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## Sensor and Simulation Notes

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SENSOR AND SIMULATION NOTES

NOTE 194

Considerations in SGEMP Simulation

May 1974

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ABSTRACT

This note is a compilation of five Tank Physics Memos which were prepared to address several aspects of satellite simulation vacuum tank design requirements.

Memo #1 (prepared July 1972) presents formulae, tables, and graphs useful in determining energy loss and scattering of electrons moving through several types of material.

Memo #2 (prepared July 1972) gives analytical approximations for the spectrum of electrons resulting from the slowing down of a beam of initially mono-energetic electrons and for their bremsstrahlung spectrum.

Memo #3 (prepared August 1972) discusses the factors which determine the allowable gas pressure in the test chamber with the conclusion that it should not be greater than  $10^{-4}$  torr.

Memo #4 (prepared September 1972) presents the current status of considerations on the tank radius including d.c. effects, cavity mode effects, and satellite mode effects.

Memo #8 (prepared October 1972) covers electrical design considerations for the design of a test chamber including suppression of photoelectrons from the tank wall, Q-spoiling, and d.c. charge return.

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## ENERGY LOSS OF ELECTRONS

In this note we collect formulae, tables, and graphs relating to energy loss and scattering of electrons moving through various materials of interest.

### 1. ENERGY LOSS RATE

The loss of kinetic energy  $W$  per unit track length  $s$  is given by Bethe's formula (Reference 1),

$$-\frac{dW}{ds} = \frac{2\pi NZe^4}{mv^2} [1] \quad (1)$$

where

$$[1] = \ln \left[ \frac{(mc^2)^2 (\gamma - 1) (\gamma^2 - 1)}{2I^2} \right] - \left( \frac{2}{\gamma} - \frac{1}{\gamma^2} \right) \ln 2 \\ + \frac{1}{\gamma^2} + \frac{1}{8} \left( 1 - \frac{1}{\gamma} \right)^2 . \quad (2)$$

In these formulae,  $N$  is the density of atoms,  $Z$  the atomic number,  $e$ ,  $m$ , and  $v$  the electron charge, rest mass, and velocity,  $c$  is the velocity of light, and

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} , \quad \beta = \frac{v}{c} . \quad (3)$$

The quantity  $I$  is called the mean excitation potential, and is given in Table 4 of Reference 1 for several materials. In our Table 1 we give  $I$  for some materials of interest to us. Note that in this table, the composite materials  $\text{CH}_2$  and air are treated differently, in that

the parameters given for air are for the average air atom N or O, whereas for CH<sub>2</sub> one molecule of CH<sub>2</sub> is to be regarded as an atom. Hence Z for air is the average 7.2, while for CH<sub>2</sub> it is the total 6 + 1 + 1 = 8.

Table 1. Energy loss and scattering parameters.

| Z   | MATERIAL        | I(eV) | $\Gamma$ | Z $\Gamma$ | (2Z $\Gamma$ ) <sup>-1</sup> |
|-----|-----------------|-------|----------|------------|------------------------------|
| 1   | H               | 15.6  |          |            |                              |
| 6   | C               | 76.4  |          |            |                              |
| 8   | CH <sub>2</sub> | 51.2  | 0.155    | 1.24       | 0.40                         |
| 7.2 | air             | 80.5  | 0.269    | 1.94       | 0.26                         |
| 13  | Al              | 150   | 0.276    | 3.59       | 0.14                         |
| 26  | Fe              | 243   | 0.278    | 7.23       | 0.069                        |
| 82  | Pb              | 737   | 0.296    | 24.3       | 0.021                        |

## 2. MULTIPLE SCATTERING OF ELECTRONS

The angular width of a beam of electrons is gradually increased in transitting material, principally by nuclear scattering. If  $\overline{\theta^2}$  is the mean square angle of deviation from the beam axis, then the increase of  $\overline{\theta^2}$  per unit track length is given by Williams' theory (Reference 2),

$$\frac{d\overline{\theta^2}}{ds} = \frac{8\pi NZ^2 e^4}{\gamma^2 m^2 v^4} [2] \quad (4)$$

where, for the Thomas-Fermi model of atomic screening,

$$[2] = \ln \left( \frac{131 \sqrt{\gamma^2 - 1}}{Z^{1/3}} \right) . \quad (5)$$

Equation 4 is valid when  $\overline{\theta^2} \ll 1$ . In Reference 3, Longmire and Longley extend the model by introducing the obliquity factor

$$\eta \equiv \frac{1}{(\cos\theta)} \quad (6)$$

$$\approx 1 + 1/2 \overline{\theta^2} \text{ for small } \overline{\theta^2} .$$

Thus the mean forward progress  $dx$  of the beam is related to the increment of track length  $ds$  by

$$dx = \frac{1}{\eta} ds . \quad (7)$$

The equation for  $\eta$  is taken over from Equation 6

$$\frac{d\eta}{ds} = \frac{1}{2} \frac{d\overline{\theta^2}}{ds} . \quad (8)$$

The advantage of  $\eta$  is that it can be allowed to become large, unlike  $\theta$ .

### 3. MEAN RANGE OF ELECTRONS

The differential equations for energy loss and multiple scattering can be expressed in terms of mean forward progress  $x$  instead of track length. We then have

$$\frac{d\eta}{dx} = f(W)\eta \quad (9)$$

$$\frac{dW}{dx} = -g(W)\eta \quad (10)$$

where

$$f(W) = \frac{4\pi NZ^2 e^4 \gamma^2}{(mc^2)^2 (\gamma^2 - 1)^2} [2] \quad (11)$$

$$g(W) = \frac{2\pi NZ e^4 \gamma^2}{mc^2 (\gamma^2 - 1)} [1] . \quad (12)$$

Dividing Equation 9 by Equation 10 and using  $W = (\gamma - 1)mc^2$ , we find

$$d\eta = - \frac{2Z}{(\gamma^2 - 1)} \frac{[2]}{[1]} d\gamma . \quad (13)$$

It turns out that the ratio

$$\Gamma \equiv \frac{[2]}{[1]} \quad (14)$$

is very nearly independent of energy  $\gamma$ . In Table 1 values of  $\Gamma$  are

given for some substances. If  $\Gamma$  is assumed constant, Equation 13 can be integrated, with the result

$$\eta = 1 + Z\Gamma\lambda n \left[ \frac{(\gamma_0 - 1)(\gamma + 1)}{(\gamma_0 + 1)(\gamma - 1)} \right]. \quad (15)$$

This result shows how the obliquity factor  $\eta$  increases as the energy  $\gamma$  decreases from its initial value  $\gamma_0$ .

When Equation 15 is used for  $\eta$  in Equation 10, integration yields the mean range as a function of initial energy. On the other hand, if the factor  $\eta$  in Equation 10 is replaced by unity, integration yields the extreme range. These ranges are graphed versus energy in Figure 1 for  $\text{CH}_2$ , air, Al, and Pb.

A comparison of the theoretical model used above with experimental data on the transmission of electrons through aluminum foils is given in Reference 3. The comparison is very favorable. The fraction of electrons of given initial energy transmitting the foil as a function of foil thickness is sketched in Figure 2, in which the mean and extreme ranges are indicated. There is a large spread in the forward ranges of individual electrons, due to the multiple scattering.

#### 4. SCATTERING RANGE OF ELECTRONS

For some purposes it is useful to know the mean distance electrons travel before being scattered substantially. From Equation 15 we can find the kinetic energy  $W = (\gamma - 1)mc^2$  at which the obliquity factor has increased to the value  $\eta = 1 + \Delta\eta$ ,

$$\frac{W}{W_0} = \left[ 1 + A \left( 1 + \frac{1}{2} \frac{W_0}{mc^2} \right) \right]^{-1}. \quad (16)$$

where

$$A = \exp \left( \frac{\Delta\eta}{Z\Gamma} \right) - 1. \quad (17)$$



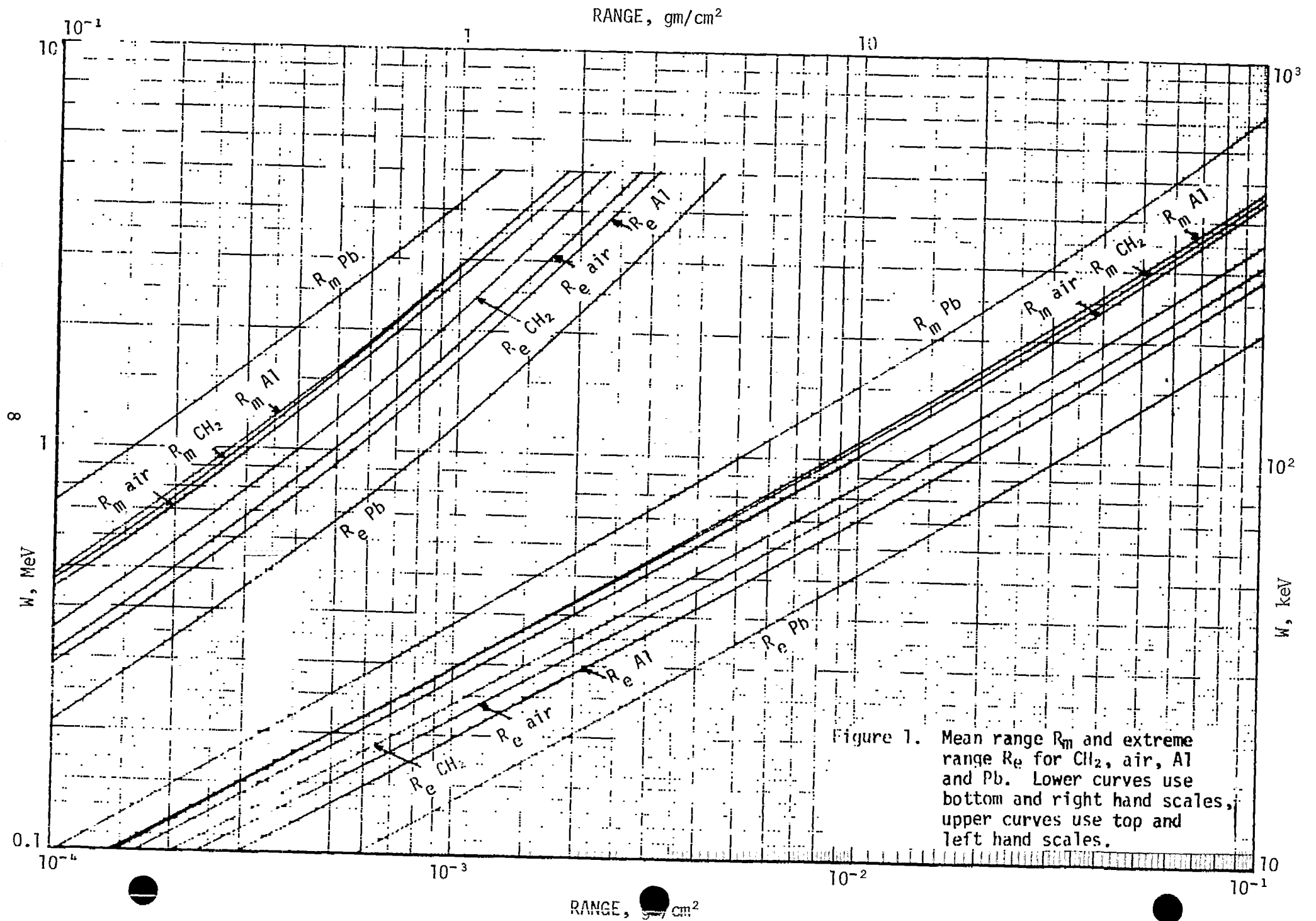


Figure 1. Mean range  $R_m$  and extreme range  $R_e$  for CH<sub>2</sub>, air, Al and Pb. Lower curves use bottom and right hand scales, upper curves use top and left hand scales.

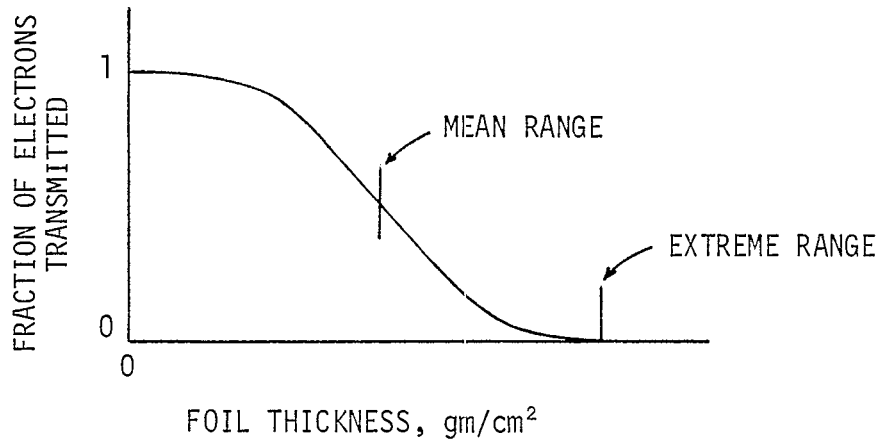


Figure 2. Fraction of electrons transmitted through foils as a function of foil thickness.

For example, the scattering effect is beginning to be substantial when

$$\sqrt{\theta^2} \approx 0.7 \text{ radians} \approx 40 \text{ degrees.} \quad (18)$$

At this point we have

$$\Delta\eta \approx \frac{\theta^2}{2} \approx \frac{1}{4}. \quad (19)$$

Thus the parameter  $\Delta\eta/Z\Gamma$  is small compared with unity, and we may expand the exponential in Equation 17, with the result

$$A \approx \frac{\Delta\eta}{Z\Gamma}. \quad (20)$$

In addition if  $W_0 \ll 2mc^2 \approx 1 \text{ MeV}$ , we may approximate Equation 16 by

$$\frac{W}{W_0} \approx 1 - \frac{\Delta\eta}{Z\Gamma}. \quad (21)$$

This approximation is quite good up to  $W \approx 100 \text{ keV}$ . Figure 1 shows that for  $W < 100 \text{ keV}$ , the extreme range  $R_e$  is approximately proportional to  $W_0^2$ . Therefore if  $\Delta R$  is the range for substantial scattering, we have

$$\frac{\Delta R}{R_e} \approx - \frac{2\Delta W}{W_0} \approx \frac{2\Delta\eta}{Z\Gamma}. \quad (22)$$

If we use the conditions 18 and 19 to define substantial scattering, the range for substantial scattering is

$$\Delta R \approx R_e / 2Z\Gamma. \quad (23)$$

The quantity  $(2Z\Gamma)^{-1}$  is listed in Table 1. The ratio of scattering range to energy loss range is smaller for elements with higher Z. Note that for aluminum and higher Z elements, the scattering becomes substantial well before much energy is lost.

### 5. IONIZATION BY ELECTRONS

Figure 3 is a graph of the product  $\sigma_i v$  for electrons on air atoms, taken from Reference 3. Here  $\sigma_i$  is the cross section, per air atom, for producing an additional free electron, and  $v$  is the velocity of the incident electron. The rate of production of secondary electrons, per primary electron, is  $N\sigma_i v$ , where  $N$  is the density of air atoms (not molecules). The rate  $N\sigma_i v$  includes the production of more than one electron in ionization events followed by Auger effect.

The average energy lost by the primary electron per secondary electron produced is given in Table 2 (taken from Reference 3) for various primary energies.

Table 2. Energy loss in air per secondary electron.

|                             |   |      |      |      |      |      |
|-----------------------------|---|------|------|------|------|------|
| PRIMARY ENERGY, MeV         | = | 0.05 | 0.25 | 0.5  | 1    | 1.5  |
| <u>eV LOST</u><br>SECONDARY | = | 80.0 | 84.4 | 85.4 | 86.4 | 86.9 |

Many of the secondary electrons have sufficient energy to make further ionization. If time is available for all secondaries, tertiaries, etc., to slow down, the total ionization is approximately one electron per 34 eV lost by the primary electron. The rate of

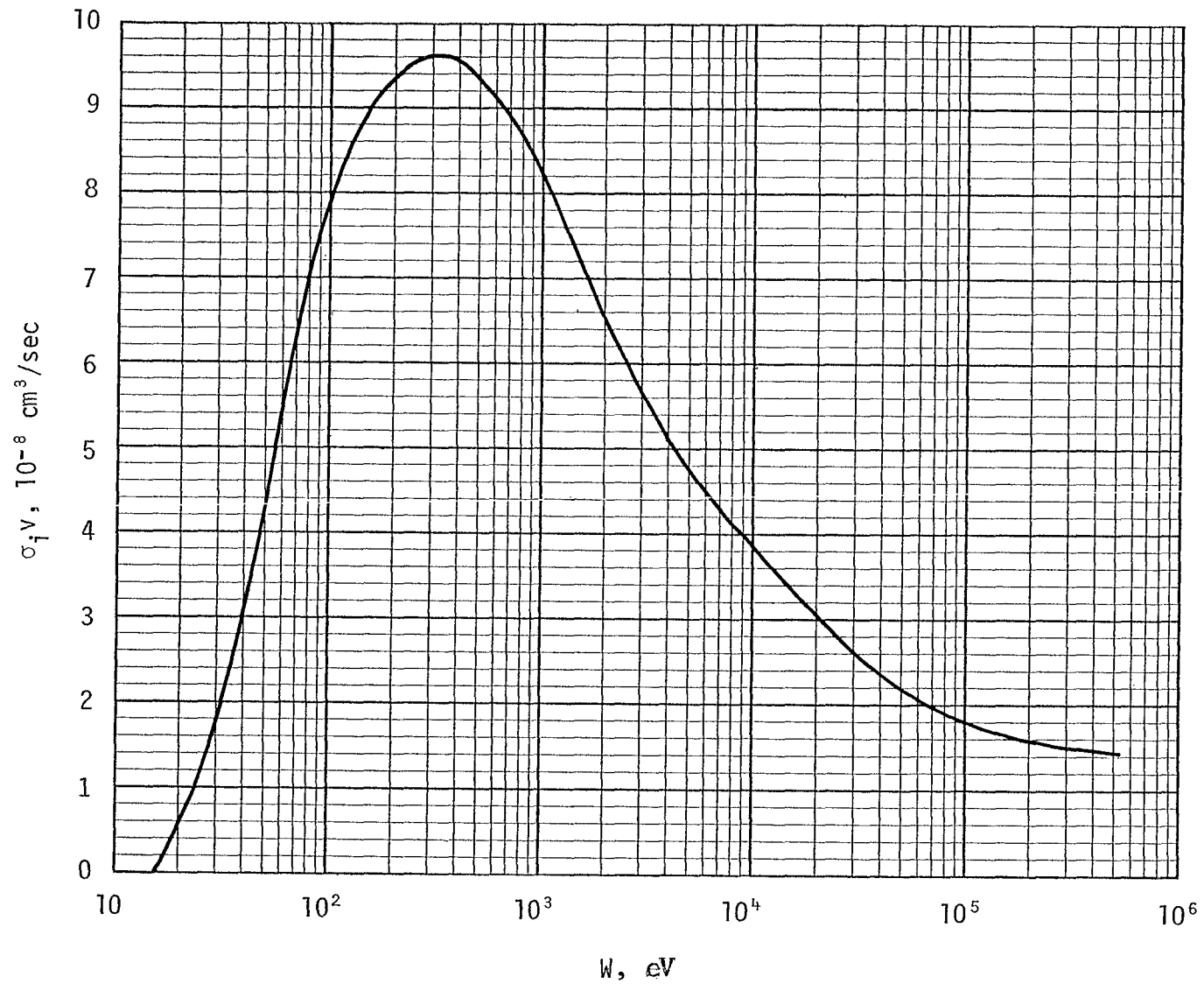


Figure 3. Rate of production of secondary electrons, per primary electron of energy  $W$  moving through unit density of air atoms.

multiplication of the secondaries can be estimated from Figure 3, and is discussed in Reference 3. The initial energy spectrum of the secondary electrons is such that there are approximately equal amounts of secondary energy in each decade of electron energy, from a few eV up to the primary energy.

## REFERENCES

1. See the review article by Bethe and Ashkin in "Experimental Nuclear Physics," E. Segre, editor, Vol. I, Wiley, 1953; p. 166.
2. See the discussion in Mott and Massey, "The Theory of Atomic Collisions," Third Edition, Oxford, 1965; p. 467.
3. Longmire, C. L., and H. J. Longley, Improvements in the Treatment of Compton Current and Air Conductivity in EMP Problems, MRC-N-2, Mission Research Corporation, October 1971 (Unclassified). Also AFWL Theoretical Note 126.

## THE BREMSSTRAHLUNG SPECTRUM FROM A SLOWING DOWN SPECTRUM OF ELECTRONS

In this memo we find analytical approximations for the spectrum of electrons resulting from the slowing down of a beam of initially monoenergetic electrons, and for the spectrum of bremsstrahlung emitted by the electrons.

### 1. THE SLOWING DOWN SPECTRUM OF ELECTRONS

We assume a steady source injects electrons with kinetic energy  $W_0$  into a material medium. As a result of energy loss, the electrons will establish a continuous distribution of energies from zero to  $W_0$ . If  $S_e(W)$  is the number of electrons per unit energy  $W$  in this spectrum, conservation of electrons requires that

$$-S_e(W) \frac{dW}{dt} = \text{constant}, \quad (1)$$

where  $-\frac{dW}{dt}$  is the rate of energy loss by an electron of energy  $W$ ,\* which has been discussed in Memo 1 of this series. Since for bremsstrahlung production we are interested in heavy elements, we use the curve for the extreme range  $R_e$  for Pb. From Figure 1 of Memo 1 we see that for electron energies between 10 keV and 200 keV,

$$R_e \sim W^{1.7} \quad (2)$$

---

\* It is assumed here that energy is lost in small amounts per collision.

within a few percent. Therefore

$$\frac{dW}{dt} = v \frac{dW}{dR_e} \sim \sqrt{W} / \left( \frac{dR_e}{dW} \right)$$

$$\sim W^{-0.2}.$$
(3)

Then from Eq. 1 we find

$$S_e(W) \sim W^{0.2} \text{ for } W \leq W_0,$$

$$= 0 \text{ for } W \geq W_0.$$
(4)

As stated above, this spectrum is for a steady source of electrons of energy  $W_0$ . If the source is pulsed, Eq. (4) gives the time-integrated electron spectrum.

The slowing down time of 100 keV electrons in Pb is of the order of  $10^{-13}$  second, if the electron spends all of its time in the Pb. If the electron passes back and forth through a thin foil, the slowing down time will be determined mainly by its track length outside the foil.

## 2. THE BREMSSTRAHLUNG SPECTRUM FOR MONOENERGETIC ELECTRONS

The rate of emission of x-ray energy in the quantum energy range  $du$  by an electron with kinetic energy  $W$  moving through material with atomic number  $Z$  is given in Reference 1 as

$$S_x(u, W) du \sim du \frac{Z^2}{\sqrt{W}} \ln \left[ \frac{(\sqrt{W} + \sqrt{W-u})^2}{u} \right] f(\xi, \xi_0),$$
(5)



where

$$f(\xi, \xi_0) = \frac{\xi}{\xi_0} \frac{1 - e^{-2\pi\xi_0}}{1 - e^{-2\pi\xi}},$$

$$\xi = \frac{Z}{137} \left( \frac{c}{v} \right), \quad \xi_0 = \frac{Z}{137} \left( \frac{c}{v_0} \right). \quad (6)$$

Here  $v_0$  and  $v$  are the velocities of the electron just before and just after emission of an x ray of energy  $u$ , and  $c$  is the velocity of light. We shall be dealing with high  $Z$ , of the order of 80. In addition,  $c/v_0$  and  $c/v$  are not less than about 2. Therefore  $\xi$  and  $\xi_0$  are not less than about unity, and the exponentials  $\exp(-2\pi\xi_0)$  and  $\exp(-2\pi\xi)$  are negligible compared to unity. Therefore

$$f(\xi, \xi_0) \approx \frac{v_0}{v} = \sqrt{\frac{W}{W-u}}. \quad (7)$$

The constant of proportionality in Eq. (5) is independent of  $W$ ,  $u$ , and  $Z$ . Once  $Z$  has been chosen we may also drop the factor  $Z^2$  from spectral considerations. Using the approximation (7) and defining

$$x \equiv u/W, \quad (8)$$

we find

$$S_x(u, W) \sim \frac{1}{\sqrt{W}} g(x),$$

$$g(x) = \frac{1}{\sqrt{1-x}} \ln \left[ \frac{1 + \sqrt{1-x}}{\sqrt{x}} \right]. \quad (9)$$

The function  $g(x)$  is graphed in Figure 1 for  $x$  in the range 0.05 to 1. An approximation adequate for our purposes is

$$g(x) \approx x^{-0.3} \quad . \quad (10)$$

With this approximation, the x-ray energy spectrum from electrons of energy  $W$  is

$$\begin{aligned} S_x(u, W) &\sim \frac{1}{W^{0.2} u^{0.3}} \quad , \quad u < W \\ &= 0, \quad u > W \quad . \end{aligned} \quad (11)$$

### 3. THE BREMSSTRAHLUNG SPECTRUM FROM SLOWING ELECTRONS

For the slowing-down spectrum of electrons, Eq. (4), the x-ray energy spectrum  $\bar{S}(u)$  is

$$\begin{aligned} \bar{S}(u) &= \int_u^W S_x(u, W) S_e(W) dW \\ &\sim (W_0 - u)/u^{0.3} \quad . \end{aligned} \quad (12)$$

To normalize  $\bar{S}(u)$  so that

$$\int_0^W \bar{S}(u) du = A \quad \text{energy units/unit area,} \quad (13)$$

we need

$$\begin{aligned} \bar{S}(u) &= \frac{1.19A}{W_0^{1.7}} \frac{(W_0 - u)}{u^{0.3}} \\ &= \frac{1.19A}{W_0} \left[ \left( \frac{W_0}{u} \right)^{0.3} - \left( \frac{u}{W_0} \right)^{0.7} \right] \quad . \end{aligned} \quad (14)$$

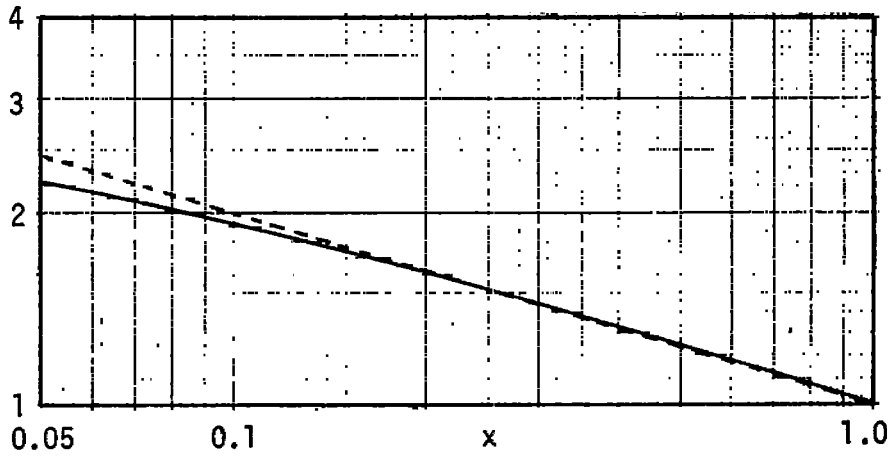


Figure 1. The solid curve is the function  $g(x)$ . The dashed line is  $x^{-0.3}$ .

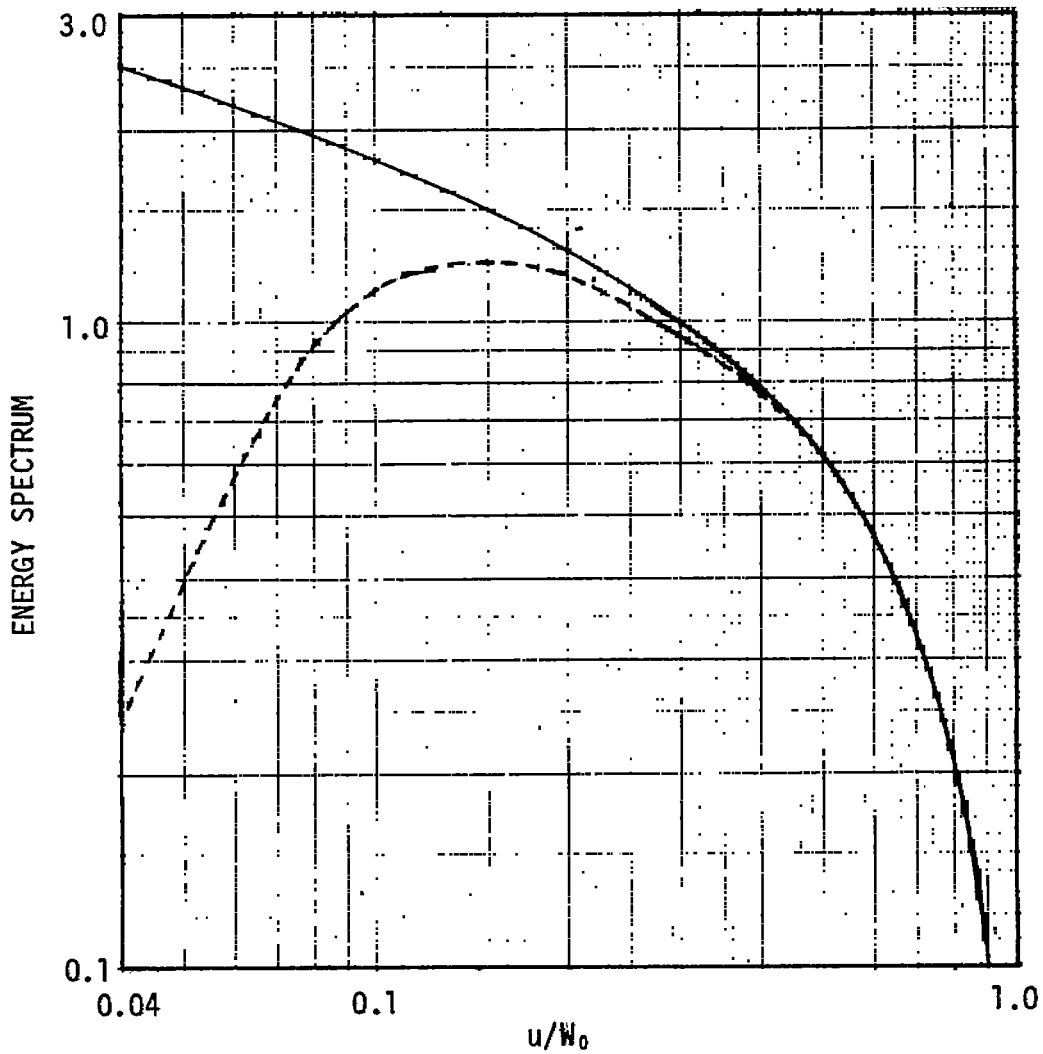


Figure 2. The solid curve is  $W_0 \bar{S}(u)/1.19A$ . The dashed curve is corrected for target absorption for the case  $u_1 = 0.1W_0$ .

We repeat that  $\bar{S}(u)$  is the energy spectrum of x rays. To find the photon spectrum, i.e., the number of photons per unit photon energy  $u$ , divide  $\bar{S}(u)$  by  $u$ .

The function  $W_0 \bar{S}(u)/1.19A$  is graphed in Figure 2.

#### 4. ABSORPTION IN X-RAY TARGET

For lower energy x rays, absorption in the target is a serious effect. We assume that the target is thin compared with the electron range, so that the x rays are produced uniformly in depth in the target. For x rays leaving in the direction normal to the target foil, the mean attenuation factor  $\eta_1$  is

$$\eta_1 = \frac{1}{d} \int_0^d e^{-\mu x} dx = \frac{1}{\mu d} \left[ 1 - e^{-\mu d} \right]. \quad (15)$$

Here  $d$  is the thickness of the target foil and  $\mu$  is the absorption coefficient at the x-ray energy considered. In case x rays leaving the foil in other directions than the normal contribute appreciably,  $d$  should be increased by a mean obliquity factor.

The attenuation factor  $\eta_1$  has the asymptotic forms

$$\begin{aligned} \eta_1 &= 1 - \frac{\mu d}{2} + \frac{(\mu d)^2}{6} + \dots, \quad \mu d \ll 1, \\ &= \frac{1}{\mu d}, \quad \mu d \gg 1. \end{aligned} \quad (16)$$

The latter form indicates that when the target is thick for x rays, only the fraction made in the first absorption length from the surface can escape.

It is convenient to express  $\mu$  in  $\text{cm}^2/\text{gm}$  and the thickness  $d$  in  $\text{gm}/\text{cm}^2$ .

In heavy elements  $\mu$  is approximately proportional to  $u^{-2.5}$ , except near absorption edges, the locations of which depend on the particular element chosen for the target. We shall assume that

$$\mu d \approx (u_1/u)^{2.5} \quad . \quad (17)$$

Here  $u_1$  is the photon energy for which the target is one absorption length thick. In designing the x-ray source, effort will be made to make  $u_1$  as small as possible compared with  $W_0$ . For example,  $u_1 = 0.1 W_0$  may be achievable.

To find the spectrum of x rays leaving the target, the spectrum  $\bar{S}(u)$  must be multiplied by the attenuation factor  $\eta_1$ . If  $\bar{S}(u)$  is normalized as in Eq. (14), the energy in the emerging spectrum will be less than A. For the case where  $u_1 \ll W_0$ , integration leads to the approximate result

$$\int_0^W \eta_1 \bar{S}(u) du \approx A \left[ 1 - 1.6 \left( \frac{u_1}{W_0} \right)^{0.7} \right] \quad (18)$$

$$\approx 0.7A \text{ if } u_1 = 0.1 W_0 .$$

Thus A would have to be increased to maintain a given energy flux. For some uses the correction factor in Eq. (18) may be ignored.

The spectrum corrected for target attenuation is graphed in Figure 2.

## 5. ATTENUATION IN WINDOW

If the x-ray source target is separated from the test chamber by a thin window, lower energy x rays will also be absorbed by the window. To minimize window absorption, beryllium is an ideal window

material. A 1-mm plate of Be, supported by a rib structure if necessary, would give an attenuation factor

$$\eta_{\text{Be}} \approx \exp [ - (5.3\text{keV}/u)^3 ]. \quad (19)$$

The attenuation for  $u \approx 10 \text{ keV}$  is practically negligible. Approximately the same attenuation factor would result from an aluminum foil 1 mil thick, or from a titanium, iron, or molybdenum foil 0.1 mil thick.

## VACUUM REQUIREMENTS

In this memo we discuss the factors which determine the allowable gas pressure in the test chamber. It is concluded that the pressure should not be greater than  $10^{-4}$  Torr.

### 1. GENERAL CONFIGURATION AND CONDITIONS

We envision a spherical tank with radius of the order of 10 meters, with a test object with effective radius of the order of 2 meters at its center. Following a burst of x rays, photoelectrons are ejected from the test object. Some of these strike and become stuck in the chamber walls, while others (the lower energy ones) return to the test object. The potential difference developed between the test object and the wall ranges from a few kilovolts to a few tens of kilovolts.

Limitations are placed on the tank pressure by the following considerations:

- a. Scattering and slowing of the photoelectrons by the residual gas must be negligible;
- b. Photoelectrons created in the gas by the x rays must be negligible compared with those ejected from the test object;
- c. Ionization produced by the photoelectrons in the gas must not short out the electric fields too quickly;
- d. The electric field must not cause breakdown of the residual gas.

## 2. SCATTERING AND SLOWING OF PHOTOELECTRONS

From the estimates of the x-ray spectrum in Memo 2 of this series, it is clear that there will be few photons with energies less than 2 or 3 keV. It is therefore clear that the principal drivers of the electric fields in the cavity at large will be the photoelectrons with energies greater than a few keV. Let us then choose the pressure so that the mean range of a 1-keV electron is equal to the tank radius, or  $10^3$  cm. From Memo 1, we find (by extrapolation) that the mean range in air of a 1-keV electron is about  $2 \times 10^{-6}$  gm/cm<sup>2</sup>. We may therefore have an air density of  $2 \times 10^{-9}$  gm/cm<sup>3</sup>. Since air density at 20°C and 1 Torr is  $1.6 \times 10^{-6}$  gm/cm<sup>3</sup>, this range requirement would be met by choosing the pressure

$$p \leq 10^{-3} \text{ Torr} . \quad (1)$$

Note that for 10-keV electrons the mean range is  $1.7 \times 10^{-4}$  gm/cm<sup>2</sup>, or about 100 times the tank radius. Since  $R_m$  is roughly proportional to  $W^2$ , 10-keV electrons would therefore lose approximately 0.5 percent of their energy to the gas in moving through the tank radius.

The 1-keV electrons will not move as far as the tank radius, since they will be turned by the electric field set up by the more energetic electrons. Actually, they move through the order of 10 percent of the tank radius, and so lose only the order of 10 percent of their energy to the gas.

The multiple scattering of electrons with energy 10 keV and higher is small under the condition (1). From Equation 15 of Memo 1 we find the obliquity factor

$$\eta \approx 1 + Z\Gamma \ln\left(\frac{W_0}{W}\right) . \quad (2)$$

Using  $Z\Gamma = 1.94$  for air and  $W = 0.995 W_0$ , we have

$$1 + \frac{\overline{\theta^2}}{2} \approx \eta \approx 1 + (1.94)(0.005)$$



or

$$\sqrt{\theta^2} \approx 0.14 \text{ radians} . \quad (3)$$

For the 1-keV electrons, the scattering would be substantial.

### 3. PHOTOELECTRONS PRODUCED IN GAS

In Memo 2 of this series we found an approximate expression for the x-ray energy spectrum produced in a high-Z target. The energy in photon energy interval  $du$  is

$$\bar{S}(u)du = 1.19A \left[ \left( \frac{W_0}{u} \right)^{0.3} - \left( \frac{u}{W_0} \right)^{0.7} \right] \frac{du}{W_0} . \quad (4)$$

The total energy in the spectrum is

$$\int_0^{W_0} \bar{S}(u)du = A \frac{\text{ergs}}{\text{cm}^2} . \quad (5)$$

In Equation 4 the units of  $u$  and  $W_0$  are arbitrary as long as they are the same; we shall use keV as the units for  $u$  and  $W_0$ . The photon spectrum is (since  $1 \text{ keV} = 1.6 \times 10^{-9} \text{ ergs}$ )

$$\begin{aligned} \phi(u) &= \bar{S}(u) / (1.6 \times 10^{-9} u) \\ &= \frac{7.4 \times 10^8 A}{W_0 u} \left[ \left( \frac{W_0}{u} \right)^{0.3} - \left( \frac{u}{W_0} \right)^{0.7} \right] \frac{\text{photons}}{\text{keV cm}^2} . \end{aligned} \quad (6)$$

To correct for absorption in the x-ray source target, we multiply by the attenuation factor,

$$\eta_1 = \left( \frac{u}{u_1} \right)^{2.5} \left\{ 1 - \exp \left[ - \left( \frac{u_1}{u} \right)^{2.5} \right] \right\} . \quad (7)$$

Here  $u_1$  is the photon energy for which the target is one absorption length thick.

To correct for absorption in the window between the x-ray source and the test chamber, we multiply by a second attenuation factor,

$$\eta_2 = \exp\left[-\left(\frac{u_2}{u}\right)^3\right]. \quad (8)$$

Here  $u_2$  is the photon energy at which the window is one absorption length thick.

For a gold foil 0.1 mil or 0.005 gm/cm<sup>2</sup> in thickness,  $u_1 \approx 10$  keV. For a beryllium window 40 mils (1 mm) or 0.18 gm/cm<sup>2</sup> thick,  $u_2 \approx 5$  keV.

In the neighborhood of  $u = 10$  keV, where most of the air absorption will occur, the absorption coefficient of air is

$$\mu_a \approx 4.5 \left(\frac{u_a}{u}\right)^3 \frac{\text{cm}^2}{\text{gm}}, \quad u_a = 10 \text{ keV}. \quad (9)$$

We shall adjust the constant A in Equation 4 to give the desired flux at the center of the sphere. Since the x-ray source will be placed at the edge of the sphere, and since the air is distributed throughout the volume of the sphere, the air will see a different average flux from that seen at the center. In fact, for a point isotropic source at the edge of a sphere, the flux averaged over the volume is 3/2 times the flux at the center.

Thus, if M is the total mass (grams) of air in the tank, the total number of photoelectrons produced in the air is

$$N_{ea} = \frac{3}{2} M \int_0^{W_0} \Phi(u) \eta_1 \eta_2 \mu_a du. \quad (10)$$

To evaluate this integral approximately, we may drop the term  $\left(\frac{u}{W_0}\right)^{0.7}$  in Equation 6, since it is only ~ 10 percent of the other term in the region of interest. We assume  $u_1 \approx 10$  keV,  $u_2 \approx 5$  keV. Then the integrand  $I_a$  in Equation 10 has the approximate  $u$  dependence:

$$\begin{aligned}
I_a &\sim u^{-4.3} \text{ for } u > u_1 \\
&\sim u^{-1.8} \text{ for } u_2 < u < u_1 .
\end{aligned}
\tag{11}$$

An overestimate of  $N_{ea}$  is therefore given by

$$N_{ea} \approx \frac{3M}{2} \left( \frac{7.4 \times 10^8 A}{W_0^{0.7}} \right) \left( \frac{4.5 u_a^3}{u_1^{2.5}} \right) \int_0^\infty e^{-(u_2/u)^3} \frac{du}{u^{1.8}} .
\tag{12}$$

The value of the integral here is  $1.16/u_2^{0.8}$ . Thus, collecting numerical factors, we have

$$N_{ea} = 5.7 \times 10^9 \frac{Au^3 M}{W_0^{0.7} u_1^{2.5} u_2^{0.8}} \text{ photoelectrons} .
\tag{13}$$

(A in erg/cm<sup>2</sup>;  $W_0$ , u in keV; M in grams)

As an example we take:

$$\left.
\begin{aligned}
A &= (10^{-3} \text{ cal/cm}^2) = 4.2 \times 10^4 \text{ erg/cm}^2, \\
W_0 &= 100 \text{ keV}, u_1 = 10 \text{ keV}, u_2 = 5 \text{ keV}, \\
u_a &= 10 \text{ keV}, \\
M &= \left( \frac{4\pi}{3} 10^9 \text{ cm}^3 \right) \left( 2 \times 10^{-9} \frac{\text{gm}}{\text{cm}^3} \right) = 8.4 \text{ grams} .
\end{aligned}
\right\}
\tag{14}$$

We then find

$$N_{ea} = 7.2 \times 10^{13} \text{ photoelectrons} .
\tag{15}$$

The photoelectron spectrum has very nearly the same dependence on energy as the integrand  $I_a$ . For each photoelectron emitted, there will be approximately one Auger electron emitted with energy of a few hundred eV.

#### 4. PHOTOELECTRONS FROM THE TEST OBJECT

We need to compare the numbers of electrons emitted by the air and by the test object. We assume the test object is represented by an exposed area

$$\mathcal{P} = 10^5 \text{ cm}^2 \text{ of aluminum .} \quad (16)$$

The number of photoelectrons emitted from the surface of a given material per photon incident is approximately (Reference 1)

$$y \approx \frac{3}{8} \mu R_m \text{ electrons/photon .} \quad (17)$$

Here  $\mu$  is the absorption coefficient ( $\text{cm}^2/\text{gm}$ ) of the material for x rays of energy  $u$ , and  $R_m$  is the mean range in the material of electrons with energy  $u - w_b$ , where  $w_b$  is the binding energy for the relevant atomic shell. For aluminum,  $w_b \approx 1.6 \text{ keV}$ , and can be neglected compared with  $u$  for the dominant x rays.

For aluminum in the neighborhood of  $u = 10 \text{ keV}$ ,

$$\left. \begin{aligned} \mu &= 27 \left( \frac{u_a}{u} \right)^{2.75} \text{ cm}^2/\text{gm} , \\ R_m &= 1.5 \times 10^{-4} \left( \frac{u}{u_a} \right)^{1.75} \text{ gm/cm}^2 , \\ u_a &= 10 \text{ keV (reference energy) .} \end{aligned} \right\} \quad (18)$$

The photoelectric yield for aluminum is therefore

$$y \approx 1.5 \times 10^{-3} \left( \frac{u_a}{u} \right) \text{ electrons/photon .} \quad (19)$$

This yield does not include a rather uncertain number of low-energy (few eV) electrons, which are not important in the problem at hand.

Since  $R_m$  does not depend strongly on the type of material,  $y$  varies with material in approximately the same way that  $\mu$  does; i.e.,

$\gamma$  increases quite rapidly with increasing atomic number.

The expression (17) is correct for photons arriving at right angles to the surface. For oblique rays,  $\mu$  should be replaced by  $\mu/\cos\theta$ , where  $\theta$  is the angle between the ray and the normal, since the photoelectrons are made closer to the surface in this case. The result is that the yield per unit area (not projected) of exposed surface is approximately independent of the angle at which the x-ray flux strikes the surface. Thus the effective area of a sphere of radius  $r$  is  $2\pi r^2$  rather than  $\pi r^2$ .

We can now calculate the total number of electrons  $N_{\text{et}}$  ejected from the test object. Using results from Section 3, we find

$$N_{\text{et}} = \mathcal{P} \int_0^{W_0} \Phi(u) n_1 n_2 \gamma du . \quad (20)$$

Under the same assumptions made in Section 3 concerning the relation of  $u_1$  and  $u_2$ , the  $u$ -dependence of the integrand  $I_t$  here is as follows:

$$\left. \begin{aligned} I_t &\sim u^{-2.3} \text{ for } u > u_1 , \\ &\sim u^{0.2} \text{ for } u_2 < u < u_1 . \end{aligned} \right\} \quad (21)$$

For an underestimate of the integral based on these dependences we take only the interval  $u \geq u_1$ ,

$$\begin{aligned} N_{\text{et}} &\approx \mathcal{P} \left( \frac{7.4 \times 10^8 \text{A}}{W_0^{0.7}} \right) \left( 1.5 \times 10^{-3} u_a \right) \left( \frac{1}{1.3 u_1^{1.3}} \right) \\ &\approx 8.6 \times 10^5 \frac{\mathcal{P} \text{Au}_a}{W_0^{0.7} u_1^{1.3}} \text{ photoelectrons} . \end{aligned} \quad (22)$$

As an example, we take

$$\left. \begin{aligned}
A &= (10^{-3} \text{ cal/cm}^2) = 4.2 \times 10^4 \text{ erg/cm}^2, \\
W_0 &= 100 \text{ keV}, u_1 = 10 \text{ keV}, \\
u_a &= 10 \text{ keV}, \\
\mathcal{P} &= 10^5 \text{ cm}^2.
\end{aligned} \right\} \quad (22)$$

We then find

$$N_{\text{et}} = 7.2 \times 10^{13} \text{ photoelectrons.} \quad (23)$$

This is (accidentally) the same number as  $N_{\text{ea}}$ . The ratio  $N_{\text{ea}}/N_{\text{et}}$  is expressed by the formula

$$\frac{N_{\text{ea}}}{N_{\text{et}}} = 6.8 \times 10^3 \frac{u_a^2 M}{u_1^{1.2} u_2^{0.8} \mathcal{P}}. \quad (24)$$

This ratio is properly independent of the total x-ray flux  $A$ , but is proportional to the mass of air in the tank per unit exposed surface area of the test object and also depends on the break points in the x-ray spectrum. The numerical factor is composed of the constants

$$6.8 \times 10^3 = \frac{\frac{3}{2}(4.5)(1.16)}{(1.5 \times 10^{-3})/1.3} \quad (25)$$

associated with the absorption coefficient of air (4.5), the photoelectric yield of aluminum ( $1.5 \times 10^{-3}$ ), and other factors near unity which come from integrals.

We have overestimated  $N_{\text{ea}}$ , not only in the approximations, but also in not allowing for shadowing of some air by the test object and for some collimation of the x-ray flux, which will be stronger at the test object than at its sides. We have underestimated  $N_{\text{et}}$  in the approximations, in choosing a rather small value for  $\mathcal{P}$ , in not counting x rays and photoelectrons that come out the back side of the test object, and in choosing the low-Z element aluminum as the typical material.

Furthermore, the photoelectron spectrum from the test object, which has approximately the energy dependence of  $I_t$ , Equation 21, is somewhat harder than the spectrum from the air, which is like  $I_a$ , Equation 11.

However, it is clear that the air pressure used above, Equation 1, is marginal. If collimation limiting the x rays to essentially the air column defined by the test object is not used, the pressure should be reduced to

$$p \leq 10^{-4} \text{ Torr.} \quad (26)$$

Furthermore, this limit should be reexamined if very small test objects are considered.

#### 5. IONIZATION OF THE GAS BY PHOTOELECTRONS

According to data collected in Reference 2, the ionization cross section  $\sigma_i$  of molecular nitrogen and oxygen for electrons of several energies  $w$  are as given in Table 1. The table also gives the mean free path of the electrons for making an ion pair, at the pressure  $10^{-4}$  Torr. The mean free path is greater than the assumed radius of the tank in all cases.

Table 1. Ionization cross section and mean free path for electrons in air at  $10^{-4}$  Torr.

| $w$ , eV         | $\sigma_i$ , $10^{-16}$ cm <sup>2</sup> | mfp, cm           |
|------------------|---|-------------------|
| $10^2$ (maximum) | 2.7                                     | $1.1 \times 10^3$ |
| $10^3$           | 0.9                                     | $3.3 \times 10^3$ |
| $10^4$           | 0.1                                     | $3.0 \times 10^4$ |

Most of the photoelectrons will start with energies of the order of 5 to 10 keV. Each of these electrons will make of the order of 0.1 to 0.2 secondary electrons before striking the wall or returning to the test object. The secondary electrons, which start with energies of a few tens of eV, will be accelerated by the electric field toward the test object. On the average, each secondary electron will make of the order of 0.1 tertiaries, and so on. Each successive generation of electrons is born closer to the test object. It is clear that the series converges quite rapidly for the pressure chosen.

Positive ions with energies of the order of 10 keV move at velocities of the order of  $3 \times 10^7$  cm/sec. The time for the positive ions to move to the wall is therefore of the order of  $10^{-5}$  second, which is much longer than the relevant period ( $10^{-6}$  sec) of the experiment.

We conclude that ionization made by the photoelectrons in the gas will reduce the net electric fields by about 10 percent at  $10^{-4}$  Torr, which seems acceptable. Electrical breakdown of the gas will not occur in the relevant time span.

## 6. CONCLUSIONS

We see that various considerations above lead to the following upper limits on the pressure:

- a. photoelectron scattering and energy loss  $10^{-3}$  Torr
- b. photoelectrons from gas/test object  $10^{-4}$  Torr
- c. secondary ionization in gas and breakdown  $10^{-4}$  Torr

The limit (b) could be raised by eliminating the low-energy photons from the spectrum, i.e., by raising  $u_1$  and  $u_2$  in Equation 24. This works because the air absorption falls off as  $u^{-3}$ , whereas the photoelectric yield falls off as  $u^{-1}$ . However, we would still have the same limit (c), and it is probably more realistic to keep the lower energy photons.



## REFERENCES

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2. Blank, C. A. (Project Officer), Reaction Rate Handbook, DASA 1948, July 1967.

## CONSIDERATIONS ON TANK SIZE

This memo discusses the present status of considerations on the tank radius.

There are several factors which affect the choice of tank radius. All of these, except the cost factor, are such that increasing the tank radius produces a desirable result. Thus the final choice will have to balance cost versus quality of simulation.

Not all of the factors can be assessed accurately at present. Uncertainties, however, are not so large that one could not choose a radius, with some risk either in simulation quality or in costs which are higher than the cost-effective optimum.

### 1. "DC" EFFECTS

For the satellite in space, a certain amount of charge in photoelectrons escapes to infinity. These electrons are eventually replaced by electrons from the ambient plasma which fall into the satellite. This neutralization occurs at the plasma frequency, which is of the order of  $10^5$  Hz; thus neutralization occurs in a few microseconds. Most of the photoelectrons that did not escape to infinity returned to the satellite in a few tenths of a microsecond.

In the tank, more photoelectrons will hit the wall (and become stuck in it) than escape to infinity in the real case. These electrons will be allowed to return to the satellite through d.c. conductors connecting the satellite to the wall. By choosing the electrical

circuit (R, L, and C) of this return path, it should be possible to make the net charge on the satellite be the desired function of time.

For typical spectra, the amount of charge  $Q(r)$  that can reach a given radius  $r$  from the satellite is approximately proportional to

$$Q(r) \sim \left( \frac{1}{r_e} - \frac{1}{r} \right)^{0.7}. \quad (1)$$

Here  $r_e$  is the effective radius of the satellite, which in cm is approximately equal to its capacitance to infinity in pf. The ratio of  $Q(r)$  to  $Q(\infty)$  is

$$\frac{Q(r)}{Q(\infty)} = \left( 1 - \frac{r_e}{r} \right)^{-0.7}. \quad (2)$$

For a test object contained within a cylinder 10 meters long and 3 meters in diameter, the effective radius cannot be larger than about 4 meters.

For a tank radius of 10 meters, we would have

$$\frac{Q(r)}{Q(\infty)} = (1 - 0.4)^{-0.7} = 1.43. \quad (3)$$

Thus the d.c. return circuit should be designed to return  $0.43/1.43 = 30$  percent of the charge in times of a few tenths of microseconds, and the remainder in a few microseconds. This appears to be easily achievable. A d.c. resistor of the order of  $10^4$  ohms is needed to give relaxation times of a few microseconds.

Note that to make this correction one would have to rely on theory for the difference in behavior of the photoelectrons in the finite and infinite sphere cases. However, even if the correction is in error by 30 percent, the late return current would be in error only by 10 percent. The return current will have to be distributed over the area of the satellite, which means several d.c. return paths will be required.

From the standpoint of the d.c. effects, a 10-meter radius seems adequate. Improvement or degradation with increasing or decreasing radius occur only slowly.

## 2. CAVITY MODE EFFECTS

An extraneous effect due to enclosing the satellite in a metal tank is the appearance of oscillations in the fields at the natural frequencies of the cavity (modified, of course, by the presence of the test object). For example the fundamental (electric) mode in a sphere oscillates at frequency

$$f_1 = 0.438 \text{ c/R Hz } , \quad (4)$$

where  $c$  is the velocity of light and  $R$  is the sphere radius. For  $R = 10$  meters,  $f_1 = 13.1$  MHz. Other modes have higher frequencies.

With respect to these extraneous oscillations, we need to understand two aspects:

- a. how strongly they are excited;
- b. how rapidly they can be damped by practical Q-spoiling techniques.

We have studied both of these problems, and are about ready to write up our findings. With some risk in being premature, we may summarize the results as follows:

- a. Cavity mode excitation. In the first place, it is clear that the fields developed at the test object cannot be affected by the tank walls until an EM wave has had time to run out to the wall and back—a time of six or seven shakes in a tank of 10-meters radius. At later times, the ratio of the electric field  $E_{osc}$ , due to the fundamental mode, to the electrostatic field  $E_{stat}$ , at the test object is

$$\frac{E_{\text{osc}}}{E_{\text{stat}}} \approx \left(\frac{r_e}{R}\right)^2 \frac{v}{c} . \quad (5)$$

Here  $r_e$  is the effective radius of the test object,  $R$  is the tank radius,  $v$  is the mean velocity of the photoelectrons, and  $c$  is the velocity of light. For  $r_e = 4$  meters,  $R = 10$  meters, and  $v/c = 1/3$  (30 keV photoelectrons), we find

$$\frac{E_{\text{osc}}}{E_{\text{stat}}} \approx 0.05 . \quad (6)$$

This ratio seems acceptable.

- b. Q-spoiling. It appears possible to damp the oscillations in a few cycles, for example by stringing a grid of resistance wires in the outer 20 percent of the tank radius. This would leave a clear space of radius 8 meters (for a tank radius of 10 meters), which should provide adequate working room for the test object. While the damping grid will raise the frequencies of the natural modes a little, it does not appear necessary to increase the tank radius beyond 10 meters.

### 3. SATELLITE MODE EFFECTS

The satellite in space has several modes of oscillation, in addition to the d.c. mode in which the satellite is charged with respect to infinity. These oscillatory modes will be excited to varying amplitudes by the ejection of photoelectrons, but will be damped fairly strongly by radiating their energy into space.

When the satellite is placed in the tank, the frequencies of the modes will be shifted. Also the radiation damping would be removed if the cavity had high  $Q$ . However, the Q-spoiling mentioned

in Section 2 reduces the (amplitude) reflectivity of the cavity walls to about 0.5, so that the damping rate is not drastically reduced by the presence of the tank.

We cannot be precise in the absence of a particular satellite model. In general, however, the modes of the satellite structure will run between two extremes, which we shall call modes of low and high aspect ratio.

- a. Low aspect ratio. These are the modes of structures with non-convoluted shape, like the oscillations on the outside of a sphere. The frequency of a general mode can be expressed by the formula

$$\omega = k c/D \text{ radians/sec ,} \quad (7)$$

where  $c$  is the velocity of light,  $D$  is the overall dimension of the structure, and  $k$  is a numerical factor. For modes of low aspect ratio,  $k$  is of the order of 2 or 3. For the outside of a sphere of radius  $r$ ,

$$\omega \approx 1.7 c/(2r) , \quad (8)$$

so that  $k = 1.7$ . If we let  $T$  be the transit time of the photoelectrons across the structure,

$$T = D/v , \quad (9)$$

we have

$$\omega T = k c/v \approx 3 k \approx 5 \text{ to } 10 . \quad (10)$$

These modes are therefore not excited strongly by the photoelectron ejection, since their frequencies are 5 to 10 times the driving frequency. These modes would be strongly affected by the presence of the tank, were it not for the low reflectivity of the Q-spoiling grid. With the grid in place they will be damped in a few cycles, and their frequencies shifted only moderately.

- b. High aspect ratio. These are modes of highly convoluted structures, such as two large plates connected by a thin wire. For such modes  $k$  in Equation 7 tends to be small compared with unity, so that it is possible to have

$$\omega T \approx 1 . \quad (11)$$

These modes could therefore be excited strongly, and in fact, they are the ones of primary concern for vulnerability. These modes are not affected much by the presence of the tank, provided there is reasonable separation from the wall, since their capacitance and inductance must be mainly internal.

For these modes there is some doubt as to whether a 10-meter radius for the tank is large enough. One thinks of solar cell paddles mounted by slender rods to the main body of the satellite. While it is my guess that 10 meters is large enough to yield an acceptably reliable test, one cannot be certain in the absence of quantitative estimates of the effect on the relevant modes of an actual structure. There is therefore some risk in choosing 10 meters for the radius. A radius of 15 meters, which doubles the minimum distance from the satellite extremities to the wall, would surely be large enough, but such a tank would cost about 3 times as much as a tank with 10-meter radius.

In conclusion, it is this writer's opinion that choice of 10-meters radius is an acceptable risk, if the choice needs to be made at this time.

## ELECTRICAL DESIGN CONSIDERATIONS

In this memo we discuss the electrical design of the test chamber.

### 1. SUPPRESSION OF PHOTOELECTRONS FROM TANK WALL

X rays hitting the tank wall would generate more photoelectrons than those ejected from the satellite if no precautions are taken. The photoelectric yield  $y$  (number of photoelectrons leaving surface per incident photon) is about  $3/8$  of the probability that the photon is absorbed within one electron range from the surface, i.e.,

$$y \approx \frac{3}{8} \mu R_m . \quad (1)$$

Here  $\mu$  is the mass absorption coefficient ( $\text{cm}^2/\text{gm}$ ) of x-rays of given energy and  $R_m$  ( $\text{gm}/\text{cm}^2$ ) is the mean range of the resulting photoelectrons.

For x-rays with energy just above an absorption edge, the photoelectrons have small energy compared with the x-ray and hence small mean range. For x-ray energies far above the absorption edge, the difference between x-ray and photoelectron energies may be neglected for our purposes. In iron, the presumed tank material, the K-edge is at about 7 keV, rather low in the x-ray spectrum likely to be available. Aluminum (~1.6 keV) and carbon (~0.3 keV) are still lower.

The mean range  $R_m$  ( $\text{gm}/\text{cm}^2$ ) of electrons of a given energy is roughly independent of the material in which they are stopped, as is illustrated by Figure 1 of Memo 1. The photoelectric yields of various



materials are therefore roughly proportional to their photoelectric absorption coefficients for x-rays of the given energy. For x-rays with 10-keV energy the absorption coefficients of CH<sub>2</sub>, aluminum, and iron are listed in Table 1. For x-rays of higher energies the ratios of absorption coefficients are approximately the same. The relative photoelectric yields are therefore nearly independent of x-ray energy.

Table 1. Absorption coefficients and photoelectric yields for 10 keV x-rays.

| MATERIAL        | $\mu_a \frac{\text{cm}^2}{\text{gm}}$ | y/y(CH <sub>2</sub> ) |
|-----------------|---------------------------------------|-----------------------|
| CH <sub>2</sub> | 2.2                                   | 1                     |
| Al              | 27                                    | 12                    |
| Fe              | 160                                   | 70                    |

### 1.1 The CH<sub>2</sub> Layer

Since aluminum is typical of the satellite materials, it is clear that leaving the iron tank walls bare would be very bad. It is also clear that the emission per unit area of the wall can be reduced to about 10 percent of that from the satellite by placing a layer of CH<sub>2</sub> on the wall. The lower Z materials LiH and Be would be even better, but present severe practical problems. The value of low-Z coatings in this connection has been stressed by Richard Schaefer.

In order to stop a 100-keV electron (assumed to be the largest energy) the CH<sub>2</sub> layer would need to be about 0.015 gm/cm<sup>2</sup> in thickness. The writer understands that materials are available that do not lead to insurmountable outgassing problems. It would be convenient if the material had a weak electrical conductivity to remove static charge effects;  $\sigma = 10^{-5}$  mhos/meter gives a charge relaxation time of about 10<sup>-6</sup> seconds. Conductivity is not necessary for the simulation quality, since capacitive coupling from charges in the CH<sub>2</sub> layer to the iron wall is very good.

## 1.2 The Electron Repelling Grid

Since the exposed area of the tank wall may be ten times that of the satellite, use of the CH<sub>2</sub> coating by itself does not appear to be a sufficient cure for the problem of photoelectrons from the walls. We therefore envision in addition a wire grid standing off the wall, charged to a sufficiently negative potential to return to the wall most of the photoelectrons that emerge from the CH<sub>2</sub> layer. Let us see what fraction of the electrons need to be returned.

The spectrum of photoelectrons leaving aluminum was calculated in Memo 6, for an X-ray exposure of 10<sup>-5</sup> calories/cm<sup>2</sup>. In Figure 1, this spectrum has been integrated to give the amount of charge Q(>w) in coulombs per square meter of exposed aluminum surface carried by electrons with energy greater than w keV. If A(m<sup>2</sup>) is the exposed area of the satellite and C is the capacitance between satellite and tank, the charge that reaches the tank wall is determined by

$$\frac{A Q(>w)}{C} = 10^3 w$$

or

$$\frac{Q(>w)}{10^3 w} = \frac{C}{A} . \quad (2)$$

The function Q(>w)/10<sup>3</sup>w is graphed in Figure 2. If A = 30 m<sup>2</sup> and C = 3 × 10<sup>-10</sup> farads, then C/A = 1 × 10<sup>-11</sup>. From Figure 2 we see that electrons with energy greater than about 6 keV will reach the wall. From Figure 1 we then see that about 90 percent of all electrons emitted from the satellite reach the wall.

Now if we assume the maximum X-ray exposure of 10<sup>-3</sup> calories/cm<sup>2</sup>, the ordinates in Figures 1 and 2 are 100 times larger. Thus for the same C/A we read w = 20 keV from Figure 2, and from Figure 1 we see that only about 12 percent of the electrons leaving the satellite reach the wall.

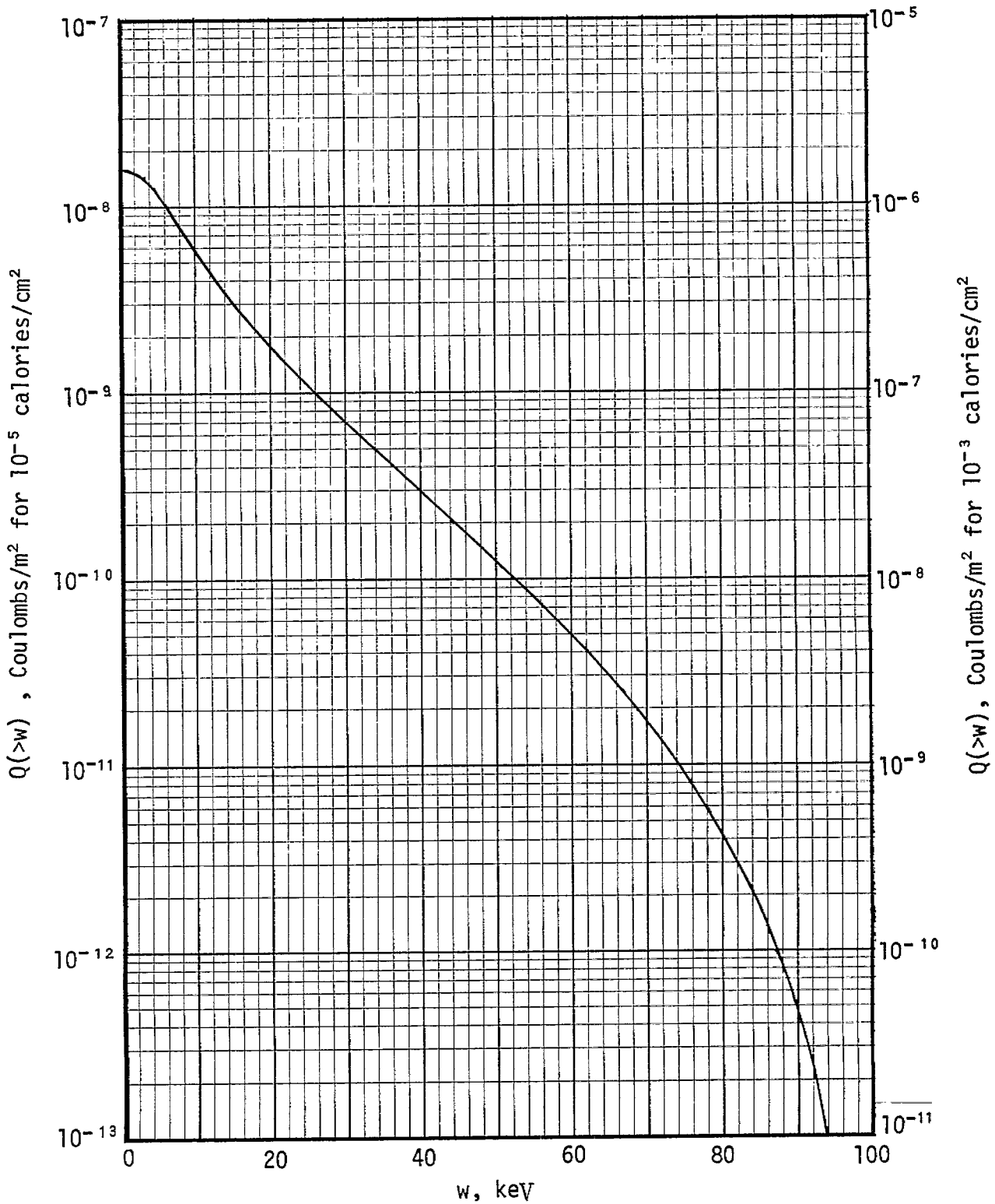


Figure 1. The charge  $Q(>w)$  carried by electrons with initial kinetic energy greater than  $w$ .

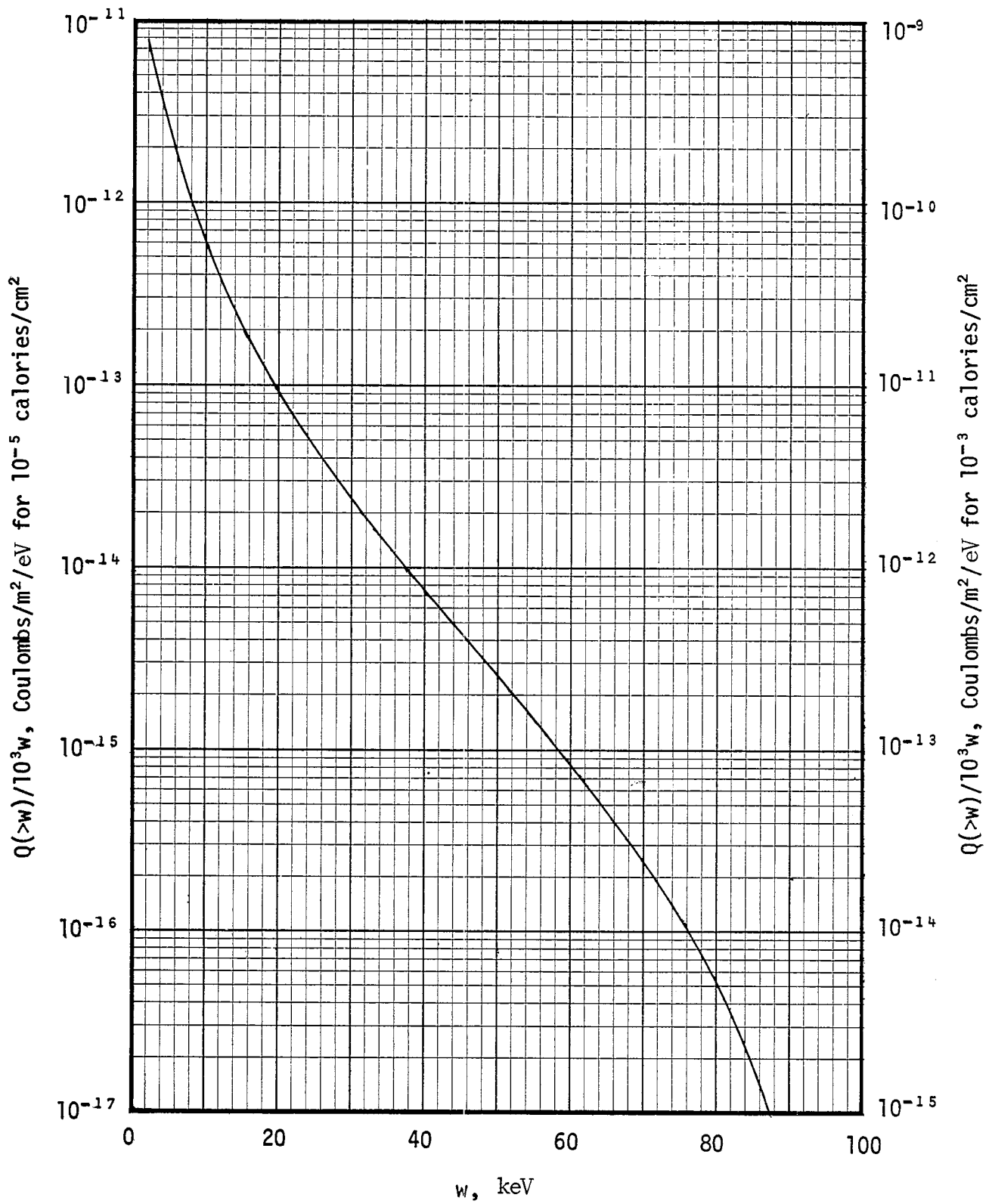


Figure 2. The function  $Q(>w)/10^3w$ .

The repelling grid must be designed so that the number of electrons getting past it into the tank is small compared with the number of electrons from the satellite reaching the tank wall.

By coating the wall with  $\text{CH}_2$  we have reduced its emission per unit area to about one-tenth of that from the satellite. However, since the exposed area of the wall may be ten times larger than that of the satellite, the total emission from the wall and satellite will be comparable. Considering the high-exposure case, let us say that we want no more than  $1/30$  (12 percent) = 0.4 percent of the electrons emitted from the wall to escape through the grid into the tank. From Figure 1 we see that the potential of the grid must be about -60 kilovolts.

This estimate neglects help from space charge limiting by the wall electrons themselves, but this effect is small because of the reduced emission per unit area from the wall. It also neglects a slight discharging of the grid by electrons from the satellite passing it and entering the wall.

This grid should be mounted fairly close to the tank wall—within a few tens of cm—to prevent the wall electron currents from radiating into the tank. The capacitance from the grid to the tank wall is of the order of  $0.03 \mu\text{F}$  (30 cm spacing from wall). When charged to -60 kilovolts it carries  $\sim 2 \times 10^{-3}$  coulombs of negative charge. This may be compared with the  $5 \times 10^{-5}$  coulombs of electrons ejected from the wall in the high-exposure case.

The potential on the grid should be brought up slowly so as not to induce oscillations in the tank. This writer sees no need to have more than one feed through the tank wall for charging purposes, and this feed can be high impedance. During the pulse the potential is maintained by the low impedance capacitive coupling from grid to wall.

The satellite is envisioned as floating inside the sphere formed by this grid. It is not grounded to the tank wall prior to the x-ray pulse. Thus its potential is initially the same as that of the grid. Needless to say, there are no wire lines connecting the satellite to the outside world.

The actual design of the grid is not critical. Suppose the grid is placed a distance  $D$  from the wall, and is made up of parallel wires of radius  $a$  spaced a distance  $d$  between centers, as illustrated in Figure 3. Let  $V_w$  be the potential on the grid wires, and  $V_s$  be the asymptotic potential of the space to the right of the grid. Then if  $D \gg d \gg a$ , a measure of the effectiveness of the grid as compared with a continuous sheet is given by the ratio

$$\frac{V_w}{V_s} \approx 1 + \frac{d}{2\pi D} \ln \left( \frac{d}{2\pi a} \right). \quad (3)$$

For example, we may choose

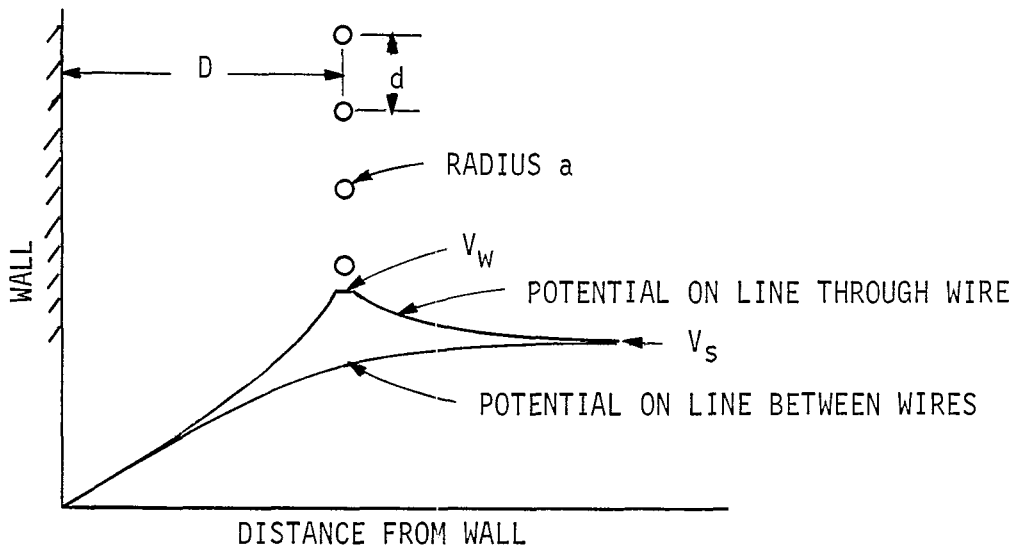


Figure 3. Schematic diagram of grid and potential.

$$D = 30 \text{ cm}$$

$$d = 3 \text{ cm } (\rightarrow 6 \text{ cm later}) \quad (4)$$

$$a = 0.05 \text{ cm}$$

In this case

$$\frac{V_W}{V_S} = 1.036 \quad (5)$$

This grid presents an effective area for intercepting x-rays of 1/30 of the total wall area, which seems adequately small, provided it is coated with CH<sub>2</sub>. Actually the grid should be made of crossed wires spaced twice as far, or 6 cm apart, so that it conducts well in all directions. The crossed wires are welded at all intersections, and it is supported by thin insulating rods fastened to the wall.

This grid will also return to the wall any secondary electrons ejected from the wall by electrons striking it.

## 2. Q-SPOILING

There needs to be at least one other grid for the purpose of reducing the reflectivity of the wall. In Memo 7, we showed how a conducting membrane could be used to reduce reflectivity for all waves which do not have a node near the membrane. Here we need to convert the membrane into a grid. It will be recalled that a membrane located 2 meters from the wall and having a resistance

$$Z = \frac{1}{2} (377) = 188 \text{ ohms/square} \quad (6)$$

reduces the reflectivity quite effectively, except at frequencies such that the number of half wavelengths between the wall and membrane is very nearly an integer, which we consider later. If we make the grid with resistance wire having resistance R ohms/cm, with the wires spaced d cm apart on centers, we need

$$R = Z/d \quad (7)$$

For example, if the spacing of the wires is 6 cm, we would need

$$R = 31 \text{ ohms/cm} \quad (d = 6 \text{ cm}) \quad (8)$$

A wire grid has an inductive reactance not present in a continuous membrane, due to flux linkages of the individual wires. The inductance per unit length of each grid wire is

$$L = \frac{60}{c} \ln \left( \frac{d}{2\pi a} \right) \text{ henrys/cm} \quad (9)$$

where  $a$  is again the radius of the grid wire and  $c = 3 \times 10^{10}$  cm/sec is the velocity of light. The inductive reactance of each wire is then

$$X = \omega L = 60 k_0 \ln \left( \frac{d}{2\pi a} \right) \text{ ohms/cm} \quad (10)$$

where  $k_0$  is the free space wave number at frequency  $\omega$ . We wish to keep the reactance  $X$  small compared with the resistance  $R$ , i.e.,

$$k_0 d \ll Z/60 \ln \left( \frac{d}{2\pi a} \right) \quad (11)$$

Let us again choose

$$\frac{d}{2a} = 60 \quad (12)$$

so that in a crossed-wire grid only about 3 percent of the total area is not open to the X-rays. Then using  $Z$  from Equation 6, Equation 11 becomes

$$k_0 d \ll 1 \quad (13)$$

For the fundamental tank mode

$$k_0 = 2.74/(10 \text{ meters}) \quad (14)$$

so that we need

$$d \ll 10^3/2.74 = 360 \text{ cm} \quad (15)$$

For this frequency,  $d = 6$  cm as chosen in Equation 8 is adequately small. For this value of  $d$ ,  $k_0 d$  equals unity for wavelengths of about 36 cm



or frequencies of the order of  $10^9$  Hz. The 6 cm spacing should therefore be small enough for frequencies up to  $10^8$  Hz, and it seems unlikely that frequencies higher than this would be excited in the simulation experiments.

Thus we choose for the crossed-grid parameters

$$\begin{aligned}
 \text{Spacing from tank wall} &= D = 2 \text{ meters} \\
 \text{Spacing of parallel wires} &= d = 6 \text{ cm} \\
 \text{Radius of wires} &= a = 0.05 \text{ cm} \\
 \text{Resistance of wires} &= R = 31 \text{ ohms/cm}
 \end{aligned}
 \tag{16}$$

From the values of R and a here we can deduce the conductivity of the wire material

$$\sigma = (\pi a^2 R)^{-1} = 4.1 \text{ mho/cm} .
 \tag{17}$$

The skin depth in this material at frequency  $f$ (Hz) is

$$\delta = \frac{5 \times 10^3}{\sqrt{\sigma f}} \approx 0.25 \text{ cm at } f = 10^8 \text{ Hz} .
 \tag{18}$$

This is adequately large compared with the radius of the wire.

Since this grid is spaced 2 meters from the wall, it will not absorb waves of length 4, 2, 1.33, 1, etc., meters, corresponding to frequencies of 75, 150, 225, 300, etc., MHz. Perhaps none of these frequencies will be prevalent in the experimental situation. They could probably be absorbed, however, by making the electron-repelling grid of Section 1 out of the same resistance wire as used here. Some analysis needs to be done to verify this point, and to make sure that the finite resistivity does not allow the wall electron currents to radiate into the volume. It appears likely that the outer grid can be used for both purposes.

The inner (Q-spoiling) grid is mounted on posts attached to the supports of the outer (electron repelling) grid. The writer is uncertain at this time as to whether these posts should be insulators, conductors, or resistors for optimum performance.

### 3. DC CHARGE RETURN

In space at synchronous satellite radius the plasma frequency is about  $10^5$  Hz, so that the satellite charge would be neutralized in (1/4 cycle) 2.5 microseconds. It is not clear that such low frequencies are important in vulnerability considerations. Note, however, that neutralization in about this time could be achieved by tying high-resistance wires between the satellite and the outer grid. Since the capacitance between the satellite and the tank is about  $3 \times 10^{-10}$  farads the total resistance between outer grid and satellite should be

$$R_n = 2.5 \times 10^{-6} \text{ sec} / 3 \times 10^{-10} \text{ fd} = 8 \times 10^3 \text{ ohms} . \quad (19)$$

The high resistance wires should be tied to the centers of the larger satellite areas, and not of course to small sensitive points.