Combining Voltage or Current Dividers with Sensor Cables

I. Introduction

In measurements of electromagnetic fields associated with the nuclear EMP (or simulation of the same) we often encounter a situation in which the voltages or currents produced on our sensors by these fields are too large to be fed directly into active electronic devices. In certain cases, however, such devices may be desirable or even necessary. The cathode follower is often used with a dipole electric field probe, to present a very high impedance to the probe. Thus, some sort of voltage divider network is needed to reduce the voltage to the cathode follower. Similarly, an active device may be needed to present a very low impedance to another type of electromagnetic field probe, such as a short-circuited loop, in which case a current divider network may be appropriate.

In addition, there are situations in which the active devices (as above) should be placed some distance from the field sensor for shielding from nuclear radiation. The connection of the sensor with its associated active electronics would normally be done with some sort of transmission line. In Sensor and Simulation Note VI (SSN VI) we described some techniques for appropriately placing discrete resistors in sensor cables to avoid ringing in the unterminated sensor cables. However, in the cases considered in SSN VI there were no voltage or current dividers included. The purpose of this note is to describe some techniques whereby voltage (or current) dividers and unterminated transmission lines can be combined for use with appropriate electromagnetic field sensors without introducing ringing in the unterminated sensor cables.

In this note we will develop circuit techniques for use with voltage divider networks and later consider the dual circuits for use with current dividers. Thus, the discussion will center around methods for use with voltage dividers as might be used with an electric field dipole and cathode follower. Basically, the techniques employ lumped resistors for both the cable termination and the divider at high frequencies and lumped capacitors which combine with the cable capacitance to give the same voltage divider action for low frequencies. The dual current divider circuits involve resistors and inductors.

II. Development of Divider-Cable Circuits

As a first step in the development of combination cable and voltage divider circuits consider the two resistive voltage divider circuits with cables as in figure 1. Each circuit is driven by a voltage source in series with a capacitance, representing an electric field probe. The simplest circuit is that of figure 1A in which the cable is terminated in its characteristic impedance at the signal output of the cable. The terminated cable (of arbitrary length) is then used in combination with R_1 to give the voltage divider ciruict. We have a voltage divider ratio

$$t_o = \frac{Z}{R_1+Z} \qquad (1)$$

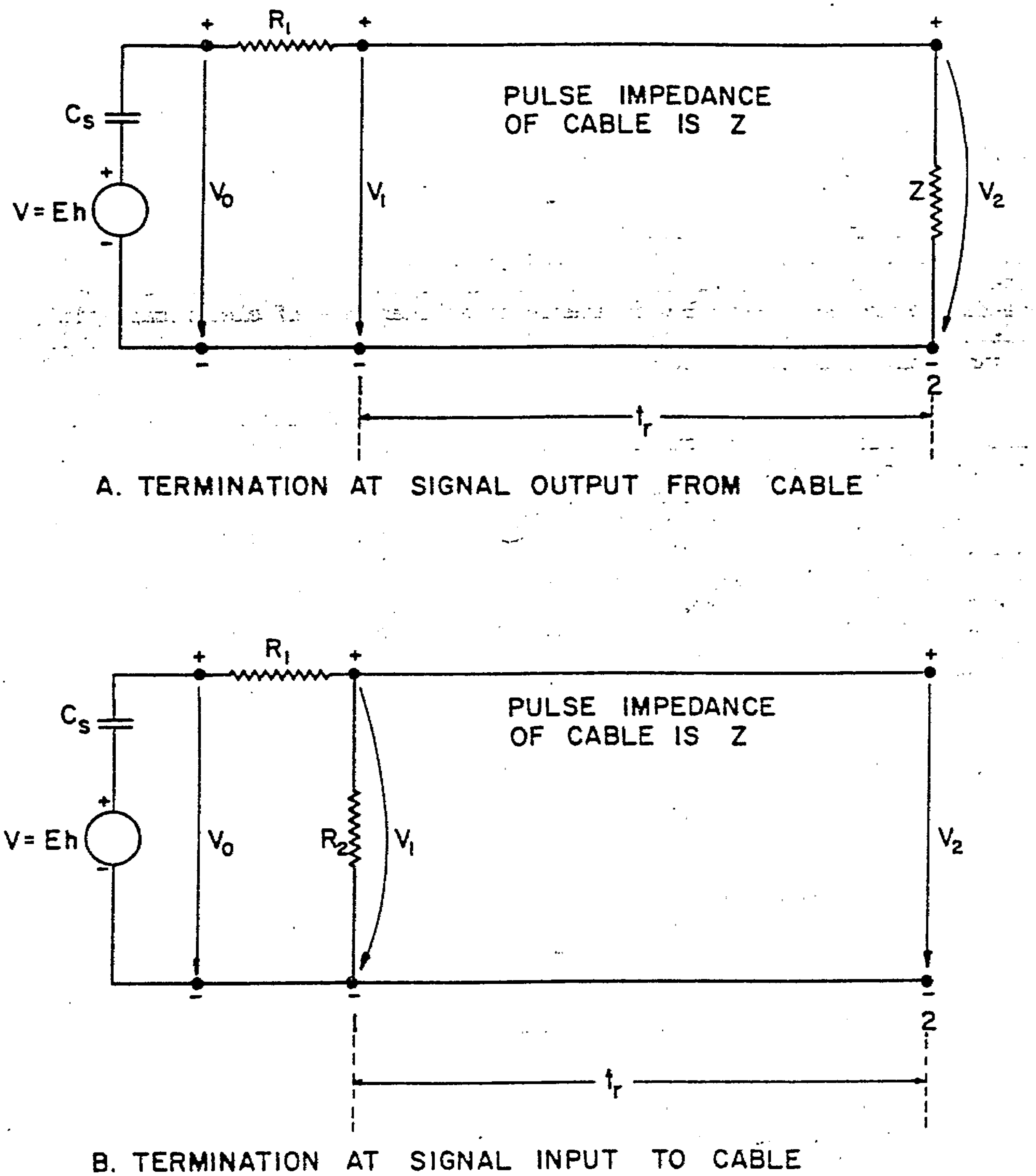


Figure 1. RESISTIVE DIVIDERS WITH TRANSMISSION LINES

which represents the ratio of V_1 to V_o. Including the time delay, t_r, due to the transmission line then equation (1) also gives the ratio of V_2 to V_o.

An alternate method for combining a resistive voltage divider and a transmission line is given in figure 1B. In this case, the cable is terminated at its signal input which requires that

$$Z = \frac{R_1 R_2}{R_1 + R_2} \tag{2}$$

This presumes that

$$R_1 C_s >> 2t_r \tag{3}$$

or that the sensor capacitance is large enough to allow the reflection from point 2 to terminate at point 1 in the voltage divider. The voltage divider ratio is now

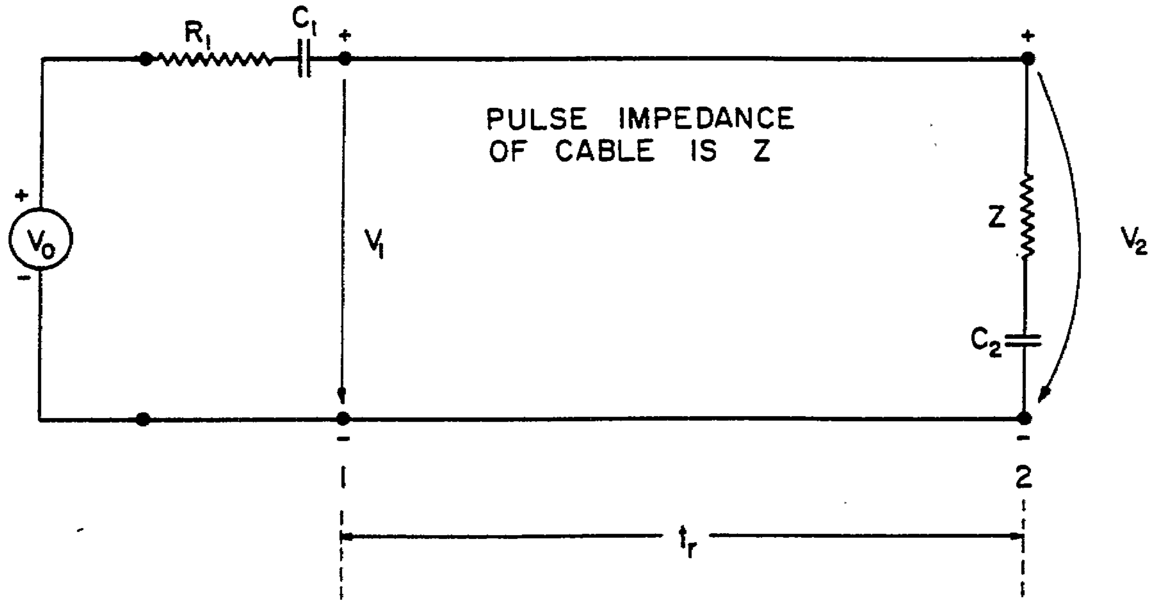
$$t_o = \frac{R_2}{R_1 + R_2} = \frac{Z}{R_1} \tag{4}$$

For small t_o, R_2 approaches Z and the restriction of equation (3) does not apply. Actually, for a pulse less than $2t_r$ wide, the ratio of V_1 to V_o is $t_o/2$ but at point 2 there is a +1 voltage reflection which doubles the voltage so that, including the delay time of the cable, t_o represents the ratio of V_2 to V_o.

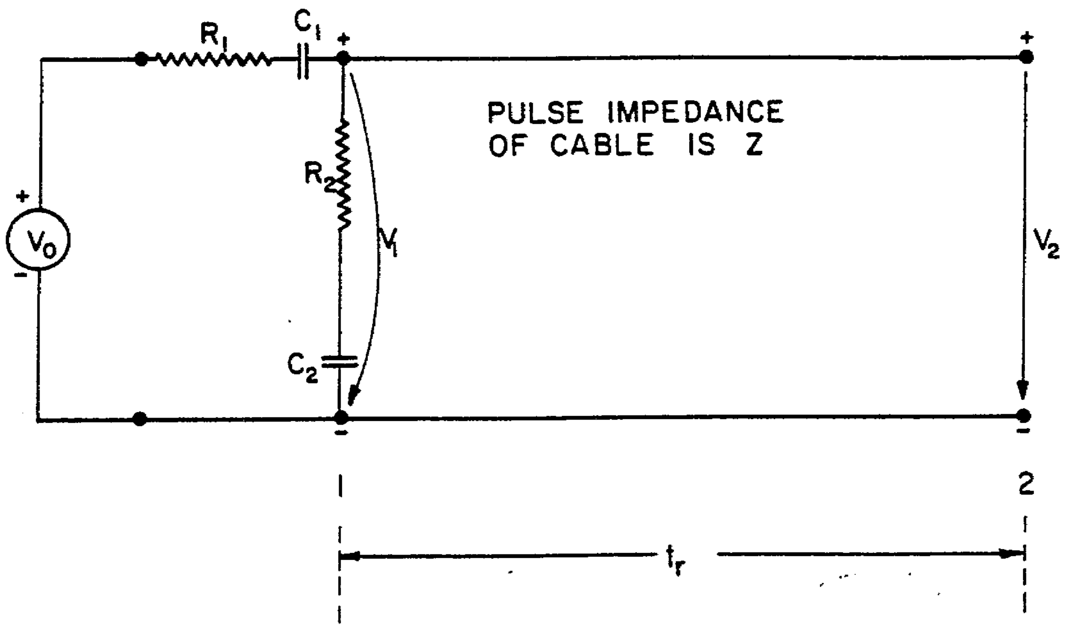
However, a resistive voltage divider, when operated with a capacitive electric field probe, must have large resistance values so that the probe is not loaded, or, as in the cases in figure 1, so that V_o is essentially Eh for all times of interest. Since we are limited by the practical maximum impedance, Z, of transmission lines, then for a given voltage divider ratio, t_o, the maximum resistance values are determined. If these maximum resistance values are not large enough for our particular application we will have to look for another technique for combining voltage dividers with transmission lines.

Another type of voltage divider which we might consider is a capacitive divider. Imagine that all the resistors in figure 1 were replaced with capacitors. Then if the capacitor replacing R_1 were much less than C_s there would be a negligible voltage drop across C_s, a desirable situation in certain cases (e.g., see SSN XV). However, the transmission lines are no longer terminated, meaning that, for pulses with changes on the order of t_r or faster, ringing will occur. On the other hand, for comparatively longer time components in a signal we would have a capacitive divider with the cable capacitance, t_r/Z, contributing to the capacitive divider.

As illustrated in figure 2 we can combine the features of a resistive divider with those of a capacitive divider. For simplicity in the analysis, we will assume that C_1 is much less than C_s so that V_o is essentially Eh and we can use V_o as the voltage source, thus restricting our attention to the divider-cable networks. For high frequencies the networks of figure 2 reduce



A. HIGH FREQUENCY TERMINATION AT SIGNAL OUTPUT FROM CABLE



B. HIGH FREQUENCY TERMINATION AT SIGNAL INPUT TO CABLE

Figure 2. RC DIVIDERS WITH TRANSMISSION LINES

to those of figure 1, i.e., we have resistive dividers, while for low frequencies we have capacitive dividers. For these circuits to behave as ideal voltage divider networks it is necessary that the low frequency voltage divider ratio be the same as for high frequencies. Thus,

$$t_o = \frac{C_1}{C_1+C_2+t_r/Z} \tag{5}$$

Note that the cable capacitance is added in parallel with C_2 for low frequencies where the resistors have no effect. This holds for both types of divider-cable networks in figure 2. Also note that these circuits are illustrated in unbalanced or single-ended form, whereas they can be easily changed to a balanced form.

With the voltage divider ratios the same for both high and low frequencies, it is important to know how smoothly one case transitions into the other. Thus, we need to know how these divider ratios vary with frequency and/or the response of these circuits to a convenient pulse shape such as a unit step. From such analyses we can obtain error estimates for various combinations of parameter values which we can in turn use for design purposes.

III. RC Divider: Termination at Signal Output from Cable

Consider the characteristics of the divider-cable circuit of figure 2A. First, we define some parameters to simplify the equations. Thus, let

$$\tau = R_1C_1 = ZC_2+t_r \tag{6}$$

This is the time constant for the resistive-capacitive divider and is consistent with the appropriate voltage divider ratio, t_o, expressed in equations (1) and (5). Another important parameter is

$$\eta = \frac{t_r}{\tau} = \left[1 + \frac{ZC_2}{t_r}\right]^{-1} \tag{7}$$

which represents the fraction of the capacitance (on the signal output side of the divider) which is contributed by the transmission line.

Using the Laplace transform notation (from which we can obtain both frequency response and unit step response information) we first define a voltage transfer function

$$t_v = \frac{V_2}{V_o e^{-st_r}} \tag{8}$$

Note that by this definition we are subtracting the cable delay from our results, i.e., for a pulse V_o we refer all times to time of arrival at point 2. Noting that we have a voltage reflection coefficient at point 2 of

$$r = [1 + 2\,sZC_2]^{-1} = [1 + 2s\tau(1-\eta)]^{-1} \tag{9}$$

we can determine the impedance looking into the cable at point 1 from

$$Z_1 = Z \frac{1+re^{-2st_r}}{1-re^{-2st_r}} \tag{10}$$

Likewise we can use two intermediate voltage ratios

$$\frac{V_2}{V_1} = \frac{(1+r)e^{-st_r}}{1+re^{-2st_r}} \tag{11}$$

and

$$\frac{V_1}{V_o} = \frac{Z_1}{Z_1+R_1+\frac{1}{sC_1}} \tag{12}$$

Thus,

$$t_v = \frac{Z_1}{Z_1+R_1\frac{(1+s\tau)}{s\tau}} \quad \frac{1+r}{1+re^{-2st_r}} \tag{13}$$

If we normalize t_v by defining

$$t_v' = \frac{t_v}{t_o} \tag{14}$$

and then combine the results of equations (6) through (13) into equation (14) we have

$$t_v' = 2(1+s\tau(1-\eta))s\tau \left\{(1+2s\tau(1-\eta))(1-t_o+s\tau) - e^{-2s\tau\eta}(1-t_o+(1-2t_o)s\tau)\right\}^{-1} \tag{15}$$

This last result simplifies somewhat if we take the limiting case of $t_o = 0$, in which case

$$t_v' = \left\{\frac{1+s\tau}{2(1+s\tau(1-\eta))s\tau}\left[1+2s\tau(1-\eta) - e^{-2s\tau\eta}\right]\right\}^{-1} \tag{16}$$

Conveniently t_v' is defined so that if s is replaced by $j\omega$ for a frequency analysis, then t_v' goes to unity amplitude and zero phase for both high and low frequencies.

Next, let us consider the response of this circuit to a step function input of amplitude $1/t_o$, again so that the output will be normalized. Define this as a normalized response function, $R'(t)$. To calculate this we should first recognize that we would expect a unity step rise in the output, followed by a relatively simple function for times less than 2 t_r when the first reflections will arrive at the output. However, since the high frequency components of any reflections have been reduced by the high frequency resistive termination at the output, then reflections returning to the output will not have sharp rises, but will be rather smooth. Eventually the output will settle down at unity amplitude due to the capacitive divider action.

Now, we shall calculate the response for times less than $2t_r$ and use the maximum deviation from unity during this time as an indication of the overall quality of the divider-cable circuit. With this restriction on the times of interest we can remove the exponential from equation (15) because this term is associated with signals arriving after $2t_r$. Multiplying by 1/s (for a unit step) we then have for $0 \leq t \leq 2t_r$ that

$$\frac{t_v'}{s} = \frac{2\tau(1+s\tau(1-\eta))}{(1-t_o+s\tau)(1+2s\tau(1-\eta))} \qquad (17)$$

or

$$\frac{t_v'}{s} = \frac{1}{s+\frac{1-t_o}{\tau}} \left\{ 1 + \frac{\frac{1}{2\tau(1-\eta)}}{s+\frac{1}{2\tau(1-\eta)}} \right\} \qquad (18)$$

Inverting this transform we have

$$R'(t) = e^{-\frac{1-t_o}{\tau}t} + \frac{e^{-\frac{1-t_o}{\tau}t} - e^{-\frac{t}{2\tau(1-\eta)}}}{1-2(1-\eta)(1-t_o)} \qquad (19)$$

which holds for $0 \leq t \leq 2t_r$. Considering the limiting case of $t_o = 0$ we have for $0 \leq t \leq 2t_r$

$$R'(t) = e^{-\frac{t}{\tau}} + \frac{e^{-\frac{t}{\tau}} - e^{-\frac{t}{2\tau(1-\eta)}}}{2\eta-1} \qquad (20)$$

For small η (the ideal case of small cable capacitance) equation (20) has a minimum, over its range of validity at $t = 2t_r$ where

$$R'(2t_r) = e^{-2\eta} + \frac{e^{-2\eta} - e^{-\frac{\eta}{1-\eta}}}{2\eta-1} \qquad (21)$$

For $\eta \ll 1$ this is approximately

$$R'(2t_r) \approx 1-\eta \qquad (22)$$

so that the fractional deviation from an ideal response is approximately the fraction of the capacitance (loading the signal output) from the transmission line. This would seem to be a rather simple and reasonable error criterion for such a divider-cable circuit.

IV. RC Divider: Termination at Signal Input to Cable

The other divider-cable circuit, that of figure 2B, will have results similar to the previous case but first we must redefine the parameters of equations (6) and (7) to be consistent with the definitions for this circuit contained in equations (2), (4), and (5). Thus, the time constant of the resistive-capacitive divider is now

$$\tau = R_1C_1 = R_2[C_2 + \frac{t_r}{Z}] \qquad (23)$$

and similarly

$$\eta = \frac{t_r}{\tau} = \frac{Z}{R_2}\left[1 + \frac{ZC_2}{t_r}\right]^{-1} \qquad (24)$$

Note, however, that for this case η is not in general the fraction of the capacitance (on the signal output side of the divider) contributed by the cable. It is only the fractional capacitance in the limit $t_o = 0$ in which case $R_2 = Z$.

As before, we define a voltage transfer function

$$t_v = \frac{V_2}{V_o e^{-st_r}} \qquad (25)$$

Since for this case we have a plus one voltage reflection coefficient at point 2 (open circuit) we have an impedance looking into the cable at point 1 of

$$Z_1 = Z\,\frac{1 + e^{-2st_r}}{1 - e^{-2st_r}} \qquad (26)$$

Our intermediate voltage ratios are now

$$\frac{V_2}{V_1} = \frac{2\,e^{-st_r}}{1 + e^{-2st_r}} \qquad (27)$$

and

$$\frac{V_1}{V_o} = \frac{(R_1 + \frac{1}{sC_1})^{-1}}{(R_1 + \frac{1}{sC_1})^{-1} + (R_2 + \frac{1}{sC_2})^{-1} + Z_1^{-1}} \qquad (28)$$

Thus,

$$t_v = \left\{\frac{1+e^{-2st_r}}{2}\left[1 + \frac{R_1 + \frac{1}{sC_1}}{R_2 + \frac{1}{sC_2}} + \frac{R_1 + \frac{1}{sC_1}}{Z_1}\right]\right\}^{-1} \qquad (29)$$

Again, normalizing t_v by

$$t_v' = \frac{t_v}{t_o} \tag{30}$$

and combining the results of equations (23) through (29) into equation (30) we have

$$t_v' = \left\{ \left[\frac{t_o}{2} + \frac{1-t_o}{2} \; \frac{1+\frac{1}{s\tau}}{1+\frac{1}{s\tau\left(1-\frac{\eta}{1-t_o}\right)}} \right] (1+e^{-2s\tau\eta}) + \frac{1}{2}\left(1+\frac{1}{s\tau}\right)(1-e^{-2s\tau\eta}) \right\}^{-1} \tag{31}$$

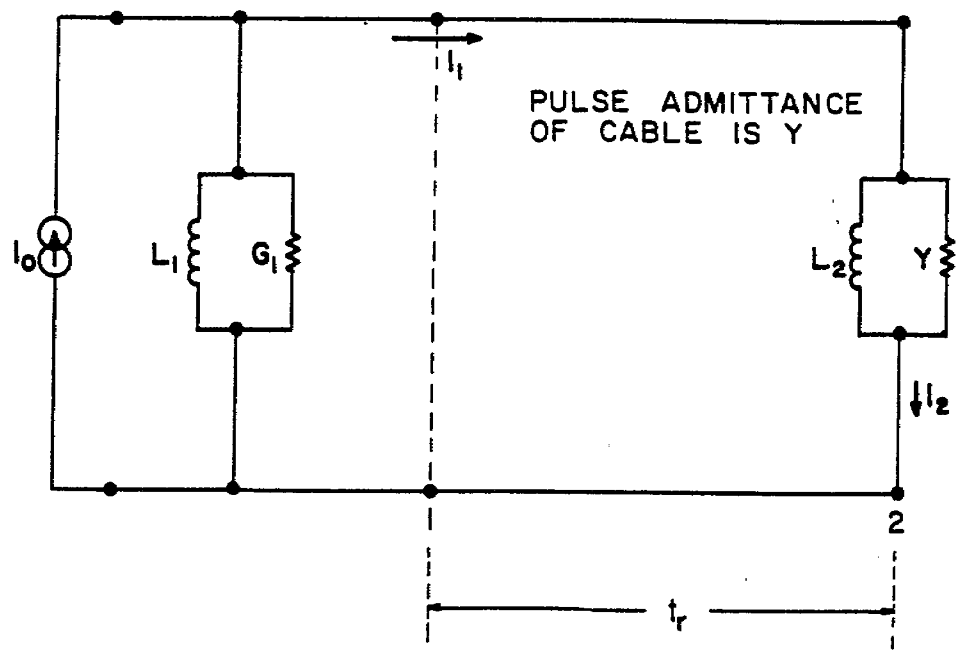
Again, if we take the limiting case of $t_o = 0$ we have

$$t_v' = \left\{ \frac{1+s\tau}{2(1+s\tau(1-\eta))s\tau} \left[1+2s\tau(1-\eta) - e^{-2s\tau\eta} \right] \right\}^{-1} \tag{32}$$

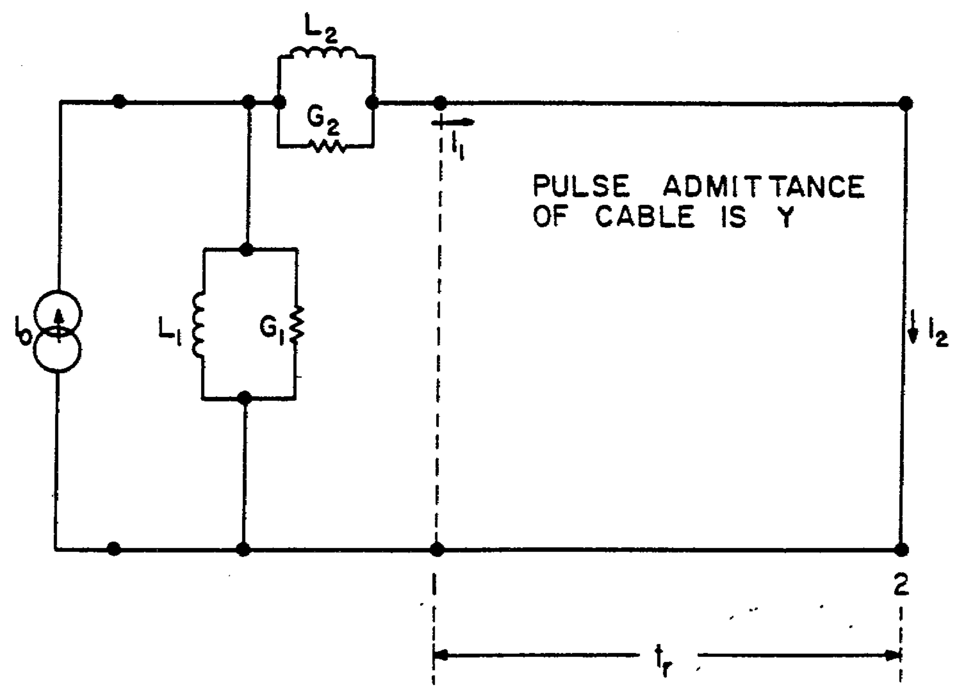
Note that this is the same result as equation (16). Thus in the limit of $t_o = 0$ the two types of divider-cable networks have identical voltage transfer functions. This means that in the limiting case the two networks have the same frequency response characteristics and the same pulse response characteristics. Thus, this circuit can be characterized by the same normalized step response function, $R'(t)$, as developed in equations (17) through (22). The fractional deviation from an ideal step response is again approximately the fraction of the capacitance (loading the signal output) from the transmission line.

V. Analogous GL Dividers with Cable

We now turn from the resistive-capacitive voltage dividers of Sections III and IV to the conductive-inductive current dividers of figure 3. The circuits of figures 3A and 3B are respectively the electrical duals of the previously considered circuits of figures 2A and 2B. Thus, the circuit elements of figure 2 are replaced by their dual elements (conductance for resistance, inductance for capacitance, cable admittance for cable impedance, and current source for voltage source), the network hookup is arranged in a dual manner (parallel elements for series elements and vice versa, and short circuits for open circuits), and as a result the dual circuit response quantities have the same characteristics (e.g., current reflection coefficients replace voltage reflection coefficients and current transfer ratios replace voltage transfer ratios). When the circuit element values of the voltage divider networks are transferred to be the circuit element values of the dual elements (noting that the units then change to the units of the dual elements) in the current divider networks the results derived for the voltage divider networks can be transferred directly to the current divider networks.



A. HIGH FREQUENCY TERMINATION AT SIGNAL OUTPUT FROM CABLE



B. HIGH FREQUENCY TERMINATION AT SIGNAL INPUT TO CABLE

Figure 3. GL DIVIDERS WITH TRANSMISSION LINES

Thus, let us first consider the current divider of figure 3A, which terminates the signal cable at the signal output, the results for the dual circuit having been derived in Section III. Redefining some parameters as

$$t_o = \frac{Y}{G_1+Y} = \frac{L_1}{L_1+L_2+t_r/Y} \tag{33}$$

and

$$\tau = G_1 L_1 = YL_2+t_r \tag{34}$$

and

$$\eta = \frac{t_r}{\tau} = \left[1 + \frac{YL_2}{t_r}\right]^{-1} \tag{35}$$

and replacing t_v' by t_I' as

$$t_I' = \frac{t_I}{t_o} = \frac{1}{t_o} \frac{I_2}{I_o e^{-st_r}} \tag{36}$$

we can use the results of equations (15) and (16) for the current transfer function. The response of this circuit to a step input of current of amplitude $1/t_o$ is contained in equations (17) through (22), showing that the fractional deviation from an ideal response is approximately the fraction of the inductance (loading the signal output) from the transmission line.

Next consider the current divider of figure 3B which terminates the signal cable at the divider network. The results for the dual of this circuit have been derived in Section IV. Redefining the important parameters as

$$t_o = \frac{G_2}{G_1+G_2} = \frac{Y}{G_1} = \frac{L_1}{L_1+L_2+t_r/Y} \tag{37}$$

and

$$\tau = G_1 L_1 = G_2 \left[L_2 + \frac{t_r}{Y}\right] \tag{38}$$

and

$$\eta = \frac{t_r}{\tau} = \frac{Y}{G_2}\left[1 + \frac{YL_2}{t_r}\right]^{-1} \tag{39}$$

and replacing t_v' by t_I' as in equation (36) we can use the results of equations (31) and (32) for the current transfer function. As with the two voltage divider circuits the two current divider circuits have the same transfer functions and step responses in the limit of $t_o = 0$. Thus, the step response characteristics of this circuit are also given by equations (17) through (22).

Thus, from our analyses we have the characteristics of two conductive-inductive current divider circuits which could be used with a "short circuited" loop (magnetic field probe). However, the inductance, L_1, shunting the loop should be much less than the loop inductance so that the loop can be represented by a current source as in figure 3. This restriction is similar to that discussed in Section II regarding C_1 in the resistive-capacitive voltage dividers.

VI. Summary

In circumstances which require that active electronic devices be used with voltage dividers and transmission lines (e.g., to get away from a region of high nuclear radiation in which an E field sensor is placed) the divider-cable network can be made to give minimum signal distortion. In these circuits we terminate the cable for high frequencies and incorporate the cable capacitance into the capacitance of a resistive-capacitive divider network for low frequencies. These techniques can be combined, if desired, with the circuits of SSN VI as in figure 4 (providing $R_1 >> Z$) to remove the divider circuit some distance from an electric field probe.

As discussed in the note, there are at least two networks suitable for the voltage divider-cable application. Likewise, there are two current divider-cable networks, electrical duals of the voltage divider-cable networks. With all of these networks we should be careful to recognize the assumptions regarding the impedance or admittance of the sources driving the networks for our results to be valid, e.g., an electric field probe need not be a simple lumped capacitance for sufficiently high frequencies. This last comment, of course, applies to any circuits which we might use in conjunction with EM field probes.

CARL E. BAUM, 1/LT, USAF
3 November 1965

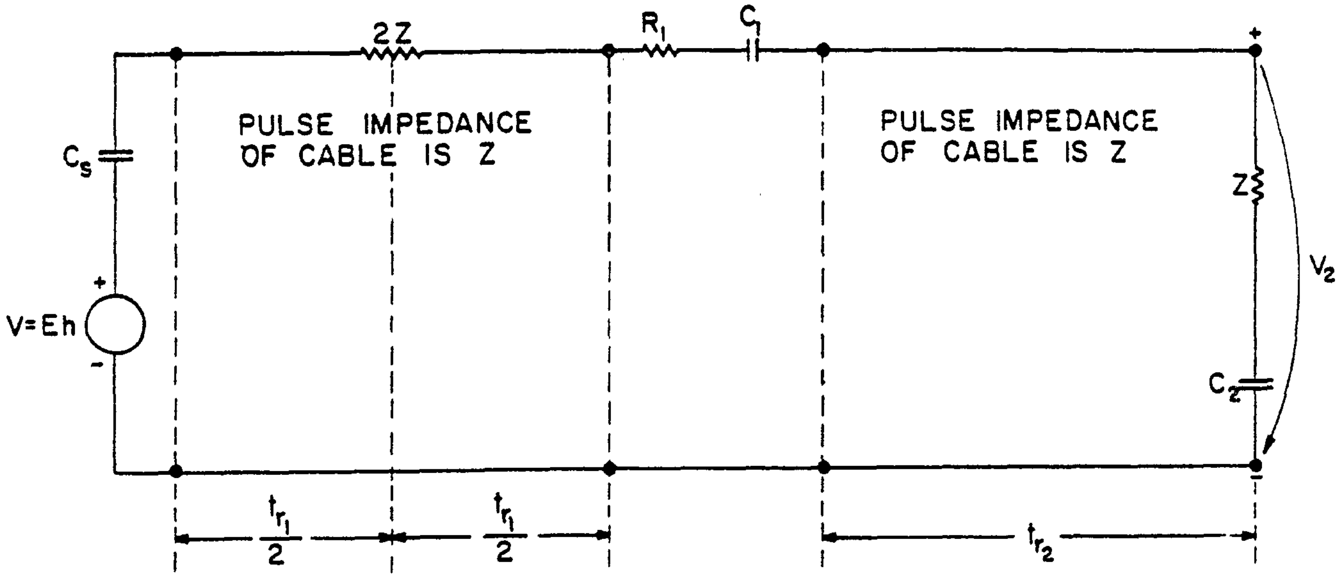


Figure 4. A POSSIBLE COMBINATION OF AN ELECTRIC FIELD DIPOLE WITH A VOLTAGE DIVIDER AND TRANSMISSION LINES