Sensor and Simulation Notes

Note 260

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The PARTES Concept in EMP Simulation

Carl E. Baum
Air Force Weapons Laboratory

Abstract

EMP simulation has been developing steadily for almost two decades. In the case of large EMP simulators intended for illumination of complete systems with correct spatial dependence, and their sometimes threat-like characteristics including correct frequency spectrum, the state of the art is rather mature. Various possibilities have been explored consistent with what Maxwell's equations allow one to do, leading to a somewhat logical list of simulator types. Some new thrusts in EMP simulation are pointed toward small EMP simulators which in some cases could be thought of as partial EMP simulators in that they illuminate a part of the system or synthesize part of the relevant sources for an EMP test. One of the promising concepts for special applications is the PARTES concept. Relying on integral equation descriptions of the electromagnetic interaction processes, and on the field equivalence principle for the synthesis of the electromagnetic fields, one can define a set of electric and magnetic dipoles which approximately synthesize proper EMP excitation by the use of superposition which implies an assumption of linearity. By varying the amplitude and phase of the sampling dipoles one can approximate any direction of incidence and polarization for a free-space plane wave over some limited volume of space.

A very important result concerns the use of vector norms and associated matrix norms in conjunction with the PARTES concept. In particular, one can obtain an approximate tight bound of the response of the systems at an internal failure port for all angles of incidence and polarization through the uses of such concepts. While the PARTES concept is limited by the assumption of linearity and by the number of samples, it appears to have significant potential for EMP hardness maintenance of complex electronic systems.

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I. Introduction

EMP simulation has been reviewed and defined in a previous paper [3].

> "(EMP) simulation is an experiment in which the postulated (EMP) exposure situation is replaced by a physical situation in which:
>
> 1. the (EMP) sources are replaced by a set of equivalent sources which to a good approximation produce the same excitation (including reconstruction by superposition to the extent feasible) to the total system under test or some portion thereof as would exist in the postulated nuclear environment, and
>
> 2. the system under test is configured so that it reacts to sources (has the same Green's function) in very nearly the same way and to the same degree as it would in the postulated nuclear environment.
>
> A(n) (EMP) simulator is a device which provides the excitation used for (EMP) simulation without significantly altering the response of the system under test by the simulator presence."

An important assumption that is often made is that of linearity. One can design threat-like simulators which avoid making such an assumption. However, introducing an assumption of linearity introduces a range of possibilities based on constructing a response (say at some "pin" or "failure port" [2,4]) in the system as a linear combination of responses to a set of excitations. The same linear combination of the excitations is chosen so that the desired excitation is obtained by superposition.

This principle of superposition has been commonly used with respect to temporal superposition in EMP applications, i.e., waveforms with different time variation can be linearly combined to construct some desired waveform, this construction applying to both excitation and (linear) response. Often one thinks of this temporal superposition in terms of the response to a delta function excitation, but this requires the additional assumption of time-translation invariance, an assumption which is often valid and very useful. In this case the superposition takes the form of a convolution integral. Time-translation invariance also allows one the convenience of temporal Laplace (Fourier) transforms into the complex frequency domain where convolution is replaced by simpler multiplication. Of course inverse transformation is typically subsequently required back to the time variable.

Let us now generalize our traditional concept of temporal superposition to the concept of spatial superposition. One can decompose the sources into a set of spatial functions such that a linear combination of these gives (at least to an approximation for a finite set of functions) the desired source function (in general a vector function) in space. The same linear combination of the individual responses then gives the desired response to the original postulated source distribution in space.

There are many ways to define electromagnetic sources. Such sources might be constrained current distributions in space. They can be associated with the electromagnetic fields incident on a scatterer. They may be short-circuit surface current and charge densities on shorted apertures. In general, any quantities which are forcing functions for any set of equations which provide an exact or approximate description of the electromagnetic response of all or part of a scatterer (system) of interest can be thought of as sources. A general approach to EMP simulation then views all of these sources as potentially reproducable (approximately, including use of superposition) by an EMP simulator [3,9].

Of the many kinds of spatial source construction one might consider, let us restrict our attention to the reconstruction of the incident electromagnetic fields. Specifically let us consider the approximate reconstruction of appropriate incident electromagnetic quantities on some closed surface S surrounding a volume V which contains the system to be tested. This reconstruction of incident-field quantities on (or near) such boundary surfaces by a set of sources (at some set of locations on or near S) is referred to as PARTES (Piecewise Application of Radiation Through an EMP Simulator) [3,9]. Present considerations are restricted to cases that have no sources in V and no currents passing through S. While the present development considers the incident field to be in free space, cases of more general types of linear media in which the incident field propagates can be considered.

For the present development of the PARTES theory the incident fields are reconstructed using the field equivalence principle. The equivalent electric and magnetic surface current densities on S are then approximated by a set of electric and magnetic dipoles on S. The relative contributions of these two types of equivalent sources are interesting, and some interpretation of their roles can be made in the case that the system outer boundary surface S_s is (approximately) perfectly conducting and approaches S from the inside (i.e., within V).

Another approach to developing the equivalent sources is based on some integral-equation representation of the scattering process in which the incident field assumes the role of a source. If the scatterer is (approximately) perfectly conducting, its outer boundary S_s is the domain of integration. Elementary sources (equivalent magnetic dipoles) on S_s can approximate the incident field within certain limitations. This approach is compared to the field-equivalence form and interpreted.

Considering the response of the system as a linear combination of the PARTES sources, or as a linear operator on the incident field, and considering the incident field as a plane wave, some interesting bounds on the system response can be found. In particular these bounds can give approximate worst case results for all angles of incidence and polarization of an incident plane wave.

II. Field Equivalence Principle

Beginning with the Maxwell equations in free space

$$\nabla \times \vec{E} = -\mu_o \frac{\partial}{\partial t} \vec{H} - \vec{J}_h$$
$$\nabla \times \vec{H} = \varepsilon_o \frac{\partial}{\partial t} \vec{E} + \vec{J} \tag{2.1}$$

these can be put into the convenient combined-field form [7]

$$\left[\nabla \times - \frac{qj}{c} \frac{\partial}{\partial t}\right] \vec{E}_q = qjZ_o\vec{J}_q$$

$\vec{E}_q \equiv \vec{E} + qjZ_o\vec{H}$ (combined field)

$\vec{J}_q \equiv \vec{J} + \frac{qj}{Z_o} \vec{J}_h$ (combined current density) (2.2)

$q = \pm 1$ (separation index)

$j = + \sqrt{-1}$ (unit imaginary)

$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}$ (wave impedance of free space)

$c = \frac{1}{\sqrt{\mu_o\varepsilon_o}}$ (speed of light in free space)

There is also the combined charge density

$$\rho_q = \rho + \frac{qj}{Z_o} \rho_h \tag{2.3}$$

with the combined continuity equation

$$\nabla \cdot \vec{J}_q = - \frac{\partial}{\partial t} \rho_q$$
$$\nabla \cdot \vec{E}_q = \frac{1}{\varepsilon_o} \rho_q \tag{2.4}$$

Referring to figure 2.1, if there is some (infinitesimally thin) sheet on surface Σ with tangential surface current densities (electric and/or magnetic), there is a boundary condition

$$\vec{1}_{\Sigma} \times \left[\vec{E}_q^{(+)} - \vec{E}_q^{(-)}\right] = qjZ_o\vec{J}_{s_q}$$

$$\vec{J}_{s_q} = \vec{J}_s + \frac{qj}{Z_o}\vec{J}_{s_h} \quad \text{(combined surface current density)} \tag{2.5}$$

This combined boundary condition relates the tangential fields on the two sides of Σ, indicated by + and - superscripts, to the combined surface current density (units A/m) on Σ. Note that the unit surface normal $\vec{1}_{\Sigma}$ points to the + side from Σ.

Consider now in figure 2.2 the volume V bounded by the closed surface S ($V \cap S = 0$). V need not be simply connected for the following considerations. Note that V is free space (contains no scatterer) initially for this discussion.

Suppose that some incident field is defined over all space as

$$\vec{E}_q^{(inc)}(\vec{r},t) = \vec{E}^{(inc)}(\vec{r},t) + qjZ_o\vec{H}^{(inc)}(\vec{r},t)$$

$$\vec{J}_q^{(inc)}(\vec{r},t) = \vec{J}^{(inc)}(\vec{r},t) + \frac{qj}{Z_o}\vec{J}_h^{(inc)}(\vec{r},t) \tag{2.6}$$

where $\vec{J}_q^{(inc)}$ accounts for the sources of the incident field as required. Constraining such sources to be zero inside V, let us construct a field in the form

$$\vec{E}_q(\vec{r},t) = \begin{cases} \vec{E}_q^{(inc)}(r,t) & , \quad \vec{r} \in V \\ 0 & , \quad \vec{r} \notin [V \cup S] \end{cases}$$

$$\vec{J}_{s_q}^{(inc)}(\vec{r}_s,t) = \frac{qj}{Z_o}\vec{1}_S(\vec{r}_s) \times \vec{E}_q^{(inc)}(\vec{r}_s,t)$$

$$\vec{r}_s \equiv \vec{r} \text{ for } \vec{r} \in S \tag{2.7}$$

$$\vec{1}_S(\vec{r}_s) \equiv \text{outward pointing normal to S at } \vec{r}_s$$

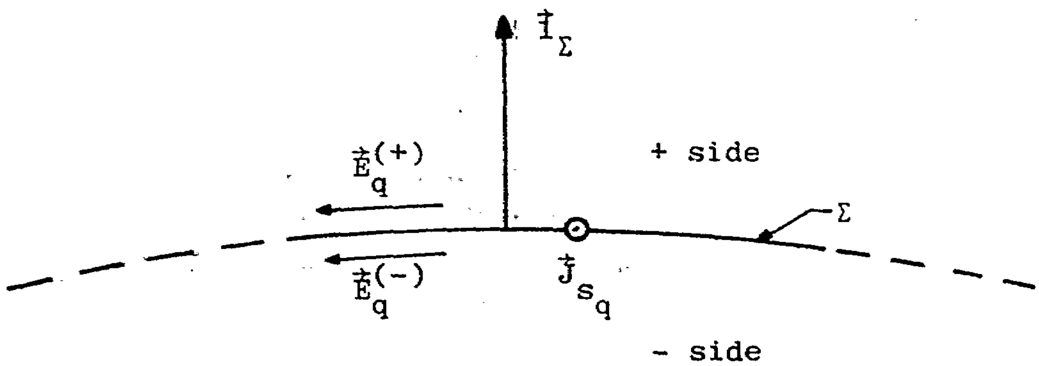


Fig. 2.1. Combined Boundary Condition at a Sheet

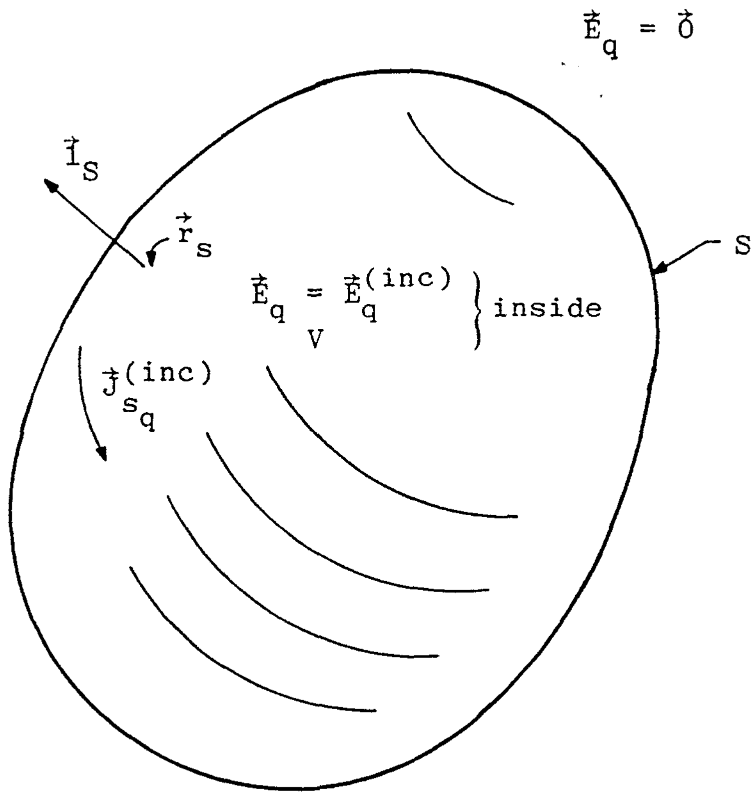


Fig. 2.2. Field Equivalence Principle for Constructing Incident Fields Inside a Volume

The use of the combined boundary condition (2.5) with + for outside S and - for inside S thus gives us a formula for sources on S such that the desired incident field is reproduced inside S. Note that source currents as in (2.6) can be included inside V and hence in (2.7) if desired. However the present development is not for such a case. Separating these out we have two equivalent sources on S

$$\vec{J}_s^{(inc)}(\vec{r}_s,t) = -\vec{1}_S(\vec{r}_s) \times \vec{H}^{(inc)}(\vec{r}_s,t)$$
$$\vec{J}_{s_h}^{(inc)}(\vec{r}_s,t) = \vec{1}_S(\vec{r}_s) \times \vec{E}^{(inc)}(\vec{r}_s,t) \qquad (2.8)$$

In this form (2.7) gives a very compact statement of Love's [10] field equivalence principle (for fields interior to a volume).

III. Approximation of Surface-Current-Density Sources on S by Discrete Sources

Figure 3.1 indicates the subdivision of S into elementary zones designated S_n with

$$S = \bigcup_{n=1}^{N_s} S_n \tag{3.1}$$

neglecting the boundary curves of the S_n (depending on whether or not the S_n include the boundary curves). In principle the number of surface zones or patches N_s should be large enough that each S_n is approximately flat (at least for most S_n). Furthermore the S_n each have area A_n with roughly the same A_n for each n, and with roughly the same transverse dimensions so that the maximum linear dimension tends to zero as $N_s \to \infty$. The total area of S is A with

$$A = \sum_{n=1}^{N_s} A_n \tag{3.2}$$

Let $\vec{r}_s = \vec{r}_{s_n}$ be the approximate center of S_n and write the unit outward normal there as $\vec{1}_S(\vec{r}_{s_n})$. Construct a local coordinate system based on a right-handed set of unit vectors $\vec{1}_{S_m}(\vec{r}_{s_n})$ as

$$\begin{aligned}\vec{1}_{S_1}(\vec{r}_{s_n}) \times \vec{1}_{S_2}(\vec{r}_{s_n}) &= \vec{1}_{S_3}(\vec{r}_{s_n}) \equiv \vec{1}_S(\vec{r}_{s_n})\\ \vec{1}_{S_2}(\vec{r}_{s_n}) \times \vec{1}_{S_3}(\vec{r}_{s_n}) &= \vec{1}_{S_1}(\vec{r}_{s_n})\\ \vec{1}_{S_3}(\vec{r}_{s_n}) \times \vec{1}_{S_1}(\vec{r}_{s_n}) &= \vec{1}_{S_2}(\vec{r}_{s_n})\end{aligned} \tag{3.3}$$

Consider next the surface current densities (electric and magnetic) on S_n. At large distances and long wavelengths (low frequencies) compared to the linear dimensions of S_n the fields are well approximated by the electric and magnetic dipole terms, the leading terms in the multipole expansion [1]. Now the combined dipole moment is

$$\vec{p}_q \equiv \vec{p} + \frac{qj}{c} \vec{m} \tag{3.4}$$

so that for some volume of finite dimensions (say V') which contains all of the currents of interest (none passing through V') we can write

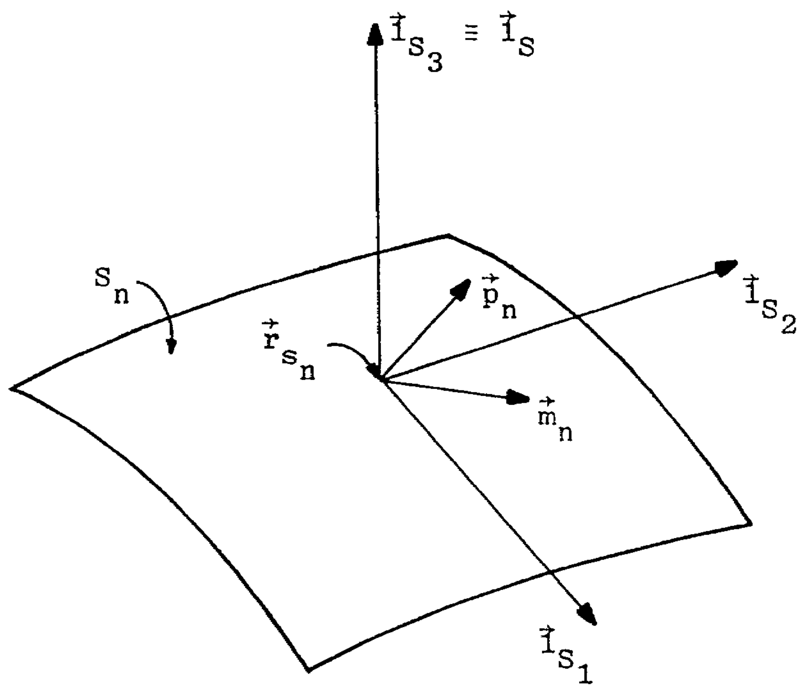


Fig. 3.1. Surface Patch with Local Coordinates

$$\vec{p}_q(t) = \int_{V'} \rho_q(\vec{r},t)\, \vec{r}\, dV = \int_{-\infty}^{t}\int_{V'} \vec{J}_q(\vec{r},t')\, dV\, dt'$$

$$\tilde{\vec{p}}_q(s) = \int_{V'} \tilde{\rho}_q(\vec{r},s)\, \vec{r}\, dV = \frac{1}{s}\int_{V'} \tilde{\vec{J}}_q(\vec{r},s)\, dV \tag{3.5}$$

where ~ indicates the Laplace transform (two-sided) and s is the complex frequency. Here as $t \to -\infty$ the combined charge density etc. is assumed zero (zero initial conditions). One can also have an $\vec{r} \times \vec{J}_q$ term in the integrand, but the contribution of this is made negligible in the present application by the choice of the location of the coordinate center for the computation of $\vec{p}_q$ in the approximate center of V'.

So consider the dipole moments associated with the surface current densities on a particular S_n. The volume integrals reduce to surface integrals as

$$\vec{p}_{n_q}(t) = \int_{S_n} \rho_{s_q}^{(inc)}(\vec{r}_s,t)\, dV = \int_{-\infty}^{t}\int_{S_n} \vec{J}_{s_q}^{(inc)}(\vec{r}_s,t')\, dV\, dt'$$

$$\tilde{\vec{p}}_{n_q}(s) = \int_{S_n} \tilde{\rho}_{s_q}^{(inc)}(\vec{r}_s,t)\, dV = \frac{1}{2}\int_{S_n} \tilde{\vec{J}}_{s_q}^{(inc)}(\vec{r}_s,s)\, dV \tag{3.6}$$

with the combined dipole located at $\vec{r}_{s_n}$. Note that the above integrals over S_n must include the edge (in the case of ρ_{s_q}) because of the current discontinuity there and resulting line charge density (singularity in the surface charge density) when considering only the contribution of the S_n patch.

If we assume S_n sufficiently small and approximately flat we can use the value of the current density at $\vec{r}_{s_n}$ giving

$$\tilde{\vec{p}}_{n_q}(s) \simeq \frac{A_n}{s}\, \tilde{\vec{J}}_{s_q}^{(inc)}(\vec{r}_{s_n},s)$$

$$= \frac{qj}{Z_o}\, \frac{A_n}{s}\, \vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{E}}_q^{(inc)}(\vec{r}_{s_n},s) \tag{3.7}$$

thereby relating our elementary dipole sources to the desired incident field.

Separating out the electric and magnetic dipole moments we have

$$\tilde{\vec{p}}_n(s) \simeq \frac{A_n}{s}\,\tilde{\vec{J}}_s^{(inc)}(\vec{r}_{s_n},s)$$

$$= -\frac{A_n}{s}\,\vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{H}}^{(inc)}(\vec{r}_{s_n},s)$$

$$\tilde{\vec{m}}_n(s) \simeq \frac{A_n}{s\mu_o}\,\tilde{\vec{J}}_{s_h}^{(inc)}(\vec{r}_{s_n},s) \tag{3.8}$$

$$= \frac{A_n}{s\mu_o}\,\vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{E}}^{(inc)}(\vec{r}_{s_n},s)$$

These electric and magnetic dipole moments are parallel to S_n and located at $\vec{r}_{s_n}$ as indicated in figure 3.1. Noting that the $\vec{1}_{S_m}$ for m = 1,2 are the unit vectors parallel to S_n as $\vec{r}_{s_n}$ we have the components

$$\tilde{p}_{n,m}(s) = \frac{A_n}{s}\,\tilde{J}_{s_m}^{(inc)}(\vec{r}_{s_n},s) = \begin{cases} \frac{A_n}{s}\,\tilde{H}_2^{(inc)}(\vec{r}_{s_n},s) & \text{for } m = 1 \\ -\frac{A_n}{s}\,\tilde{H}_1^{(inc)}(\vec{r}_{s_n},s) & \text{for } m = 2 \end{cases}$$

$$\tilde{m}_{n,m}(s) = \frac{A_n}{s\mu_o}\,\tilde{J}_{s_{h_m}}^{(inc)}(\vec{r}_{s_n},s) = \begin{cases} -\frac{A_n}{s\mu_o}\,\tilde{E}_2^{(inc)}(\vec{r}_{s_n},s) & \text{for } m = 1 \\ \frac{A_n}{s\mu_o}\,\tilde{E}_1^{(inc)}(\vec{r}_{s_n},s) & \text{for } m = 2 \end{cases} \tag{3.9}$$

Given some specified incident field and some subdivision of S into a set of S_n with "centers" at a set of $\vec{r}_{s_n}$, one can then specify the two components (m = 1,2) of the two kinds of dipoles (electric and magnetic) which will approximately reproduce the incident field inside V. Of course N_S must be chosen sufficiently large that the dipoles approximate the continuous ideal distributions of surface current densities. The spacing should be small compared to radian wavelength and small compared to the distance of the observer away from (and to the inside of) S, at least for an accurate approximation.

IV. Perfectly Conducting Scatterer with S_s Inside and Near S

So far the nature of the scatterer in V can be quite general. An interesting case, however, is that of a perfectly conducting scatterer with outer surface S_s in V as in figure 4.1. With S_s as the outer surface bounding V_s the fields now become

$$\vec{E}_q(\vec{r},t) = \begin{cases} \vec{E}^{(inc)}(\vec{r},t) + \vec{E}^{(sc)}(\vec{r},t) \text{ for } \vec{r} \in \left[V - [V_s \cup S_s]\right] = V \cap \overline{[V_s \cup S_s]} \\ \vec{E}^{(sc)}(\vec{r},t) \text{ for } \vec{r} \notin [V \cup S] \\ \vec{0} \quad \text{for } \vec{r} \in V_s \end{cases} \tag{4.1}$$

with the boundary condition on S_s

$$\overleftrightarrow{1}_{t'}(\vec{r}_s{}') \cdot \left[\vec{E}^{(inc)}(\vec{r}_s{}',t) + \vec{E}^{(sc)}(\vec{r}_s{}',t)\right] = \vec{0}$$

$$\vec{r}_s{}' \equiv \vec{r} \text{ for } \vec{r} \in S_s \equiv \text{coordinates on } S_s$$

$$\overleftrightarrow{1}_{t'}(\vec{r}_s{}') \equiv \overleftrightarrow{1} - \vec{1}_{S_s}(\vec{r}_s{}')\, \vec{1}_{S_s}(\vec{r}_s{}') \equiv \text{transverse identity on } S_s \tag{4.2}$$

$$\vec{1}_{S_s}(\vec{r}_s{}') \equiv \text{unit outward normal on } S_s$$

Note that the scattered fields are continuous through S since the discontinuity associated with the equivalent-source surface current densities is satisfied by the incident-field-term discontinuity.

Figure 4.2 indicates what happens when S and S_s are close, separated by a spacing h. The local unit vectors $\vec{1}_S$ and $\vec{1}_{S_s}$ become parallel as $h \to 0$ and $\vec{r}_s \to \vec{r}_s{}'$. First a local portion of the diagram in figure 4.1 is given in figure 4.2A. Second the response to the source electric-surface-current-density sheet $\vec{J}_s^{(inc)}$ is indicated in figure 4.2B. For small h there is an approximate solution for the response to such an excitation for the magnetic field as

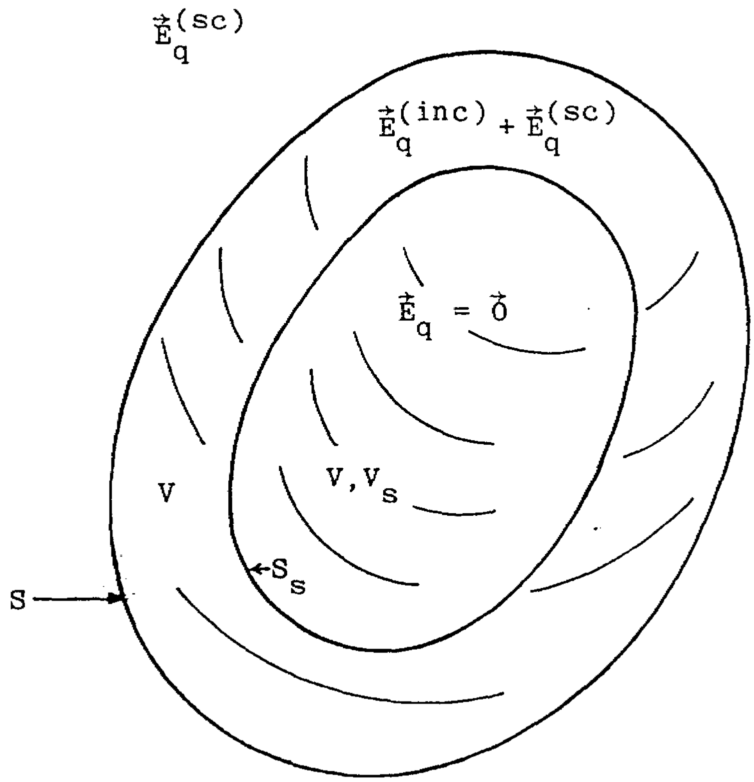
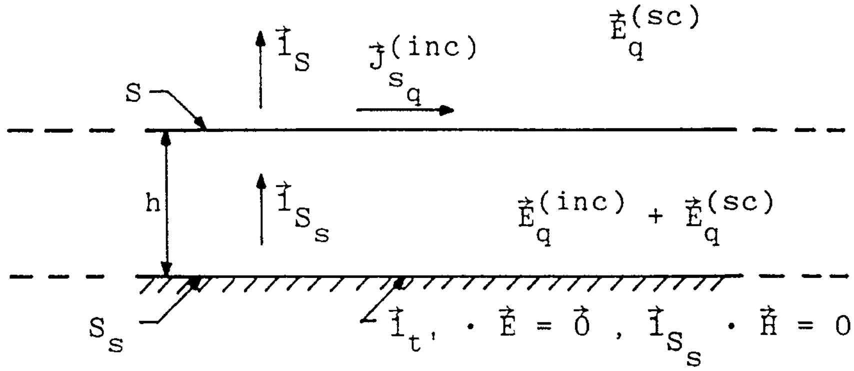
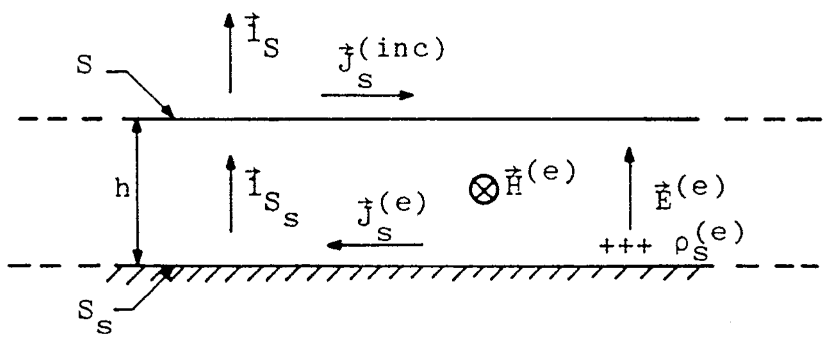


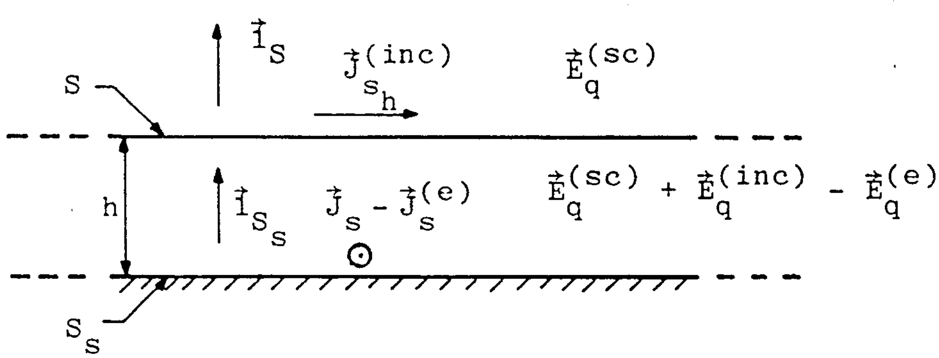
Fig. 4.1. Inclusion of Perfectly Conducting Scatterer Inside Incident-Field Volume



A. Combined Fields



B. Response to Electric Current Sheet



C. Response to Magnetic Current Sheet

Fig. 4.2. S_s Close to S

$$\vec{J}_s^{(e)}(\vec{r}_s') \simeq -\vec{J}_s^{(inc)}(\vec{r}_s)$$

$$\vec{r}_s \simeq \vec{r}_s' + h\, \vec{1}_{S_s}(\vec{r}_s')$$

$$\vec{r} \simeq \vec{r}_s' + ah\, \vec{1}_{S_s}(\vec{r}_s) \tag{4.3}$$

$$\vec{H}^{(e)}(\vec{r}) \simeq \begin{cases} \vec{1}_S(\vec{r}_s) \times \vec{J}_s^{(inc)}(\vec{r}_s) & \text{for } 0 < a < 1 \\ \vec{0} & \text{for } a < 0 \text{ or } a > 1 \text{ (inside } S_s \text{ or outside } S) \end{cases}$$

where the superscript e denotes this part of the response. The result for the magnetic field is rather simple based on a locally quasi-static calculation which gives no field outside S. Noting that $\vec{J}_s^{(inc)}$ is given by (2.8) we have

$$\vec{H}^{(e)}(\vec{r}) \simeq -\vec{1}_S(\vec{r}_s) \times \left[\vec{1}_S \times \vec{H}^{(inc)}(\vec{r}_s,t)\right]$$

$$= \overleftrightarrow{1}_t(\vec{r}_s) \cdot \vec{H}^{(inc)}(\vec{r}_s,t) \qquad \text{for } 0 < a < 1$$

$$\overleftrightarrow{1}_t(\vec{r}_s) \equiv \overleftrightarrow{1} - \vec{1}_S(\vec{r}_s)\, \vec{1}_S(\vec{r}_s) \tag{4.4}$$

Note that $\vec{H}^{(e)}$ between S_s and S is then just the tangential components of $\vec{H}^{(inc)}$. Observe that the normal magnetic field must be zero on S_s and as $S \to S_s$ the same must be true on S, giving $\vec{H}^{(e)} \to \vec{0}$ outside S as $S \to S_s$. If $\vec{H}^{(e)} \to 0$ outside S then so must $\vec{E}^{(e)}$ by (2.1). We may compute $\vec{E}^{(e)}$ between S_s and S via the surface charge density giving

$$\tilde{\vec{E}}^{(e)}(\vec{r},s) \simeq \frac{1}{\epsilon_o}\, \tilde{\rho}_s^{(e)}(\vec{r}_s',s)\, \vec{1}_{S_s}(\vec{r}_s') = -\frac{1}{\epsilon_o s}\, \nabla_s \cdot \tilde{\vec{J}}_s^{(e)}(\vec{r}_s',s)\, \vec{1}_{S_s}(\vec{r}_s')$$

$$\simeq \frac{1}{\epsilon_o s}\, \nabla_s \cdot \tilde{\vec{J}}_s^{(inc)}(\vec{r}_s,s)\, \vec{1}_S(\vec{r}_s)$$

$$= \frac{1}{\epsilon_o s}\, \nabla_s \cdot \left[-\vec{1}_S(\vec{r}_s) \times \tilde{\vec{H}}^{(inc)}(\vec{r}_s,s)\right] \vec{1}_S(\vec{r}_s)$$

$$\simeq \frac{1}{\epsilon_o s}\, \nabla_s' \cdot \left[-\vec{1}_{S_s}(\vec{r}_s') \times \tilde{\vec{H}}^{(inc)}(\vec{r}_s',s)\right] \vec{1}_{S_s}(\vec{r}_s') \tag{4.5}$$

Moving on to figure 4.2C we can find the solution to the source surface-magnetic-current-density sheet by subtracting the situation in figure 4.2B from that in 4.2A (i.e., apply superposition). From this we observe that the scattered field outside S is due only to the source surface-magnetic-current density. Summarizing we have

$$\vec{J}_s^{(h)}(\vec{r}_s',t) \simeq \vec{J}_s(\vec{r}_s',t) - \vec{J}_s^{(e)}(\vec{r}_s',t) = \vec{J}_s(\vec{r}_s',t) + \vec{J}_s^{(inc)}(\vec{r}_s',t)$$

$$\vec{E}_q^{(e)}(\vec{r},t) \simeq \begin{cases} \vec{E}_q^{(sc)}(\vec{r},t) & \text{for } a > 1 \text{ (outside S)} \\ \vec{E}_q^{(sc)}(\vec{r},t) + \vec{E}_q^{(inc)}(\vec{r},t) - \vec{E}_q^{(e)}(\vec{r},t) & \text{for } 0 < a < 1 \\ \vec{0} & \text{for } a < 0 \text{ (inside } S_s) \end{cases} \tag{4.6}$$

V. Approach Via Impedance Integral Equation for Perfectly Conducting Scatterer

Another way to view the results as $S \to S_S$ in section 4 is to consider the E-field or impedance integral equation for a perfectly conducting scatterer in the form

$$\left\langle \tilde{\overleftrightarrow{Z}}_{t'}(\vec{r}_s\,', \vec{r}_s\,''; s) \; ; \; \tilde{\vec{J}}_s(\vec{r}_s\,'', s) \right\rangle = \tilde{\vec{E}}_{t'}^{(inc)}(\vec{r}_s\,', s)$$

$$\vec{r}_s\,' \; , \; \vec{r}_s\,'' \in S_s \tag{5.1}$$

$$\tilde{\vec{E}}_{t'}^{(inc)}(\vec{r}_s\,', s) \equiv \overleftrightarrow{1}_{t'}(\vec{r}_s\,') \cdot \tilde{\vec{E}}^{(inc)}(\vec{r}_s\,', s)$$

$$\tilde{\overleftrightarrow{Z}}_{t'}(\vec{r}_s\,', \vec{r}_s\,''; s) \equiv \overleftrightarrow{1}_{t'}(\vec{r}_s\,') \cdot \tilde{\overleftrightarrow{Z}}(\vec{r}_s\,', \vec{r}_s\,''; s) \cdot \overleftrightarrow{1}_{t'}(\vec{r}_s\,'')$$

where the integration (denoted by <,>) is with respect to the common spatial coordinates ($\vec{r}_s\,''$ in this case) over the domain S_s. The dyadic impedance kernel is [5]

$$\begin{aligned}
\tilde{\overleftrightarrow{Z}}(\vec{r}_s\,', \vec{r}_s\,''; s) &= s\mu_o \tilde{\overleftrightarrow{G}}_o(\vec{r}_s\,', \vec{r}_s\,''; s) \\
&= \frac{s^2\mu_o}{4\pi c} \left\{ \left[-2\zeta^{-3} - 2\zeta^{-2} \right] e^{-\zeta} \, \vec{1}_R \vec{1}_R \right. \\
&\quad \left. + \left[\zeta^{-3} + \zeta^{-2} + \zeta^{-1} \right] e^{-\zeta} \left[\overleftrightarrow{1} - \vec{1}_R \vec{1}_R \right] \right\} \\
&\qquad \text{for } \vec{r}_s\,'' \neq \vec{r}_s\,'
\end{aligned} \tag{5.2}$$

$$\zeta \equiv \frac{s}{c} \left| \vec{r}_s\,' - \vec{r}_s\,'' \right|$$

$$\vec{1}_R \equiv \frac{\vec{r}_s\,' - \vec{r}_s\,''}{\left| \vec{r}_s\,' - \vec{r}_s\,'' \right|}$$

with appropriate care taken for $\vec{r}_s\,''$ near $\vec{r}_s\,'$.

This impedance integral equation is developed from the representation of the scattered (or radiated) electric field in terms of the surface current density as

$$\tilde{\vec{E}}^{(sc)}(\vec{r},s) = -\left\langle \tilde{\overleftrightarrow{Z}}(\vec{r},\vec{r}_s\,';s) \; ; \; \tilde{\vec{J}}_s(\vec{r}_s\,',s) \right\rangle \tag{5.3}$$

Enforcing the boundary condition that the tangential components of the total electric field be zero on S_s gives (5.1). This condition is enforced on S in the limit as $S \to S_s$ because of the problem in evaluating the integral at $\vec{r}_s\,'' = \vec{r}_s\,'$. Note that this procedure is closely related to the development in section 4.

In section 4 it is shown that as $S \to S_s$ the scattered field is produced only by the source magnetic surface-current density on S. This is equivalent to the tangential incident electric field by (2.8). So the scattered field computed by (5.3) is the total field computed outside S; this is then associated with only the equivalent magnetic surface current density on S. The impedance integral equation and the magnetic surface current density on S from the field equivalence theorem are then closely related.

From (4.6) we note that the surface current density on S_s associated with only the surface-magnetic-current-density source differs from $\vec{J}_s$. However the scattered field outside S is the same as that obtained by integrating over $\vec{J}_s$ in (5.3). This indicates that the external field (outside S) is correctly produced by only the magnetic current sources in the presence of the scatterer, but between S_s and S the situation is more complicated.

Now one can try to view the PARTES concept of EMP simulation directly from (5.1) via the moment method (MoM) [11]. One way to do this is to divide S_s into patches as was done to S in section 3, and convert (5.1) into a matrix equation relating the surface current density in each patch (in terms of the "expansion" functions) to the incident electric field in each patch (in terms of the "testing" functions). The incident electric field in a patch can be thought of as a surface magnetic current density (via (2.8)) which can be approximated by a magnetic dipole (via (3.8)).

However, an elementary magnetic dipole on S_s (and parallel to it) gives a very singular field distribution near the dipole (at $\vec{r}_{s_n}\,'$) corresponding to an equivalent magnetic dipole with twice the strength of $\vec{m}_n$ as computed by (3.8). This intense nonuniform field near $\vec{r}_{s_n}\,'$ can give excessive excitation to anything responding to such fields (such as small apertures, etc.). Furthermore the resulting surface current density on S_s does not include the $\vec{J}_s^{(e)}$ term.

It is interesting to note that since the $\vec{J}_s^{(e)}$ term is directly proportional to the incident field with no additional function of the complex frequency s, then this term does not contain the scatterer resonances (natural frequencies) used in the singularity expansion method (SEM) representation of the scatterer response [12]. The natural frequencies then must appear in the $\vec{J}_s^{(h)}$ term associated with the response to the equivalent magnetic-current sources. Remember that the scatterer is perfectly conducting for these results.

VI. Incident Plane Wave

The development of the PARTES simulation technique is quite general in that any incident field (with no sources in V) can be approximately reproduced by electric and magnetic sources on S. Often, however, one is interested in a specific type of incident field described as a plane wave in the form

$$\vec{E}^{(inc)}(\vec{r},t) = E_o f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)\vec{1}_e$$

$$\vec{H}^{(inc)}(\vec{r},t) = \frac{E_o}{Z_o} f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)\vec{1}_h$$

$$\tilde{\vec{E}}^{(inc)}(\vec{r},s) = E_o \tilde{f}(s)\, e^{-\gamma \vec{1}_1 \cdot \vec{r}}\, \vec{1}_e$$

$$\tilde{\vec{H}}^{(inc)}(\vec{r},s) = \frac{E_o}{Z_o} \tilde{f}(s)\, e^{-\gamma \vec{1}_1 \cdot \vec{r}}\, \vec{1}_h \tag{6.1}$$

$\vec{1}_e \times \vec{1}_h = \vec{1}_1$ (direction of incidence)

$\vec{1}_h \times \vec{1}_1 = \vec{1}_e$ (electric polarization, or just polarization)

$\vec{1}_1 \times \vec{1}_e = \vec{1}_h$ (magnetic polarization)

$\gamma = \frac{s}{c}$ (complex propagation constant of free space)

The waveform function f is typically chosen in some canonical form, such as for the high-altitude EMP below the source region [6]. Such a plane wave is appropriate for an in-flight system such as a missile or an aircraft. For systems on or near the ground two such plane waves (because of the ground reflection) would be appropriate.

As indicated in figure 6.1 we can define a set of reference polarization vectors 1_p for p = 1,2 with

$$\vec{1}_1 \times \vec{1}_2 = \vec{1}_3 \;,\; \vec{1}_2 \times \vec{1}_3 = \vec{1}_1 \;,\; \vec{1}_3 \times \vec{1}_1 = \vec{1}_2 \tag{6.2}$$

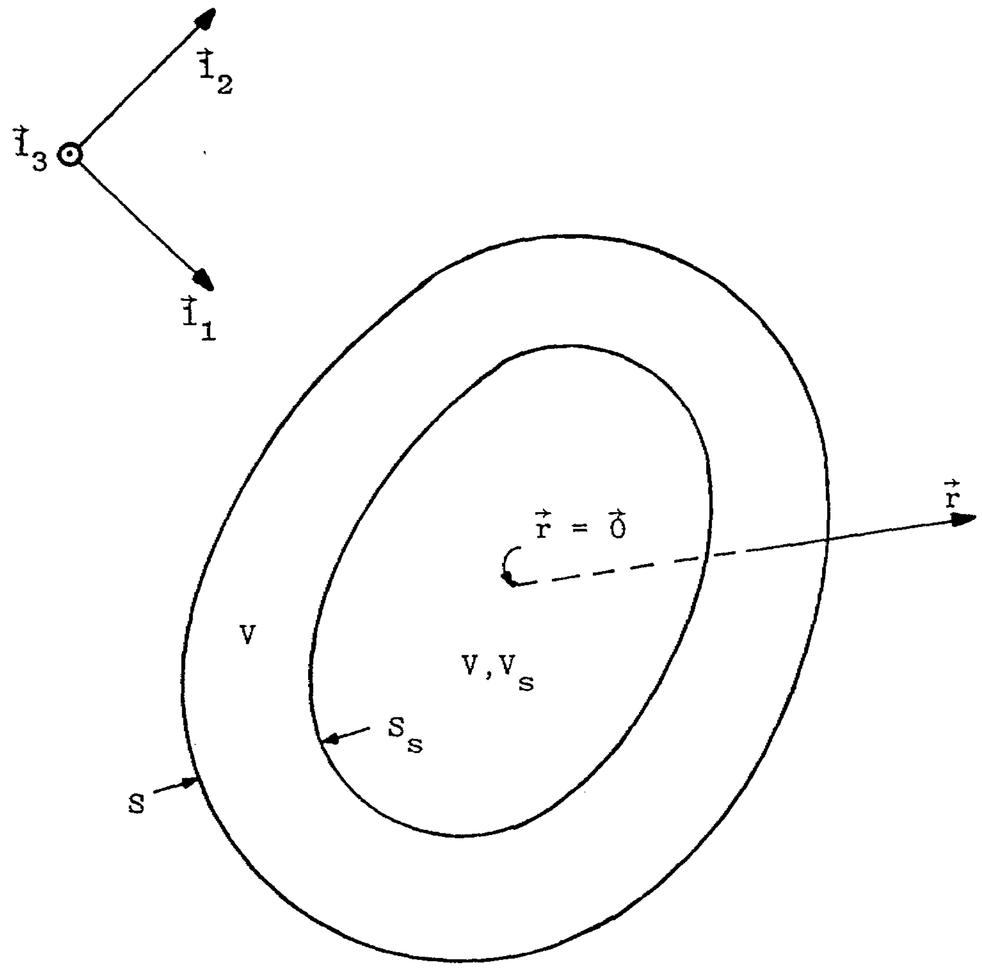


Fig. 6.1. Coordinates for Incident Plane Wave

Linear combinations of $\vec{1}_1$ and $\vec{1}_2$ can be used to give any desired $\vec{1}_e, \vec{1}_h$ combination. One can take two canonical cases as

$$\begin{Bmatrix} \vec{1}_e \\ \vec{1}_h \end{Bmatrix} = \begin{Bmatrix} \vec{1}_1 \\ \vec{1}_2 \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} \vec{1}_e \\ \vec{1}_h \end{Bmatrix} = \begin{Bmatrix} \vec{1}_2 \\ -\vec{1}_1 \end{Bmatrix} \tag{6.3}$$

as well as appropriate linear combinations of these two. Note that the direction of incidence $\vec{1}_1$ takes on all possible real directions (varies over 4π steradians).

In terms of the combined field our incident plane wave is

$$\vec{E}_q^{(inc)}(\vec{r},t) = E_o f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right)\left\{\vec{1}_e + qj\vec{1}_h\right\}$$

$$\tilde{\vec{E}}_q^{(inc)}(\vec{r},s) = E_o \tilde{f}(s)\, e^{-\gamma \vec{1}_1 \cdot \vec{r}} \left\{\vec{1}_e + qj\vec{1}_h\right\} \tag{6.4}$$

The equivalent combined surface current density on S is

$$\begin{aligned} \vec{J}_{s_q}^{(inc)}(\vec{r}_s,t) &= \frac{qj}{Z_o}\, \vec{1}_S(\vec{r}_s) \times \vec{E}_q^{(inc)}(\vec{r}_s,t) \\ &= qj\, \frac{E_o}{Z_o}\, f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right) \vec{1}_S(\vec{r}_s) \times \left\{\vec{1}_e + qj\vec{1}_h\right\} \\ &= \frac{E_o}{Z_o}\, f\left(t - \frac{\vec{1}_1 \cdot \vec{r}}{c}\right) \vec{1}_S(\vec{r}_s) \times \left\{qj\vec{1}_e - \vec{1}_h\right\} \end{aligned} \tag{6.5}$$

The combined dipole for patch S_n, centered on $\vec{r}_{s_n}$, is then

$$\begin{aligned} \tilde{\vec{p}}_{n_q}(s) &\simeq \frac{A_n}{s}\, \tilde{\vec{J}}_{s_q}^{(inc)}(\vec{r}_{s_n},s) \\ &= \frac{qj}{Z_o}\, \frac{A_n}{s}\, \vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{E}}_q^{(inc)}(\vec{r}_{s_n},s) \\ &= qj\, \frac{A_n}{s}\, \frac{E_o}{Z_o}\, \tilde{f}(s)\, e^{-\gamma \vec{1}_1 \cdot \vec{r}}\, \vec{1}_S(\vec{r}_{s_n}) \times \left\{\vec{1}_e + qj\vec{1}_h\right\} \\ &= \frac{A_n}{s}\, \frac{E_o}{Z_o}\, \tilde{f}(s)\, e^{-\gamma \vec{1}_1 \cdot \vec{r}}\, \vec{1}_S(\vec{r}_{s_n}) \times \left\{qj\vec{1}_e - \vec{1}_h\right\} \end{aligned} \tag{6.6}$$

This is split into electric and magnetic dipole moments as

$$\tilde{\vec{p}}_n(s) \simeq -\frac{A_n}{s}\frac{E_o}{Z_o} e^{-\gamma\vec{1}_1\cdot\vec{r}} \vec{1}_S(\vec{r}_{s_n}) \times \vec{1}_h$$

$$\tilde{\vec{m}}_n(s) \simeq \frac{A_n}{s}\frac{E_o}{\mu_o} e^{-\gamma\vec{1}_1\cdot\vec{r}} \vec{1}_S(\vec{r}_{s_n}) \times \vec{1}_e \tag{6.7}$$

Further decomposition into components depends on S and the outward normal $\vec{1}_S(\vec{r}_s)$ associated with it.

Noting that $\vec{p}_n$ and $\vec{m}_n$ are both parallel to S, and hence perpendicular to $\vec{1}_S(\vec{r}_{s_n})$, consider the expressions

$$\vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{p}}_{n_q}(s) \simeq \frac{A_n}{s} \vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{J}}_{s_q}^{(inc)}(\vec{r}_{s_n},s)$$

$$= -\frac{qj}{Z_o}\frac{A_n}{s} \overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \tilde{\vec{E}}^{(inc)}(\vec{r}_{s_n},s)$$

$$= \frac{A_n}{s}\frac{E_o}{Z_o}\tilde{f}(s)\, e^{-\gamma\vec{1}_1\cdot\vec{r}} \overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \left\{-qj\vec{1}_e + \vec{1}_h\right\}$$

$$\vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{p}}_n(s) = \frac{A_n}{s}\frac{E_o}{Z_o}\tilde{f}(s)\, e^{-\gamma\vec{1}_1\cdot\vec{r}} \overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \vec{1}_h \tag{6.8}$$

$$\vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{m}}_n(s) = -\frac{A_n}{s}\frac{E_o}{\mu_o}\tilde{f}(s)\, e^{-\gamma\vec{1}_1\cdot\vec{r}} \overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \vec{1}_e$$

in terms of the tangential components of $\vec{1}_e$ and $\vec{1}_h$ on S. This allows one to view the source elements as proportional to the projections of the incident fields on S at the $\vec{r}_{s_n}$.

VII. Bounds with Respect to Incident-Field Parameters

Now we are in a position to discuss a remarkable aspect of PARTES, its use to bound the signals at various places in the system. Consider a set of positions in the system where one wishes to know the signal, say $V(t)$ or $\tilde{V}(s)$, here taken as a voltage, but it could be a current, field, etc. Give these positions or "failure ports" [2] an index $u = 1,2,\ldots,N_f$ where N_f is the total number of signals $V_u(t)$ in which one is interested. Note that a particular "failure port" may have more than one value of u assigned to it if there is more than one signal of interest to be measured (or calculated) there.

Under an assumption of a linear, time-invariant system one can write a transfer function from one of the equivalent dipoles (index v) on S to the failure port (index u) as $\tilde{T}_{u,v}$ with

$$\tilde{V}_u(s) = \sum_{v=1}^{N_v} \tilde{T}_{u,v}(s)\, \tilde{d}_v(s) \tag{7.1}$$

Here $\tilde{d}_v$ is a quantity related to a particular component ($m = 1,2$) of an electric or magnetic dipole at a particular $\vec{r}_{s_n}$ on S. For convenience let us define $\tilde{d}_v$ as

$$\tilde{d}_v(s) \equiv \begin{cases} \left(\vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{p}}_n(s)\right)_m & \text{for electric dipoles} \\ \left(\frac{1}{c}\, \vec{1}_S(\vec{r}_{s_n}) \times \tilde{\vec{m}}_s(s)\right)_m & \text{for magnetic dipoles} \end{cases} \tag{7.2}$$

Here the mth component of the cross-product form in (6.8) is used because of the simpler form it takes. The 1/c with the magnetic moment puts it into the same units as the electric moment so that the $\tilde{T}_{u,v}$ all have the same dimensions ($F^{-1}m^{-1}$ in this case). Various other convenient normalizations (such as one that makes the $\tilde{T}_{u,v}$ dimensionless) may also be chosen.

An indexing scheme is needed to generate the v values for $v = 1,2,\ldots,N_v$ where

$$\begin{aligned} N_v &= N_s \,\{2 \text{ values of } m\}\{2 \text{ kinds of dipoles}\} \\ &= 4N_s \end{aligned} \tag{7.3}$$

Four v values are associated with each $\vec{r}_{s_n}$ and a table can be constructed from

$$v = 4(n - 1) + v' \tag{7.4}$$

where

$$v' = \begin{cases} & \underline{\text{m value}} & \underline{\text{dipole type}} \\ 1 \text{ for} & 1 & , \text{ electric} \\ 2 \text{ for} & 2 & , \text{ electric} \\ 3 \text{ for} & 1 & , \text{ magnetic} \\ 4 \text{ for} & 2 & , \text{ mangetic} \end{cases} \tag{7.5}$$

The transfer function may now be accumulated to form a matrix equation (with now dummy indices) as

$$\left(\tilde{V}_n(s)\right) = \left(\tilde{T}_{n,m}(s)\right) \cdot \left(\tilde{d}_n(s)\right) \tag{7.6}$$

The transfer-function matrix is now a complex rectangular $(N_f \times N_v)$ matrix.

Consider one of the responses $\tilde{V}_u$ and ask how large it might become if the $\tilde{d}_v$ are not specified. The $\tilde{T}_{u,v}$ can be measured, in general. However, the $\tilde{d}_v$ are dependent on a particular incident wave (with S and the S_n and $\vec{r}_{s_n}$ specified).

There are various bounds one can obtain for $\tilde{V}_u$ from (7.1) based on the concept of vector and matrix norms. In the general case for a norm we can write with say u fixed as u_o

$$\|\tilde{V}_{u_o}(s)\| \leq \left\|\left(\tilde{T}_{u_o,v}(s)\right)\right\| \; \left\|\left(\tilde{d}_v(s)\right)\right\| \tag{7.7}$$

where $(\tilde{T}_{u_o,v})$ is regarded as a single-row matrix or, equivalently, as a vector (with N_v components). There are many possible vector and matrix norms one may define [8]. A common example is the vector magnitude (or 2 norm) applied to (7.7) giving

$$|\tilde{V}_{u_o}(s)| \leq \left|\left(\tilde{T}_{u_o,v}(s)\right)\right| \; \left|\left(\tilde{d}_v(s)\right)\right| \tag{7.8}$$

where the left side is the special case of the magnitude of a scalar. Another interesting norm is the infinity norm or maximum norm

$$|\tilde{V}_{u_o}(s)| = \|\tilde{V}_{u_o}(s)\|_\infty \leq \|(\tilde{T}_{u_o,v}(s))\|_\infty \ \|(\tilde{d}_v(s))\|_\infty$$

$$\|(\tilde{d}_v(s))\|_\infty = \max_{v=1,2,\ldots,N_v} |\tilde{d}_v(s)| \quad (7.9)$$

$$\|(\tilde{T}_{u_o,v}(s))\|_\infty = \sum_{v=1}^{N_v} |\tilde{T}_{u_o,v}(s)|$$

Thus the question of a bound on $|\tilde{V}_u|$ can be asked in terms of the maximum $|\tilde{d}_v|$.

These results are generalizable to the case of the vector of responses $(\tilde{V}_u)$ as

$$\|(\tilde{V}_u(s))\| \leq \|(\tilde{T}_{u,v})\| \ \|(\tilde{d}_v(s))\| \quad (7.10)$$

Now if the norm is magnitude (or 2 norm or euclidean norm) we have

$$|(\tilde{V}_u(s))| \leq |(\tilde{T}_{u,v}(s))| \ |(\tilde{d}_v(s))|$$

$$|(\tilde{T}_{u,v}(s)| = \left[\lambda_{max}\left(((\tilde{T}_{u,v}(s))^\dagger \cdot (\tilde{T}_{u,v}(s))\right)\right]^{\frac{1}{2}} \quad (7.11)$$

λ_{max} ≡ maximum eigenvalue

† ≡ adjoint ≡ T*

T ≡ transpose

* ≡ conjugate

This type of norm bounds the root mean square of the $|\tilde{V}_u|$.

Using the ∞ norm gives

$$\max_{u=1,2,\ldots,N_f} |\tilde{V}_u(s)| = \|(\tilde{V}_u(s))\|_\infty \leq \|(\tilde{T}_{u,v}(s))\|_\infty \ \|(\tilde{d}_v(s))\|_\infty$$

$$\|(\tilde{d}_v(s))\|_\infty = \max_{v=1,2,\ldots,N_v} |\tilde{d}_v(s)|$$

$$\|(\tilde{T}_{u,v}(s))\|_\infty = \max_{u=1,2,\ldots,N_f} \sum_{v=1}^{N_v} |\tilde{T}_{u,v}(s)| \qquad (7.12)$$

$$\equiv \text{maximum row magnitude sum}$$

In this norm we have the maximum of all the $|\tilde{V}_u|$. If one has a criterion (say for system survivability) that all the $|\tilde{V}_n(j\omega)|$ be less than some value, this is equivalent to requiring that $\|(\tilde{V}_u)\|_\infty$ be less than some value. This can be assured if $\|(\tilde{T}_{u,v})\|_\infty$ multiplied by $\|(\tilde{d}_v)\|_\infty$ is sufficiently small since this product is an upper bound.

But now look at the $\tilde{d}_v(s)$ for our incident plane wave. Furthermore set

$$s \equiv j\omega$$
$$\gamma \equiv jk \qquad (7.13)$$

with ω,k real. Then from (7.2) with substitution from (6.8) we have

$$|\tilde{d}_v(j\omega)| = \begin{cases} \frac{A_n}{s}\frac{E_o}{Z_o} |\tilde{f}(j\omega)| \; |\overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \vec{1}_h| \\ \frac{A_n}{s}\frac{E_o}{Z_o} |\tilde{f}(j\omega)| \; |\overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \vec{1}_e| \end{cases} \qquad (7.14)$$

Note that since

$$\left| e^{-jk\vec{1}_1\cdot\vec{r}} \right| = 1 \qquad (7.15)$$

the exponential terms are removed by the magnitude operation. Next maximize $|\tilde{d}_v|$ with respect to $\vec{1}_e$ and $\vec{1}_h$. Allowing all possible directions (4π steradians) for $\vec{1}_1$, and all possible $\vec{1}_e$ perpendicular to $\vec{1}_1$ (2π radians), results in both $\vec{1}_e$ and $\vec{1}_h$ taking on all possible directions. Thus

$$\max_{\text{all } \vec{1}_1, \vec{1}_e} |\overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \vec{1}_e| = 1$$

$$\max_{\text{all } \vec{1}_1, \vec{1}_e} |\overleftrightarrow{1}_t(\vec{r}_{s_n}) \cdot \vec{1}_h| = 1 \tag{7.16}$$

giving

$$\max_{\text{all } \vec{1}_1, \vec{1}_e} |\tilde{d}_v(j\omega)| = \frac{A_n}{|\omega|} \frac{E_o}{Z_o} |\tilde{f}(j\omega)| \tag{7.17}$$

Now if all A_n are chosen to be equal as

$$A_n = \frac{A}{N_s} \equiv \frac{1}{N_s} \{\text{area of } S\} \text{ , for } n = 1,2,\ldots,N_s \tag{7.18}$$

then

$$\max_{\text{all } \vec{1}_1, \vec{1}_e} |\tilde{d}_v(j\omega)| = \frac{1}{|\omega|} \frac{A}{N_s} \frac{E_o}{Z_o} |\tilde{f}(j\omega)| \tag{7.19}$$

which is independent of n and hence of v. The maximum of the infinity norm of $(\tilde{d}_v)$ is then

$$\max_{\text{all } \vec{1}_1, \vec{1}_e} \|(\tilde{d}_v(j\omega))\|_\infty = \max_{\text{all } \vec{1}_1, \vec{1}_e, v} |\tilde{d}_v(j\omega)|$$

$$= \frac{1}{|\omega|} \frac{A}{N_s} \frac{E_o}{Z_o} |\tilde{f}(j\omega)| \tag{7.20}$$

Putting this all together gives the bound

$$\max_{\text{all } \vec{1}_1, \vec{1}_e} \|(\tilde{V}_u(j\omega))\|_\infty = \max_{\text{all } \vec{1}_1, \vec{1}_e, u} |(\tilde{V}_u(j\omega))|$$

$$\leq \frac{1}{|\omega|} \frac{A}{N_s} \frac{E_o}{Z_o} |\tilde{f}(j\omega)| \; \|(\tilde{T}_{u,v}(j\omega))\|_\infty \tag{7.21}$$

where the matrix norm is as given in (7.12), and is in general measurable. Note the simple form of the result which allows one to bound the maximum signal inside the system for any (simulated) direction of incidence and polarization in terms of the matrix of transfer functions from the elementary dipoles. The result of (7.9) for individual signals is also generalized as

$$\max_{\text{all } \vec{1}_1, \vec{1}_e} |\tilde{V}_{u_o}(j\omega)| \leq \frac{1}{|\omega|} \frac{A}{N_s} \frac{E_o}{Z_o} |\tilde{f}(j\omega)| \;\|(\tilde{T}_{u_o,v}(j\omega)\|_\infty \tag{7.22}$$

which uses only a vector of transfer functions.

In the limit as $N_s \to \infty$ with spacing between dipoles $\to 0$ the above results tend to those for a continuous source distribution on S. The sums in the ∞ norms for vectors and matrices become integrals (of complex magnitudes) over $\vec{r}_S$ with summation over dipole type and orientation. The vector and matrix norms can then be reinterpreted as functional and operator norms in the problem of bounding the signals inside a complex system over a range of incident-wave parameters.

VIII. Summary

This note has covered much territory in developing the PARTES concept for EMP simulation. The incident fields have been related to equivalent sources on a surface surrounding the system of interest. These equivalent sources have been approximated by sets of electric and magnetic dipoles. These sources have been related to the impedance integral equation when matricized in MoM form. Choosing the incident field as an incident plane wave, it is shown that the transfer functions from each of the dipoles to a failure port inside the system can be used to bound the response at that failure port for all angles of incidence and polarization by the use of vector/matrix norms.

This type of simulator is more complex than the commonly used variety because of its use of spatial superposition. However, the individual sources are small and might be capable of inclusion with a system of interest in its operational mode (e.g., on a flying aircraft). As such it has the potential application of monitoring hardness maintenance. Added to this is the advantage of getting around the angle-of-incidence and polarization variation, at least in a bounding sense. A limitation of PARTES, on the other hand, is that with small sources and measurement of transfer functions from individual dipoles it does not address the nonlinear system response problems that can be addressed in a criterion-like pulse EMP simulator. In principle PARTES can be operated in a pulsed mode with many high-amplitude pulsed dipoles timed together; however, one might expect such a criterion-like pulsed version to be difficult to build and operate.

There are various questions to be investigated concerning PARTES. The individual dipoles need to be designed and optimum positions and spacings established. Any practical implementation of PARTES will surely have simulation errors; these need to be quantified. The PARTES concept can be applied to other EMP simulation problems, such as that of a plane wave incident on a system on or near the earth surface. The general electromagnetic theoretic aspects of PARTES and their implications for other types of problems (such as EMP interaction) need further exploration and extension.

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