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Electromagnetic Surface Wave Propagation Over a Rectangular Bonded Wire Mesh

DAVID A. HILL

Institute for Telecommunication Sciences
Office of Telecommunications
U.S. Department of Commerce
Boulder, Colorado 80302

JAMES R. WAIT*

Cooperative Institute for Research in Environmental Sciences
University of Colorado/NOAA
Boulder, Colorado 80309

Abstract-The electromagnetic surface wave which can propagate over a rectangular wire mesh of infinite extent, is considered. The propagation constant is determined both from a rigorous Floquet formulation and an approximate method using averaged boundary conditions. The agreement is fairly good for sufficiently small mesh dimensions. The rectangular mesh is found to be highly anisotropic, and the possibility of an effective anisotropic transfer inductance representation for the mesh is discussed briefly.

INTRODUCTION

Wire mesh screens are often employed in electromagnetic shielding applications. Plane wave reflection and transmission coefficients are normally utilized to characterize the shielding effectiveness and, of course, a good shield should have a low transmission coefficient for a wide range of incidence angles for any wave polarization. The method of averaged boundary conditions [1] has been used to analyze the

*Consultant to ITS/OT

INTRODUCTION

Wire mesh screens are often employed in electromagnetic shielding applications. Plane wave reflection and transmission coefficients are normally utilized to characterize the shielding effectiveness and, of course, a good shield should have a low transmission coefficient for a wide range of incidence angles for any wave polarization. The method of averaged boundary conditions [1] has been used to analyze the reflecting and transmitting properties of both square [2] and rectangular [3] meshes. The method gives good results for both bonded and unbonded junctions, but is restricted to mesh dimensions which are small compared to a wavelength. General solutions have been given for plane wave scattering from separated wire grids in free space [4] and for unbonded wire grids over a half-space [5] and for bonded wire grids in free space [6]. These solutions generally involve matrix inversion to solve for the wire currents, but fortunately, large matrices are not required.

A major difference between a wire mesh and a continuous metal sheet is the ability of the wire mesh to support a trapped surface wave. When the source and observer are located near the mesh, this surface wave is quite important. For the case of a square bonded mesh, surface wave propagation has been analyzed [7], and the propagation constant of the surface wave is closely related to the shielding effectiveness of the mesh. Here we extend the previous analysis to the more general case of a rectangular bonded mesh. For comparison, the approximate method of averaged boundary conditions is also applied to the rectangular mesh geometry.

FORMULATION

The geometry of the infinite rectangular bonded mesh in free space (permittivity ε_0 and permeability μ_0) is illustrated in Fig. 1. Arrays of wires parallel to the x axis with spacing b and parallel to the y axis with spacing a are centered in the plane $z = 0$, and perfect contacts are made at the junctions. The wire radius c is small compared to both the spacings a and b and the free space wavelength λ. Consequently, only the axial wire currents are important and the usual thin wire approximations are valid.

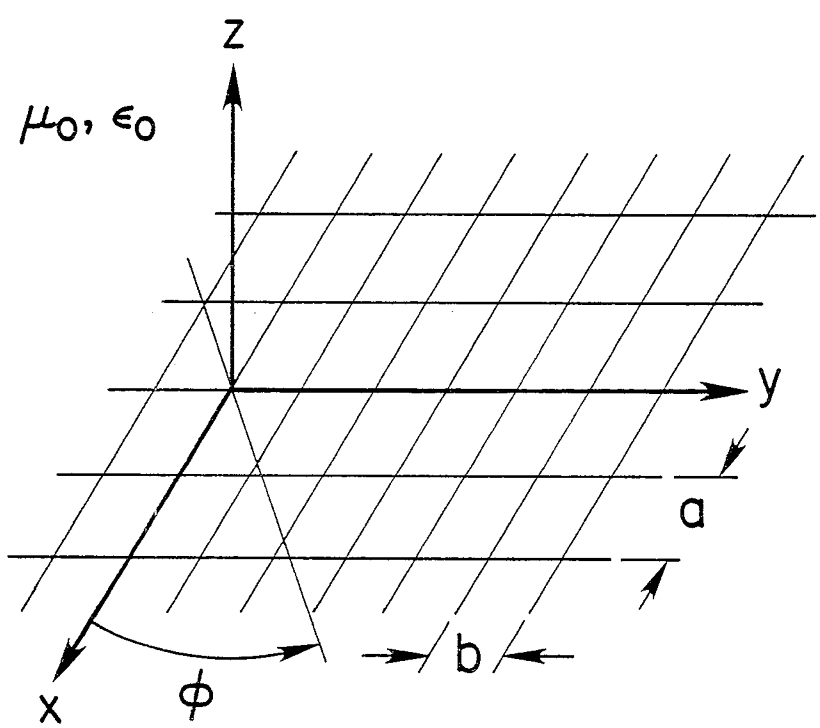

Figure 1. Geometry for a surface wave propagating on a rectangular eitr mrdh eiyh bonded junctions. Wire radius equals c.

tions are valid.

Since the mesh structure has a plane of symmetry at $z = 0$, the electromagnetic field can be decomposed into symmetric and antisymmetric parts which are uncoupled [8]. The electromagnetic surface wave of interest here is symmetric, and the rectangular components of the electric field satisfy the following [7]:

$$E_{\substack{x\\y}}(x,y,z) = E_{\substack{x\\y}}(x,y,-z)$$

and

$$E_z(x,y,z) = -E_z(x,y,-z). \tag{1}$$

The analysis closely follows that for the square mesh [7]. Again, we invoke Floquet's theorem [9] in order to express the relevant electromagnetic quantity as an exponential function multiplied by a function which is periodic in x and y. Thus, for a surface wave propagating at an angle ϕ to the negative x axis, the current on the qth x-directed wire I_{xq} and the current on the mth y-directed wire I_{ym} can be written:

$$I_{xq} = \exp[\gamma(x\cos\phi + qb\sin\phi)] \sum_m A_m \exp(i2\pi mx/a) \tag{2}$$

and

$$I_{ym} = \exp[\gamma(ma\cos\phi + y\sin\phi)] \sum_q B_q \exp(i2\pi qy/b). \tag{3}$$

Here A_m and B_q are the unknown Fourier coefficients and γ is the propagation constant which we seek. The m and q summations are over all integers including zero from $-\infty$ to ∞. The calculation of the fields produced by these currents is straight-forward [5] and will not be repeated here. The thin wire boundary condition for perfectly conducting wires is the following: $E_x(x,o,c) = E_y(o,y,c) = 0$. This condition need only be applied at the $m = 0$ and $q = 0$ wires; the periodic Floquet form of (2) and (3) assures they will be subsequently satisfied at all wires.

Actually, the expressions for the current in (2) and (3) are identical to those in the plane wave scattering case [5] except that γ has replaced ikS where k was the free space wave number $(=2\pi/\lambda)$ and S $(=\sin\theta)$ was the sine of the incidence angle. Thus the previous equations for A_m and B_q [5] can be used with the following modifications: 1) set the incident fields equal to zero (source-free problem), 2) set the grid separation h equal to zero (bonded grids in the same plane), 3) set the wire impedances equal to zero (perfectly conducting wires), and 4) set the half space parameters equal to those of free space. As a result, equations (24) and (26) in [5] reduce to the following:

$$A_m \frac{(k^2-k_x^2)P_m}{2ikb} + \frac{ik_x}{2ka} \sum_q B_q k_y \frac{\exp(-\Gamma c)}{\Gamma} = 0, \tag{4}$$

$$B_q \frac{(k^2-k_y^2)Q_q}{2ika} + \frac{ik_y}{2kb} \sum_m A_m k_x \frac{\exp(-\Gamma c)}{\Gamma} = 0, \tag{5}$$

where

$$P_m = \frac{b}{\pi}\left\{-\ell n\left[1 - \exp\left(\frac{-2\pi c}{b}\right)\right] + \Delta_m\right\} + \frac{\exp(-\Gamma_{mo}c)}{\Gamma_{mo}} , \tag{6}$$

$$\Delta_m = \frac{1}{2} \sum_q{}' \left[\frac{2\pi}{b} \frac{\exp(-\Gamma c)}{\Gamma} - \frac{\exp(-2\pi|q|c/b)}{|q|}\right] , \tag{7}$$

$$Q_q = \frac{a}{\pi} \left\{-\ell n\left[1 - \exp\left(\frac{-2\pi c}{a}\right)\right] + \delta_q\right\} + \frac{\exp(-\Gamma_{oq}c)}{\Gamma_{oq}} , \tag{8}$$

$$\delta_q = \frac{1}{2} \sum_m{}' \left[\frac{2\pi}{a} \frac{\exp(-\Gamma c)}{\Gamma} - \frac{\exp(-2\pi|m|c/a)}{|m|}\right] , \tag{9}$$

$$\Gamma_{mq}(=\Gamma) = (k_x^2+k_y^2-k^2)^{\frac{1}{2}} \tag{10}$$

$$k_x = (2\pi m/a) + kS\cos\phi , \tag{11}$$

and

$$k_y = (2\pi q/b) + kS\sin\phi . \tag{12}$$

A superscripted prime $'$ over the summation sign indicates omission of the $q = 0$ (or $m = 0$) term. S is now defined as $\gamma/(ik)$.

The doubly infinite set of linear equations (4) and (5) for A_m and B_q is numerically inefficient in the present form because A_m and B_q decay slowly for large $|m|$ and $|q|$. The difficulty arises because the current expansions in (2) and (3) are slowly convergent for the discontinuous current that occurs at the wire junctions in the bonded meshes. However, we can circumvent this convergence problem by modifying the current expansions to allow for a jump discontinuity at the origin. This procedure was adopted previously for square bonded meshes [6], [7]. As we indicate below, the method requires only slight modifications for rectangular bonded meshes.

We now rewrite the current expressions in (2) and (3) in the following equivalent forms:

$$I_{xq} = \exp[\gamma(x\cos\phi + qb\sin\phi)] \cdot [f_{\Delta a}(x) + \sum_m{}' A_m \exp(i2\pi mx/a)] \tag{13}$$

$$I_{ym} = \exp[\gamma(ma\cos\phi + y\sin\phi)]$$

$$\cdot \left[-f_{\Delta b}(y) + \sum_q B'_q \exp(i2\pi qy/b)\right] \tag{14}$$

where A'_m and B'_q are the modified coefficients. The sawtooth function $f_{\Delta a}$ is chosen to have a jump Δ at the origin and with a width a; it is defined by

$$f_{\Delta a}(x) = \frac{\Delta}{2\pi i} \sum_n{}' \frac{\exp(i2\pi nx/a)}{n} \tag{15}$$

$$= \Delta \left[U(x) - \frac{x}{a} - \frac{1}{2}\right]; \; -\frac{a}{2} < x < \frac{a}{2} \; ,$$

where

$$U(x) = \begin{cases} 1; & x > 0 \\ 0; & x < 0 \end{cases}$$

As before, the superscripted prime $'$ over the summation sign indicates omission of the $n = 0$ term. The function $f_{\Delta b}(y)$ is defined in an exactly analogous manner.

From (2), (3), and (13) - (15), it is clear that A_m, A'_m, B_q and B'_q are related by

$$A_m = A'_m + \frac{\Delta(1-\delta_{mo})}{2\pi im} \tag{16}$$

and

$$B_q = B'_q - \frac{\Delta(1-\delta_{qo})}{2\pi iq} \tag{17}$$

where

$$\delta_{mo} = \begin{cases} 1; & m = 0 \\ 0; & m \neq 0 \end{cases}$$

Then, by substituting (16) and (17) into (4) and (5), we obtain the following equivalent set of equations for the modified coefficients:

$$A_m' \frac{(k^2-k_x^2)P_m}{2ikb} + \frac{ik_x}{2ka} \sum_q B_q' k_y \frac{\exp(-\Gamma c)}{\Gamma}$$
$$+ \Delta \left\{ \frac{(k^2-k_x^2)P_m}{2kb} \frac{(\delta_{mo}-1)}{2\pi m} - \frac{k_x}{2ka} \left[\frac{P_m'}{b} + \frac{kS\sin\phi}{2\pi} P_{1m} \right] \right\} = 0 \quad (18)$$

and

$$B_q' \frac{(k^2-k_y^2)Q_q}{2ika} + \frac{ik_y}{2kb} \sum_m A_m' k_x \frac{\exp(-\Gamma c)}{\Gamma}$$
$$+ \Delta \left\{ \frac{k^2 - k_y^2}{2ka} \frac{(1-\delta_{qo})}{2\pi q} + \frac{k_y}{2kb} \left[\frac{Q_q'}{a} + \frac{kS\cos\phi}{2\pi} Q_{1q} \right] \right\} = 0 \quad (19)$$

where

$$P_{1m} = {\sum_q}' \frac{\exp(-\Gamma c)}{q\Gamma} \quad (20)$$

$$P_m' = P_m - \frac{\exp(-\Gamma_{mo} c)}{\Gamma_{mo}} \quad (21)$$

$$Q_{1q} = {\sum_m}' \frac{\exp(-\Gamma c)}{m\Gamma} \quad (22)$$

and

$$Q_q' = Q_q - \frac{\exp(-\Gamma_{oq} c)}{\Gamma_{oq}} \quad (23)$$

Again, the superscript prime $'$ on the summation indicates omission of the $q = 0$ (or $m = 0$) term. Now, all summations are in a rapidly convergent form.

Since we have introduced an unknown Δ in the modified current expansions in (13) and (14), another equation is required to have an equal number of equations and unknowns (A_m', B_q', and Δ). The most convenient equation is obtained from charge continuity [6]:

$$\frac{1}{2}\left[\left.\frac{\partial I_{xo}}{\partial x}\right|_{x=0^-} + \left.\frac{\partial I_{xo}}{\partial x}\right|_{x=0^+}\right] = \frac{1}{2}\left[\left.\frac{\partial I_{yo}}{\partial y}\right|_{y=0^-} + \left.\frac{\partial I_{yo}}{\partial y}\right|_{y=0^+}\right] \tag{24}$$

By substituting (13) and (14) into (24), we obtain the following auxiliary condition:

$$\begin{aligned} &- \frac{\Delta}{2\pi}\left(1 + \frac{b}{a}\right) + \sum_m A_m'\left(im\,\frac{b}{a} + \frac{\gamma b}{2\pi}\cos\phi\right) \\ &- \sum_q B_q'\left(iq + \frac{\gamma b}{2\pi}\sin\phi\right) = 0 \end{aligned} \tag{25}$$

Since the current expansions in (13) and (14) are rapidly convergent, the doubly infinite set of equations, (18) and (19), can be truncated with the m ranging from $-M$ to M and q ranging from $-Q$ to Q where M and Q are small integers. Thus (18), (19), and (25) yield a set of $T(=2N+2Q+3)$ linear, homogeneous equations in A_m', B_q', and Δ:

$$\begin{bmatrix} \\ \\ T \times T \\ \\ \text{coefficient} \\ \\ \text{matrix} \\ \\ \end{bmatrix} \begin{bmatrix} A'_{-M} \\ \vdots \\ A_o \\ \vdots \\ A_M \\ B_{-Q} \\ \vdots \\ B_o \\ \vdots \\ B_Q \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \tag{26}$$

A nontrivial solution to (26) exists only if the determinant, which is a function of $\gamma(=ikS)$, vanishes. Thus, the mode equation to be solved for γ is written

$$\begin{vmatrix} T \times T \\ \text{coefficient} \\ \text{matrix} \end{vmatrix} = 0 \tag{27}$$

The z dependence of the Floquet harmonics is given by the factor $\exp(-\Gamma_{mq}|z|)$. For sufficiently small ka and kb, there are no grating lobes (Γ_{mq} real) and thus there is no loss mechanism. Consequently, γ is purely imaginary and $|\gamma| > k$ (or $S > 1$). Equivalently, the equations (18), (19), and (20) can be normalized so that all coefficients are real functions of the real variable S. This real form has been programmed and (27) has been solved numerically for S by the bisection method [10].

AVERAGED BOUNDARY CONDITIONS

Plane wave scattering from rectangular meshes has been analyzed by Astrakhan [3] by the method of averaged boundary conditions. For a bonded rectangular mesh, Astrakhan derives the following expression for the vertically polarized reflection coefficient, $R_{\shortparallel}^{e}$:

$$R_{\shortparallel}^{e} = C\{1 - kC[\gamma_2\cos^2\phi + (\delta_2-\delta_1)\sin\phi\cos\phi - \gamma_1\sin^2\phi]\}/I_o \tag{28}$$

where

$$I_o = C(1+k^2\delta_1\delta_2-k^2\gamma_1\gamma_2) + kS^2[\gamma_2\cos^2\phi + (\delta_2-\delta_1)\sin\phi\cos\phi - \gamma_1\sin^2\phi] + k(\gamma_1-\gamma_2) \tag{29}$$

$$\gamma_1 = \alpha_1(1 - \frac{a/b}{1 + a/b} S^2 \cos^2\phi)$$

$$\gamma_2 = -\alpha_2(1 - \frac{b/a}{1 + b/a} S^2 \sin^2\phi)$$

$$\delta_1 = \alpha_1 \frac{a/b}{1 + a/b} S^2 \sin\phi \cos\phi$$

$$\delta_2 = -\alpha_2 \frac{b/a}{1 + b/a} S^2 \sin\phi \cos\phi$$

$$\alpha_1 = \frac{ib}{\pi} \ln\left(\frac{b}{2\pi c}\right)$$

$$\alpha_2 = \frac{ia}{\pi} \ln\left(\frac{a}{2\pi c}\right)$$

and

$$C = (1-S^2)^{\frac{1}{2}}$$

[Astrakhan's paper contains a misprint, and $\gamma_2 \sin^2\phi$ should be $\gamma_1 \sin^2\phi$ in his (14)]. The propagation constant $(\gamma = ikS)$ of the surface wave is obtained from the pole of $R^e_{\shortparallel}$ in the S plane. Thus the mode equation for S is

$$\left(R^e_{\shortparallel}\right)^{-1} = 0 \quad \text{or} \quad I_o = 0. \tag{30}$$

Since the averaged boundary condition formulation is valid for only electrically small meshes and does not include the possibility of grating lobes, there is no possible loss mechanism for any values of the parameters. Thus the solution of (30) for $S(=\gamma/ik)$ is always real and greater than one. For the general case, no analytical solution for (30) was found, so it was solved numerically by the bisection method [10].

For the special case of propagation along one of the grids (ϕ = 0° or 90°), the expression for $R_{\shortparallel}^{e}$ in (28) simplifies considerably. For $\phi = 0°$, we have

$$R_{\shortparallel}^{e}\Big|_{\phi=0°} = \left[1 + \frac{k\alpha_1}{C}(1-R_1S^2)\right]^{-1} \tag{31}$$

where

$$R_1 = \frac{(a/b)}{1 + (a/b)}$$

Thus the mode equation for $\phi = 0°$ is

$$0 = \left(R_{\shortparallel}^{e}\right)^{-1} = 1 + \frac{k\alpha_1}{C}(1-R_1S^2) \tag{32}$$

By using the quadratic formula, we find

$$S\Big|_{\phi=0°} = \left\{\frac{2R_1k^2\alpha_1^2 - 1 + [1 + 4k^2\alpha_1^2(R_1^2-R_1)]^{\frac{1}{2}}}{2R_1^2k^2\alpha_1^2}\right\}^{\frac{1}{2}} \tag{33}$$

For the special case of small $|k\alpha_1|$, (33) reduces to

$$S\Big|_{\phi=0°} \simeq 1 - \frac{1}{2}k^2\alpha_1^2(R_1-1)^2 \tag{34}$$

For $\phi = 90°$, the results are quite similar.

$$R_{\shortparallel}^{e}\Big|_{\phi=90°} = \left[1 + \frac{k\alpha_2}{C}(1-R_2S^2)\right]^{\frac{1}{2}} \tag{35}$$

where

$$R_2 = \frac{b/a}{1 + b/a}$$

$$S\Big|_{\phi=90^\circ} = \left\{ \frac{2R_2k^2\alpha_2^2 - 1 + [1 + 4k^2\alpha_2^2(R_2^2-R_2)]^{\frac{1}{2}}}{2R_2^2k^2\alpha_2^2} \right\}^{\frac{1}{2}} \qquad (36)$$

$$S\Big|_{\phi=90^\circ} \simeq 1 - \frac{1}{2}\, k^2\alpha_2^2(R_2-1)^2 \qquad (37)$$

For the special case of a square bonded mesh (a=b), the results become independent of ϕ and the mesh can be characterized by an isotropic surface transfer impedance which is inductive [7]. The situation is much more complicated for the rectangular mesh as can be seen from the complicated ϕ dependence for $R_{\shortparallel}^{e}$ in (28). However, by noting the simple expressions for $R_{\shortparallel}^{e}$ for $\phi = 0^\circ$ and 90° in (31) and (34), we can postulate an approximate surface transfer impedance representation for the rectangular mesh which is anisotropic. The anisotropic thin sheet boundary condition has the form [11], [12]:

$$H_{1y} - H_{2y} = -M_xE_x \qquad (38)$$

and

$$H_{1x} - H_{2x} = M_yE_y \qquad (39)$$

where the subscript 1 denotes the region above the mesh and the subscript 2 denotes the region below. In a similar manner to the square mesh case 7 , we can infer M_x from $R_{\shortparallel}^{e}\Big|_{\phi=0^\circ}$ in (31) and M_y from $R_{\shortparallel}^{e}\Big|_{\phi=90^\circ}$ in (35) with the following result

$$M_x = \frac{2/\eta}{k\alpha_1(1-R_1S^2)} \qquad (40)$$

and

$$M_y = \frac{2/\eta}{k\alpha_2(1-R_2S^2)} \tag{41}$$

where $\eta = (\mu_o/\varepsilon_o)^{\frac{1}{2}}$. For angles near grazing, including the surface wave case $(S\simeq1)$, M_x and M_y can be approximated by

$$M_x \simeq \frac{2/\eta}{k\alpha_1(1-R_1)} \tag{42}$$

and

$$M_y \simeq \frac{2/\eta}{k\alpha_2(1-R_2)} \tag{43}$$

We can also define an effective transfer inductance from

$$M_{\substack{x\\y}} = (i\omega\ell_{\substack{x\\y}})^{-1} \tag{44}$$

Thus, ℓ_x and ℓ_y are given by

$$\ell_x = \frac{(1-R_1)\mu_o b}{2\pi} \ \ell n\left(\frac{b}{2\pi c}\right) \tag{45}$$

and

$$\ell_y = \frac{(1-R_2)\mu_o a}{2\pi} \ \ell n\left(\frac{a}{2\pi c}\right) \tag{46}$$

For normal incidence $(S=0)$, the surface transfer inductances inferred from (40) and (41) are

$$\ell_x = \frac{\mu_o b}{2\pi} \ \ell n\left(\frac{b}{2\pi c}\right) \tag{47}$$

and

$$\ell_y = \frac{\mu_o a}{2\pi} \ \ell n\left(\frac{a}{2\pi c}\right) \tag{48}$$

In this limiting case of normal incidence $(S=0)$, ℓ_x is independent of a and ℓ_y is independent of b because the two grids are uncoupled. Also, we see that (47) and (49) are consistent with the analysis of a single grid for normal incidence [13]. In the grazing case, the factor

$(1-R_1)$ in (45) or $(1-R_2)$ in (46) represents the coupling between the grids.

NUMERICAL RESULTS

The mode equation (27) was solved numerically by the bisection method [10], and the convergence was examined by increasing M and Q until the value of $S(=\gamma/ik)$ did not change significantly. Even for the rectangular mesh $(a \neq b)$, no advantage was found in making $M \neq Q$. Also, for $M = Q$, it was found that S remained essentially constant beyond $M = Q = 2$ (T=11) which is consistent with previous results [6], [7]. All results shown here are for $M = Q = 2$, and the required determinant calculation is fairly rapid for the resultant 11×11 matrix. For comparison, the approximate results from the method of averaged boundary conditions have been calculated from (30). These results were checked further with the analytical solution in (33) for $\phi = 0°$ and in (36) for $\phi = 90°$.

In Figs. 2-4, we illustrate the ϕ dependence of S for different values of a/b. Because of symmetry, only the range of ϕ from 0° to 90° need be shown. As indicated in Fig. 2, the value of S for the square mesh (i.e. a = b), is nearly independent of ϕ for the b/λ values considered. However, when a/b is increased, as seen clearly in Figs. 2 and 3, the ϕ dependence of S becomes significant.

As discussed previously [7], the departure of S from unity is a measure of the degradation of the shielding effectiveness of the mesh for grazing propagation. Also, as expected, the method of averaged boundary conditions tends to underestimate S, particularly as b/λ is increased [7].

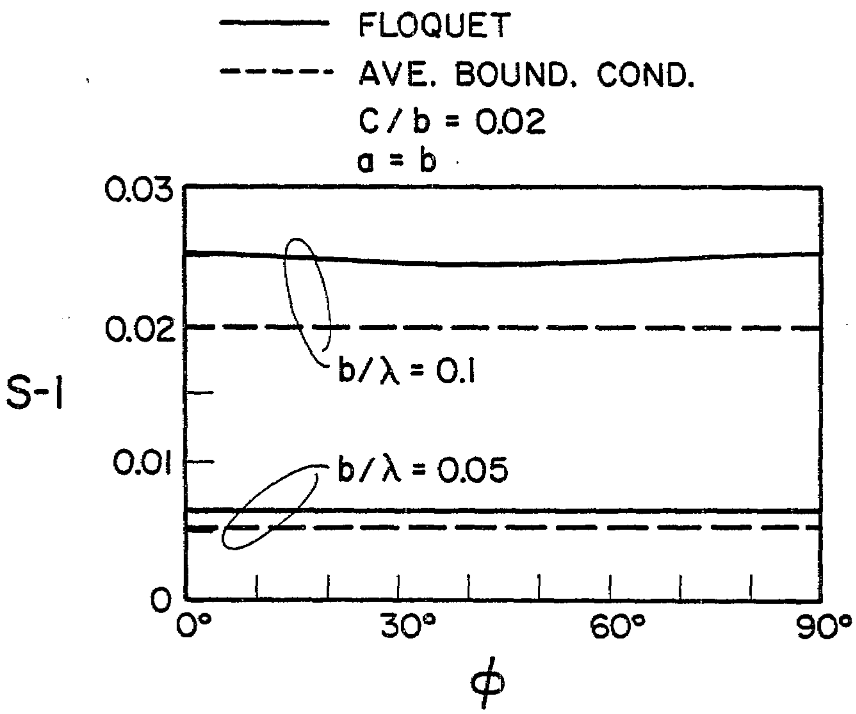

Figure 2. Normalized propagation constant $S(=\gamma/ik)$ for a square mesh as a function of direction.

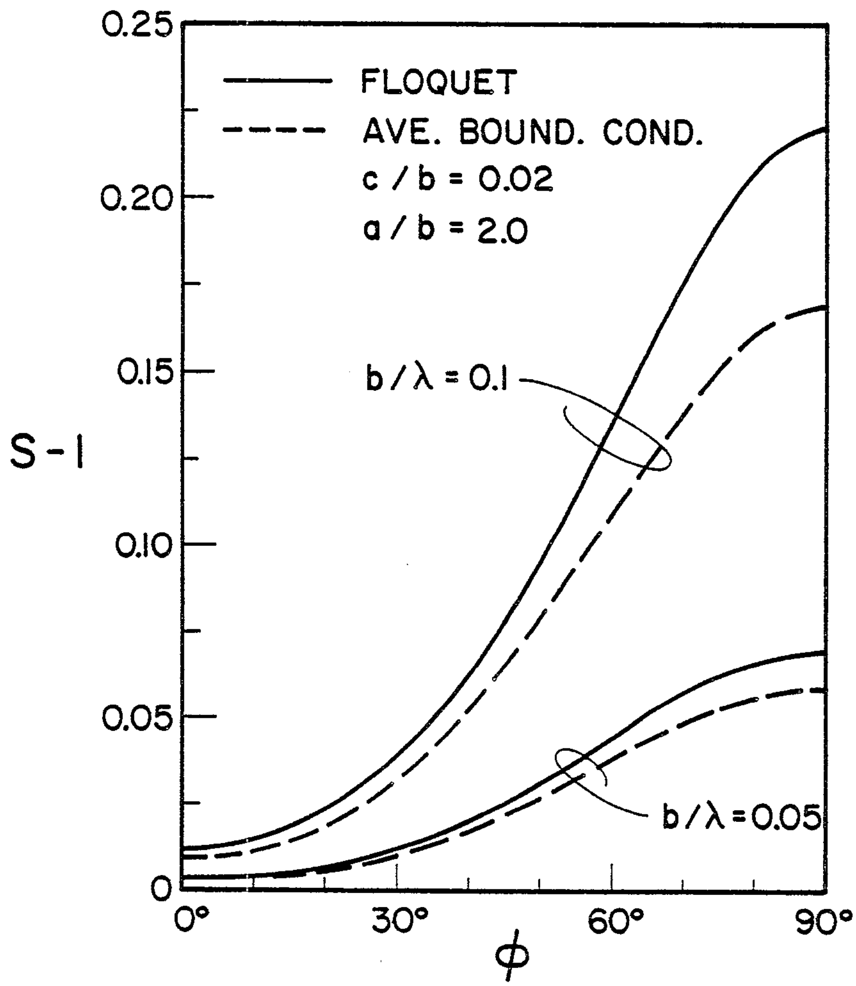

Figure 3. Normalized propagation constant S(=γ/ik) for a 2 to 1 mesh as a function of direction.

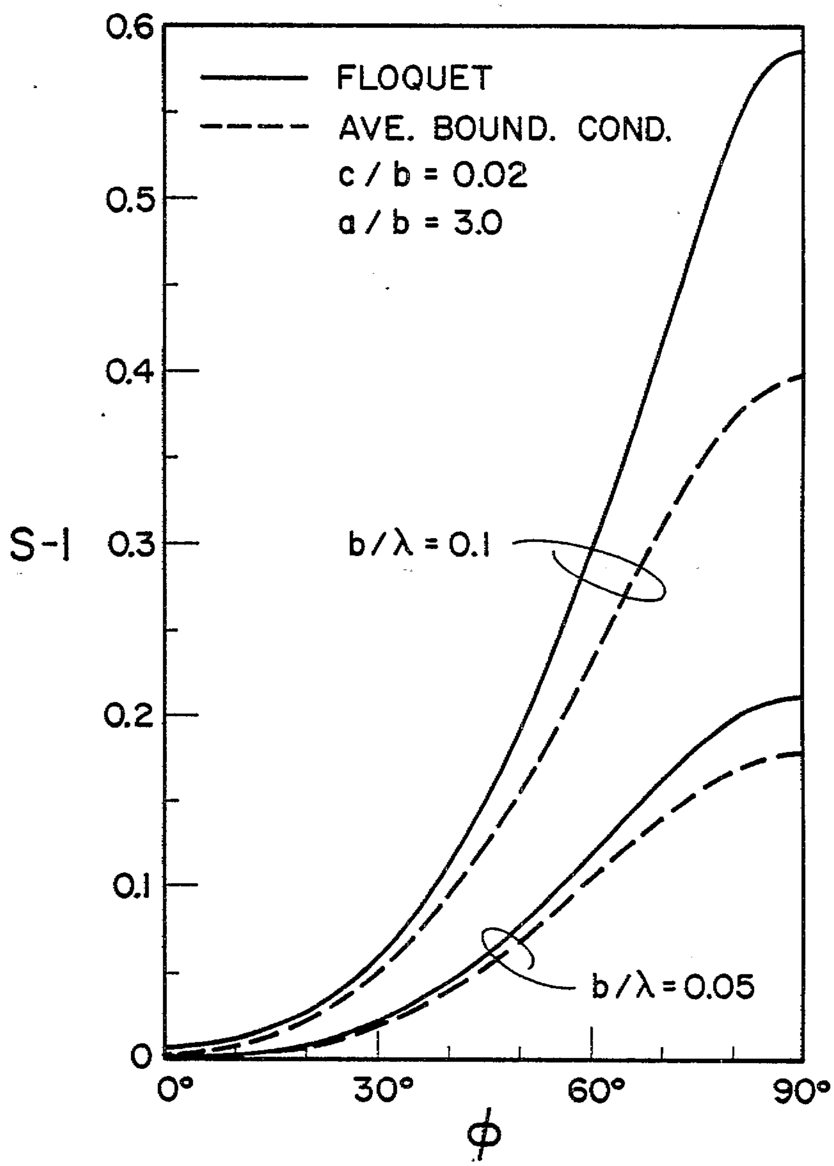

Figure 4. Normalized propagation constant $S(=\gamma/ik)$ for a 3 to 1 mesh as a function of direction.

In some applications, such as in parallel plate waveguide simulators [14], propagation at $\phi = 0°$ is of particular interest. In Fig. 5, the frequency dependence for a square mesh and a 3 to 1 mesh is shown. The 3 to 1 ratio has been used in some EMP simulators (private communication, C.E. Baum). Note that S is smaller for the 3 to 1 mesh than for the square mesh in all cases. In the limit $a/b = \infty$ (no crossed wires) TEM propagation is possible and S goes to one. Analytically, the limit of large a/b is difficult to obtain because for sufficiently large a/λ (or b/λ), Γ_{mq} in (10) is no longer real and grating lobes can result. The mode equation (27) for S is still valid, but S is expected to be complex under such conditions. Then, the mesh would act as a radiator rather than a simple slow-wave structure (i.e. S is real and greater than one). We have not investigated the grating lobe case because the mesh is not an effective shield under such conditions. Furthermore, the method of averaged boundary conditions does not include the possibility of grating lobes.

CONCLUDING REMARKS

The propagation constant of a surface wave propagating along a rectangular mesh in free space has been determined numerically from a general Floquet formulation. For comparison, an approximate solution, from the method of averaged boundary conditions, is also presented. The agreement between the two methods is fairly good for sufficiently small mesh dimensions. In contrast to the square mesh which has a fairly isotropic behavior [7], the rectangular mesh is highly anisotropic.

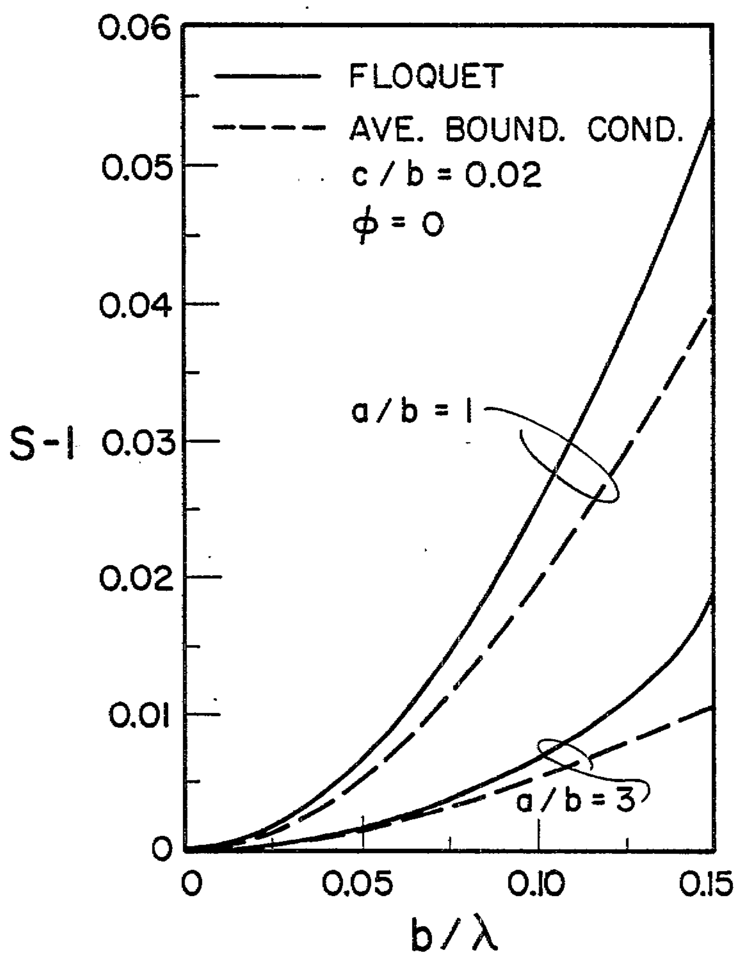

Figure 5. Frequency dependence of propagation constant for propagation along the x-directed wires.

A worthwhile extension is the introduction of imperfect conductivity in the wires by use of an impedance boundary condition at the wire surface rather than imposing zero tangential electric field. The introduction of a lossy half-space [5], [15] would also be useful in modeling ground screens for antennas. Either of the above extensions would introduce a loss mechanism and result in a complex propagation constant γ. Finally, a second mesh (as in a parallel plate waveguide) can be introduced and this configuration has already been treated by the method of averaged boundary conditions [16].

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