel plate transmission line of spacing, $2b_1$, with a width much greater than this spacing.[3] If the two wire case with a closest spacing of 2(b-c) and the

3. See reference 1.

same potentials is considered, however, the electric field in the center is less. Thus, an electric field efficiency may be defined as

$$f_E = \frac{b-c}{b_1} = \frac{2}{\operatorname{arccosh}\left(\frac{b}{c}\right)} \left(\frac{\frac{b}{c} - 1}{\frac{b}{c} + 1} \right)^{1/2} \tag{27}$$

Then the electric field in the center of the two wire transmission line is calculated from

$$E_y \Bigg|_{\substack{x=0\\y=0}} = \frac{\Delta V}{2b_1} = \frac{\Delta V}{2(b-c)} f_E \tag{28}$$

where ΔV is the potential between the wires.

Considering the magnetic field, define an effective half-width, a_1, for the transmission line relating the current in the wires to the magnetic field between the two wires. Then, in normalized form

$$\frac{1}{2a_1'} = -\frac{1}{2\pi} \frac{\partial v}{\partial x} \Bigg|_{\substack{x=0\\y=0}} = \frac{1}{2\pi} \frac{\partial u}{\partial y} \Bigg|_{\substack{x=0\\y=0}} \tag{29}$$

or

$$a_1' = \frac{\pi}{2} \tag{30}$$

or

$$a_1 = \frac{\pi}{2} b_o = \frac{\pi}{2} \sqrt{b^2 - c^2} \tag{31}$$

We can calculate the magnetic field in the center of the two wire transmission line from

$$H_x \Bigg|_{\substack{x=0\\y=0}} = \frac{I}{2a_1} \tag{32}$$

where I is the current out of the upper wire and into the lower wire. From equations (20), (22), and (29) we can relate these effective distances by

$$\frac{b_1}{a_1} = \frac{b_1'}{a_1'} = f_g \tag{33}$$

The normalized field distribution is given by (using equations (4) and (24))

$$E_{y_{rel}} = \frac{1}{2} \frac{\partial u}{\partial y} = \frac{1}{2} \left[\frac{1+y}{x^2+(1+y)^2} + \frac{1-y}{x^2+(1-y)^2} \right] \tag{34}$$

and

$$E_{x_{rel}} = \frac{1}{2}\frac{\partial u}{\partial x} = \frac{1}{2}\left[\frac{x}{x^2+(1+y)^2} - \frac{x}{x^2+(1-y)^2}\right] . \tag{35}$$

The normalized magnetic field is equal to the normalized electric field but perpendicular to it. This normalized field is defined so that $E_{y_{rel}}$ is unity at the origin of the coordinates. We have the interesting special cases along the coordinate axes

$$E_{y_{rel}}\bigg|_{x=0} = \left[1-y^2\right]^{-1} \tag{36}$$

for $|y| < 1$ and

$$E_{y_{rel}}\bigg|_{y=0} = \left[1+x^2\right]^{-1} \tag{37}$$

while $E_{x_{rel}}$ is zero along these axes. Equations (36) and (37) show the symmetry of the field distribution in the center of the transmission line. By symmetry the odd derivatives (first, third, etc.) of $E_{y_{rel}}$ with respect to x or y are zero at $(x,y) = (0,0)$. This gives a somewhat uniform field distribution there.

III. Symmetrical Four Wire Transmission Line.

Second, consider the four wire transmission line as in figure 1B. In normalized form, construct a conformal transformation by taking the field distribution for two identical two wire transmission lines which are shifted $\pm$ a' in the x direction. Thus, using equation (1), let

$$w'(z) = w(z+a') + w(z-a') \tag{38}$$

where

$$w' = u'+jv' \tag{39}$$

This is valid only for $c' \ll a'$ and $c' \ll 1$. The finite size of, for example, the wires at $x = +a'$ distort the field distribution from the wires at $x = -a'$ in their immediate vicinity. For line charges and/or currents, this type of superposition is exact. The equipotentials and magnetic field lines are given by constant u' and the electric field lines by constant v'. These are illustrated for positive x and y in figure 3 but can be extended to the remaining quadrants by symmetry. In the figure a' is taken as $\sqrt{\frac{1}{3}}$. The reasons for this particular choice are discussed later.

Using equations (4), (5), and (38) gives for the potential functions

FOR $u' < \pi$, u' AND v' ARE IN INCREMENTS OF $.05\,\pi$.

FOR $u' > \pi$, u' AND v' ARE IN INCREMENTS OF $.1\,\pi$.

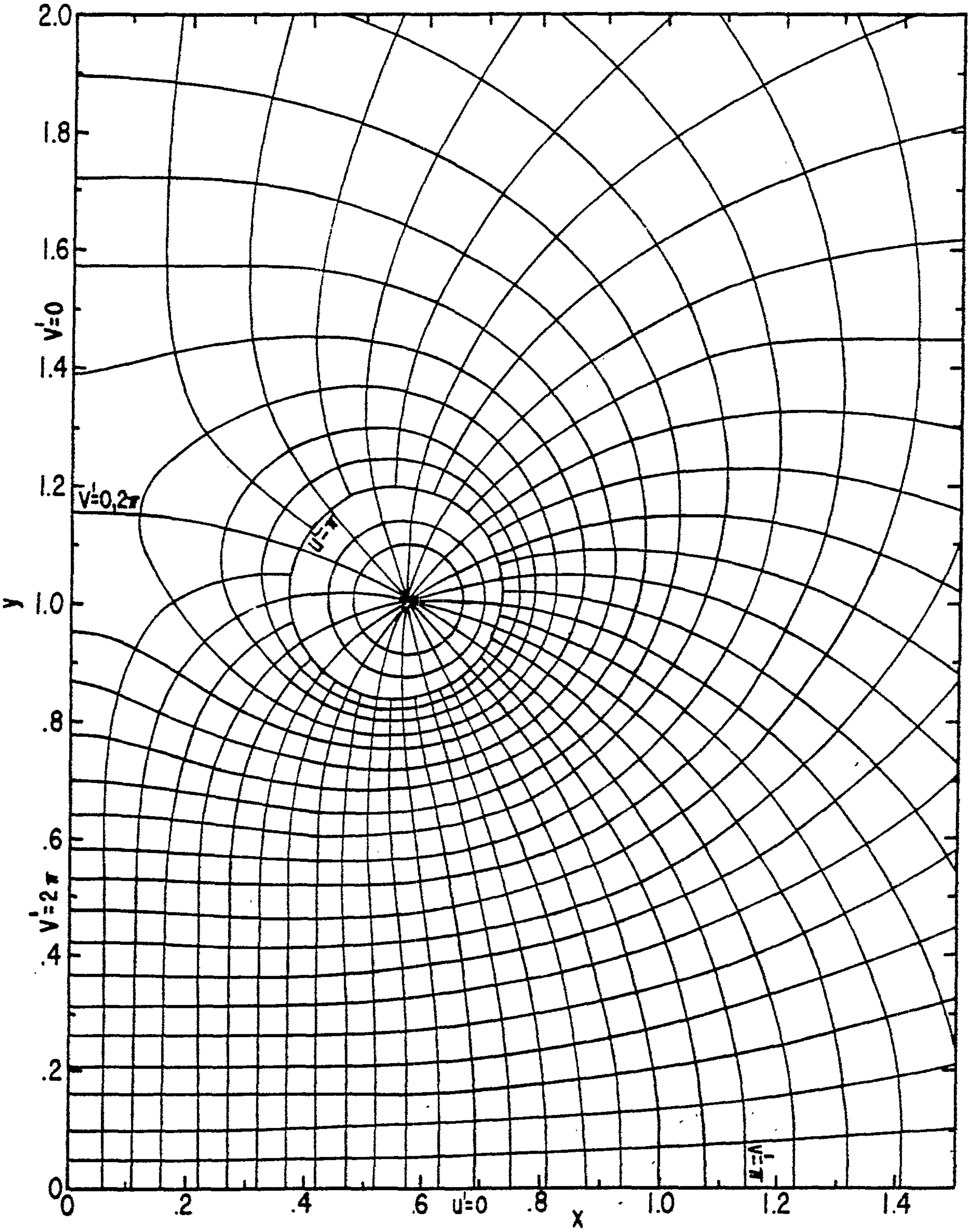


FIGURE 3. FIELD AND POTENTIAL DISTRIBUTION FOR SYMMETRICAL FOUR WIRE TRANSMISSION LINE

$$u' = \frac{1}{2} \ln\left[\frac{(x+a')^2+(1+y)^2}{(x+a')^2+(1-y)^2}\right] + \frac{1}{2} \ln\left[\frac{(x-a')^2+(1+y)^2}{(x-a')^2+(1-y)^2}\right] \tag{40}$$

and

$$v' = \arctan\left[\frac{2(x+a')}{(x+a')^2+y^2-1}\right] + \arctan\left[\frac{2(x-a')}{(x-a')^2+y^2-1}\right] \tag{41}$$

These are used for the contours of constant u' and v' in figure 3.

To calculate the geometric factor, f_g, in the transmission line impedance we use

$$f_g = \frac{2u_o'}{4\pi} \tag{42}$$

Here $2u_o'$ is the difference in u between the wires for positive and negative y as illustrated in figure 1B and 4π is the change in v for a path circling the two wires for either positive or negative y. Define

$$u' = u_L'+u_R' \tag{43}$$

where

$$u_L' = \frac{1}{2} \ln\left[\frac{(x+a')^2+(1+y)^2}{(x+a')^2+(1-y)^2}\right] \tag{44}$$

is the contribution from the left pair of wires. The contribution from the right pair of wires is

$$u_R' = \frac{1}{2} \ln\left[\frac{(x-a')^2+(1+y)^2}{(x-a')^2+(1-y)^2}\right] \tag{45}$$

Also define

$$u_o' = u_{L_o}' + u_{R_o}' \tag{46}$$

where u_{L_o}' and u_{R_o}' are evaluated for the wire centered at $(x,y) = (a',b')$ from equations (44) and (45), respectively. Then equation (45) represents the potential due to the charge on the right pair of wires which contributes a potential on this wire. Evaluating the potential at $(x,y) = (a',b'-c')$ gives

$$u_{R_o}' = \ln\left[\frac{1+(b'-c')}{1-(b'-c')}\right] \tag{47}$$

Note that u_{R_o}' could have been evaluated at any position of the wire circumference with an equivalent result. Another form for this potential, which comes from the two wire line (equation (15)), is

$$u_{R_o}' = \text{arccosh}\left(\frac{b'}{c'}\right) = \text{arccosh}\left(\frac{b}{c}\right) \tag{48}$$

There is no single value of u_{L_o}' on the surface of the upper right wire, however; so we take an average by evaluating it at the center of the wire. This gives

$$u_{L_o}' = \frac{1}{2} \ln \left[\frac{4a'^2+(b'+1)^2}{4a'^2+(b'-1)^2} \right] \tag{49}$$

or

$$u_{L_o}' = \frac{1}{2} \ln \left[\frac{4a^2+(b+\sqrt{b^2-c^2})^2}{4a^2+(b-\sqrt{b^2-c^2})^2} \right] \tag{50}$$

This approximation is valid for $c' \ll a'$ and $c' \ll 1$. Thus,

$$f_g = \frac{1}{2\pi} \left\{ \operatorname{arccosh}\left(\frac{b}{c}\right) + \frac{1}{2} \ln \left[\frac{4a^2+(b+\sqrt{b^2-c^2})^2}{4a^2+(b-\sqrt{b^2-c^2})^2} \right] \right\} \tag{51}$$

This is plotted in figure 4 with a' as a parameter and compared with the results for the two wire line (equation (21)).

Again relate the electric and magnetic fields at the center of the transmission line to the voltages and currents. Define b_1 as the effective half-spacing for the relationship between the wire potential and the electric field. Then, in normalized form,

$$\frac{1}{b_1'} = \frac{1}{u_o'} \left. \frac{\partial u'}{\partial y} \right|_{\substack{x=0 \\ y=0}} \tag{52}$$

Since, from equations (44) through (46),

$$\left. u' \right|_{x=0} = \ln \left[\frac{a'^2+(1+y)^2}{a'^2+(1-y)^2} \right] \tag{53}$$

and thus

$$\left. \frac{\partial u'}{\partial y} \right|_{x=0} = \frac{2(1+y)}{a'^2+(1+y)^2} + \frac{2(1-y)}{a'^2+(1-y)^2} \tag{54}$$

and

$$\left. \frac{\partial u'}{\partial y} \right|_{\substack{x=0 \\ y=0}} = \frac{4}{a'^2+1} \tag{55}$$

Then

$$b_1' = \frac{a'^2+1}{4} \left\{ \operatorname{arccosh}\left(\frac{b'}{c'}\right) + \frac{1}{2} \ln \left[\frac{4a'^2+(b'+1)^2}{4a'^2+(b'-1)^2} \right] \right\} \tag{56}$$

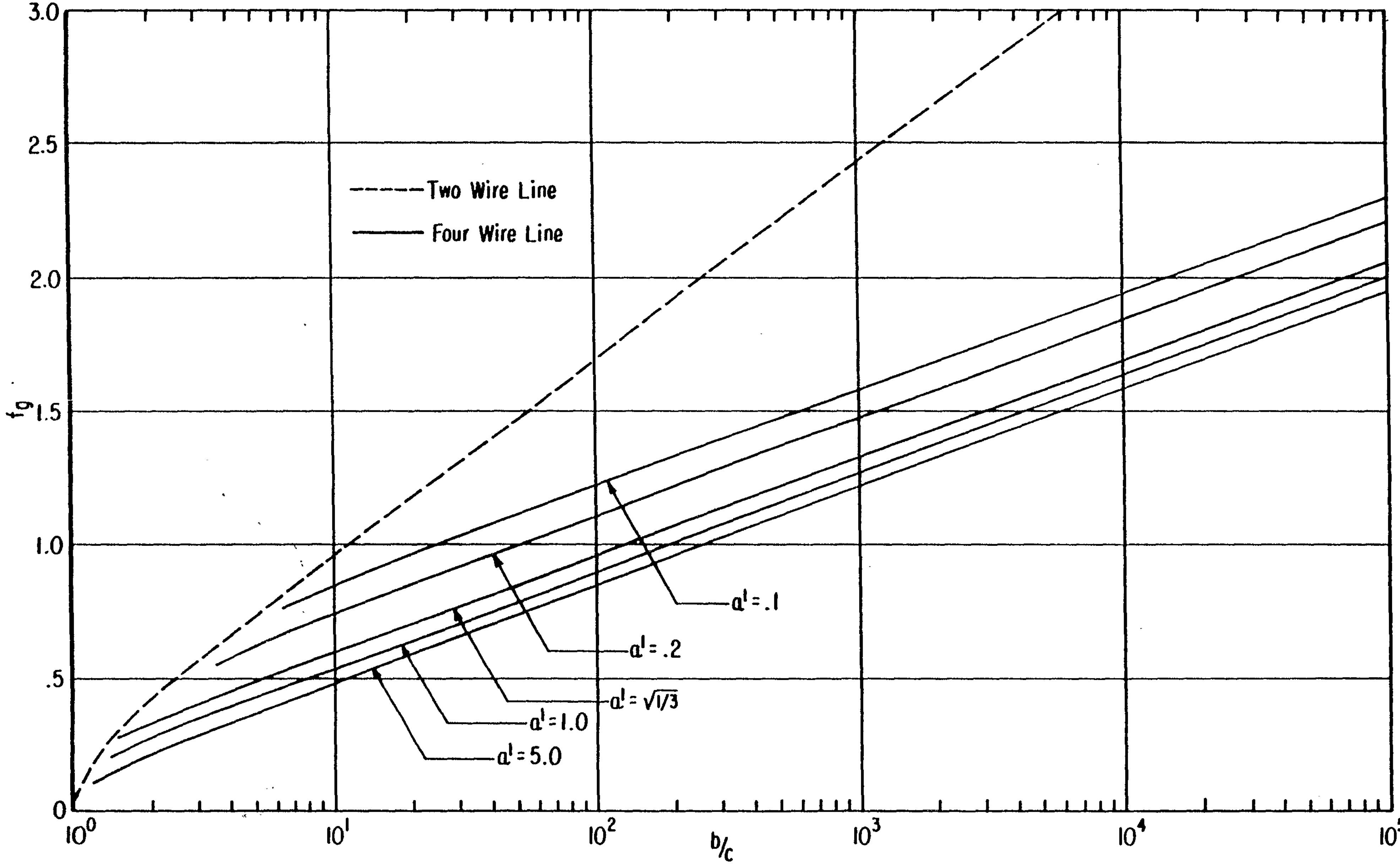


FIGURE 4. GEOMETRIC IMPEDANCE FACTOR FOR SYMMETRICAL TWO WIRE AND FOUR WIRE TRANSMISSION LINES

or

$$b_1 = \frac{\sqrt{b^2-c^2}}{4} \left[\frac{a^2}{b^2-c^2} + 1\right] \left\{ \operatorname{arccosh}\left(\frac{b}{c}\right) + \frac{1}{2} \ln \left[\frac{4a^2+(b+\sqrt{b^2-c^2})^2}{4a^2+(b-\sqrt{b^2-c^2})^2}\right] \right\} \quad (57)$$

As with the symmetrical two wire transmission line, let us define an electric field efficiency as

$$f_E = \frac{b-c}{b_1} = 4\left(\frac{\frac{b}{c}-1}{\frac{b}{c}+1}\right)^{1/2} \left[\frac{a^2}{b^2-c^2} + 1\right]^{-1} \left\{ \operatorname{arccosh}\left(\frac{b}{c}\right) + \frac{1}{2} \ln \left[\frac{4a^2+(b+\sqrt{b^2-c^2})^2}{4a^2+(b-\sqrt{b^2-c^2})^2}\right] \right\}^{-1} \quad (58)$$

This is plotted in figure 5 with a' as a parameter and compared with the results for the two wire line (equation (27)). Using equations (28) and (58) the electric field may be related to the potential between the wires (for positive and negative y).

Relating the current in the wires to the magnetic field at the center of the coordinate system, again define an effective half-width, a_1. In normalized form

$$\frac{1}{2a_1'} = -\frac{1}{4\pi} \left.\frac{\partial v'}{\partial x}\right|_{\substack{x=0\\y=0}} = \frac{1}{4\pi} \left.\frac{\partial u'}{\partial y}\right|_{\substack{x=0\\y=0}} \quad (59)$$

or

$$a_1' = \frac{\pi}{2} \left[a'^2+1\right] \quad (60)$$

or

$$a_1 = \frac{\pi}{2} \sqrt{b^2-c^2} \left[\frac{a^2}{b^2-c^2} + 1\right] \quad (61)$$

Note the similarity in form to equation (31) for the two wire line. Using equations (32) and (61) we can relate the current and magnetic field. These effective distances, b_1 and a_1, for the four wire transmission line have the same relationship as the two wire case (equation (33)).

The normalized field distribution is given by (using equations (44), (45), and (55))

$$E_{y_{rel}} = \frac{a'^2+1}{4} \frac{\partial u'}{\partial y}$$
$$= \frac{a'^2+1}{4} \left\{ \frac{1+y}{(x+a')^2+(1+y)^2} + \frac{1-y}{(x+a')^2+(1-y)^2} + \frac{1+y}{(x-a')^2+(1+y)^2} + \frac{1-y}{(x-a')^2+(1-y)^2} \right\} \quad (62)$$

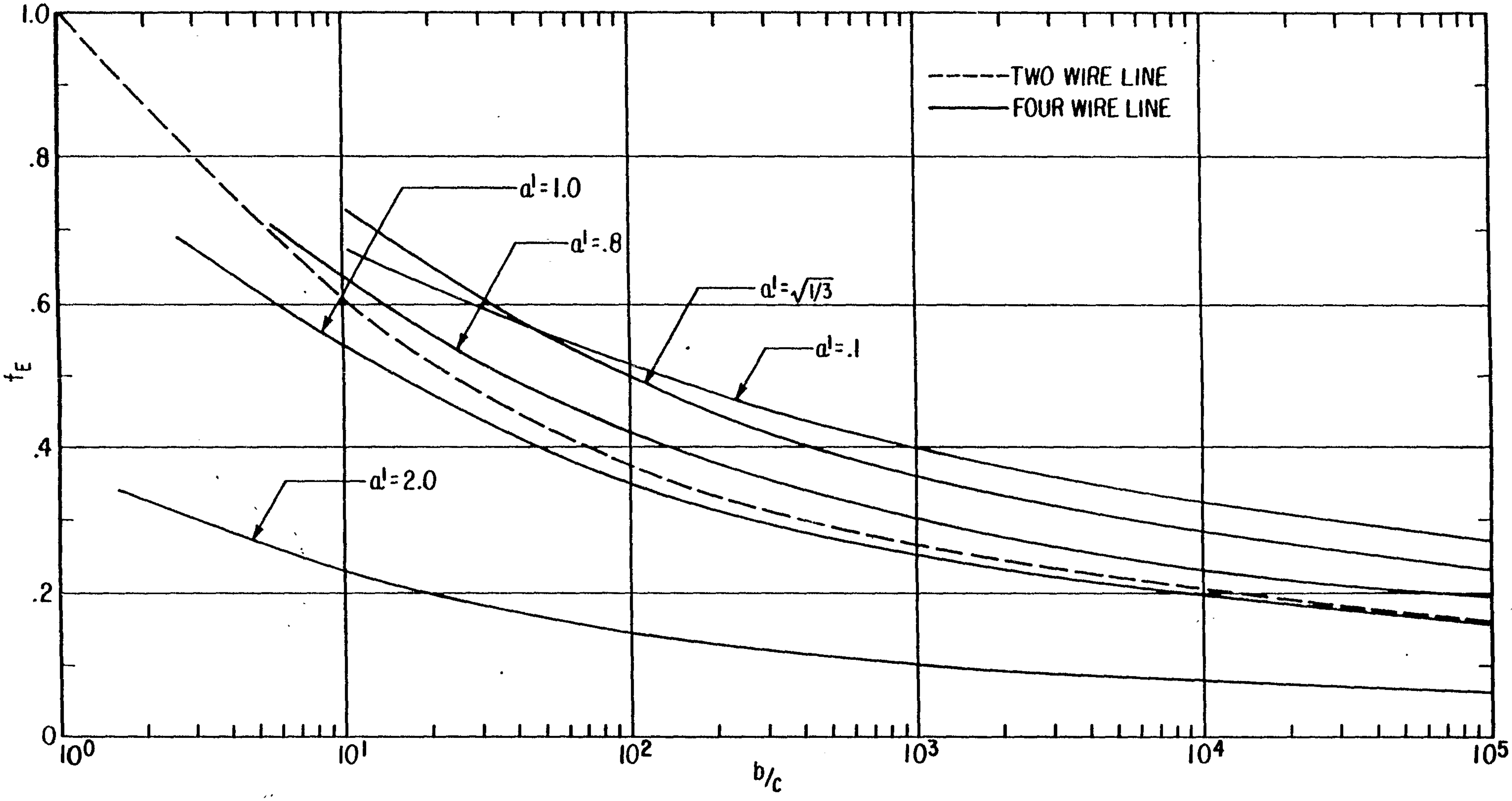


FIGURE 5. ELECTRIC FIELD EFFICIENCY FOR SYMMETRICAL TWO WIRE AND FOUR WIRE TRANSMISSION LINES

and

$$E_{x_{rel}} = \frac{a'^2+1}{4}\frac{\partial u'}{\partial x}$$

$$= \frac{a'^2+1}{4}\left\{\frac{x+a'}{(x+a')^2+(1+y)^2} - \frac{x+a'}{(x+a')^2+(1-y)^2} + \frac{x-a'}{(x-a')^2+(1+y)^2} - \frac{x-a'}{(x-a')^2+(1-y)^2}\right\} \quad (63)$$

Again $E_{y_{rel}}$ is equal to unity at the origin of the coordinates. Along the coordinate axes the normalized field distribution reduces to

$$E_{y_{rel}}\Big|_{x=0} = \frac{a'^2+1}{2}\left\{\frac{1+y}{a'^2+(1+y)^2} + \frac{1-y}{a'^2+(1-y)^2}\right\} \quad (64)$$

and

$$E_{y_{rel}}\Big|_{y=0} = \frac{a'^2+1}{2}\left\{\frac{1}{1+(x+a')^2} + \frac{1}{1+(x-a')^2}\right\} \quad (65)$$

Again $E_{x_{rel}}$ is zero along these axes. These normalized field distributions along the coordinate axes are plotted in figures 6 and 7 with a' as a parameter and are compared with the results for the two wire transmission line (equations (36) and (37)).

As with the two wire transmission line the field distribution for the four wire case is symmetric about the x and y axes. The odd derivatives of $E_{y_{rel}}$ with respect to x or y are zero at (x,y) = (0,0) giving a somewhat uniform field distribution near the center of the coordinates. Conveniently, however, the parameter, a', can be chosen to optimize the field distribution. Varying a' changes the geometry of the structure since for $c' \ll 1$, a' is very nearly the ratio of the width to the height of the structure (in the orientation of figure 1B). Figures 6 and 7 show the effect of varying a' on the uniformity of the field distribution near the coordinate origin.

To optimize the field distribution we can choose the value of a'. There are various criteria by which an optimum field distribution can be defined. The one to be discussed here involves the field distribution at (x,y) = (0,0). Let us choose a' to make the second derivatives of $E_{y_{rel}}$ with respect to x and y zero at the coordinate center. The odd derivatives (first, third, etc.) are already zero by symmetry. Using equation (64) for derivatives with respect to y gives

$$\frac{\partial E_{y_{rel}}}{\partial y}\Bigg|_{x=0} = \frac{a'^2+1}{2}\left\{\frac{a'^2-(1+y)^2}{[a'^2+(1+y)^2]^2} - \frac{a'^2-(1-y)^2}{[a'^2+(1-y)^2]^2}\right\} \quad (66)$$

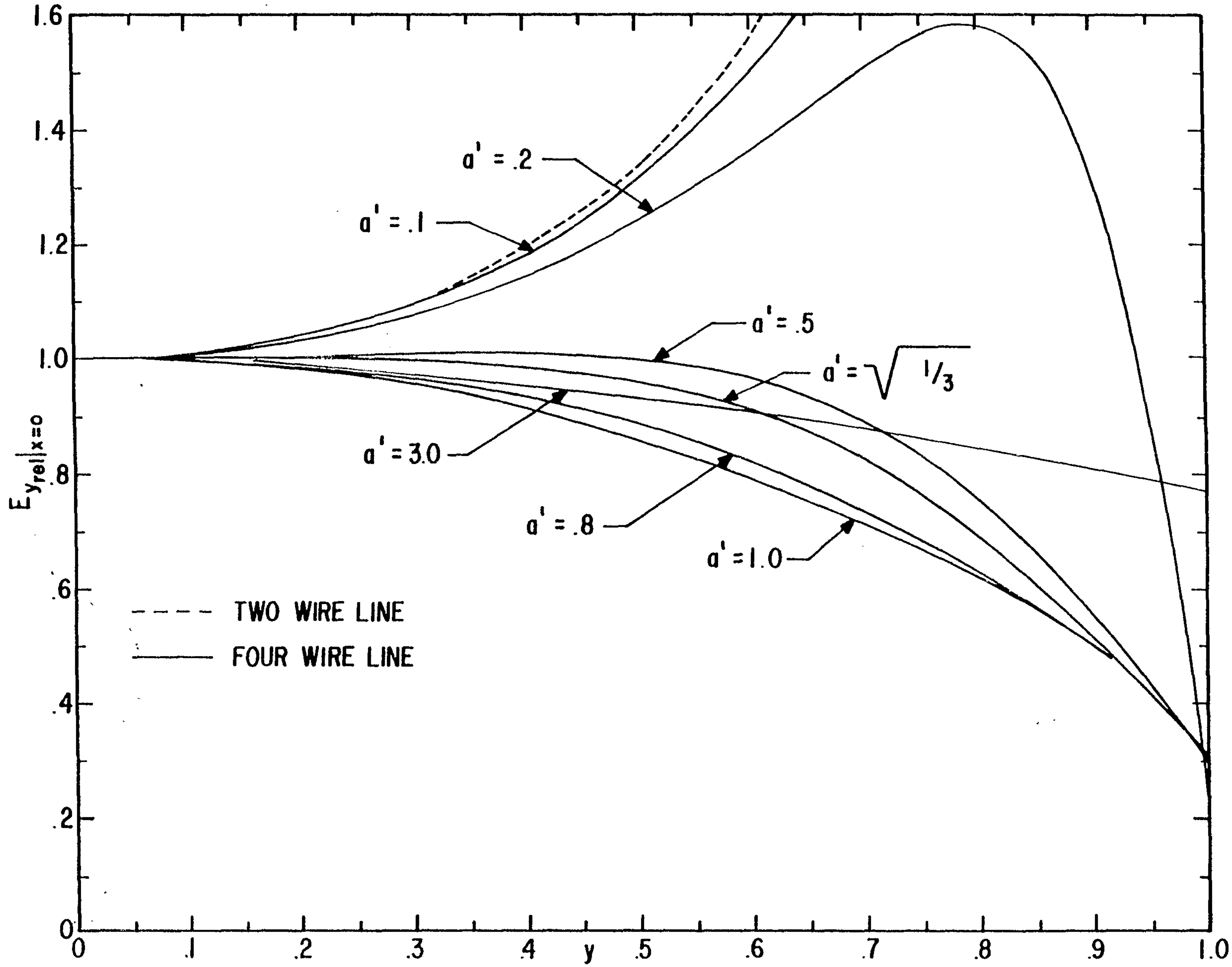


FIGURE 6. NORMALIZED E_y ALONG y AXIS FOR SYMMETRICAL TWO WIRE AND FOUR WIRE TRANSMISSION LINES

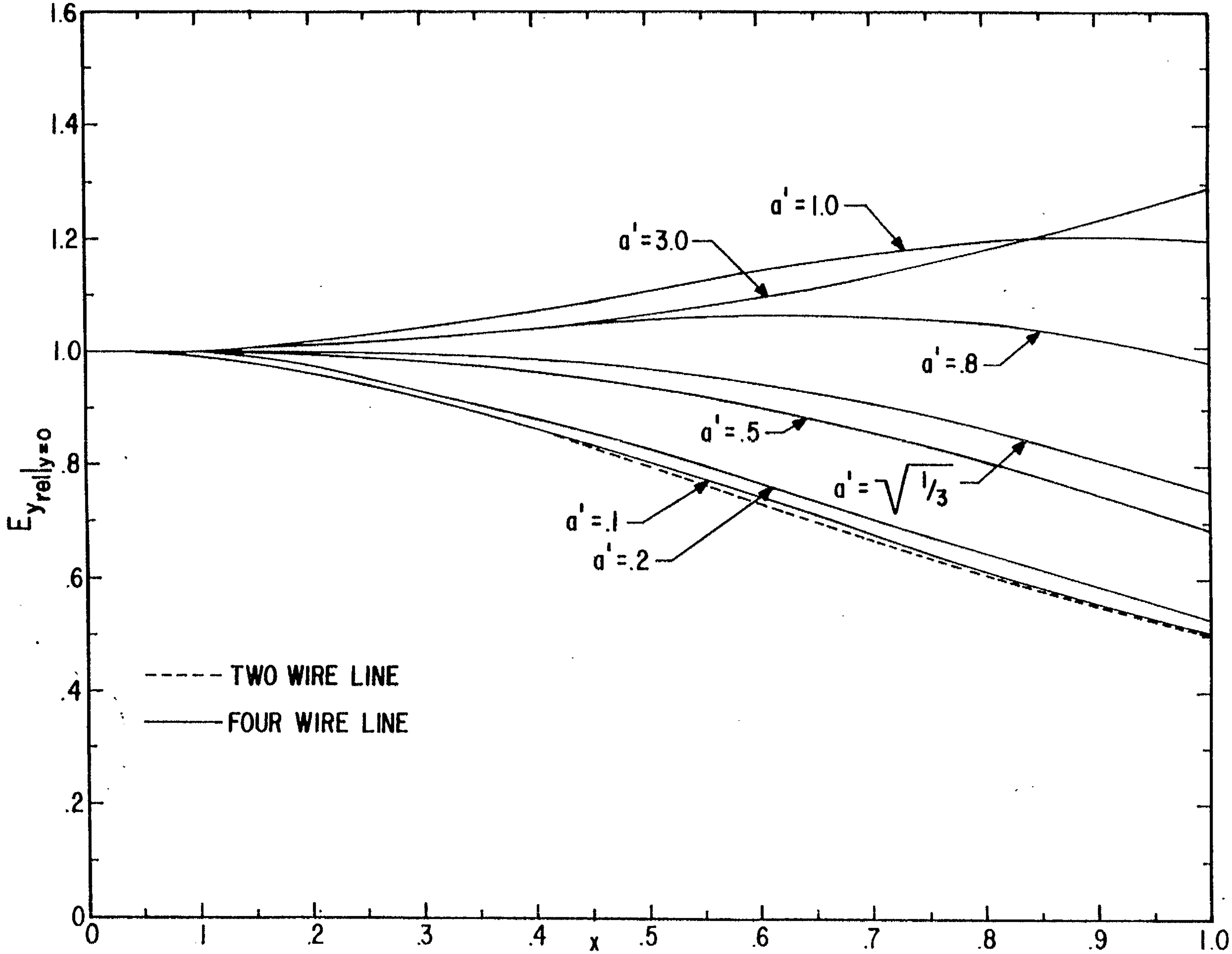


FIGURE 7. NORMALIZED E_y ALONG x AXIS FOR SYMMETRICAL TWO WIRE AND FOUR WIRE TRANSMISSION LINES

and

$$\left.\frac{\partial^2 E_{y_{rel}}}{\partial y^2}\right|_{x=0} = -\,[a'^2+1]\left\{\frac{(1+y)[3a'^2-(1+y)^2]}{[a'^2+(1+y)^2]^3} + \frac{(1-y)[3a'^2-(1-y)^2]}{[a'^2+(1-y)^2]^3}\right\} \tag{67}$$

Using equation (65) for derivatives with respect to x gives

$$\left.\frac{\partial E_{y_{rel}}}{\partial x}\right|_{y=0} = -\,[a'^2+1]\left\{\frac{x+a'}{[1+(x+a')^2]^2} + \frac{x-a'}{[1+(x-a')^2]^2}\right\} \tag{68}$$

and

$$\left.\frac{\partial^2 E_{y_{rel}}}{\partial x^2}\right|_{y=0} = -\,[a'^2+1]\left\{\frac{1-3(x+a')^2}{[1+(x+a')^2]^3} + \frac{1-3(x-a')^2}{[1+(x-a')^2]^3}\right\} \tag{69}$$

Then from equations (67) and (69)

$$\left.\frac{\partial^2 E_{y_{rel}}}{\partial y^2}\right|_{\substack{x=0\\y=0}} = -\left.\frac{\partial^2 E_{y_{rel}}}{\partial x^2}\right|_{\substack{x=0\\y=0}} = 2\,\frac{1-3a'^2}{[1+a'^2]^2} \tag{70}$$

These second derivatives are zero for the special case of

$$a' = \sqrt{\frac{1}{3}} \tag{71}$$

For this special case the first three derivatives of the field distribution with respect to x and y are zero at the origin of the coordinates. This choice of dimensions for a uniform field distribution is similar to that for Helmholtz coils which approximate a uniform magnetic field. The normalized field distribution (equations (62) and (63)) for this special case of the symmetrical four wire transmission line is plotted in figures 8 through 11. These figures show the extent in x and y of a given degree of field uniformity.

IV. Summary.

The symmetrical two wire and four wire transmission lines then have a somewhat uniform field distribution in the center of the respective structures. For the four wire case there is a particular ratio of the structural dimensions which gives a particularly uniform field distribution in that the first three derivatives of the field with respect to x and y, at the coordinate origin, are zero.

These transmission lines have less of a structure of conducting surfaces than do parallel plate type of transmission lines of similar size. Thus, they may have application for simulators of the nuclear EMP where such mechanical problems may be important.

We would like to thank A2C Anthony Regal and Mr. John Vogel for the plots contained in this note.

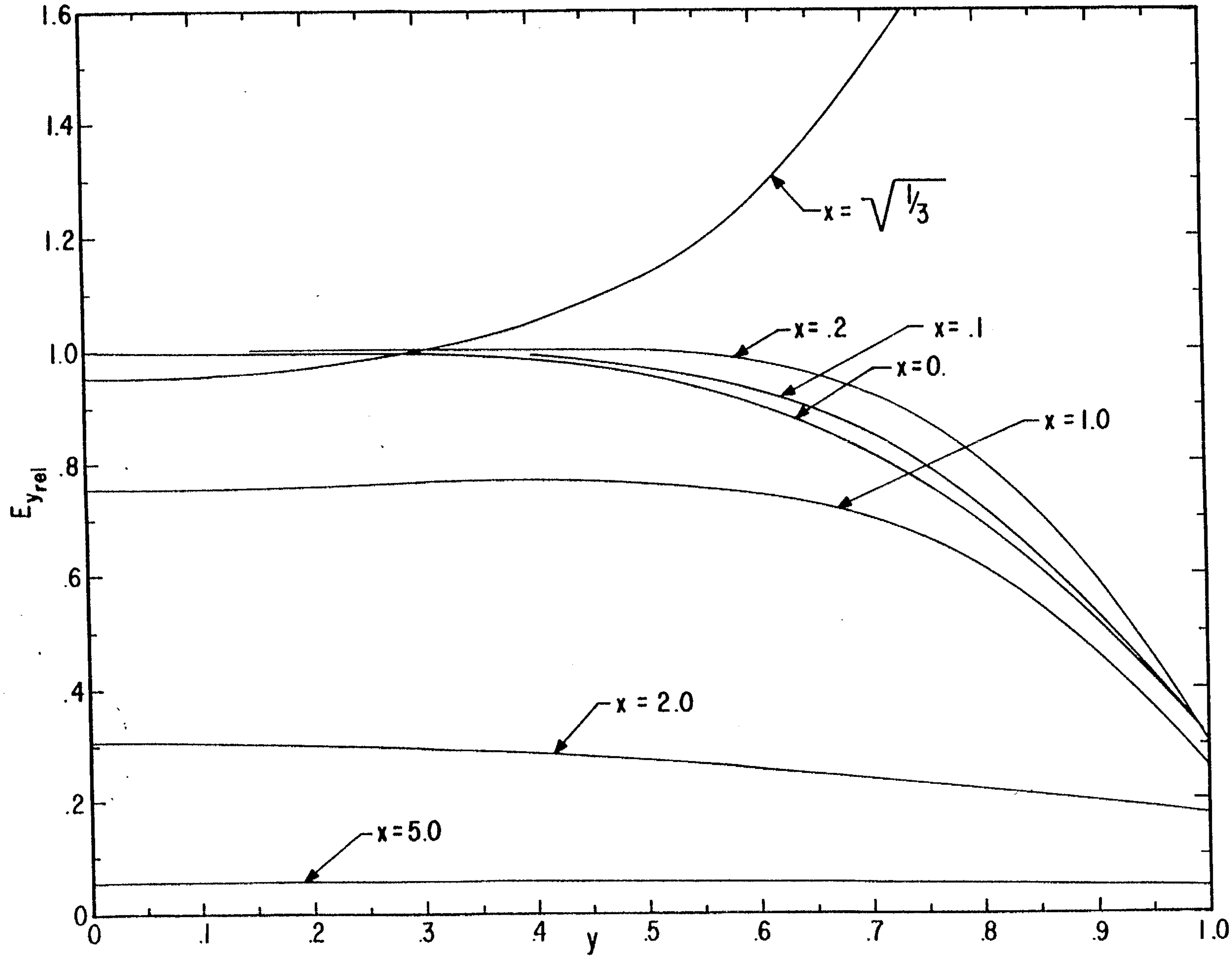


FIGURE 8. NORMALIZED E_y FOR CONSTANT x FOR SYMMETRICAL FOUR WIRE TRANSMISSION LINE: $a' = \sqrt{1/3}$

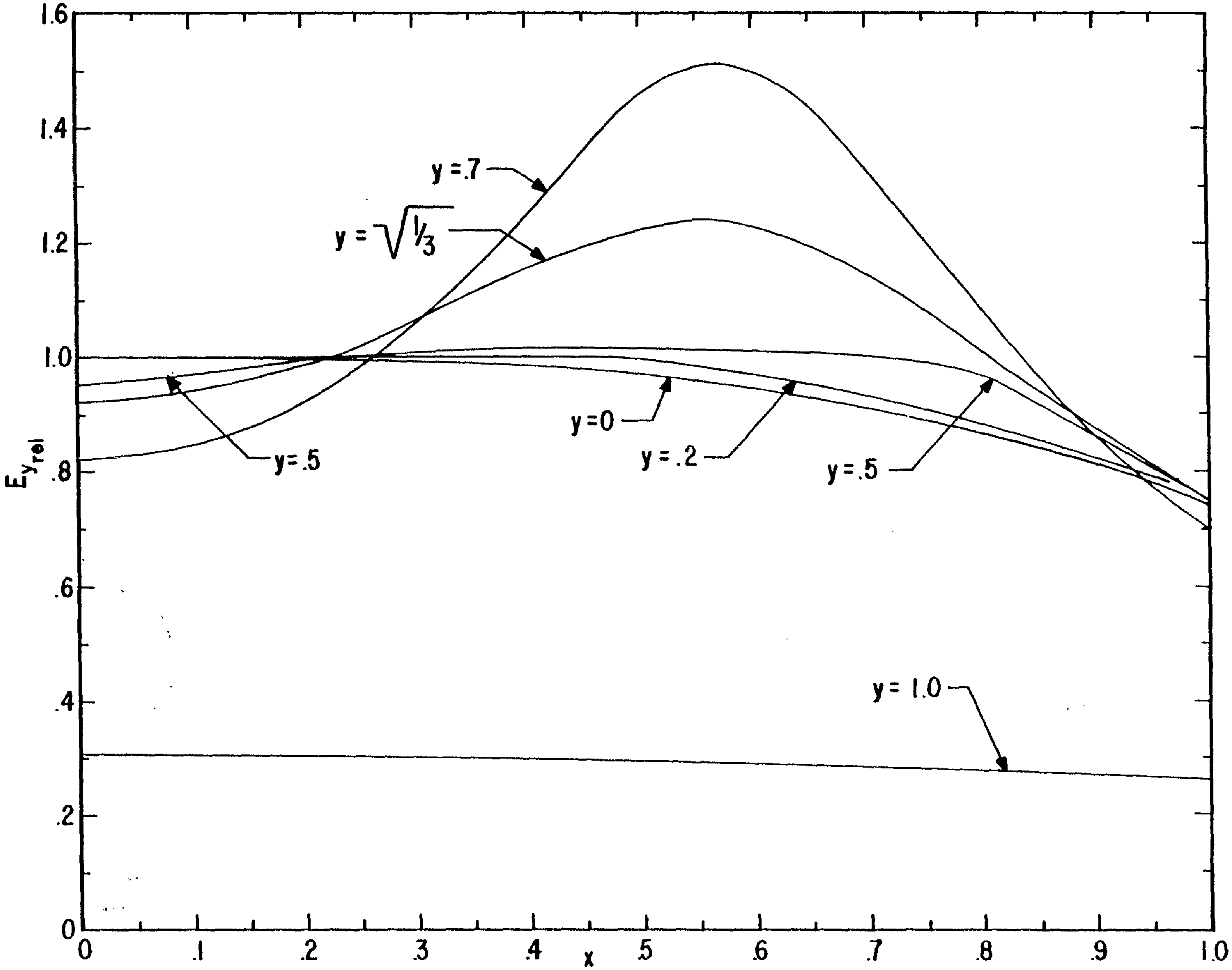


FIGURE 9. NORMALIZED E_y FOR CONSTANT y FOR SYMMETRICAL FOUR WIRE TRANSMISSION LINE: $a' = \sqrt{1/3}$

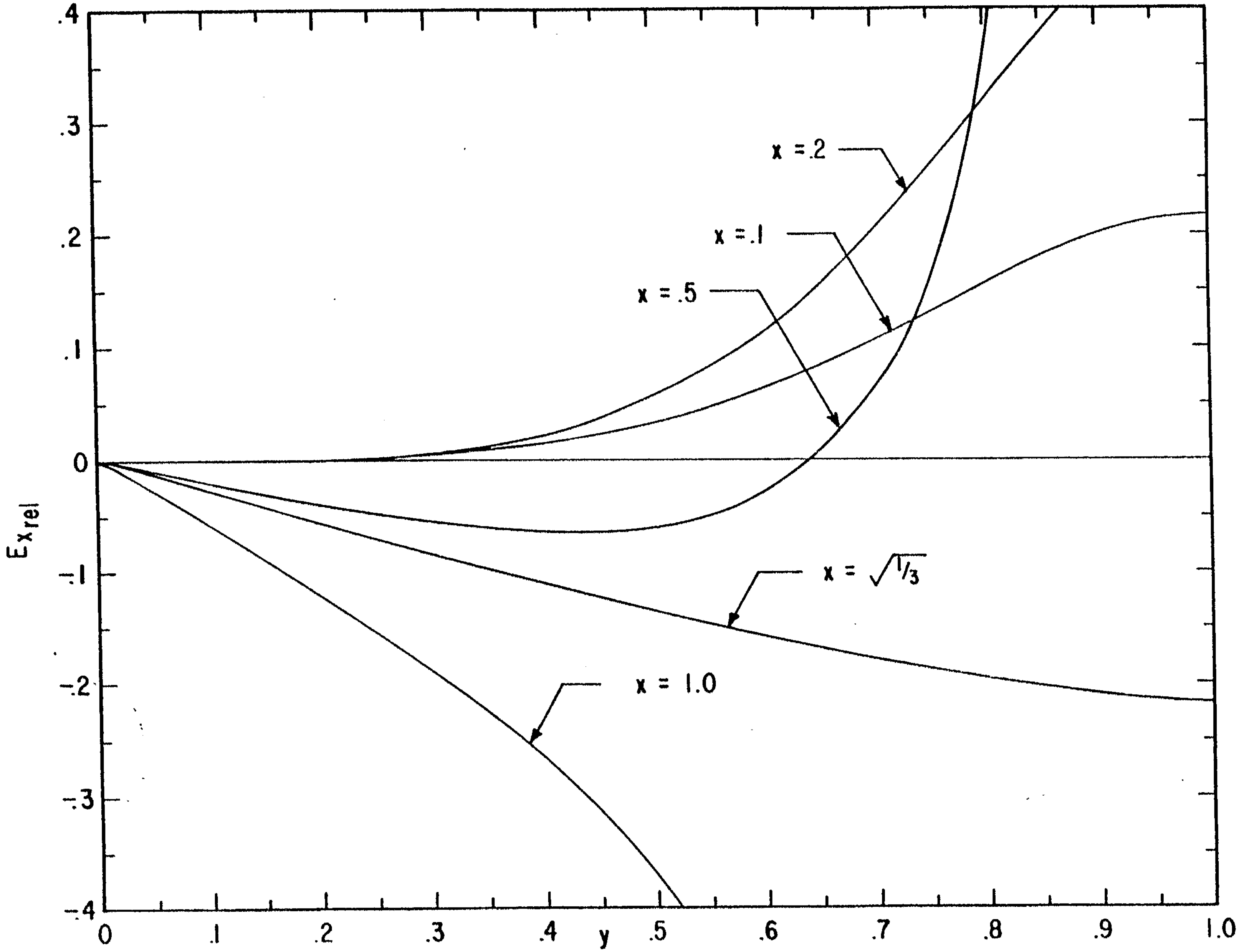


FIGURE 10. NORMALIZED E_x FOR CONSTANT x FOR SYMMETRICAL FOUR WIRE TRANSMISSION LINE: $a' = \sqrt{1/3}$

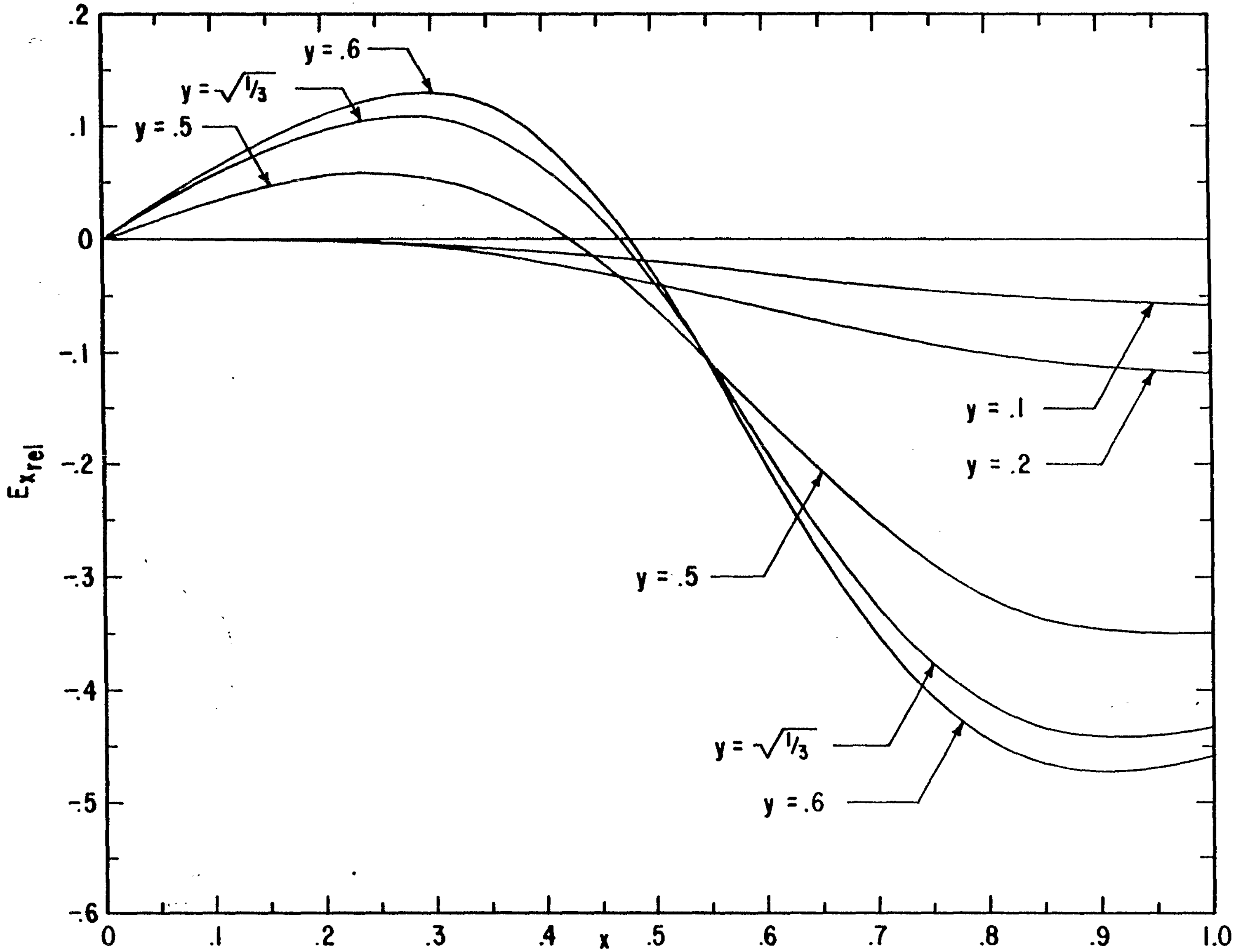


FIGURE 11. NORMALIZED E_x FOR CONSTANT y FOR SYMMETRICAL FOUR WIRE TRANSMISSION LINE: $a' = \sqrt{1/3}$