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Space-Time Domain Magnetic Field Integral Equation in the Solution of an Infinite Cylindrical Antenna With a Biconical Feed

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Abstract

In this report, the space-time domain magnetic field integral equation is used to investigate the transient behaviors of an infinite cylindrical antenna with a biconical feed. The formulation, and numerical methods are presented, and the difficulty in dealing with the source term is pointed out.

I. Introduction

Certain EMP simulators have the radiating elements in the form of a cylindrical antenna with a biconical wave launcher as shown in Fig. 1. The effect of the junction between the biconical section and the cylindrical section on the behavior of the transient radiation field have been studied by some workers [1]-[3]. So far, however, there is no rigorous study of this problem. Methods [2], [3], using the geometric theory of diffraction, are valid only for very early times. In this note, we present the study of this problem using the space-time domain magnetic field integral equation (MFIE) with a proposed numerical procedure for obtaining the solution.

The direct space-time domain integral equation methods have recently been studied more thoroughly [4], [5], [6]. They offer advantages over the space-frequency domain methods with Fourier inversion insofar as the early time accuracy and savings in computational time are concerned. The newly developed singularity expansion method [7], although having many advantages, is not particularly suitable for early time computation. Most space-time domain integral equation methods are devoted to the problems of both radiating and scattering from thin wires and yield extremely reliable results. These methods usually employ the electric field or the magnetic vector potential formulation. Bennett and Weeks [4] studied the scattering by finite bodies using the magnetic field formulation. Here, we study the MFIE as applied to the radiation problem; this has not yet been reported elsewhere. It turns out that the difference between the scattering case and the radiating case, i.e. the source term, causes many problems in this study.

The mathematical formulation will be outlined in the next section, and the proposed numerical procedures described in section III. In section IV, we detail the problems associated with the source term, using the cylindrical antenna as the example.

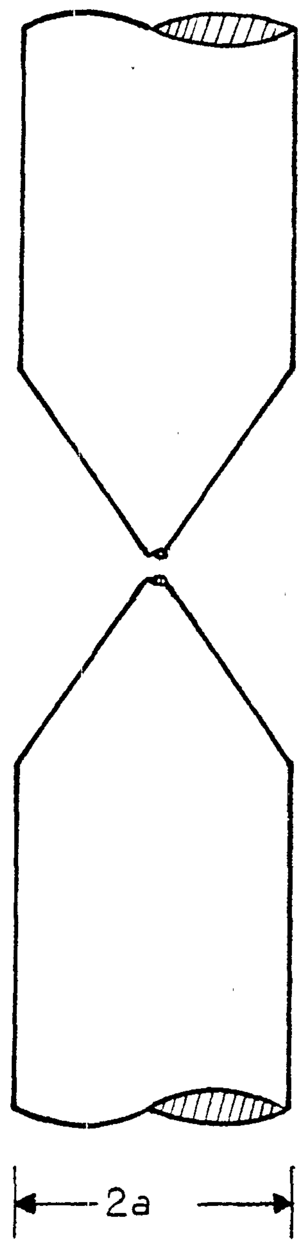

Figure 1. Cylindrical antenna with a biconical wave launcher.

II. Mathematical Formulation

The space-time domain MFIE has been derived elsewhere [6]. In this rotationally symmetrical case, we have

$$\underline{J}(\underline{r},t) = \frac{1}{2\pi}\hat{n} \times \int_{\text{Source}} \varepsilon \frac{\partial E^{inc}(\underline{r}',\tau)}{\partial \tau} \hat{\phi}' \frac{1}{|\underline{r}-\underline{r}'|} dS'$$

$$+ \frac{1}{2\pi}\hat{n} \times \int_{\text{Antenna}} \left(\frac{1}{|\underline{r}-\underline{r}'|} + \frac{1}{c}\cdot\frac{\partial}{\partial \tau}\right)\underline{J}(\underline{r}',\tau) \times \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|^2} dS' \qquad (1)$$

where $\underline{J}(\underline{r},t)$ is the surface current density on the surface of the antenna at the position $\underline{r}$ and time t, E^{inc} is the z-directed excitation electric field in the source region, $\hat{n}$ is the outward-directed normal to the surface at position $\underline{r}$, $\int$ denotes the principal value integral over the indicated surface. The quantities ε and c have the usual meaning of dielectric constant and velocity of light in the medium and

$$\tau = t - \frac{1}{c}\,|\underline{r} - \underline{r}'|, \qquad (2)$$

is the retarded time between the observation and source points.

We have to choose a coordinate system for this problem. Although the spherical coordinate system is best suited to the biconical section, it is not particularly convenient for the cylindrical section. On the other hand, the cylindrical coordinate system is suitable for the rotationally symmetrical structures and this is chosen for this problem.

There are three parts to this problem: the interaction of the current densities within the cylindrical section, within the biconical section and between these two sections. The basic methods of solution are the same for these parts, and they differ only in the details of the geometry. We shall, therefore, concentrate on the cylindrical case. Due to the rotationally symmetrical excitation, the surface current density is ϕ-independent, and is given by

$$J(z,t) = (2\pi)^{-1} \int_0^{2\pi} \underline{J}(\underline{r},t)\cdot\hat{z}\,d\phi,$$

which, when (1) is specialized to the cylindrical coordinates, satisfies the following integral equation:

$$J(z,t) = -\frac{\varepsilon a}{\pi}\int_0^{\pi}\int_{-b}^{b}\frac{\partial E^{inc}(z',\tau)}{\partial\tau}\frac{\cos\phi'}{\sqrt{(z-z')^2+4a^2\sin^2(\phi'/2)}}dz'd\phi'$$

$$+\frac{2a^2}{\pi}\int_0^{\pi}\int_{L_1}^{L_2}\left[\frac{1}{\sqrt{(z-z')^2+4a^2\sin^2(\phi'/2)}}+\frac{1}{c}\frac{\partial}{\partial\tau}\right]J(z',\tau)$$

$$\cdot\frac{\sin^2(\phi'/2)}{(z-z')^2+4a^2\sin^2(\phi'/2)}dz'd\phi', \qquad (3)$$

where 2b is the gap width, a is the radius of the cylinder, and $z = L_1$ to L_2 is the length of interest along the antenna.

The corresponding equation in the space-frequency domain as given by Latham and Lee [8], is

$$J(z,\omega) = (\pi a)^{-1}\int_{-b}^{b}Y(z-z',\omega)E^{inc}(z',\omega)dz' - 2\int_{L_1}^{L_2}K(z-z',\omega)J(z',\omega)dz' \qquad (4)$$

where

$$Y(z-z',\omega) = \frac{i\omega a^2}{Z_o c}\int_0^{\pi}\frac{\exp[i\omega c^{-1}\sqrt{(z-z')^2+4a^2\sin^2(\phi'/2)}]}{\sqrt{(z-z')^2+4a^2\sin^2(\phi'/2)}}\cos\phi' d\phi',$$

and

$$K(z-z',\omega) = \frac{a}{4\pi}\frac{\partial}{\partial a}\int_0^{\pi}\frac{\exp[i\omega c^{-1}\sqrt{(z-z')^2+4a^2\sin^2(\phi'/2)}]}{\sqrt{(z-z')^2+4a^2\sin^2(\phi'/2)}}d\phi'.$$

In Appendix A, we show that (3) can be obtained by taking the inverse Fourier transform of (4) with zero initial conditions. We also show that it is possible to have alternative forms of integral equations which involves double integrations containing the time convolution at the expense of the ϕ' integration in (3). It is concluded that (3) is superior than the other forms and will be considered in this note.

In the present problem, we consider only a cylindrical antenna of infinite extent. Therefore, we do not include the reflections from the ends of the antenna.

III. Numerical Procedures for a Cylindrical Antenna

Equation (3) can be solved by means of numerical methods. The unknown $J(z,t)$ is expressed in terms of the given excitation electric field E^{inc}, and the surface current density on the antenna at time prior to t, which are known. It is seen that the solution procedure involves no matrix inversions. The solution can be carried out step-by-step, i.e., in a time-marching manner.

It has to be pointed out here that the evaluation of the source term, i.e., the first term on the right hand side of (3) is very complicated and we devote the next section to describe this term. For the moment, we assume that the source term is evaluated in some convenient manner and concentrate on the scattered term, i.e., the second term in (3).

The antenna along the z-direction can be marked in increments Δz in length. The time increment Δt is defined by

$$\Delta t = \frac{1}{c}\,\Delta z. \tag{5}$$

The half circumference, i.e., from $\phi' = 0$ to π, is divided into $m + 1$ points, so that

$$\Delta\phi = \pi/m,$$

and the chord between two neighboring points is

$$2a\,\sin(\Delta\phi/2) \simeq \Delta z. \tag{6}$$

This sampling scheme is illustrated in Fig. 2.

We follow the usual numerical method by taking the quantity $J(z',\tau)$ constant over a cell, i.e., over a range of Δz and $\Delta\phi$. The scattered term over this cell is thus equal to the product of this J value and the kernel in the double integral over this cell. Denoting the scattered field produced from the source in the region $z' = z_1 - \Delta z/2$ to $z_1 + \Delta z/2$ and $\phi' = \phi_1 - \Delta\phi/2$ to $\phi_1 + \Delta\phi/2$ by $J^{sc}(z/z_1,\phi_1)$, we have the following expression

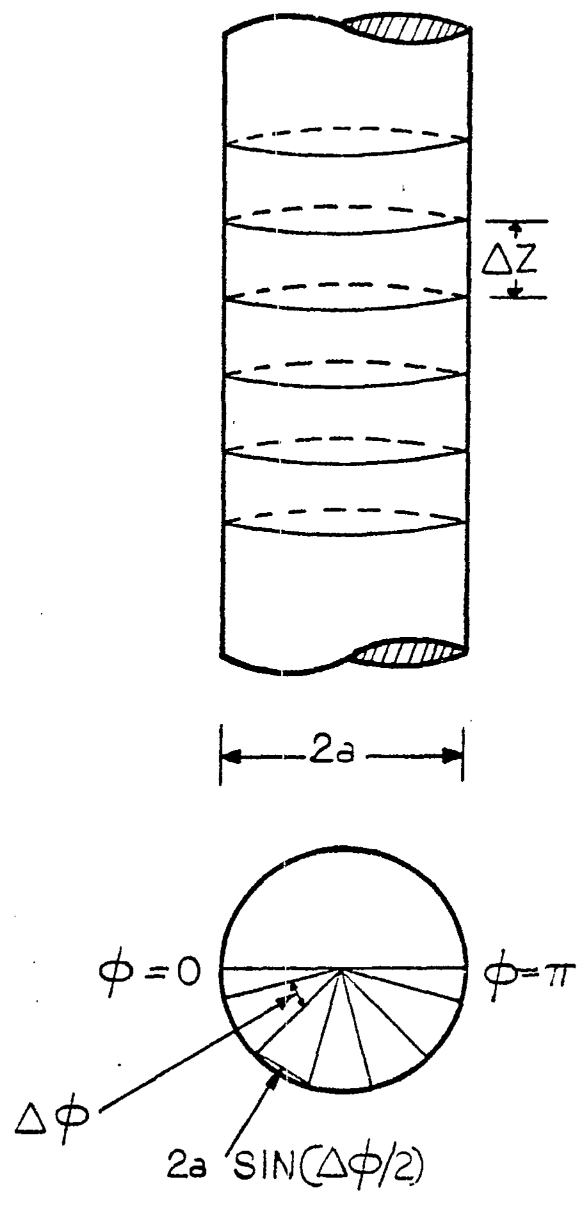

Figure 2. Sampling scheme for a cylindrical antenna. In this case m = 6 and $\Delta\phi = \pi/6$.

$$J^{sc}(z/z_1,\phi_1) \simeq \frac{2a^2}{\pi}\int_{\phi_1-\Delta\phi/2}^{\phi_1+\Delta\phi/2}\int_{z_1-\Delta z/2}^{z_1+\Delta z/2}\left\{\frac{J(z_1,\tau_1)}{[(z-z')^2+4a^2\sin^2(\phi'/2)]^{3/2}}\right.$$

$$\left.+\frac{J(z_1,\tau_1)-J(z_1,\tau_1-\Delta t)}{c\Delta t[(z-z')^2+4a^2\sin^2(\phi'/2)]}\right\}\sin^2(\phi'/2)dz'd\phi'. \qquad (7)$$

Integrating with respect to z', we get

$$J^{sc}(z/z_1,\phi_1) \simeq \frac{1}{4\pi}J(z_1,\tau_1)\int_{\phi_1-\Delta\phi/2}^{\phi_1+\Delta\phi/2}\left\{\frac{z-z_1+\Delta z/2}{\sqrt{(z-z_1+\Delta z/2)^2+4a^2\sin^2(\phi'/2)}}\right.$$

$$\left.-\frac{z-z_1-\Delta z/2}{\sqrt{(z-z_1-\Delta z/2)^2+4a^2\sin^2(\phi'/2)}}\right\}d\phi'$$

$$+\frac{a^2}{\pi\Delta z}\left\{J(z_1,\tau_1)-J(z_1,\tau_1-\Delta t)\right\}\int_{\phi_1-\Delta\phi/2}^{\phi_1+\Delta\phi/2}\sin^2(\phi'/2)$$

$$\cdot\left\{\tan^{-1}\frac{z-z_1+\Delta z/2}{2a\sin(\phi'/2)}-\tan^{-1}\frac{z-z_1-\Delta z/2}{2a\sin(\phi'/2)}\right\}d\phi'. \qquad (8)$$

The quantity τ_1 is defined by

$$\tau_1 = t - c^{-1}\sqrt{(z-z_1)^2+4a^2\sin^2(\phi_1/2)}.$$

The two single integrals in (8) can be conveniently integrated numerically. For $z - z_1 = (i-1)\Delta z$ and $\phi_1 = (k-1)\Delta\phi$, we denote

$$T^1_{i,k} = \frac{1}{2\pi}\int_{(k-3/2)\Delta\phi}^{(k-1/2)\Delta\phi}\left\{\frac{(i-1/2)\Delta z}{\sqrt{[(i-1/2)\Delta z]^2+4a^2\sin^2(\phi'/2)}}\right.$$

$$\left.-\frac{(i-3/2)\Delta z}{\sqrt{[(i-3/2)\Delta z]^2+4a^2\sin^2(\phi'/2)}}\right\}d\phi', \qquad (9)$$

$$T^2_{i,k} = \frac{2a^2}{\pi\Delta z}\int_{(k-3/2)\Delta\phi}^{(k-1/2)\Delta\phi} \sin^2\frac{\phi'}{2}\left\{\tan^{-1}\frac{(i-1/2)\Delta z}{2a\sin(\phi'/2)}\right.$$

$$\left. - \tan^{-1}\frac{(i-3/2)\Delta z}{2a\sin(\phi'/2)}\right\}d\phi', \qquad (10)$$

equation (8) becomes

$$J^{sc}(z/z_1,\phi_1) = T^1_{i,k}J(z_1,\tau_1) + T^2_{i,k}\{J(z_1,\tau_1) - J(z_1,\tau_1 - \Delta t)\}. \qquad (11)$$

In the special condition that $z_1 = z$ and $\phi_1 = 0$, i.e., at the singular cell, the integrand in the first term of (7) is singular but is integrable. We use the method of auxiliary integral [5] for this evaluation. This method enables the singular part be subtracted out which is analytically integrable; the rest of the integration is carried out numerically. Let

$$S = \frac{2a^2}{\pi}\int_o^{\Delta\phi/2}\int_{z-\Delta z/2}^{z+\Delta z/2}\frac{\sin^2(\phi'/2)}{[(z-z')^2+4a^2\sin^2(\phi'/2)]^{3/2}}dz'd\phi'$$

$$= \frac{2a^2}{\pi}\int_o^{\Delta\phi/2}\int_{z-\Delta z/2}^{z+\Delta z/2}\frac{\sin^2(\phi'/2)[\cos(\phi'/2)+1-\cos(\phi'/2)]}{[(z-z')^2+4a^2\sin^2(\phi'/2)]^{3/2}}dz'd\phi'$$

$$= S^a + S^b,$$

where

$$S^a = \frac{2a^2}{\pi}\int_o^{\Delta\phi/2}\int_{z-\Delta z/2}^{z+\Delta z/2}\frac{\sin^2(\phi'/2)\cos(\phi'/2)}{[(z-z')^2+4a^2\sin^2(\phi'/2)]^{3/2}}dz'd\phi'$$

$$= \frac{4a^2}{\pi}\int_o^{\Delta\phi/2}\int_{z}^{z+\Delta z/2}\frac{\sin^2(\phi'/2)\cos(\phi'/2)}{[(z-z')^2+4a^2\sin^2(\phi'/2)]^{3/2}}dz'd\phi'$$

$$= \frac{\Delta z}{2a\pi}\log\left[\frac{4a\sin(\Delta\phi/4)}{\Delta z} + \sqrt{1+[\frac{4a\sin(\Delta\phi/4)}{\Delta z}]^2}\right]$$

and

$$S^b = \frac{4a^2}{\pi} \int_o^{\Delta\phi/2} \int_z^{z+\Delta z/2} \frac{\sin^2(\phi'/2)(1-\cos(\phi'/2))}{[(z-z')^2+4a^2 \sin^2(\phi'/2)]^{3/2}} dz' d\phi'$$

$$= \frac{1}{\pi} \int_o^{\Delta\phi/2} (1 - \cos \frac{\phi'}{2}) \frac{\Delta z}{\sqrt{\Delta z^2+16a^2 \sin^2(\phi'/2)}} d\phi'$$

This integrand vanishes when $\phi' = 0$ and hence the integration can be carried out numerically. The quantity $T^1_{1,1}$ is then defined by

$$T^1_{1,1} = S^a + S^b.$$

At the singular cell, the second term in (7) is not singular. In fact, as $z_1 = z$ and $\phi_1 = 0$,

$$T^2_{1,1} = \frac{1}{2\pi} \Delta\phi.$$

The solution of (3) in the discretized form, is written as

$$\begin{aligned} J(z,t) = (1 - T^1_{1,1} - T^2_{1,1})^{-1} \Big\{ & \text{Source Term} - T^2_{1,1} J(z,t - \Delta t) \\ & + \sum_{i=2} \sum_{k=2}^{m} [(T^1_{i,k} + T^2_{i,k}) J\{z - (i - 1)\Delta z, \tau_{i,k}\} \\ & - T^2_{i,k} J\{z - (i - 1)\Delta z, \tau_{i,k} - \Delta t\}] \\ & + \sum_{i=2} \sum_{k=2}^{m} [(T^1_{i,k} + T^2_{i,k}) J\{z + (i - 1)\Delta z, \tau_{i,k}\} \\ & - T^2_{i,k} J\{z + (i - 1)\Delta z, \tau_{i,k} - \Delta t\}] \end{aligned} , \quad (12)$$

where

$$\tau_{i,k} = t - c^{-1}\sqrt{[(i - 1)\Delta z]^2 + 4a^2 \sin^2[(k - 1)\Delta\phi/2]}.$$

It has to be pointed out that $\tau_{i,k}$ may not be a multiple of Δt, i.e., the value $J[z \pm (i - 1)\Delta z, \tau_{i,k}]$ may not be at a sampled point and interpolation (or extrapolation) has to be used. In this case, we use a simple t-wise linear interpolation (or extrapolation).

In Table I, we tabulate the values of $T^1_{i,k}$ and $T^2_{i,k}$ for the case $\Delta z = 0.02$, $a = 0.1$, $\Delta\phi = \pi/6$.

Table I. $T^1_{i,k}$, $T^2_{i,k}$ values for $a = 0.1$, $\Delta z = 0.02$ and $\Delta\phi = \pi/6$

i \ k	1	2	3	4	5	6	7	
1	1.184(-1)	7.248(-2)	4.042(-2)	2.901(-2)	2.381(-2)	2.139(-2)	2.067(-2)	← $T^1_{i,k}$
	8.333(-2)	3.070(-4)	6.238(-4)	8.911(-4)	1.095(-3)	1.223(-3)	1.267(-3)	← $T^2_{i,k}$
2	0	3.234(-2)	2.979(-2)	2.454(-2)	2.121(-2)	1.946(-2)	1.892(-2)	
	0	1.763(-4)	5.074(-4)	7.961(-4)	1.013(-3)	1.248(-3)	1.194(-3)	
3	0	8.409(-3)	1.507(-2)	1.615(-2)	1.565(-2)	1.511(-2)	1.491(-2)	
	0	7.215(-5)	3.216(-4)	6.015(-4)	8.270(-4)	9.693(-4)	1.018(-3)	
4	0	2.918(-3)	7.281(-3)	9.626(-3)	1.046(-2)	1.068(-2)	1.071(-2)	
	0	3.583(-5)	1.981(-4)	4.260(-4)	6.319(-4)	7.686(-4)	8.160)-4)	
5	0	1.301(-3)	3.797(-3)	5.739(-3)	6.805(-3)	7.279(-3)	7.409(-3)	
	0	2.097(-5)	1.285(-4)	3.018(-4)	4.744(-4)	5.953(-4)	6.383(-4)	

The values in Table I are written in the form of magnitude (power of 10). E.g. $1.184(-1) = 1.184\times10^{-1}$.

IV. Source Term for a Cylindrical Antenna

As far as we are aware, there have not been any attempts to use the space-time domain MFIE for a radiation problem. The evaluation of the source term for the radiation case has found to be troublesome and we report this in more detail here. We shall, however, compare this term with Wu's result [9] for an infinitely long cylindrical antenna with a delta gap when the excitation electric field is a time step.

When we have a time step excitation electric field, i.e.,

$$E^{inc}(z,t) = E_o U(t) f(z), \qquad (13)$$

the time derivative of E^{inc}, as occurs in the source term of (3), is a delta function in time,

$$\frac{\partial E^{inc}(z,t)}{\partial t} = E_o \delta(t) f(z).$$

The function f(z) specifies the space-distribution of the source. The source term now is

$$J^{inc}(z, t) = -\frac{\varepsilon a E_o}{\pi} \int_0^{\pi} \int_{-b}^{b} \delta(\tau) f(z') \frac{\cos \phi'}{\sqrt{(z-z')^2 + 4a^2 \sin^2(\phi'/2)}} dz' d\phi'. \qquad (14)$$

It is interesting to observe that with this MFIE formulation, the time step excitation electric field excites the antenna in a time delta function manner, i.e., a "one-pass" process. After this impulse passes an observation point, the current density at this point is substained only by the interaction of current densities at other points, with the contributions given by the scattered term. In the EFIE formulation, the time step excitation electric field contibutes to the current density continuously. It is now clear that the evaluation of this MFIE source term is more critical than the EFIE.

For a delta gap, i.e., $f(z) = \delta(z)$, (14) becomes

$$J^{inc}(z, t) = -\frac{\varepsilon a E_o}{\pi} \int_0^{\pi} \delta\left(t - \frac{1}{c}\sqrt{z^2 + 4a^2 \sin^2(\phi'/2)}\right) \frac{\cos \phi'}{\sqrt{z^2 + 4a^2 \sin^2(\phi'/2)}} d\phi'. \qquad (15)$$

To carry out the integration, the δ-function is first treated as a function in ϕ'. This is done using the following property:

$$\delta[p(x)] = \left| \left(\frac{dp(x_o)}{dx} \right)^{-1} \right| \delta(x - x_o),$$

where (16)

$$p(x_o) = 0 \quad \text{and} \quad \frac{dp(x_o)}{dx} \neq 0.$$

The δ-function in (15) is thus rewritten as

$$\delta\left(t - \frac{1}{c}\sqrt{z^2 + 4a^2 \sin^2(\phi'/2)}\right) = \frac{c\sqrt{z^2+4a^2 \sin^2(\phi_o/2)}}{a^2 \sin \phi_o}\, \delta(\phi' - \phi_o) \tag{17}$$

where ϕ_o is the value of ϕ' so that the argument of the left hand side vanishes, i.e.,

$$\sin(\phi_o/2) = (2a)^{-1}\sqrt{(ct)^2 - z^2}. \tag{18}$$

Now (15) becomes

$$J^{inc}(z, t) = -\frac{E_o}{Z_o \pi a} \frac{\cos \phi_o}{\sin \phi_o}$$

$$= -\frac{E_o}{Z_o \pi a} \frac{1-2 \sin^2(\phi_o/2)}{2 \sin(\phi_o/2) \cos(\phi_o/2)} . \tag{19}$$

The quantity Z_o is the intrinsic impedance of the medium. At the very early time, i.e., $ct \to z$, $\sin(\phi_o/2) \to 0$ and $\cos(\phi_o/2) \to 1$

$$J^{inc}(z, t) \to -\frac{E_o}{Z_o \pi a} \cdot \frac{1}{2 \sin(\phi_o/2)}$$

$$= -\frac{E_o}{Z_o 2\pi a} \frac{2a}{\sqrt{(ct)^2-z^2}}$$

At this very early time, the total surface current density J is contributed only from the source term. The total current, being $2\pi aJ$, is

$$I(z,t) = -E_o 2aZ_o^{-1}[(ct)^2 - z^2]^{-1/2}. \qquad (20)$$

Apart from the amplitude of the electric field $-E_o$, this is identical to Wu's result.

For a finite gap, the obvious step from the numerical point of view is to divide the source region into cells. The method is similar to that outlined in the last section and quantities similar to $T^1_{i,k}$ are defined. When $z' = z$ and $\phi' = 0$, the integrand is singular but integrable, and the method of auxiliary integral is used.

One serious drawback of this method is the fact that the retarded time may not be a multiple of Δt. As the δ-function has zero time width, this would produce a "miss" phenomenon. One could, of course, introduce the criterion that if $\tau \leq |\Delta t/2|$, the δ-function is considered to be present. However, this introduces discretization errors which are large for this present purpose. In fact, computation shows that these errors are of the same order as the scattered term and the resultant accuracy is doubtful.

Another way is to utilize the δ-function integration property to reduce the source term to a single integral with a defined trajectory. This method introduces no discretization error and gives more insight into the source term.

With a uniform source distribution, $f(z') = 1$ in (14). Using the δ-function property as given by (16), the δ-function is rewritten as

$$\delta\left(t - \frac{1}{c}\sqrt{(z - z')^2 + 4a^2 \sin^2(\phi'/2)}\right) = \delta(z - z_o)c|z - z_o|^{-1}\sqrt{(z - z_o)^2 + 4a^2 \sin^2(\phi'/2)},$$

where,

$$t - \frac{1}{c}\sqrt{(z - z_o)^2 + 4a^2 \sin^2(\phi'/2)} = 0,$$

or,

$$|z - z_o| = \sqrt{(ct)^2 - 4a^2 \sin^2(\phi'/2)}. \qquad (21)$$

Equation (14) becomes

$$J^{inc}(z,t) = -\frac{aE_o}{Z_o\pi}\int \frac{\cos\phi'}{|z-z_o|}\,d\phi', \tag{22}$$

and from (21)

$$J^{inc}(z,t) = -\frac{aE_o}{Z_o\pi}\int \frac{\cos\phi'}{\sqrt{(ct)^2-4a^2\sin^2(\phi'/2)}}\,d\phi'. \tag{23}$$

The integration limits are subject to the following conditions

$$-b \le z_o \le b$$
$$0 \le \phi' \le \pi. \tag{24}$$

For a given value of t, (21) and (24) define the trajectory along which the δ-function contributes to $J^{inc}(z,t)$. This is illustrated in Fig. 3. When the observation point is outside the source region, the integration is from $\phi' = \phi_A$ to ϕ_B as shown in Fig. 3(a). When the observation point is within the source region, the integration is from $\phi' = \phi_A$ to ϕ_B plus from $\phi' = \phi_C$ to ϕ_B, as shown in Fig. 3(b). At ct = 2a, $\phi_B = \pi$.

The evaluation of (23) is straightforward when the observation point is outside the source region. The integration limits are from ϕ_A to ϕ_B, as shown in Fig. 3(a). When the observation point is inside the source region, the denominator of the integrand, i.e., $|z - z_o|$, could vanish. If $ct \ne 2a$, it is integrable. As shown in Fig. 3(b), the integration is from ϕ_A to ϕ_B plus that from ϕ_C to ϕ_B. The point B is such that $z_o = z$. It is convenient to carry out the following transformation

$$\sin\theta = (2a/ct)\sin(\phi'/2), \tag{25}$$

and (23) becomes

$$J^{inc}(z,t) = -\frac{E_o}{Z_o\pi}\int_{\theta_L}^{\pi/2} \frac{1-2k^2\sin^2\theta}{\sqrt{1-k^2\sin^2\theta}}\,d\theta \tag{26}$$

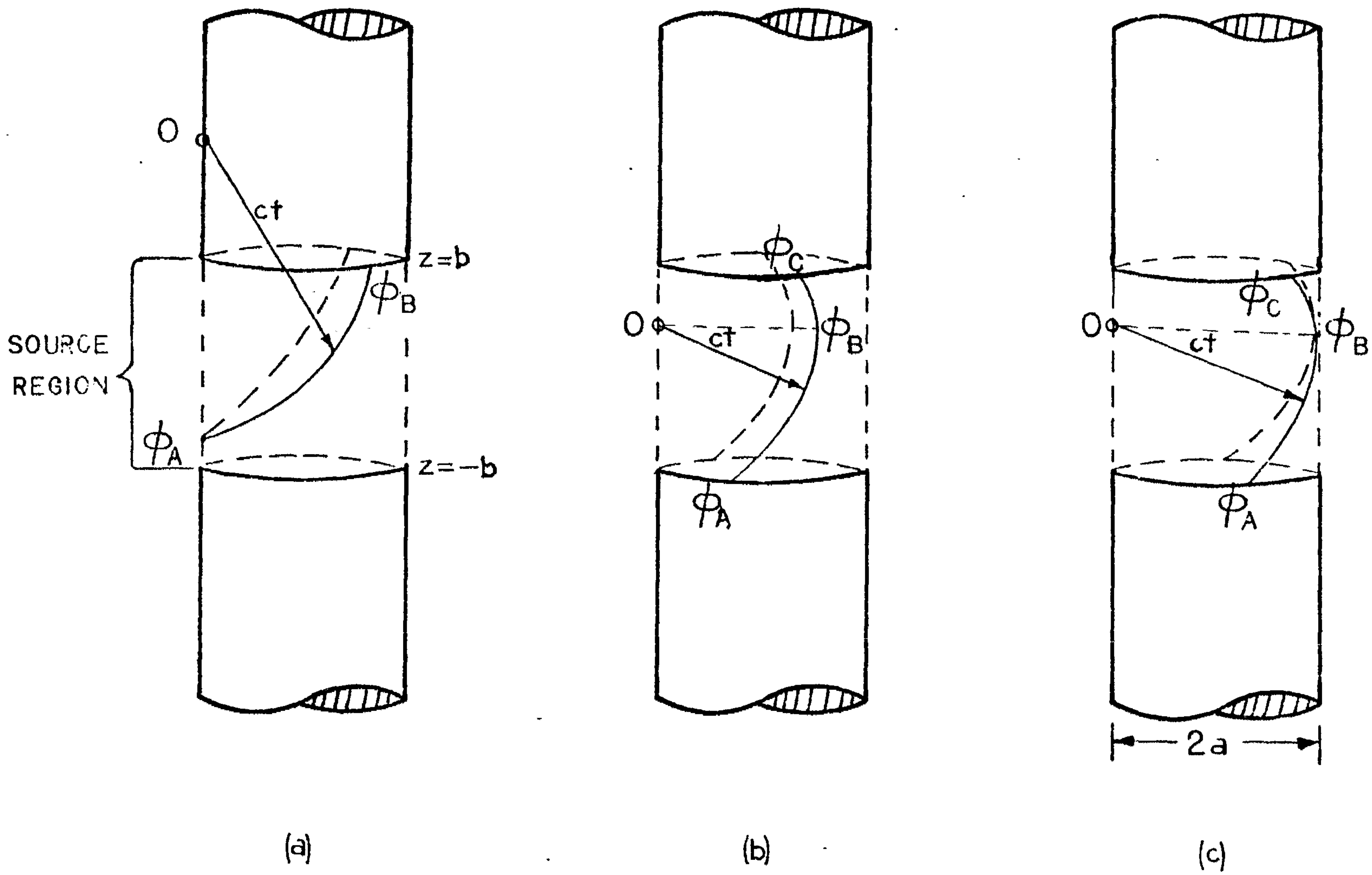

Figure 3. Trajectories of δ-function contribution to an observation point.
(a) The observation point is outside the source region,
(b) the observation point is inside the source region, and
(c) the case ct = 2a.

where

$$k = ct/2a$$

and θ_L is the lower limit of integration corresponding to

$$\theta_L = \sin^{-1}[k^{-1} \sin(\phi_L/2)]$$

where ϕ_L is either ϕ_A or ϕ_C. Equation (26) is in the form of elliptic integrals and can be readily evaluated except for the special case $k = 1$.

One interesting case is when $ct \to 0$. In this case, the range of the angle ϕ' in (14) is very small. Equation (14) can now be looked upon as the contribution from a plane semi-circumference with radius ct, as is given by Fig. 4.

$$\lim_{ct \to 0} J^{inc}(z, t) = -\frac{\varepsilon E_o}{\pi} \int_o^{\pi} \int_{-b}^{b} \delta(t - \rho_t/c)(\cos \phi'/\rho_t) dS'$$

$$= -\frac{\varepsilon E_o}{\pi} \iint \delta(t - \rho_t/c)(\cos \phi'/\rho_t) \cdot \rho_t d\rho_t d\theta_t$$

$$= -\frac{E_o}{Z_o} .$$

This result has also been obtained by some other means.

The case that $k = 1$, i.e., $ct = 2a$, occurs when the contribution comes from, among other parts, the point diametrically opposite the observation point. This is illustrated in Fig. 3(c). In this case, the integrands in (23) or (26) are not integrable, and as $ct \to 2a$, the integral tends to infinity asymptotically as $\ln[1/(1-k^2)]$. Thus, within the source region, the current density could be infinite at this particular instance of time. However, this type of asymptotic behavior is integrable and would not give rise to infinite values to the scattered terms at later time.

For the scattered term, the integration across cells with this logarithmic singularity is difficult. This singularity is expressed as a function of t,

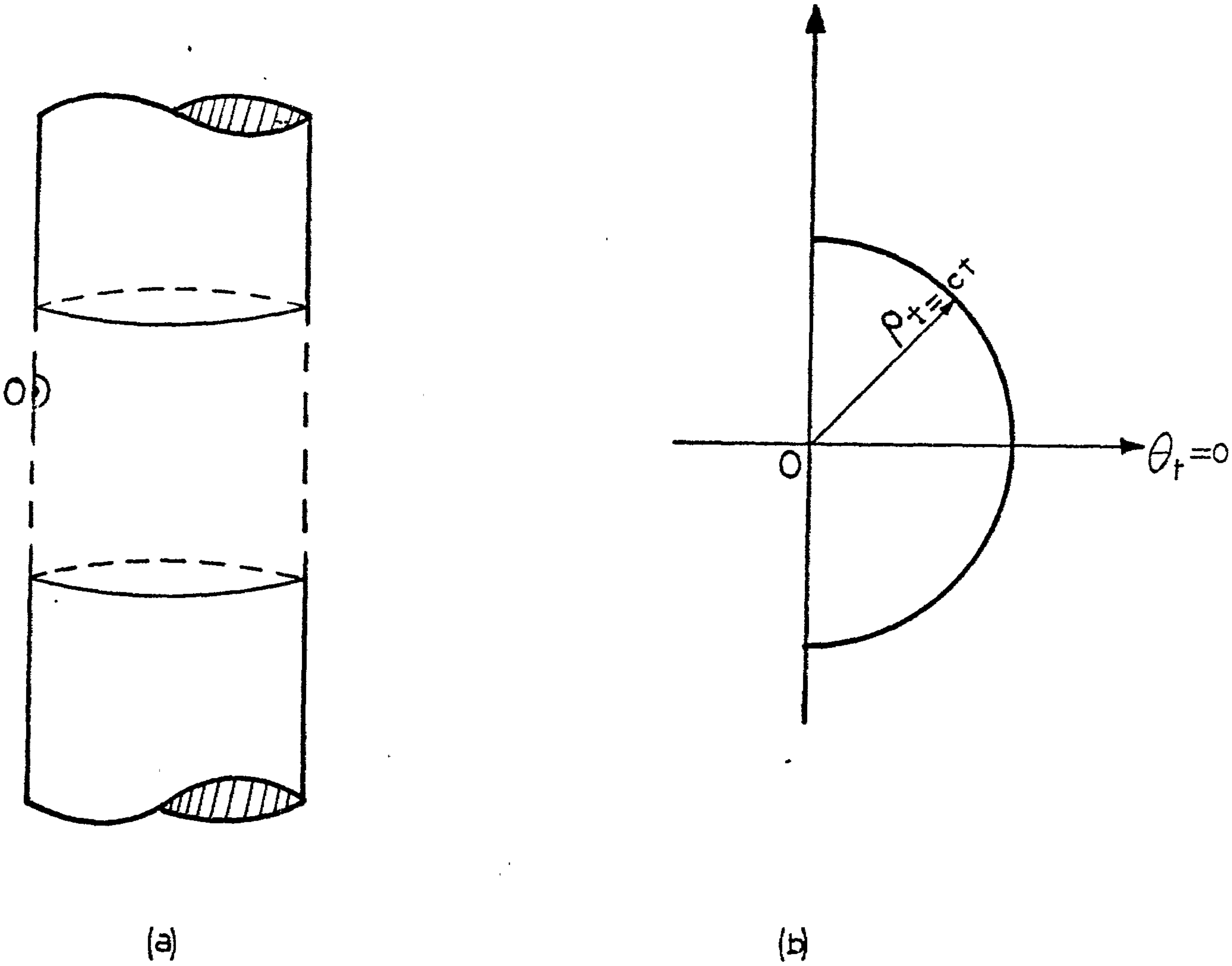

Figure 4. The case $ct \to 0$.

(a) The trajectory of contribution and

(b) the planar trajectory of contribution approximation.

and we have to convert the integrated parameters, i.e., z' and ϕ', as time parameters. This process is tedious, time consuming and offers sources of inaccuracy. This procedure has to be applied for each individual cell because each cell has different (z',ϕ'-time) relationships. Because of the singularity nature, this procedure is necessary and cannot be easily approximated.

V. Infinite Biconical Antenna and Cylindrical Antenna with a Biconical Feed

In the previous two sections, we detail the formulation and treatment for an infinite cylindrical antenna. Here, we point out the same for an infinite biconical antenna. The space-time domain MFIE, in the cylindrical coordinates, is

$$J_r(z,t) = -\frac{\varepsilon a}{2\pi}\int_{-\pi}^{\pi}\int_{\text{Source}} \frac{\partial E^{inc}(z',\tau)}{\partial\tau} \frac{\cos\phi'}{\sqrt{[(z-z')^2+4zz'\sin^2(\phi'/2)]T+(z-z')^2}}\,dz'd\phi'$$

$$+\frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{L_1}^{L_2}\left\{\frac{1}{[\{(z-z')^2+4zz'\sin^2(\phi'/2)\}T+(z-z')^2]^{3/2}}\right.$$

$$\left.+\frac{1}{[(z-z')^2+4zz'\sin^2(\phi'/2)]T+(z-z')^2}\frac{1}{c}\frac{\partial}{\partial\tau}\right\}$$

$$\cdot\, J_r(z',\tau)2zz'T\sin^2(\phi'/2)dz'd\phi', \tag{27}$$

where θ_o is the angle of the biconical antenna, as shown in Fig. 5, J_r is the radially directed surface current density,

$$T = \tan^2\theta_o,$$

and

$$\tau = t - c^{-1}\{[(z - z')^2 + 4zz'\sin^2(\phi'/2)]T + (z - z')^2\}$$

The method of solution of (27) is similar to that of (3), as outlined in section III and IV. However, in the present case, the coefficient of $\sin^2(\phi'/2)$ involves z' and this makes the singular integration much more difficult. Reasonable approximation has to be introduced to reduce the complexity of the integration.

Of particular interests is the source term. This term is strongly dependent on the source geometry assumed. For a point source, i.e., the electric field is applied only at the vertex, the source term contributes a singularity to the current density, which can be seen from (27); this is clearly

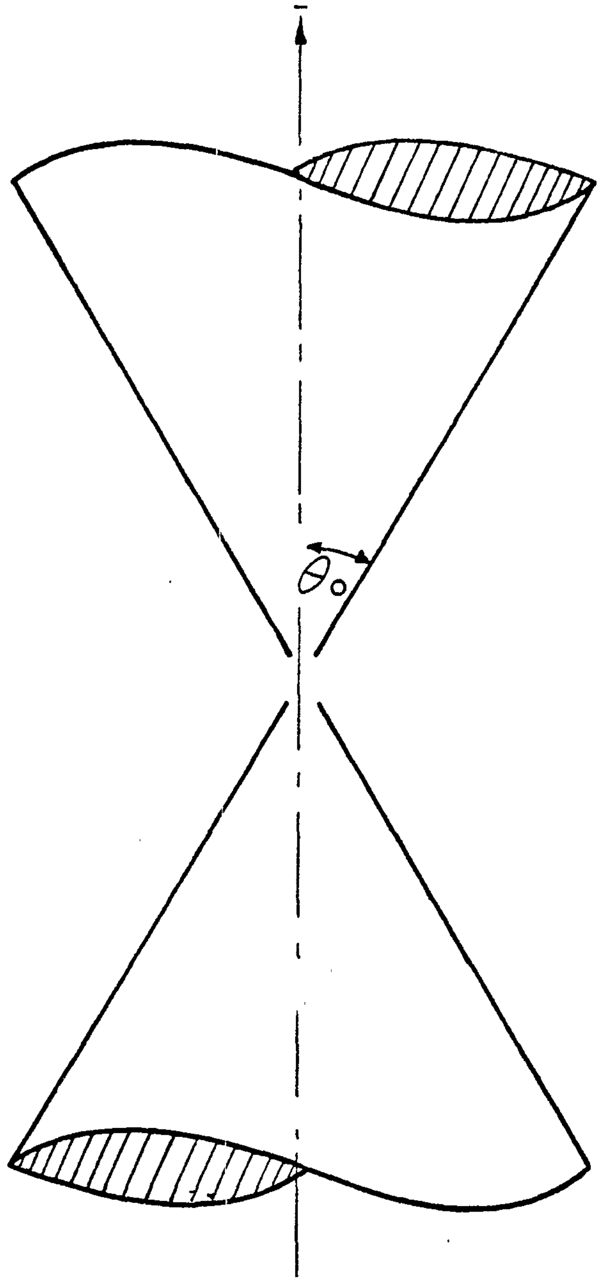

Figure 5. The infinite biconical antenna.

not a suitable model. One could truncate part of the vertex and model the source as a small cylindrical region, similar to that outlined in the last section. This model has the same property as that outlined in the last section and the logarithmic singularity nature makes the evaluation of the scattered term complicated. Other models, such as a spherical feed, complicate the overall geometry and also have similar kind of trouble.

This process of formulation and numerical method can be extended to the infinite cylindrical antenna with a biconical feed. However, the problem lies on the satisfactory modeling of the source region and the likely occurence of the singular values of the current density.

VI. Conclusions

We have presented the space-time domain MFIE and the numerical procedures of solution, with the infinite cylindrical antenna as a specific example. The method is a step-by-step time marching technique and involves no inversion of matrices. However, numerical results have not been presented, due to difficulties inherent in this type of formulation.

The main difficulties in the solution can be summarized as:

(1) The source term is strongly dependent on the source geometry. As pointed out in section V, all the possible source geometries have a certain kind of singularity problem. The most realistic geometries are the ones with the vertex of the biconical section truncated, and the ones with a spherical feed in place of the vertex. Both of these have the singularity described below.

(2) If the source region is in the form of a circular cylinder with radius a, then, at ct = 2a, the current density within the source region is infinite. This singular current density is integrable and does not contribute to further singular values of the current density at later time. As pointed out, however, the evaluation of the scattered term with this singular current density is critical, the procedure is tedious and the accuracy is not reliable.

(3) This above trouble arises from the time step excitation electric field, as its time derivative is a delta time function. One alternative is to use a narrow pulse, such as a Gaussian pulse, to represent the δ-function. However, this suffers some disadvantages. Such a pulse can not be integrated as conveniently as the δ-function and hence discretization error is large and is comparable to the value of the scattered term. This error can be reduced by taking a "slow" pulse, but this leads to an inaccurate representation of the excitation waveform.

(4) It is worth noting that in the scattering problem using space-time domain MFIE [4], the source term is relatively easy to handle. It has been pointed out, however, that the current density may exhibit instabilities and the solution may grow with time. This may be attributed to the numerical procedure used.

In view of the above difficulties resulting from this initial investigation into the feasibility of using the space-time domain MFIE in solving this type of simulator problems, it is strongly recommended that such formulations should not be used. In the following, we briefly outline the alternative method of solution -- the electric field formulation.

In applying the space-time domain Hallén integral equation to solving thin wire problems [5], the source term presents no difficulty at all, neither does the scattered term give rise to instability. The equation is of the following form

$$\int_{-\pi}^{\pi}\int_{L_1}^{L_2} \frac{I(z',t-|z-z'|/c)}{8\pi^2\sqrt{(z-z')^2+4a^2\sin^2(\phi'/2)}}\,dz'd\phi' = \frac{1}{2Z_o}\int_{L_1}^{L_2} E^{inc}(z',t-|z-z'|/c)dz' + f_1(ct-z)+f_2(ct+z), \qquad (28)$$

where the current I(z,t) is z-directed, a is the radius of the wire, E^{inc} is the excitation electric field, f_1 and f_2 are two functions to take care of the effects of the reflections of the current by the ends of the antenna. For an infinitely long wire, $f_1 = f_2 = 0$. It is observed that the source term, i.e., the integral on the right hand side of the equation, involves the electric field directly. This has nicer property than the $(\partial/\partial\tau)E^{inc}$ quantity of the MFIE, and (28) would not give rise to the type of singularity the MFIE gives.

For a thick wire, (28) is not appropriate and we use the space-time domain EFIE, which has been applied to the scattering problem by Davis et. al. [6]. The equation is

$$\hat{n}\times\underline{E}^{inc}(\underline{r},t) = \frac{1}{4\pi}\hat{n}\times\int_S\left\{\frac{\mu}{|\underline{r}-\underline{r}'|}\frac{\partial\underline{J}(\underline{r}',\tau)}{\partial\tau} + \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|}\left(\frac{1}{|\underline{r}-\underline{r}'|^2}+\frac{1}{|\underline{r}-\underline{r}'|}\frac{1}{c}\frac{\partial}{\partial\tau}\right)\frac{1}{\varepsilon}\int_{-\infty}^{\tau}\nabla\cdot\underline{J}\,d\tau'\right\}dS', \qquad (29)$$

and

$$\tau = t - c^{-1}|\underline{r} - \underline{r}'|.$$

Here, the source term contains $\underline{E}^{inc}(\underline{r},t)$ and again, this has nicer property than the MFIE. The numerical process in solving (29) would be similar to that of (3).

From the above description, the electric field formulation has a better-behaved source term than the MFIE. The evaluation of this term is less critical and discretization process of solution can be applied with higher accuracy. It is suggested that future investigation into this problem be made using the space-time domain EFIE.

Appendix A.

Alternative Forms of Space-Time Domain MFIE

In this appendix we show that the inverse Fourier transform of (4) into the space-time domain yields two triple integrals with respect to z', ϕ' and t'. Taking different orders of integration, various forms of integral equations are obtained, which have their own physical interpretations. We shall judge their relative merits from the point of view of numerical evaluation.

The integrands of (4) consists of products of two functions of frequency, and hence the inverse Fourier transform contains convolution integrals in time, i.e.,

$$J(z,t) = (\pi a)^{-1} \iint Y(z - z',t - t')E^{inc}(z',t')dt'dz'$$
$$-2 \iint K(z - z',t - t')J(z',t')dt'dz'. \tag{A.1}$$

The integration limits are determined by initial conditions and causality. It can be readily shown that

$$Y(z - z',t) = -a^2(Z_o c)^{-1} \frac{\partial}{\partial(t-c^{-1}R)} \int_o^{\pi} R^{-1} \cos \phi' \delta(t - c^{-1}R)d\phi', \tag{A.2}$$

and

$$K(z - z',t) = (4\pi)^{-1} a \frac{\partial}{\partial a} \int_o^{\pi} R^{-1} \delta(t - c^{-1}R)d\phi', \tag{A.3}$$

where

$$R = \sqrt{(z - z')^2 + 4a^2 \sin^2(\phi'/2)}. \tag{A.4}$$

We shall consider the following cases arising from taking different orders of integration.

Case 1. Integrating (A.1) first with respect to t', we obtain (3). This equation describes the contribution to J(z,t) from $E^{inc}(z',\tau)$ and the values of $J(z',\tau)$ from all the points on the surface of the antenna.

Case 2. Integrating (A.1) first with respect to ϕ', we obtain an integral equation describing the contribution to $J(z,t)$ from rings of current densities on the antenna. The integration limits are either for all time up to t with a restricted range of z', or for the whole length of the antenna with a restricted range of t'.

To carry out this ϕ' integration, we make use of (16), such that

$$\delta(t - c^{-1}R) = \delta(\phi' - \phi_o)cR(a^2 \sin \phi_o)^{-1}. \tag{A.5}$$

where

$$\sin (\phi_o/2) = (2a)^{-1} \sqrt{c^2t^2 - (z - z')^2}. \tag{A.6}$$

It is the condition

$$0 \leq \sin^2(\phi_o/2) \leq 1$$

that restricts the integration limits on t' and z'. Equation (A.1) can now be written as:

$$J(z,t) = a(Z_o\pi)^{-1} \int_{-\Delta}^{\Delta} \int_{t-c^{-1}\sqrt{4a^2+(z-z')^2}}^{t-c^{-1}|z-z'|} \frac{2a^2-[c^2(t-t')^2-(z-z')^2]}{\sqrt{c^2(t-t')^2-(z-z')^2}\sqrt{4a^2-[c^2(t-t')^2-(z-z')^2]}} \frac{\partial E^{inc}(z',t')}{\partial t'} dt'dz'$$

$$-(c\pi)^{-1} \int_{L_1}^{L_2} \int_{t-c^{-1}\sqrt{4a^2+(z-z')^2}}^{t-c^{-1}|z-z'|} \left[\frac{c^2(t-t')^2-(z-z')^2}{4a^2-[c^2(t-t')^2-(z-z')^2]}\right]^{1/2}$$

$$\left[\frac{1}{(t-t')^2} + \frac{1}{(t-t')} \frac{\partial}{\partial t'}\right] \cdot J(z',t')dt'dz' \tag{A.7}$$

This equation is more involved than (3). The integration limits with respect to t' are functions of z'. This is more complicated numerically and requires more computer memories.

Alternatively, (A.7) can be expressed as:

$$J(z,t) = a(Z_o\pi)^{-1} \int_o^t \left[\int_{z-c(t-t')}^{z-\sqrt{c^2(t-t')^2-4a^2}} + \int_{z+\sqrt{c^2(t-t')^2-4a^2}}^{z+c(t-t')} \right] I_1(z',t')dz'dt'$$

$$-(c\pi)^{-1} \int_o^t \left[\int_{z-c(t-t')}^{z-\sqrt{c^2(t-t')^2-4a^2}} + \int_{z+\sqrt{c^2(t-t')^2-4a^2}}^{z+c(t-t')} \right] I_2(z',t')dz'dt', \tag{A.8}$$

where $I_1(z',t')$ is the integrand of the first double integral in (A.7), and $I_2(z',t')$ is of the second one. The z' integration of the first double integral in (A.8) is limited to the gap, and the second is limited to the antenna. Equation (A.8) is even more complicated than (A.7). Furthermore, the t' integration is from 0 to t, i.e., it changes with increasing t and requires large computer memories at large value of t.

Case 3. Integrating (A.1) first with respect to z', we would obtain integral equations similar to (A.7) and (A.8) with similar complexities.

It is concluded that (3) is most appropriate for the numerical solution and hence is treated exclusively in the text.

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