Sensor and Simulation Notes

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Extrapolation Techniques for Interpreting the Results of Tests in EMP Simulators in Terms of EMP Criteria

Carl E. Baum
Air Force Weapons Laboratory

Abstract

In general, an EMP simulation test as carried out in practice is not perfect. There are various differences from the criterion situation, for which differences one would like to correct. Such corrections and determination of associated errors are denoted as extrapolation. Depending on the types of error present in a given simulator relative to a given criterion, there are various types of extrapolation which one can apply. These types of extrapolation are also dependent on the degree of detail one is willing to utilize in making the corrections. This note introduces some important types of extrapolation and points to some directions of potential future development.

Contents

I. Introduction

EMP simulation is a rather extensive technical area due to the variety of EMP environments to be simulated, the kinds of systems to be tested in simulators, and the kinds of EMP simulators that can be used for each combination of EMP environment and system to be tested [1]. Let us make some working definitions.

(EMP) simulation is:

an experiment in which the postulated (EMP) exposure situation is replaced by a physical situation in which

1. the (EMP) sources are replaced by a set of equivalent sources which to a good approximation produce the same excitation (including reconstruction by superposition to the extent feasible) to the total system under test or some portion thereof as would exist in the postulated (nuclear) environment, and
2. the system under test is configured so that it reacts to sources (has the same Green's function) in very nearly the same way and to the same degree as it would in the postulated (nuclear) environment.

A(n) (EMP) simulator is:

a device which provides the excitation used for (EMP) simulation without significantly altering the response of the system under test by the simulator presence.

Hence EMP simulation can be thought of as being comprised of two parts, the simulator and the system under test (including anything required to obtain and/or observe the test results (instrumentation)). Furthermore these two parts should be matched to each other given the type of EMP environment to be simulated and the operational conditions of the system.

The problem of concern in this note stems ultimately from the fact that experiments, in general, have errors, and some experiments have more error than others. In some cases the ratio of the

errors to the true signal (which would be present under ideal conditions including corrections or extrapolation) is greater than unity. Such cases are only marginally simulation at best; for very large errors one must eventually not regard the experiment as simulation at all.

This note considers some aspects of simulation error, specifically simulator error, and how it can be experimentally (as well as theoretically) quantified. This error is related to the concept of extrapolation and is what is left after extrapolation is performed.

Simulator extrapolation is:

an extension of the simulator in which the response of the system undergoing a simulation test is corrected to some degree for differences of its response from those under criterion conditions associated with

1. differences in the simulator environment from the criterion environment, and
2. proximity of the simulator to the system changing its response characteristics (Green's function) from those existing under criterion conditions. (Note that local earth, water, etc. in the context of an EMP simulator is part of the simulator.)

Note that this definition addresses the simulator related deficiencies and this is the aspect considered in this note. To generalize this to simulation extrapolation one might add another point as:

3. differences in the system configuration changing its response characteristics from those existing under criterion (operational) conditions.

These differences include electrical switch positions, physical configuration such as landing gear position on aircraft, etc. For this note let us assume that there are no differences as in item 3 so that the system electromagnetic topology is not altered

[5, 6, 10]. Let us also assume that there exists some approximate highly conducting boundary (although with various penetrations) which can be thought of as approximately isolating the system interior from its exterior; we thereby have the interior exerting negligible influence on the electromagnetic response of the system exterior.

An essential part of the extrapolation process is establishing "extrapolation to what?", i.e., what is the criterion of concern.

> A(n) (EMP) criterion is:
>
> a quantitative statement of the physical parameters of the (EMP) environment relevant to the (EMP) response of a system of interest in a volume of space and region of time and/or frequency extended to contain all physical parameters having a non-negligible influence on any of the (EMP) response parameters (e.g., as in the case of EMP (plane wave) a particular direction of incidence and a particular polarization and proximity to other scatterers).

In the case of the nuclear EMP this involves detailed statements concerning the fields, source current density, and conductivities in the vicinity of the system [4]. In this note only the case without local sources (non-source region) is considered; this simplifies the response at the system penetrations and removes an important type of nonlinearity associated with the nuclear-radiation-induced local conductivity and/or source current density which is a function of the fields. To some extent the extrapolation concepts discussed in this note can likely be extended to source region conditions, but this is a matter for further investigation.

An EMP environment of widespread interest is that often referred to as the high-altitude EMP, both for in-flight systems and ground-based systems. For in-flight systems (below the source region) one can characterize this environment as a plane wave [3]. As such one must in general specify waveform and/or associated

frequency spectrum, polarization, and angle of incidence. Near the earth surface there is an additional reflected wave which interacts with the system while the system is still interacting with the first wave; hence the high-altitude EMP environment is more complicated near the ground. The corresponding EMP simulators for accurately addressing these two quite different "high-altitude" EMP environments should consequently be dissimilar. One should not expect a simulator for systems on the ground to perform well for in-flight conditions, and conversely.

For this note we assume some particular criterion EMP environment (non-source-region) has been specified. This can be extended to a set of criterion environments (including different polarizations and angles of incidence) by repeating the extrapolation process for as many pairs of criterion environment vs. simulation conditions as desired. Different extrapolation functions and errors (to be discussed later) may result in each case.

It is important to note that the extrapolation formulas and associated error formulas developed in this note are meant to apply to any one given pair of criterion and simulation conditions. Such extrapolation can be performed for any criterion/simulation pair (or criterion/simulator pair) or for any number of such pairs. However, in general a different extrapolation is performed for each such pair. One might subscript the extrapolation and error functions and/or include functional dependences to indicate the dependence of such functions on the choice of criterion/simulation pair. Such variables might include, for criteria, direction of incidence, polarization, incident waveform, proximity to the earth surface, etc.; for simulation they might include which simulator, position and orientation of the test object (system) in the simulator, which pulser, what test-object configuration, etc. For this note such dependences are not discussed; future notes may address such questions.

II. Types of Simulation Performance Factors

In considering the problem of extrapolating test results to compensate (in part) for differences between a simulation and a criterion it is convenient to separate out the different types of possible simulation deficiencies. Our starting point is the description of the response of an antenna or scatterer by an integral equation of the general form [9]

$$\left\langle \tilde{\vec{\vec{I}}}(\vec{r},\vec{r}';s) \cdot \tilde{\vec{J}}_s(\vec{r}',s) \right\rangle = \tilde{\vec{I}}(\vec{r},s) \quad , \quad \vec{r}\in S \tag{2.1}$$

where the domain of integration (over $\vec{r}'$) is taken to be the surface of an object designated by S, although volume integration is appropriate since for our development we assume the system of interest to be approximately describable, at least in part (for separating inside from outside), as a perfectly conducting body. The general integral equation (2.1) can be specialized to the impedance (or E-field) integral equation as

$$\left\langle \tilde{\vec{\vec{Z}}}(\vec{r},\vec{r}';s) \cdot \tilde{\vec{J}}_s(\vec{r}',s) \right\rangle = \tilde{\vec{E}}_{so}(\vec{r},s) \quad , \quad \vec{r}\in S \tag{2.2}$$

Here $\tilde{\vec{E}}_{so}$ is some "source" electric field such as an incident field or a field specified at some antenna gap. The impedance kernel for free-space conditions is

$$\tilde{\vec{\vec{Z}}}(\vec{r},\vec{r}';s) \equiv s\mu_o \tilde{\vec{\vec{G}}}_o(\vec{r},\vec{r}';s) \tag{2.3}$$

where $\tilde{\vec{\vec{G}}}_o$ is the dyadic Green's function of free space [8] given by

$$\tilde{\tilde{G}}_o(\vec{r},\vec{r}';s) = \frac{\gamma}{4\pi}\left\{[-2\zeta^{-3} - 2\zeta^{-2}]e^{-\zeta}\vec{1}_R\vec{1}_R + [\zeta^{-3} + \zeta^{-2} + \zeta^{-1}]e^{-\zeta}[\vec{\tilde{1}} - \vec{1}_R\vec{1}_R]\right\} + \frac{1}{3\gamma^2}\delta(\vec{r} - \vec{r}')\vec{\tilde{1}} \tag{2.4}$$

$$\gamma = \frac{s}{c}$$

$$\zeta = \gamma R$$

$$R = |\vec{r} - \vec{r}'|$$

$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

where the principal value is implied for integration except for distributions with their usual rules for integration over volumes [2, 7]; for surfaces proper limits should be taken.

For these considerations we have the complex frequency

$$s \equiv \Omega + j\omega \tag{2.5}$$

which is the Laplace transform variable in the two-sided Laplace transform (designated by a tilde ~ above the quantity) as

$$\tilde{f}(s) \equiv \int_{-\infty}^{\infty} f(t)e^{-st}dt$$

$$f(t) = \frac{1}{2\pi j}\int_{\Omega_o-j\infty}^{\Omega_o+j\infty} \tilde{f}(s)e^{st}ds \tag{2.6}$$

where the Bronwich inversion contour in the complex s plane is defined in a strip of convergence $\Omega_- < \mathrm{Re}[s] < \Omega_+$ of the Laplace transform integral. Frequency domain and its extension as a complex frequency domain have an important role in techniques for extrapolating simulation test results to EMP criteria.

Let us now divide the simulation performance into several factors based on general mathematical description(s) of the response. For this discussion let us take an integral equation of the general form of (2.1), or more specifically (2.2). For this purpose let us assume:

1. The system of interest can be approximated as having a perfectly conducting outer surface as far as it affects the external surface current and charge densities, both in the simulation test and in the EMP criterion. This is interpreted in terms of the short-circuit surface current and charge densities at the assumed closed penetrations.

2. The penetrations through the outer surface are electrically small. This allows one to characterize the excitation of the penetration by quasi-static processes characterized by the short-circuit surface current and charge densities.

3. The penetrations through the outer surface are small compared to their distances from additional scatterers (such as ground planes) introduced by the simulator. This avoids a change in the distribution of the quasi-static modes near the penetrations as compared to the EMP criterion situation of interest.

As a notational matter let us distinguish by superscripts the various electromagnetic parameters for various situations as

$$\begin{aligned} C &\equiv \text{criterion} \\ S &\equiv \text{simulation} \\ E &\equiv \text{extrapolated} \end{aligned} \tag{2.7}$$

These are the parameters in respectively the criterion or "ideal" situation to which comparison is being made, the situation in the simulation test, and the extrapolated situation or that after extrapolation ("correction") has been made. The extrapolated situation is an artificial one representing some approximation of the

criterion situation based on the results of certain simulation measurements with certain correction (or extrapolation).

Let us choose a set of functions chosen as a scalar measure of the accuracy of a particular aspect of the simulation. In general increasing value of the function is used to indicate better (more accurate) simulation, at least for the particular aspect concerned. One might have each of these vary as real values between 0 and 1; in this case one might define an error function as 1 minus the first function. Define:

$$f_s \equiv 1 - \varepsilon_s \equiv \text{source function}$$

$$\varepsilon_s \equiv \text{source error} \qquad (2.8)$$

This measures how closely the incident field $\tilde{\vec{I}}^{(S)}(\vec{r},s)$ approximates $\tilde{\vec{I}}^{(C)}(\vec{r},s)$. This might be, for example, some average over frequencies and space of interest (say rms) of the normalized magnitude of the difference of these functions.

$$f_k \equiv 1 - \varepsilon_k \equiv \text{kernel function}$$

$$\varepsilon_k \equiv \text{kernel error} \qquad (2.9)$$

This measures how criterion-space like the test volume in the simulator is, i.e., how the Green's function at $\vec{r}$ from a source at $\vec{r}'$ is like that which would occur under criterion conditions. Said another way this measures how closely $\tilde{\vec{\Gamma}}^{(S)}(\vec{r},\vec{r}';s)$ approximates $\tilde{\vec{\Gamma}}^{(C)}(\vec{r},\vec{r}';s)$ over the test volume of interest. A related consideration which can be used is the change (measurable) of the external natural frequencies of a system and/or the introduction of additional external natural frequencies due to the simulator/test-object interaction (insofar as it differs from the criterion situation).

Note that $\tilde{\vec{\Gamma}}^{(C)}(\vec{r},\vec{r}';s)$ is defined for the criterion space of interest, including the presence or absence of nearby soil, concrete, water, conducting posts, etc.

$$f_{sys} \equiv 1 - \varepsilon_{sys} \equiv \text{system function}$$

$$\varepsilon_{sys} \equiv \text{system error} \qquad (2.10)$$

This measures how closely the system under test is configured like the criterion situation for the system. It relates to the integration over the system in (2.1) denoted by $\langle , \rangle$ which can be distinguished as $\langle , \rangle^{(S)}$ for the simulation and $\langle , \rangle^{(C)}$ for the criterion situation.

Now let us combine these simulation performance factors in some ways that exhibit the total simulation performance. First define a two-component vector

$$\begin{aligned} (S_{r_n}) &\equiv \text{simulator vector} \\ &= f_s \oplus f_k = (f_s, f_k) \\ &= (\text{source function}) \oplus (\text{kernel function}) \end{aligned} \qquad (2.11)$$

and then a three-component vector

$$\begin{aligned} (S_{n_n}) &\equiv \text{simulation vector} \\ &= (S_{r_n}) \oplus f_{sys} = f_s \oplus f_k \oplus f_{sys} = ((S_{r_n}), f_{sys}) = (f_s, f_k, f_{sys}) \\ &= (\text{simulator vector}) \oplus (\text{system function}) \\ &= (\text{source function}) \oplus (\text{kernel function}) \oplus (\text{system function}) \end{aligned} \qquad (2.12)$$

The first of these considers the simulator including some volume (the test volume) in (or in some specified position with respect to) the simulator. The second of these includes the system in the definition so as to complete the definition of simulation. Here the direct sum $\oplus$ has been introduced; for vectors (including one-component vectors) this combines the vectors in an ordered manner (non-commutative) so as to form a vector of a number of components equal to the sum of the numbers of components of the original vectors; for square matrices this operation creates a block diagonal matrix with the original matrices as the blocks (in ordered fashion down the diagonal).

With these definitions we can consider a simulator as representing a point in simulator space (two dimensional) which we might denote by $\{(S_{r_n})\}$, i.e., the set of all (S_{r_n}). Note the inclusion of some reference test volume in the definition. Then we can write

$$\begin{aligned}\{(S_{r_n})\} &\equiv \text{simulator space} \\ &= \{f_s \oplus f_k\} \\ &= \{(f_s, f_k)\}\end{aligned} \tag{2.13}$$

Similarly we have an actual simulation (a test including a system) as a point in simulation space (three dimensional) given by

$$\begin{aligned}\{(S_{n_n})\} &\equiv \text{simulation space} \\ &= \{(S_{r_n}) \oplus f_{sys}\} = \{f_s \oplus f_k \oplus f_{sys}\} \\ &= \{((S_{r_n}), f_{sys})\} = \{(f_s, f_k, f_{sys})\}\end{aligned} \tag{2.14}$$

Here simulator and simulation spaces have been defined as two and three dimensional, respectively. Perhaps these forms should be referred to as basic simulator space and basic simulation space. More elaborate forms might split f_s, f_k, and/or f_{sys} into smaller parts thereby replacing one or more of these functions by vectors and correspondingly increasing the dimensionality of the spaces.

As a measure of the performance of a particular simulator one might use a simulator function which we take for present purposes as

$$\begin{aligned}S_r &\equiv \text{simulator function} \\ &= f_s f_k\end{aligned} \tag{2.15}$$

and similarly for a simulation test we have

$$\begin{aligned}S_n &\equiv \text{simulation function} \\ &= S_r f_{sys} = f_s f_k f_{sys}\end{aligned} \tag{2.16}$$

These have the desirable property that if any of the two or three functions in these products are zero (indicating the total absence of an important property) then the simulator and/or simulation function, as appropriate, are also zero. Of course, one could define other scalar combinations of f_s, f_k, and f_{sys}, depending perhaps on how these three functions are themselves quantitatively defined. Note that the definitions of equations (2.15) and (2.16) are not Euclidean metrics on simulator and simulation spaces respectively.

An alternate approach is to define a measure of the simulator error which we could take as

$$\begin{aligned} \varepsilon_{sr} &\equiv \text{simulator error} \\ &\equiv |(1,1) - (S_{r_n})| = |(1,1) - (f_s, f_k)| \\ &= |(\varepsilon_s, \varepsilon_k)| = [\varepsilon_s^2 + \varepsilon_k^2]^{1/2} \end{aligned} \tag{2.17}$$

and similarly for a simulation test we have

$$\begin{aligned} \varepsilon_{sn} &\equiv \text{simulation error} \\ &\equiv |(1,1,1) - (S_{n_n})| = |(1,1,1) - (f_s, f_k, f_{sys})| \\ &= |(\varepsilon_s, \varepsilon_k, \varepsilon_{sys})| = [\varepsilon_s^2 + \varepsilon_k^2 + \varepsilon_{sys}^2]^{1/2} = [\varepsilon_{sr}^2 + \varepsilon_{sys}^2]^{1/2} \end{aligned} \tag{2.18}$$

As defined here these errors are the vector magnitudes of

$$\begin{aligned} (\varepsilon_{sr_n}) &\equiv \text{simulator error vector} \\ &= (\varepsilon_s, \varepsilon_k) \\ (\varepsilon_{sn_n}) &\equiv \text{simulation error vector} \\ &= (\varepsilon_s, \varepsilon_k, \varepsilon_{sys}) \end{aligned} \tag{2.19}$$

respectively so that ε_{sr} and ε_{sn} do represent Euclidean norms on corresponding error spaces. This type of error measure is

appropriate for a high quality simulator and high quality simulation. Such errors being small can be taken as a definition of a criterion simulator and simulation, i.e.

$$(\varepsilon_{sr} \text{ small}) \equiv (\text{approximate criterion simulator})$$
$$(\varepsilon_{sn} \text{ small}) \equiv (\text{approximate criterion simulation}) \tag{2.20}$$

Let us now define

$$(f_{s_n}) \equiv N_s \text{ component source vector}$$
$$(f_{k_n}) \equiv N_k \text{ component kernel vector} \tag{2.21}$$
$$(f_{sys_n}) \equiv N_{sys} \text{ component system vector}$$

Each of these vectors is a set of discrete values of the corresponding function ranging from 0 to 1 in increasing order, say in uniform increments (given by the reciprocal of a positive integer). Each of these vectors represents the set of assumed possible values of the corresponding function.

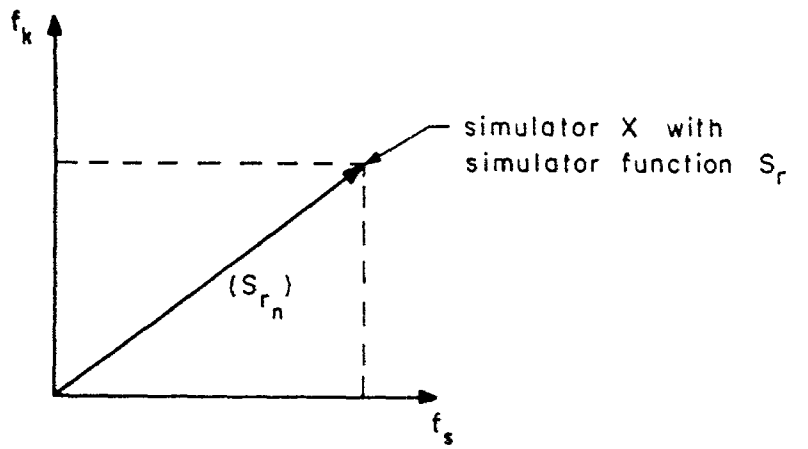
Now we can construct

$$\begin{aligned}(S_{r_{\ell,m}}) &\equiv \text{simulator 2-tensor (matrix, } N_s \times N_k) \\ &= (f_{s_n}) \otimes (f_{k_n}) \\ &= (f_{s_\ell} f_{k_m})\end{aligned} \tag{2.22}$$

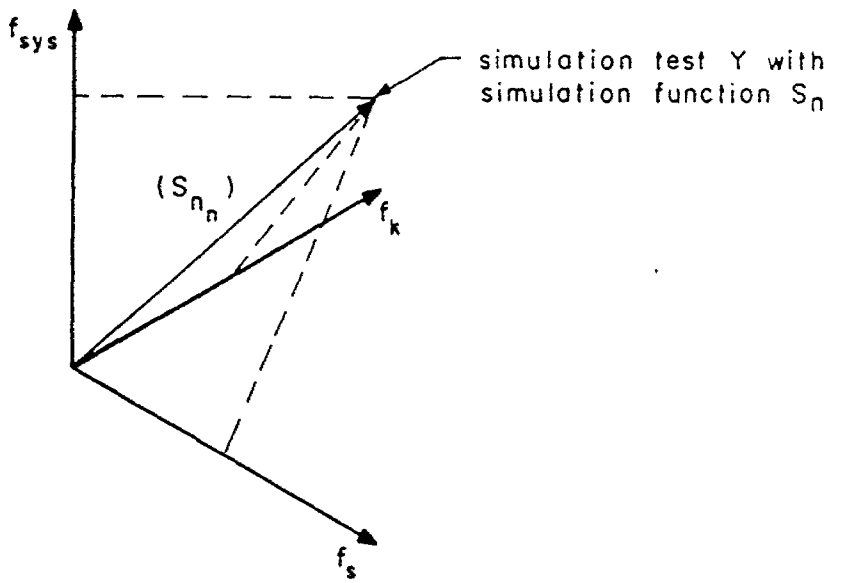
$$\begin{aligned}(S_{n_{\ell,m,n}}) &\equiv \text{simulation 3-tensor } (N_s \times N_k \times N_{sys}) \\ &= (S_{r_{\ell,m}}) \otimes (f_{sys_n}) \\ &= (S_{r_{\ell,m}} f_{sys_n}) \\ &= (f_{s_n}) \otimes (f_{k_n}) \otimes (f_{sys_n}) \\ &= (f_{s_\ell} f_{k_m} f_{sys_n})\end{aligned}$$

These are arrays of simulator and system functions respectively corresponding to points in simulator and simulation space ((2.13) and (2.14)) respectively. One can consider the numbers of components, N_s, N_k, and N_{sys}, as tending to infinity so that we might define (2.22) in continuous form. Note the use of the direct product $\otimes$; this is a generalization of the concept of the dyadic or outer product of vectors to arbitrary numbers and ranks of tensors.

Figure 2.1 illustrates the simulator and simulation spaces. Points in these spaces represent specific simulators or simulation tests. Associated with each point is a simulator or simulation function, and collectively these give the simulator 2-tensor and simulation 3-tensor.



A. Simulator space $\{(S_{r_n})\}$



B. Simulation space $\{(S_{n_n})\}$

Figure 2.1 Simulator and Simulation Spaces

III. A Topological View of the Simulation and Extrapolation Processes

In order to categorize the different kinds of extrapolation techniques, one may place them in the context of the different parts of the process beginning from an EMP environment and ending at signals at various places of interest in the system of interest. Consider the transfer function from an environment to an interior system position as being approximately representable by a product of transfer functions. More generally, there may be a set of transfer functions to a position of interest giving a transfer vector, each component of which is approximately representable as a product of transfer functions. For various transfer vectors for intermediate steps in the interaction sequence [5, 6, 10] into the system there may be matrix relations to efficiently characterize relations among successive transfer vectors.

Consider the diagram in figure 3.1. This schematically indicates some of the electromagnetic processes occurring in a criterion environment and a simulation test. Without the test object present the simulator environment has some spatial and temporal (or frequency) characteristics designated by the incident fields. Such incident fields are controlled by the various sources and local media (simulator structure, local ground, etc.) present in the simulator. These incident fields may or may not be a good approximation to the criterion incident fields which exist in the presence of various possible scatterers (which can be referred to as primary scatterers and may include local ground, water, etc.).

Placing the test object with assumed approximately closed perfectly conducting surface into the incident fields introduces some changes. Under criterion conditions the test object scatters fields to the primary scatterer(s) (scatterers such as ground, water, etc. present under criterion conditions), and the primary scatterer(s) in turn return scatter to the test object in a manner consistent with the boundary conditions. The combination of incident fields, fields scattered from the pimary scatterer(s), and

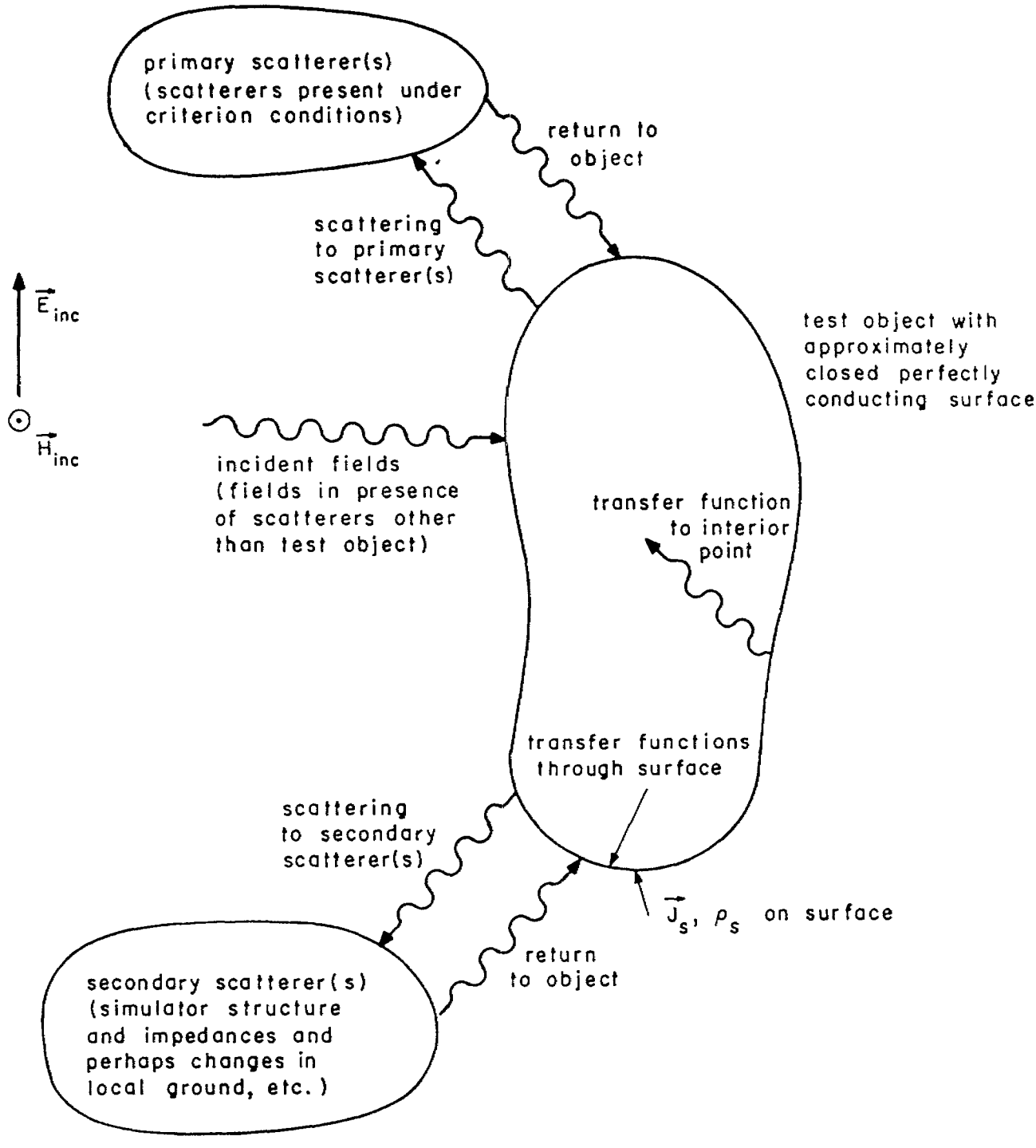


Figure 3.1 Diagramatic Form of Signal Flow in Criterion Environment and Simulation Test

fields scattered from the test object satisfies the Maxwell equations (using the assumed perfectly conducting boundary of the test object), thereby determining the surface current density $\vec{J}_s$ and surface charge density ρ_s on the test object.

Compare the criterion situation to the simulation test. In the latter case the scatterers (other than the test object) may have important differences including the presence of a simulator structure (antenna) with various impedances and perhaps local ground, water, etc. different from the criterion situation (say an aircraft being tested on the ground for an in-flight criterion condition). Besides different incident fields there is different scattering back and forth between the test object and these different scatterers (referred to as secondary scatterers) in the simulation test. This results in different $\vec{J}_s$ and ρ_s on the test object (different from the criterion situation).

Let us now define a surface response function as

$$\tilde{F}_{s_m}(\vec{r}_s,s) \equiv \begin{cases} Z_o \tilde{\vec{J}}_s(\vec{r}_s,s) \cdot \vec{1}_m & \text{for } m = 1,2 \\ \frac{1}{\varepsilon_o} \tilde{\rho}_s(\vec{r}_s,s) = \tilde{\vec{E}}_s(\vec{r}_s,s) \cdot \vec{1}_m & \text{for } m = 3 \end{cases} \tag{3.1}$$

where for each position $\vec{r}_s$ on the object surface (not including singularities such as edges) there are three mutually orthogonal unit vectors $\vec{1}_m$ for m = 1,2,3 in a right-handed sense, i.e.

$$\vec{1}_1 \times \vec{1}_2 = \vec{1}_3 \ , \ \vec{1}_2 \times \vec{1}_3 = \vec{1}_1 \ , \ \vec{1}_3 \times \vec{1}_1 = \vec{1}_2 \tag{3.2}$$

Here m = 1,2 are used for orthogonal components of the surface current density (or equivalently the surface magnetic field) tangential to the object surface which we designate by S. The surface outward pointing normal is $\vec{1}_3$ which also pertains to the normal surface electric field $\vec{E}_s$ or equivalently the surface charge density. For convenience the surface current density is multiplied by Z_o and the surface charge density is multiplied by $1/\varepsilon_o$ to put

both types of quantities into a set of common units (volts/meter or electric field). If the incident field is specified in electric-field units then the surface transfer function is dimensionless.

Let us also consider the surface response function in a more discrete form. Suppose that there is some number of discrete penetrations N_p through S, each penetration located at $\vec{r}_{s_n}$ for $n = 1,2,\cdots,N_p$. Then one can define

$$\tilde{F}_{s_{n,m}}(s) \equiv \tilde{F}_{s_m}(\vec{r}_{s_n},s) \equiv \begin{cases} Z_o \tilde{\vec{J}}_s(\vec{r}_{s_n},s) \cdot \vec{1}_m & \text{for } m = 1,2 \\ \frac{1}{\varepsilon_o} \tilde{\rho}_s(\vec{r}_{s_n},s) & \text{for } m = 3 \end{cases} \tag{3.3}$$

as a set of discrete surface response functions corresponding to the penetrations. Note that the penetrations are assumed closed (shorted) for the above definition of the surface response function. The three scalar surface quantities in (3.1) and (3.3) correspond to the set of magnetic and electric excitations appropriate to small penetrations, such as small apertures and small antennas on S. By small we mean both electrically small (small compared to radian wavelengths of interest) and physically small compared to local radii of curvature of S and distances to other perturbations on S.

If we have defined an incident-field function $\tilde{F}_{inc}$ in electric-field units then we have

$$\tilde{T}_{s_m}(\vec{r}_s,s) \equiv \frac{\tilde{F}_{s_m}(\vec{r}_s,s)}{\tilde{F}_{inc}(s)}$$

$$\tilde{T}_{s_{n,m}}(s) \equiv \frac{\tilde{F}_{s_{n,m}}(s)}{\tilde{F}_{inc}(s)} \tag{3.4}$$

as continuous and discrete forms, respectively, of a dimensionless surface transfer function. For this purpose the incident-field

function is defined as any convenient scalar related to the incident field such as some particular vector component of the incident field at some particular point, or some appropriate average of the incident field component(s) over many points.

Next one transports signals from the surface current densities and surface charge densities through the various penetrations to some interior position of the test object. This interior position might be some pin on a connector into some black box. This position is of interest because one uses it for referencing signals associated with permanent damage or temporary functional disruption (upset). As such one can refer to this position as a failure port.

In propagating to a failure port the signals from the exterior of S must pass through S (via one or more penetrations with appropriate parameters such as aperture polarizabilities, antenna height or area, etc.). Such signals may pass through various topological layers inside the test object and finally arrive at the failure port. At the n'th failure port we have a response to the incident environment as

$$\tilde{F}_{i_{n',m'}}(s) \equiv \begin{cases} \frac{1}{L}\,\tilde{V}_p(s) & \text{for } m' = 1 \\ \frac{Z_o}{L}\,\tilde{I}_p(s) & \text{for } m' = 2 \end{cases} \tag{3.5}$$

where L is some characteristic length and m' designates whether voltage or current (normalized) is being considered. Corresponding to this failure-port response function we have a failure-port transfer function from the incident environment as

$$\tilde{T}_{i_{n',m'}}(s) \equiv \frac{\tilde{F}_{i_{n',m'}}(s)}{\tilde{F}_{inc}(s)} \tag{3.6}$$

The next step is to factor the failure port response or transfer function as

$$\tilde{F}_{i_{n',m'}}(s) = \sum_{n,m} \tilde{T}_{n',m';n,m}(s)\, \tilde{F}_{s_{n,m}}(s)$$

$$\tilde{T}_{i_{n',m'}}(s) = \sum_{n,m} \tilde{T}_{n',m';n,m}(s)\, \tilde{T}_{s_{n,m}}(s) \qquad (3.7)$$

This merely states the assumption that the signal reaching the failure port can be considered as a linear combination of the surface fields at the (shorted) penetrations. This also assumes no significant interaction between the penetrations (n values) and independent contributions from the different surface field quantities (m values). This gives a set of surface to failure-port transfer functions $\tilde{T}_{n',m';n,m}(s)$.

An important point for this development is an assumption that each surface-to-failure-port transfer function of relevance is unchanged in going from criterion conditions to the simulation test. This will help us in developing some extrapolation formulas and associated errors. This assumption that the system configuration is unchanged is stated as $f_{sys} = 1$ or $\varepsilon_{sys} = 0$ in the notation introduced in section II.

IV. Types of Extrapolation

By extrapolation we mean here the correction of simulation test results to some criterion situation as well as possible determination of some of the errors in this process. Note for the present development that only surface-response-function differences are being considered; the system under test is assumed to have the appropriate criterion configuration. The types of extrapolation to be discussed are based on the kinds of differences in the simulation as compared to criterion, and the kinds of corrections employed. These types of extrapolation are summarized in figure 4.1. Note in addition that only linear processes are considered here.

The extrapolation process centers around an extrapolation function $\tilde{f}_e(s)$ which is applied to the results in a simulation test to given an "extrapolated" result as

$$\tilde{F}^{(E)}_{i_{n',m'}}(s) = \tilde{f}_e(s)\tilde{F}^{(S)}_{i_{n',m'}}(s) \tag{4.1}$$

for some interior-failure-port response, and

$$\begin{aligned} \tilde{F}^{(E)}_{s_{n,m}}(s) &= \tilde{f}_e(s)\tilde{F}^{(S)}_{s_{n,m}}(s) \\ \tilde{F}^{(E)}_{s_m}(\vec{r}_s,s) &= \tilde{f}_e(s)\tilde{F}^{(S)}_{s_m}(\vec{r}_s,s) \end{aligned} \tag{4.2}$$

for some surface response. Here $\tilde{f}_e(s)$ is left somewhat ambiguous in that it might be a single function applied to all positions on the test object (for a specific criterion situation and a specific simulation test) or it might conceivably be a set of functions, each function applied to different portions of the test object. The definition of the extrapolation function takes the general form

$$\tilde{f}_e(s) \equiv \left\{ \frac{\tilde{F}^{(C)}}{\tilde{F}^{(S)}} \right\}\Bigg|_{avg} \tag{4.3}$$

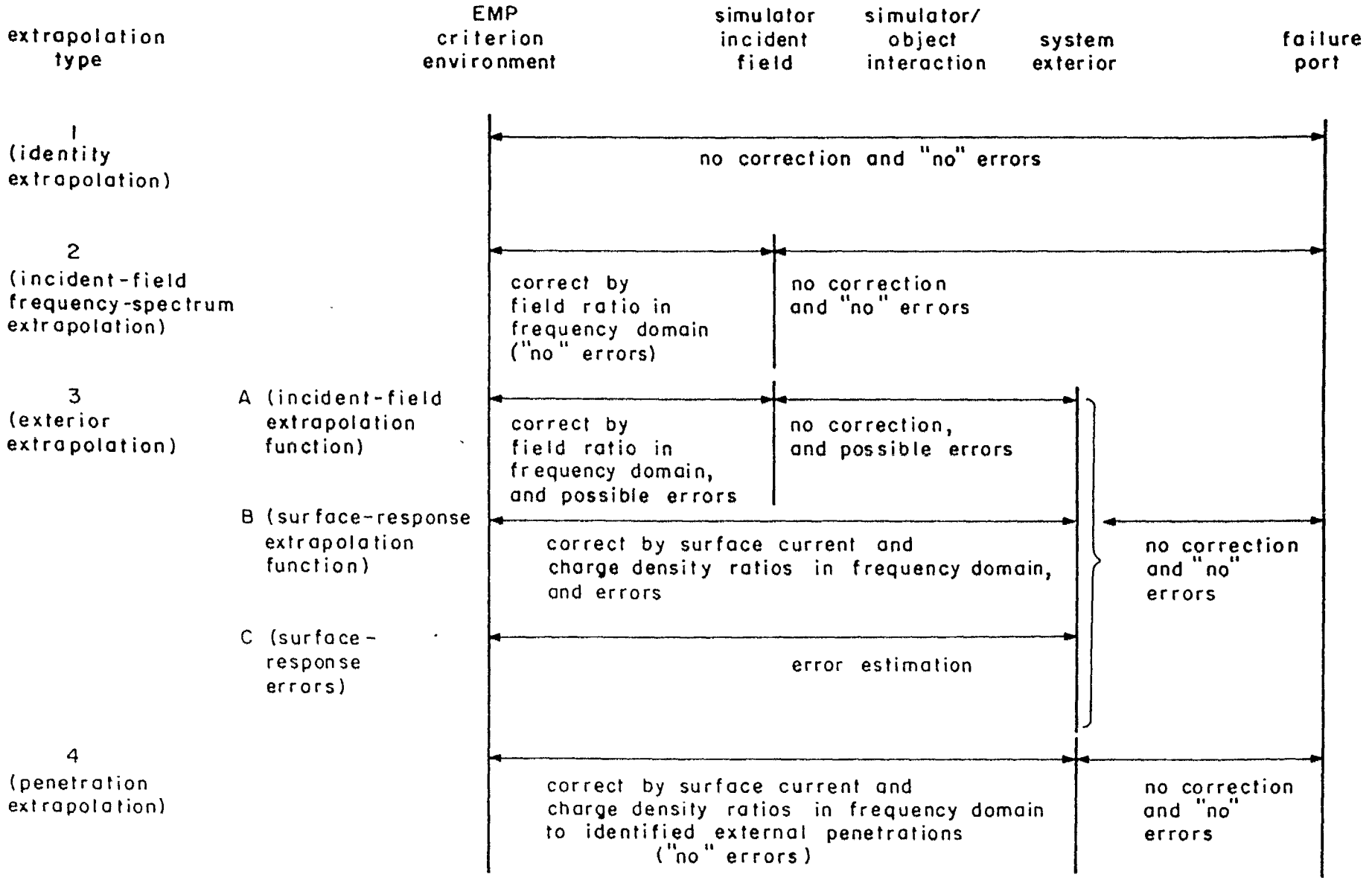


Figure 4.1 Extrapolation Sequence Diagram

where the general term $\tilde{F}$ is ratioed for criterion and simulator conditions and is perhaps averaged over many such ratios in some way. Here $\tilde{F}$ may refer to any of the incident and response functions discussed previously.

A. Type 1: Identity Extrapolation

This type of extrapolation is only included for completeness. It applies to the case that the incident fields in the simulator match those in the criterion situation in both spatial and temporal (frequency) characteristics, and the simulator/object interaction is negligible. This is the trivial case with

$$\tilde{f}_e(s) = 1 \tag{4.4}$$

As indicated in figure 4.1 this corresponds to no correction (extrapolation) and no (or negligible) errors in this process. This means that for each interior failure port

$$\tilde{F}^{(S)}_{i_{n',m'}}(s) = \tilde{F}^{(C)}_{i_{n',m'}}(s) = \tilde{F}^{(E)}_{i_{n',m'}}(s) \tag{4.5}$$

and for the surface response

$$\begin{aligned} \tilde{F}^{(S)}_{s_{n,m}}(s) &= \tilde{F}^{(C)}_{s_{n,m}}(s) = \tilde{F}^{(E)}_{s_{n,m}}(s) \\ \tilde{F}^{(S)}_{s_m}(\vec{r}_s,s) &= \tilde{F}^{(C)}_{s_m}(\vec{r}_s,s) = \tilde{F}^{(E)}_{s_m}(\vec{r}_s,s) \end{aligned} \tag{4.6}$$

Stated another way this case corresponds to a criterion (or very nearly criterion) simulator. In this special case even nonlinearities are in principle accounted for if the system has criterion configuration.

B. Type 2: Incident-Field Frequency-Spectrum Extrapolation

This type of extrapolation is applicable to the situation in which the only simulation deficiency is the frequency spectrum of the incident field. In time domain this is manifested as waveform and/or amplitude differences from criterion. The extrapolation function can then be defined by

$$\tilde{f}_e(s) \equiv \frac{\tilde{F}^{(C)}_{inc}(s)}{\tilde{F}^{(S)}_{inc}(s)} \tag{4.7}$$

where the incident-field function can be defined in a variety of ways, such as

$$\tilde{F}_{inc}(s) = \tilde{\vec{E}}_{inc}(\vec{r}_o, s) \cdot \vec{1}_o \tag{4.8}$$

where $\vec{r}_o$ is some spatial position where the sample of the incident field is taken and $\vec{1}_o$ is a unit vector in some specified direction at $\vec{r}_o$. If desired the form in (4.8) can be extended to include other than electric field and to averages over various positions and orientations. For a single sample, as in (4.8), the position and orientation should be chosen to minimize the impact of measurement error (i.e., keep away from nulls). In principle any choice of $\vec{r}_o$ and $\vec{1}_o$, including averaging, will give exactly the same result for $\tilde{f}_e(s)$ since, by hypothesis, the criterion and simulation differ only by a spatially independent factor.

Since there is, also by hypothesis, no simulator/object interaction to alter the object response, then the object response (both exterior and interior) is scaled by the same frequency factor everywhere. Hence the object response can also be used (except for measurement problems) to determine the same extrapolation function, i.e.,

$$\tilde{f}_e(s) = \frac{\tilde{F}^{(C)}_{s_m}(\vec{r}_s, s)}{\tilde{F}^{(S)}_{s_m}(\vec{r}_s, s)} = \frac{\tilde{F}^{(C)}_{s_{n,m}}(s)}{\tilde{F}^{(S)}_{s_{n,m}}(s)} = \frac{\tilde{F}^{(C)}_{i_{n',m'}}(s)}{\tilde{F}^{(S)}_{i_{n',m'}}(s)} \tag{4.9}$$

Using (4.1) and (4.2) to define the extrapolated quantities we find that they are the same as the corresponding criterion quantities, i.e.,

$$\tilde{F}^{(E)}_{s_m}(\vec{r}_s,s) = \tilde{F}^{(C)}_{s_m}(\vec{r}_s,s)$$

$$\tilde{F}^{(E)}_{s_{n,m}}(s) = \tilde{F}^{(C)}_{s_{n,m}}(s) \qquad (4.10)$$

$$\tilde{F}^{(E)}_{i_{n',m'}}(s) = \tilde{F}^{(C)}_{i_{n',m'}}(s)$$

Stated another way this type of extrapolation has no errors in principle although various errors will generally appear in actual implementation. Refering to figure 4.1 this type of extrapolation is characterized by a correction in the frequency spectrum of the incident field with no other corrections performed and no resulting errors.

C. Type 3: Exterior Extrapolation

Suppose now that the simulation is complicated by some characteristics which make the object response have different variations from the criterion response for different positions on the object. Still assuming a criterion system configuration and a linear response (as characterized by transfer functions in (3.4), (3.6), and (3.7)) we would like to define an extrapolation function which will be used in a common way over the entire test object. Of course there will be errors in so doing because, by hypothesis, the different portions of the test object will vary from criterion in quantitatively different ways.

The different variations in the object response at different positions can be attributed to at least one of two different mechanisms. First the spatial variation of the incident field may not be the same in the simulator as in criterion (such as different angles of incidence, different polarizations, addition of non-criterion reflected waves from non-criterion secondary scatterers,

etc.). Second there may be simulator/object interaction changing the test-object resonances and related Green's function parameters.

This type of extrapolation then applies to situations in which there is significant error remaining after application of an extrapolation function to the test data. Note that if one knew how much each penetration was contributing to a given failure-port response, and if one measured the surface-response at these penetrations and measured the surface to failure-port transfer functions, then he could use (3.7) to avoid these errors. However, by hypothesis, such corrections are not employed in this type of extrapolation because the relevant data is unknown (i.e., which penetrations, etc.) and one does not wish to go to the trouble of obtaining such data.

Figure 4.1 shows that this third type of extrapolation involves a use of an extrapolation function determined by either the incident field (ignoring some aspects of the test-object response), or by the surface response of the test object. In addition one can then for either or both kinds of extrapolation function determine some quantitiative aspects of the errors remaining by use of the surface response of the test object.

1. Type 3A: Incident-Field Extrapolation Function

One way to define an incident field extrapolation function is of the simple form

$$\tilde{f}_e^{(A)}(s) \equiv \frac{\tilde{F}_{inc}^{(C)}(s)}{\tilde{F}_{inc}^{(S)}(s)} \qquad (4.11)$$

$$\tilde{F}_{inc}(s) = \tilde{\vec{E}}_{inc}(\vec{r}_o, s) \cdot \vec{1}_o$$

which is as in type 2 ((4.7) and (4.8)). There is a difference, however, in this case in that the extrapolation function may be dependent on the choice of the sample position $\vec{r}_o$ and sample direction $\vec{1}_o$; this would be due to any differences in the spatial

dependence of the incident field between the criterion and the simulation. Note that one may also use

$$\tilde{f}_e^{(A)}(s) \equiv \left\{ \frac{\tilde{F}_{inc}^{(C)}(s)}{\tilde{F}_{inc}^{(S)}(s)} \right\} \Bigg|_{\substack{avg \\ \vec{r}_o, \vec{1}_o}} \tag{4.12}$$

if averaging the incident-field ratio at various locations, orientations, field type, etc. gives better results. Various types of weighted averages might be used. Note the use of the superscript A to distinguish this type of extrapolation function and its depiction in figure 4.1.

2. Type 3B: Surface-Response Extrapolation Function

Another way to define an extrapolation function is to use the surface response as

$$\tilde{f}_e^{(B)}(s) \equiv \frac{\tilde{F}_{s_m}^{(C)}(\vec{r}_{s_o}, s)}{\tilde{F}_{s_m}^{(S)}(\vec{r}_{s_o}, s)} \tag{4.13}$$

where $\vec{r}_{s_o}$ is a sample position on the test object surface. The surface quantity of interest is the surface response function as in (3.1) where m in (4.13) can assume the value 1, 2, or 3 corresponding to which surface field component is being considered. A generalization of (4.13) in continuous or discrete form is

$$\tilde{f}_e^{(B)}(s) \equiv \left\{ \frac{\tilde{F}_{s_m}^{(C)}(\vec{r}_s, s)}{\tilde{F}_{s_m}^{(S)}(\vec{r}_s, s)} \right\} \Bigg|_{\substack{avg \\ \vec{r}_s, m}} \tag{4.14}$$

$$\tilde{f}_e^{(B)}(s) \equiv \left\{ \frac{\tilde{F}_{s_{n,m}}^{(C)}(s)}{\tilde{F}_{s_{n,m}}^{(S)}(s)} \right\} \Bigg|_{\substack{avg \\ n,m}}$$

using the forms in (3.1) and (3.3), respectively. Note the superscript B used to distinguish this type of extrapolation function and its depiction in figure 4.1.

An advantage of this type of extrapolation function is the bypassing of the incident-field and simulator-object interaction processes, at least formally. There is still a problem of what $\vec{r}_{s_o}$ and m to choose or what kind of averaging to choose. In addition the criterion surface response functions must be obtainable, at least approximately; these may be obtained from accurate surface response calculations, or from accurate measurements on a scale model of the test object (with properly scaled local media, etc.) with frequency spectral correction (type 2 extrapolation), or from similar measurements on the test object in a different simulator with proper local media and spatial field distribution and correction for frequency spectrum (type 2 extrapolation).

For treating the question of averaging let us introduce a penetration density function or penetration weight function, $P_{s_m}(\vec{r}_s)$ in continuous form or $P_{s_{n,m}}$ in discrete form, with the usual normalization property of probability density functions as

$$\sum_{m=1}^{3} \int_S P_{s_m}(\vec{r}_s)dS = 1 \qquad (4.15)$$

$$\sum_{m=1}^{3} \sum_{n=1}^{N_p} P_{s_{n,m}} = 1$$

where N_p is the number of penetrations or some set of chosen positions on the object exterior. Physically a penetration density function weights the relative importance of the various penetrations as far as they may contribute to the interior response functions $\tilde{F}^{(C)}_{i_{n',m'}}(s)$ under criterion conditions. As such, a penetration density function may also be considered as a function of s but such dependence on s is not utilized here and so not explicitly indicated. Since, by hypothesis, we do not know which

penetration contributes which amount to the interior response, and perhaps do not even know some of the penetration locations, then we are justified in treating the penetration density function as some general density function over the surface s.

There are various types of averages of ratios as in (4.14) that one might use. One could use a simple linear weighted average as

$$\tilde{f}_e^{(B)}(s) \equiv \sum_{m=1}^{3} \int_S \frac{\tilde{F}_{s_m}^{(C)}(\vec{r}_s,s)}{\tilde{F}_{s_m}^{(S)}(\vec{r}_s,s)} P_{s_m}(\vec{r}_s)dS \tag{4.16}$$

$$\tilde{f}_e^{(B)}(s) \equiv \sum_{m=1}^{3} \sum_{n=1}^{N_p} \frac{\tilde{F}_{s_{n,m}}^{(C)}(s)}{\tilde{F}_{s_{n,m}}^{(S)}(s)} P_{s_{n,m}}$$

This has the sometimes unfortunate property that large ratios of criterion-to-simulation values (in magnitude) get weighted much more than small magnitude ratios. Furthermore some of the ratios may have similar magnitude but phase differences approximately π, thereby cancelling. One could also define the averages in the sense of the reciprocal of the linear weighted average of the reciprocal ratios (i.e., simulator to criteria), but with similar shortcomings.

Another type of average with interesting properties is a weighted logarithmic average which we can write as

$$\tilde{f}_e^{(B)}(s) \equiv \exp\left\{\sum_{m=1}^{3} \int_S \ln\left[\frac{\tilde{F}_{s_m}^{(C)}(\vec{r}_s,s)}{\tilde{F}_{s_m}^{(S)}(\vec{r}_s,s)}\right] P_{s_m}(\vec{r}_s)dS\right\} \tag{4.17}$$

$$\tilde{f}_e^{(B)}(s) \equiv \exp\left\{\sum_{m=1}^{3} \sum_{n=1}^{N_p} \ln\left[\frac{\tilde{F}_{s_{n,m}}^{(C)}(s)}{\tilde{F}_{s_{n,m}}^{(S)}(s)}\right] P_{s_{n,m}}\right\}$$

As in the case of (4.16) this form allows $\tilde{f}_e^{(B)}(s)$ to be an analytic function of the complex frequency s. Note that a linear weighted average of the magnitudes of the ratios of criterion-to-simulation surface response functions is not in general an analytic function of s. However note from (4.17) for the continuous case we have

$$|\tilde{f}_e^{(B)}(s)| = \exp\left\{ \sum_{m=1}^{3} \int_S \ell n \left[\left| \frac{\tilde{F}_{s_m}^{(C)}(\vec{r}_s, s)}{\tilde{F}_{s_m}^{(S)}(\vec{r}_s, s)} \right| \right] P_{s_m}(\vec{r}_s) dS \right\} \tag{4.18}$$

$$\arg(\tilde{f}_e^{(B)}(S)) = \sum_{m=1}^{3} \int_S \arg\left[\frac{\tilde{F}_{s_m}^{(C)}(\vec{r}_s, s)}{\tilde{F}_{s_m}^{(S)}(\vec{r}_s, s)} \right] P_{s_m}(\vec{r}_s) dS$$

and for the discrete case we have

$$|\tilde{f}_e^{(B)}(s)| = \exp\left\{ \sum_{m=1}^{3} \sum_{n=1}^{N_p} \ell n \left[\left| \frac{\tilde{F}_{s_{n,m}}^{(C)}(s)}{\tilde{F}_{s_{n,m}}^{(S)}(s)} \right| \right] P_{s_{n,m}} \right\} \tag{4.19}$$

$$\arg(\tilde{f}_e^{(B)}(S)) = \sum_{m=1}^{3} \sum_{n=1}^{N_p} \arg\left[\frac{\tilde{F}_{s_{n,m}}^{(C)}(s)}{\tilde{F}_{s_{n,m}}^{(S)}(s)} \right] P_{s_{n,m}}$$

Here only real values of the penetration density function are assumed in deriving the results. Note that the magnitudes of the extrapolation functions in (4.17) are found to be logarithmic averages of the criterion-to-simulator surface response function magnitude ratios, so that here we have a way of averaging magnitudes that is consistent with having $\tilde{f}_e^{(B)}(s)$ an analytic function of s. This analytic aspect can be important because of its association with the physically realizability of physical quantities, and of the possibility of representing the extrapolation in terms

of its singularities in the s (complex frequency) plane according to the singularity expansion method (SEM) [9, 11].

A simplified form of this weighted logarithmic average is found by choosing a penetration density function of the form

$$P_{s_\ell} = \frac{1}{N'_p} \quad , \quad \ell = 1,2,\cdots,N'_p \tag{4.20}$$

where the index ℓ is chosen to randomly sample over combinations of $\vec{r}_s$ and m together (i.e., the ℓth "penetration" consists of some particular m at particular $\vec{r}_{s_n}$). Then $\tilde{F}_{s_n,m}(s)$ is rewritten as $\tilde{F}_{s_\ell}(s)$ and (4.17) becomes

$$\tilde{f}_e^{(B)}(s) = \exp\left\{ \sum_{\ell=1}^{N'_p} \ell n \left[\frac{\tilde{F}_{s_\ell}^{(C)}(s)}{\tilde{F}_{s_\ell}^{(S)}(s)} \right] \frac{1}{N'_p} \right\}$$

$$= \left\{ \prod_{\ell=1}^{N'_p} \frac{\tilde{F}_{s_\ell}^{(C)}(s)}{\tilde{F}_{s_\ell}^{(S)}(s)} \right\}^{\frac{1}{N'_p}} \tag{4.21}$$

which illustrates that the weighted logarithmic average is also a weighted geometric average.

3. Type 3C: Surface-Response Errors

Since a scalar extrapolation function cannot in general completely correct for the simulation errors involving differences in the spatial dependence of the incident field and/or the presence of simulator/object interaction, then one would like to have some quantitative understanding of the remaining errors. Comparing the extrapolated to criterion responses at the failure ports we have (using (3.7)) a set of ratios $\tilde{R}_{i_{n'},m'}(s)$ as

$$\tilde{R}_{i_{n',m'}}(s) \equiv \tilde{E}^{-1}_{i_{n',m'}}(s) \equiv \frac{\tilde{F}^{(E)}_{i_{n',m'}}(s)}{\tilde{F}^{(C)}_{i_{n',m'}}(s)}$$

$$= \frac{\sum_{m=1}^{3}\sum_{n=1}^{N_p} \tilde{T}_{n',m';n,m}(s)\, \tilde{F}^{(E)}_{s_{n,m}}(s)}{\sum_{m=1}^{3}\sum_{n=1}^{N_p} \tilde{T}_{n',m';n,m}(s)\, \tilde{F}^{(C)}_{s_{n,m}}(s)} \qquad (4.22)$$

where the ratio of criterion-to-extrapolated response is designate by $\tilde{E}_{i_{n',m'}}(s)$ and can be referred to as error (in a ratio sense). In another form $\ell n[\tilde{E}_{i_{n',m'}}(s)]$ can be referred to as the error so that the value of zero corresponds to no error. Note that extrapolation functions $\tilde{f}_e(s)$ which are analytic functions of s give analytic extrapolated responses and thereby analytic ratios and analytic errors. These errors are indicated as part C under the third type of extrapolation in figure 4.1.

Let us now implement an important approximation. Let us assume that the signal at some failure port with indices n',m' is attributable to one surface response with indices n,m. This is equivalent to saying that for each s of interest only one surface-to-failure-port transfer function $\tilde{T}_{n',m';n,m}(s)$ is important because of its large magnitude and/or the corresponding surface-response-function magnitude. Under this assumption (4.22) reduces to

$$\tilde{R}_{i_{n',m'}}(s) = \tilde{R}_{s_{n,m}}(s) \qquad (4.23)$$

$$\tilde{R}_{s_{n,m}}(s) \equiv \frac{\tilde{F}^{(E)}_{s_{n,m}}(s)}{\tilde{F}^{(C)}_{s_{n,m}}(s)}$$

For this result it is important that the surface-to-failure-port transfer function be the same for criterion as for simulation (and hence extrapolated) conditions. The importance of the approximation of one important penetration (n) with corresponding penetration mode (m) is that the failure-port extrapolated-to-criterion ratio is the same as the surface extrapolated-to-criterion ratio, albeit perhaps for an unknown penetration and mode (n,m).

It may be the case that for some range of frequencies the ith failure port is dominantly excited by one surface response labelled as say n_1,m_1, while for another range of frequencies of interest another surface response n_2,m_2 is dominant. In such a case the n,m indices in the first equation of (4.23) can be considered as a function of s. However, since (by hypothesis) the index set n,m for a given n',m' is unknown, and since we are going to select n,m randomly in the intended application, then the fact that the true n,m may be different for different frequencies will not affect our procedure.

Generalizing these results we have

$$\tilde{R}_{s_m}(\vec{r}_s,s) \equiv \frac{\tilde{F}^{(E)}_{s_m}(\vec{r}_s,s)}{\tilde{F}^{(C)}_{s_m}(\vec{r}_s,s)} \equiv \tilde{E}^{-1}_{s_m}(\vec{r}_s,s) \tag{4.24}$$

for continuous surface positions and for various penetration modes (m). Since one may not know where some important penetrations are located, a continuous form of the ratio is appropriate. For experimental purposes one might randomly select a set of position-mode pairs $(\vec{r}_{s_n},m)$ giving

$$\tilde{R}_{s_\ell}(s) \equiv \frac{\tilde{F}^{(E)}_{s_\ell}(s)}{\tilde{F}^{(C)}_{s_\ell}(s)} \equiv \tilde{E}^{-1}_{s_\ell}(s) \tag{4.25}$$

for use in a manner similar to (4.21). One can take $|\tilde{R}_{s_\ell}(j\omega)|$ and plot it on a graph as a function of ω or $f(=\omega/(2\pi))$. Magnitudes larger than 1 indicate overextrapolation and magnitudes less than 1 indicate underextrapolation. Cases of underextrapolation are of concern because they may indicate that the signal at a failure port is less than some value which would give failure, whereas correct (accurate) extrapolation might indicate failure.

If $\tilde{R}_{s_m}(\vec{r}_s,s)$ is near 1 in the complex plane (defining accurate extrapolation) for the range of $\vec{r}_s$ and m of interest we can appropriately define

$$\begin{aligned} \tilde{\Delta}_{s_m}(\vec{r}_s,s) &\equiv \tilde{R}_{s_m}(\vec{r}_s,s) - 1 \\ \tilde{\Delta}_{s_{n,m}}(s) &\equiv \tilde{R}_{s_{n,m}}(s) - 1 \\ \tilde{\Delta}_{s_\ell}(s) &\equiv \tilde{R}_{s_\ell}(s) - 1 \end{aligned} \tag{4.26}$$

where $\tilde{\Delta}_s$ near zero defines accurate extrapolation. This type of error measure can be used with high quality simulation and extrapolation. For cases that $\tilde{\Delta}_s$ has magnitude large compared to one (defining inaccurate extrapolation) then the ratio form seems more appropriate.

An alternate form of the ratios uses the surface transfer functions in (3.4). These are equivalent to and derivable from (4.23) through (4.25) (using the type 3A (incident field) extrapolation as in say (4.11))

$$\begin{aligned} \tilde{R}_{s_m}(\vec{r}_s,s) &= \tilde{f}_e^{(A)}(s)\, \frac{\tilde{F}_{s_m}^{(S)}(\vec{r}_s,s)}{\tilde{F}_{s_m}^{(C)}(\vec{r}_s,s)} \\ &= \tilde{f}_e^{(A)}(s)\, \frac{\tilde{F}_{inc}^{(S)}(s)}{\tilde{F}_{inc}^{(C)}(s)}\, \frac{\tilde{T}_{s_m}^{(S)}(\vec{r}_s,s)}{\tilde{T}_{s_m}^{(C)}(\vec{r}_s,s)} \\ &= \frac{\tilde{T}_{s_m}^{(S)}(\vec{r}_s,s)}{\tilde{T}_{s_m}^{(C)}(\vec{r}_s,s)} \end{aligned} \tag{4.27}$$

and similarly

$$\tilde{R}_{s_{n,m}}(s) = \frac{\tilde{T}^{(S)}_{s_{n,m}}(s)}{\tilde{T}^{(C)}_{s_{n,m}}(s)}$$

$$\tilde{R}_{s_{\ell}}(s) = \frac{\tilde{T}^{(S)}_{s_{\ell}}(s)}{\tilde{T}^{(C)}_{s_{\ell}}(s)} \qquad (4.28)$$

Having defined the deviations of the extrapolated response functions from the criterion response functions in terms of ratios of the surface response functions we can consider some of the properties of this type of ratio. Let us consider an average of the $\tilde{R}_s$ over the surface of the object. Recalling the introduction of a penetration density function in (4.15) and its subsequent use in constructing averages in defining $\tilde{f}_e^{(B)}(s)$ let us similarly average the various forms of $\tilde{R}_s$. Specifically define the weighted logarithmic average (consistent with the form in (4.17) for the extrapolation function) in continuous form as

$$\left\{\tilde{R}_{s_m}(\vec{r}_s,s)\right\}\Big|_{\substack{\text{avg}\\ \vec{r}_s,m}} \equiv \exp\left\{\sum_{m=1}^{3}\int_S \ell n\left[\tilde{R}_{s_m}(\vec{r}_s,s)\right]P_{s_m}(\vec{r}_s)dS\right\}$$

$$= \exp\left\{\sum_{m=1}^{3}\int_S \ell n\left[\frac{\tilde{F}^{(E)}_{s_m}(\vec{r}_s,s)}{\tilde{F}^{(C)}_{s_m}(\vec{r}_s,s)}\right]P_{s_m}(\vec{r}_s)dS\right\}$$

$$= \exp\left\{\sum_{m=1}^{3}\int_S \left[\ell n\left[\tilde{f}_e(s)\right] + \ell n\left[\frac{\tilde{F}^{(S)}_{s_m}(\vec{r}_s,s)}{\tilde{F}^{(C)}_{s_m}(\vec{r}_s,s)}\right]\right]P_{s_m}(\vec{r}_s)dS\right\}$$

$$= \tilde{f}_e(s)\exp\left\{\sum_{m=1}^{3}\int_S \ell n\left[\frac{\tilde{F}^{(S)}_{s_m}(\vec{r}_s,s)}{\tilde{F}^{(C)}_{s_m}(\vec{r}_s,s)}\right]P_{s_m}(\vec{r}_s)dS\right\}$$

$$= \tilde{f}_e(s)\left[\tilde{f}_e^{(B)}(s)\right]^{-1}$$

$$= 1 \quad \text{for } \tilde{f}_e(s) = \tilde{f}_e^{(B)}(s) \tag{4.29}$$

Stated another way the average of $\ell n[\tilde{R}_{S_m}(\vec{r}_s,s)]$ weighted by $P_{S_m}(\vec{r}_s)$ is 0, provided we take the weighted logarithmic average definition of $\tilde{f}_e(s)$ as in (4.17). The "errors" $\tilde{E}_{S_m}(\vec{r}_s,s)$ then have a weighted logarithmic average of 1. Note this does not mean no error but no average error. Other discrete forms of the ratios including $\tilde{R}_{S_{n,m}}(s)$ and $\tilde{R}_{S_\ell}(s)$ also have one for the weighted logarithmic average. Note that if $\tilde{f}_e(s)$ is chosen different from $\tilde{f}_e^{(B)}(s)$ as in (4.17), then the weighted logarithmic average of the ratios $\tilde{R}_s$ can be expressed using both $\tilde{f}_e(s)$ and $\tilde{f}_e^{(B)}(s)$ as above.

Considering the case of ratios near to unity or small $\tilde{\Delta}_s$ then (4.29) can be written in approximate form as

$$\left\{1 + \tilde{\Delta}_{S_m}(\vec{r}_s,s)\right\}\Bigg|_{\substack{\text{avg} \\ \vec{r}_s,m}} \equiv \exp\left\{\sum_{m=1}^{3}\int_S \ell n\left[1 + \tilde{\Delta}_{S_n}(\vec{r}_s,s)\right]P_{S_m}(\vec{r}_s)dS\right\}$$

$$\simeq \exp\left\{\sum_{m=1}^{3}\int_S \tilde{\Delta}_{S_m}(\vec{r}_s,s)P_{S_m}(\vec{r}_s)dS\right\} \tag{4.30}$$

Using the result of (4.29) for $\tilde{f}_e(s) \equiv \tilde{f}_e^{(B)}(s)$ we have

$$\sum_{m=1}^{3}\int_S \tilde{\Delta}_{S_m}(\vec{r}_s,s)P_{S_m}(\vec{r}_s)dS \simeq 0 \tag{4.31}$$

which is accurate to order of $\tilde{\Delta}_s^2$. This result indicates that for ratios $\tilde{R}_s$ near unity the weighted logarithmic average of $\tilde{R}_s$ is approximately equivalent to the weighted average of $\tilde{\Delta}_s$ if the extrapolation function is chosen as $\tilde{f}_e^{(B)}(s)$ as in (4.17).

Having defined the extrapolation function and resulting ratios (extrapolated divided by criteria), these ratios $\tilde{R}_s$ (or correspondingly "errors" $\tilde{E}_s$) can be considered for various statistical properties including variance, extreme values, worst case, etc. In fact the closeness of the set of all the $\tilde{R}_s$ (over some sample of $\vec{r}_s$ and m) to unity can be taken as a definition of simulation quality from the points of view of the spatial variation of the incident field and the simulator/object interaction. In the limit that the ratios tend to unity then the type 3 extrapolation with errors (3C) goes to type 2 extrapolation which only requires frequency spectrum correction of the incident field or object response with no errors (in principle).

D. Type 4: Penetration Extrapolation

In this type of extrapolation we remove the requirement of having a single scalar extrapolation function and admit a potential plethora of such functions which we might designate as $\tilde{f}_{e_{n,m}}(s)$, one for each combination of penetration (n) and penetration mode (m). This would be defined by

$$\tilde{f}_{e_{n,m}}(s) \equiv \frac{\tilde{F}^{(C)}_{s_{n,m}}(s)}{\tilde{F}^{(S)}_{s_{n,m}}(s)} \tag{4.32}$$

which gives the extrapolated results for the surface response as

$$\tilde{F}^{(E)}_{s_{n,m}}(s) \equiv \tilde{f}_{e_{n,m}}(s)\tilde{F}^{(S)}_{s_{n,m}}(s) = \tilde{F}^{(C)}_{s_{n,m}}(s) \tag{4.33}$$

which might be interpreted as "perfect" extrapolation.

Noting that the extrapolation function(s) is (are) intended to be applied to failure port signals inside the test object, then we have from (3.7)

$$\tilde{F}^{(E)}_{i_{n',m'}}(s) = \sum_{m=1}^{3}\sum_{n=1}^{N_p} \tilde{T}_{n',m';n,m}(s)\tilde{F}^{(E)}_{s_{n,m}}(s)$$

$$= \sum_{m=1}^{3}\sum_{n=1}^{N_p} \tilde{T}_{n',m';n,m}(s)\tilde{f}_{e_{n,m}}(s)\tilde{F}^{(S)}_{s_{n,m}}(s)$$

$$= \sum_{m=1}^{3}\sum_{n=1}^{N_p} \tilde{T}_{n',m';n,m}(s)\tilde{F}^{(C)}_{s_{n,m}}(s) = \tilde{F}^{(C)}_{i_{n',m'}}(s) \qquad (4.34)$$

Note now that the $\tilde{f}_{e_{n,m}}(s)$ does not in general factor out of the summation. Thus the accuracy of this type of extrapolation brings with it a significant increase in complexity. Stated another way the ratios of extrapolated to criteria responses at the failure ports as in (4.22) become

$$\tilde{R}_{i_{n',m'}}(s) = 1 \qquad (4.35)$$

In implementing this type of extrapolation one must then generate a set of extrapolation functions corresponding to each relevant penetration exciting a particular failure port (n',m'). Starting with a particular failure port then one must determine which penetrations and associated penetration modes are exciting the failure port. This might be accomplished by a variety of procedures including:

1. tracing observed signals from the failure port to the penetrations and then identifying and quantifying the coupling modes at the penetrations
2. exciting all the penetrations one at a time with special devices to produce the separate penetration modes (short circuit current and charge densities and thereby measuring for a given n',m' all the $\tilde{T}_{n',m';n,m}(s)$)

3. driving the failure port of interest with a source (voltage or current), and measuring what penetrations (and modes) the signals come out from (essentially applying reciprocity to 2).

Thus one can see that while this fourth type of extrapolation (penetration extrapolation) in principle gives more accurate results, this result is gained at the expense of increased complexity in the test because the failure port signals must be traced to specific penetrations. However, as indicated in figure 4.1 it does get around the incident-field and simulator/object interaction errors in the test.

V. Limitations of Extrapolation

Extrapolation, to the degree that it is developed in this note, does not solve or quantify all the problems in EMP simulation testing. The reader should note some of the remaining problems. Listing some of these limitations we have for all but identity extrapolation (type 1 or no extrapolation) some significant problems.

A. Linearity of the system response is assumed, an assumption which is not always justified.

B. There is difficulty in identifying relevant failure ports for response measurements there.

C. One cannot necessarily measure the important EMP signal components at failure ports because of:

1. system or background noise
2. EMP signal components that are emphasized by the test at the expense of other relevant EMP signal components
3. instrumentation deficiencies

D. In comparing simulation results to criterion conditions the criterion quantities will in general have some error. These include in particular the criterion surface response functions since these are removed from the actual statement of the criterion and only connected to it via the system under test. Errors will come in via the technique used to determine the criterion surface response including:

1. measurement of the system under more criterion-like conditions
2. calculation of the system's surface response
3. measurement of the surface response on an imperfect scale model under imperfect excitation conditions

E. The surface response functions measured on the system in the simulation test will in general have measurement errors.

F. In determining ratios in type 3 extrapolation for an extrapolation function and for error quantification, only a finite number of such ratios can be determined, thereby introducing errors.

G. There are various ways that a penetration density function can be defined for use in type 3 extrapolation. Various phyiscal considerations and probability theory need to be brought to bear to further develop this matter.

H. Changes of the system configuration in the test away from criterion need to be accounted for in some way. In part one may get around this if the outer surface of the system can be redefined by excluding some elementary volume (with an approximately perfectly conducting boundary) from our consideration and refering the surface transfer functions to the new system "outer" boundary. However determination of the appropriate criterion surface response functions may be significantly more difficult due to the presence of various conductors penetrating the new "outer" boundary.

VI. Summary

This note has explored some possibilities for the extrapolation of simulation test data on test objects (systems) to criterion conditions. Various types of extrapolation are defined based on the kinds of simulation deficiencies and the degree of detail one is willing to perform in making the extrapolation.

An important concept motivates the development of error quantification and reduction algorithms (or methodologies) such as those discussed here. It is the need for statements concerning system vulnerability (or hardness) to be based on experimental data. Such data should be chosen in a way that minimizes personal and organizational biases about what the simulation test results "ought to say." A more scientific approach is to define an experiment and let the experimental results give the verdict with a minimum of interpretation after the fact. The present results should help in this quest, but should only be considered as a shot (or at best a barrage) in the war against fuzzy thinking about system test results, and even (shudder) assessments.

These extrapolation techniques are potentially generalizable to include other test considerations. Such other considerations include perturbations of the system configuration, measurement errors, statistical sampling of failure ports, statistical treatment of penetration density functions, signal-to-noise ratio, determination of criterion surface response functions, nonlinearities, combinations of results from more than one simulation test, etc. The reader should note, however, that the subject of extrapolation (and simulation) is only a part (albeit an important part) of the more general subject of assessment.

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