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The Source Excitation of
a Finite-Width, Parallel-Plate Waveguide

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Abstract

The fields excited by a vertical current sheet source inside a parallel-plate waveguide are studied in this work. The analytical expressions for the fields have been derived, and the results of the numerical evaluation are presented. The field distributions have been obtained as functions of the longitudinal propagation constant and the transverse coordinates of the guide. The numerical results are presented in graphical forms, and the computer program used to obtain the results is contained in the Appendix.

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I. INTRODUCTION

The finite-width parallel-plate waveguide is used as an EMP simulator for guiding electromagnetic waves. There have been a number of studies devoted to the analysis of the guide. Rushdi et al. [1] and Marin [2], [3] decompose the field in the guide into two components (for the $\exp(j\omega t)$ time convention):

$$\vec{E}(\vec{r}) = \vec{E}_1 + \vec{E}_2$$

where

$$\vec{E}_1 = \sum_n \vec{A}_n(x,y) e^{-j\gamma_n z} \quad ,$$

$$\vec{E}_2 = \int \int_\Gamma \vec{B}(\alpha,\beta) e^{-j(\alpha x + \beta y + \gamma z)} d\alpha d\beta \quad .$$

The first term $\vec{E}_1$ describes the contribution from the discrete spectrum, i.e., the so-called "leaky-wave" contribution. The second term $\vec{E}_2$ is a superposition of plane waves, the contribution from the continuous spectrum. In references [1] – [3], only $\vec{E}_1$ is studied in detail. This information is useful, of course, only if $\vec{E}_2$ is negligible (for certain guide geometries and source configurations). To test this assumption, we can either evaluate $\vec{E}_2$ directly or calculate the total field $\vec{E}$ instead. We take the second approach. In references [4], [5], Krichevsky and Mittra determine $\vec{E}$ due to a current sheet in the guide by the Wiener-Hopf technique. Their results, however, are valid only if the cross-section of the guide is large in terms of wavelength. In the present report, we again calculate $\vec{E}$ by a different technique moment method which is suitable for a guide with small-to-moderate cross-section.

To calculate the field, we can first write the current source in the form of

$$\vec{J}(\vec{r}) = \int A(\beta) \overset{\approx}{\vec{J}}(x,y) e^{-j\beta z} d\beta \quad .$$

We can then calculate the field due to the integrand in the above equation for different β values, and obtain the total field by superposition. In this report, we investigate the electromagnetic fields due to the current source in the form of $\vec{J}(x,y) e^{-j\beta z}$ for different values of the longitudinal propagation constant β. The step involving the integration over the spectral variable β is not carried out in that work.

II. STATEMENT OF THE PROBLEM

The geometry of the problem is shown in Fig. 1, the open parallel waveguide of width 2L, height 2H is infinitely long in the z-direction, and is excited by a y-direction oriented current source J^i defined as:

$$\vec{J}^i = \hat{y} J^i(x,y) e^{-j\beta z}$$

$$= \hat{y} \delta(x - x_0) \begin{Bmatrix} \sin \alpha_p y \\ \cos \alpha_p y \end{Bmatrix} \text{rect}\left(\frac{y}{2h_0}\right) e^{-j\beta z} \quad , \tag{1}$$

where

$$\alpha_p = \frac{p\pi}{2h_0} \quad ; \quad p = 1, 2, 3, \cdots \tag{2}$$

and δ is the Dirac delta function. Rect is the rectangular function defined as:

$$\text{rect } (t) = \begin{cases} 1 & ; \quad |t| \leq 1/2 \\ 0 & ; \quad \text{elsewhere} \end{cases} \tag{3}$$

In Equation (1) and all the following equations in this report, we have adopted a convention in which all equations are given in both cases of odd and even mode current excitation with the upper equation for the odd case, the lower, the even case.

The total electromagnetic field can be decomposed into two parts: $\vec{E}^t = \vec{E}^i + \vec{E}^s$, $\vec{H}^t = \vec{H}^i + \vec{H}^s$. $\vec{E}^i$ and $\vec{H}^i$ are the incident fields produced by the current source in free space with the waveguide removed. $\vec{E}^s$ and $\vec{H}^s$ are the scattered fields generated by the induced current on the waveguide when illuminated by $\vec{E}^i$ and $\vec{H}^i$. The incident field is first examined in the next section.
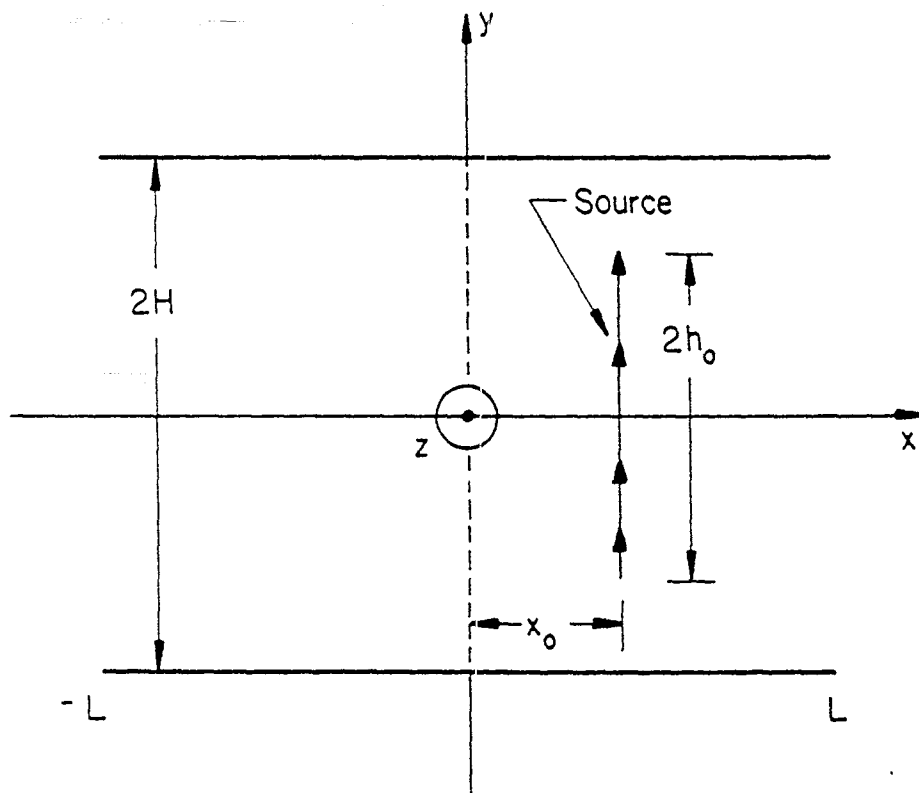
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Figure 1. Geometry of the problem.

III. INCIDENT FIELD

The incident fields $\vec{E}^i$ and $\vec{H}^i$ can be computed via the $\hat{y}$-oriented magnetic vector potential $\vec{A}^i$ produced by the impressed current source.

$$\vec{A}^i = \hat{y} A^i(x,y) e^{-j\beta z} \quad .$$

The z-direction variation for all fields is $e^{j\beta z}$. Therefore, it is suppressed in the subsequent equations. Also, the subscript p in α will be omitted in the following equations for convenience of notation.

The vector potential $\vec{A}^i$ satisfies the Helmholtz equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_t^2 \right) A^i(x,y) = -\mu J^i(x,y) \quad , \tag{5}$$

where $k_t^2 = k^2 - \beta^2 \neq 0$, $\tag{6}$

in which $k = \omega\sqrt{\mu\varepsilon}$ is the wavenumber. The solution to (5) is

$$A^i(x,y) = \frac{\mu}{4j} \int_{-h_o}^{h_o} \left\{ \begin{array}{c} \sin \alpha y' \\ \cos \alpha y' \end{array} \right\} G(x,y \; ; \; x_o, y') dy' \quad , \tag{7}$$

where G is the Green's function defined as

$$G(x,y \; ; \; x',y') = \left\{ \begin{array}{ll} H_0^{(2)}(k_t^s) & ; \quad k^2 > \beta^2 \\ \\ \frac{j2}{\pi} K_0(k_t^s) & ; \quad k^2 < \beta^2 \end{array} \right. \quad , \tag{8}$$

and $s = \sqrt{(x - x')^2 + (y - y')^2}$. $\tag{9}$

$H_0^{(2)}$ and K_0 are zero-order Hankel's function of the second kind and modified Bessel's function of the second kind, respectively.

$\vec{E}^i$ and $\vec{H}^i$ can be derived from A^i by using the following equations

$$E_x^i(x,y) = \frac{1}{j\omega\mu\varepsilon} \frac{\partial}{\partial x} \frac{\partial A^i(x,y)}{\partial y} \quad , \tag{10a}$$

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$$E_y^i(x,y) = \frac{1}{j\omega\mu\epsilon}\left[\frac{\partial^2}{\partial y^2} + k^2\right]A^i(x,y) \quad , \tag{10b}$$

$$E_z^i(x,y) = \frac{-j\beta}{j\omega\mu\epsilon}\frac{\partial A^i(x,y)}{\partial y} \quad , \tag{10c}$$

$$H_x^i(x,y) = \frac{(-j\beta)}{\mu}A^i(x,y) \quad , \tag{10d}$$

$$H_y^i(x,y) = 0 \quad , \tag{10e}$$

$$H_z^i(x,y) = \frac{1}{\mu}\frac{\partial A^i(x,y)}{\partial x} \quad . \tag{10f}$$

Now, we define $B^i(x,y) \equiv \dfrac{\partial A^i(x,y)}{\partial x}$.

From Equation (7), we have

$$B^i(x,y) = -\frac{\mu}{4j}\left\{ \begin{Bmatrix} \sin \alpha h_o \\ \cos \alpha h_o \end{Bmatrix} [G(x,y \; ; \; x_o,h_o) \pm G(x,y \; ; \; x_o,-h_o)] \right.$$
$$\left. \mp \alpha \int_{-h_o}^{h_o} \begin{Bmatrix} \cos \alpha y' \\ \sin \alpha y' \end{Bmatrix} G(x,y \; ; \; x_o,y')dy' \right\} \quad . \tag{11}$$

The partial derivatives of A^i and B^i are obtained from

$$\frac{\partial A^i(x,y)}{\partial x} = \frac{\mu}{4j}(x - x_o)\int_{-h_o}^{h_o} \begin{Bmatrix} \sin \alpha y' \\ \cos \alpha y' \end{Bmatrix} G'(x,y \; ; \; x_o,y')dy' \quad , \tag{12}$$

where $G'(x,y \; ; \; x',y') = \begin{cases} -\dfrac{k_t}{s} H_1^{(2)}(k_t s) \; ; \; k^2 > \beta^2 \\[4mm] -\dfrac{k_t}{s}\dfrac{js}{\pi} K_1(k_t s) \; ; \; k^2 < \beta^2 \end{cases}$. $\tag{13}$

$H_1^{(2)}$ and K_1 are, respectively, the first-order Hankel's function of the second kind and modified Bessel's function of the second kind.

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Also, we have

$$\frac{\partial B^i(x,y)}{\partial x} = -\frac{\mu}{4j}(x - x_o)\left\{ \begin{Bmatrix} \sin \alpha h_o \\ \cos \alpha h_o \end{Bmatrix} [G'(x,y \; ; \; x_o,h_o) \pm G'(x,y \; ; \; x_o,-h_o)] \right.$$

$$\left. \mp \alpha \int_{-h_o}^{h_o} \begin{Bmatrix} \cos \alpha y' \\ \sin \alpha y' \end{Bmatrix} G'(x,y \; ; \; x_o,y')dy' \right\} \quad . \qquad (14)$$

$$\frac{\partial B^i(x,y)}{\partial y} = -\frac{\mu}{4j}\left\{ \begin{Bmatrix} \sin \alpha h_o \\ \cos \alpha h_o \end{Bmatrix} [(y - h_o)G'(x,y \; ; \; x_o,h_o) \right.$$

$$\pm (y + h_o)G'(x,y \; ; \; x_o,-h_o)]$$

$$\pm \alpha \begin{Bmatrix} \cos \alpha h_o \\ \sin \alpha h_o \end{Bmatrix} [G(x,y \; ; \; x_o,h_o) \mp G(x,y \; ; \; x_o,-h_o)]$$

$$\left. + \alpha^2 \int_{-h_o}^{h_o} \begin{Bmatrix} \sin \alpha y' \\ \cos \alpha y' \end{Bmatrix} G(x,y \; ; \; x_o,y')dy' \right\} \quad . \qquad (15)$$

Substituting Equations (7), (11), (12), (14), (15) together with Equations (8) and (13) into Equation (10), we can obtain $\vec{E}^i$ and $\vec{H}^i$.

IV. SCATTERED FIELD

In this section we examine the scattered fields $\vec{E}_s$ and $\vec{H}_s$. Since the induced current $\vec{J}^s$ on the waveguide has two components,

$$\vec{J}^s = \hat{x} J_x^s(x) + \hat{z} J_z^s(x) \quad ; \tag{16}$$

the magnetic vector potential $\vec{A}^s$ due to the induced current $\vec{J}^s$ also has two components,

$$\vec{A}^s = \hat{x} A_x^s(x,y) + \hat{z} A_z^s(x,y) \quad . \tag{17}$$

Both A_x^s and A_z^s satisfy the Helmholtz equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_t^2 \right) A_u^s(x,y) = -\mu J_u^s(x) \quad , \tag{18}$$

where $u = x$ or z, and the solutions are

$$A_x^s(x,y) = \frac{\mu}{4j} \int_{-L}^{L} J_x^s(x') [G(x,y \; ; \; x',h) \pm G(x,y \; ; \; x',-h)] dx' \quad , \tag{19}$$

$$A_z^s(x,y) = \frac{\mu}{4j} \int_{-L}^{L} J_z^s(x') [G(x,y \; ; \; x',h) \pm G(x,y \; ; \; x',-h)] dx' \quad . \tag{20}$$

$\vec{E}^s$ and $\vec{H}^s$ can be derived from $\vec{A}^s$ by using the equations given below

$$E_x^s(x,y) = -j\omega A_x^s(x,y) + \frac{1}{j\omega\mu\epsilon} \left[\frac{\partial^2}{\partial x^2} A_x^s(x,y) - j\beta \frac{\partial}{\partial x} A_z^s(x,y) \right] \quad , \tag{21a}$$

$$E_y^s(x,y) = \frac{1}{j\omega\mu\epsilon} \left[\frac{\partial^2}{\partial x \partial y} A_x^s(x,y) - j\beta \frac{\partial}{\partial y} A_z^s(x,y) \right] \quad , \tag{21b}$$

$$E_z^s(x,y) = -j\omega A_z^s(x,y) + \frac{(-j\beta)}{j\omega\mu\epsilon} \left[\frac{\partial}{\partial x} A_x^s(x,y) - j\beta A_z^s(x,y) \right] \quad , \tag{21c}$$

$$H_x^s(x,y) = \frac{1}{\mu} \frac{\partial}{\partial y} A_z^s(x,y) \quad , \tag{21d}$$

$$H_y^s(x,y) = \frac{1}{\mu} [-j\beta A_x^s(x,y) - \frac{\partial}{\partial x} A_z^s(x,y)] \quad , \tag{21e}$$

$$H_z^s(x,y) = - \frac{1}{\mu} \frac{\partial}{\partial y} A_x^s(x,y) \quad . \tag{21f}$$

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Note that J_x^s and J_z^s are still unknown quantities; however, they can be determined by enforcing the boundary condition that the total tangential electric field be zero on the surface of the waveguide, viz.,

$$E_x^i(x,h) + E_x^s(x,h) = 0 \quad ; \quad |x| \le L \quad , \tag{22}$$

$$E_z^i(x,h) + E_z^s(x,h) = 0 \quad ; \quad |x| \le L \quad . \tag{23}$$

Due to the symmetry of the problem, we do not have to enforce the boundary condition on the lower plate ($y = -h$). Equations (22) and (23) can be explicitly written as

$$\left(\frac{\partial}{\partial x^2} + k^2\right) A_x^s(x,h) - j\beta \frac{\partial}{\partial x} A_z^s(x,h) = -j\omega\mu\varepsilon E_x^i(x,h) \quad ; \quad |x| \le L \quad , \tag{24}$$

$$(-\beta^2 + k^2) A_z^s(x,h) - j\beta \frac{\partial}{\partial x} A_x^s(x,h) = -j\omega\mu\varepsilon E_z^i(x,h) \quad ; \quad |x| \le L \quad . \tag{25}$$

We then substitute Equations (19) and (20) into Equations (24) and (25) and numerically solve for J_x^s and J_z^s to obtain $\vec{E}_s$ and $\vec{H}_s$.

V. NUMERICAL COMPUTATION

To solve Equations (24) and (25) numerically, we first transform them into finite difference equations:

$$A_x^S(x + \Delta, h) + (k^2\Delta^2 - 2)A_x^S(x,h) + A_x^S(x - \Delta, h)$$

$$- j\frac{\beta\Delta}{2}[A_z^S(x + \Delta, h) - A_z^S(x - \Delta, h)] = -j\frac{k^2\Delta^2}{\omega}E_x^i(x,h) \quad ; \quad |x| \leq L \quad , \quad (26)$$

and $\quad -j\frac{\beta\Delta}{2}[A_x^S(x + \Delta, h) - A_x^S(x - \Delta, h)] + k_t^2\Delta^2 A_z^S(x,h)$

$$= -j\frac{k^2\Delta^2}{\omega}E_z^S(x,h) \quad ; \quad |x| \leq L \quad , \quad (27)$$

where Δ is a finite increment in x. Equations (26) and (27) can be solved by the method of moments. We expand J_x^S and J_z^S in sets of N subdomain basis functions as shown in Figures 2a and 2b. We write

$$J_x^S(x) = \sum_{n=1}^{N} a_n \, \text{rect}\left(\frac{x - x_n}{\Delta}\right) \tag{28}$$

where $x_n = n\Delta - L \tag{29}$

$$\Delta = \frac{2L}{N + 1} \quad , \tag{30}$$

and (28) satisfies the end condition

$$J_x^S(\pm L) = 0 \quad . \tag{31}$$

$$J_z^S(x) = \sum_{n=1}^{N} b_n P_n(x) \quad , \tag{32}$$

where $P_1(x) = \dfrac{\sqrt{\Delta}}{\sqrt{L + x}} \quad ; \quad -L \leq x \leq x_1 + \dfrac{\Delta}{2} \quad , \tag{33a}$

$$P_n(x) = \text{rect}\left(\frac{x - x_n}{\Delta}\right) \quad ; \quad n = 2, 3, \cdots (N - 1) \quad , \tag{33b}$$

$$P_N(x) = \frac{\sqrt{\Delta}}{\sqrt{L - x}} \quad ; \quad x_N - \frac{\Delta}{2} \leq x \leq L \quad . \tag{33c}$$
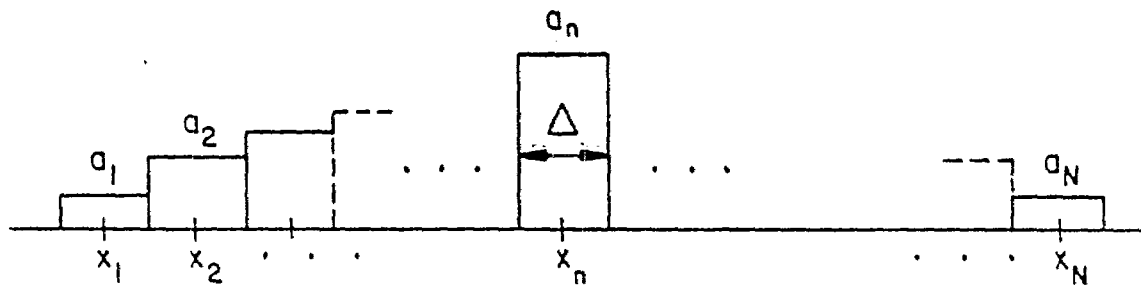
(a)
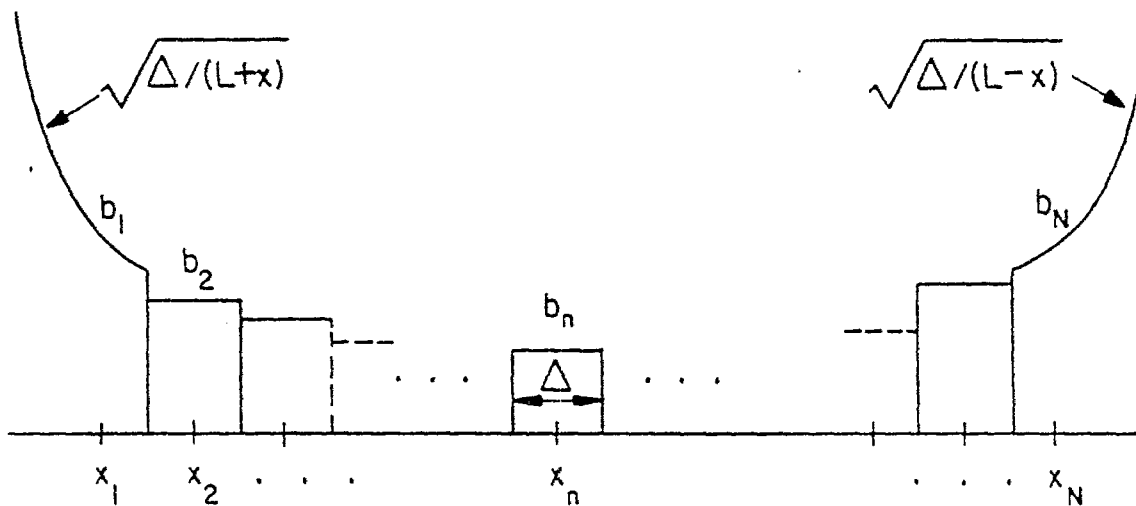
(b)

Figure 2. Expansion functions for (a) J_x^s and (b) J_z^s .

It should be pointed out that the Δ's in Equations (28) and (32) are not necessarily the same as those in Equations (26) and (27); we have chosen the same symbol for the sake of convenience. Next, we enforce (26) and (27) simultaneously at a set of N match points $\{x_m$'s$\}$. Again, $\{x_m$'s$\}$ are chosen to coincide with those points $\{x_n$'s$\}$ in (29) for the sake of simplicity. We then have

$$A_x^s(x_{m+1},h) + (k^2\Delta^2 - 2)A_x^s(x_m,h) + A_x^s(x_{m-1},h)$$

$$-j\frac{\beta\Delta}{2}[A_z^s(x_{m+1},h) - A_z^s(x_{m-1},h)] = -j\frac{k^2\Delta^2}{\omega}E_x^i(x_m,h) \qquad (34)$$

and $-j\frac{\beta\Delta}{2}[A_x^s(x_{m+1},h) - A_x^s(x_{m-1},h)] + k_t^2\Delta^2 A_z^s(x_m,h)$

$$= -j\frac{k^2\Delta^2}{\omega}E_z^i(x_m,h) \qquad (35)$$

where $m = 1, 2, \cdots, N$.

By substituting Equations (28) and (32), the expansion function expressions for $\vec{J}_x^s$ and $\vec{J}_z^s$, into Equations (19) and (20), $A_x^s(x_m,h)$ and $A_z^s(x_m,h)$ become

$$A_x^s(x_m, h) = \frac{\mu}{4j}\sum_{n=1}^{N}a_n S_{m,n}^a \qquad , \qquad (36)$$

where $S_{m,n}^a = \int_{x_n-\Delta/2}^{x_n+\Delta/2}[G(x_m,h ; x',h) \pm G(x_m,h ; x',-h)]dx' \qquad ; \qquad (37)$

$$n = 1, 2, \cdots, N,$$

$$m = 0, 1, \cdots, (N+1),$$

and $A_z^s(x_m,h) = \frac{\mu}{4j}\sum_{n=1}^{N}b_n S_{m,n}^b \qquad , \qquad (38)$

where $S_{m,1}^b = \int_{-L}^{x_1+\Delta/2}\sqrt{\frac{\Delta}{L + x'}}[G(x_m,h ; x',h) \pm G(x_m,h ; x',-h)]dx' \qquad , \quad (39a)$

$$S_{m,n}^b = S_{m,n}^a \qquad ; \qquad n = 2, 3, \cdots, (N-1), \qquad (39b)$$

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$$S_{m,N}^{b} = \int_{x_N - \Delta/2}^{L} \sqrt{\frac{\Delta}{L - X'}} \, [G(x_m,h \; ; \; x',h) \pm G(x_m,h \; ; \; x',-h)]dx' \qquad (39c)$$

$$m = 0, 1, \cdots, (N+1) \; .$$

Substituting Equations (36) through (39) into Equations (34) and (35), we arrive at

$$\begin{bmatrix} [S_{m+1,n}^{a} + (k^2\Delta^2 - 2)S_{m,n}^{a} + S_{m-1,n}^{a}] & [-j\frac{\beta\Delta}{2}(S_{m+1,n}^{b} - S_{m-1,n}^{b})] \\ [-j\frac{\beta\Delta}{2}(S_{m+1,n}^{a} - S_{m-1,n}^{a})] & [k_t^2\Delta^2 S_{m,n}^{b}] \end{bmatrix} \begin{bmatrix} [a_n] \\ [b_n] \end{bmatrix}$$

$$= -\Delta^2 \frac{4j}{\mu} \begin{bmatrix} [\frac{\partial}{\partial x} B^i(x_m,h)] \\ [-j\beta B^i(x_m,h)] \end{bmatrix} \qquad (40)$$

The above equation can be readily solved for a_n and b_n. The unknowns A_x^s and A_z^s can then be calculated using Equations (36) through (39), and the scattered field can be determined from Equation (21) and, therefore, the total field. The equations for calculating $S_{m,n}^{a}$'s and $S_{m,n}^{b}$'s are given in the following:

(A). Evaluation of $S_{m,n}^{a}$'s

(i) If $m = n$,

$$S_{m,n}^{a} = I_0 \pm image \qquad (41)$$

where $image = \int_{x_n - \Delta/2}^{x_n + \Delta/2} G(x_m,h \; ; \; x',-h)dx'$, $\qquad (42)$

and $I_0 = \int_{x_m - \Delta/2}^{x_m + \Delta/2} G(x_m,h \; ; \; x',h)dx'$ $\qquad (43)$

$$= 2 \int_{x_m}^{x_m + \Delta/2} G(x_m,h \; ; \; x',h)dx' \qquad (44)$$

$$= \begin{cases} 2 \displaystyle\int_{x_m}^{x_m+\Delta/2} H_0^{(2)}[k_t(x'-x_m)]dx' \; ; \; k^2 > \beta^2 \\[2em] \dfrac{4j}{\pi} \displaystyle\int_{x_m}^{x_m+\Delta/2} K_0[k_t(x'-x_m)]dx' \; ; \; k^2 < \beta^2 \end{cases} \qquad (45)$$

$$= \begin{cases} \dfrac{2}{k_t} \displaystyle\int_0^{k_t\Delta/2} H_0^{(2)}(t)dt \; ; \; k^2 > \beta^2 \\[2em] \dfrac{4j}{\pi k_t} \displaystyle\int_0^{k_t\Delta/2} K_0(t)dt \; ; \; k^2 < \beta^2 \end{cases} \qquad (46)$$

Since simple algorithms have been developed to evaluate the integrals of $H_0^{(2)}$ and K_0 from zero to any positive number [6], Equation (46) can be evaluated efficiently and accurately.

(ii) If $m \neq n$,

$$S_{m,n}^a = \begin{cases} \displaystyle\int_{x_n-\Delta/2}^{x_m} G(x_m,h\;;\;x',h)dx' - \int_{x_n+\Delta/2}^{x_m} G(x_m,h\;;\;x',h)dx' \pm \text{image} \; ; \; m > n \\[2em] \displaystyle\int_{x_m}^{x_n+\Delta/2} G(x_m,h\;;\;x',h)dx' - \int_{x_m}^{x_n-\Delta/2} G(x_m,h\;;\;x',h)dx' \pm \text{image} \; ; \; m < n \end{cases} \qquad (47)$$

where the image is given in Equation (42).

(B) Evaluation of $S_{m,n}^b$'s

First consider $S_{m,1}^b$, which is expressed as:

$$S_{m,1}^b = I_1 + I_2 \quad , \qquad (48)$$

where $I_1 = \displaystyle\int_{-L}^{-L+\Delta/2} \sqrt{\dfrac{\Delta}{L+x'}} \; [G(x_m,h\;;\;x',h) \pm G(x_m,h\;;\;x',-h)]dx' \qquad (49)$

and $I_2 = \displaystyle\int_{x_1-\Delta/2}^{x_1+\Delta/2} \sqrt{\dfrac{\Delta}{L+x'}} \; [G(x_m,h\;;\;x',h) \pm G(x_m,h\;;\;x',-h)]dx' \; . \qquad (50)$

(i) If $m \neq 0$,

$$I_1 = \int_{-L}^{-L+\Delta/2} \left\{ \sqrt{\frac{\Delta}{L + x'}} \left[G(x_m,h \; ; \; x',h) \pm G(x_m,h \; ; \; x',-h) \right] \right.$$

$$\left. - \sqrt{\frac{\Delta}{L + x'}} \left[G(x_m,h \; ; \; -L,h) \pm G(x_m,h \; ; \; -L,-h) \right] \right\} dx'$$

$$+ \left[G(x_m,h \; ; \; -L,h) \pm G(x_m,h \; ; \; -L,-h) \right] \int_{-L}^{-L+\Delta/2} \sqrt{\frac{\Delta}{L + x'}} \; dx'$$

(51)

where $\int_{-L}^{-L+\Delta/2} \sqrt{\frac{\Delta}{L + x'}} \; dx'$ can be evaluated analytically,

which is $\int_{-L}^{-L+\Delta/2} \sqrt{\frac{\Delta}{L + x'}} \; dx' = \sqrt{2} \, \Delta$.   . (51a)

(ii) If $m = 0$,

first, the small argument behavior of function G is obtained:

$$\lim_{x' \to -L} G(-L,h \; ; \; x',h) = - \frac{2i}{\pi} \, \ell n \left[\frac{\gamma k_t (L + x')}{2} \right] \quad , \tag{52}$$

where $\gamma = 0.5772156649$. Then, in evaluating I_1 in Equation (49),

the singularity at $x' = -L$ is first subtracted from, then added

to the integrand, and by using Equation (52), we have

$$I_1 = \int_{-L}^{-L+\Delta/2} \sqrt{\frac{\Delta}{L + x'}} \left\{ G(-L,h \; ; \; x',h) \pm G(-L,h \; ; \; x',-h) \right.$$

$$\left. + \frac{2i}{\pi} \, \ell n \left[\frac{\gamma k_t (L + x')}{2} \right] \right\} dx'$$

$$- \frac{2i}{\pi} \int_{-L}^{-L+\Delta/2} \sqrt{\frac{\Delta}{L + x'}} \left\{ \ell n(L + x') + \ell n \left(\frac{\gamma k_t}{2} \right) \right\} dx' \quad . \tag{53}$$

The first integral in the above equation is amenable to numerical

integration, and the second integral can be evaluated analytically:

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$$-\frac{2i}{\pi}\int_{-L}^{-L+\Delta/2}\sqrt{\frac{\Delta}{L+x'}}\,\ell n\left[\frac{\gamma k_t(L+x')}{2}\right]dx'$$

$$= -\frac{2i}{\pi}\sqrt{2}\,\Delta\left\{\ell n\left[\frac{\gamma k_t\Delta}{4}\right]-2\right\} . \tag{54}$$

On the other hand, I_2 can be evaluated as

$$I_2 = \int_{x_1-\Delta/2}^{x_1+\Delta/2}\left\{\sqrt{\frac{\Delta}{L+x'}}\,[G(x_m,h\,;\,x',h)\pm G(x_m,h\,;\,x',-h)]\right.$$

$$\left. -\,[G(x_m,h\,;\,x',h)\pm G(x_m,h\,;\,x',-h)]\right\}dx' + S^a_{m,1} , \tag{55}$$

$$m = 0,\ 1,\ \cdots,\ (N+1) .$$

Substituting the above expressions for I_1 and I_2 into Equation (48), we then obtain $S^b_{m,1}$. Following similar procedures, we obtain the equations for $S^b_{m,N}$:

$$S^b_{m,N} = I_3 + I_4 ,$$

where $I_3 =$

$$I_3 = \int_{L-\Delta/2}^{L}\left\{\sqrt{\frac{\Delta}{L-x'}}\,[G(x_m,h\,;\,x',h)\pm G(x_m,h\,;\,x',-h)]\right.$$

$$-\,\sqrt{\frac{\Delta}{L-x'}}\,[G(x_m,h\,;\,L,h)\pm G(x_m,h\,;\,L,-h)]$$

$$\left. +\,[G(x_m,h\,;\,L,h)\pm G(x_m,h\,;\,L,-h)]\sqrt{2}\,\Delta\right\}dx' , \tag{56}$$

when $m = 0,\ 1,\ \cdots,\ N$

or $I_3 =$

$$I_3 = \int_{L-\Delta/2}^{L}\sqrt{\frac{\Delta}{L-x'}}\left\{G(L,h\,;\,x',h)\pm G(L,h\,;\,x',-h)\right\}dx' - \frac{2i}{\pi}\,\ell n\,\frac{\gamma k_t(L-x')}{2}$$

$$+ \frac{2i}{\pi}\,\ell n\,\frac{\gamma k_t(L-x')}{2}\,dx' - \frac{2i}{\pi}\sqrt{2}\,\Delta\left\{\ell n\left[\frac{\gamma k_t\Delta}{4}\right]-2\right\} ; \tag{57}$$

when $m = N+1$,

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$$\text{and } I_4 = \int_{x_N - \Delta/2}^{x_N + \Delta/2} \left\{ \sqrt{\frac{\Delta}{L - x'}} \, [G(x_m, h ; x', h) \pm G(x_m, h ; x', -h)] \right.$$

$$\left. - [G(x_m, h ; x', h) \pm G(x_m, h ; x', -h)] \right\} dx' + S_{m,N}^a \quad ; \quad (58)$$

$$m = 0, 1, \cdots (N + 1) \quad .$$

Using the equations obtained above to compute $S_{m,n}^a$'s and $S_{m,n}^b$'s in Equation (40) and inversing the matrix, we then are able to determine the a_n's and b_n's.

VI. NUMERICAL RESULTS AND DISCUSSION

The results evaluated by the methods discussed in the last section are now presented in graphical form. The physical dimensions of the waveguide and the current source for which we perform the computation are (see Fig. 1 and Equations (1) and (2))

$L = 12.5$ m

$H = 12.75$ m

$h_o = 5$ m

$x_o = 0$ m

$p = 1$

The frequency used is 25 MHz, and the current source is of even mode.

The magnitudes of the induced-current components, $|\vec{J}_x^s|$ and $|\vec{J}_x^s|$, are plotted in Figs. 3(a) and 3(b) as functions of the transverse coordinate of the waveguide. They are presented for different β/k values in the range between 0.86 and 0.99.

The plots of the magnitudes of the electrical field components, $|\vec{E}_x|$, $|\vec{E}_y|$, and $|\vec{E}_z|$, consist of two parts.

(A) Field components as functions of β/k:

E_x, E_y, and E_z are plotted for β/k in the range between 0 and 0.999 in Figs. 4(a), 4(b) and 4(c), when $x = 0.5L$, $y = 0.47H$.

It is observed from these plots that for most values of β/k between 0 and 1, $|\vec{E}_x|$, $|\vec{E}_y|$, and $|\vec{E}_z|$ are relatively small and vary rather slowly with respect to β/k; however, for β/k in the region between 0.7 and 1, the magnitude of the field component is found to have a sharp peak. Therefore, the major contributions to the total fields should come from this portion of the spectrum.
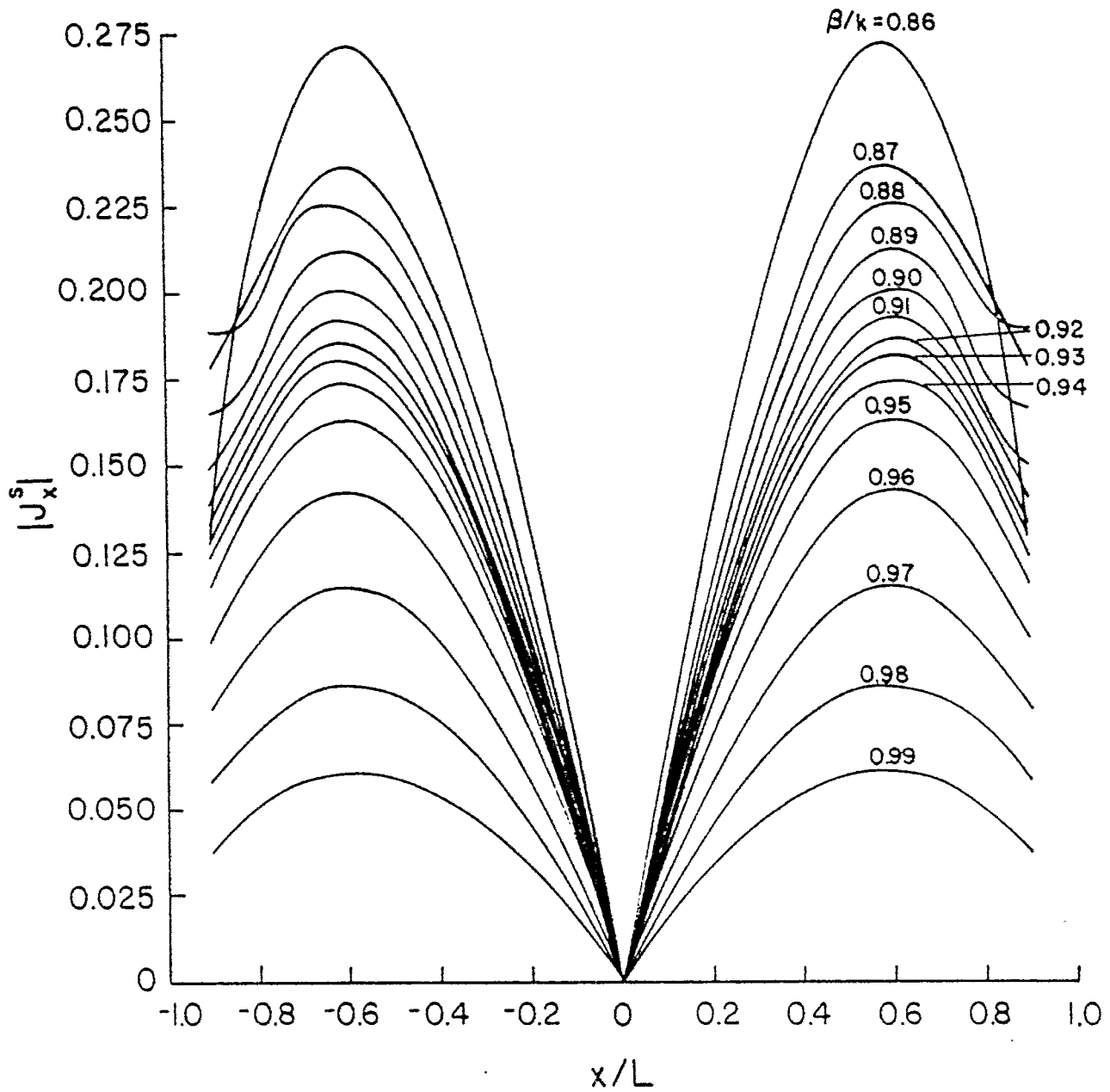
Figure 3(a). $|J_x^s|$ as a function of x coordinate.

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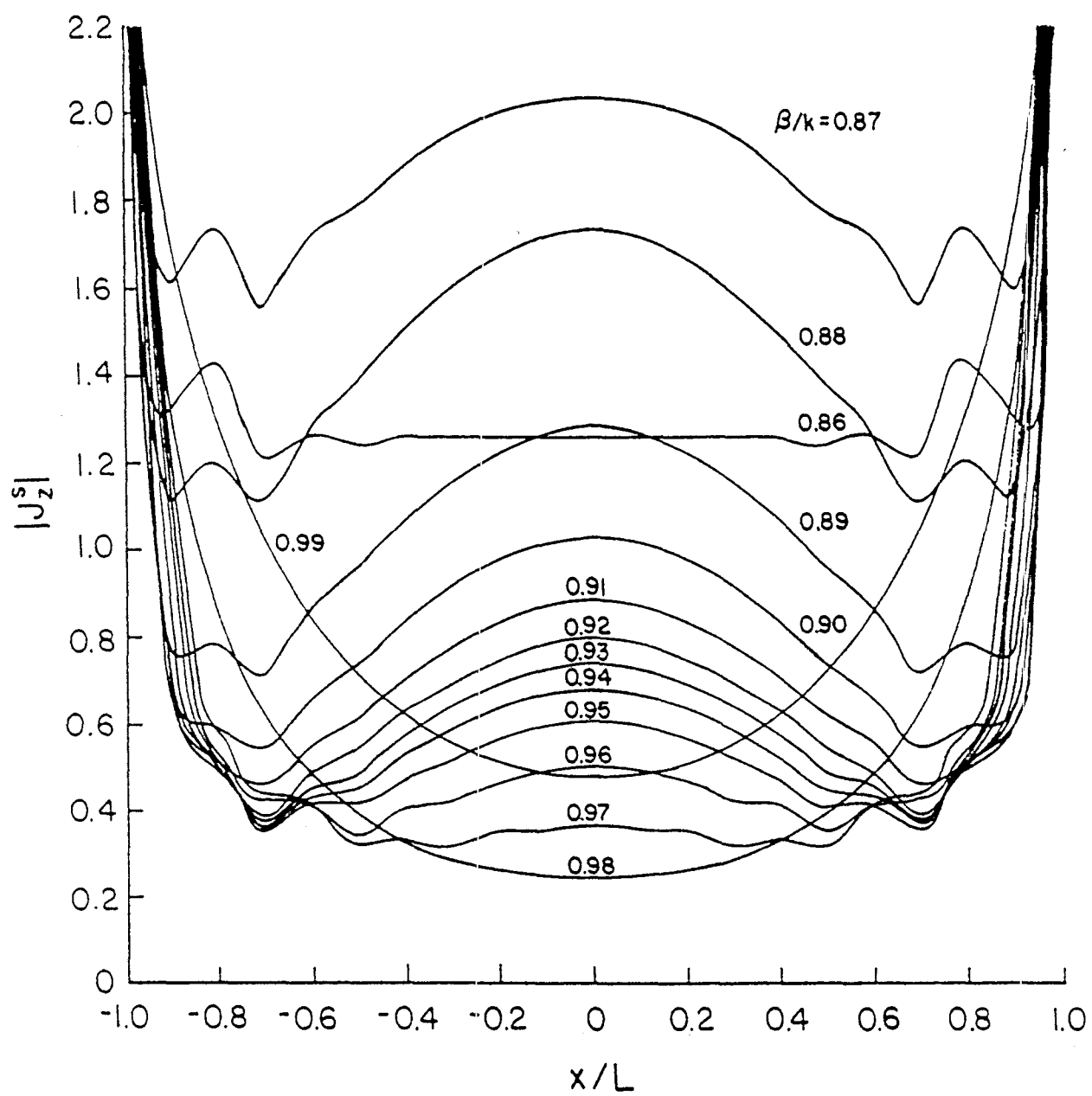
Figure 3(b). $|J_z^s|$ as a function of x coordinate

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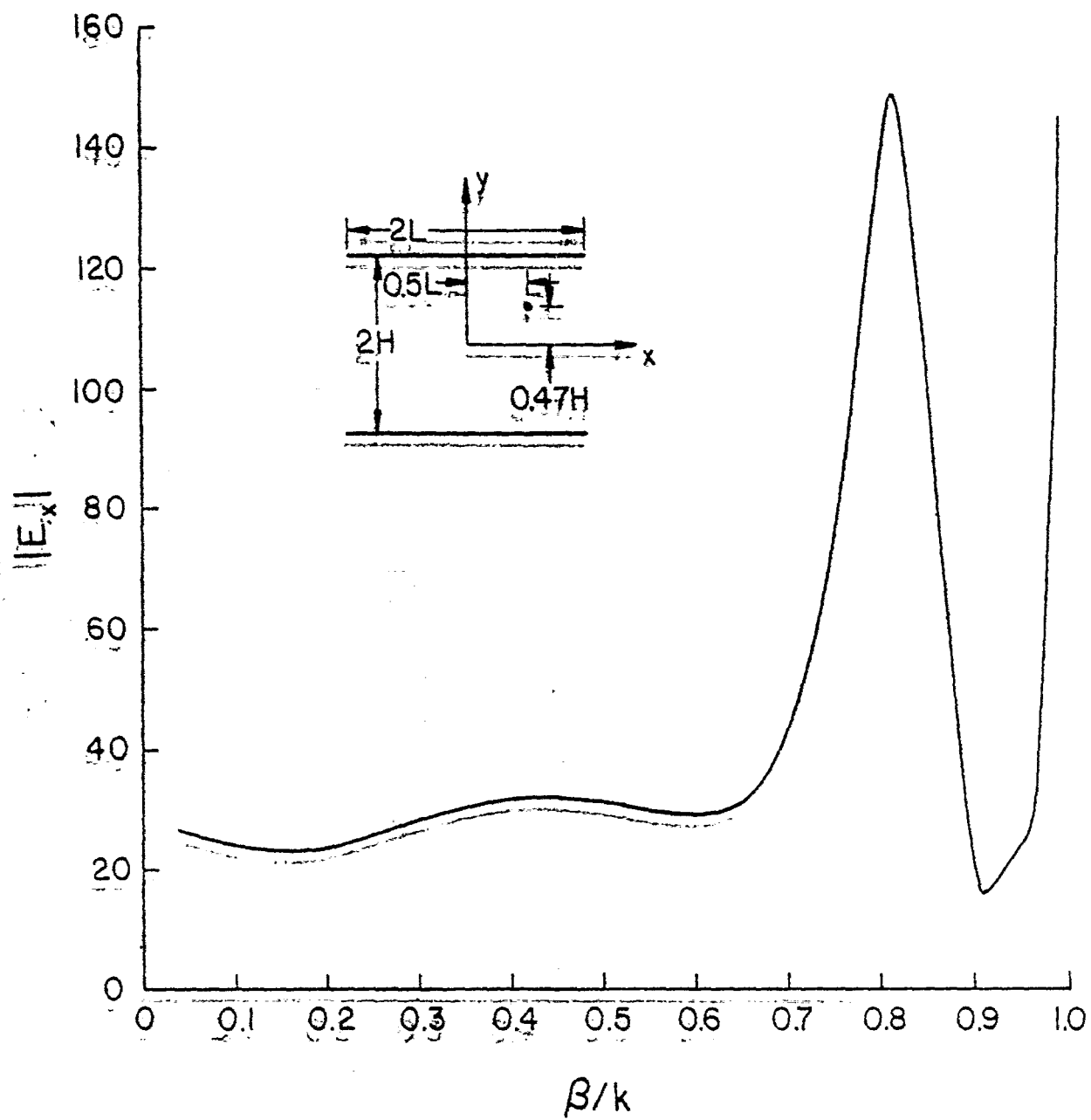
Figure 4(a). $|E_x|$ as a function of β/k for $x/L = 0.5$, $y/H = 0.47$.
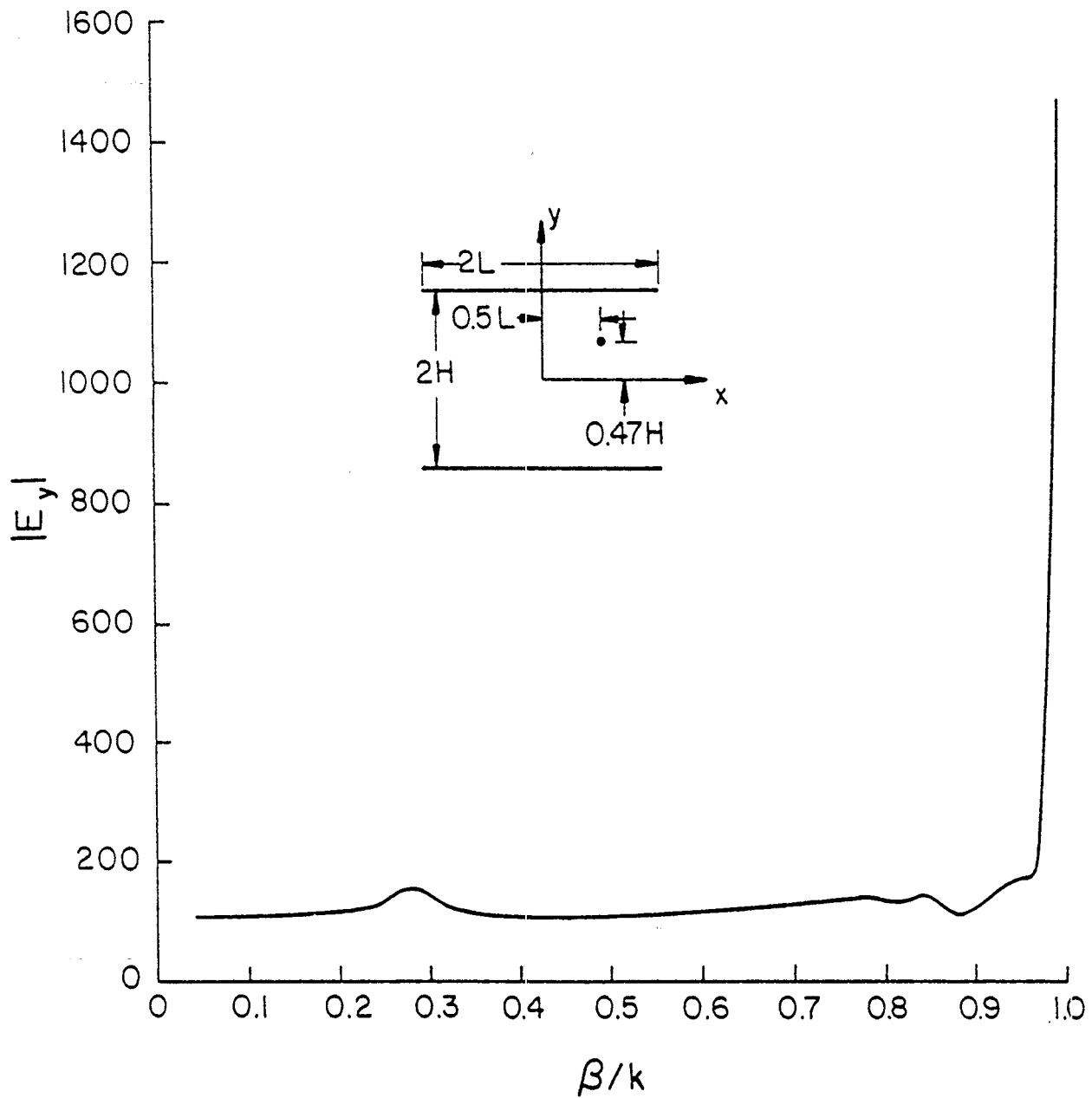
Figure 4(b). $|E_y|$ as a function of β/k for x/L = 0.5, y/H = 0.47.
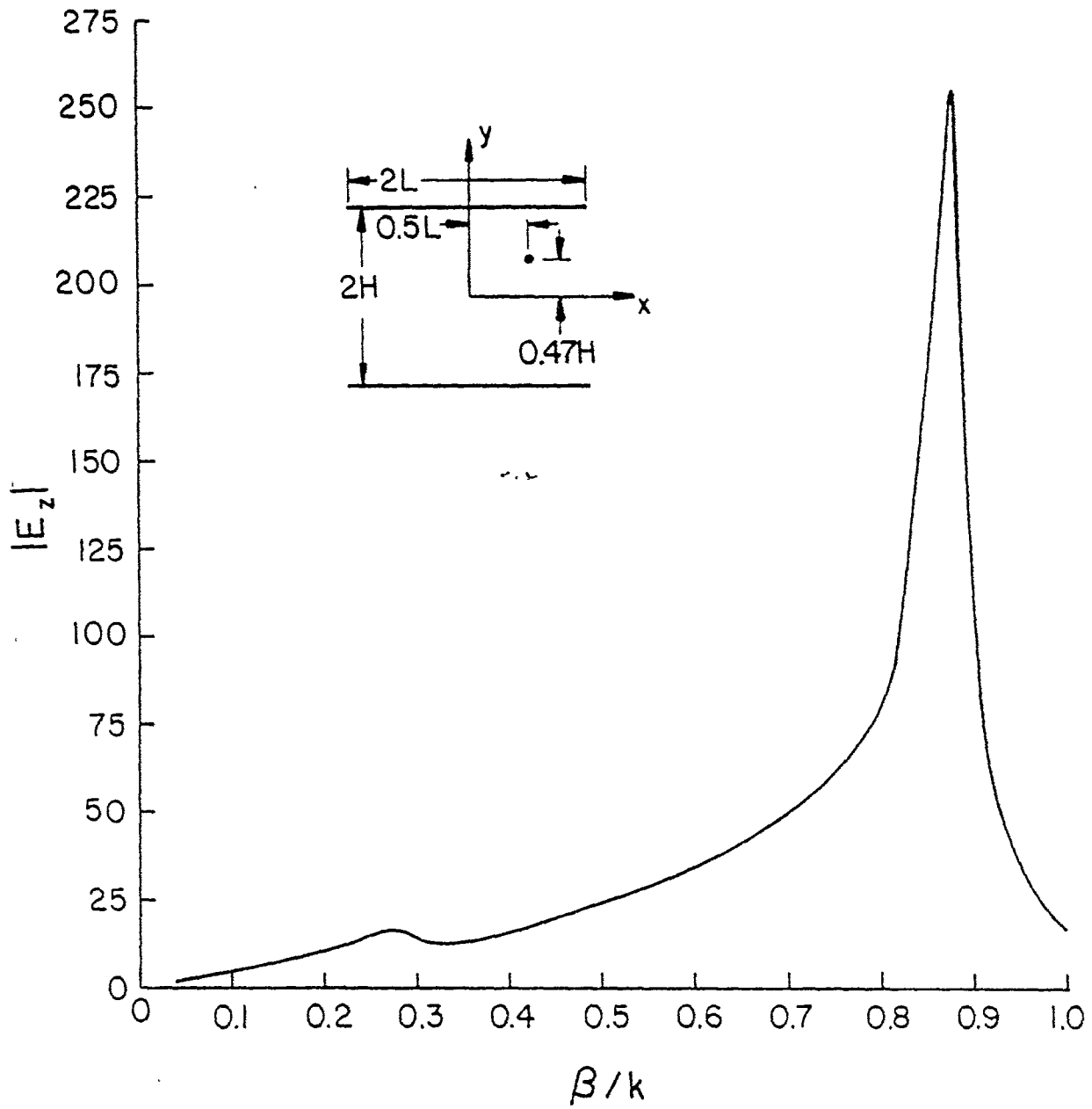
Figure 4(c). $|E_z|$ as a function of β/k for x/L = 0.5, y/H = 0.47.

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(B) Field components as functions of the transverse coordinate of

the waveguide.

In Figs. 5(a), 5(b) and 5(c), $|\vec{E}_x|$, $|\vec{E}_y|$ and $|\vec{E}_z|$ are presented as functions of the x-coordinate for $y = 0.47H$. They are plotted for different β/k values in the region between 0.879 and 0.999. When β/k approaches 1, $\vec{E}_y$ becomes the dominant component and $|\vec{E}_z|$ diminishes to zero as we would expect.
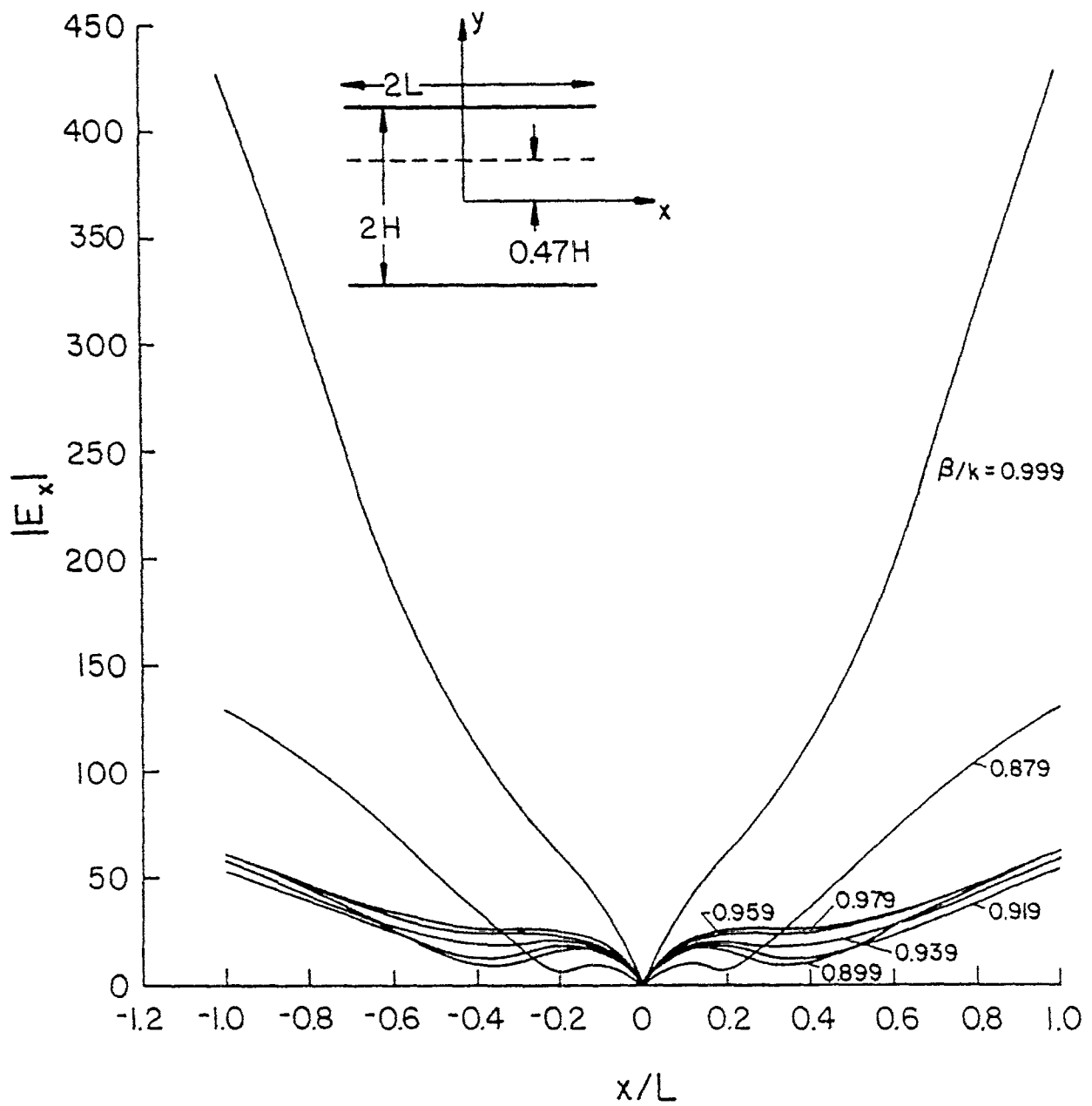
Figure 5(a). $|E_x|$ as a function of x coordinate for y/H = 0.47.
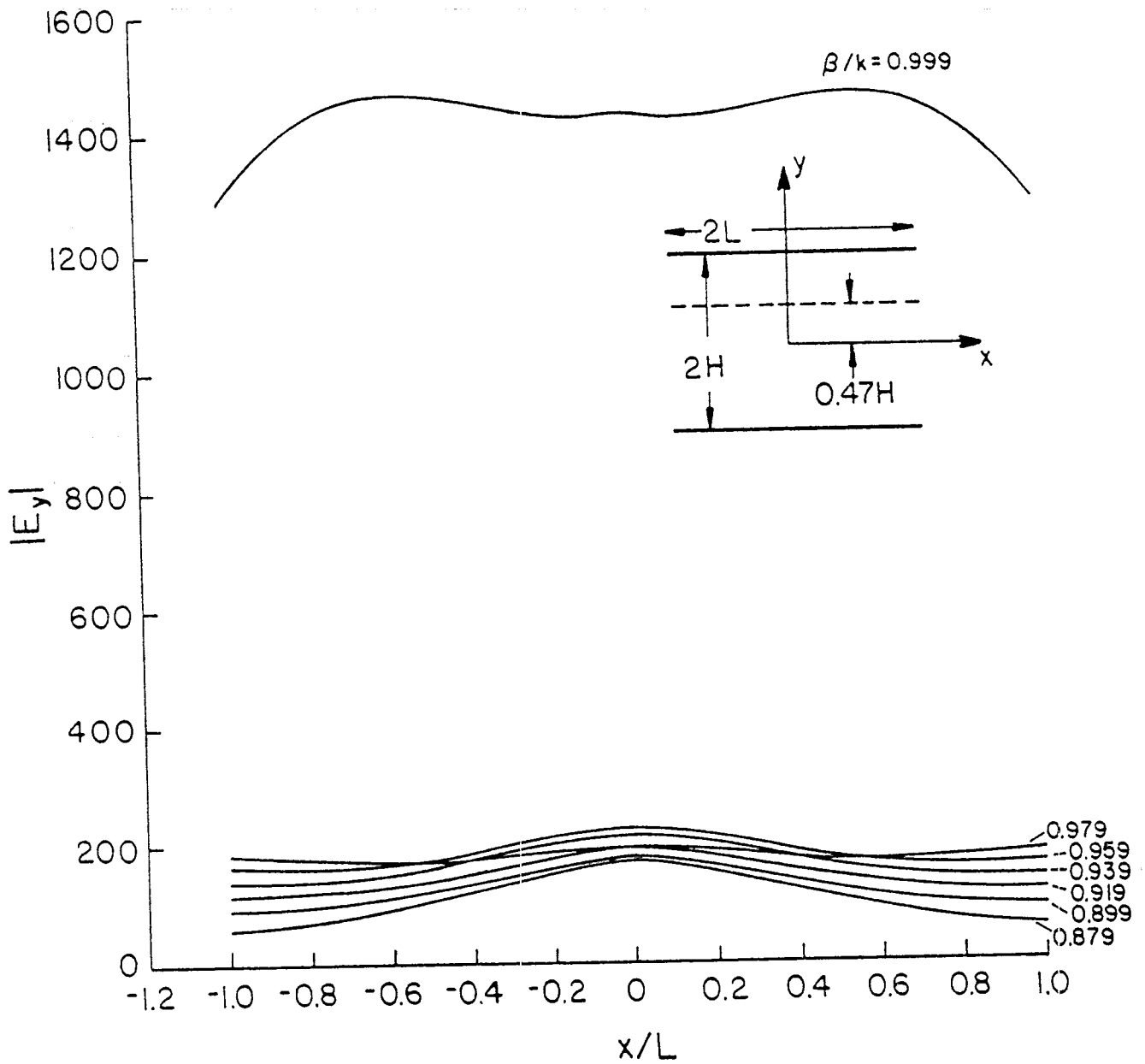
Figure 5(b). $|E_y|$ as a function of x coordinate for $y/H = 0.47$.
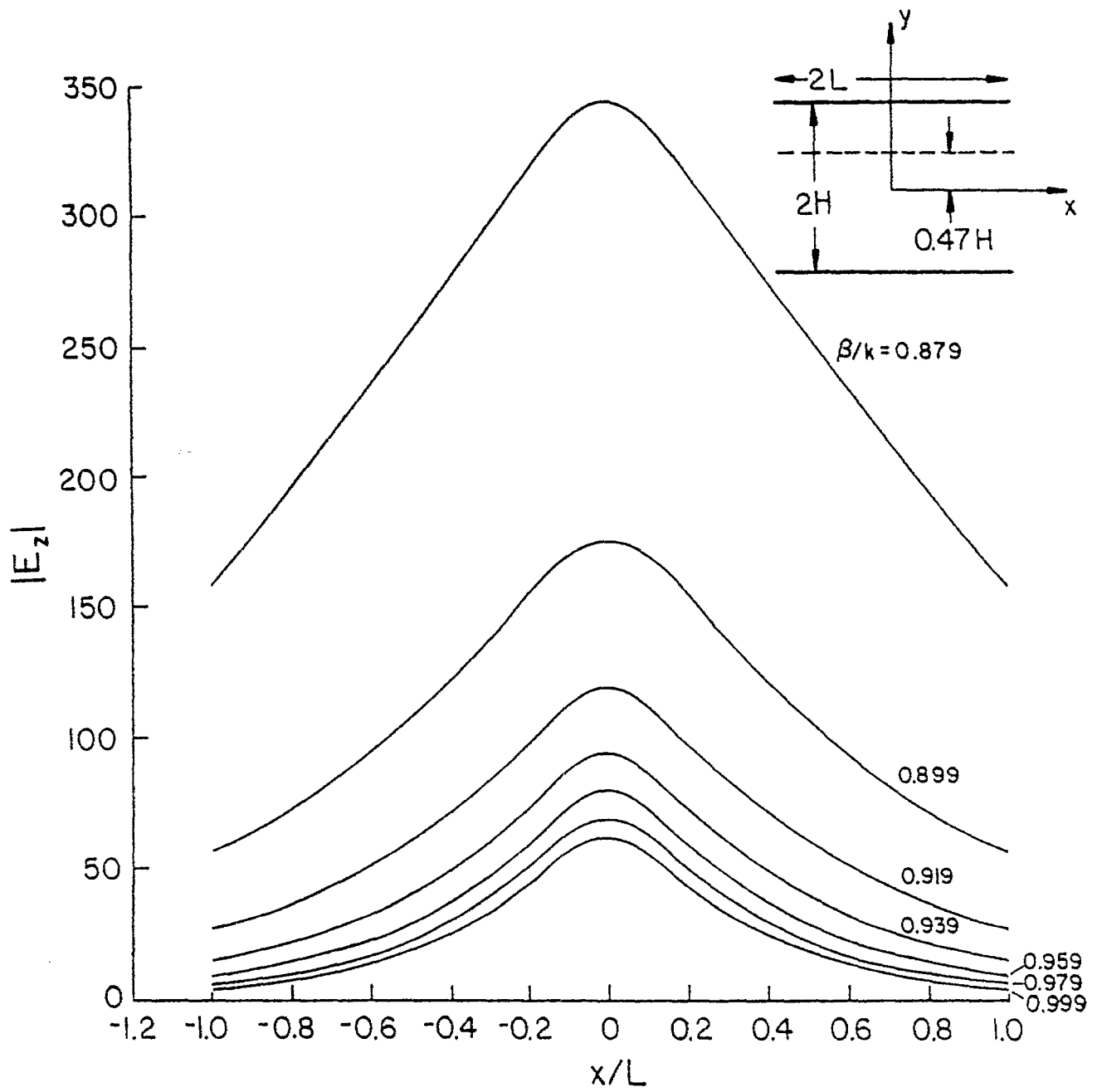
Figure 5(c). $|E_z|$ as a function of x coordinate for y/H = 0.47.

VII. REFERENCES

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APPENDIX

A computer program for determining the magnitude of all six components of the EM field is presented here. Data were obtained for x/L between 0 - 1 with step 0.05, β between 0.879 - 0.999 with step 0.02, and y at any given value. The program can be readily modified to obtain the real and imaginary parts of all field components as functions of the x-coordinate and β.

```
      PROGRAM MAIN ( INPUT,OUTPUT,TAPE3 )
      COMPLEX FLDI(5),FLDS(5),FLD
      COMPLEX S(41,42),JX,JZ,SA,SB,BI,BIX,POLAR,CONM
      COMPLEX TA(22,23),TB(22,23)
      LOGICAL ODD,EVEN
      COMMON /FREC  / AK,BETA,AKT,INDEX,PI
      COMMON /GUIDE / H,AL,XA(23),XB(23),DEL,NC
      COMMON /SOURCE/ HO,XO,NP,ALPHA,SINA,COSA
      COMMON /LOGIC / ODD,EVEN
      COMMON /APRAY / JX(21),JZ(21)
      DIMENSION PTS(14),OPTS(14)
      DIMENSION PY(15),PX(15)
      DATA PTS/21.,23.,21.,21.,21.,21.,21./
      DATA OPTS/*LINE*,*LINE*,*LINE*,*LINE*,*LINE*,*LINE*,
     *LINE*/

      PI=3.1415926535
      PRINT *,*ENTER L, H, HO, XO*
      READ *,AL,H,HC,XO
      PRINT *,*ENTER NP, WAVELENGTH, MODE( 0 FOR EVEN, 1 FOR ODD )*
      READ *,NP,WL,ODD
      PRINT *,*ENTER NC, YO*
      READ *,NC,YO
      PRINT *,*ENTER 1 FOR COMPUTING EX, 2 FOR EY, 3 FOR EZ,
     *4 FOR HX, 5 FOR HY, 5 FOR HZ*
      READ *,NFC
      DO 999 I=1,7
      BETA=0.359+0.02*I

      ODD=L.GT.0
      AK=2.*PI/WL

      IF(HO.GT. H) STOP
      IF(XO.GT. AL) STOP
      IF(XO.LT.-AL) STOP
      BETA2=BETA*BETA
      IF(BETA2.EQ.1.) STOP
      IF(BETA2.GT.1.) GOTO 1100
      INDEX=1
      AKT=AK*SQRT(1.-BETA2)
      GOTO 1200
1100  INDEX=2
      AKT=AK*SQRT(BETA2-1.)
1200  BETA=BETA*AK
      DEL=2.*AL/FLOAT(NC+1)
      NA=NC+2
      NT=NC*NC
      NB=NT+1
      XA(1)=-AL+DEL/2.

      DO 1300 N=2,NA
      XA(N)=XA(N-1)+DEL
1300  XB(N)=XB(N-1)+DEL
      XO=XO+NP*DEL/2.
      SINA=SIN(ALPHA)
      COSA=COS(ALPHA)
      ALPHA=ALPHA/HO
      EVEN=.NOT.ODD

      DO 2001 M=1,NA
      TA(M,1)=SA(M-1,1)
2001  TB(M,1)=SB(M-1,1)
      DO 2002 M=2,NC
      DO 2002 N=2,NA
      IF(M.GT.N) TA(M,1)=TA(M-N+1,1)
```

```
      IF(N.LE.N) TA(M,N)=TA(N-M+3,1)
2002  CONTINUE
      ND=NC-1
      DO 2003 N=2,ND
      DO 2003 M=1,NA
2003  T3(M,N)=TA(M,N)
      DO 2004 M=1,NA
2004  T3(M,NC)=T3(NA-M+1,1)
      CONST=AK*AK*DEL*DEL=2.
      CONM=CMPLX( 0., -BETA*DEL/2. )
      CONB=AK*AKT*DEL*DEL
      DO 2100 M=1,NC
      S(   M,   N)=TA(M+2,N)+CONM*TA(M+1,N)+TA(M,N)
      S(NC+M,   N)=CONM*( TA(M+2,N)-TA(M,N) )
      S(   M,NC+N)=CONM*( T3(M+2,N)-T3(M,N) )
2100  S(NC+M,NC+N)=NB*T3(M+1,N)
      CONM=CONM*2.*DEL
      DO 3000 M=1,NC
      XM=X3(M+1)
      S(   M,NB)=-DEL*DEL*BIX(XM,H)
3000  S(NC+M,NB)=-CONM*BI(XM,H)

      CALL CSMEQ ( S,NT,41,42 )
      DO 4100 M=1,NC
      JX(M)=S(M,NB)
4100  JZ(M)=S(M+NC,NB)
      DO 4200 M=1,NA
      CALL FIELDS(FLDX,X3(M),YO)
      CALL FIELDS(FLDS,X3(M),YC)
      FLD=FLDI(NFC)+FLDS(NFC)
      K=(M-1)*NA+N
      PX(K)=X3(N)/IL
4200  PY(K)=CABS(FLD)
999   CONTINUE
      CALL USTART
      CALL USET(*XE3*HLABELS*)
      CALL UPSET(*XLABEL*,*X/L;*)
      CALL USET(*YBOTHLABELS*)
      CALL UPSET(*YLABEL*,*MODULUS OF I*)
      CALL UPLOT(*X,*Y,*Z,*PTS,OPTS)
      CALL UEND

      STOP
      END
      COMPLEX FUNCTION SA(M,N)
      COMPLEX FO,S,IMAGE
      INTEGER ODD,EVEN
      COMMON /SPACE / TTEL,X3(23),X3(23),DEL,NC
      COMMON /LOGIC / ODD,EVEN
      COMMON /DATA  / XL,XU,H
      XC=X3(M)
      XC=XC+DEL
      XM=X3(M+1)
      Y=H
      X=X3M
      CALL CBOX( PO,(XU,H,IMAGE )
      IF(M.EQ.N) S1=FO(XU-XM)-FO(XL-XM)
      IF(M.EQ.N) S1=2.*FO(XU-XM)
      IF(M.EQ.N) S1=FO(XC-XM)-FO(XM-XU)
      IF(ODD ) S1=S1-IMAGE
      IF(EVEN) S1=S1-IMAGE
      RETURN
      END
```

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