A Detector Geometry for Measuring the Vertical Gamma Current

I. Introduction

The purpose of this note is to describe a possible gamma detector geometry suitable for measuring the vertical gamma current. Although we could also use this detector geometry for other gamma current components, it may be particularly suitable for measuring the vertical gamma current because the geometry is easily incorporated into a ground (or water) surface which can add to the collimator effectiveness. Whereas some previously suggested gamma current devices (Sensor and Simulation Notes (SSN) X and XVII) have used detectors placed around an attenuator, this device utilizes an attenuator (or collimator) placed around the detectors.

As illustrated in figure 1, this single component, gamma current detector consists of a long, thin, cylindrically symmetric, distributed detector which is surrounded by a cylindrically symmetric collimator, recessed into a ground (or water) surface. The distributed detector (or string of discrete detectors with outputs summed) is assumed to be cylindrical with its length much greater than its radius so that for calculational purposes we may approximate the detector by a geometrical line. Thus, we might call this detector a line detector and this geometry a collimated line detector geometry.

Since the detector geometry is assumed symmetric about the vertical (z) axis we can consider gamma rays, $\gamma(\theta)$, coming from a polar angle, θ, (independent of the azimuthal angle). These gamma rays will be detected by those portions of the line detector above a point where the collimator shadows the detector. As the angle, θ, is decreased, more of the line detector is exposed, increasing the magnitude of the electrical output of the detector. For $\theta = 0$ all of the line detector is exposed while for $\theta = \pi/2$, if the top of the line detector is on the same level as the ground plane, none of the detector is exposed. Ideally we may wish that the detector output vary as $\cos\theta$ so that we measure the vertical gamma current, γ_z.[1]

For the calculations in this note we assume that the gamma sensitivity of any incremental length of the line detector is independent of the direction from which the gamma rays illuminate the detector. Many types of detectors approximate this characteristic, including fluors, semiconductors, and SEMIRAD. However, this incremental independence of angle is not essential but would have to be included at the appropriate point in the analysis. It is necessary, however, that any angular dependence of the line detector be independent of gamma energy so that the final angular dependence of the collimated line detector system is independent of gamma energy over the range of interest. For our calculations we also assume that, relatively speaking, no gammas penetrate through the collimator to illuminate that part of the line detector which is ideally shadowed. This requires that the collimator dimensions be large compared to a gamma ray mean free path, but with the collimator built into the ground plane this restriction may be easily met. Finally we assume

1. Gamma ray notation is the same as that explained in SSN IX.

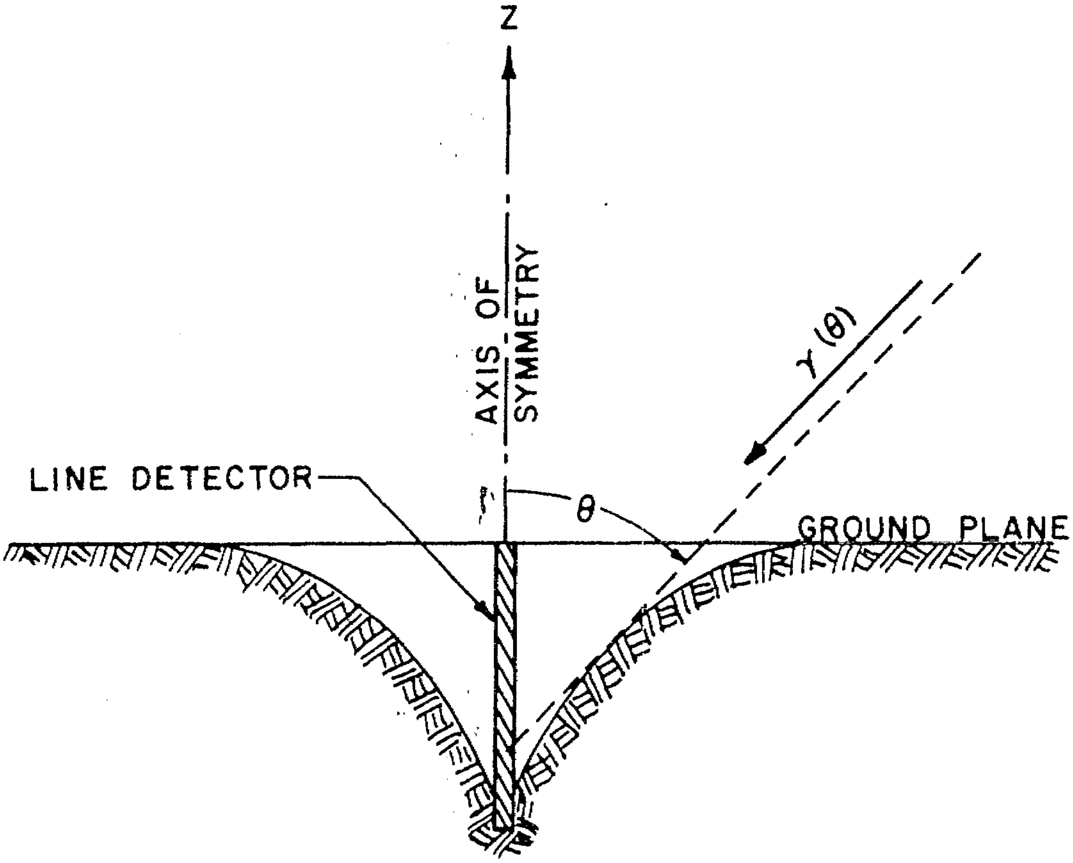

Figure 1. DETECTOR CROSS SECTION

that, relatively speaking, there are no gammas scattered near the collimator surface which add to the gamma intensity received by the illuminated portion of the line detector.

For convenience we might desire that the sensitivity per unit length of the line detector be a constant. Detectors with this characteristic might include a cylindrical rod of fluor driving a cylindrical photo diode, a long, thin cylindrical SEMIRAD, and a uniform string of semiconductor detectors wired in parallel. This case is considered in Section III.

This detector geometry has one particular advantage in measuring γ_z over a ground plane in that if we assume a point γ source at the ground plane at some distance away, then the detector will not respond to the direct gamma beam, but will respond to the scattered radiation or sky shine.[2] In this case, there is not a large common mode signal to subtract out, thus improving the signal-to-noise characteristics. This collimated line detector could be used to measure other components of the gamma current by differencing the outputs of two such detectors built into the opposite sides of a cube of attenuating material. However, such a device might prove to be awkward.

First, we consider the collimator geometry required for a given dependence of the sensitivity per unit length of the line detector. Then we calculate the collimator geometry for a line detector of uniform sensitivity per unit length. Finally, we consider the sensitivity per unit length of the line detector required by a given collimator geometry.

II. Determination of Collimator Geometry

Figure 2 illustrates the normalized collimator geometry. The line detector is taken to lie on the z axis between $z = -1$ and $z = 0$. Since the collimator is assumed to be cylindrically symmetric only a cross section need be given. This cross section is represented by the equation (independent of azimuth):

$$z = f(r) \tag{1}$$

The gamma rays illuminate the line detector between $z = z_1$ and $z = 0$.

First, let us consider a normalized sensitivity per unit length, $S(z)$, for the line detector with the constraint

$$\int_{-1}^{0} S(z)dz = 1 \tag{2}$$

The detector output varies as θ is varied and is thus called $F(\theta)$. This is given by the contribution to the signal from the illuminated region of the

2. See EMP Theoretical Note X, Prompt Gamma Effects in the Vicinity of a Ground-Air Interface, by 1/Lt Richard R. Schaefer for a discussion of this scattered radiation.

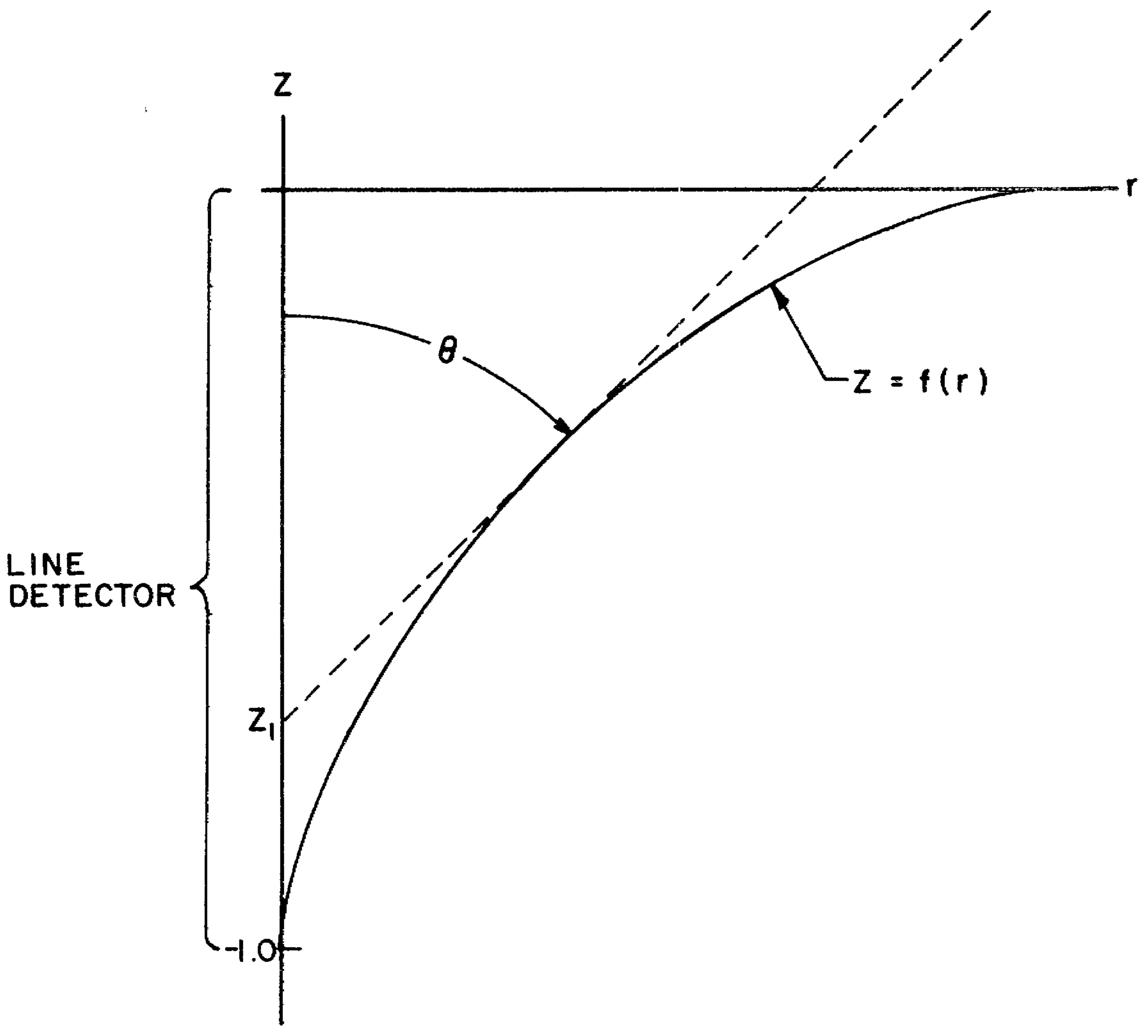

Figure 2. NORMALIZED COLLIMATOR GEOMETRY

line detector, i.e., from all of the line detector above z_1. Thus,

$$\mathcal{S}(z_1) = \int_{z_1}^{0} S(z)dz = F(\theta) \qquad (3)$$

This last equation relates the angular variation of the detector output to the portion of the line detector which is exposed. We should note that S(z) is assumed to be independent of θ which, for many detector types, is approximately correct. If we desired we could also include any angular variation of S(z) in equations (2) and (3).

The line through z_1 which is tangent to the collimator surface, $z = f(r)$, is given by

$$z = z_1 + r \cot \theta \qquad (4)$$

This line represents the limiting path of the gamma rays which just miss the collimator before reaching the line detector. The collimator surface is then defined by varying the desired z_1, θ combination of equation (3) through the range of applicability, thus parametrically varying the tangent line of equation (4). The curve, $z = f(r)$, must be tangent at least at one point to each of the tangent lines but must never cross any of these lines. Thus, all z_1, θ relationships of equation (3) are not necessarily allowed. Certainly z_1 and θ must be monotonically related but this is not a sufficient condition.

To solve for equation (1) we recognize that for a given r the tangent curve will touch the collimator surface but once as the tangent curve is varied by changing z_1 (or θ). For all other z_1 (for the given r) the tangent line will be above the collimator. Thus, we can find the minimum in the tangent line for each r. The minima will coincide with the collimator surface. Thus, calculate

$$\frac{\partial}{\partial z_1}(z_1 + r \cot \theta) = 0 \qquad (5)$$

or

$$r \frac{\partial(\cot\theta)}{\partial z_1} = -1 \qquad (6)$$

where the derivative in equation (6) can be evaluated from equation (3). The simultaneous solution of equations (4) and (6) will then determine equation (1), the collimator shape. For convenience, in equation (5) we can differentiate with respect to a well behaved, monotonic function of z_1 if it will simplify matters.

Thus, in principle, given S(z) and $F(\theta)$ we can determine that $z = f(r)$ which describes the cylindrical collimator surface. For a gamma current detector we would desire that $F(\theta) = \cos \theta$.

III. Example of Collimator Geometry

To illustrate the manner of calculating the equation describing the collimator surface let us consider the case in which the sensitivity per unit length of the line detector is a constant and the desired angular response is that required for a gamma current measurement, i.e., in which

$$S(z) = 1 \qquad (7)$$

and

$$F(\theta) = \cos\theta \qquad (8)$$

From equation (3), then,

$$\mathcal{S}(z_1) = \int_{z_1}^{0} S(z)dz = -z_1 \qquad (9)$$

and thus

$$\cos\theta = -z_1 \qquad (10)$$

The tangent line from equation (4) then becomes

$$z = z_1 - r \frac{z_1}{\sqrt{1-z_1^2}} \qquad (11)$$

From equation (6) we have

$$r \frac{\partial}{\partial z_1} \left[\frac{z_1}{\sqrt{1-z_1^2}} \right] = 1 \qquad (12)$$

or

$$\frac{r}{(1-z_1^2)^{3/2}} = 1 \qquad (13)$$

or

$$r^{2/3} = 1 - z_1^2 \qquad (14)$$

or

$$z_1 = -\sqrt{1-r^{2/3}} \qquad (15)$$

remembering that z_1 is negative.

Substituting for z_1 in equation (11) we then have

$$z = -\sqrt{1-r^{2/3}} + r^{2/3}\sqrt{1-r^{2/3}} \tag{16}$$

or

$$z = -(1-r^{2/3})^{3/2} \tag{17}$$

This last result can be written in a symmetrical form as

$$z^{2/3} + r^{2/3} = 1 \tag{18}$$

Thus, in equation (18) we have a rather simple expression for the collimator surface which is plotted in figure 3. This curve has an interesting property that the length of the tangent curve (equation (11)) between the z and r axes is unity. This can be easily seen with the help of equation (10) which gives the intercept of the tangent curve with the z axis. This constant length of the tangent line between the two axes can be used as a geometric construction of the curve describing the collimator surface.

Using this procedure we can consider other forms of sensitivity per unit length of the line detector and/or angular dependence of the detector output. We could even take these latter quantities expressed in numerical form and solve for the collimator geometry with a computer.

IV. Determination of Detector Sensitivity Characteristics

Instead of calculating the collimator shape from a given sensitivity per unit length of the line detector and a desired angular output of the detector system, we may wish to take a given collimator geometry and angular dependence of the detector output and from these calculate the required sensitivity per unit length of the line detector. Thus, we are given $f(r)$ and $F(\theta)$. In this case, we can equate the slope of the tangent line (equation (4)) to the slope of the collimator, giving at the point of tangency

$$\cot\theta = \frac{\partial f(r)}{\partial r} \tag{19}$$

With $f(r)$ and equation (4) we can relate θ and z_1 and with the desired $F(\theta)$ we can determine $S(z\ )$ from equation (3) as

$$S(z) = -\frac{\partial \mathcal{S}(z)}{\partial z} \tag{20}$$

As a result we can attack this detector geometry design problem from two directions, specifying either $S(z)$ or $f(r)$. Actually we have a third function, $F(\theta)$, to consider which we have taken as $\cos\theta$ for a gamma current detector. Hence, we really have three functions, of which we may specify two, determining the third.

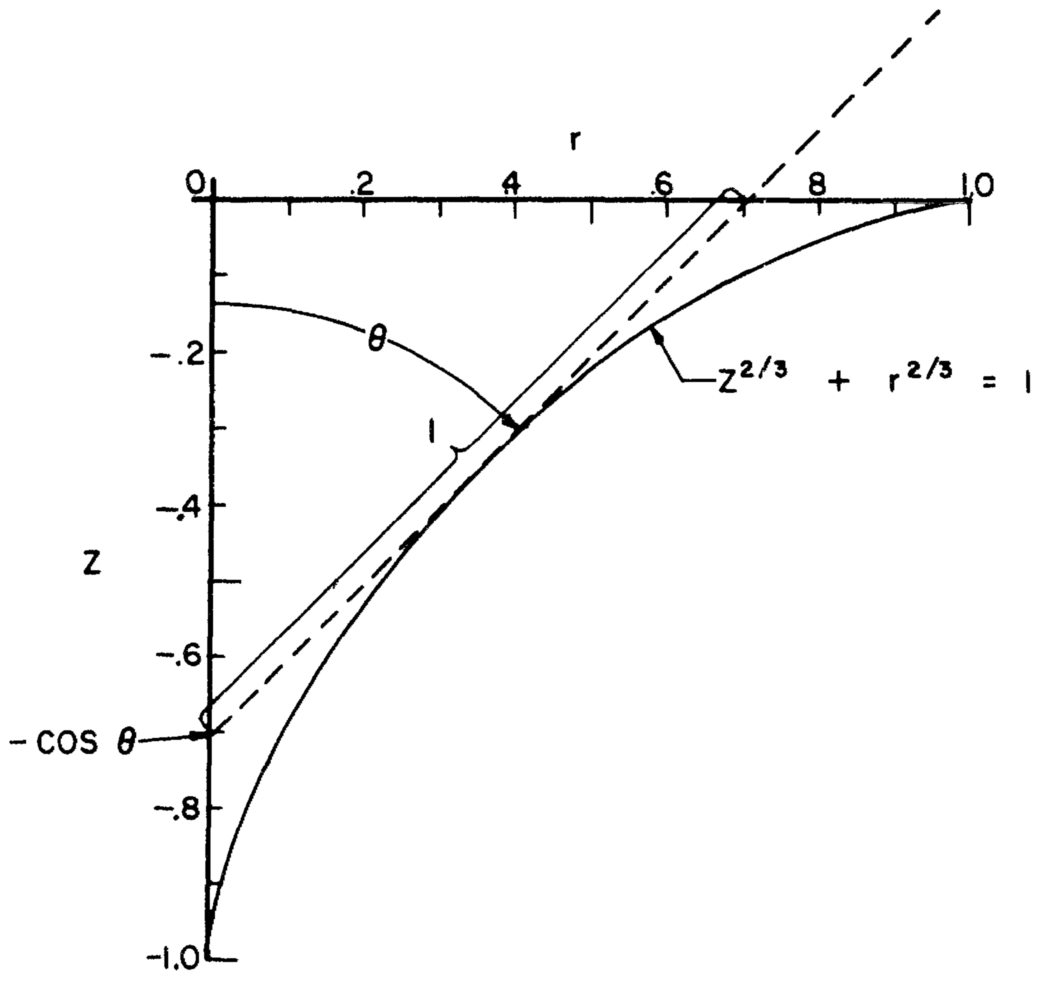

Figure 3. NORMALIZED COLLIMATOR GEOMETRY FOR LINE DETECTOR OF UNIFORM SENSITIVITY PER UNIT LENGTH.

V. Summary

In summary, then, we may be able to measure γ_z with a line detector inside a collimator recessed in the ground plane as illustrated in figure 1. Since the collimator is recessed in the ground plane it may be possible to build the collimator-detector structure of rather large size to insure that the characteristic dimensions are much larger than a gamma ray mean free path in the collimator material. For a case as illustrated in figure 3, the collimator surface is free of edges or corners giving a good collimating action. However, there will be radiation scattered from the collimator to the detector. Hopefully, this scattered radiation will not be a serious problem.

Fortunately, this detector geometry is a flexible one. For a desired angular dependence of the detector output we are still free (within certain limits) to specify either the sensitivity per unit length of the line detector or the collimator geometry and calculate the other. By judicious choice of the combination of collimator geometry and line detector sensitivity per unit length, perhaps to a certain degree various imperfections (radiation scattering, etc.) can be minimized.

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