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THE NATURAL RESONANCES OF PERPENDICULAR CROSSED WIRES PARALLEL TO AN IMPERFECT GROUND USING SEM AND FRESNEL REFLECTION COEFFICIENTS

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ABSTRACT

The singularity expansion method (SEM) has been applied to determine natural resonances of a perpendicular crossed wire structure parallel to an imperfect ground plane. To account for the finite conductivity of the ground, the Fresnel reflection coefficient method has been used to follow the trajectories of the natural resonances as the conductivity and relative permittivity of the ground vary.

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TABLE OF CONTENTS

I. INTRODUCTION

The general problem of the electromagnetic interaction of wire structures and external sources of excitation has been of interest for many years [1,2]. In a series of studies completed in recent years the progression has been as follows: the single isolated cylinder in free space in the frequency domain [3], a perpendicular crossed wire structure in free space in the frequency domain [4], a non-perpendicular set of crossed wires in free space in the frequency domain [5], a study of a cylinder [6] and a set of perpendicular crossed wires [7] using the singularity expansion method (SEM) [8], a cylinder [9] and a set of perpendicular crossed wires [10] over a perfectly conducting ground plane using image theory and SEM. Recently two studies of a cylindrical scatterer have considered different effects of the ground: in the first the effects of a close proximity perfectly conducting ground plane on the circumferential distribution of currents [11] have been studied, and in the second the effects of a finite conducting ground have been studied using SEM and Fresnel reflection coefficients [12]. Finally preliminary studies were performed to analyze the crossed wire and the cylinder over the finite conducting ground using the Sommerfeld formulation [13, 14, 15] in the frequency domain [16]. Simultaneously, others were applying transmission theory to gain information on this same problem [17].

The problem considered in this study is that of determining the behavior of the natural resonances in SEM for a set of perpendicular crossed wires over an imperfect ground using the Fresnel reflection coefficient method. While the technique has inherent limitations, trends can be established that are of value in themselves as well as providing guidelines for more correct formulations; e.g., a Sommerfeld treatment.

II. FORMULATION

This analysis deals with the interaction of a set of perpendicular crossed wires whose plane is parallel to an imperfect ground. The effect of the ground on the wire structure is approximated by the Fresnel reflection coefficient method. SEM is used to follow the trajectories of the natural resonances as the ground conductivity (σ_g) and the relative ground permittivity (ε_r) vary. In analyzing the scattering problem, thin wire approximations are utilized. That is, the wires are required to satisfy the following:

(1) the current and charge densities are constant around the periphery of the wire cross section,

(2) the axial currents are zero at the free ends of the wires,

(3) $|k|a \ll 1$

(4) $a \ll \ell$

where k is the propagation constant defined by 2π/wavelength, a is the radius of the wire, and ℓ is the length of the wire.

Graves [7] has applied Pocklington theory to a set of orthogonal crossed wires, Figure 2-1. If, in the method of moments solution to the thin wire Pocklington type integral equations, sinusoidal pulse expansions are used for the unknown current distributions and delta testing functions are used, relatively simple results are obtained that are equivalent to the following. If the current distribution on a filament is assumed sinusoidal, exact analytical expressions for the accompanying field components are [18]

$$\hat{r}_{21} \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon s} \left[I'(\vec{r}_1)\, G(\vec{r},\vec{r}_1) - I'(\vec{r}_2)\, G(\vec{r},\vec{r}_2) \right] \quad (2.1)$$

for the axial electric field where s is the complex Laplacian frequency, $s = \sigma + j\omega$; the current filament extends from $\vec{r}_1$ to $\vec{r}_2$, I' is the axial

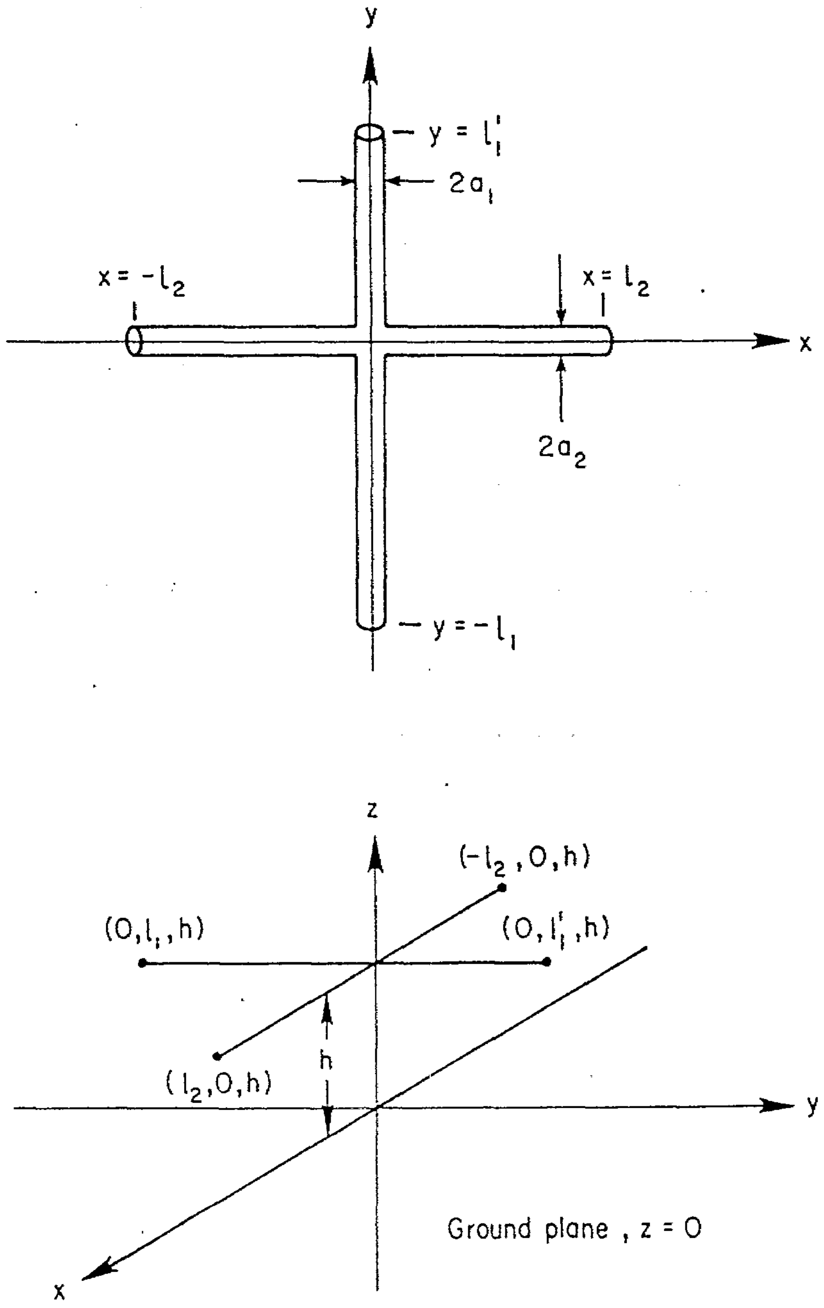

Figure 2-1. The orthogonal crossed wire configuration over a ground plane.

derivative of the current and

$$\hat{r}_{21} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \quad , \; G(\vec{r},\vec{r}_i) = \frac{e^{-s|\vec{r}-\vec{r}_i|/c}}{|\vec{r} - \vec{r}_i|}$$

The corresponding radial component of the electric field is

$$\hat{\rho}_{21} \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon s\rho_{21}} \Bigg\{ \vec{r}_{21} \cdot \Big[(\vec{r}-\vec{r}_2)I'(\vec{r}_2)\, G(\vec{r},\vec{r}_2) - (\vec{r}-\vec{r}_1)I'(\vec{r}_1)\, G(\vec{r},\vec{r}_1) \Big]$$
$$-\frac{s}{c} \Big[I(\vec{r}_2)\; (|\vec{r}-\vec{r}_2|)G(\vec{r},\vec{r}_2) - I(\vec{r}_1)(|\vec{r}-\vec{r}_1|)G(\vec{r},\vec{r}_1) \Big] \Bigg\} \qquad (2.2)$$

and ρ_{21} is the radial variable or the perpendicular distance from the axis of the current filament to the field point.

To derive the system of equations for the current distribution on the crossed wires by the method of moments the various wires are divided into segments. The current on the k^{th} segment of the m^{th} wire is

$$I_{mk}(x) = \frac{\alpha_k \sinh\left[s(x_{k+1} - x)/c\right] + \alpha_{k+1}\sinh\left[s(x-x_k)/c\right]}{\sinh\left[s(x_{k+1} - x_k)/c\right]} \qquad (2.3)$$

It follows that α_k is the axial current at the point $x = x_k$ where x is a general variable along any given wire.

For crossed wires in free space the procedure is as follows. Assume currents $I_{1k}(y)$ and $I_{2k}(x)$ flow on the wires in Figure 2-1. Substituting (2.3) into (2.1) and (2.2) leads to electric field expressions that can be evaluated at points on the conducting structure. For example, a current zone on wire 1 will induce an axial field at other zones on wire 1 through (2.1) and an axial field (as far as wire 2 is concerned) at zones on wire 2 through (2.2). The total axial field at each element due to the induced currents is set equal to $(-\vec{E}_{incident})_{tangent}$ at a discrete set of match points along each wire, and solutions for the α's may be obtained. In the remainder of this report the field contributions (2.1) and (2.2) are called primary scattered (p.s.) excitation terms.

The effects of a perfect ground plane may be accounted for by introducing image currents (parallel to the ground plane and opposite in direction to the object currents) and calculating their E-field contributions at match points on the crossed wire structure. A modification on the G functions in (2.1) and (2.2) is necessary due to the distance factor now depending on the height above the ground, but the procedure is straight forward [10]. In the perfect ground plane problem the tangential component of the incident field is now added to the two p.s. terms plus two image terms to produce a total tangential E field on the wires that equals zero. The image terms are referred to as secondary scattered (s.s.) terms.

Riggs and Shumpert [12] have recently studied the problem of a cylinder over an imperfect ground by modifying the image term. The modification is a multiplicative factor on the image term that equals the Fresnel reflection coefficient for a non-magnetic, finite conducting ground with conductivity σ_g and relative permittivity ε_r. Limitations do exist in this approach: the Fresnel reflection coefficients are derived for incident plane waves, thus the "scattered" field at the interface between the lossy ground and free space should approximate a plane wave. Sarkar and Strait [15] have studied this method in the frequency domain and observed that their results are within 10% of the Sommerfeld formulation if the dipole was at least $(0.25\,\lambda/\sqrt{\varepsilon_R})$ above the ground.

Consider the geometry of Figure 2-2. The current at s_2' contributes to the scattered field at s_2 due to the p.s. term and due to the s.s. term. Since the discussion is limited to axial currents only the axial field

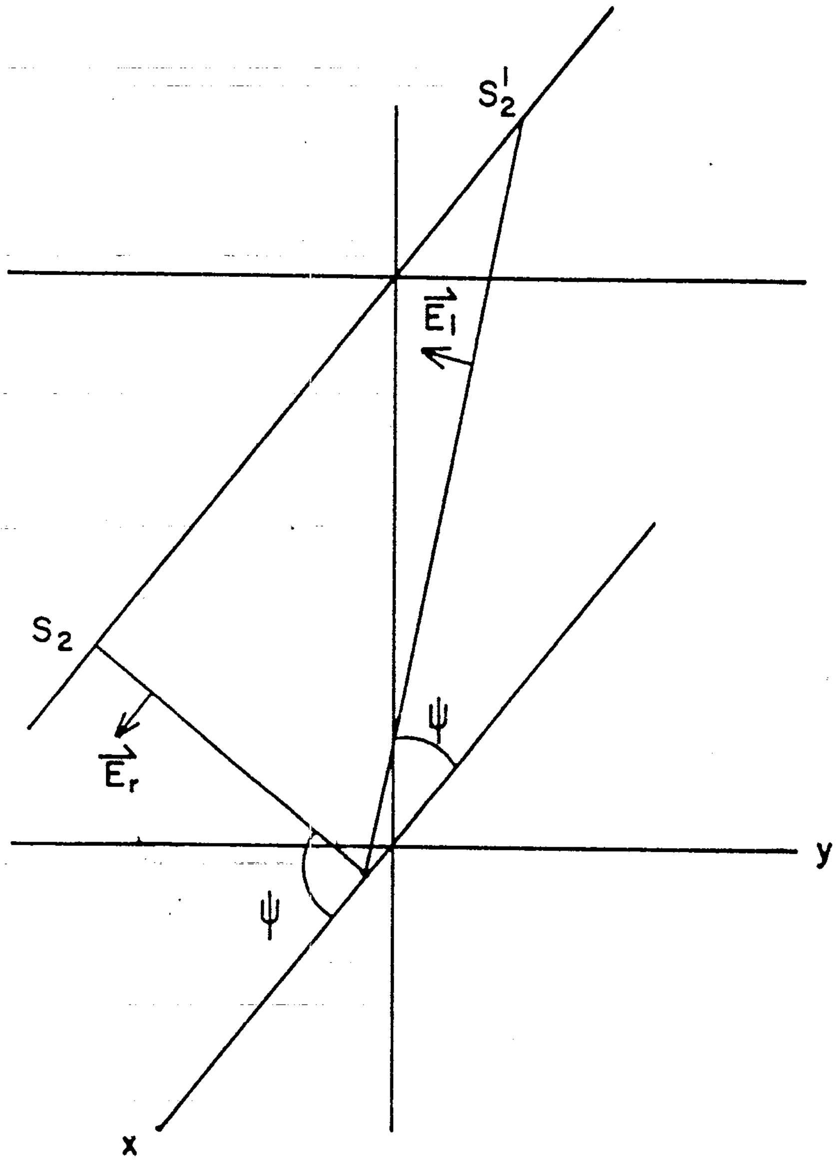

Figure 2-2. The geometry for the primary and secondary scattered fields along one wire.

component from s_2' will contribute to the p.s. and s.s. terms. It follows therefore that s.s. field lies in the plane of incidence, and the Fresnel reflection coefficient becomes (19)

$$RPA = -\frac{(\varepsilon_r + X)\sin\psi(s_2,s_2') - \sqrt{(\varepsilon_r + X) - \cos^2\psi(s_2,s_2')}}{(\varepsilon_r + X)\sin\psi(s_2,s_2') + \sqrt{(\varepsilon_r + X) - \cos^2\psi(s_2,s_2')}} \quad (2.4)$$

where $X = 120\,\frac{\pi\sigma_g}{\gamma}$, $\gamma = \frac{s}{c}$, $\sin\psi(s_2, s_2') = \frac{h^2}{\sqrt{\left(\frac{s_2-s_2'}{2}\right)^2 + h^2}}$.

The particular sign associated with RPA depends on the choice of the direction of the reflected field (Riggs and Shumpert use one convention and this study uses a different one, hence our reflection coefficient differs by a multiplicative factor, -1). In the limit as $\sigma_g \to \infty$, image theory and the reflection method become the same.

The crossed wire problem is geometrically more complex but can be treated by the reflection method. Consider the current element at s_1'; see Figure 2-3. The ground scattered field is neither parallel nor perpendicular to the incident plane. Thus the axial field $E_{s1}(s_1')$ is resolved into two terms, $E_{s1}(s_1')_{\parallel}$ and $E_{s1}(s_1')_{\perp}$, that are parallel and perpendicular to the incident plane, respectively. Similarly, the radial field component, $E_\rho(s_1')$, is resolved into parallel and perpendicular terms. First, $E_\rho(s_1')$ is projected into the s_2,s_1 plane or

$$E_\rho^{s2,s1}(s_1') = E_\rho(s_1')\sin\theta\ ;$$

see Figures 2-3 and 2-4. Then the components of $E_\rho^{s2,s1}(s_1')$ perpendicular and parallel to the incident plane are found, Figure 2-4. Finally, the components of $E_{s1}(s_1')$ are defined; see Figure 2-5.

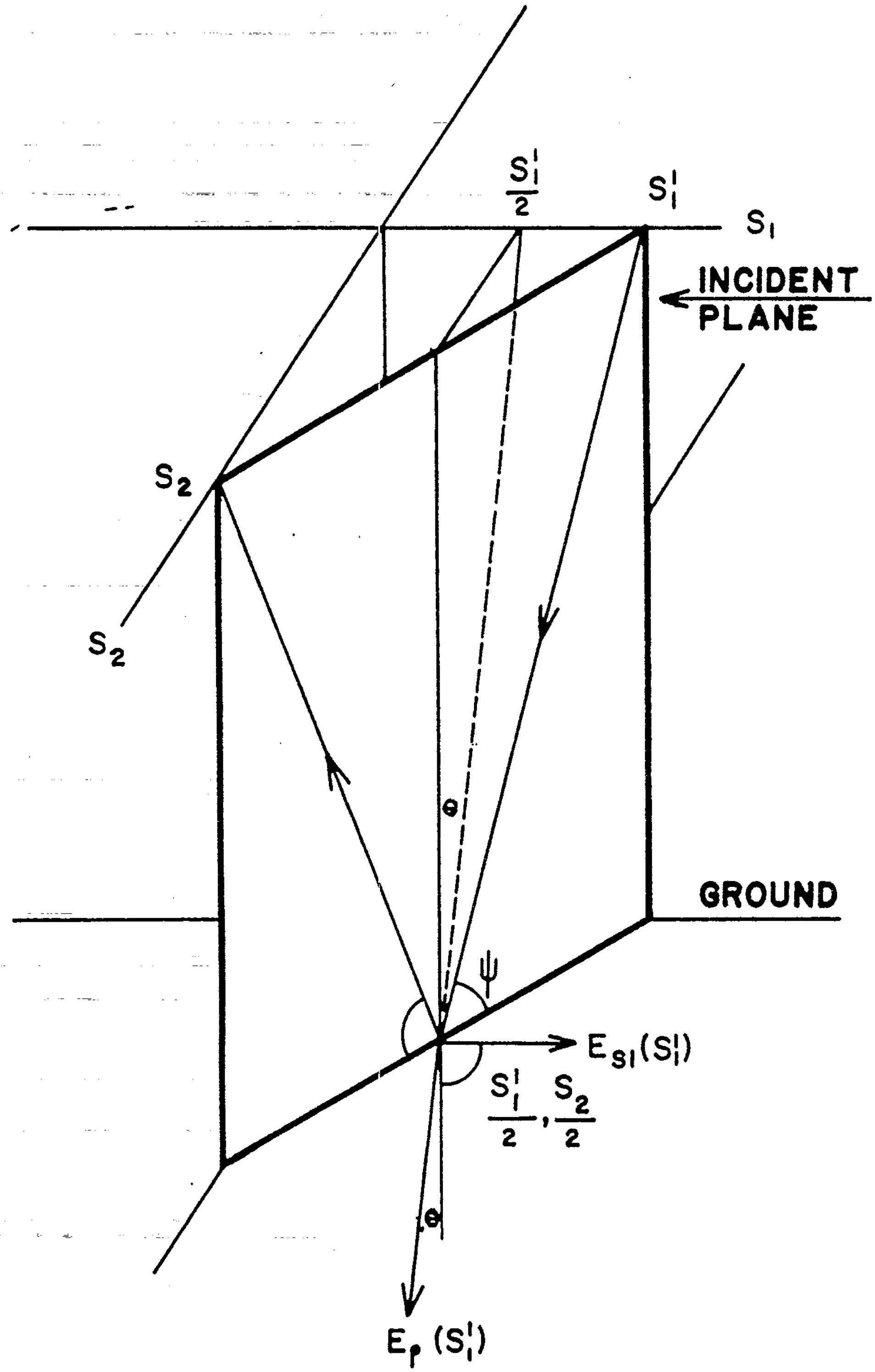

Figure 2-3. The geometry for the axial and radial field terms for the crossed wire configuration.

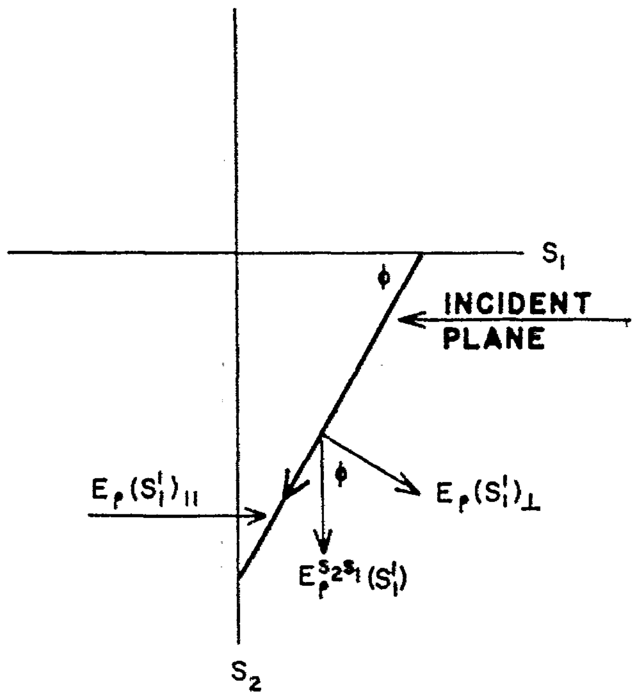

Figure 2-4. The geometry for the axial field secondary scattered term.

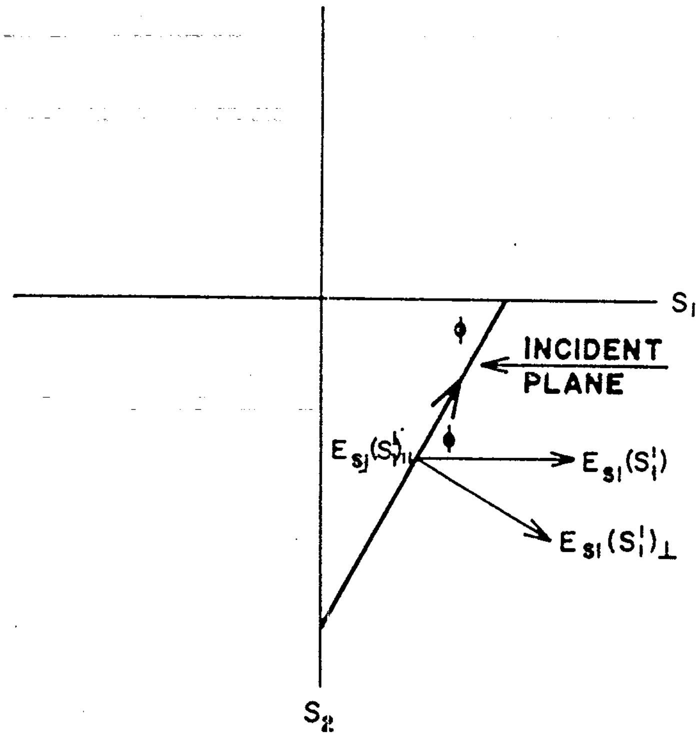

Figure 2-5. The geometry for the radial field secondary scattered term.

The total scattered field in the plane of incidence at the scattering point is

$$E_\rho(s_1')_{||} - E_{s1}(s_1')_{||}$$

and the total reflected "parallel" field at s_1 is

$$RPA\left[E_\rho(s_1')_{||} - E_{s1}(s_1')_{||}\right] \tag{2.5}$$

Similarly the total reflected "perpendicular" field at s_1 is

$$RPE\left[E_\rho(s_1')_{\perp} + E_{s1}(s_1')_{\perp}\right] \tag{2.6}$$

In (2.5) and (2.6)

$$RPA = -\frac{(\varepsilon_r + X)\sin\psi(s_2,s_1') - \sqrt{(\varepsilon_r + X) - \cos^2\psi(s_2,s_1')}}{(\varepsilon_r + X)\sin\psi(s_2,s_1') + \sqrt{(\varepsilon_r + X) - \cos^2\psi(s_2,s_1')}} \tag{2.7}$$

$$RPE = \frac{\sin\psi(s_2,s_1') - \sqrt{(\varepsilon_r + X) - \cos^2\psi(s_2,s_1')}}{\sin\psi(s_2,s_1') + \sqrt{(\varepsilon_r + X) - \cos^2\psi(s_2,s_1')}} \tag{2.8}$$

The total scattered field at a point on wire 2, e.g. s_2, can now be written as the vector sum of

1) $\vec{E}_{primary}(s_2,s_2')$ (see Figure 2-2)

2) $\vec{E}_{primary}(s_2,s_1')$ (see Figure 2-3)

3) $\vec{E}_{secondary}(s_2,s_2')$ (see Figure 2-2)

4) $\vec{E}_{secondary}(s_2,s_1')$ (see Figure 2-3)

Item 1 and 2 are simply the free space terms. Item 3 is the secondary scattered term of Riggs and Shumpert while item 4 is the combination of (2.5) and (2.6). The tangential component of (this resultant vector plus the incident field) is set to zero. Thus, a system of linear equations for the unknown current expansion coefficients is produced in the form

$$\bar{\bar{Z}}(s)\ \bar{J}(s) = \bar{E}(s) \qquad (2.9)$$

where $\bar{\bar{Z}}(s)$ is the square matrix of the system, $\bar{J}(s)$ is the column matrix of the α's(see 2.3), and $\bar{E}(s)$ is a column matrix whose elements are the particular values of the incident field components.

Since the techniques of SEM are well-documented [6,7,8,9,11,12], the details will not be repeated. The problem of interest is the behavior of the natural resonances as the ground parameters, σ_g and ε_r, vary. To find the natural resonances, the non-trivial solutions to

$$\det\left|\bar{\bar{Z}}(s)\right| = 0 \qquad (2.10)$$

must be found. Various techniques may be applied (6,7,10); in this study the Muller iteration is used.

III. RESULTS AND CONCLUSIONS

The geometry of the problem is defined in Figure 2-1. In particular, the parameters chosen are

$$h/L = 0.2,\ L/a = 20.0,\ \ell_1'/\ell_1 = 0.5,\ \ell_1' + \ell_1 = 2\ell_2 = L.$$

The trajectories of the first six natural resonances are determined as the ground conductivity varies from 120 S to 0.001 S, and the relative permittivity of the ground varies from 1 to 35. Note that since all resonances, s, are scaled by a factor L (or normalized to unit length), σ_g as it appears in (2.4), (2.7), and (2.8) is scaled by the factor L; thus the ground conductivities appearing are in S. Table 3-1 and Figure 3-1 give the comparison between the results of this study for a high conductivity (120S) and ε_r equal one (1) to a previous study of the crossed wires over a perfectly conducting ground [10]. The observed differences, mainly in the real parts, can be attributed to two factors: 1) the distance factors on the image terms [10] in the secondary scattered terms are defined in a slightly different way; 2) the conductivity is not infinite. Figures 3-2 through 3-7 present families of curves exhibiting the results of this study. The point x on each figure is the image theory [10] location of the resonance. Also on each figure is the location of the free-space resonance, ⊙ . Consider Figure 3-2: each curve on this figure represents the trajectory of sy,1,1 resonance for a particular choice of ε_r(1,5,10,20,35). The points on a given curve represent the value of the sy,1,1 resonance on the particular ε_r curve for values of σ_g (A = 120, B = 10, C = 1, D = 0.1, E = 0.01, F = 0.001 S).

Several conclusions can now be drawn. a. For a highly conducting ground (~ 120 S) the resonance location is relatively independent of the ε_r value and, in fact, approaches the image theory result in each case. b. For an ε_r = 1.0, as the ground conductivity approaches zero the resonance

TABLE 3-1

Complex Natural Resonances for Crossed Wire Structure

($\ell_1' + \ell_1 = L = 2\ell_2$, $L/a = 20$, $\ell_1'/\ell_1 = 0.5$, $h/L = 0.2$)

resonance	perfect ground	imperfect ground $\sigma_g = 120$ S $\varepsilon_r = 1$
sy,1,1	-0.0513 + j2.361	-0.0759 + j2.3420
as,1	-0.0898 + j2.592	-0.1246 + j2.5619
sy,1,2	-0.1021 + j3.769	-0.1296 + j3.7450
sy,2,2	-0.3909 + j5.740	-0.4659 + j5.7155
sy,3,1	-0.6691 + j7.581	-0.7816 + j7.5489
as,3	-0.6478 + j8.034	-0.7579 + j8.003

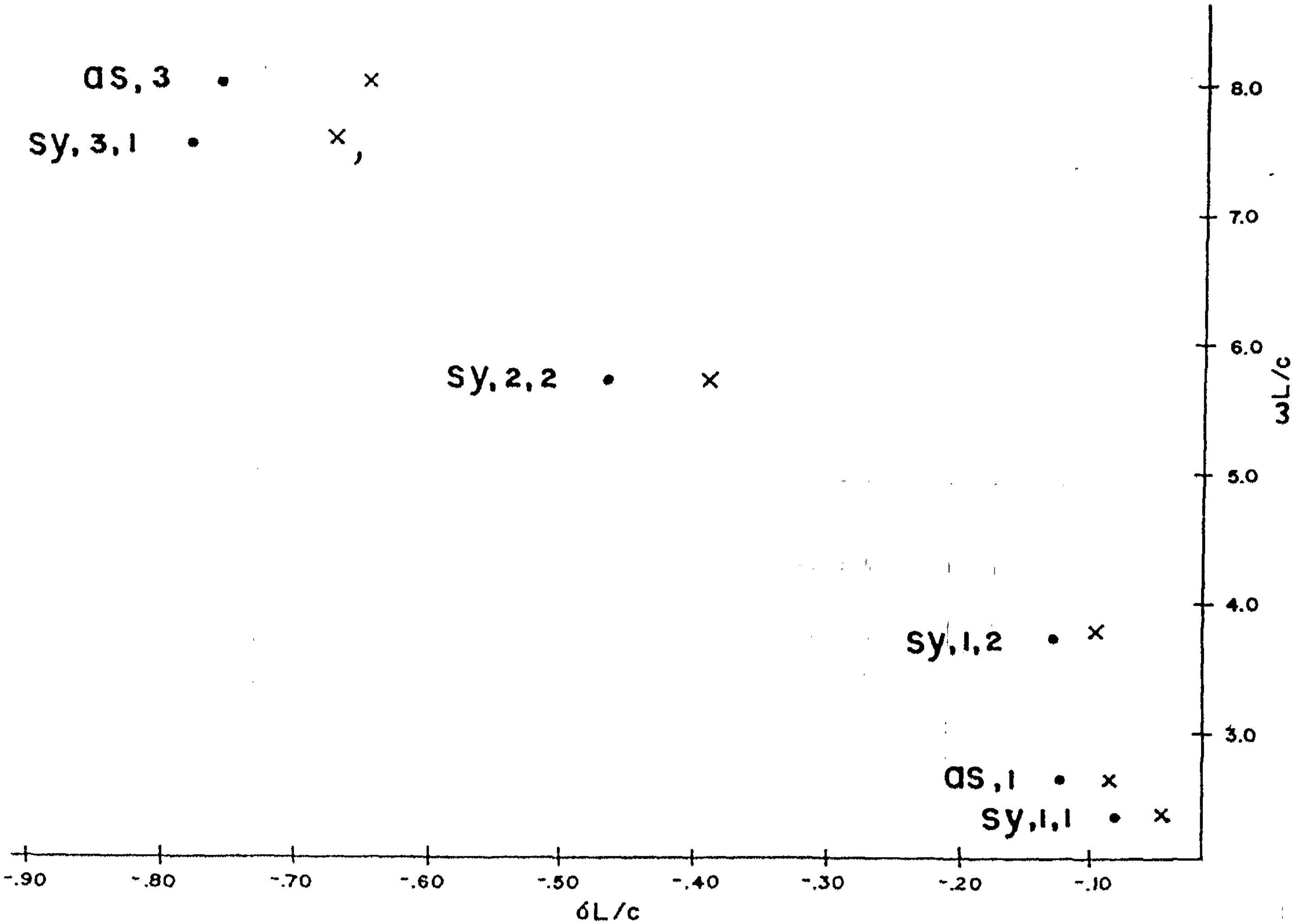

Figure 3-1. A comparison of the natural resonances by image theory and the Fresnel reflection method at high conductivity.

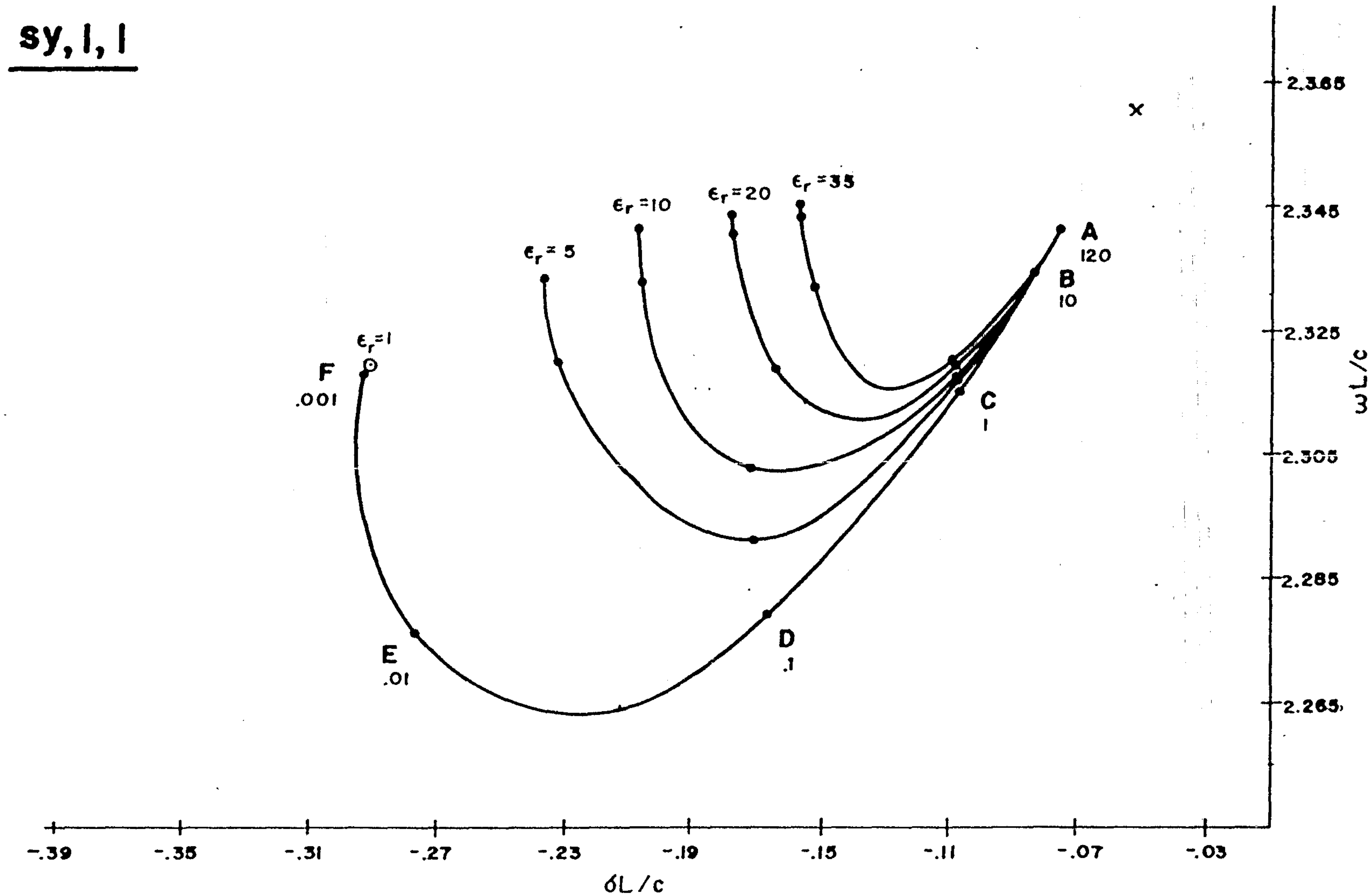

Figure 3-2. The trajectory of the natural resonance, sy,1,1 as a function of permittivity $\varepsilon_r \varepsilon_o$ and conductivity σ_g.

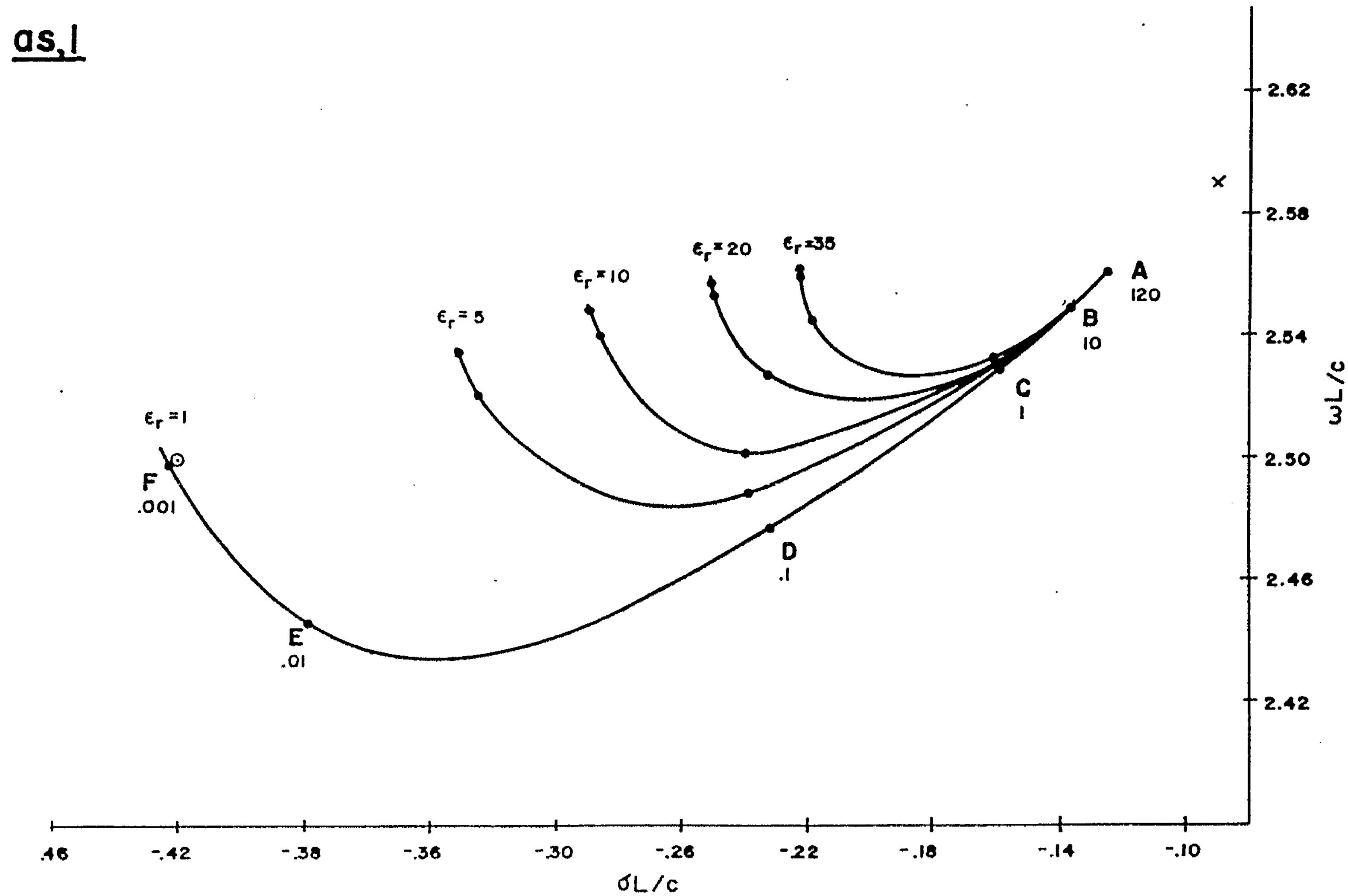

Figure 3-3. The trajectory of the natural resonance, as,l as a function of permittivity $\varepsilon_r\varepsilon_o$ and conductivity σ_g.

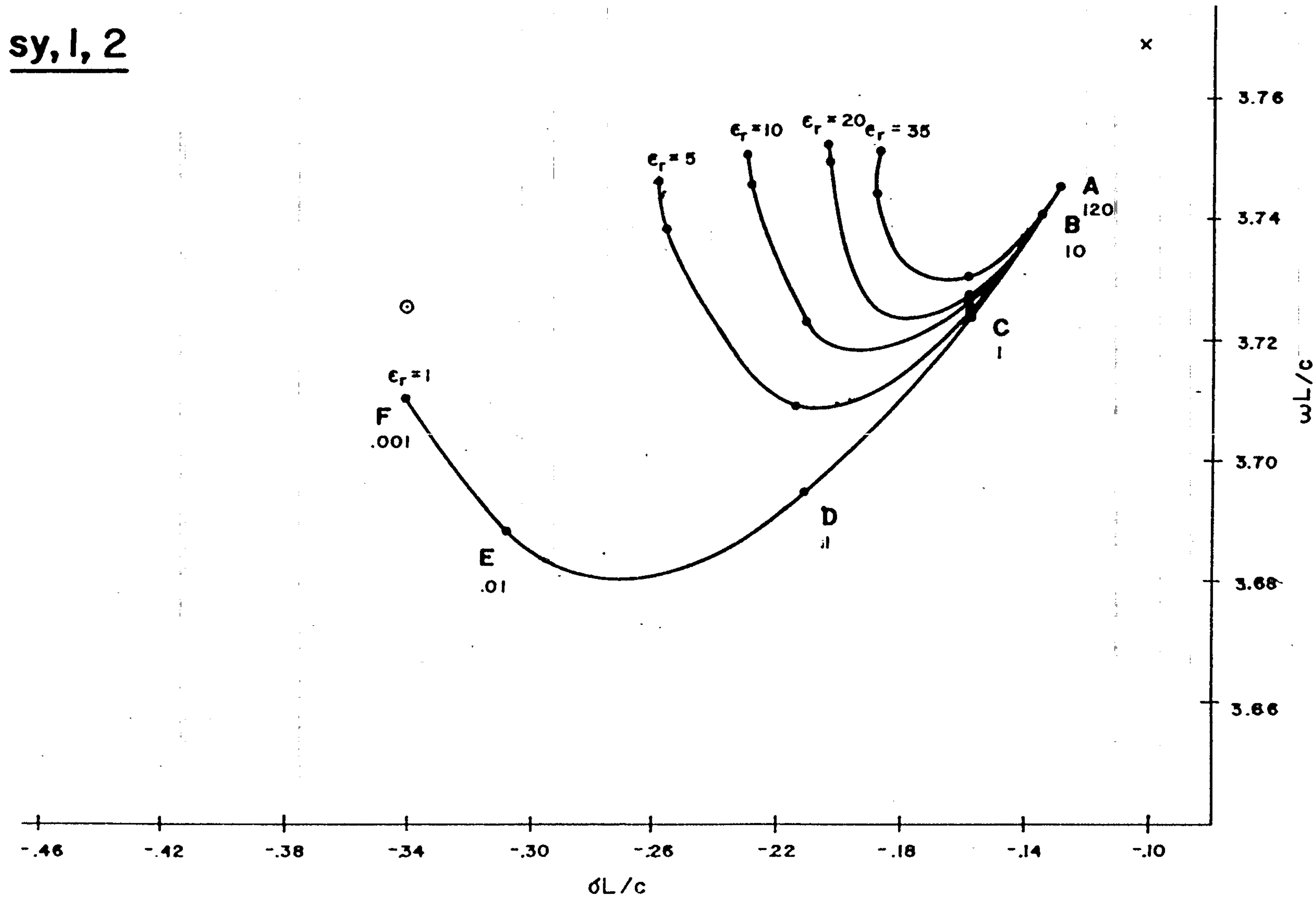

Figure 3-4. The trajectory of the natural resonance, sy,1,2 as a function of permittivity $\varepsilon_r\varepsilon_o$ and conductivity σ_g.

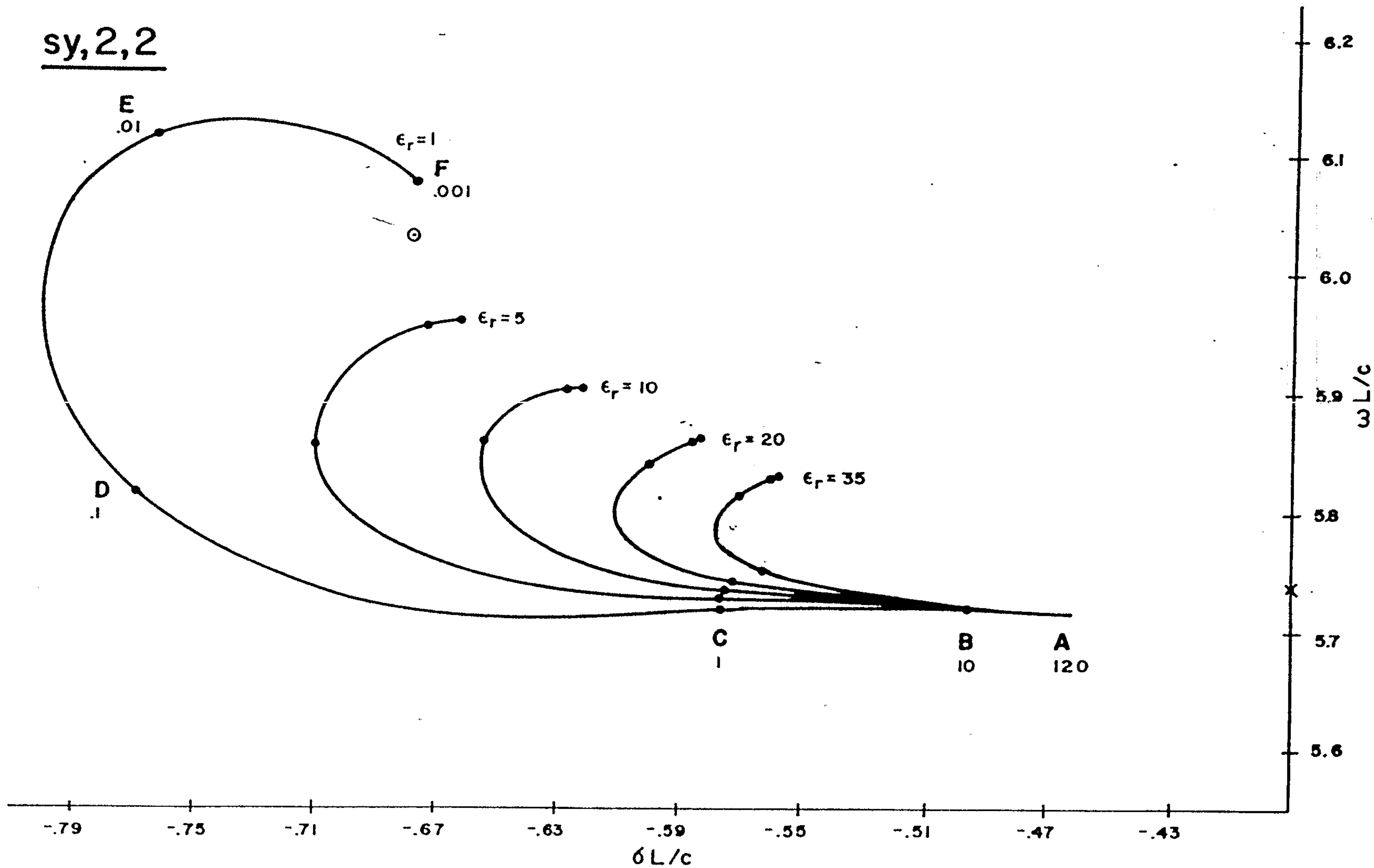

Figure 3-5. The trajectory of the natural resonance, sy,2,2 as a function of permittivity $\varepsilon_r\varepsilon_o$ and conductivity σ_g.

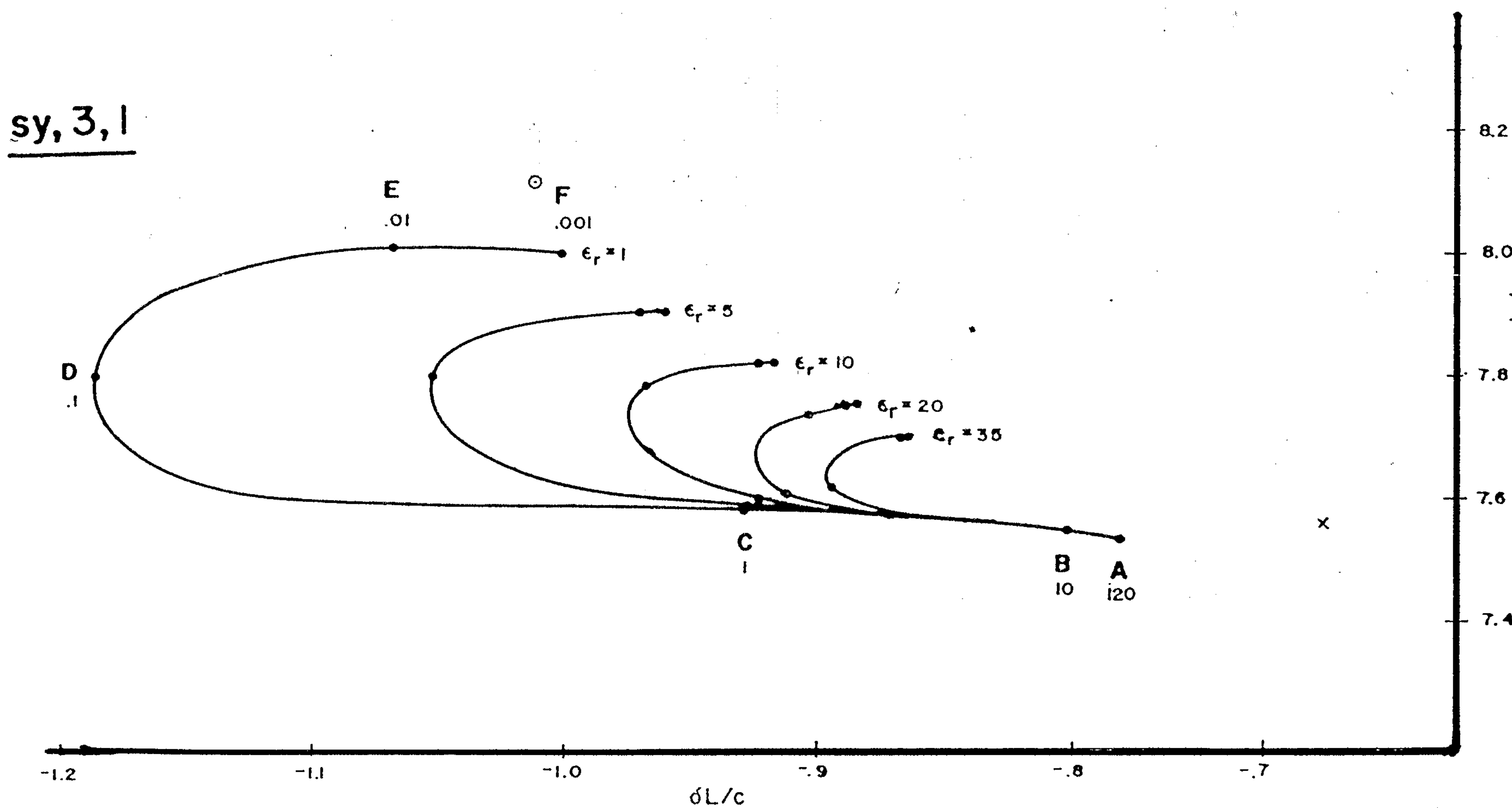

Figure 3-6. The trajectory of the natural resonance, sy,3,1 as a function of permittivity $\varepsilon_r\varepsilon_o$ and conductivity σ_g.

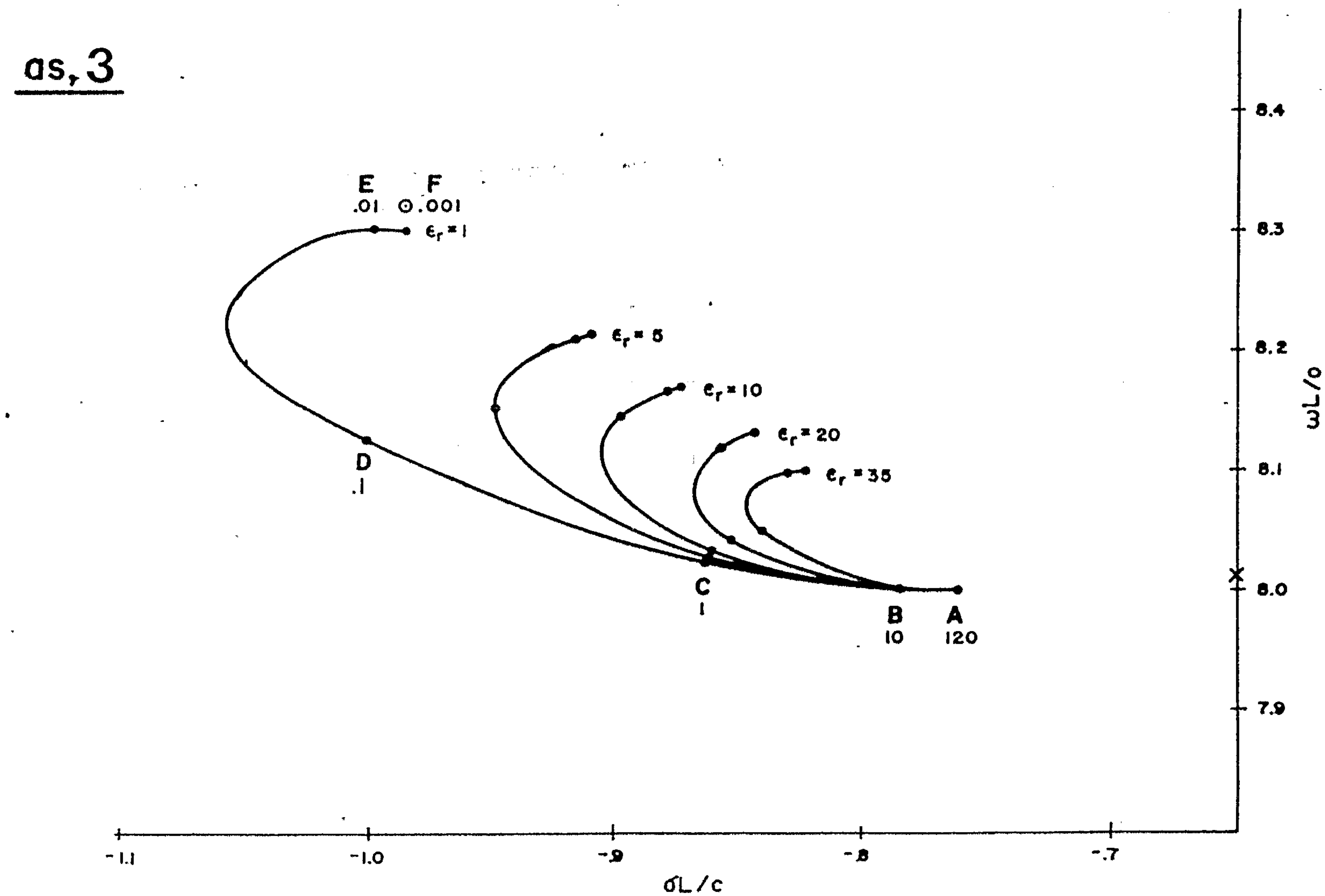

Figure 3-7. The trajectory of the natural resonance, as,2 as a function of permittivity $\varepsilon_r\varepsilon_o$ and conductivity σ_g.

location approaches the free space value. c. For large ε_r values, the trajectory of a resonance covers less of the complex plane as σ_g varies. The curves of Riggs and Shumpert exhibit the same behavior but for a very different shape parameter (L/a = 200 in their study).

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