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Sensor and Simulation Notes

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A Simple Technique for Obtaining the Near Fields of Electric Dipole Antennas from Their Far Fields

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Abstract

In this report a technique of obtaining the near fields of axially and lengthwise symmetric electric dipole antennas from their far fields is discussed. In particular it is shown that if the θ component of the far electric field is known either in time or frequency domains, all other electric and magnetic field components including their near fields can be obtained very easily. An example involving the fields of a resistively loaded dipole antenna driven by a step function generator is discussed in detail.

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I. Introduction

In the calculations of fields due to simple current distributions such as involving electric or magnetic dipoles, the far electric or magnetic field have a very simple behavior. In general these fields behave like the radiated fields and are proportional to $1/r$, r being the distance from some reference point on the antenna to the observation point. Because of the ease with which the far fields can be obtained and also because of their applications in most linear antenna problems, far field representation in general is sufficient. However, in cases such as large vertically polarized dipole simulators used in the testing of aircraft, etc., the observation point in general cannot be considered to be in the far field of the antenna (simulator). At most frequencies of interest, and within useful distances from the simulator, near field components contribute to varying degrees with their contribution at the low frequencies being predominant. As such, a simple way of calculating the near fields of such structures was desirable. This note deals with one such simple technique. An example involving an impedance loaded conical dipole over a ground plane is discussed in detail.

II. Electric and Magnetic Dipole Moments and Their Far Fields

Consider a volume V' bounded by a closed surface S'. The coordinate system is as described in figure 2.1. Let us assume that the volume V' contains electric current density $\tilde{\vec{J}}(\vec{r}\,')$ and charge density $\tilde{\rho}(\vec{r}\,')$. Here a tilde (~) over the quantity represents the bilateral Laplace transformed quantity. Assuming that the charge and current densities are zero before time $t=t_o$ and requiring no current density to pass through S', the electric dipole moment can be defined as[1,2]

$$\vec{p}(t) \equiv \int_{V'} \vec{r}\,' \rho(\vec{r}\,', t)\, dV' \tag{2.1}$$

or in terms of Laplace transformed quantities as

$$\tilde{\vec{p}} = \int_{V'} \vec{r}\,' \tilde{\rho}(\vec{r}\,')\, dV' \tag{2.2}$$

Notice that $\vec{p}(t)$ is a time-dependent dipole moment while $\tilde{\vec{p}}$ is frequency dependent. Equivalently, in terms of the current density

$$\tilde{\vec{p}} = \frac{1}{s} \int_{V'} \tilde{\vec{J}}(\vec{r}\,')\, dV' \tag{2.3}$$

s being complex radian frequency. Similarly, the magnetic dipole moment $\vec{m}$ can be written as[2]

$$\vec{m}(t) = -\frac{1}{4} \int_{V'} (\vec{r}\,' \cdot \vec{r}\,') \nabla' \times \vec{J}(\vec{r}\,', t)\, dV' \tag{2.4}$$

in the time domain and

$$\tilde{\vec{m}} = -\frac{1}{4} \int_{V'} (\vec{r}\,' \cdot \vec{r}\,') \nabla' \times \tilde{\vec{J}}(\vec{r}\,')\, dV' \tag{2.5}$$

in the Laplace domain. In the low frequency limit these dipole moments can be written as

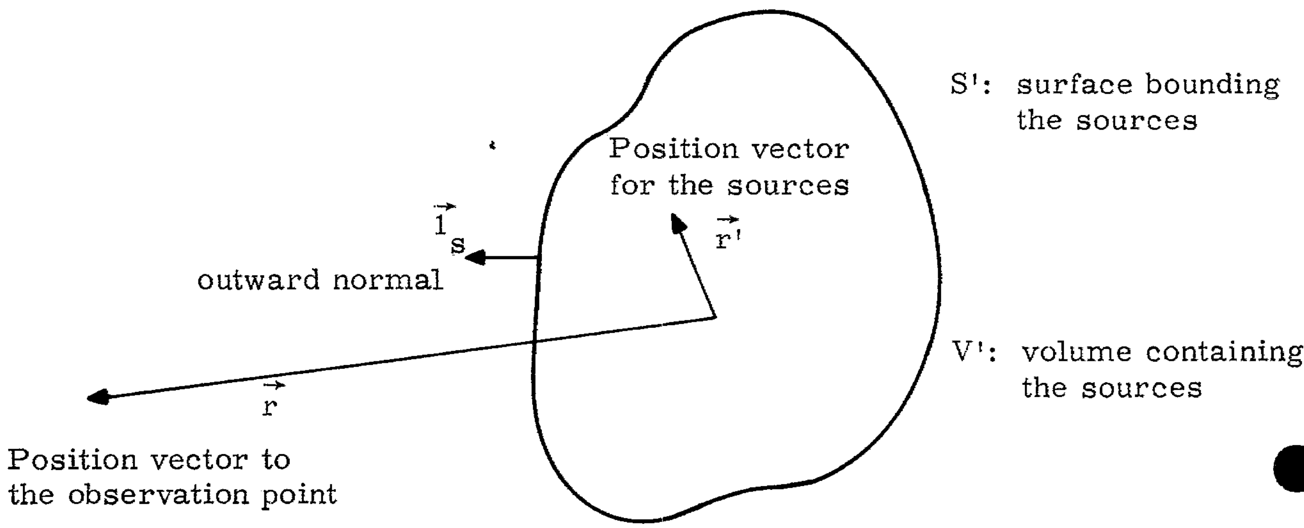

Figure 2.1. Coordinate system describing the source and radiation region

$$\tilde{\vec{p}} \equiv \int_{V'} \vec{r}' \tilde{\rho}(\vec{r}')\, dV' = \frac{1}{s} \int_{V'} \tilde{\vec{J}}(\vec{r}')\, dV' \tag{2.6}$$

where $\tilde{\rho}(\vec{r}')$ and $\tilde{\vec{J}}(\vec{r}')$ are charge and current densities respectively. The magnetic dipole moment can be written as

$$\tilde{\vec{m}} \equiv \frac{1}{2} \int \vec{r}' \times \tilde{\vec{J}}(\vec{r}')\, dV' \tag{2.7}$$

Low frequency far fields due to electric and magnetic dipoles[1] can then be written as

$$\tilde{\vec{E}}_f(\vec{r}) = \frac{e^{-\gamma r}}{4\pi r} \left\{ \mu_o s^2 \vec{1}_r \times \left[\vec{1}_r \times \tilde{\vec{p}} \right] + \frac{\mu_o}{c} s^2 \vec{1}_r \times \tilde{\vec{m}} \right\} \tag{2.9}$$

$$\tilde{\vec{H}}_f(\vec{r}) = \frac{e^{-\gamma r}}{4\pi r} \left\{ -\frac{1}{c} s^2 \vec{1}_r \times \tilde{\vec{p}} + \frac{1}{c^2} s^2 \vec{1}_r \times \left[\vec{1}_r \times \tilde{\vec{m}} \right] \right\} \tag{2.10}$$

where $\vec{1}_r$ is a unit vector in the radial direction and

μ_o = free space permeability

ϵ_o = free space permittivity

$c = 1/\sqrt{\mu_o \epsilon_o}$ = speed of light in free space

$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}} \approx 377\Omega$ = characteristic impedance of free space

$\gamma = \frac{s}{c}$ = complex propagation constant

It is clear that if the electric or magnetic fields due to an electric or magnetic dipole moment are known, other components can be simply calculated using the complementary and far field relations of the electric and magnetic quantities.

If we consider an electric dipole source term alone, electric and magnetic fields due to an electric dipole, with $1/r^2$ and $1/r^3$ terms included can be written as

$$\tilde{\vec{E}}_p(\vec{r}) = e^{-\gamma r}\left\{\frac{1}{4\pi r^2}\frac{1}{\epsilon_o}\left[\frac{1}{r}\left[3\vec{1}_r\left[\vec{1}_r\cdot\tilde{\vec{p}}\right]-\tilde{\vec{p}}\right]\right]+\frac{Z_o s}{4\pi r}\left[\frac{1}{r}\left[3\vec{1}_r\left[\vec{1}_r\cdot\tilde{\vec{p}}\right]-\tilde{\vec{p}}\right]\right]\right.$$

$$\left.+\frac{s^2\mu_o}{4\pi}\left[\frac{1}{r}\left[\vec{1}_r\left[\vec{1}_r\cdot\tilde{\vec{p}}\right]-\tilde{\vec{p}}\right]\right]\right\} \tag{2.11}$$

and

$$\tilde{\vec{H}}_p(\vec{r}) = e^{-\gamma r}\left\{-\frac{1}{4\pi r}\left[\frac{s}{r}\left[\vec{1}_r\times\tilde{\vec{p}}\right]\right]-\frac{1}{4\pi r}\frac{s^2}{c}\left[\vec{1}_r\times\tilde{\vec{p}}\right]\right\} \tag{2.12}$$

From the above equations, a general relationship exists which relates the $1/r$ term to the $1/r^2$ and $1/r^3$ terms. This relationship can be thought of as a frequency scaling along with a simple proportionality constant. It is clear that in the case of the electric field, the $1/r^3$ type field is more important at low frequencies. If $\vec{1}_r$ is perpendicular to $\tilde{\vec{p}}$ certain simplifications result in (2.11).

If we consider a magnetic dipole instead of an electric dipole, its fields can be obtained by setting

$$\vec{p} \leftrightarrow \frac{\vec{m}}{c} \tag{2.13}$$

$$\vec{E}_p(\vec{r}) \leftrightarrow Z_o\vec{H}_m(\vec{r}) \tag{2.14}$$

$$\vec{H}_p(\vec{r}) \leftrightarrow -\frac{1}{Z_o}\vec{E}_m(\vec{r}) \tag{2.15}$$

in (2.11) and (2.12).

A general source distribution may contain electric and magnetic dipoles along with their higher order counterparts. Relationships similar to (2.11) and (2.12) may be found for multipole terms. To include these multipole terms, one may expand the Green's function in series involving spherical Bessel and Hankel functions and spherical harmonics. The order of these functions is related to the order of the multipole under consideration. This procedure is quite involved and is considered outside the scope of this report.

III. Expansion of the Fields in Spherical and Cylindrical Coordinates

In practical applications it is convenient to express the electric and magnetic fields in terms of their vector components in spherical or cylindrical components. Let us consider the special case of the electric current density symmetrically situated as shown in figure 3.1 such that only a z directed dipole moment is present. In terms of spherical coordinates the electric dipole moment can be written as

$$\vec{\tilde{p}} = \tilde{p}\vec{1}_z = \tilde{p}\left(\vec{1}_r \cos(\theta) - \vec{1}_\theta \sin(\theta)\right) \tag{3.1}$$

Using this representation, the electric field given by (2.11) can be written as

$$\vec{\tilde{E}}_p(\vec{r}) = e^{-\gamma r}\left\{\frac{1}{4\pi r^3}\frac{1}{\epsilon_o}\left[2\vec{1}_r\left(\cos(\theta)\tilde{p}\right) + \vec{1}_\theta\left(\sin(\theta)\tilde{p}\right)\right] + \frac{Z_o s}{4\pi r^2}\left[2\vec{1}_r\left(\cos(\theta)\tilde{p}\right) + \vec{1}_\theta\left(\sin(\theta)\tilde{p}\right)\right] + \frac{s^2\mu_o}{4\pi r}\left[\vec{1}_\theta\left(\sin(\theta)\tilde{p}\right)\right]\right\} \tag{3.2}$$

Note that the radial electric field does not have a $1/r$ component. Suppose that the θ component of the far electric field is known, let us represent this by $\tilde{E}_{f_\theta}$ as

$$\tilde{E}_{f_\theta} \equiv \frac{s^2\mu_o}{4\pi r}\left(\sin(\theta)\tilde{p}\, e^{-\gamma r}\right) \tag{3.3}$$

then the $1/r^2$ component $\tilde{E}_{2_\theta}$ can be written as

$$\tilde{E}_{2_\theta} = \frac{Z_o s}{4\pi r^2}\left(\sin(\theta)\tilde{p}\, e^{-\gamma r}\right) = \frac{c}{sr}\tilde{E}_{f_\theta} = \frac{1}{\gamma r}\tilde{E}_{f_\theta} \tag{3.4}$$

while $1/r^3$ component can be written as

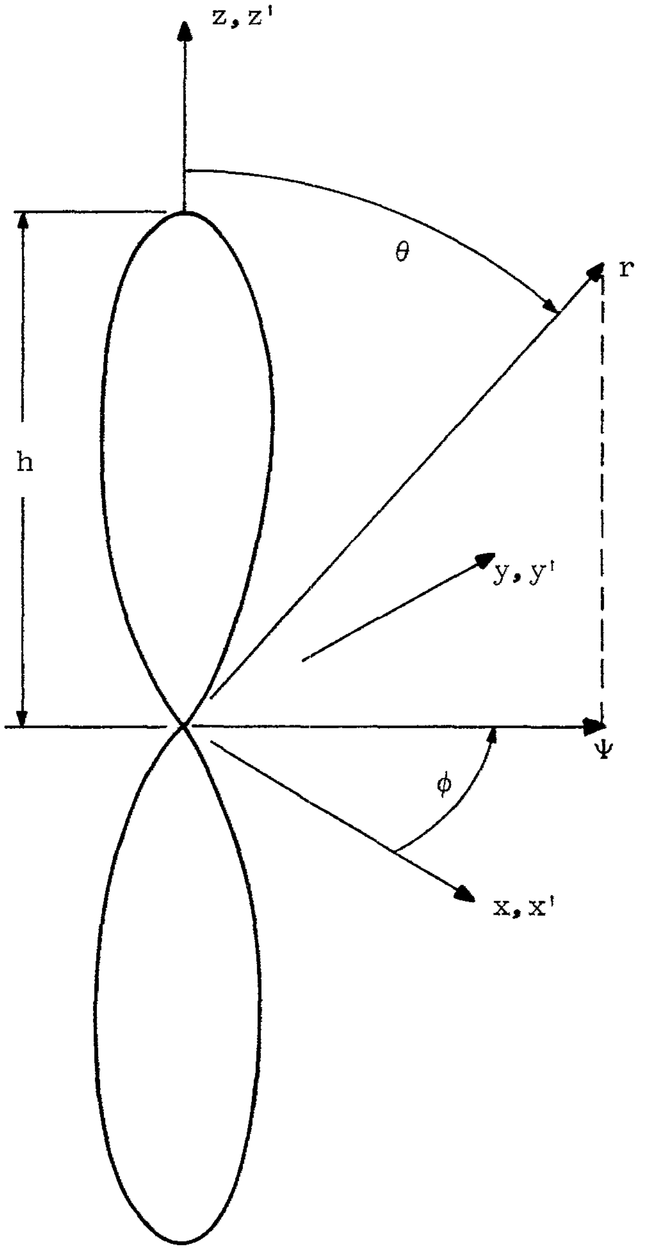

Figure 3.1. Axially and lengthwise symmetric dipole antenna

$$\tilde{E}_{3_\theta} = \frac{1}{4\pi r^3}\frac{1}{\epsilon_o}\left(\sin(\theta)\tilde{p}\,e^{-\gamma r}\right) = \frac{1}{\gamma r}\tilde{E}_{2_\theta} = \frac{1}{\gamma^2 r^2}\tilde{E}_{f_\theta} \qquad (3.5)$$

Representing the total θ component of the electric field by $\tilde{E}_\theta$, it can be written as

$$\tilde{E}_\theta = \left(1 + \frac{1}{\gamma r} + \frac{1}{\gamma^2 r^2}\right)\tilde{E}_{f_\theta} \qquad (3.6)$$

Hence in the dipole approximation of an antenna, if the θ component of the far electric field is known, the total θ component of the electric field near the antenna can be calculated with the help of (3.6).

In a similar fashion, the $1/r^2$ part of the radial electric field can be written as

$$\tilde{E}_{2_r} = \frac{Z_o s}{2\pi r^2}\left(\cos(\theta)\tilde{p}\,e^{-\gamma r}\right) = 2\cot(\theta)\tilde{E}_{2_\theta} = \frac{2\cot(\theta)}{\gamma r}\tilde{E}_{f_\theta} \qquad (3.7)$$

while the $1/r^3$ component is given by

$$\tilde{E}_{3_r} = \frac{1}{2\pi r^3}\frac{1}{\epsilon_o}\left(\cos(\theta)\tilde{p}\,e^{-\gamma r}\right) = 2\cot(\theta)\tilde{E}_{3_\theta} = \frac{2\cot(\theta)}{\gamma^2 r^2}\tilde{E}_{f_\theta} \qquad (3.8)$$

and the $1/r$ component is zero. Hence the total radial electric component of the electric field $\tilde{E}_r$ can be written as

$$\tilde{E}_r = 2\cot(\theta)\left(\frac{1}{\gamma r} + \frac{1}{\gamma^2 r^2}\right)\tilde{E}_{f_\theta} \qquad (3.9)$$

The only non-zero component of the magnetic field $\tilde{H}_\phi$ is given by

$$\tilde{H}_{f_\phi} = \frac{s^2}{4\pi r}\frac{1}{c}\left(\sin(\theta)\tilde{p}\,e^{-\gamma r}\right) = \frac{1}{Z_o}\tilde{E}_{f_\theta} \qquad (3.10)$$

$$\tilde{H}_{2_\phi} = \frac{s}{4\pi r^2}(\sin(\theta)\tilde{p}\, e^{-\gamma r}) = \frac{1}{Z_o}\tilde{E}_{2_\theta} = \frac{1}{Z_o}\frac{1}{\gamma r}\tilde{E}_{f_\theta} \tag{3.11}$$

and

$$\tilde{H}_\phi = \frac{1}{Z_o}(1 + \frac{1}{\gamma r})\tilde{E}_{f_\theta} \tag{3.12}$$

Note that the radial electric component falls off at a faster rate than the θ component. Note also that the θ component of the far electric field is sufficient to reconstruct the total radial and θ components of the electric field and the ϕ component of the magnetic field.

Let us now consider the case when the θ component of the far electric field is known in the time domain. Using basic principles of the Laplace transform we can write

$$E_\theta(\vec{r}, t) = \left[E_{f_\theta}(\vec{r}, t) + \frac{c}{r}\int_{-\infty}^{t} d\tau \left[E_{f_\theta}(\vec{r}, \tau)\right] + \frac{c^2}{r^2}\int_{-\infty}^{t} d\tau \int_{-\infty}^{\tau} dt \left[E_{f_\theta}(\vec{r}, t)\right]\right] \tag{3.13}$$

$$E_r(\vec{r}, t) = 2\cot(\theta)\left[\frac{c}{r}\int_{-\infty}^{t} dt \left[E_{f_\theta}(\vec{r}, t)\right] + \frac{c^2}{r^2}\int_{-\infty}^{t} d\tau \int_{-\infty}^{\tau} dt \left[E_{f_\theta}(\vec{r}, t)\right]\right] \tag{3.14}$$

and

$$H_\phi(\vec{r}, t) = \frac{1}{Z_o}\left[E_{f_\theta} + \frac{c}{r}\int_{-\infty}^{t} d\tau \left[E_{f_\theta}(\vec{r}, \tau)\right]\right] \tag{3.15}$$

As should be expected, if the θ component of the far electric field is known in time domain, a simple process of integration yields the total near fields.

If E_r and E_θ are known, z, ρ and ϕ components of the electric and magnetic fields in cylindrical coordinates can be written as

$$E_z(\vec{r}) = E_r(\vec{r})\cos(\theta) - E_\theta(\vec{r})\sin(\theta) \tag{3.16}$$

$$E_\rho(\vec{r}) = E_r(\vec{r})\sin(\theta) + E_\theta(\vec{r})\cos(\theta) \tag{3.17}$$

$$H_\phi(\vec{r}) = H_\phi(\vec{r}) \qquad (3.18)$$

In the case of dipole type antennas it is generally expedient to calculate the far fields. Using the procedure described above, the near fields can be very easily obtained. If the far fields are measured, it suffices to make one good measurement namely $E_{f_\theta}(\vec{r})$ either in time or frequency domain. Using this measurement, the near field calculation becomes a trivial exercise. In this connection it should be pointed out that this procedure is accurate only if $r \gg h$, i.e., the observation point is far compared to the antenna height. For distances of the order of h, one may have to include higher order multipole terms in the expansions for the field. In the high frequency regime, the reconstruction of the field is not accurate, however, the errors are small.

IV. Calculation of the Near Fields of an Impedance Loaded Dipole Antenna from its Far Field

Consider a typical axially and lengthwise symmetric dipole antenna as shown in figure 3.1. The coordinate system is as shown. The impedance loading on the antenna is assumed to be $\Lambda(z')$ per unit length.[3-7]

In our present analysis of the dipole antenna we also use the transmission line model for calculating the current distribution on the antenna. We assume that the generator feeding the antenna has a generator capacitance C_g and a voltage source $V_o U(t)$ in the time domain or V_o/s in the frequency domain. Here U(t) is the unit step while s is the complex radian frequency. In the transmission line model, the antenna in figure 3.1 can be approximated as shown in figures 4.1 and 4.2. Elements of the incremental section of the equivalent transmission line are related to the characteristic impedance of the bicone. If the bicone has cones at $\theta = \theta_1$ and $\theta = \pi - \theta_1$, the characteristic impedance Z_∞ is given by

$$Z_\infty = \frac{Z_o}{\pi} \ln\left[\cot(\theta_1/2)\right] \tag{4.1}$$

The geometric factor f_g is defined as

$$f_g \equiv \frac{Z_\infty}{Z_o} = \frac{1}{\pi} \ln\left[\cot(\theta_1/2)\right] \tag{4.2}$$

for a biconical antenna. If the biconical antenna is of half height h and of radius $a \ll h$, then the antenna can be considered to be thin and the geometric factor f_g can be approximated as

$$f_g \simeq \frac{1}{\pi} \ln\left(\frac{2h}{a}\right) \tag{4.3}$$

where the mean radius of the antenna can be used. Assuming that the biconical antenna is in free space, parameters of the incremental section of the equivalent transmission line are given by

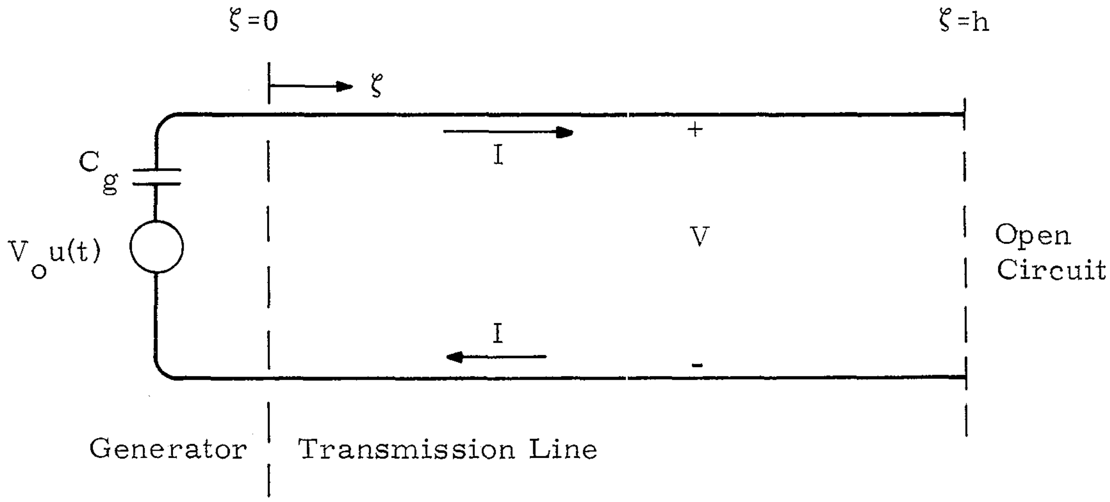

Figure 4.1. Transmission line with generator

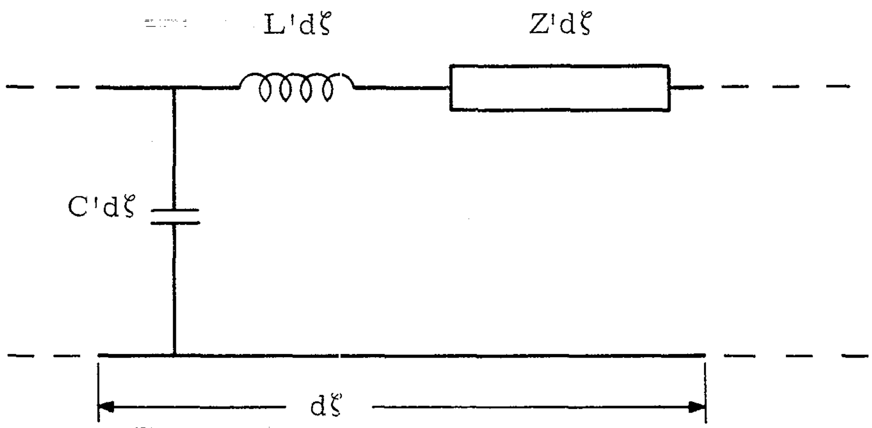

Figure 4.2. Incremental section of the transmission line

$$L' = \mu_o f_g \tag{4.4}$$

$$C' = \epsilon_o / f_g \tag{4.5}$$

and

$$Z'(\zeta) = 2\Lambda(\zeta) \tag{4.6}$$

We define normalized retarded time τ_h as

$$\tau_h = \frac{ct - r}{h} \tag{4.7}$$

where

h = half height of the antenna

r = distance to the observer from the center of the antenna

t = time measured in seconds

t_h = h/c

We also define

$s_h = \frac{sh}{c}$ = normalized radian frequency

$\alpha \equiv 1 + \frac{C_a}{C_g}$

C_a = antenna capacitance

C_g = generator capacitance

Using transmission line equations the current on the transmission line can be calculated, this current distribution can be used as the current on the antenna. For a special impedance loading of the form

$$\Lambda(\zeta) = \frac{Z_\infty}{h}\left[1 - \frac{|z'|}{h}\right]^{-1} \tag{4.8}$$

Baum[5] has calculated the θ component of the far electric field in frequency domain to be

$$\tilde{E}_{f_\theta}(\vec{r}) = \left[\frac{V_o}{2\pi f_g}\right]\frac{e^{-\gamma_o r}}{r}\,\tilde{\xi}_1'(\theta) \qquad (4.9)$$

where

$$\tilde{\xi}_1'(\theta) = \frac{\sin\theta}{(s_h+\alpha)}\left[\frac{1}{\sin^2(\theta)} - \frac{2}{s_h\sin^4(\theta)} + 2\left\{\frac{\cos^2\left(\frac{\theta}{2}\right)e^{-s_h(1-\cos(\theta))} + \sin^2\left(\frac{\theta}{2}\right)e^{-s_h(1+\cos(\theta))}}{s_h\sin^4(\theta)}\right\}\right] \qquad (4.10)$$

in the frequency domain.

It has been found very difficult to obtain the near fields of this antenna analytically. However, if we use the technique discussed in section III, the near fields can be obtained very easily. Considering frequency domain first, we can write the θ component of the electric field as

$$\begin{aligned}\tilde{E}_{2_\theta} &= \left[\frac{V_o t_h}{2\pi f_g}\right]\frac{h}{r}\frac{e^{-\gamma_o r}}{r}\left[\frac{1}{s_h}\tilde{\xi}_1'(\theta)\right] \\ &= \left[\frac{V_o t_h}{2\pi f_g}\right]\frac{h}{r}\frac{e^{-\gamma_o r}}{r}\left[\tilde{\xi}_2'(\theta)\right]\end{aligned} \qquad (4.11)$$

while

$$\begin{aligned}\tilde{E}_{3_\theta} &= \left[\frac{V_o t_h}{2\pi f_g}\right]\frac{h^2}{r^2}\frac{e^{-\gamma_o r}}{r}\left[\frac{1}{s_h^2}\tilde{\xi}_1'(\theta)\right] \\ &= \left[\frac{V_o t_h}{2\pi f_g}\right]\frac{h^2}{r^2}\frac{e^{-\gamma_o r}}{r}\left[\tilde{\xi}_3'(\theta)\right]\end{aligned} \qquad (4.12)$$

We note that the normalization factors $\tilde{\xi}_1'(\theta)$, $\tilde{\xi}_2'(\theta)$ and $\tilde{\xi}_3'(\theta)$ are all functions of normalized frequency, θ and α only. These are plotted for several values of α by varying θ and setting $s_h = j\omega_h$. Notice that at low frequencies $\tilde{\xi}_3'(\theta)$ is more predominant while at high frequencies $\tilde{\xi}_1'(\theta)$ is more important. $\tilde{\xi}_2'(\theta)$ is insignificant at most frequencies compared to the contributions of $\tilde{\xi}_1'(\theta)$ and $\tilde{\xi}_3'(\theta)$. We can write the total θ component of the electric field as

$$\tilde{E}_\theta = \left[\frac{V_o t_h}{2\pi f_g}\right] \frac{e^{-\gamma_o r}}{r} \left[\tilde{\xi}_1'(\theta) + \frac{h}{r}\tilde{\xi}_2'(\theta) + \frac{h^2}{r^2}\tilde{\xi}_3'(\theta)\right] \tag{4.13}$$

From figures 4.3 through 4.11, if α, h, r and θ are known the total θ component of the electric field can be easily visualized. If $r \gg \frac{c}{\omega}$, the contributions of $\tilde{\xi}_2'(\theta)$ and $\tilde{\xi}_3'(\theta)$ can be neglected to varying degrees.

In a similar fashion the total radial component of the electric field can be written as

$$\tilde{E}_r = 2\cot(\theta)\left[\frac{V_o t_h}{2\pi f_g}\right] \frac{e^{-\gamma_o r}}{r} \left[\frac{h}{r}\tilde{\xi}_2'(\theta) + \frac{h^2}{r^2}\tilde{\xi}_3'(\theta)\right] \tag{4.14}$$

while $\tilde{H}_\phi$ can be written as

$$\tilde{H}_\phi = \frac{1}{Z_o}\left[\frac{V_o t_h}{2\pi f_g}\right] \frac{e^{-\gamma_o r}}{r} \left[\tilde{\xi}_1'(\theta) + \frac{h}{r}\tilde{\xi}_2'(\theta)\right] \tag{4.15}$$

These can also be visualized from figures 4.3 through 4.11 given α, h, r and θ. In using $|\tilde{\xi}_1'|$, $|\tilde{\xi}_2'|$ and $|\tilde{\xi}_3'|$ graphs shown earlier care should be exercised because they should be added with the proper phase factor.

In time domain the θ component of the far electric field can be written as[5]

$$E_{f_\theta}(\vec{r}, t) = \left[\frac{V_o}{2\pi f_g}\right] \frac{1}{r}\xi_1'(\theta) \tag{4.16}$$

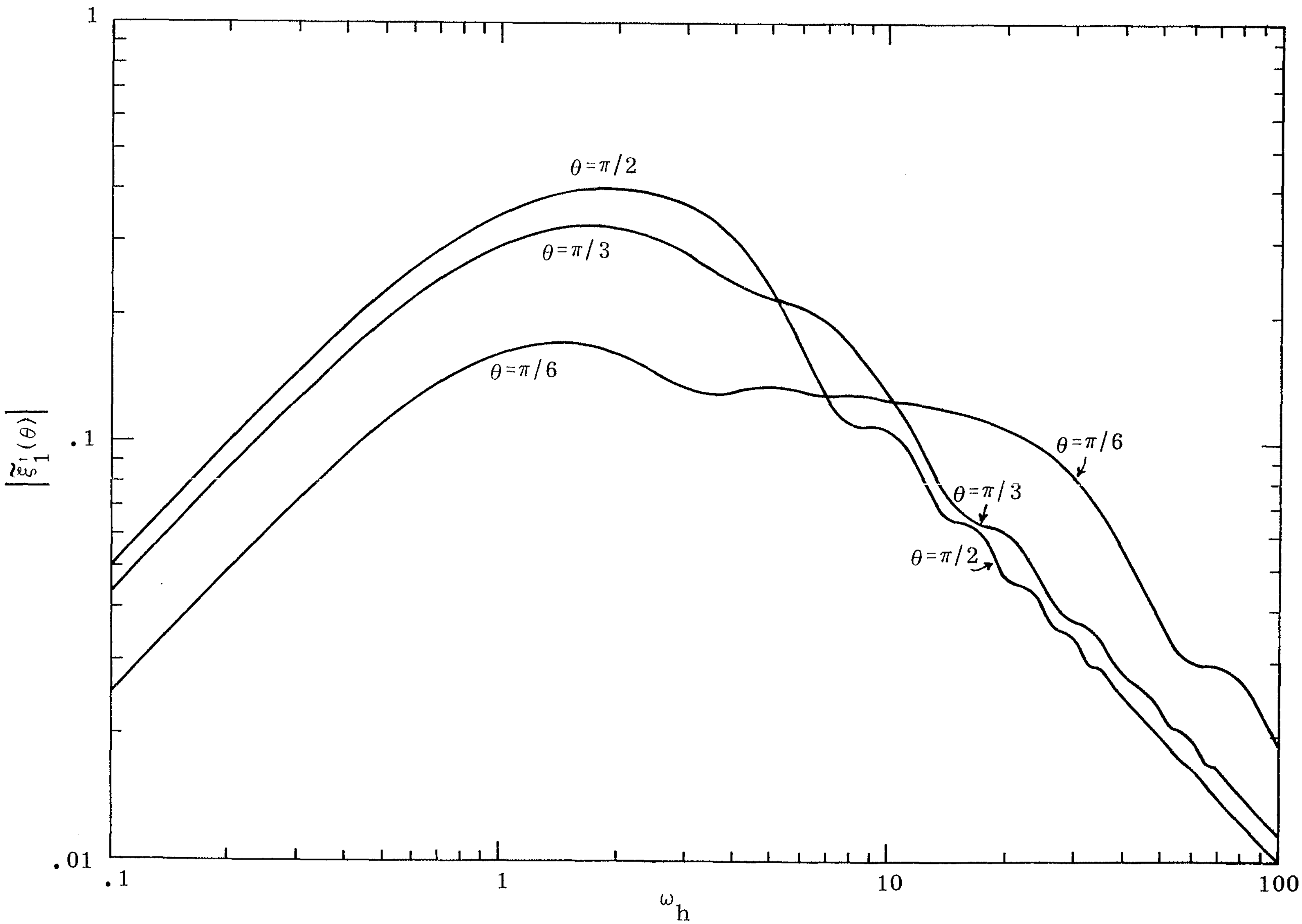

Figure 4.3. Effect of θ on $\tilde{\xi}_1'(\theta)$, $\alpha = 1$

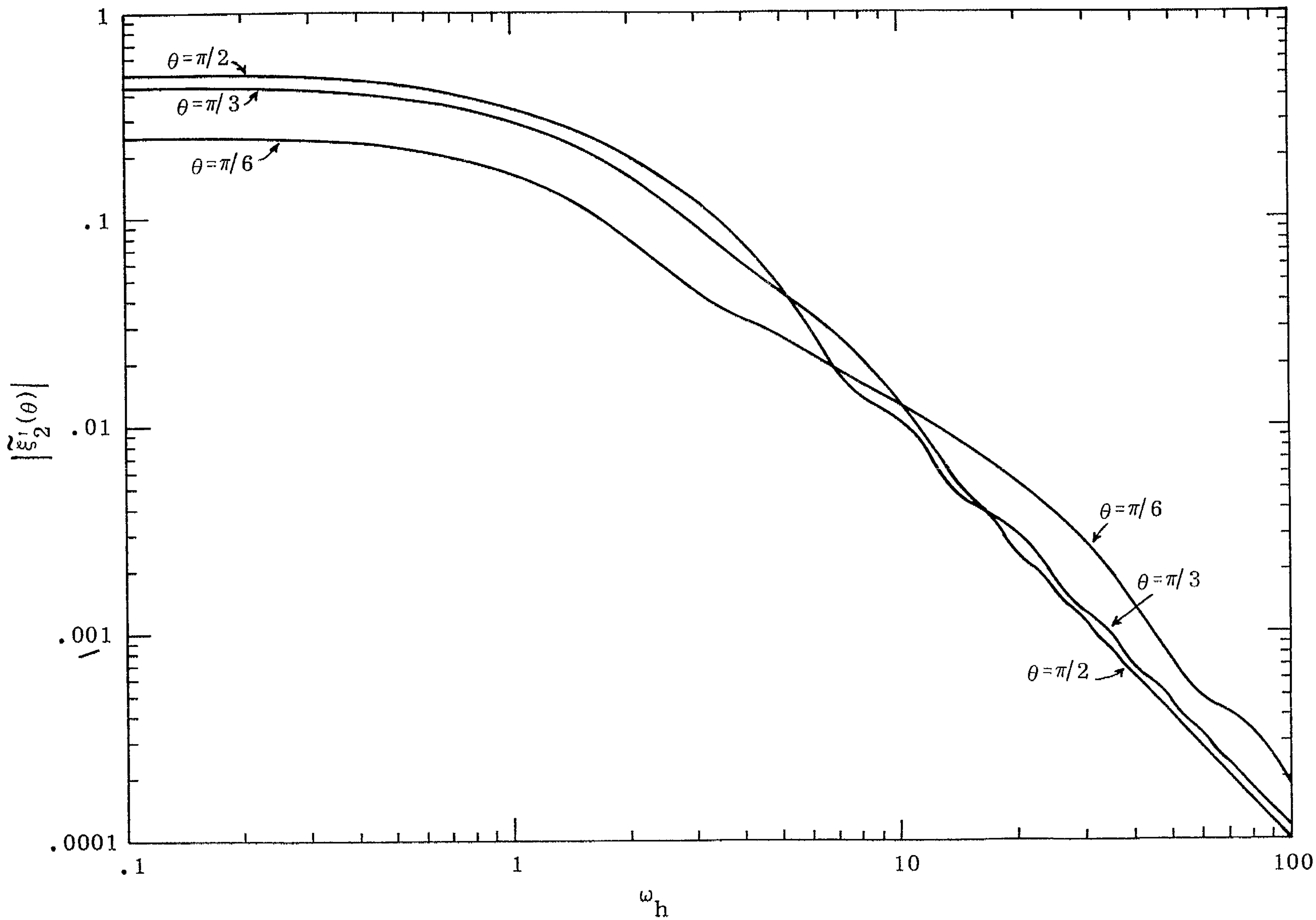

Figure 4.4. Effect of θ [illegible] $\tilde{\xi}'_2(\theta)$, $\alpha = 1$

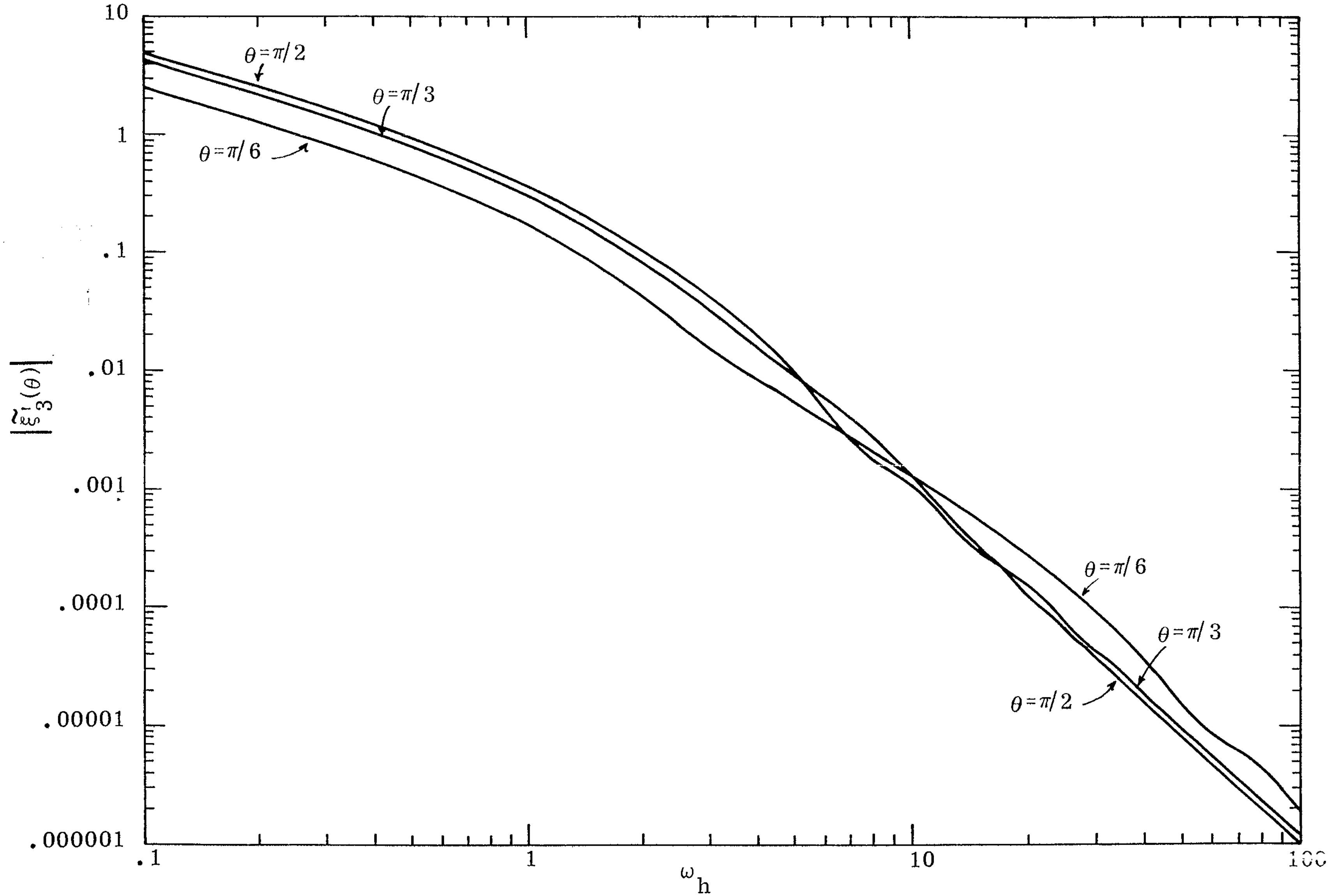

Figure 4.5. Effect of θ on $\tilde{\xi}'_3(\theta)$, $\alpha = 1$

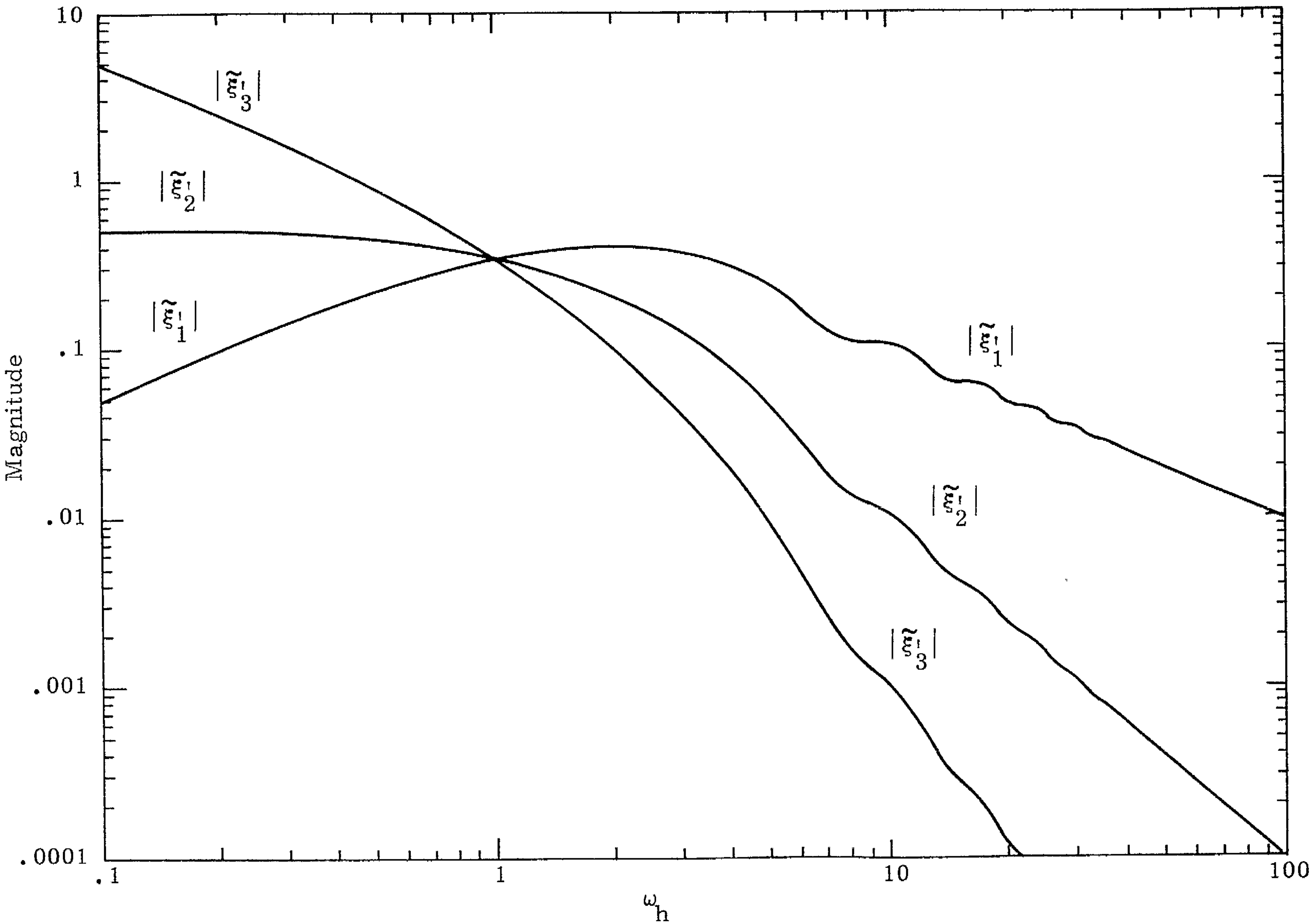

Figure 4.6. Comparison of $|\tilde{\xi}'_1|$, $|\tilde{\xi}'_2|$ and $|\tilde{\xi}'_3|$ for $\theta=\pi/2$, $\alpha = 1$

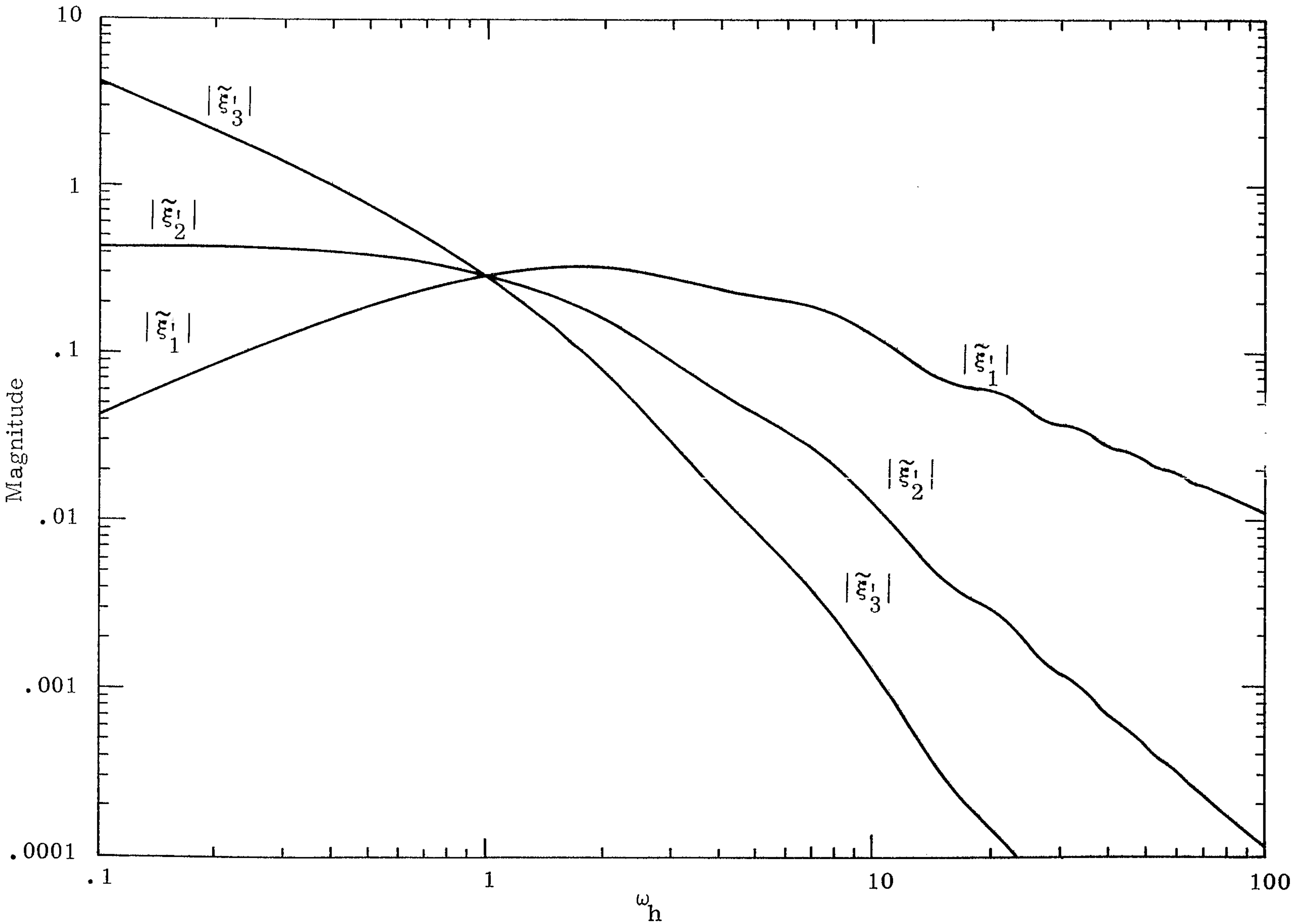

Figure 4.7. Comparison of $|\tilde{\xi}'_1|$, $|\tilde{\xi}'_2|$ and $|\tilde{\xi}'_3|$ for $\theta = \pi/3$, $\alpha = 1$

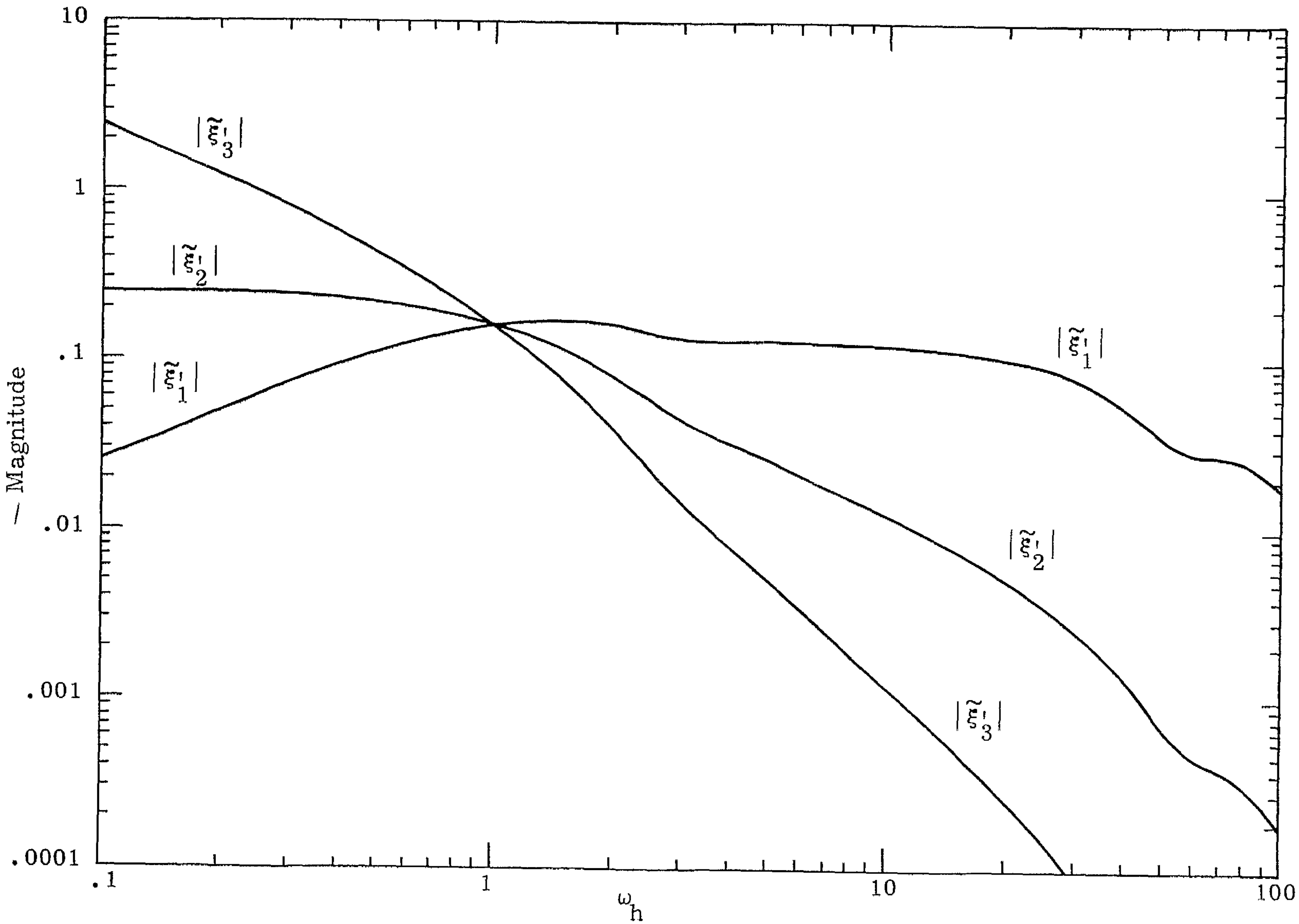

Figure 4.8. Comparison of $|\tilde{\xi}'_1|$, $|\tilde{\xi}'_2|$ and $|\tilde{\xi}'_3|$ for $\theta = \pi/6$, $\alpha = 1$

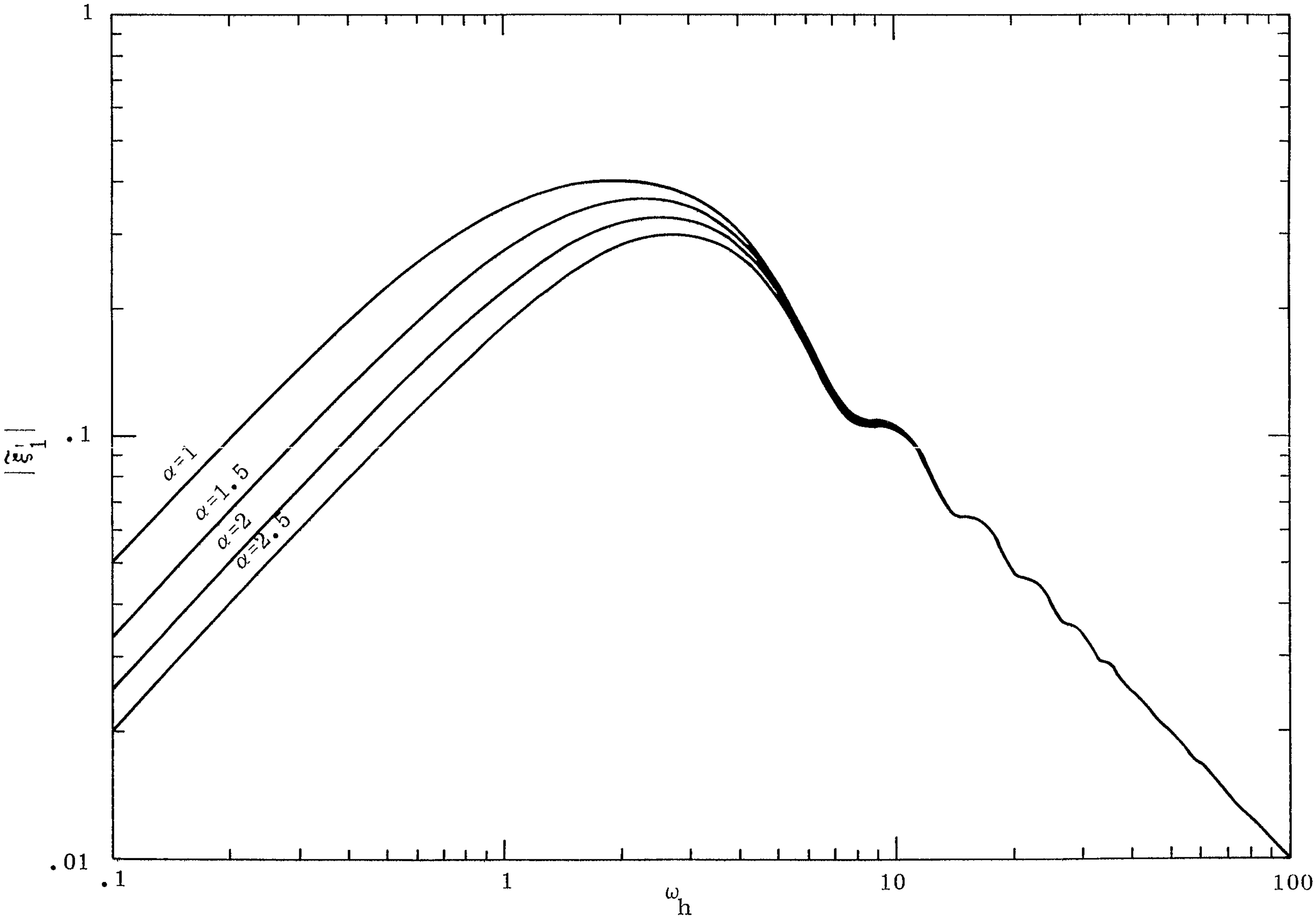

Figure 4.9. Effect of generator capacitance on $|\tilde{\xi}'_1|$, $\theta = \pi/2$

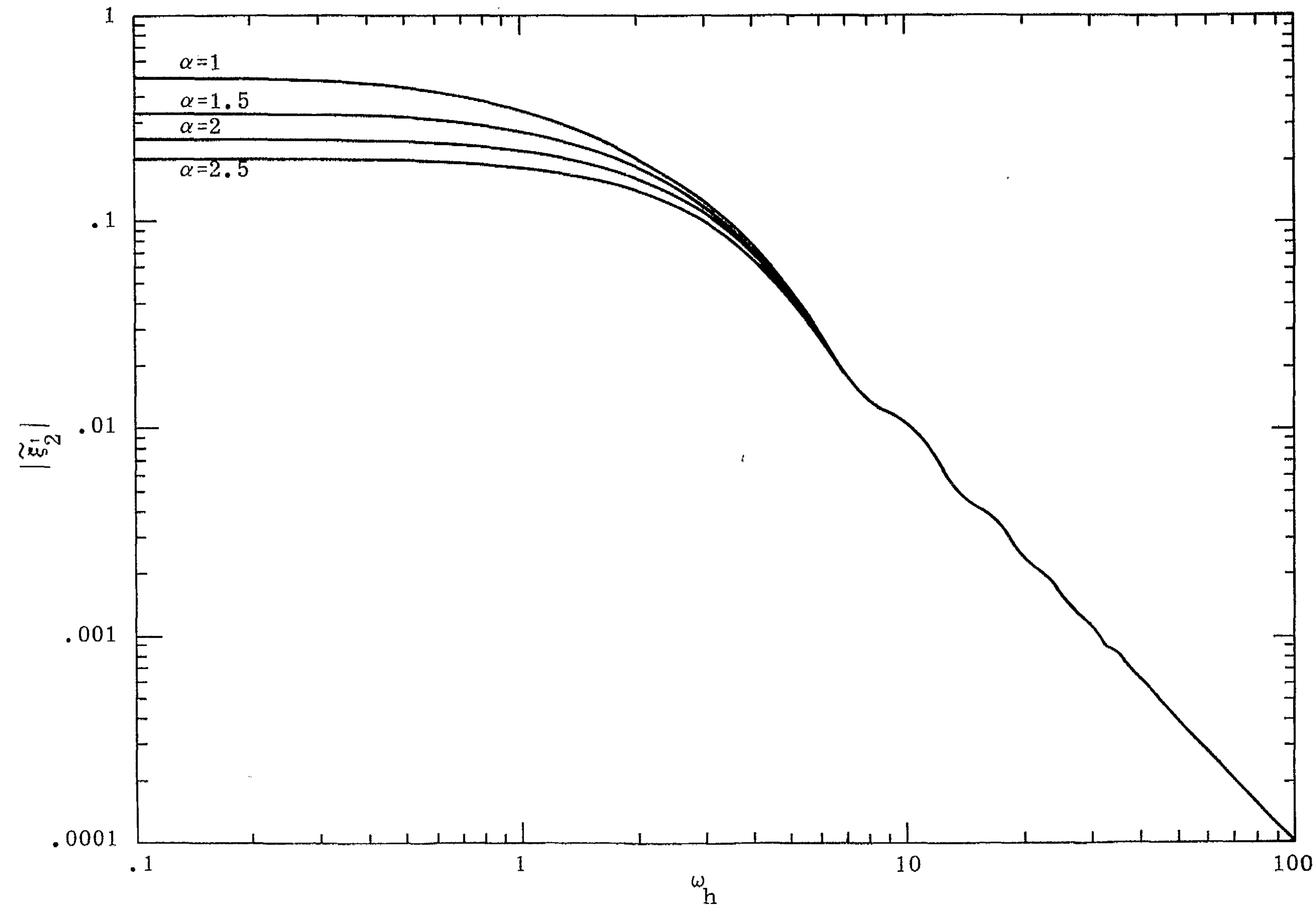

Figure 4.10. Effect of generator capacitance on $|\tilde{\xi}'_2|$, $\theta = \pi/2$

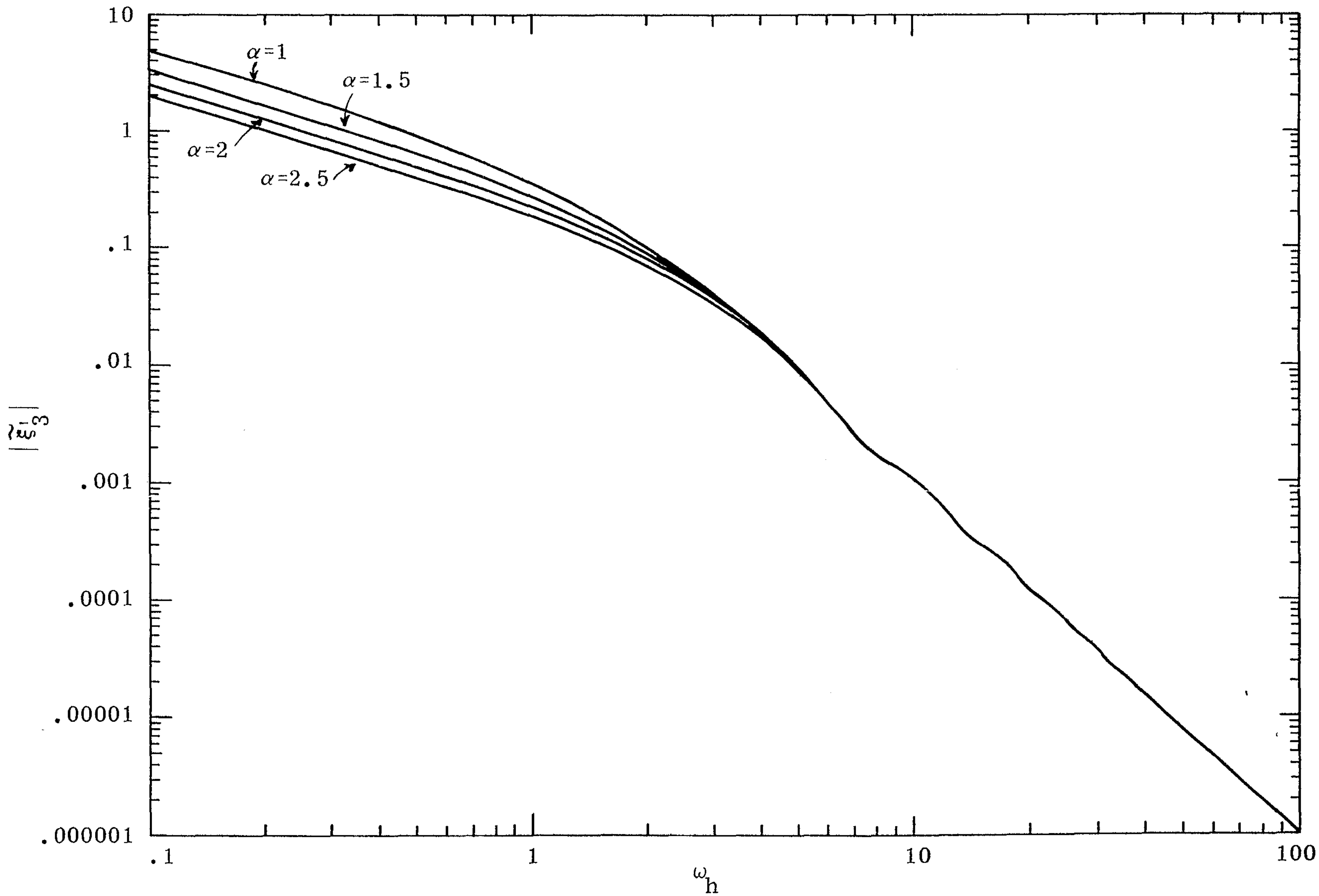

Figure 4.11. Effect of generator capacitance on $|\tilde{\xi}'_3|$, $\theta = \pi/2$

with

$$\tilde{\xi}_1'(\theta) = \sin(\theta)\left[\frac{e^{-\alpha\tau_h}}{\sin^2(\theta)} U(\tau_h) - \frac{2}{\sin^4(\theta)}\left\{\frac{1 - e^{-\alpha\tau_h}}{\alpha}\right\} U(\tau_h) + 2\cos^2\left(\frac{\theta}{2}\right)\left\{\frac{1 - e^{-\alpha\left\{\tau_h-(1-\cos(\theta))\right\}}}{\alpha}\right\} U\left(\tau_h - (1 - \cos(\theta))\right) + 2\sin^2\left(\frac{\theta}{2}\right)\left\{\frac{1 - e^{-\alpha\left\{\tau_h-(1-\cos(\theta))\right\}}}{\alpha}\right\} U\left(\tau_h - (1 + \cos(\theta))\right)\right] \tag{4.17}$$

and

$$\tau_h = \left(\frac{ct - r}{h}\right)$$

Using (3.13) we can write the $1/r^2$ part of the θ component of the electric field as

$$E_{2_\theta}(\vec{r}, t) = \frac{c}{r}\int_{-\infty}^{t} \tilde{E}_{f_\theta}(\vec{r}, \tau)\, d\tau \equiv \left[\frac{V_o}{2\pi f_g}\right]\frac{h}{r^2}\,\tilde{\xi}_2'(\theta) \tag{4.18}$$

where

$$\xi_2'(\theta) \equiv \int_0^{\tau_h} \xi_1'(\theta)\, d\tau_h$$

$$= \sin(\theta)\left[\frac{1}{\sin^2(\theta)}\left\{\frac{1 - e^{-\alpha\tau_h}}{\alpha}\right\} U(\tau_h)\right.$$

$$- \frac{2}{\sin^4(\theta)}\left\{\frac{\tau_h}{\alpha} - \left\{\frac{1 - e^{-\alpha\tau_h}}{\alpha}\right\}\right\} U(\tau_h)$$

$$+ 2\cos^2\left(\frac{\theta}{2}\right)\left\{\frac{\tau_h - (1 - \cos(\theta))}{\alpha} - \frac{1 - e^{-\alpha\{\tau_h - (1-\cos(\theta))\}}}{\alpha^2}\right\} U\left(\tau_h - (1 - \cos(\theta))\right)$$

$$\left. + 2\sin^2\left(\frac{\theta}{2}\right)\left\{\frac{\tau_h - (1 + \cos(\theta))}{\alpha} - \frac{1 - e^{-\alpha\{\tau_h - (1+\cos(\theta))\}}}{\alpha^2}\right\} U\left(\tau_h - (1 + \cos(\theta))\right)\right] \quad (4.19)$$

while the $1/r^3$ part of the θ component of the electric field as

$$E_{3_\theta}(\vec{r}, t) = \frac{c}{r}\int_{-\infty}^{t} E_{2_\theta}(\vec{r}, t)\, dt = \frac{c^2}{r^2}\int_{-\infty}^{t} d\tau \int_{-\infty}^{\tau} E_{f_\theta}(\vec{r}, t)\, dt$$

$$\equiv \left[\frac{V_o}{2\pi f_g}\right]\frac{h^2}{r^3}\,\xi_3'(\theta) \quad (4.20)$$

where

$$\xi_3'(\theta) \equiv \int_0^{\tau_h} \xi_2'(\theta)\, d\tau_h$$

$$= \sin(\theta)\Bigg[\frac{1}{\sin^2(\theta)}\left\{\frac{\tau_h}{\alpha} - \left\{\frac{1 - e^{-\alpha\tau_h}}{\alpha^2}\right\}\right\} U(\tau_h)$$

$$- \frac{2}{\sin^4(\theta)}\left\{\frac{\tau_h^2}{2\alpha} - \frac{\tau_h}{\alpha} + \left\{\frac{1 - e^{-\alpha\tau_h}}{\alpha^2}\right\}\right\} U(\tau_h)$$

$$+ 2\cos^2\left(\frac{\theta}{2}\right)\left\{\frac{1}{2\alpha}\left\{\tau_h^2 - (1 - \cos(\theta))^2\right\} - (1 - \cos(\theta))\left\{\frac{\tau_h - (1 - \cos(\theta))}{\alpha}\right\}\right.$$

$$\left. - \frac{\tau_h - (1 - \cos(\theta))}{\alpha^2} + \frac{1 - e^{-\alpha\{\tau_h - (1-\cos(\theta))\}}}{\alpha^2}\right\} U\left(\tau_h - (1 - \cos(\theta))\right)$$

$$+ 2\sin^2\left(\frac{\theta}{2}\right)\left\{\frac{1}{2\alpha}\left\{\tau_h^2 - (1 + \cos(\theta))^2\right\} - (1 + \cos(\theta))\left\{\frac{\tau_h - (1 + \cos(\theta))}{\alpha}\right\}\right.$$

$$\left. - \frac{\tau_h - (1 + \cos(\theta))}{\alpha^2} + \frac{1 - e^{-\alpha\{\tau_h - (1+\cos(\theta))\}}}{\alpha^2}\right\} U\left(\tau_h - (1 + \cos(\theta))\right)\Bigg] \quad (4.21)$$

Using (4.16 through 4.21) the total time domain θ component of the electric field can be written as

$$E_\theta(\vec{r}, t) = \left[\frac{V_o}{2\pi f_g}\right]\frac{1}{r}\left[\xi_1'(\theta) + \frac{h}{r}\xi_2'(\theta) + \frac{h^2}{r^2}\xi_3'(\theta)\right] \quad (4.22)$$

In figures 4.12 through 4.20 $\xi_1'(\theta)$, $\xi_2'(\theta)$, and $\xi_3'(\theta)$ are plotted for various angles and α. As can be seen $\xi_2'(\theta)$ and $\xi_3'(\theta)$ become more important at late times. If α, r, h and θ are known, one can reconstruct the total θ component at the given point.

The total radial component of the electric field can be obtained to be

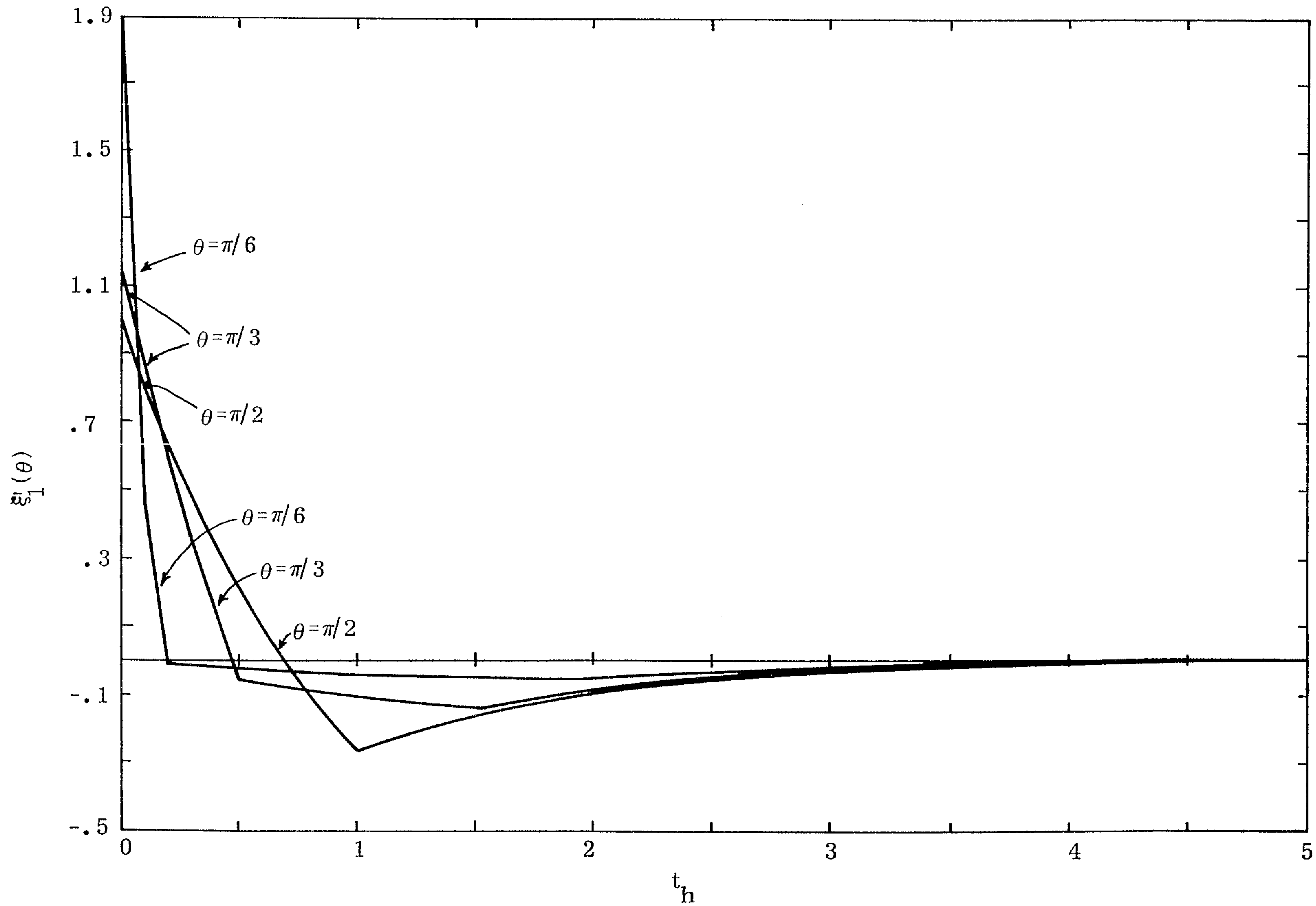

Figure 4.12. Effect of θ on $\xi_1'(\theta)$ using $\alpha = 1$

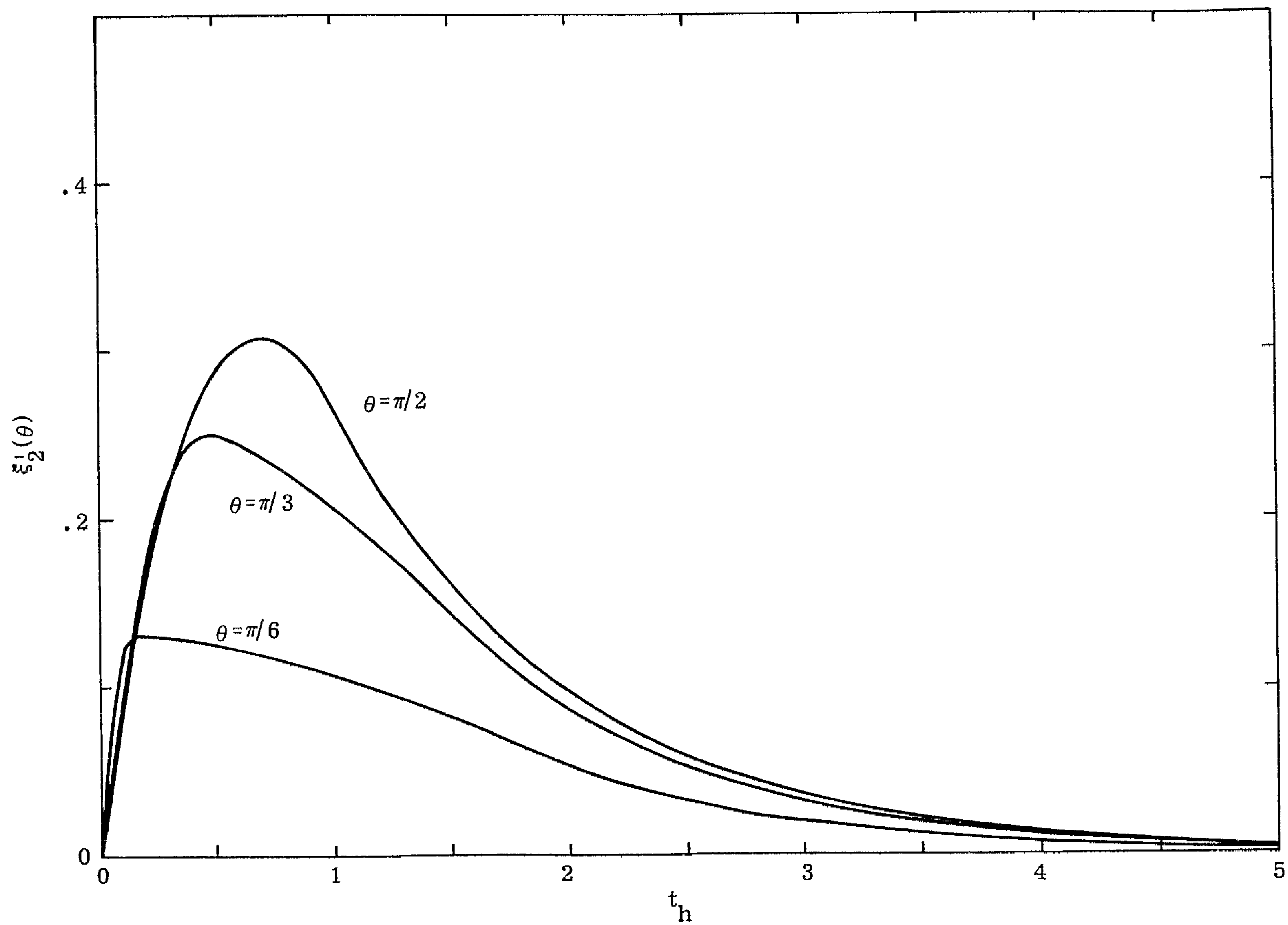

Figure 4.13. Effect of θ on $\xi_2'(\theta)$ using α = 1

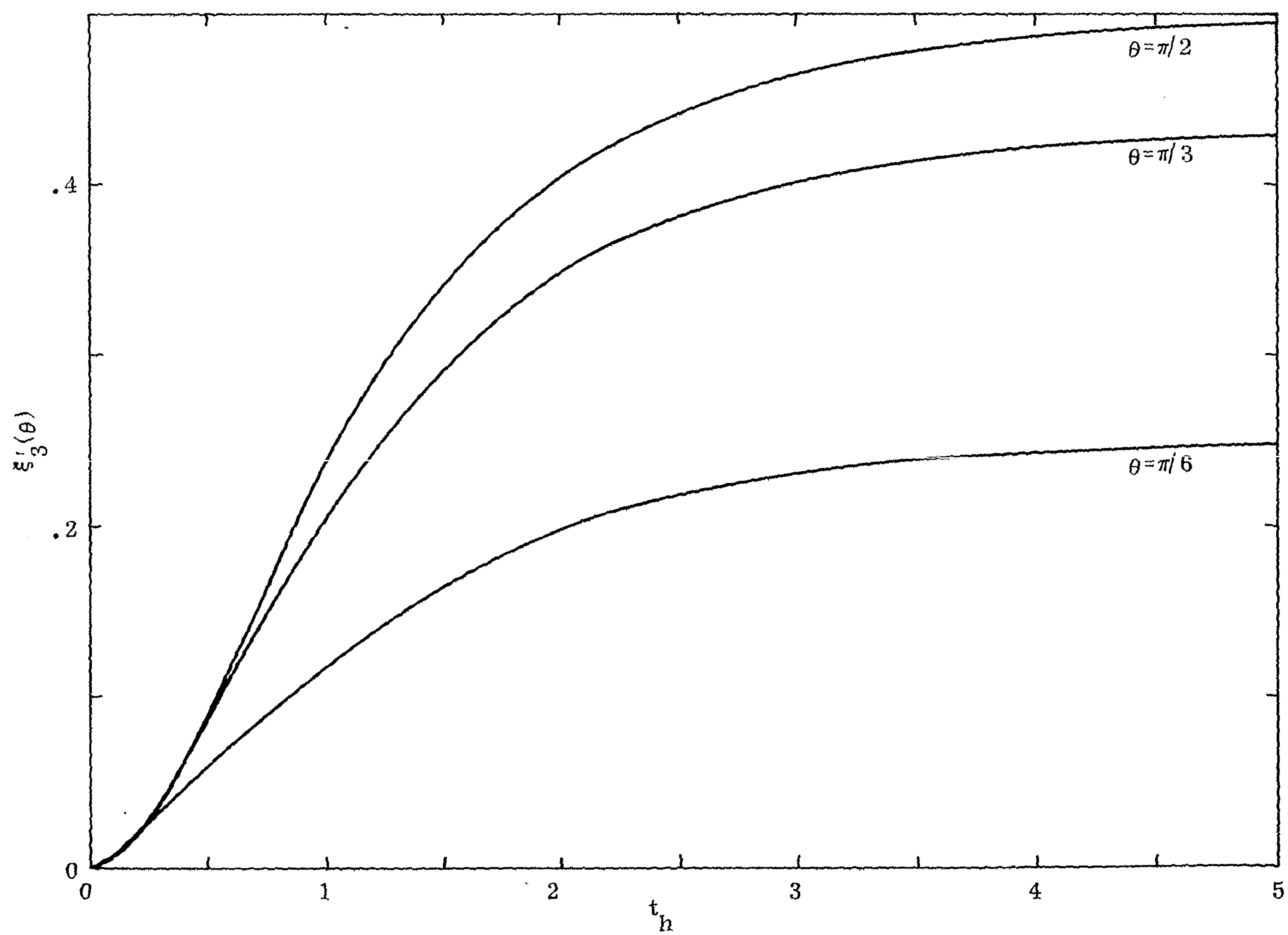

Figure 4.14. Effect of θ on $\xi_3'(\theta)$ using α = 1

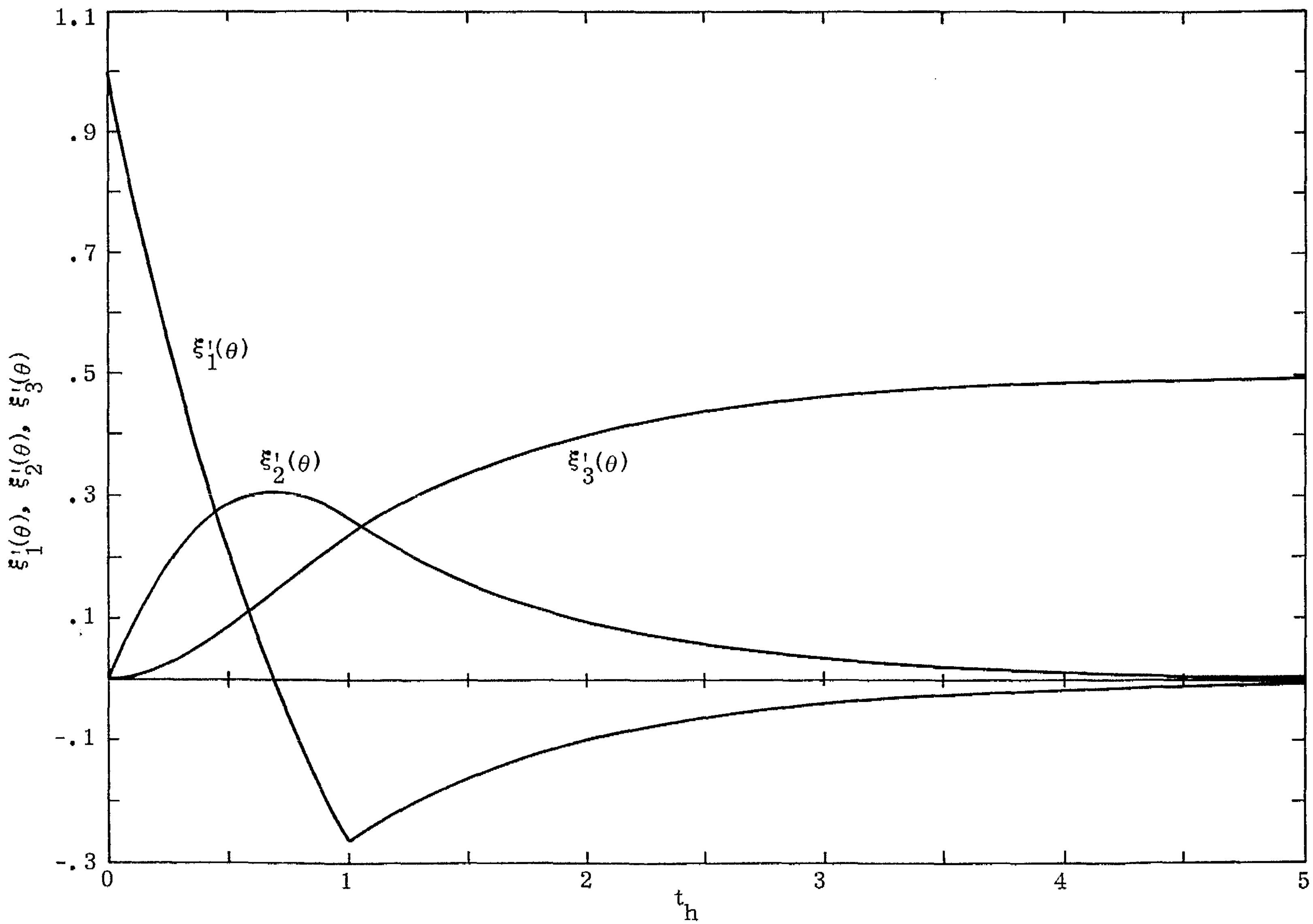

Figure 4.15. Comparison of $\xi_1'(\theta)$, $\xi_2'(\theta)$ and $\xi_3'(\theta)$ for $\theta = \pi/2$, $\alpha = 1$

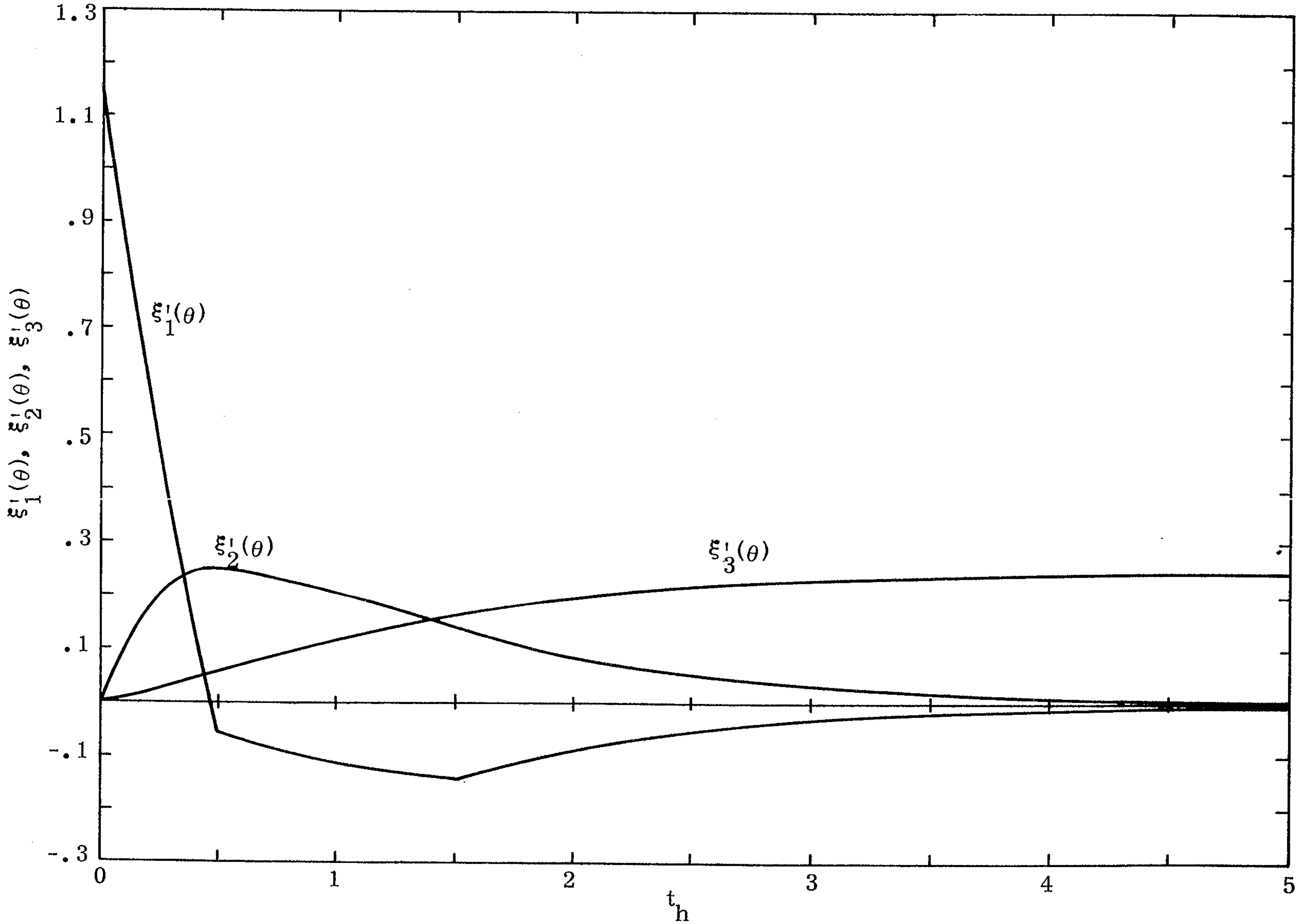

Figure 4.16. Comparison of $\xi_1'(\theta)$, $\xi_2'(\theta)$, and $\xi_3'(\theta)$ for $\theta = \pi/3$, $\alpha = 1$

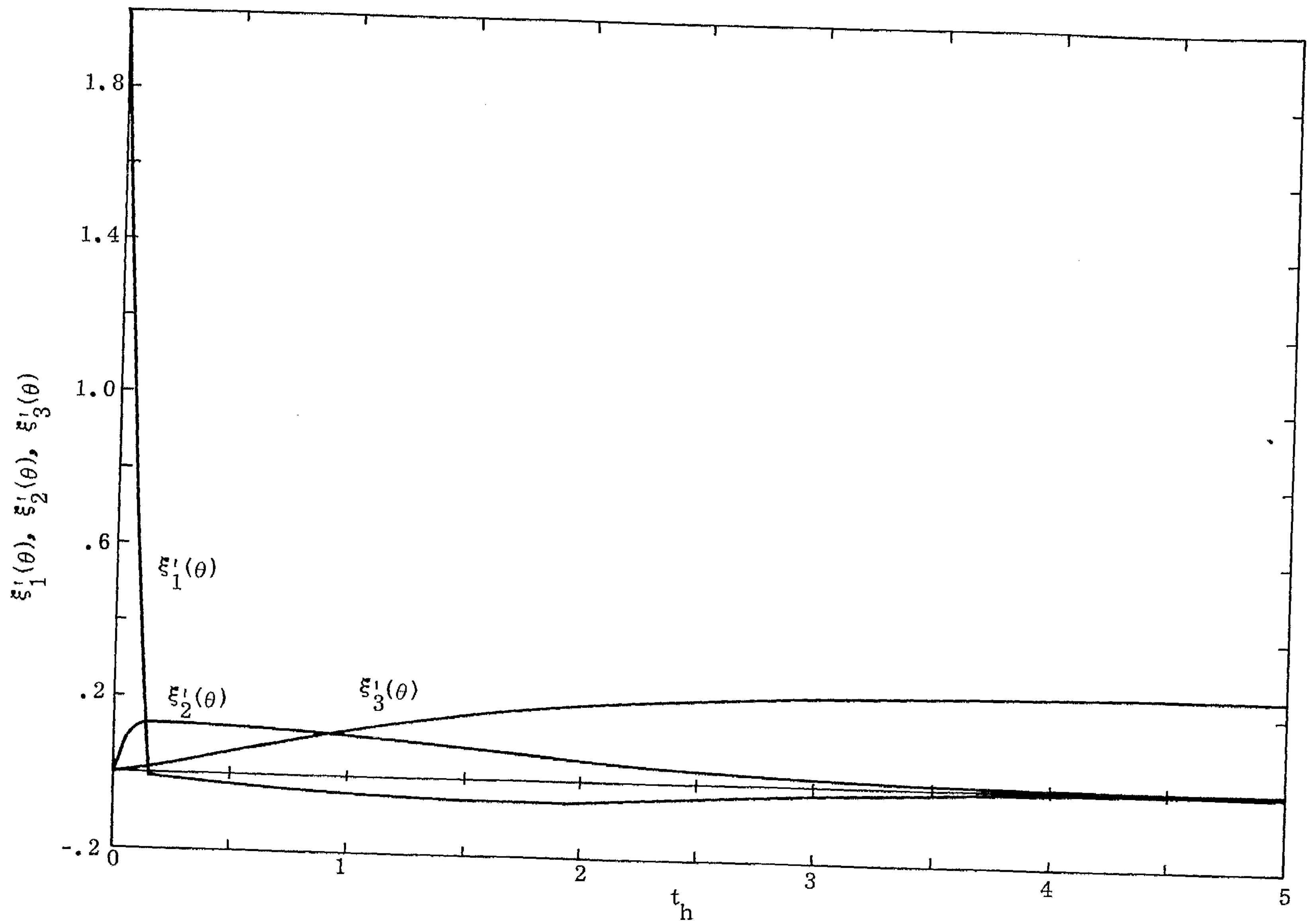

Figure 4.17. Comparison of $\xi_1'(\theta)$, $\xi_2'(\theta)$ and $\xi_3'(\theta)$ for $\theta = \pi/6$, $\alpha = 1$

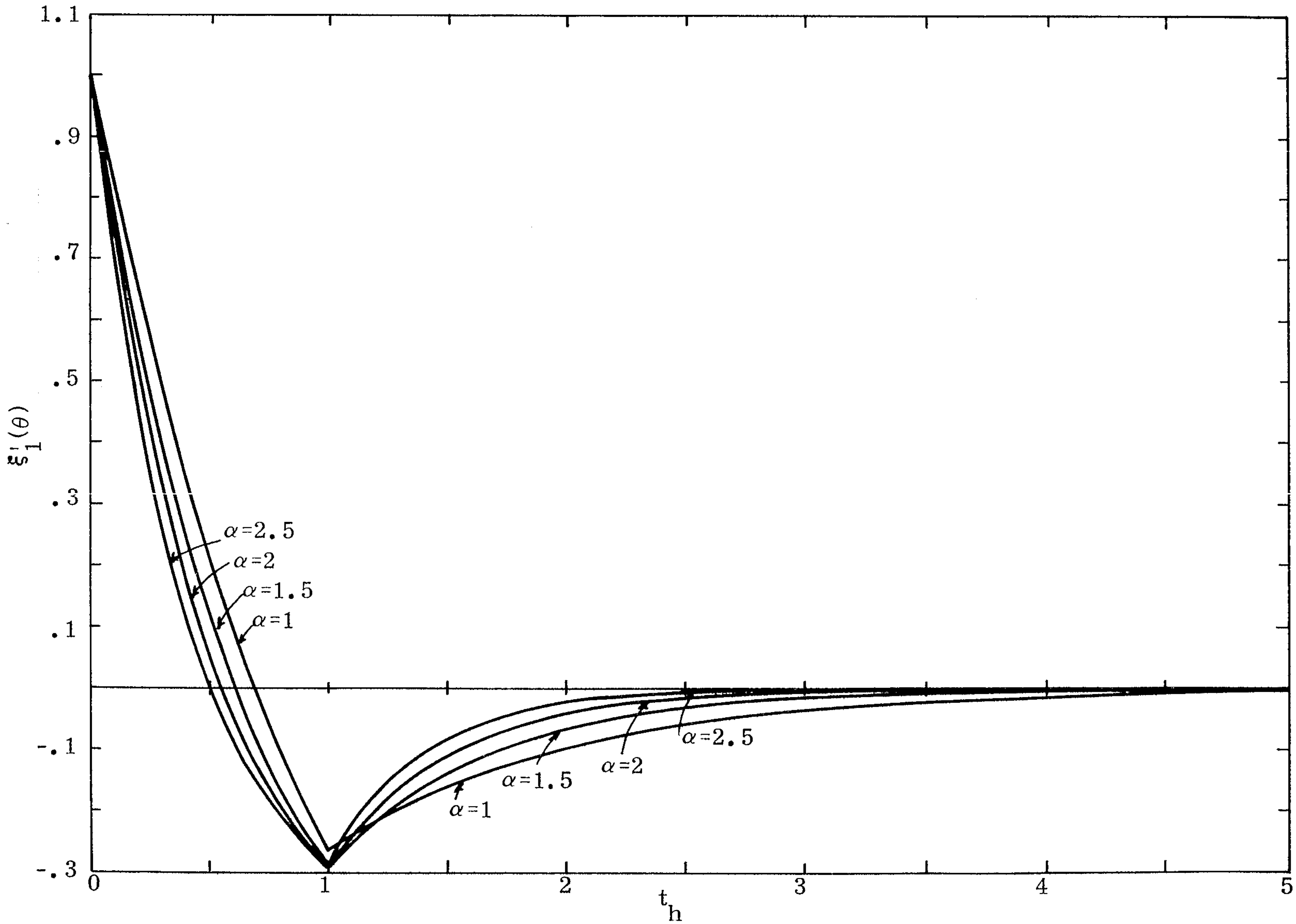

Figure 4.18. Effect of generator capacitance on $\xi_i'(\theta)$, $\theta = \pi/2$

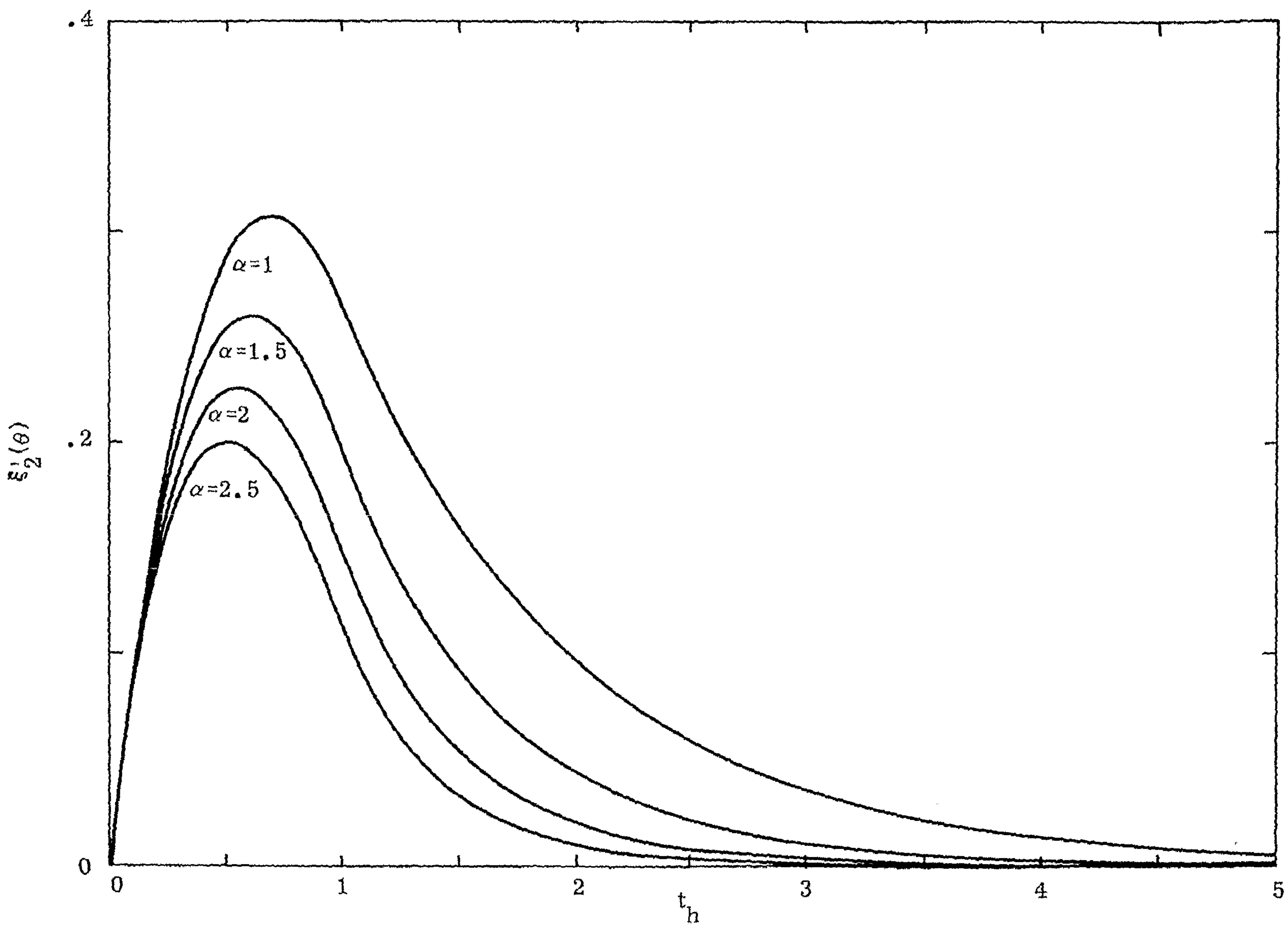

Figure 4.19. Effect of generator capacitance on $\xi_2'(\theta)$, $\theta = \pi/2$

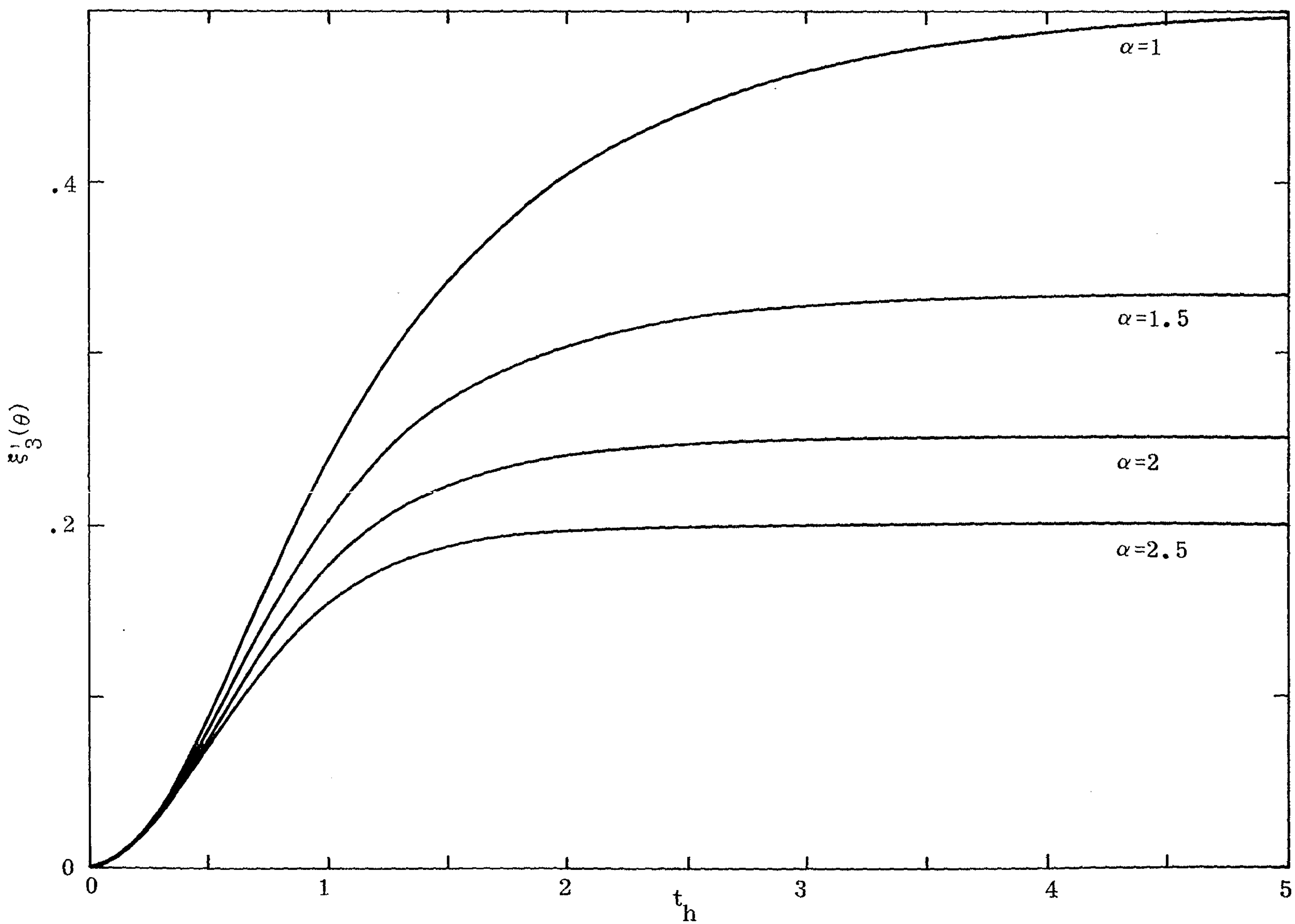

Figure 4.20. Effect of generator capacitance on $\xi_3'(\theta)$, $\theta = \pi/2$

$$E_r(\vec{r}, t) = 2\cot(\theta)\left[\frac{V_o}{2\pi f_g}\right]\frac{1}{r}\left[\frac{h}{r}\xi_2'(\theta) + \frac{h^2}{r^2}\xi_3'(\theta)\right] \tag{4.23}$$

As should be expected, the radial component becomes more important at late times. In a similar fashion the total ϕ component of the magnetic field is given by

$$H_\phi(\vec{r}, t) = \frac{1}{Z_o}\left[\frac{V_o}{2\pi f_g}\right]\frac{1}{r}\left[\xi_1'(\theta) + \frac{h}{r}\xi_2'(\theta)\right] \tag{4.24}$$

Both $E_r(\vec{r}, t)$ and $H_\phi(\vec{r}, t)$ at a given point can easily be obtained if h, r, θ and α are known.

As an example of the procedure discussed above, let us consider an impedance loaded dipole antenna having the characteristics of ATHAMAS II (large proposed vertical electric dipole). The antenna capacitance[8,9] is $\simeq$ 3.8 nF, the generator capacitance $\approx$ 4 nF, the charge equivalent height of the antenna $\approx$ 50 m while the half angle of the bicone = 40.4°. Since the antenna is assumed to be over a perfectly conducting ground plane, the generator voltage is taken as 10 MV to include the effect of perfectly conducting ground. Using these characteristics, the θ and r components of the electric field and the ϕ component of the magnetic field are plotted in figures 4.21 through 4.26 in both time and frequency domains. Since the pulser is assumed to have a step function input, the high frequency fields behave as 1/f, f being the frequency, while the late time domain fields reach a steady state value. A similar procedure can be used to obtain the field components in cylindrical coordinates. It should be noted that if $\theta < \theta_1$ or $(\pi - \theta) < \theta_1$ the formulae for the fields are not valid because of the shadowing, the diffraction from the edges, etc. If $r \gg h$ and θ satisfies the above condition the fields as calculated here should compare well with the measured quantities. For low frequencies or late times if $r \gg h$ the results are valid for all θ.

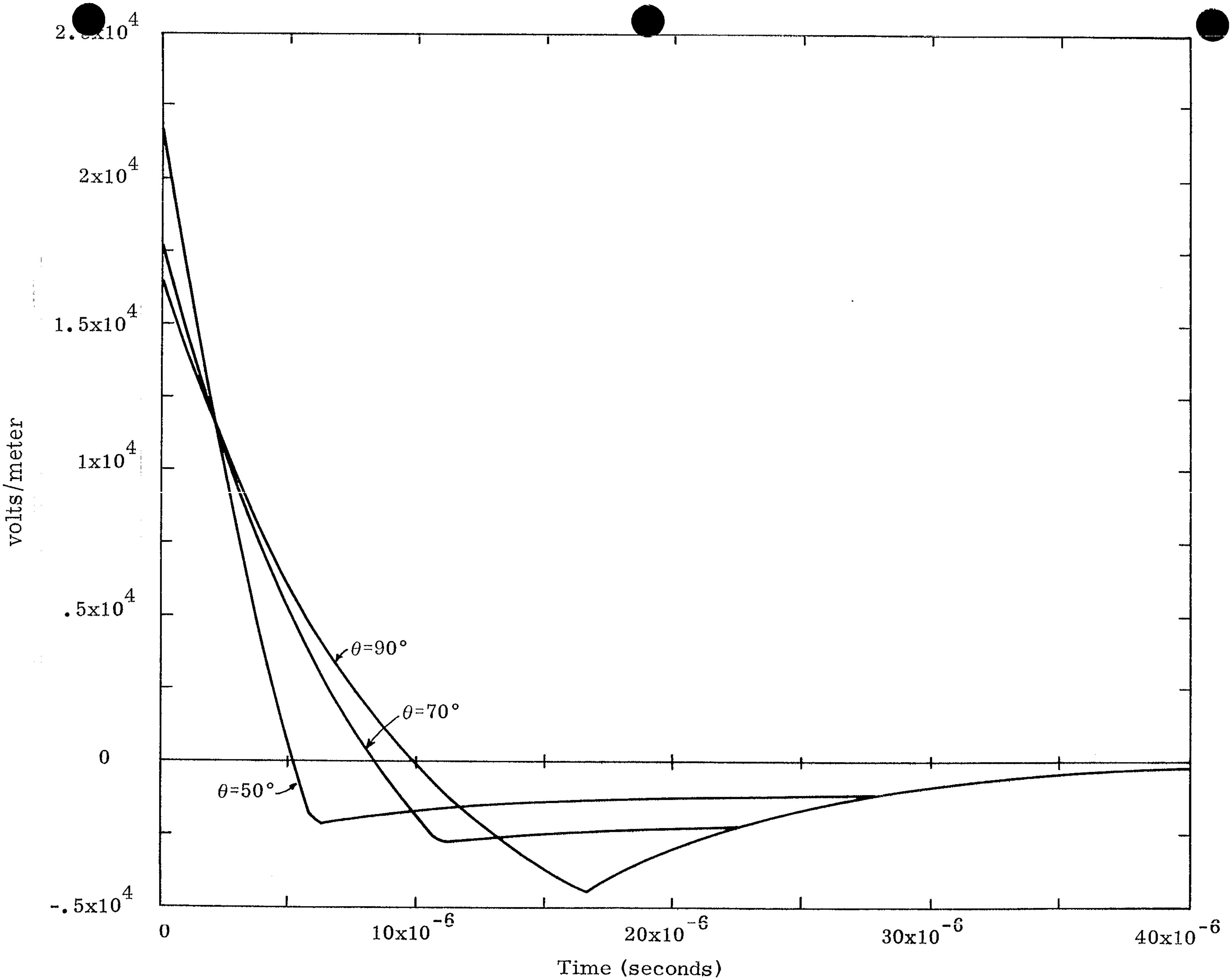

Figure 4.21. E_θ at r=300m using an ideal pulser and the antenna on a ground plane

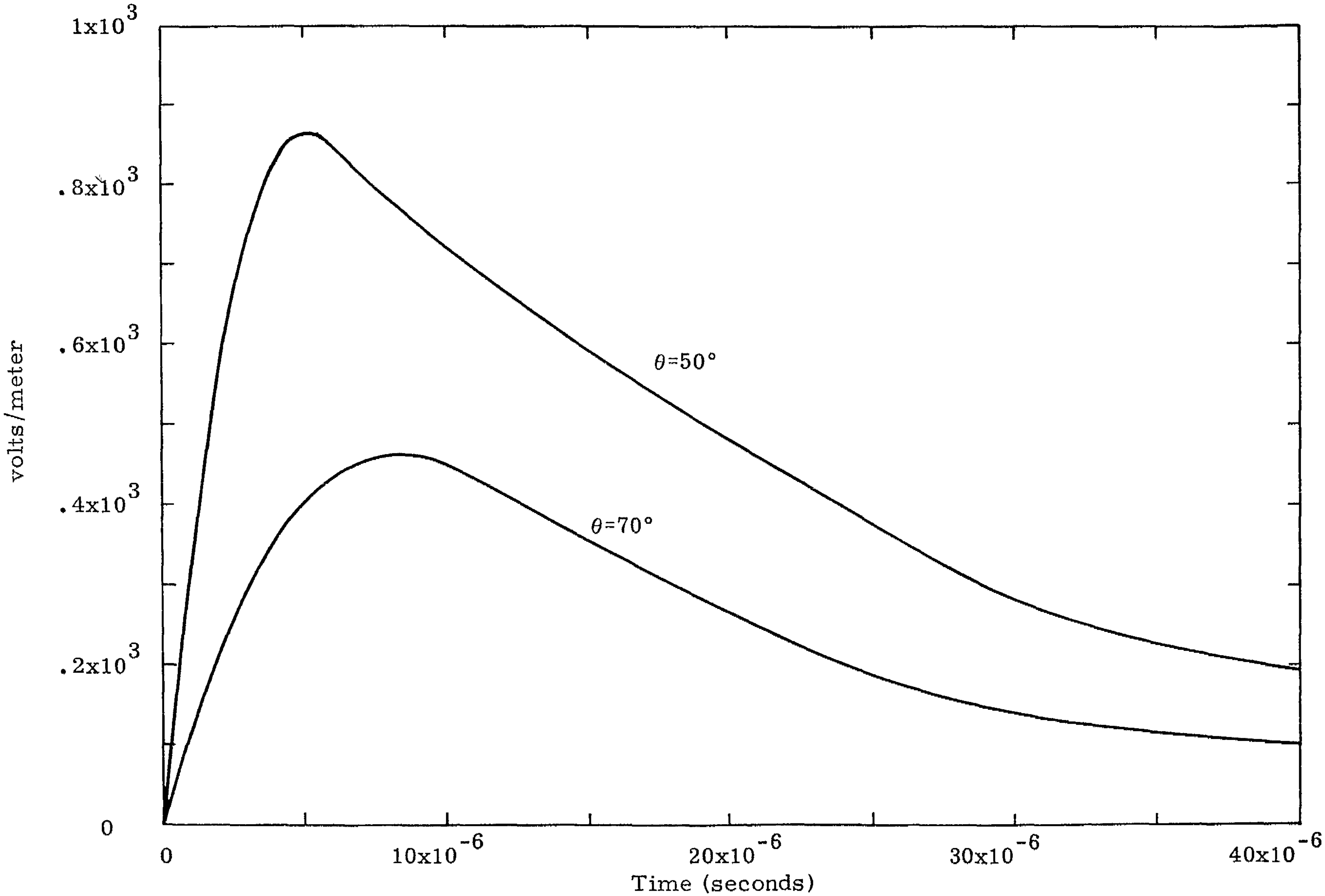

Figure 4.22. E_r at r=300m using an ideal pulser and the antenna on a ground plane

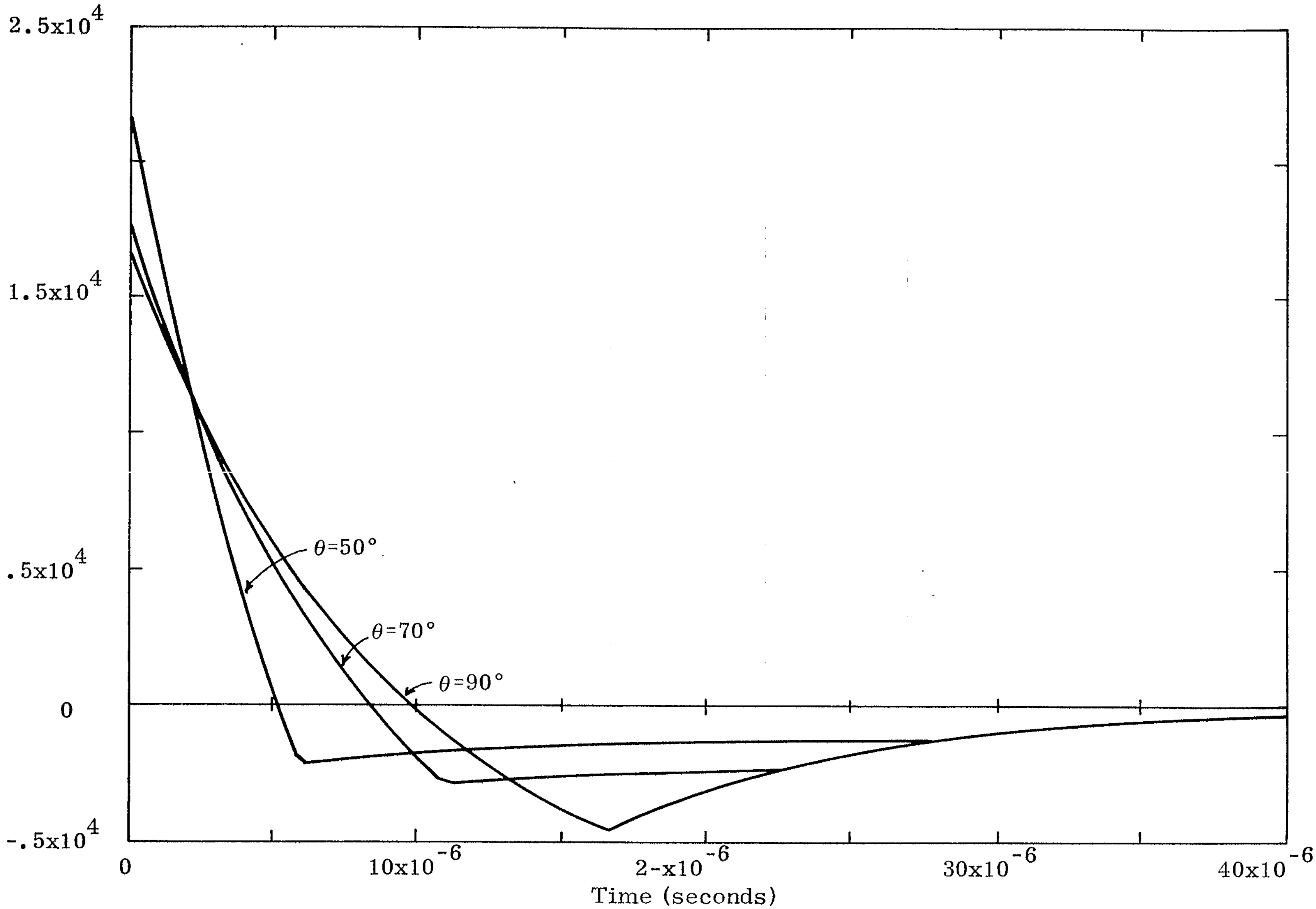

Figure 4.23. $Z_o H_\phi$ at r=300m using an ideal pulser and the antenna on a ground plane

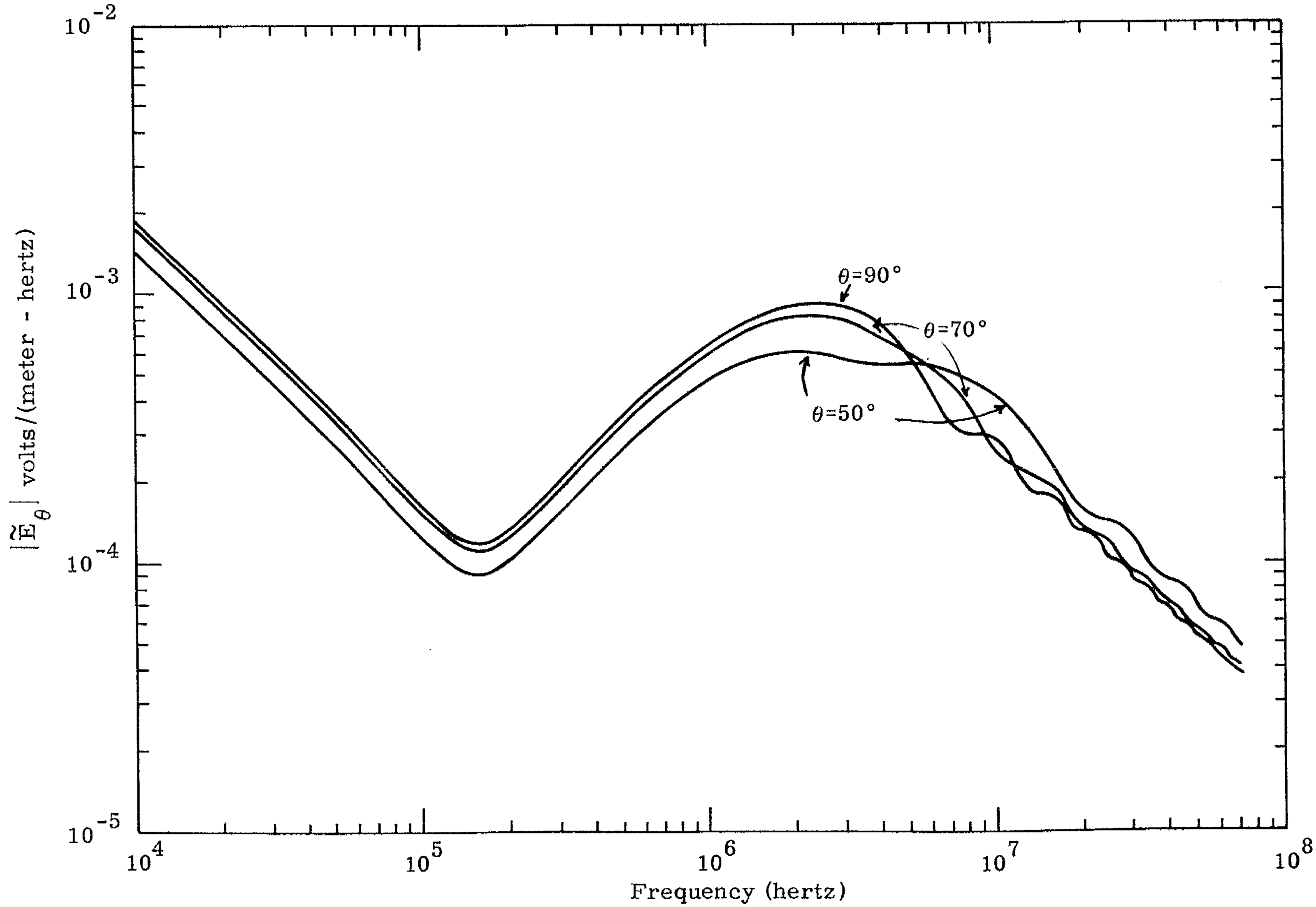

Figure 4.24. $\tilde{E}_\theta$ at r=300m using an ideal pulser and the antenna on a ground plane

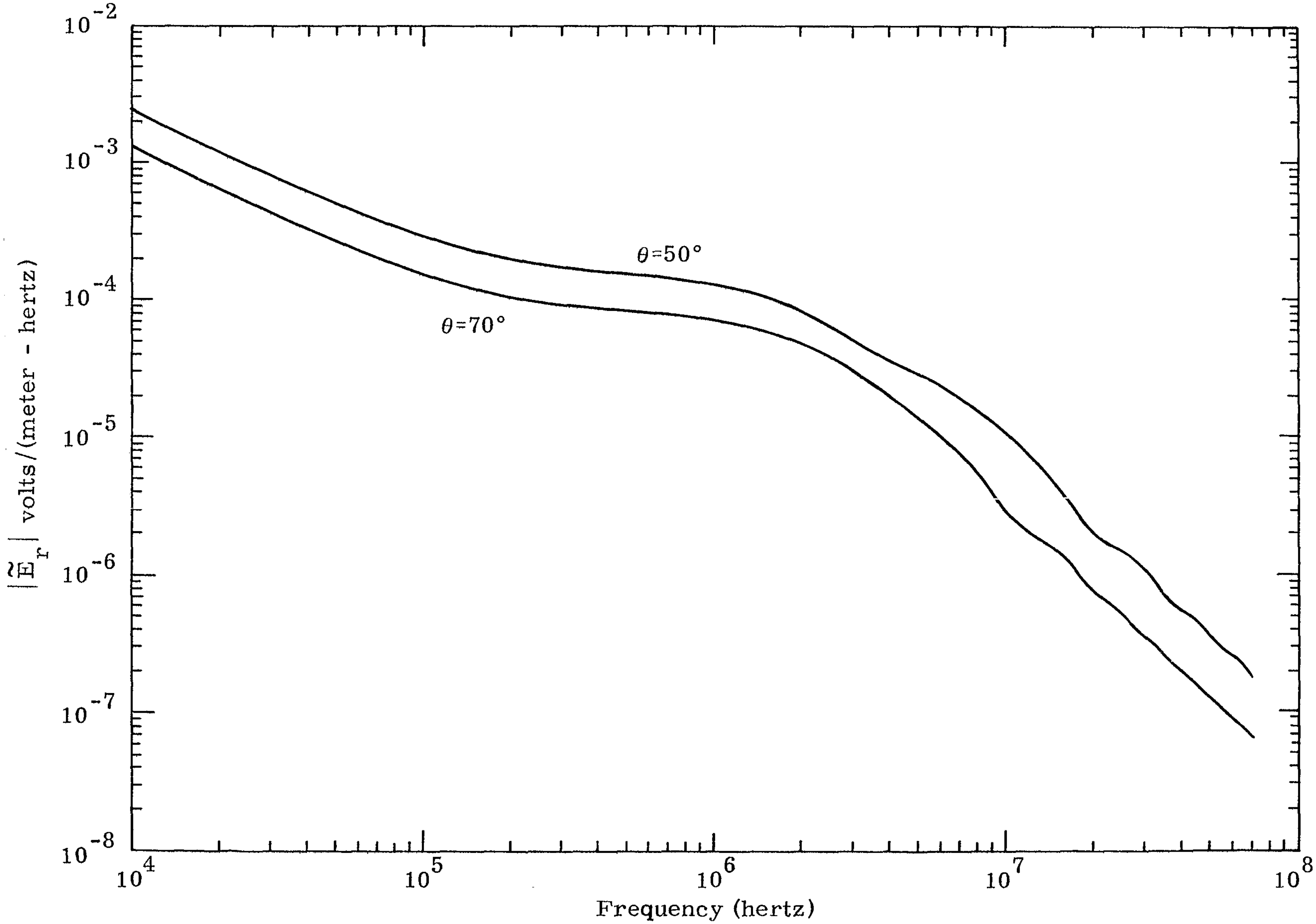

Figure 4.25. $\tilde{E}_r$ at r=300m using an ideal pulser and the antenna on a ground plane

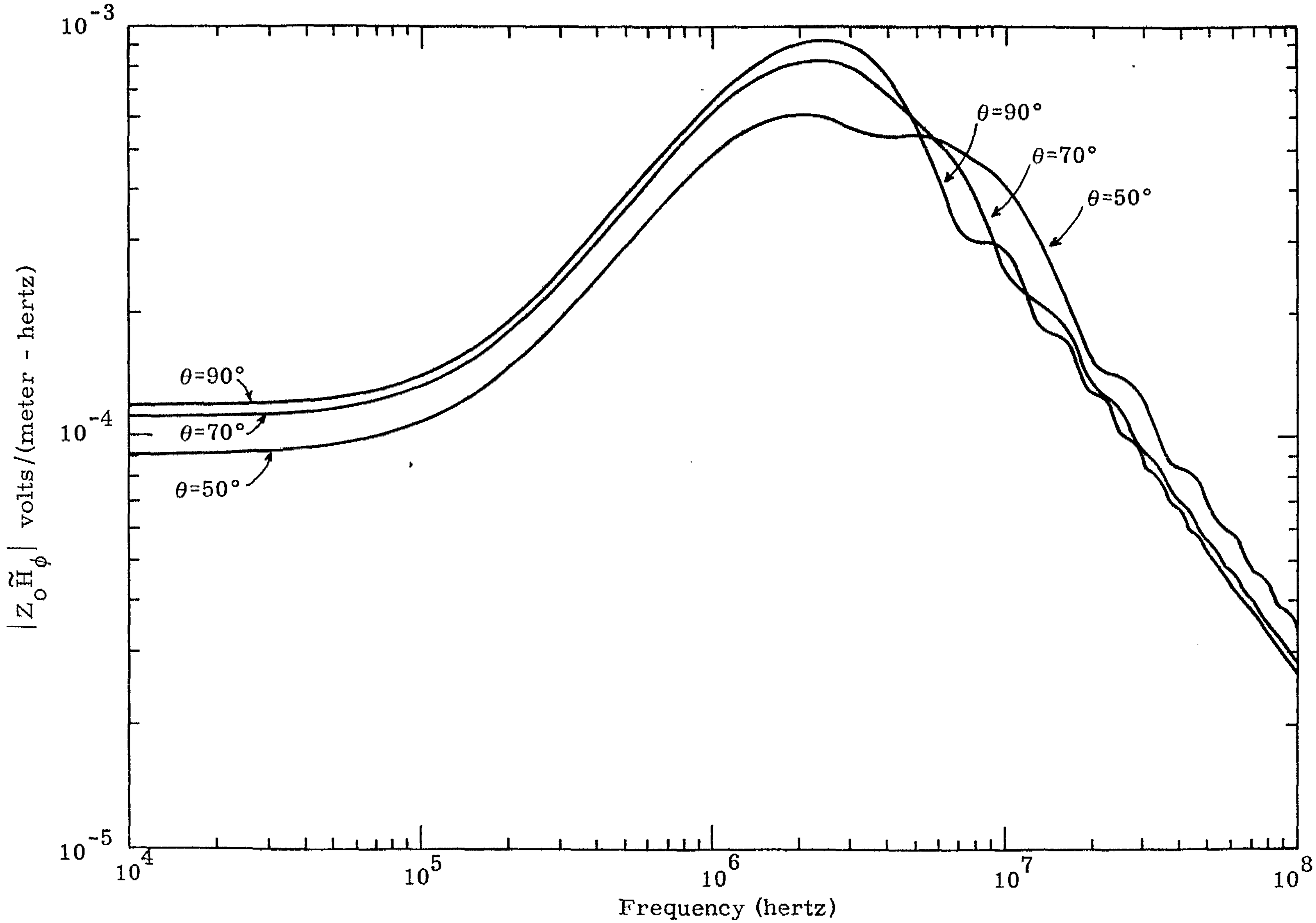

Figure 4.26. $Z_o \tilde{H}_\phi$ at r=300m using an ideal pulser and the antenna on a ground plane

V. Conclusions

A procedure has been developed which yields the near field electric and magnetic components of an axially and lengthwise symmetric electric dipole if the far field components of the same are known. In particular, it has been shown that if E_{f_θ}, i.e., the θ component of the far field is known, all other components of the electric and magnetic fields including their near fields can be determined. This procedure is shown to have been applicable both in time and frequency domains. In frequency domain the near field components can simply be obtained by frequency scaling the far field components while in time domain they can be obtained by integrating the far fields with respect to time.

Although we have only discussed the case of analytical calculation, this procedure is equally applicable to measured data. If the far field components of an electric dipole type source are known, the near fields can be obtained easily by using the procedures discussed here.

If the antenna is a magnetic dipole instead of an electric dipole, the procedures developed in this report can still be used by noting that a magnetic dipole is a complement of an electric dipole. Immediately it is clear that if H_{f_θ} , i.e., the θ component of the far magnetic field is known, all other components of the electric and magnetic fields including the near field terms can be obtained by using the principle of duality.

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