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Proximity Effect of Semi-infinite Parallel Plate Transmission Line in the Presence of a Perfectly Conducting Ground

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Abstract

The effect of ground proximity on the field distribution of semi-infinite parallel plate transmission line is investigated. Detailed numerical calculations are presented in this note.

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1. Introduction

In recent years extensive works have been carried out in the field of EMP simulators at Air Force Weapons Laboratory initiated by Captain Carl E. Baum. In particular, the field distribution and line impedance of a parallel plate transmission has been examined in detail by Baum(1) and extended by Brown and Granzow(2). This basic structure can be considered as the proto-type of simulators considered in other notes(3, 4). When a semi-infinite parallel plate transmission line is in the proximity of a perfectly conducting plane ground, one expects the field distribution and impedance of a transmission line to be modified to a greater or lesser extent, depending on the degree of the ground proximity. In this note the effect of the ground proximity on the field distribution will be discussed in order to establish such ground effects in explicit numerical terms. For imperfectly conducting ground plane, when the loss factor $\sigma/\omega\epsilon$ is large, one expects that the result would be equally applicable.

The field distribution of a semi-infinite parallel plate transmission line placed in the proximity of a perfectly conducting plane ground is solved rigorously by conformal transformation. The derivation and the essential formulas are presented in the Appendix. The theoretical results so obtained are discussed in Section 2.

In Section 3, results of numerical computations are presented for field-line distribution and electric field intensity for several ground proximities.

Based on the theoretical and numerical results presented in Sections 2 and 3, conclusions are drawn in Section 4 on the effects of the perfectly conducting plane on the distribution of the field lines and electric field intensities.

2. Theoretical Results

In this section we present a theoretical discussion on field lines and electric field intensities in a semi-infinite parallel plate transmission line system in the proximity of a perfectly conducting plane ground. The geometric configuration of such a system is shown in Fig. 2-1. The plate separation distance is designated by $2b$ and the distance between the near edge of the plates to the ground by d. If the ground and the plates are assumed to be perfectly conducting, the line can support

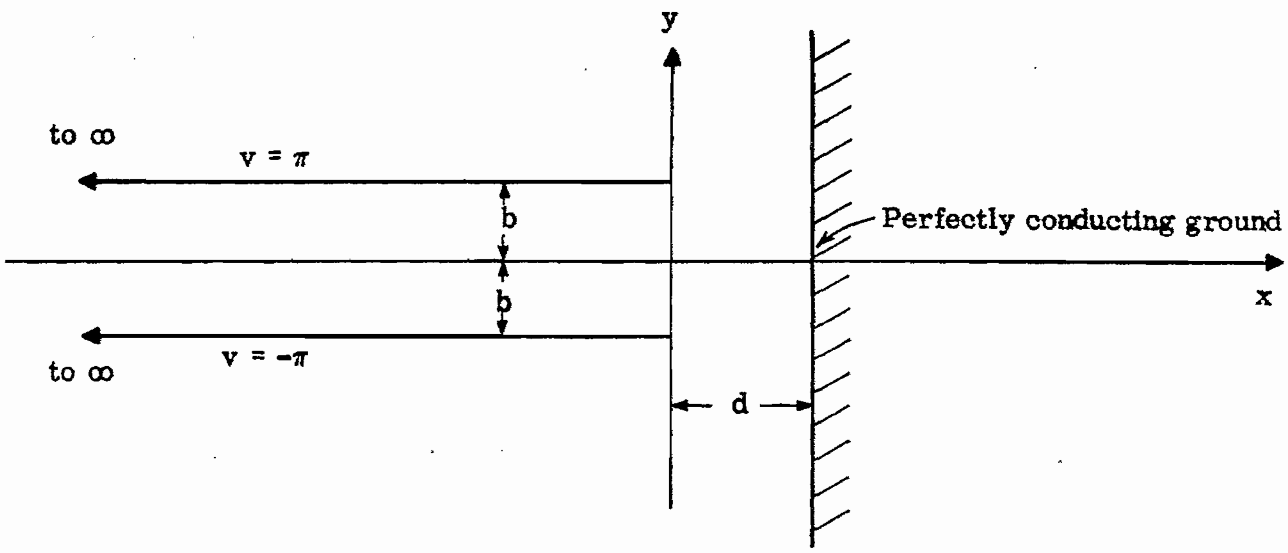

FIG. 2-1: Geometry Of Semi-Infinite Parallel Plate Transmission Line Near Perfectly Conducting Plane Ground.

a TEM wave and the field distribution can be obtained by solving an electrostatic problem.

2.1 Conformal Transformation

The conformal transformation for the geometry shown in Fig. 2-1 is derived in the Appendix. It is given by

$$\frac{z}{b} = \frac{d}{b} + \frac{2}{\pi}\left\{\frac{w}{2} - \sqrt{t_o}\sqrt{t_o - e^w} - \mathrm{Ln}\left[\sqrt{t_o} + \sqrt{t_o - e^w}\right]\right\}$$

$$= \frac{2}{\pi}\left[\frac{w}{2} + \sqrt{t_o}\left(\sqrt{t_o+1} - \sqrt{t_o - e^w}\right) - \mathrm{Ln}\left(\frac{\sqrt{t_o} + \sqrt{t_o - e^w}}{\sqrt{t_o} + \sqrt{t_o+1}}\right)\right], \qquad (1)$$

where

$$z = x + i\,y\ , \qquad (2)$$

$$w = u + i\,v \qquad (3)$$

and u and v denote, respectively, the electric and the magnetic flux function. The "proximity factor" t_o appearing in eq. (1) satisfies the implicit relation

$$\frac{d}{b} = \frac{2}{\pi}\left[\sqrt{t_o(t_o+1)} + \mathrm{Ln}\left(\sqrt{t_o} + \sqrt{t_o+1}\right)\right]. \qquad (4)$$

By use of the relation $t = e^w$, the eq. (4) can be alternatively expressed as

$$\frac{d}{b} = \frac{2}{\pi}\left[\sqrt{e^{u_o}\left(e^{u_o}+1\right)} + \mathrm{Ln}\left(\sqrt{e^{u_o}} + \sqrt{e^{u_o}+1}\right)\right]. \qquad (4a)$$

The variation of u_o as a function of d/b is presented in Table II-1 and is plotted in Fig. 2-2.

TABLE II-1: Ground Proximity Factor u_o (= Lnt_o) as a Function of the Normalized Ground Proximity Distance, d/b.

u_o	-5.0903	-3.7101	-2.9090	-2.3469	-1.9168	-1.5709	-1.2833
d/b	0.1	0.2	0.3	0.4	0.5	0.6	0.7
u_o	-1.0388	-.82681	-.64062	-.32670	-.07003	.14561	.33058
d/b	0.8	0.9	1.0	1.2	1.4	1.6	1.8
u_o	.49195	.82126	1.0786	1.4656	1.7517	1.9777	2.1641
d/b	2.0	2.5	3.0	4.0	5.0	6.0	7.0
u_o	2.3224	2.4599	2.5813	3.3514	3.7850		
d/b	8.0	9.0	10.0	20.0	30.0		

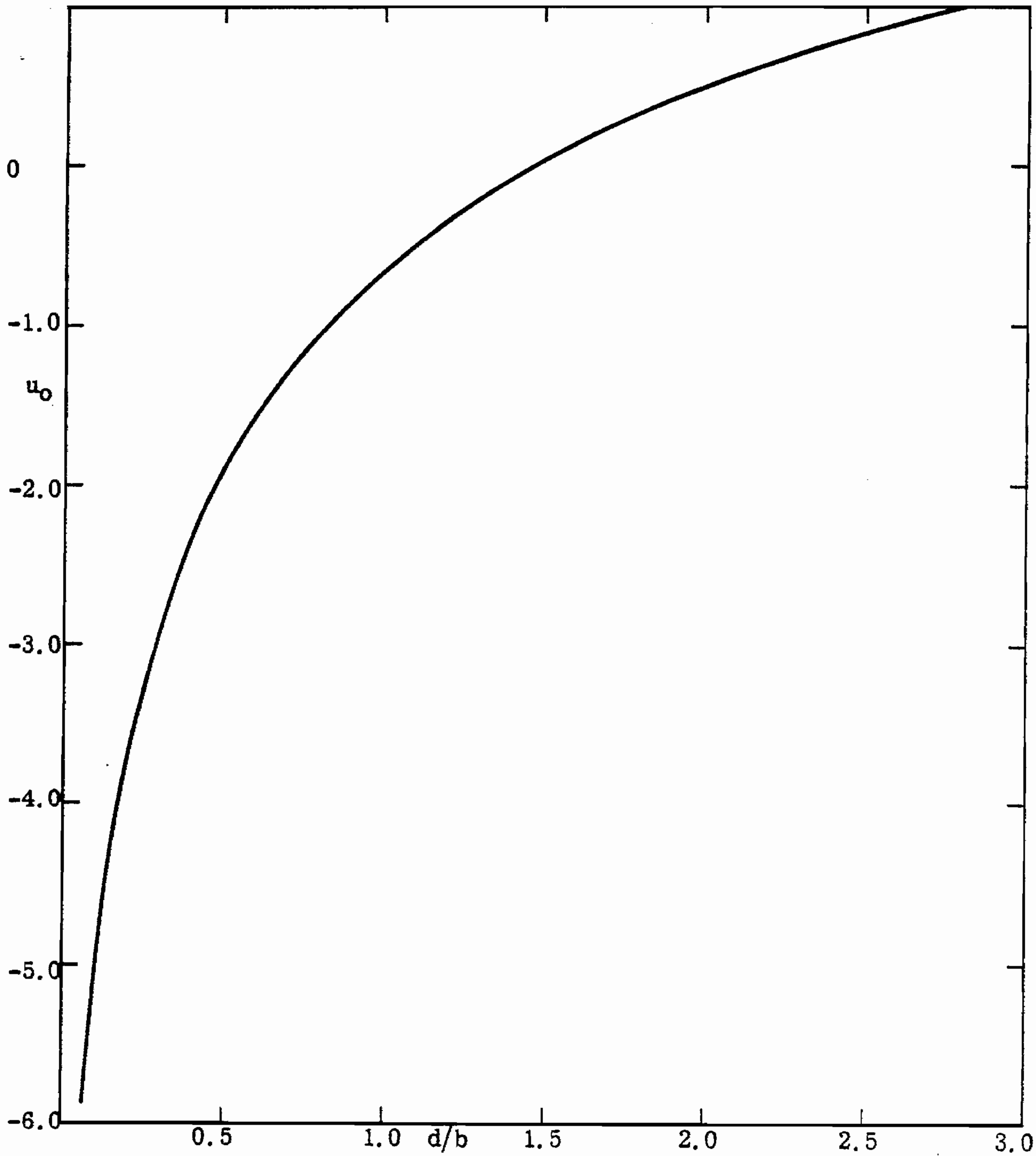

FIG. 2-2: Variation of the Ground Proximity Factor u_o as a Function of d/b, where d/b is the normalized ground proximity distance.

From eq. (4), it can be easily shown, that for $d/b \gg 1$,

$$t_o \simeq \frac{\pi d}{2b} \quad . \tag{5}$$

For this case, eq. (1) may be approximated by

$$\frac{z}{b} \simeq \frac{1}{\pi}\left[e^w + w + 1\right] + \frac{1}{4\pi t_o}\left[e^{2w} + 2e^w + 1\right] \quad . \tag{6}$$

In the limiting case where $d/b \to \infty$, i.e. t_o (or u_o)* $\to \infty$, eq. (6) degenerates to the conformal transformation of a semi-infinite parallel plate transmission line without the presence of the ground, namely,

$$\frac{z}{b} = e^w + w + 1 \; . \tag{7}$$

2.2 Electric Field Intensity

The electric field intensity of the transmission line system, E, is given by the derivative of the complex potential function w :

$$E = E_y + iE_x = -\frac{dw}{dz} \quad . \tag{8}$$

Since the right hand side of (8) is obtained from eq. (1) as

$$\frac{dz}{dw} = \frac{b}{\pi}\,\frac{\sqrt{t_o}\,(1+e^w)}{\sqrt{t_o - e^w}}, \tag{9}$$

eq. (8) becomes

$$E = -\frac{\pi}{b}\,\frac{\sqrt{t_o - e^w}}{\sqrt{t_o}\,(1+e^w)} \quad . \tag{10}$$

The uniform field in the region between the plates of the transmission can be found from (10) by letting

$u \to -\infty$ in w.

Thus

$$E_{uniform} = -\frac{\pi}{b} = E_{y_{uniform}} \quad . \tag{11}$$

It seems appropriate, therefore, to normalize the field intensity of the transmission line with respect to the uniform field.

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*In the subsequent discussion, t_o and u_o will be used interchangingly , whichever form is more convenient.

Let us define the relative electric field intensity E_{rel} as

$$E_{rel} = E \,/\, \frac{\pi}{b} \quad . \tag{12}$$

To obtain the x- and y-components of the electric field intensity, one makes use of Cauchy-Riemann relations for w , i.e.

$$E_x = -\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \ , \qquad E_y = -\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \ .$$

Thus

$$E_{rel} = E_{y_{rel}} + i\, E_{x_{rel}} = -\frac{\sqrt{t_o - e^w}}{\sqrt{t_o}\,(1+e^w)} \ , \tag{13}$$

or

$$E_{y_{rel}} = -\mathrm{Re}\left[\frac{\sqrt{e^{u_o} - e^w}}{\sqrt{e^{u_o}}\,(1+e^w)}\right], \tag{14}$$

$$E_{x_{rel}} = -\mathrm{Im}\left[\frac{\sqrt{e^{u_o} - e^w}}{\sqrt{e^{u_o}}\,(1+e^w)}\right] \ . \tag{15}$$

Equations (14) and (15) are used in a computer program to calculate the relative field components.

A systematic numerical computation of the electric field intensity of the transmission line has been carried out and the results are presented in Section 3. It may be instructive, however, to consider briefly some special cases of interest such as field variations on the upper plate, on the center plane and at the ground.

The relative electric field intensity on the upper plate can be obtained by subsituting $v = \pi$ in eq. (13). This substitution yields

$$E_{x_{rel}} = 0 \ , \tag{16}$$

$$E_{y_{rel}} = -\frac{\sqrt{e^{u_o} + e^u}}{\sqrt{e^{u_o}}\,(1-e^u)} \ . \tag{17}$$

The relation between u and x along this plate is found from eq. (1) to be

$$\frac{x}{b} = \frac{2}{\pi}\left[\frac{u}{2} + e^{u_o} \left(e^{u_o}+1 - e^{u_o}+e^{u}\right) - \mathrm{Ln}\left(\frac{e^{u_o} + e^{u_o}+e^{u}}{e^{u_o} + e^{u_o}+1} \right)\right], \tag{18}$$

where $u > 0$ corresponds to the outer surface of the upper plate, and $u < 0$ to the inner surface. Thus, by eq. (17), we have on the upper plate

$$E_{y_{rel}} = \begin{cases} \dfrac{\sqrt{e^{u_o} + e^{u}}}{\sqrt{e^{u_o}}\,(e^{u}-1)} & \text{for } u > 0 \text{ (outer surface)} \\[2ex] -\dfrac{\sqrt{e^{u_o} + e^{u}}}{\sqrt{e^{u_o}}\,(1-e^{u})} & \text{for } u < 0 \text{ (inner surface)} \end{cases} . \tag{19}$$

On the center plane, $v = 0$. From eqs. (1) and (13) we find

$$\frac{z}{b} = \frac{x}{b} = \frac{2}{\pi}\left[\frac{u}{2} + \sqrt{e^{u_o}}\left(\sqrt{e^{u_o}+1} - \sqrt{e^{u_o}-e^{u}}\right) - \mathrm{Ln}\left(\frac{\sqrt{e^{u_o}} + \sqrt{e^{u_o}-e^{u}}}{\sqrt{e^{u_o}} + \sqrt{e^{u_o}+1}}\right)\right], \tag{20}$$

and

$$E_{x_{rel}} = 0 , \tag{21}$$

$$E_{y_{rel}} = -\frac{\sqrt{e^{u_o}-e^{u}}}{\sqrt{e^{u_o}}\,(1+e^{u})} , \tag{22}$$

where $u < u_o$.

On the ground plane, $v = 0$ and $u > u_o$. Equation (1) now becomes

$$\frac{z}{b} = \frac{d}{b} + i\,\frac{2}{\pi}\left[\sqrt{e^{u_o}(e^{u}-e^{u_o})} + \phi/2\right], \tag{23}$$

where

$$\tan\frac{\phi}{2} = \sqrt{e^{u-u_o} - 1},$$

i.e.

$$\frac{x}{b} = \frac{d}{b} , \tag{24}$$

$$\frac{y}{b} = \frac{2}{\pi}\left[\sqrt{e^{u_o}(e^{u}-e^{u_o})} + \phi/2\right] . \tag{25}$$

The relative electric field intensity on the ground is found from eq. (13) to be

$$E_{x_{rel}} = -\frac{\sqrt{e^{u}-e^{u_o}}}{\sqrt{e^{u_o}}\,(1+e^{u})} \tag{26}$$

$$E_{y_{rel}} = 0 \; ; \tag{27}$$

3. Graphical Representation

In Section 2, a mathematical discussion was presented for a semi-infinite parallel plate transmission line placed in the proximity of a perfectly conducting plane ground. Since the subject of central importance posed by the problem is the effects of the ground proximity on the distribution of the field lines and electric field intensities, these quantities are presented in this section in graphical form with a view to showing the parametric effects on the field distributions.

The dimension of the length of the transmission line system (i.e. z-plane) is normalized by b, one-half of the separation distance between the upper and lower places (cf. Fig. 2-1); the ground proximity distance , d , is accordingly represented in its normalized form, i.e. in d/b. Seven different cases of d/b are considered in the present work. They are: 0.1, 0.2, 0.5, 1.0 , 2.0, 5.0 and 10.0 in addition to the case without the presence of the ground, or $d/b \rightarrow \infty$. The electric potentials of the upper and lower plates of the transmission line are normalized to $\pm\pi$ volts; those at the center plane and the ground at zero volts.

The normalized ground proximity distance, d/b, is a function of u_o as given by eq. (4a), where $u_o = Lnt_o$. A short computer program was written to calculate, by use of the Newton's method, the desired set of d/b for an appropriate corresponding set of u_o as the initial estimates. The mapping of the flux functions were also computer-calculated by feeding as an input to a program a set of appropriate values of u and v. A third computer program was developed to calculate the field intensities at a set of predetermined points in the transmission line system by use of the Newton-Raphson method for the complex function. Since the method requires an initial estimate in the w-plane (i.e. $w = u + i v$) for each point in the z-plane (i.e. $z = x + i y$) for a given ground proximity d/b, an extensive numerical computation was carried out in order to obtain a reasonable initial estimate for w as an input to the computer program for evaluation of the field intensities. Because of a large number of graphs to be presented, an individual detailed caption will not be shown on each graph. Instead we present a table showing the parameters involved and the Figure numbers related to those parameters.

3.1 Electric and Magnetic Field Lines

In this section we present the electric and magnetic field lines of a semi-infinite parallel plate transmission line near a perfectly conducting plane ground, i.e. the mapping of eq. (1). The incremental steps of $\pi/5$ are taken both for u(electric field line) and v (magnetic field line). The dotted lines superimposed on each figure represent the field lines of the similar transmission line without the ground in the Figs. 3-1-1 through 3-1-7-a.

TABLE III-1: Electric and Magnetic Field Lines of a Semi-Infinite Parallel Plate Transmission Line Near a Perfectly Conducting Ground

Parameters		Figure
Coordinates	d/b	Nos.
z-plane	0.1	3-1-1
	0.2	3-1-2
	0.5	3-1-3
	1.0	3-1-4
	2.0	3-1-5
	5.0	3-1-6
	10.0	3-1-7
	10.0	3-1-7-a

Figure 3-1-7-a is included for the case of $d/b = 10$ to show the field line variation on the ground.

As seen from the graphs, the deviation of the field lines caused by the presence of the perfectly conducting ground from those without ground is more drastic for a smaller ground proximity distance d/b, particularly in the neighborhood of the upper edge where the electric potential gradient is maximum. It is also seen that in the region $x/b \lesssim -1.5$ and $0 \leq y/b \leq 1.0$, the effect of the presence of the ground on the field lines is negligible for all cases of the ground proximities included. For $d/b \gtrsim 1.0$, the ground effect is small in the entire region between the upper plate and the center plane (i.e. in the region $x/b \leq 0$, $0 \leq y/b \leq 1.0$).

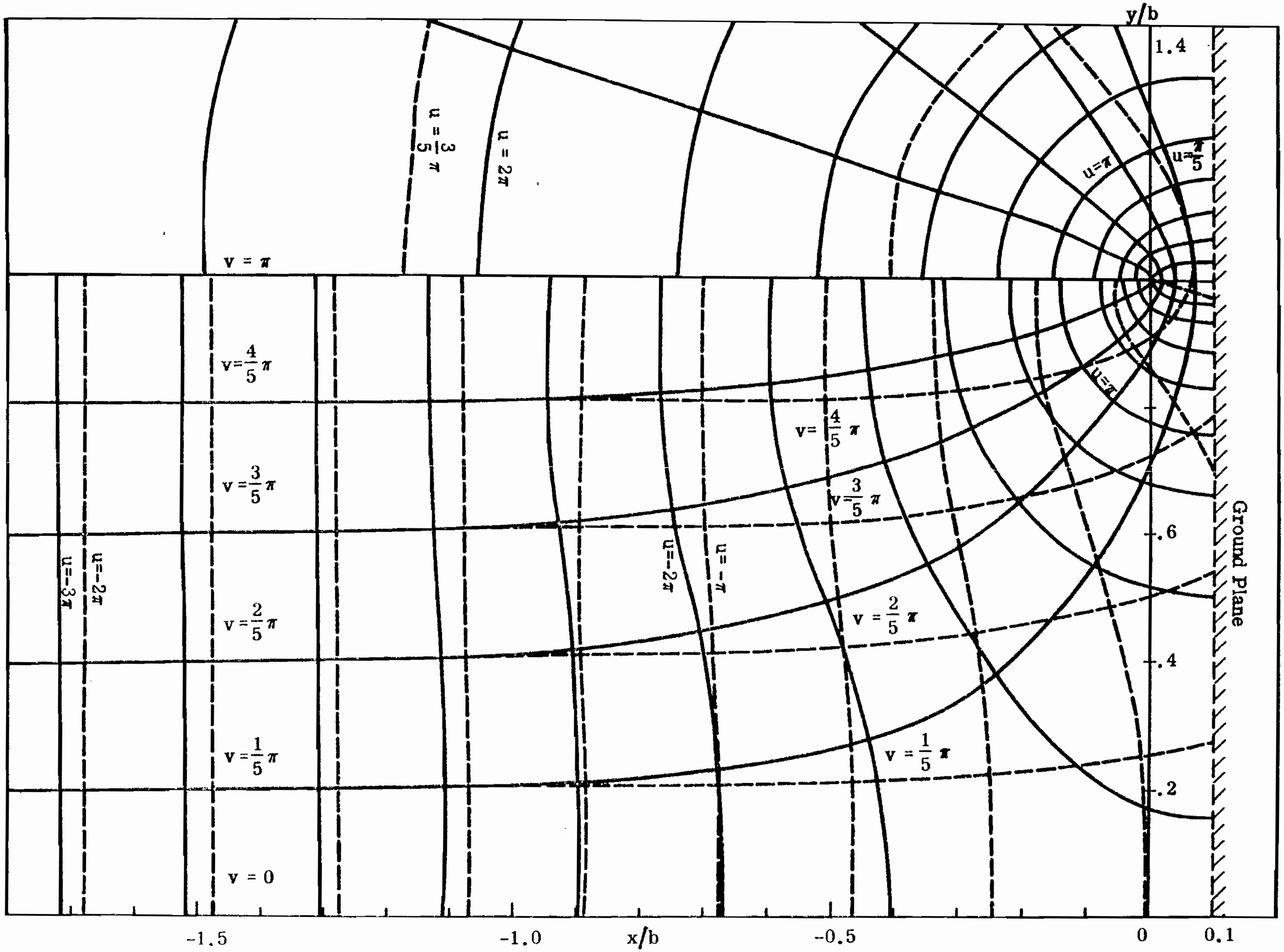

FIG. 3-1-1. Field Lines, $d/b = 0.1$.

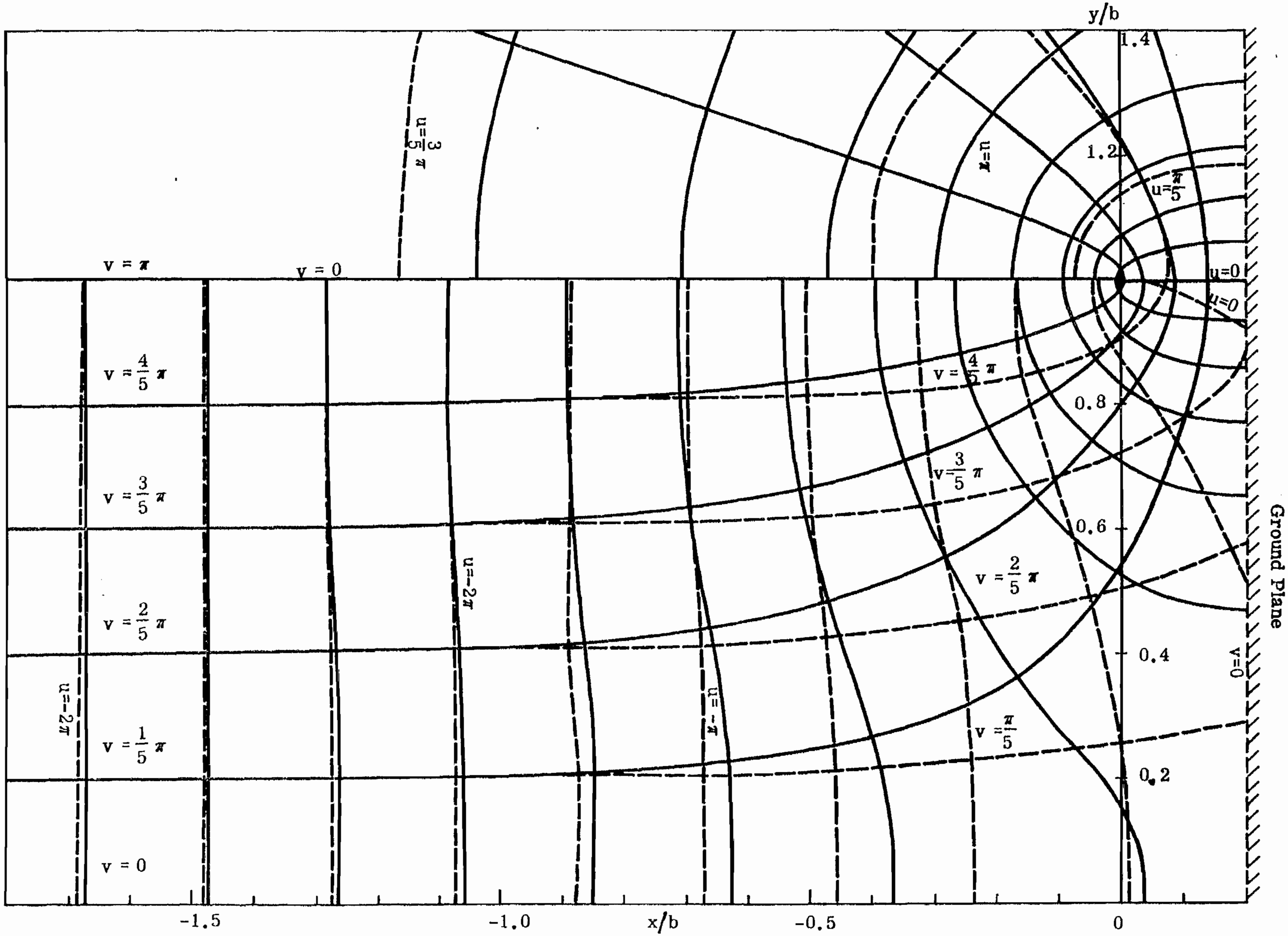

FIG. 3-1-2. Field Lines, d/b = 0.2.

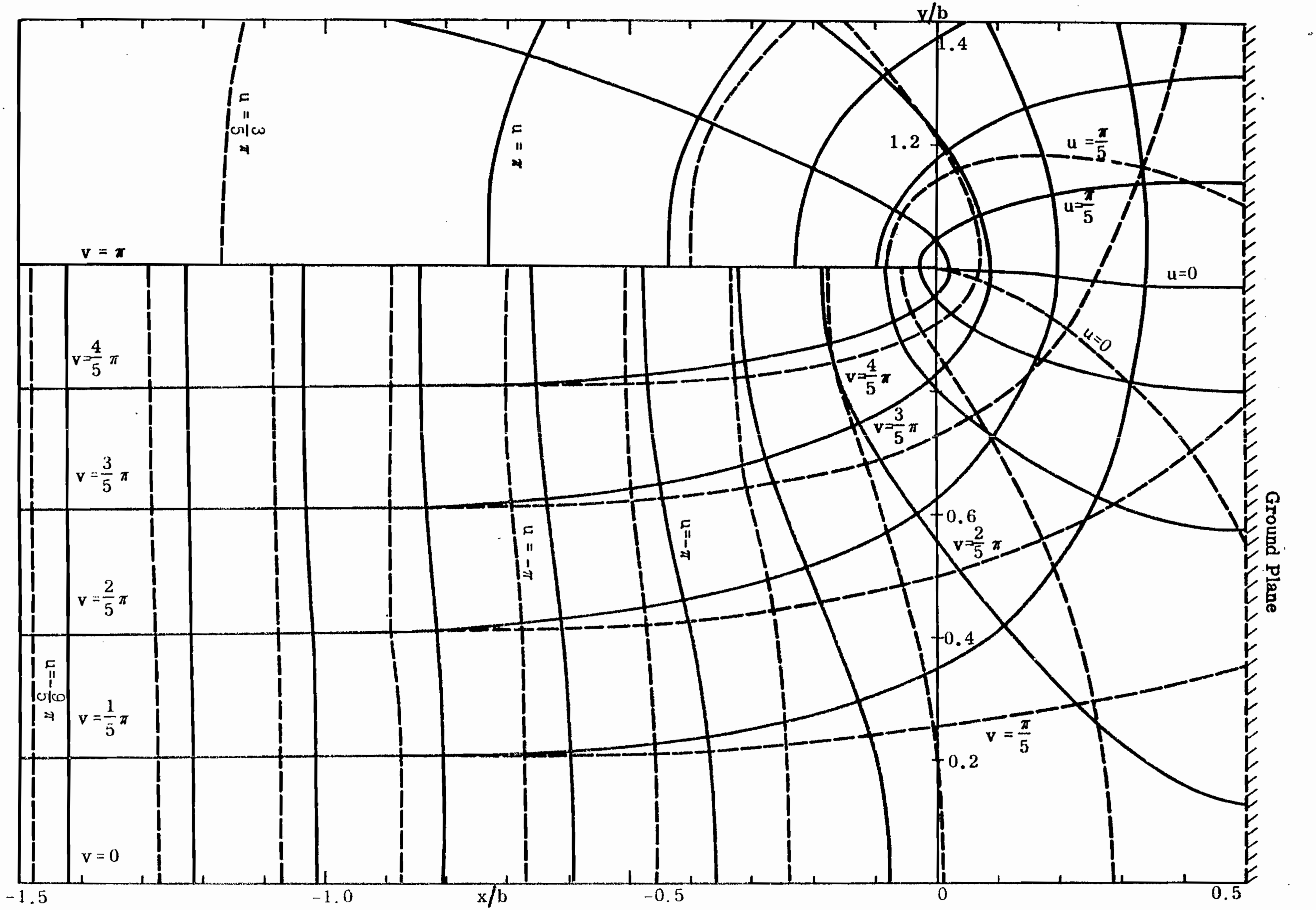

FIG. 3-1-3. Field Lines, d/b = 0.5.

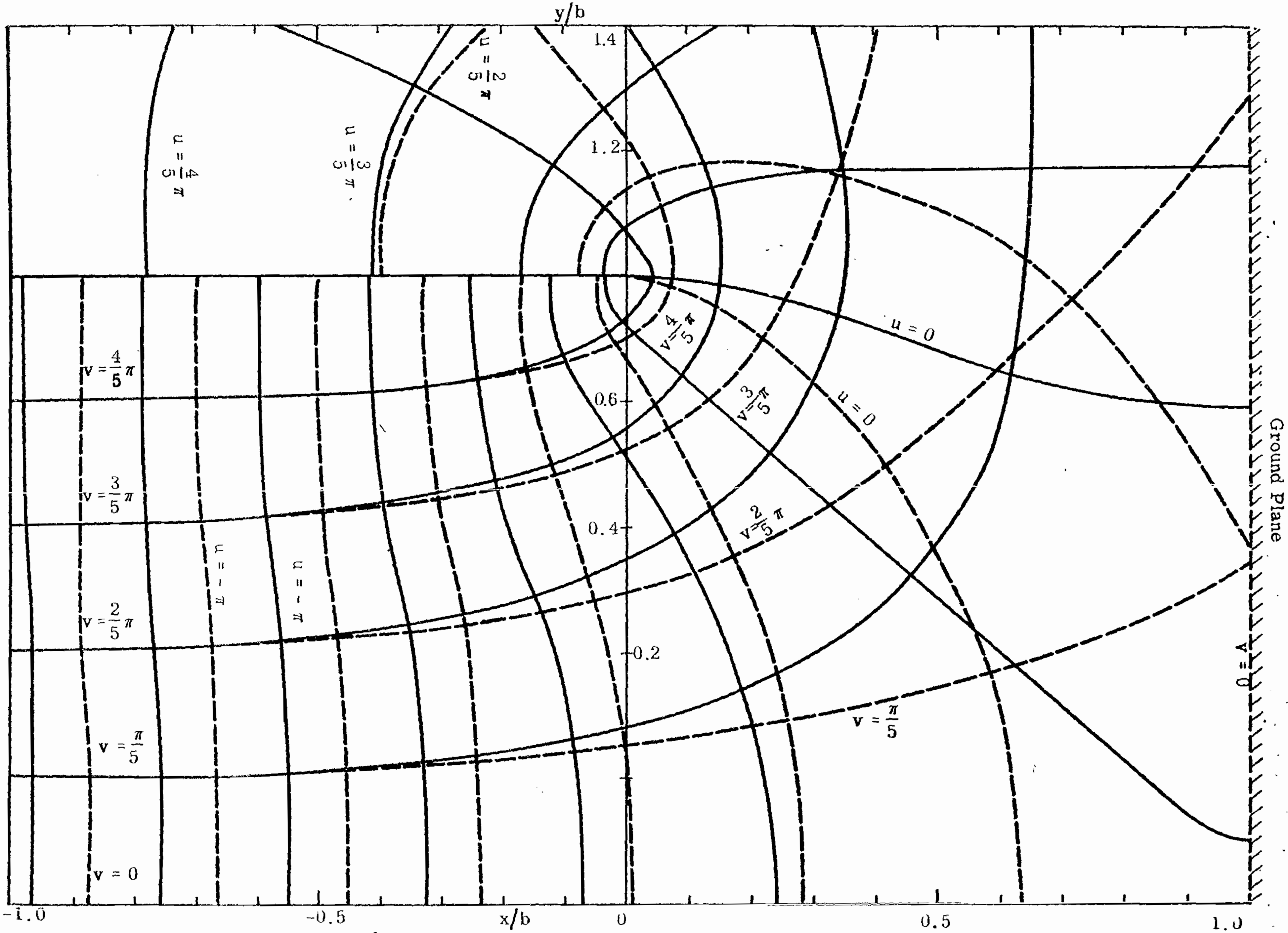

FIG. 3-1-4. Field Lines, d/b = 1.0.

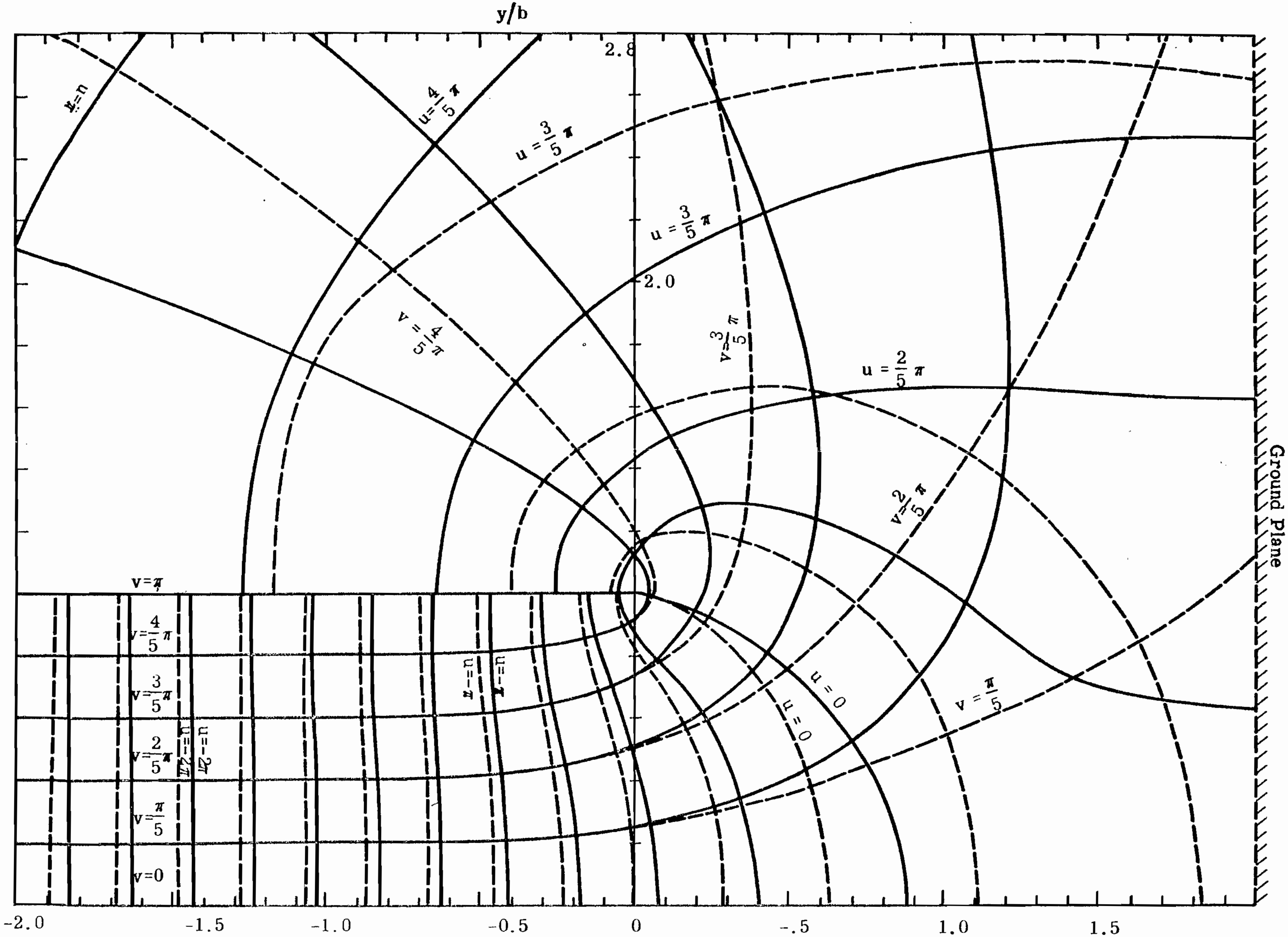

FIG. 3-1-5. Field Lines, d/b = 2.0.

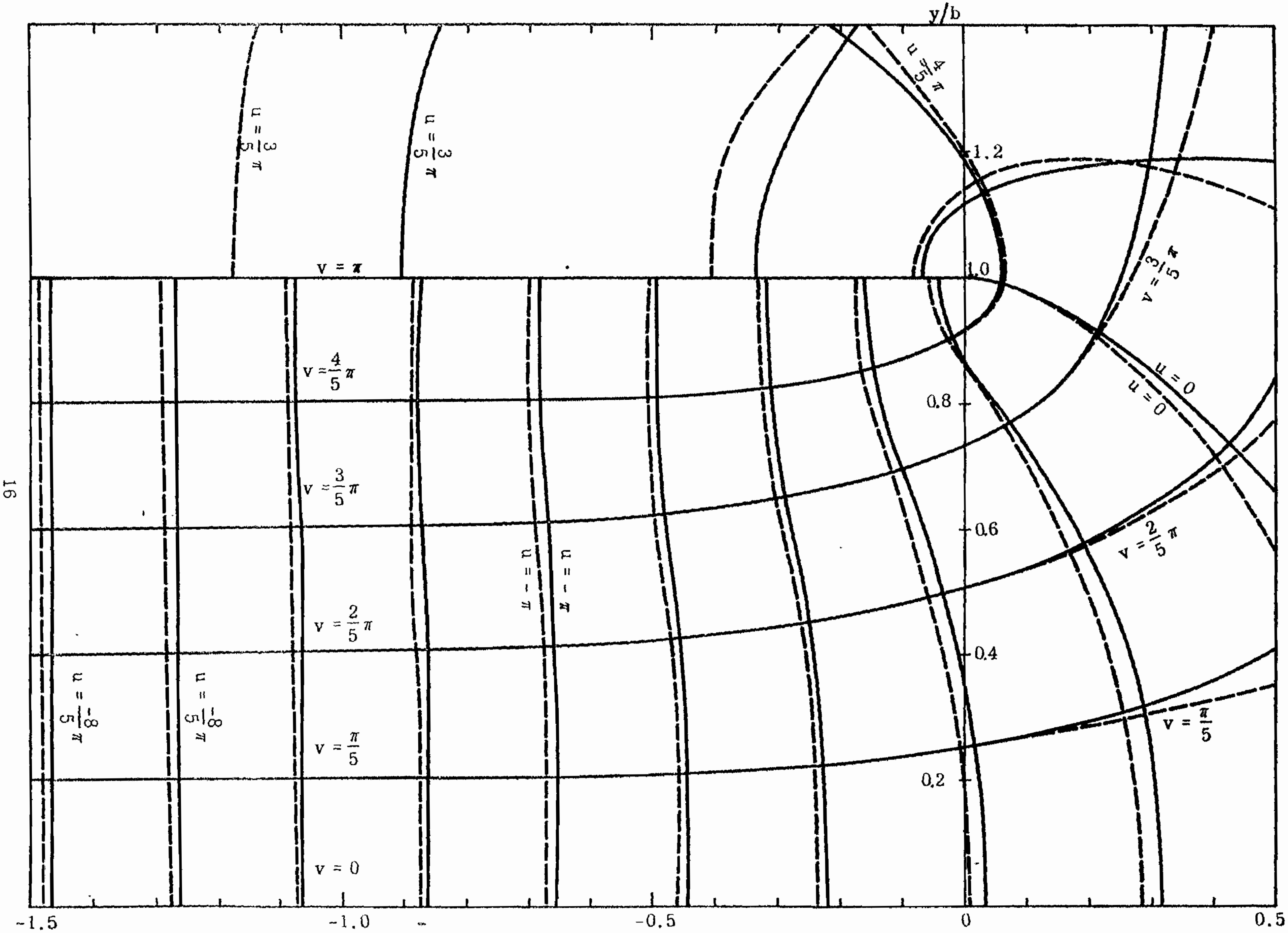

FIG. 3-1-6. Field Lines, d/b = 5.0.

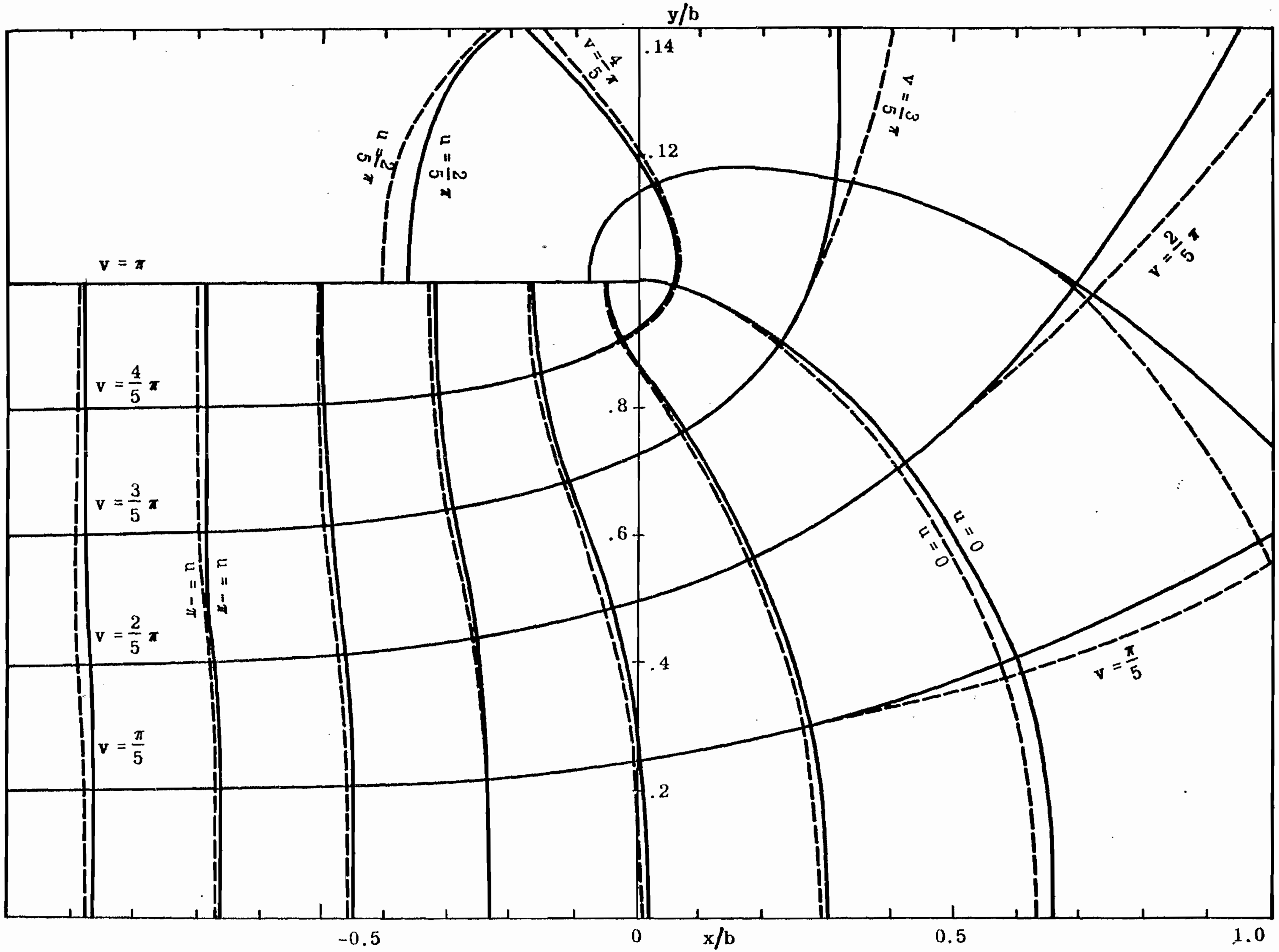

FIG. 3-1-7. Field Lines, d/b = 10.

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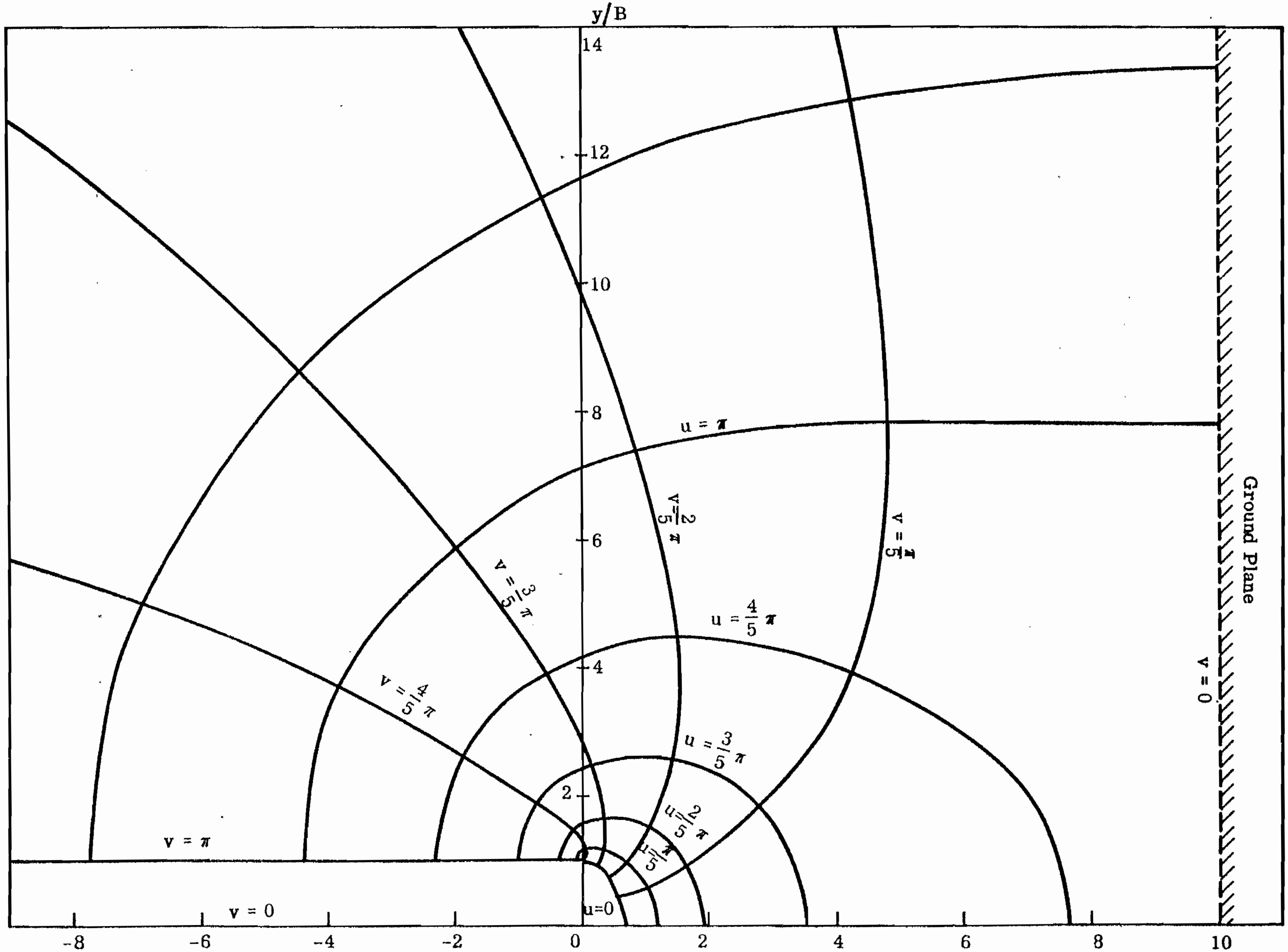

FIG. 3-1-7a. Field Lines, d/b =10.

3.2 Relative Electric Field Intensities

The electric field intensity of a semi-infinite parallel plate transmission line placed in the proximity of a perfectly conducting plane ground is given by eq. (10). The field intensity is normalized in the sense that the electric potential at the upper plate is set at +1 volt. Henceforth, by the field intensity, we mean the relative field intensity.

From eq. (13) it is seen that the field intensity E_{rel} is singular at the point $x/b = 0$, $y/b = 1$. It is not readily apparent from this equation how $E_{y_{rel}}$ and $E_{x_{rel}}$ would behave there. In order to examine the behavior of the x- and y-component of E_{rel} in a close neighborhood of the singular point, one needs to combine the eqs. (1) and (13). After a little algebraic manipulation , for the case $d/b < 1$, we obtain

$$E_{y_{rel}} \approx - \frac{\frac{1}{\sqrt{t_o}}\left(1+\frac{t_o}{2}\right) x}{\pi\left[x^2+(y-1)^2\right]}$$

and

$$E_{x_{rel}} \approx \frac{\frac{1}{\sqrt{t_o}}\left(1+\frac{t_o}{2}\right)(y-1)}{\pi\left[x^2+(y-1)^2\right]}.$$

Since $t_o \ll 1$ for small d/b, one can see from the above expressions that $E_{y_{rel}}$ and $E_{x_{rel}}$ approach infinity as $x \rightarrow 0$ and $y \rightarrow 1$.

In Figs. 3-2-1 through 3-2-7-a we present $|E_{y_{rel}}|$ as a function of x/b with y/b fixed as a parameter. Note that $|E_{y_{rel}}|$ for the case of $d/b = 0.1$, for example , is presented in two separate figures: Fig. 3-2-1 for the region $0 \leq y/b \leq 1.0$ and 3-2-1-a for the region $1.0 < y/b \leq 1.5$. This was done solely for the sake of graphical clarity. The same notational scheme applies to all other cases of d/b.

TABLE III-2: The y-Components of the Electric Field Intensities, $\left|E_{y_{rel}}\right|$.

Parameters		Figure
y/b	d/b	Nos.
$0 \leq \frac{y}{b} \leq 1.0$	0.1	3-2-1
	0.2	3-2-2
	0.5	3-2-3
	1.0	3-2-4
	2.0	3-2-5
	5.0	3-2-6
	10.0	3-2-7
$1.0 < \frac{y}{b} \leq 1.5$	0.1	3-2-1-a
	0.2	3-2-2-a
	0.5	3-2-3-a
	1.0	3-2-4-a
	2.0	3-2-5-a
	5.0	3-2-6-a
	10.0	3-2-7-a

A computational result showed that the values of $\left|E_{y_{rel}}\right|$ for $d/b = 10.0$ and $d/b \to \infty$ (i.e., without the ground) are practically identical except in the neighborhood of the singular point. For this reason, $\left|E_{y_{rel}}\right|$ for $d/b \to \infty$ is not shown in the graph.

It is observed from the graphs of $\left|E_{y_{rel}}\right|$ that the ground effect on the y-component of the electric field intensity is small in the region $x/b \lesssim -1.5$, which could have been inferred from the observation of the influence of the ground on the field lines shown in Section 3.1.

Similarly, the x-components of the electric field intensities of the transmission line are shown in Figs. 3-2-8 through 3-2-14-a.

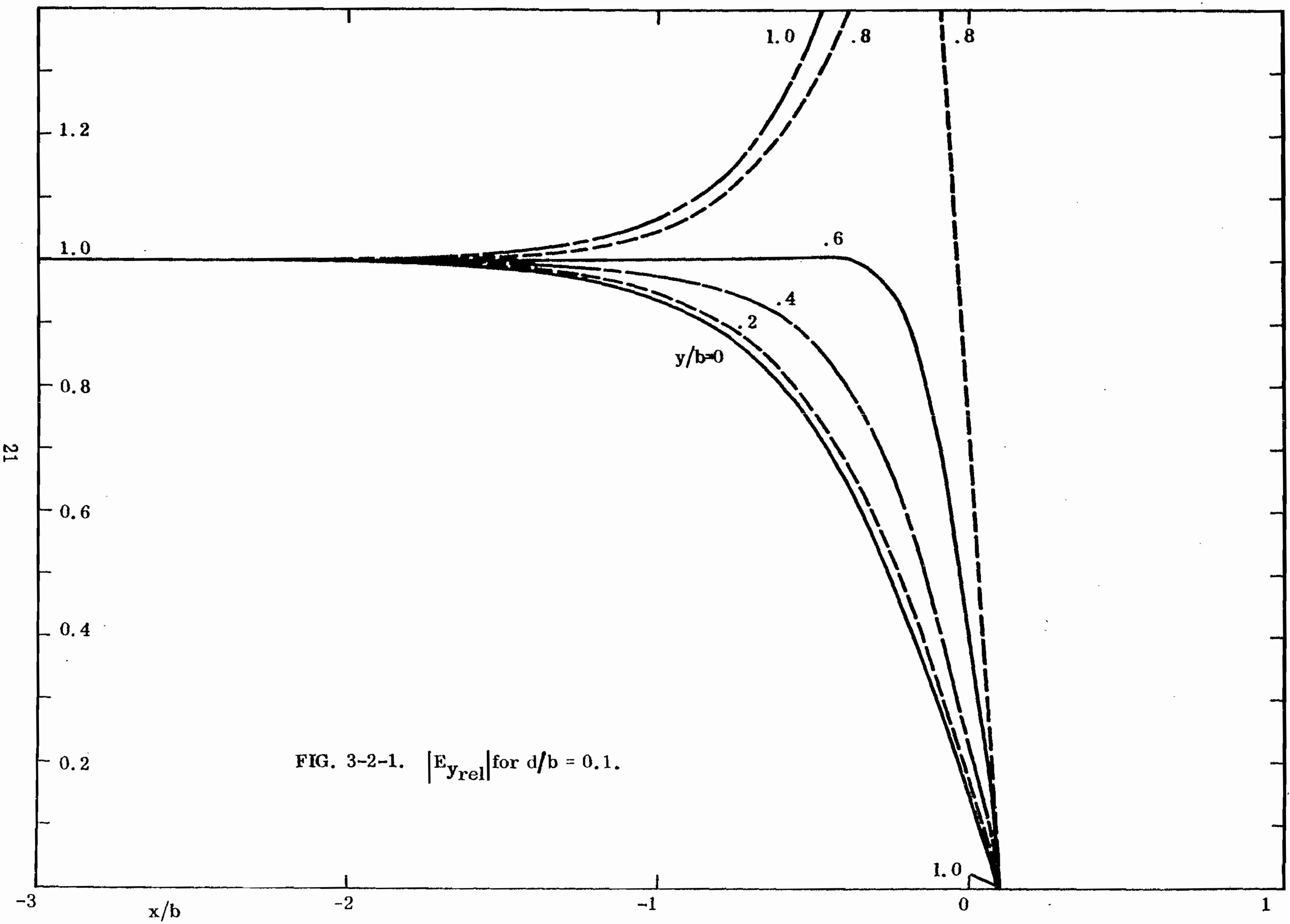

FIG. 3-2-1. $|E_{y_{rel}}|$ for $d/b = 0.1$.

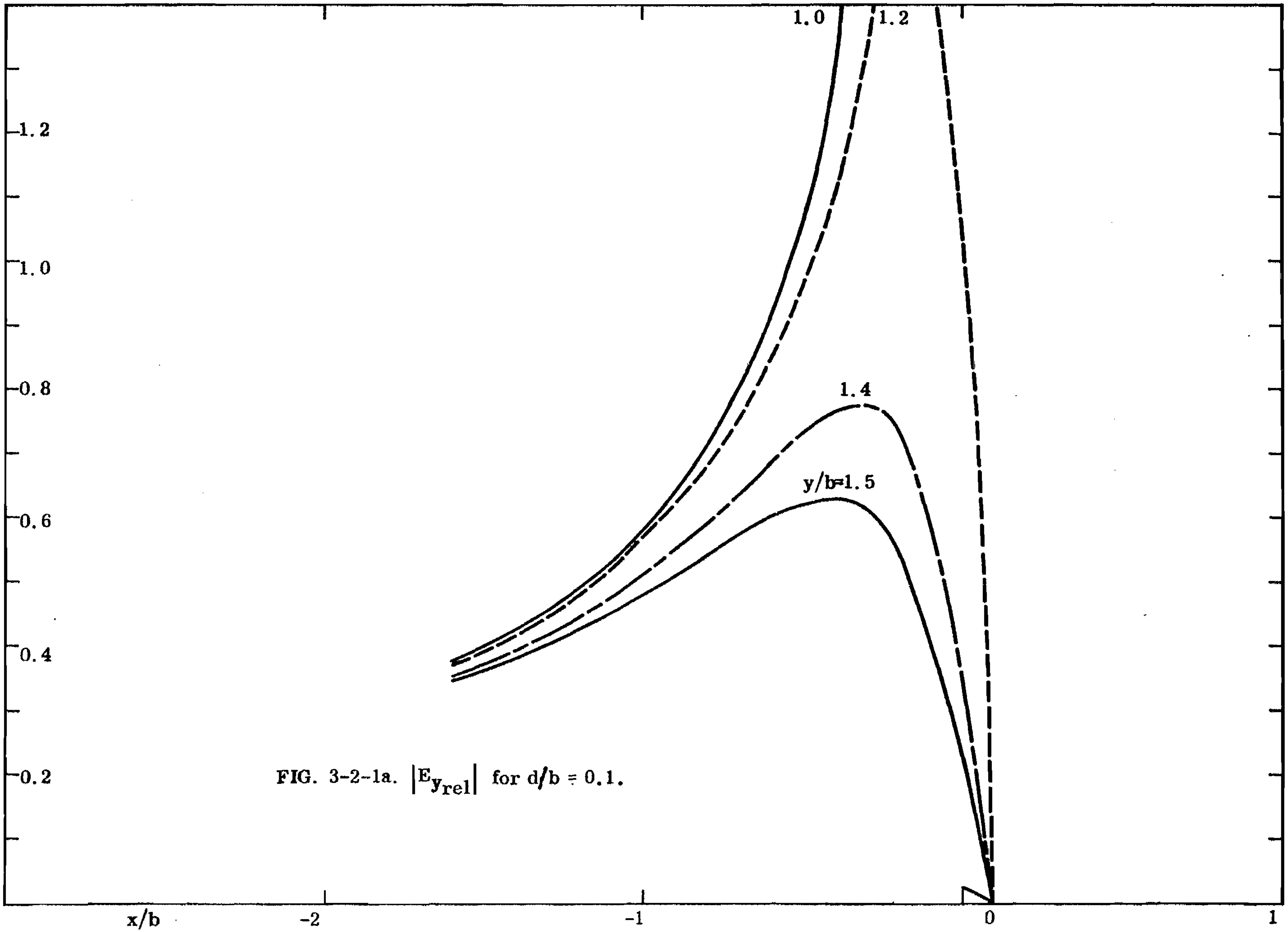

FIG. 3-2-1a. $|E_{y_{rel}}|$ for $d/b = 0.1$.

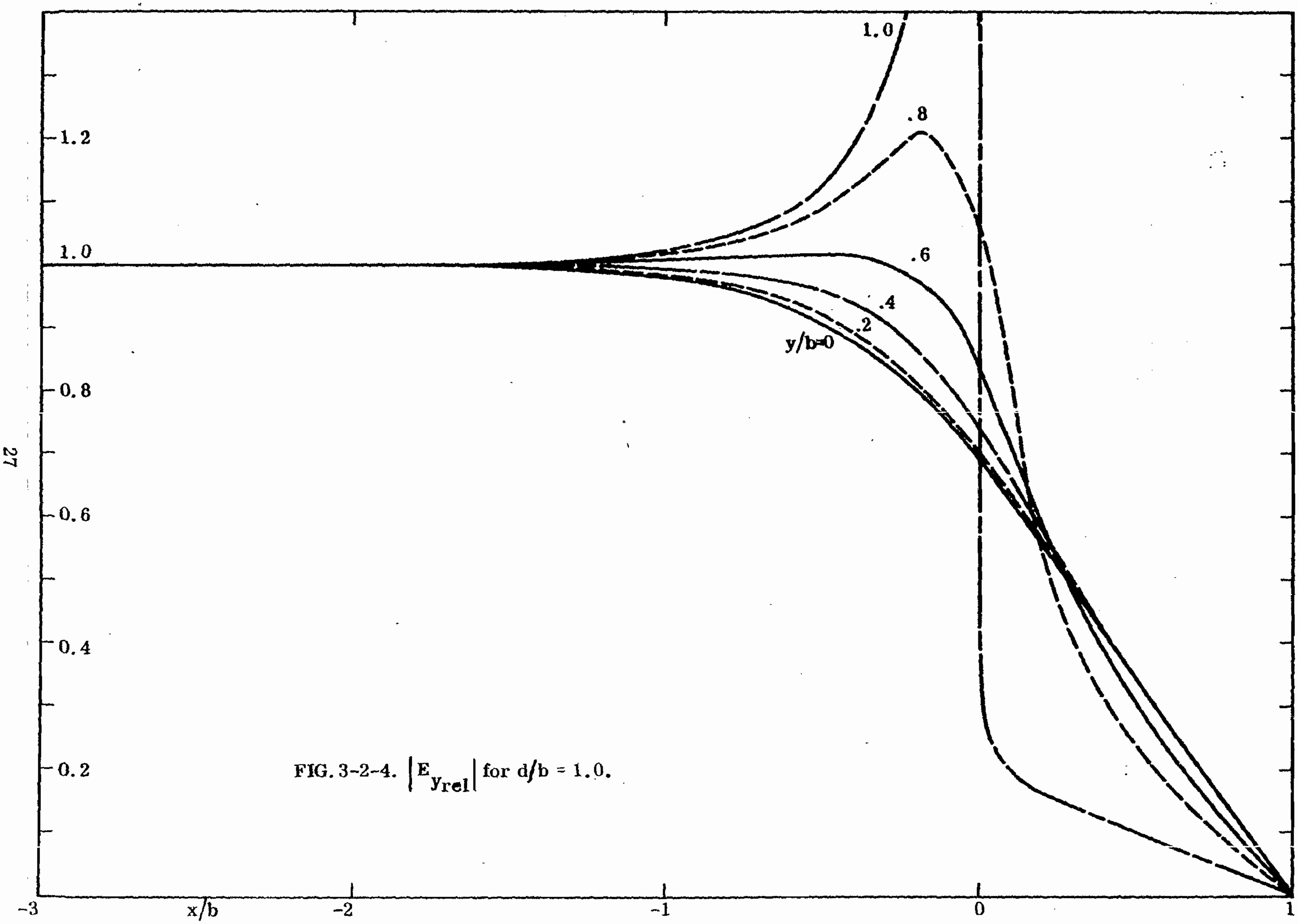

FIG. 3-2-4. $\left|E_{y_{rel}}\right|$ for d/b = 1.0.

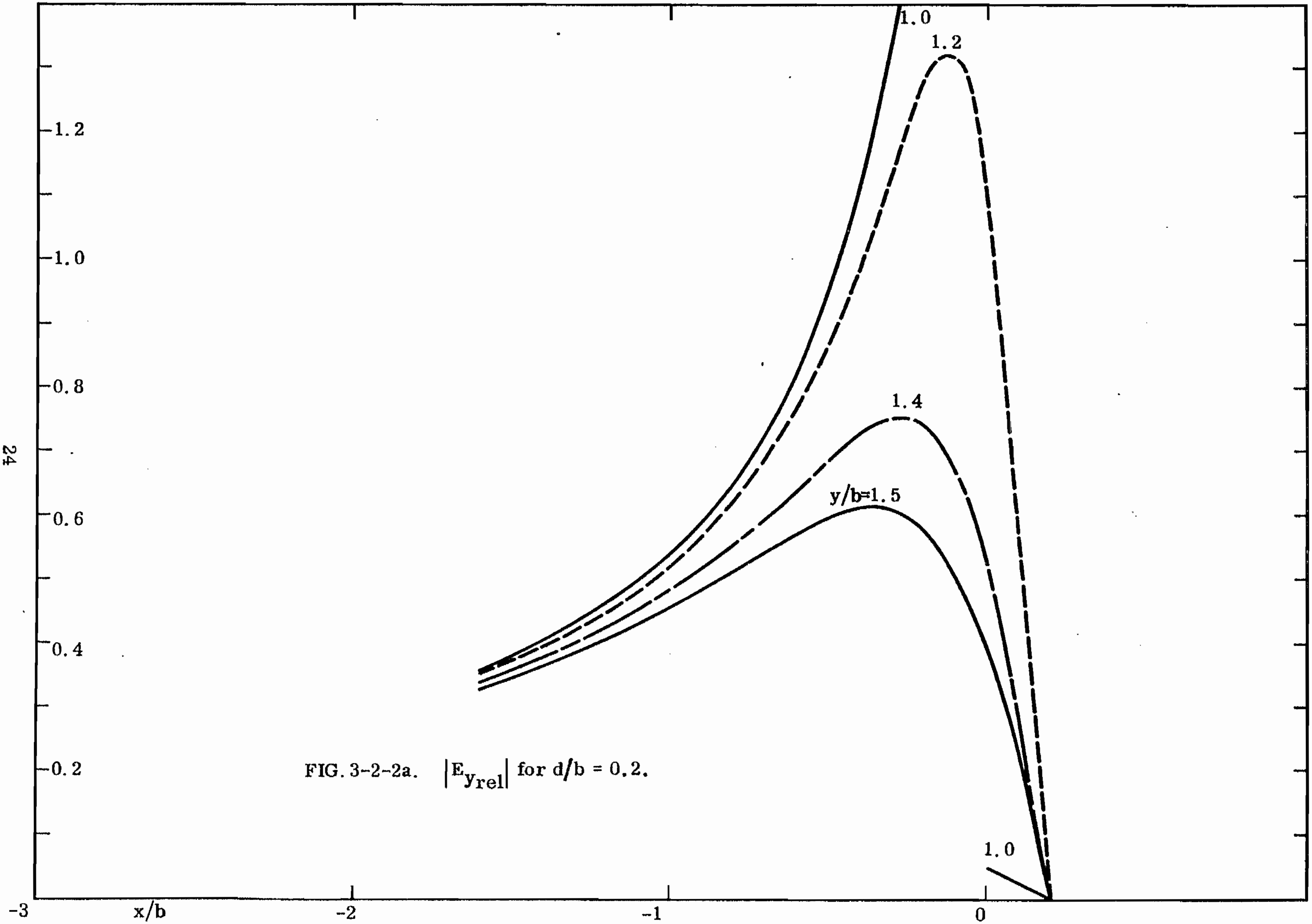

FIG. 3-2-2a. $|E_{y_{rel}}|$ for d/b = 0.2.

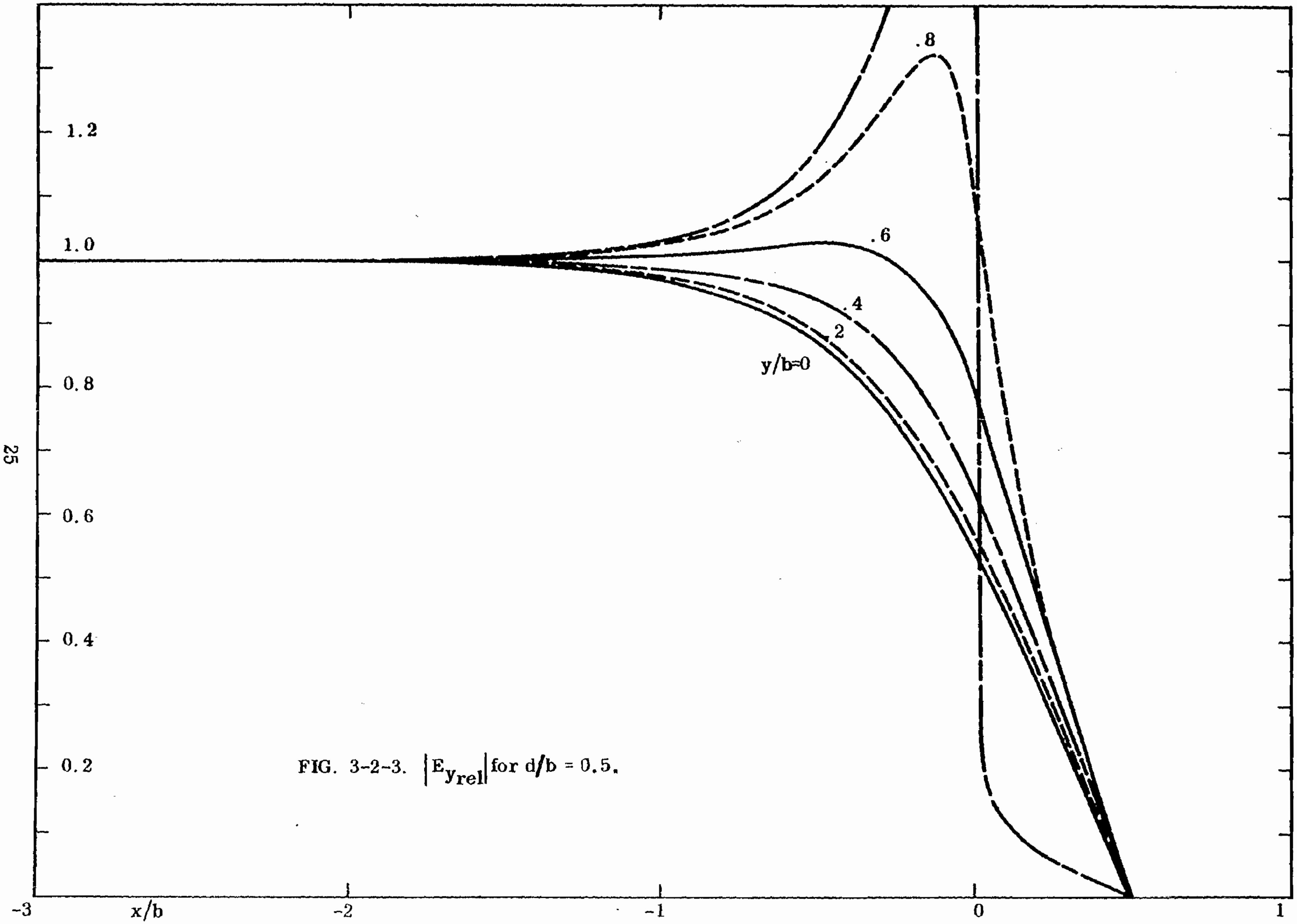

FIG. 3-2-3. $\left|E_{y_{rel}}\right|$ for $d/b = 0.5$.

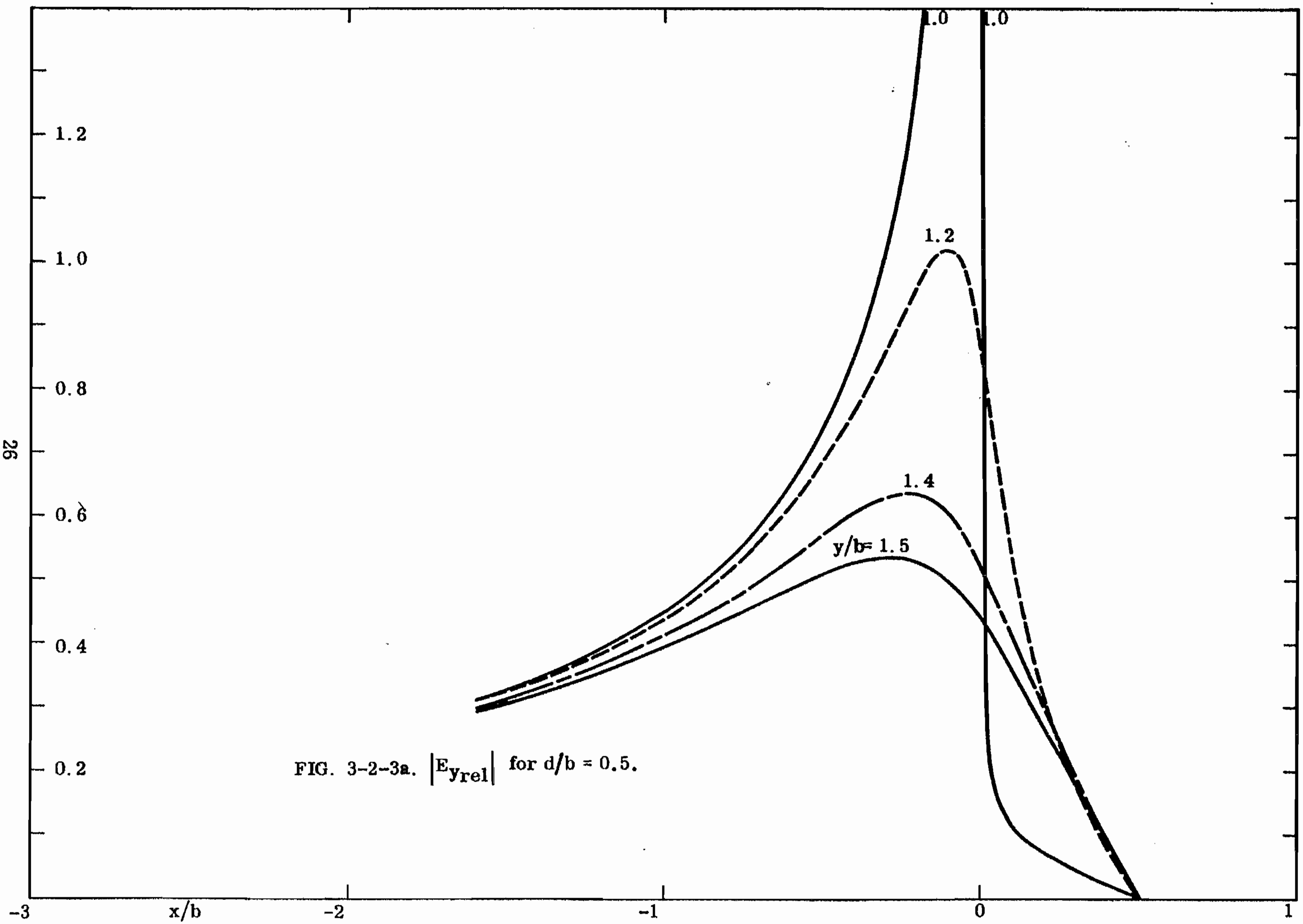

FIG. 3-2-3a. $|E_{y_{rel}}|$ for $d/b = 0.5$.

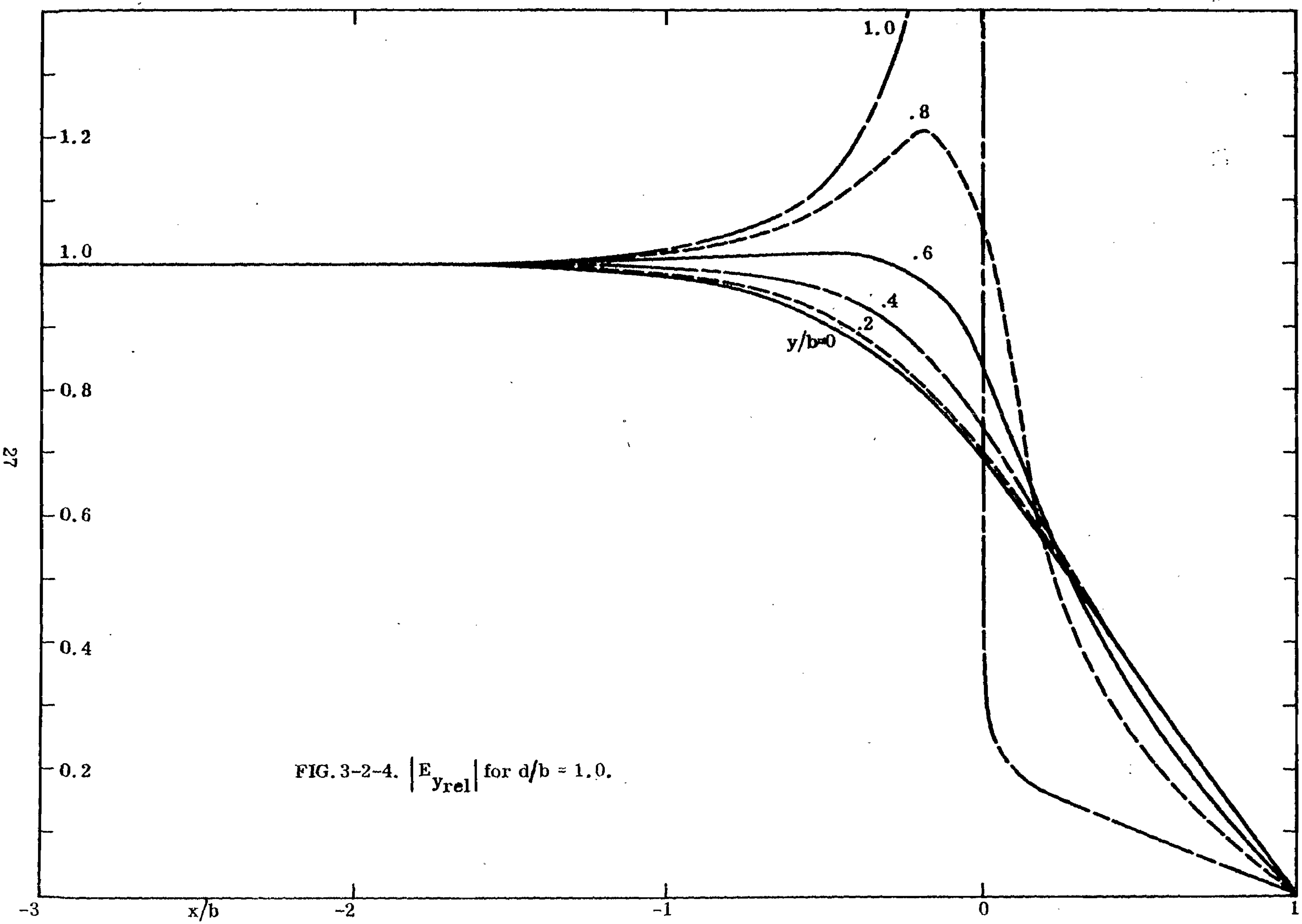

FIG. 3-2-4. $\left|E_{y_{rel}}\right|$ for d/b = 1.0.

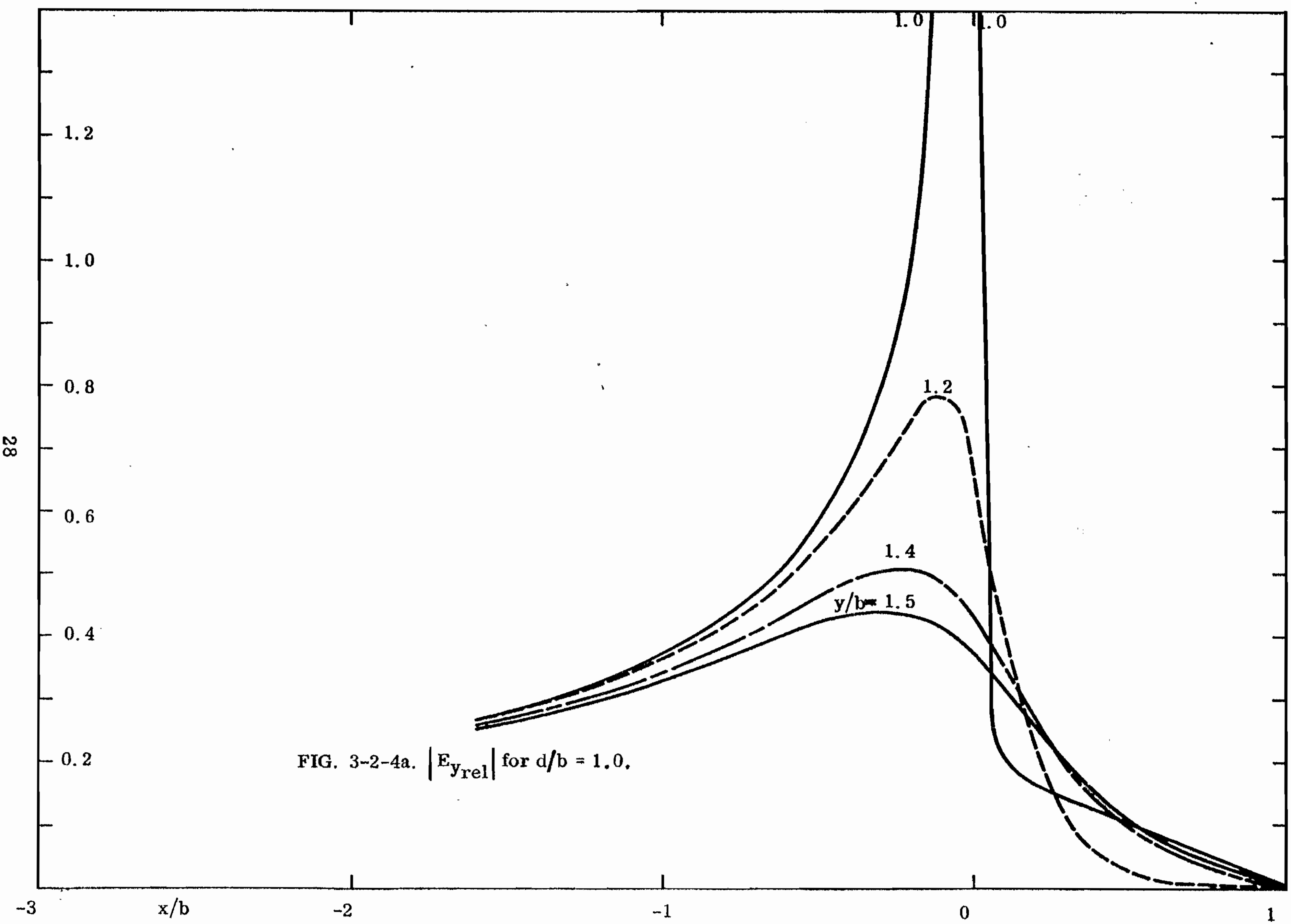

FIG. 3-2-4a. $\left|E_{y_{rel}}\right|$ for $d/b = 1.0$.

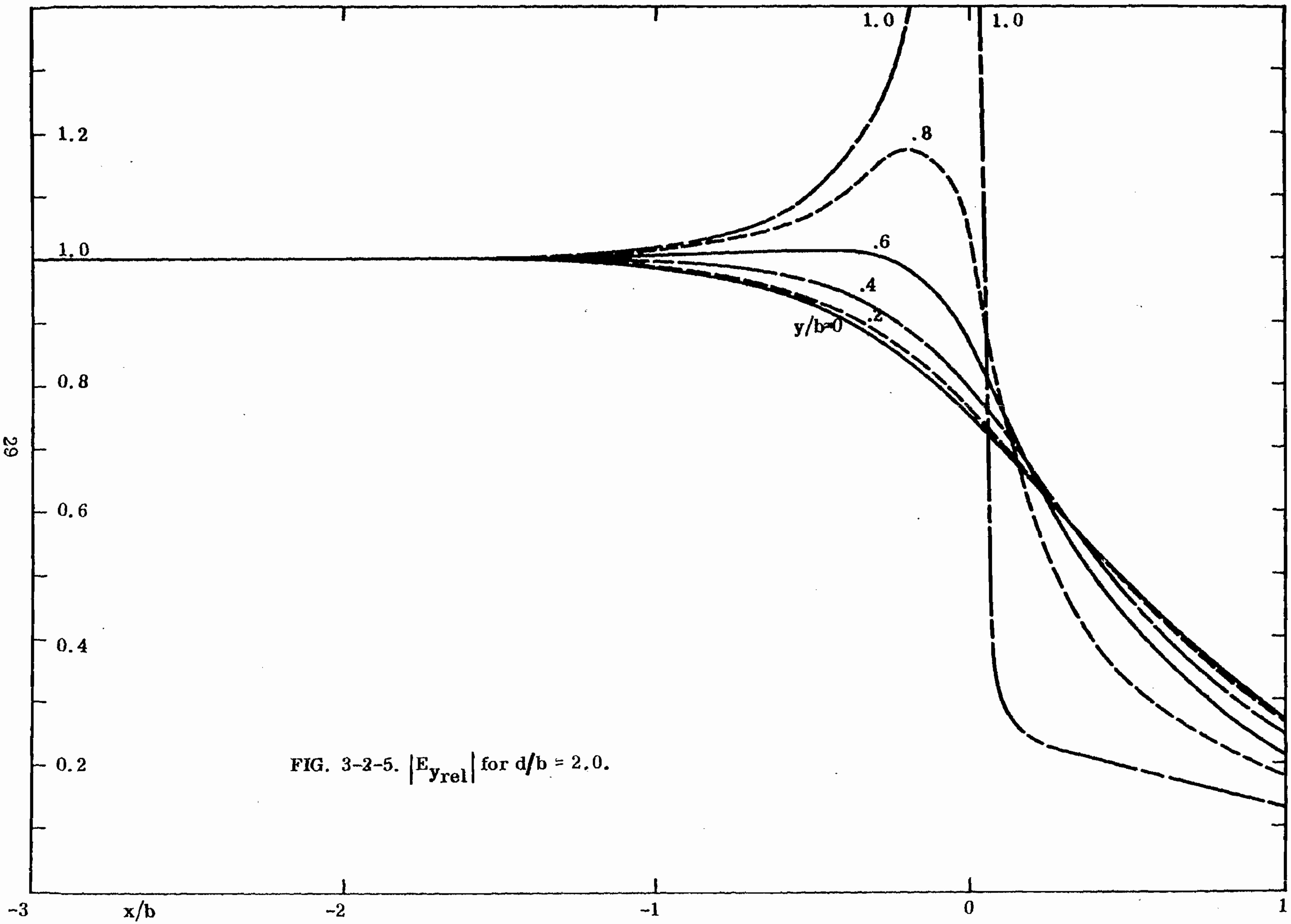

FIG. 3-2-5. $\left|E_{y_{rel}}\right|$ for $d/b = 2.0$.

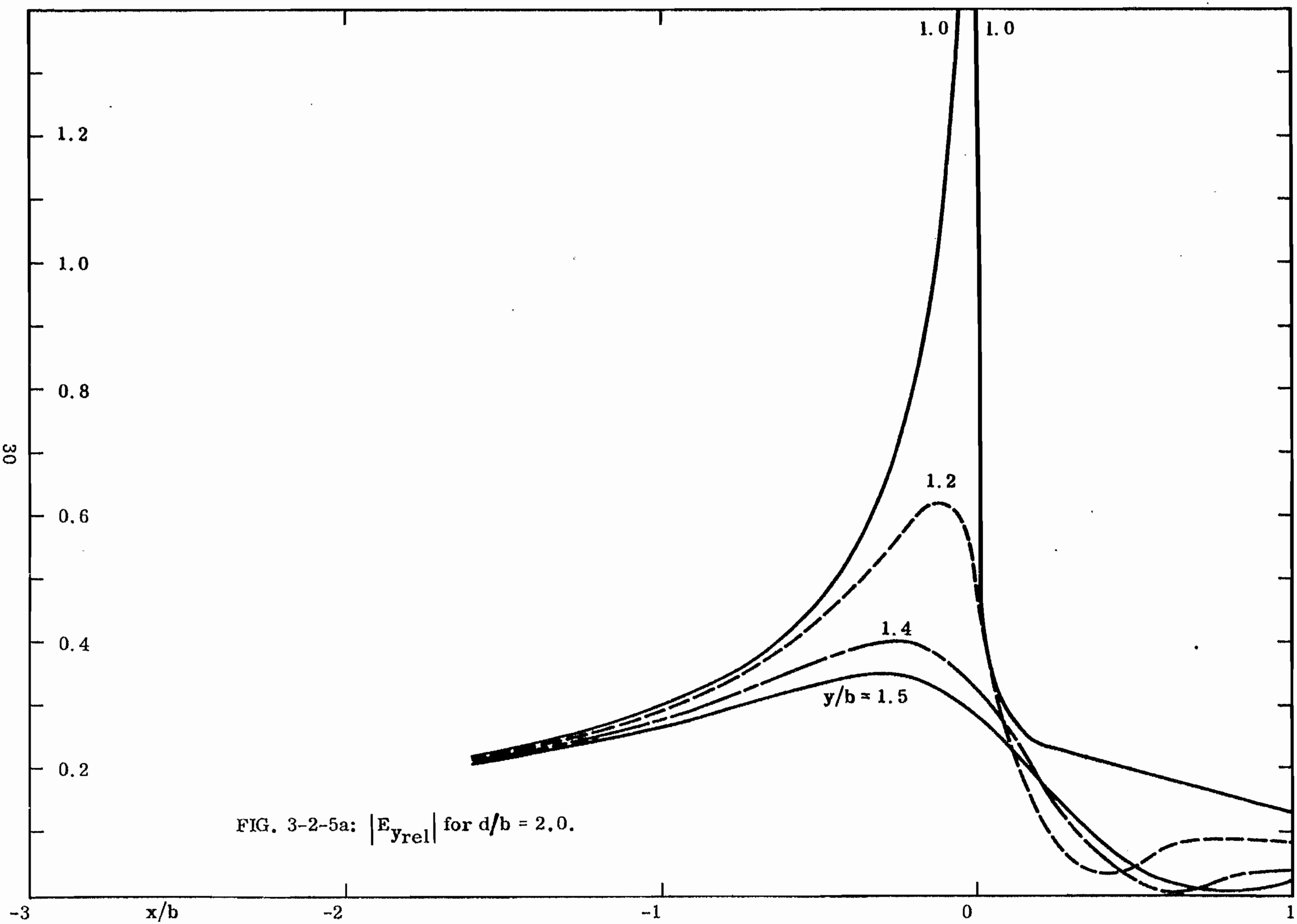

FIG. 3-2-5a: $\left|E_{y_{rel}}\right|$ for $d/b = 2.0$.

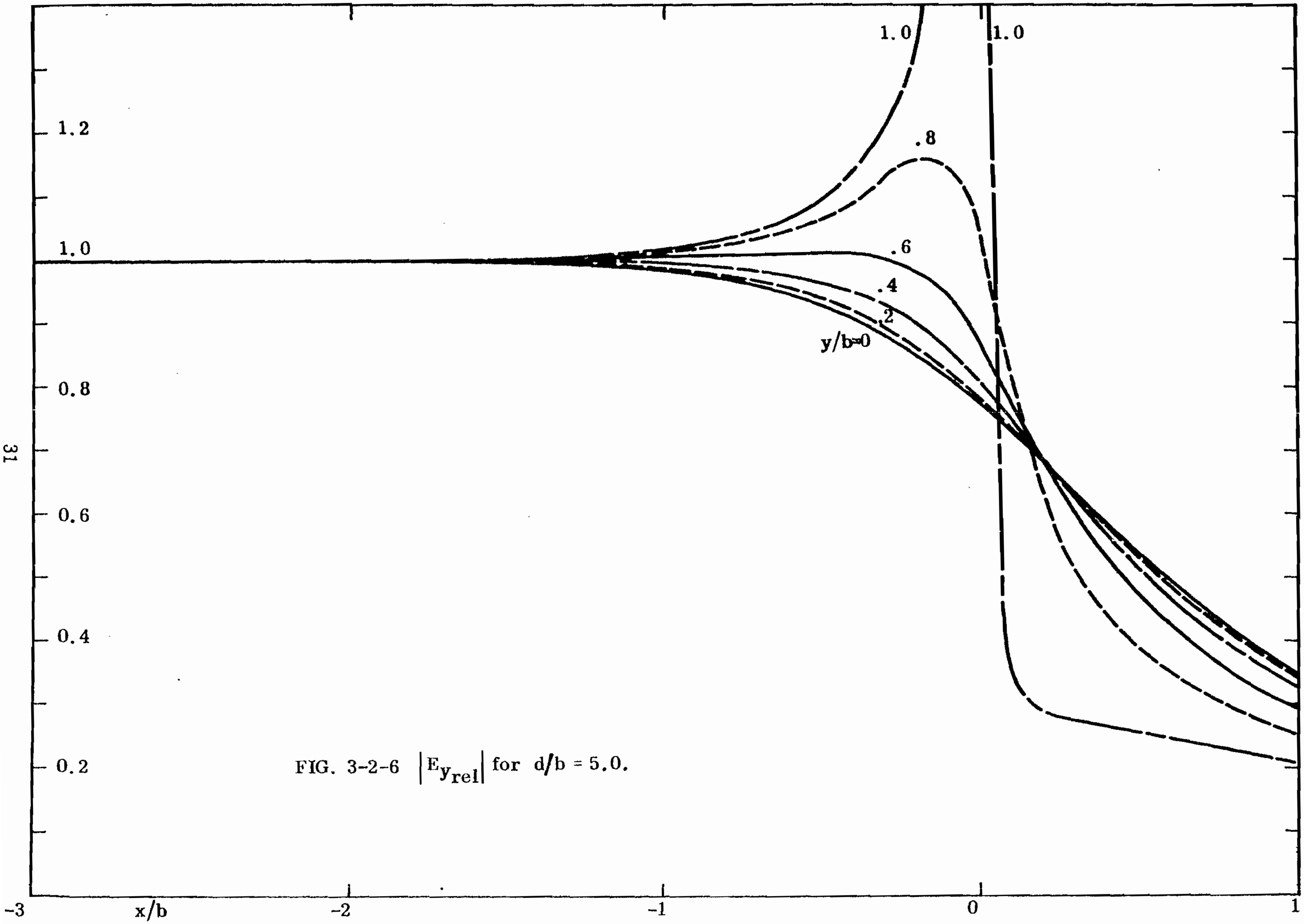

FIG. 3-2-6 $\left|E_{y_{rel}}\right|$ for $d/b = 5.0$.

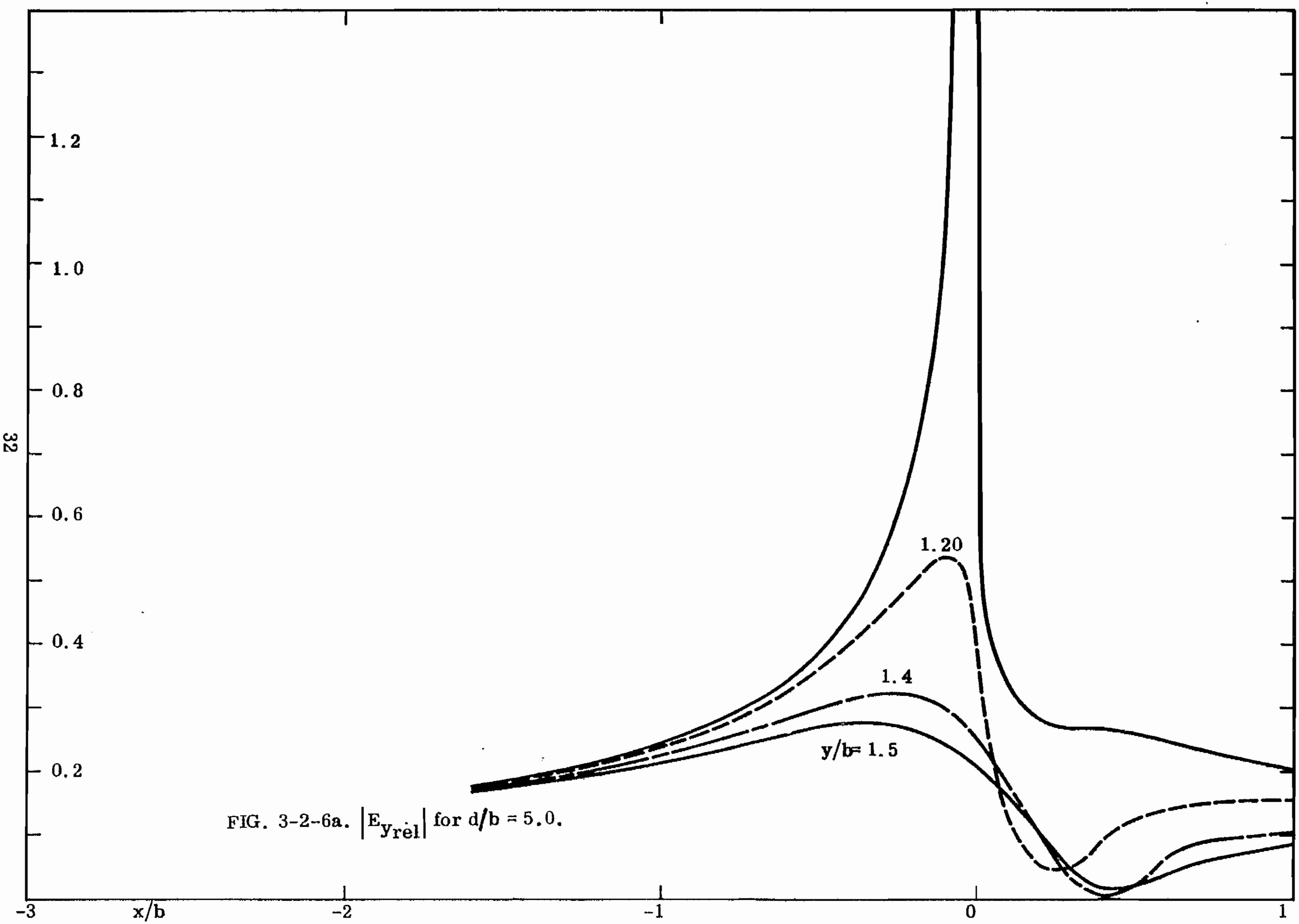

FIG. 3-2-6a. $|E_{y_{rel}}|$ for $d/b = 5.0$.

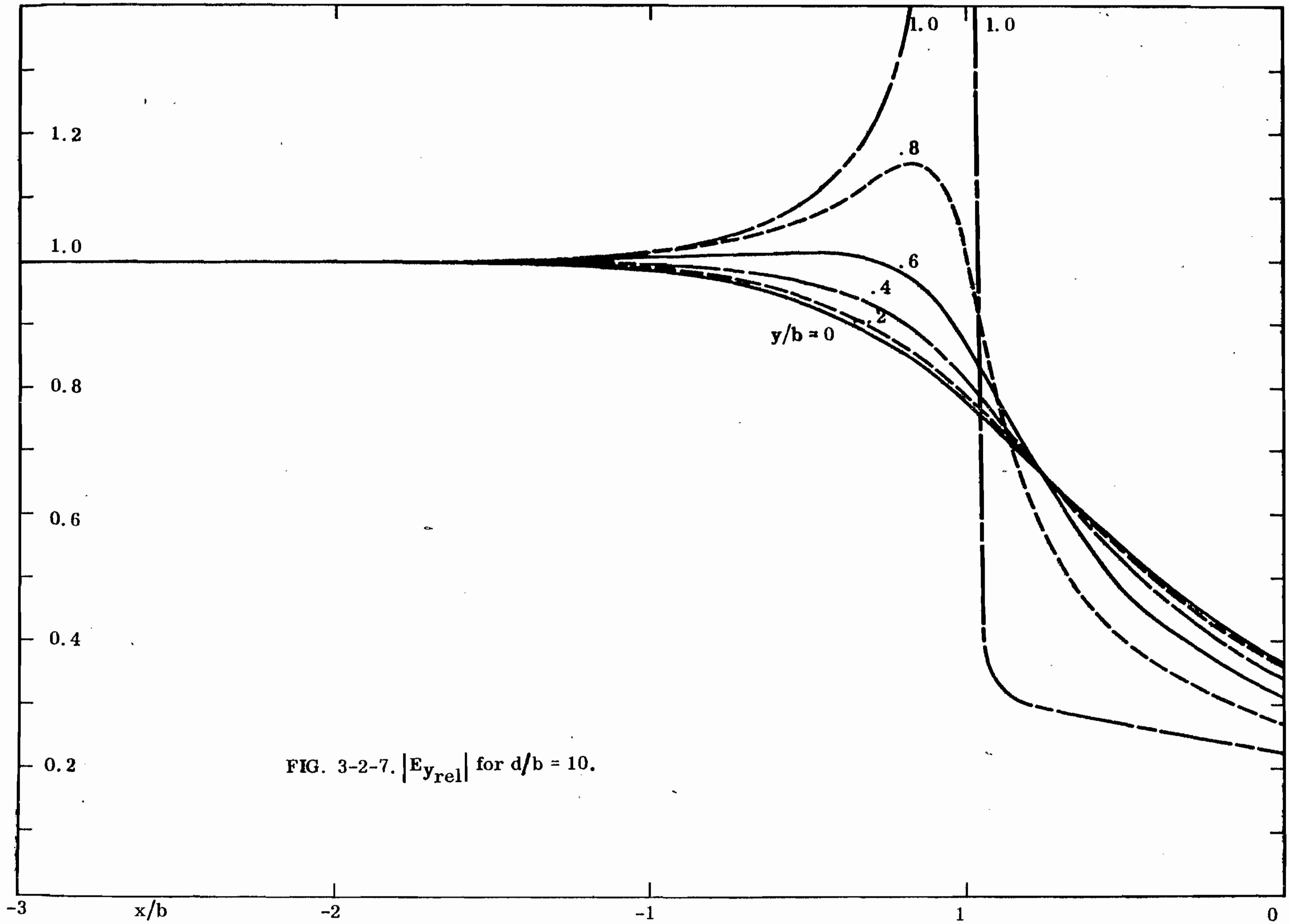

FIG. 3-2-7. $|E_{y_{rel}}|$ for d/b = 10.

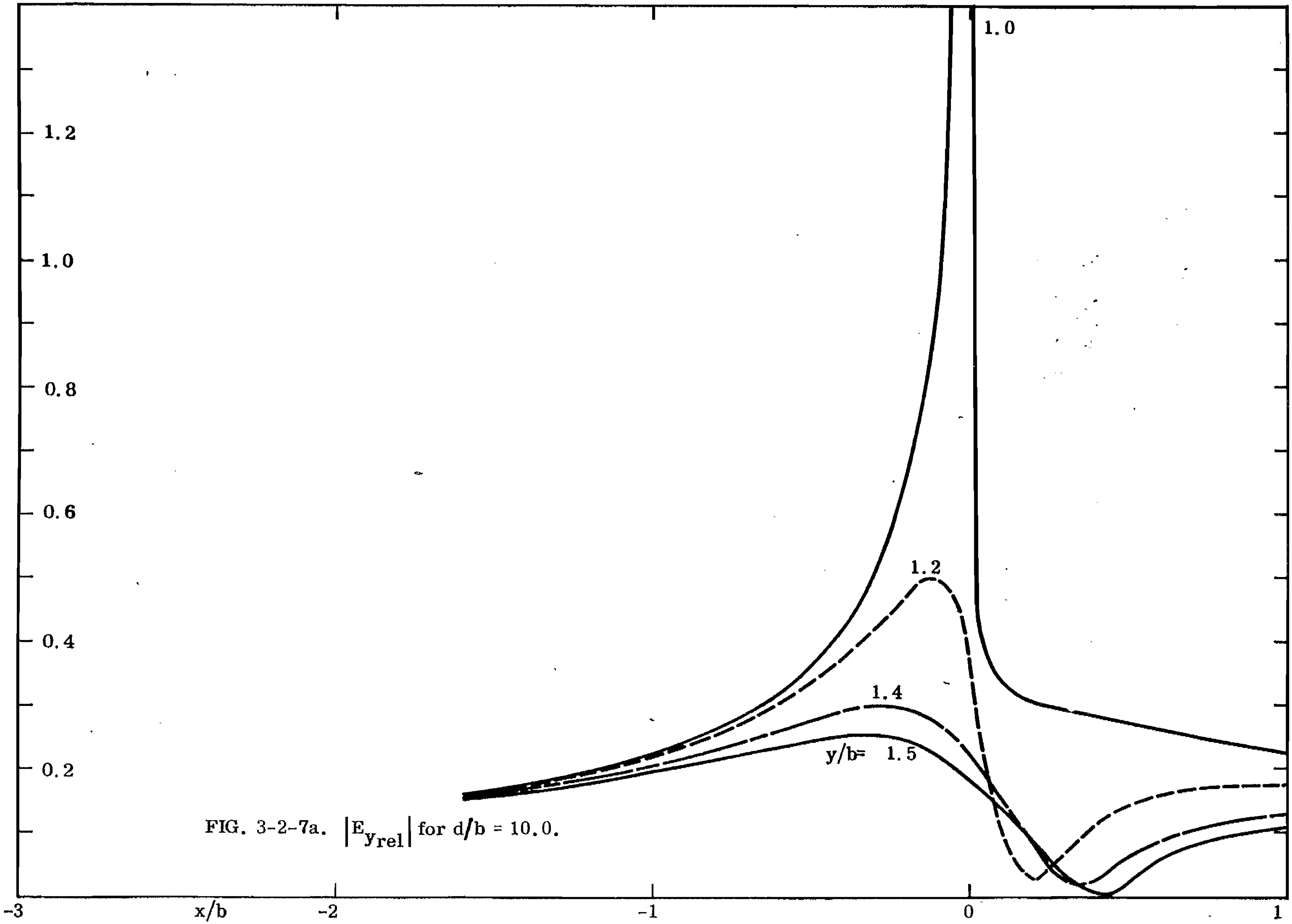

FIG. 3-2-7a. $\left|E_{y_{rel}}\right|$ for $d/b = 10.0$.

TABLE III-3: The x-Components of Relative Electric Field Intensity, $|E_{x_{rel}}|$.

Parameters y/b	d/b	Figure Nos.
$0 \le \frac{y}{b} \le 1.0$	0.1	3-2-8
	0.2	3-2-9
	0.5	3-2-10
	1.0	3-2-11
	2.0	3-2-12
	5.0	3-2-13
	10.0	3-2-14
$1.0 < \frac{y}{b} \le 1.5$	0.1	3-2-8-a
	0.2	3-2-9-a
	0.5	3-2-10-a
	1.0	3-2-11-a
	2.0	3-2-12-a
	5.0	3-2-13-a
	10.0	3-2-14-a

Figures 3-2-15 through 3-2-21-a show the relative electric field intensity, $|E_{rel}|$, as a function of x/b with y/b fixed as a parameter.

TABLE III-4: The Relative Electric Field Intensity, $|E_{rel}|$.

Parameters y/b	d/b	Figure Nos.
$0 \le \frac{y}{b} \le 1.0$	0.1	3-2-15
	0.2	3-2-16
	0.5	3-2-17
	1.0	3-2-18
	2.0	3-2-19
	5.0	3-2-20
	10.0	3-2-21
$1.0 < \frac{y}{b} \le 1.5$	0.1	3-2-15-a
	0.2	3-2-16-a
	0.5	3-2-17-a
	1.0	3-2-18-a
	2.0	3-2-19-a
	5.0	3-2-20-a
	10.0	3-2-21-a

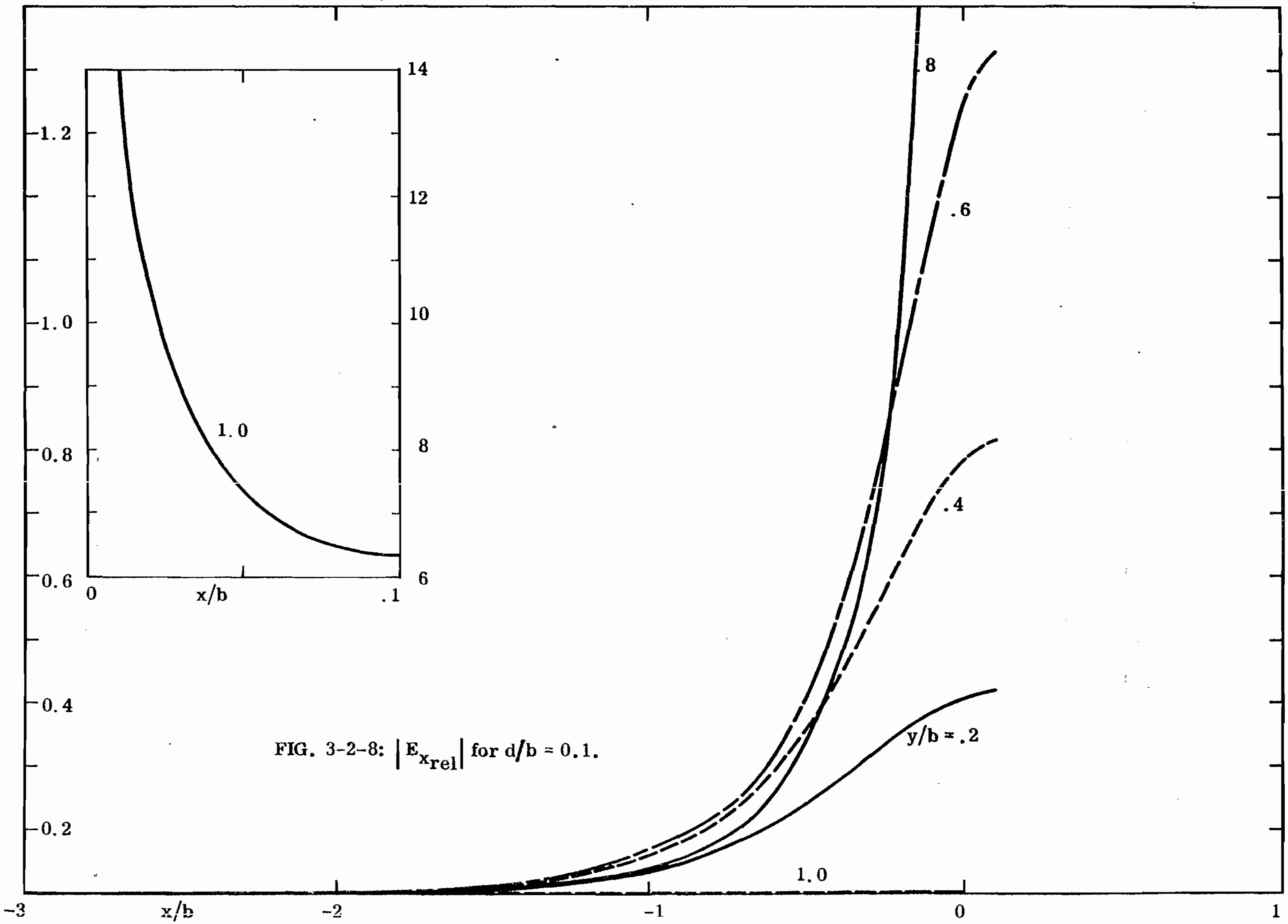

FIG. 3-2-8: $\left|E_{x_{rel}}\right|$ for d/b = 0.1.

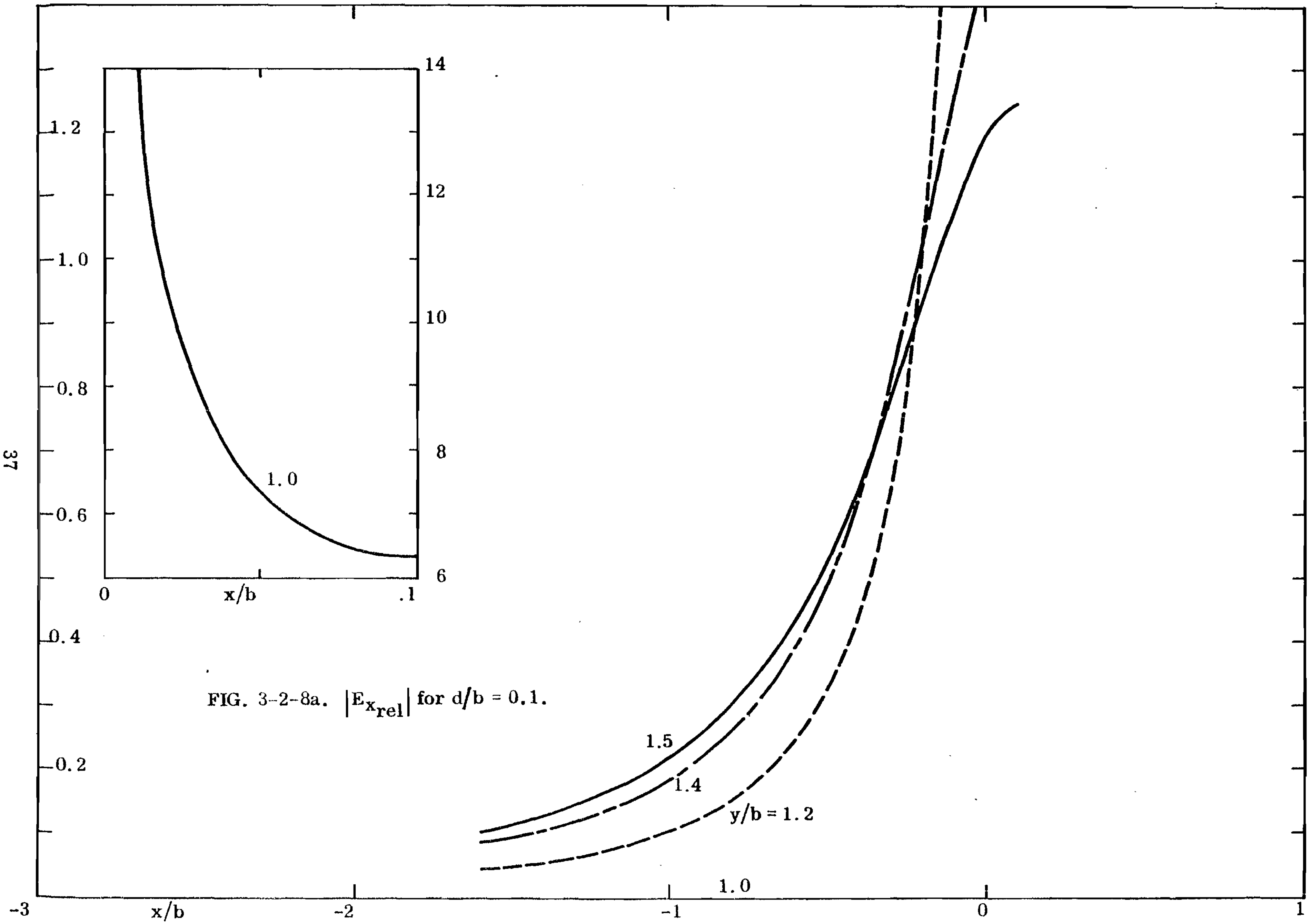

FIG. 3-2-8a. $|E_{x_{rel}}|$ for $d/b = 0.1$.

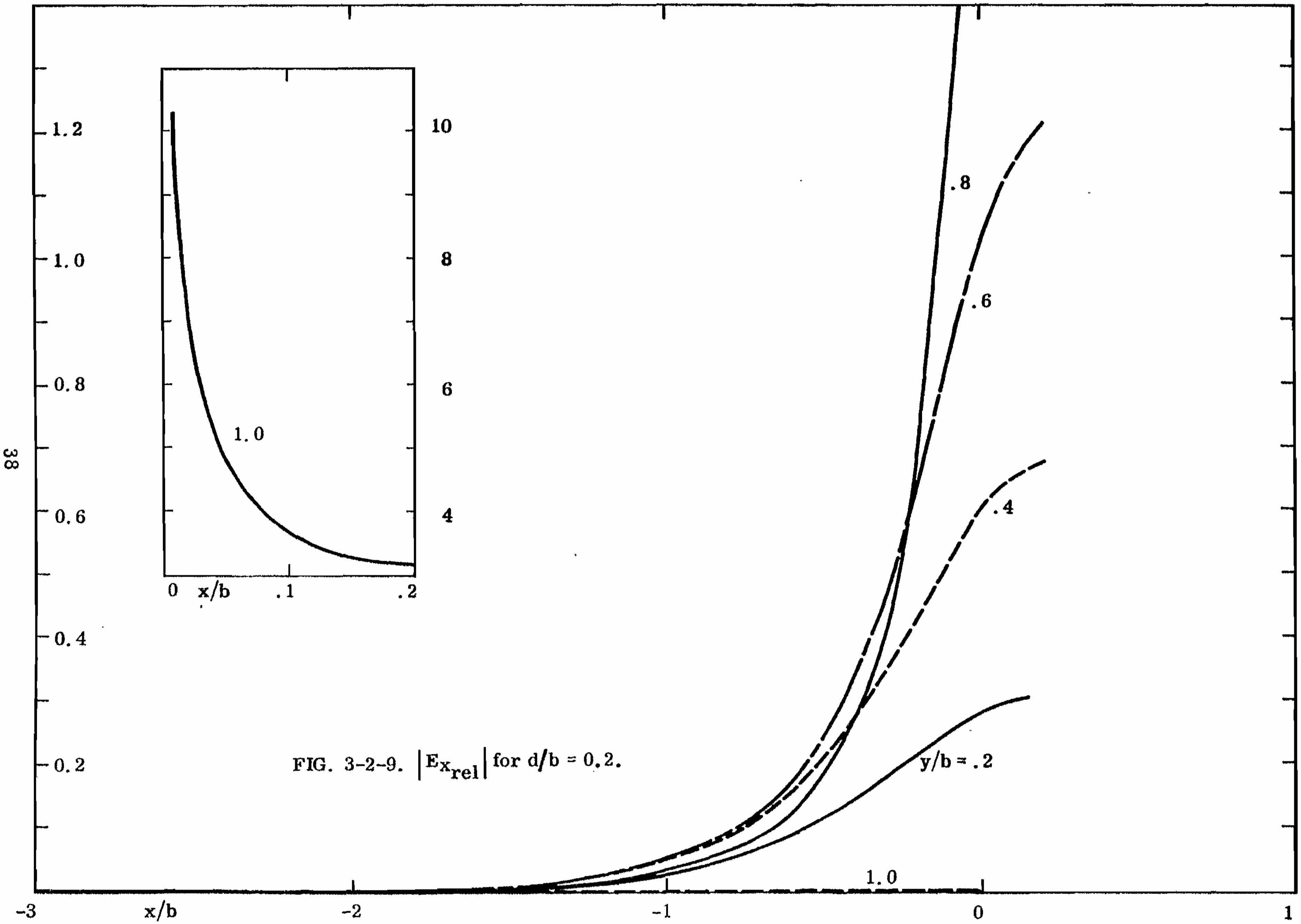

FIG. 3-2-9. $\left|E_{x_{rel}}\right|$ for $d/b = 0.2$.

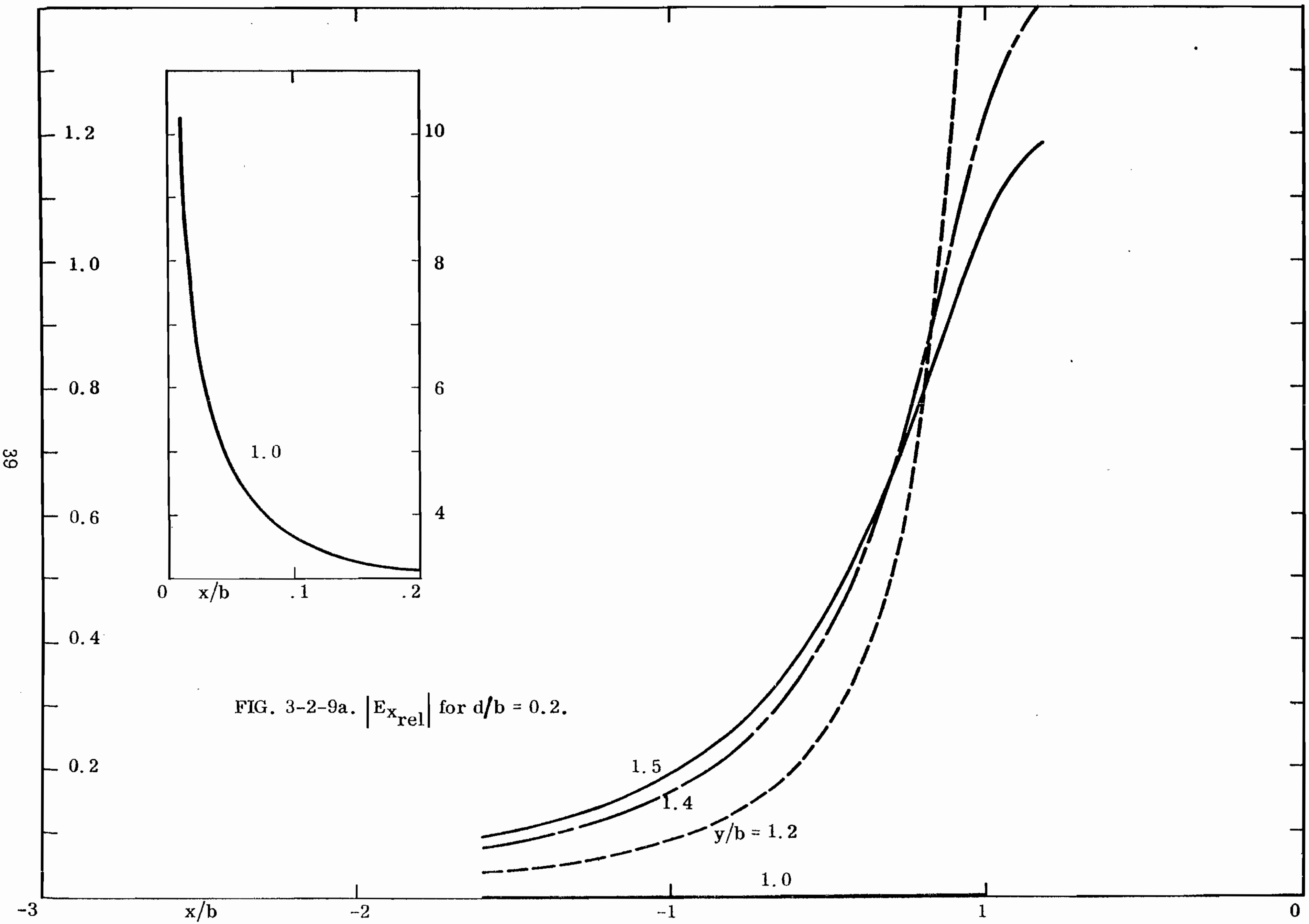

FIG. 3-2-9a. $\left|E_{x_{rel}}\right|$ for $d/b = 0.2$.

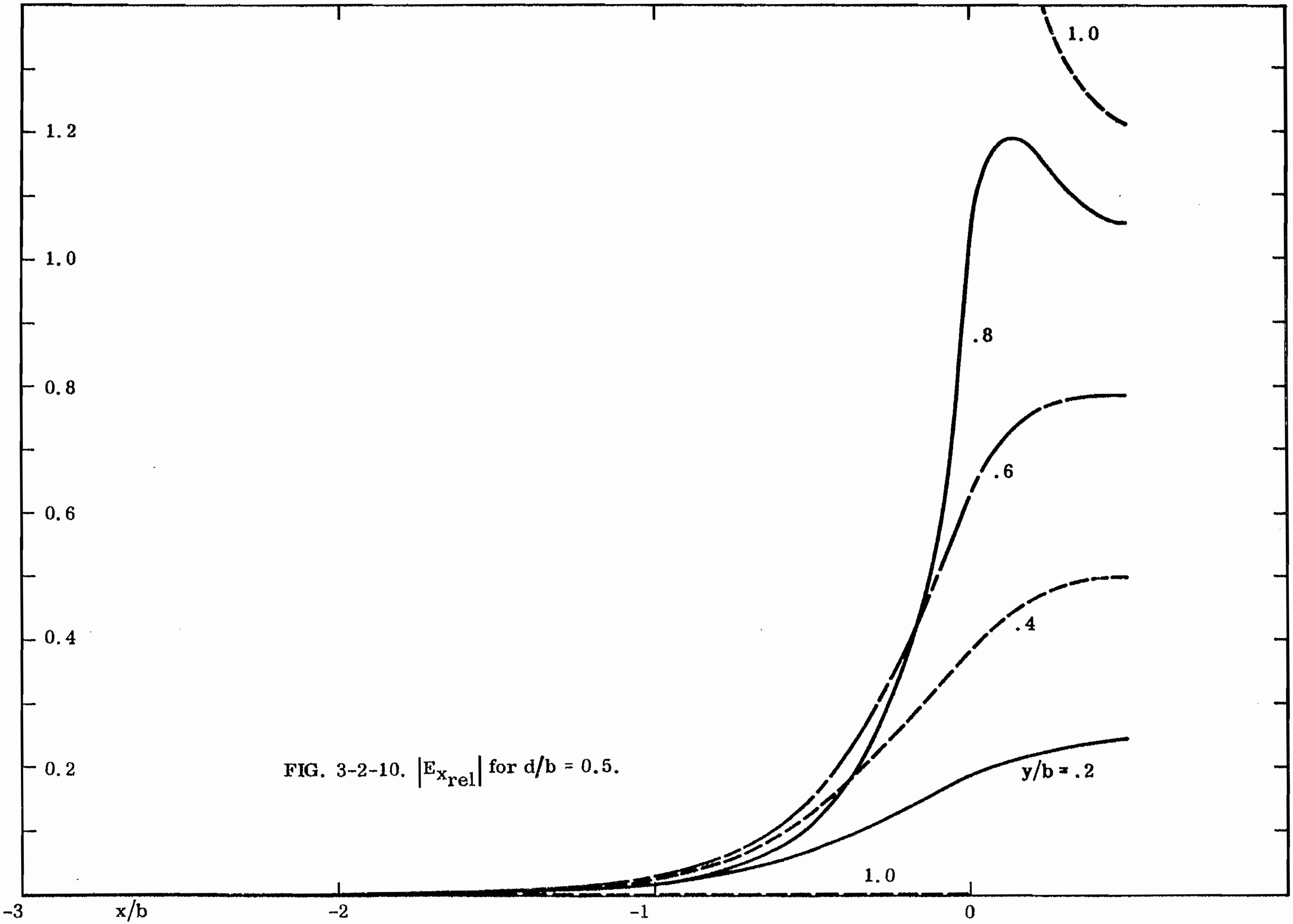

FIG. 3-2-10. $|E_{x_{rel}}|$ for d/b = 0.5.

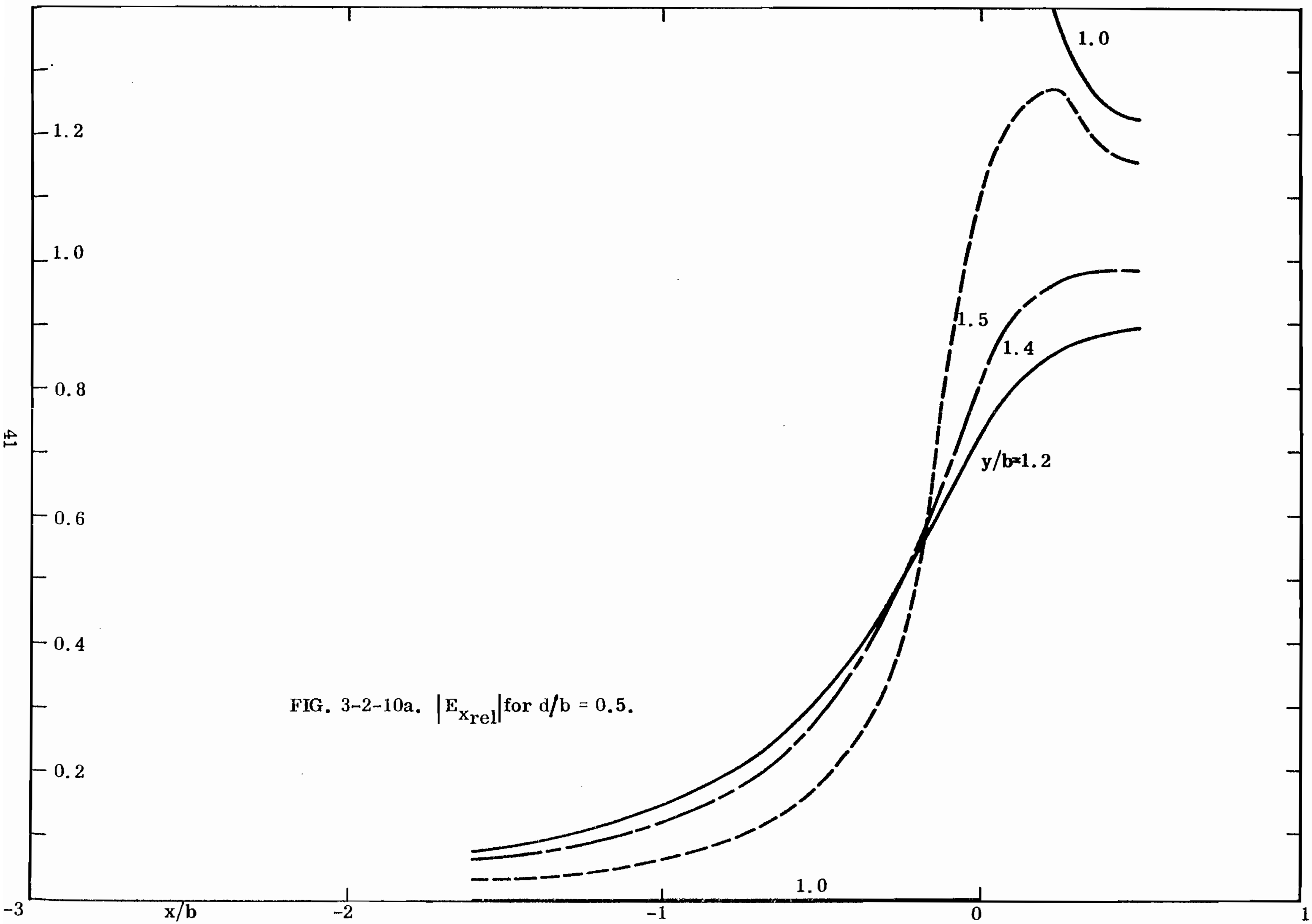

FIG. 3-2-10a. $\left|E_{x_{rel}}\right|$ for d/b = 0.5.

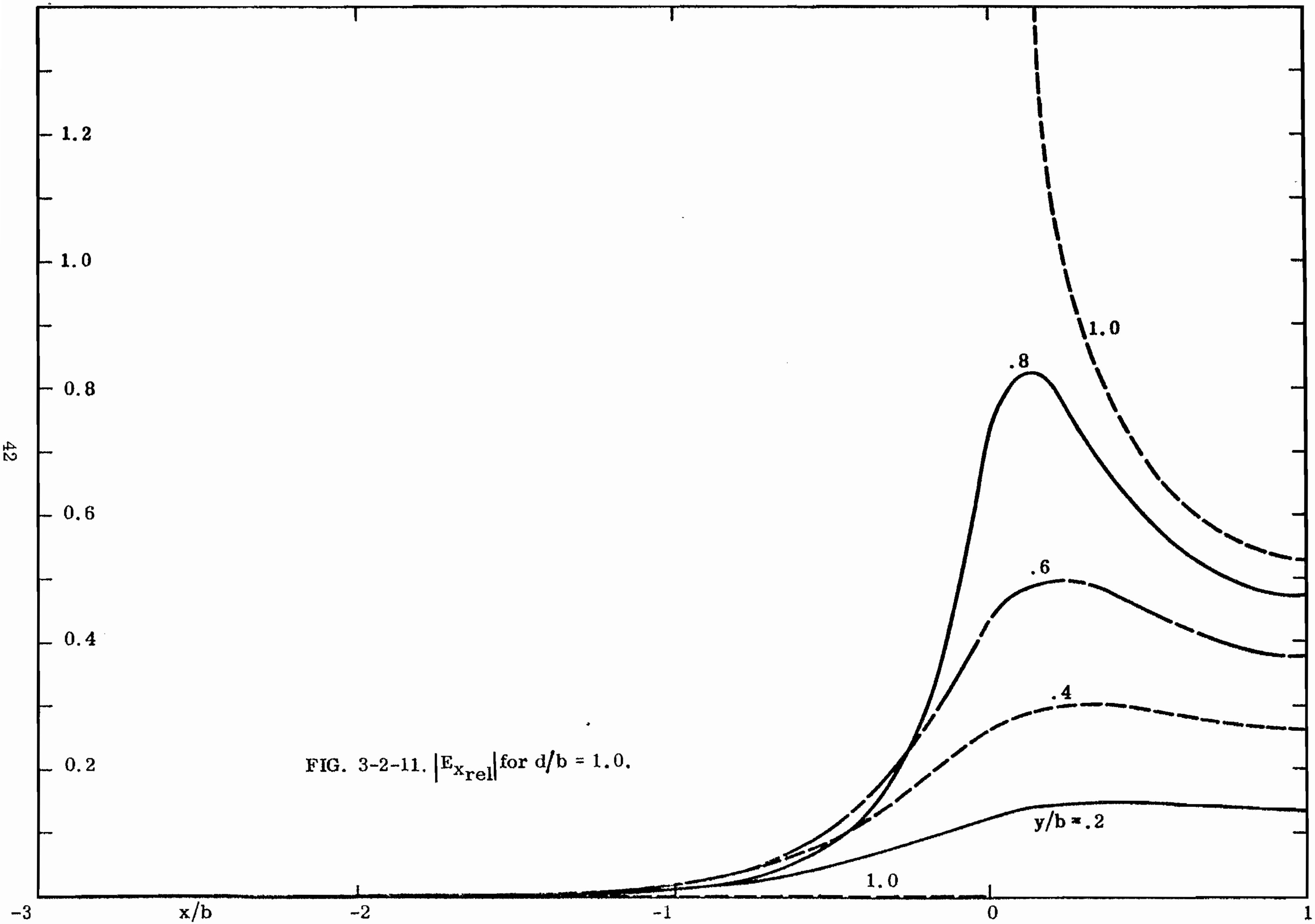

FIG. 3-2-11. $|E_{x_{rel}}|$ for $d/b = 1.0$.

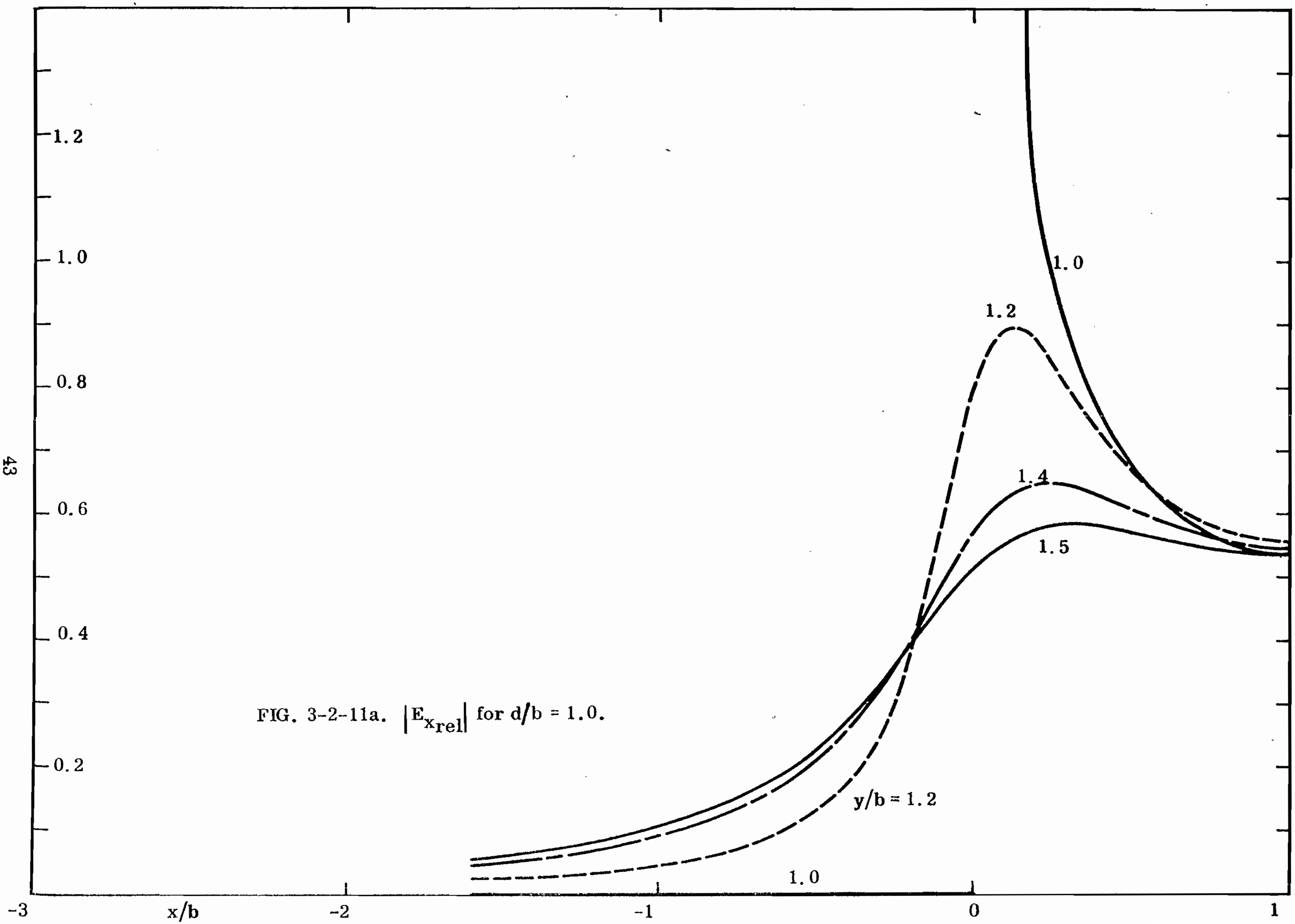

FIG. 3-2-11a. $|E_{x_{rel}}|$ for d/b = 1.0.

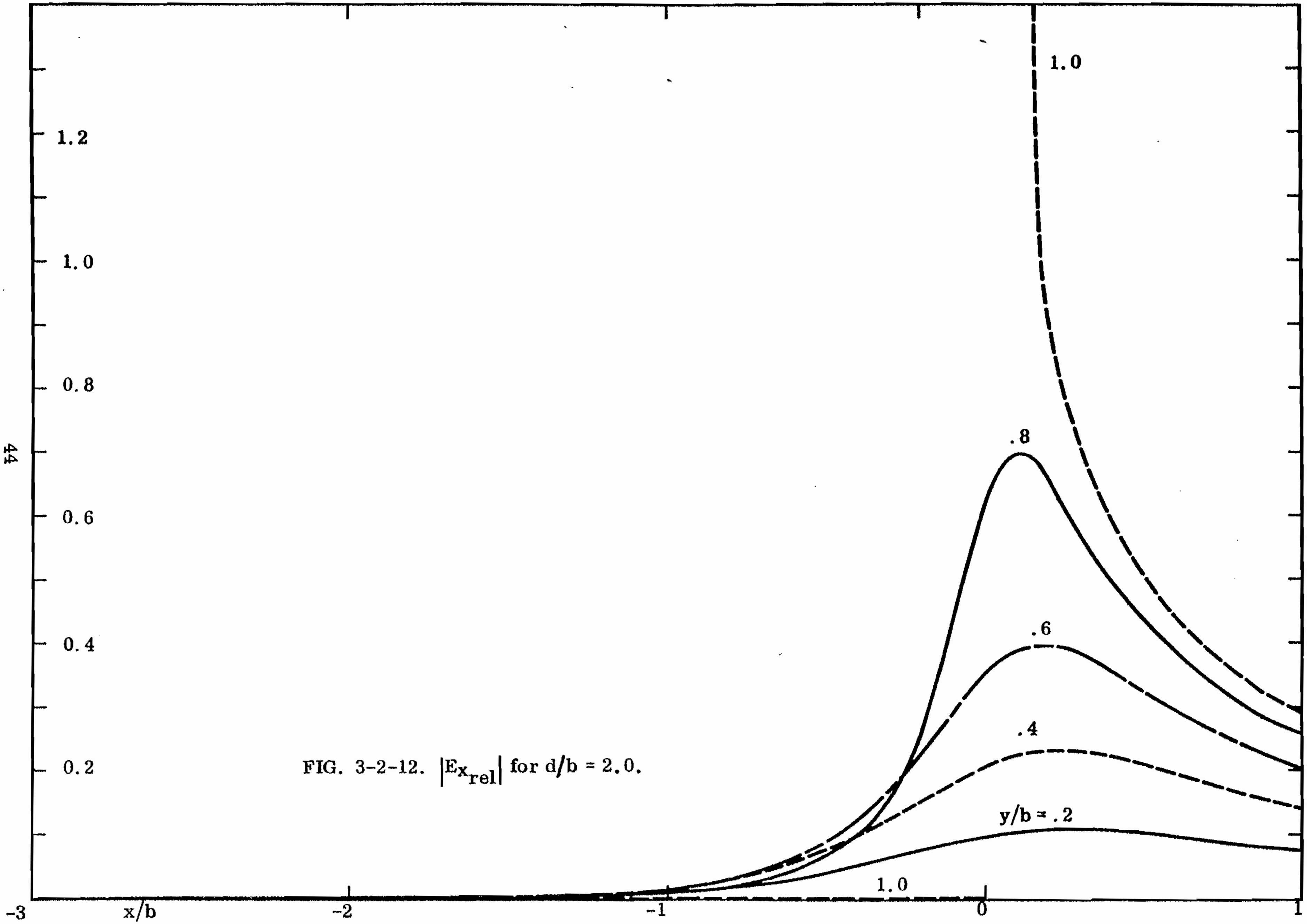

FIG. 3-2-12. $|Ex_{rel}|$ for $d/b = 2.0$.

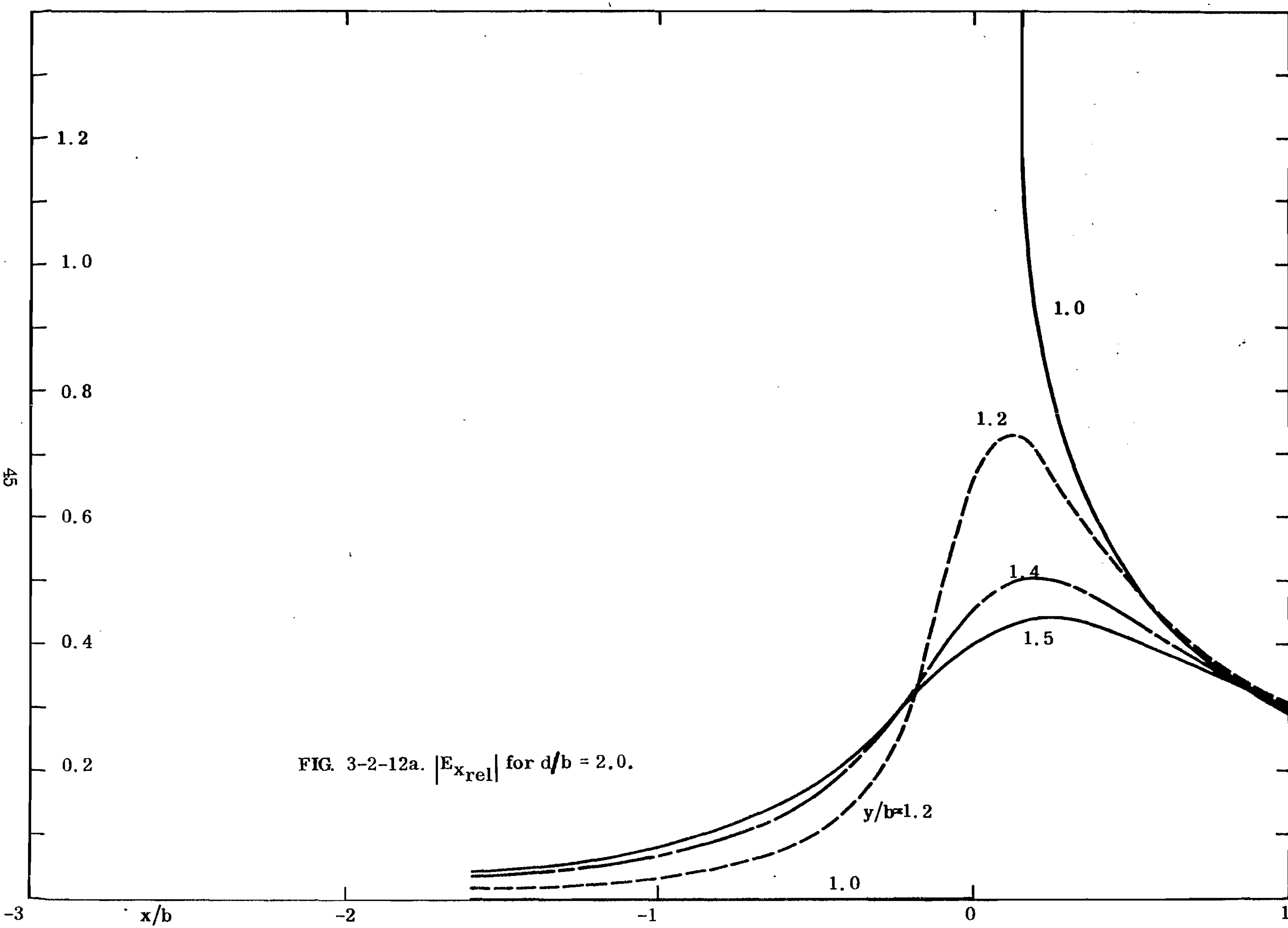

FIG. 3-2-12a. $|E_{x_{rel}}|$ for $d/b = 2.0$.

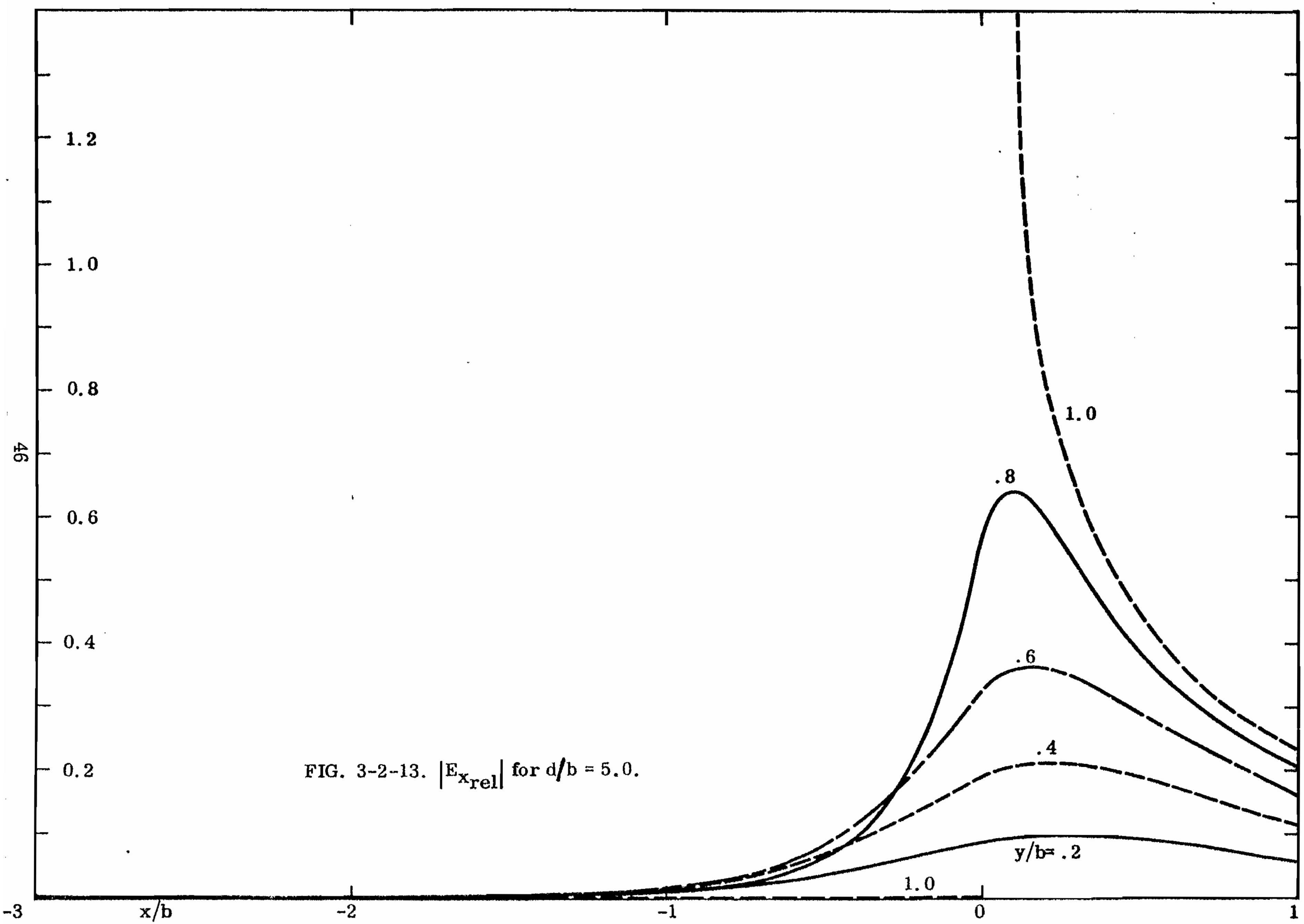

FIG. 3-2-13. $|E_{x_{rel}}|$ for $d/b = 5.0$.

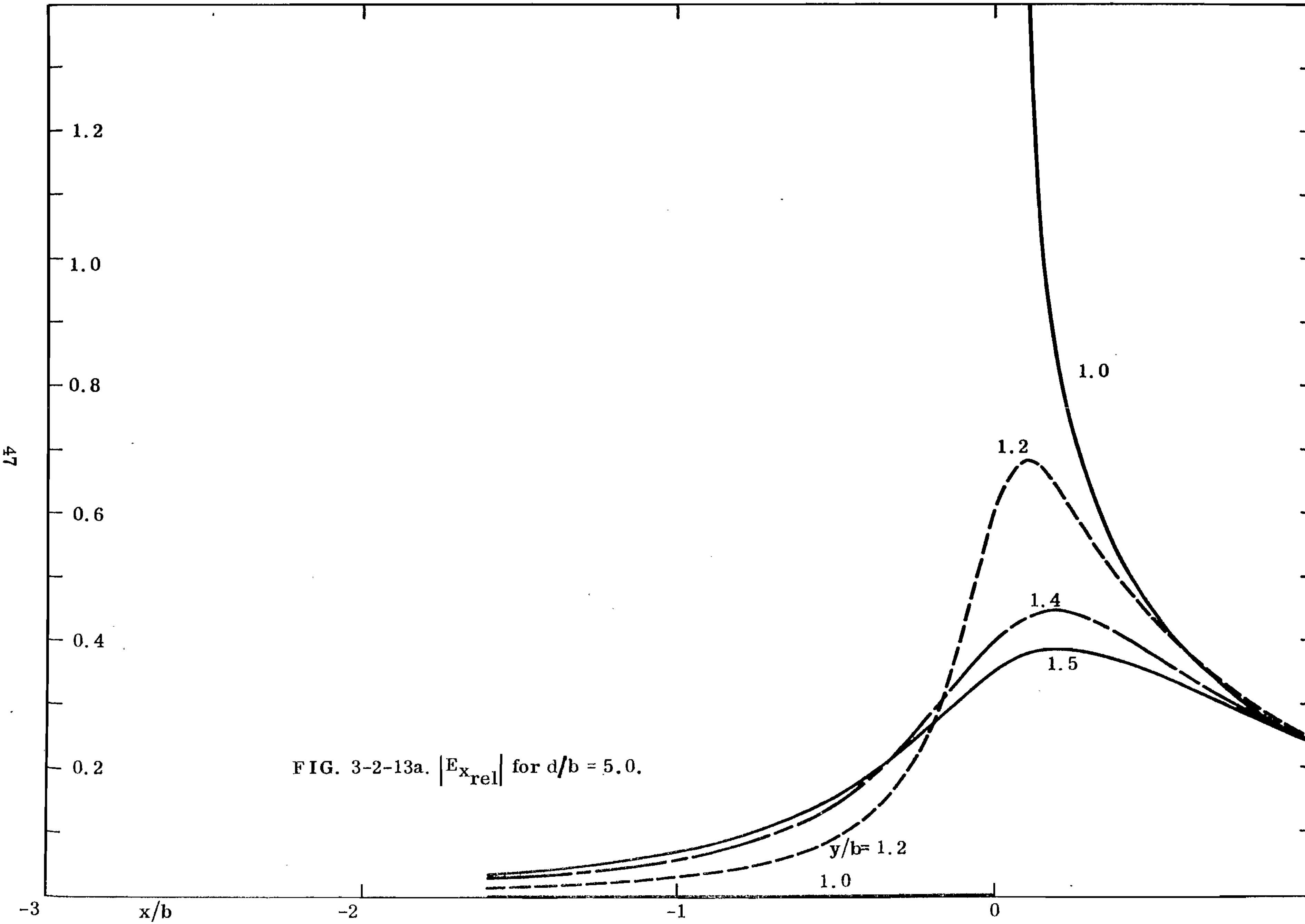

FIG. 3-2-13a. $\left|E_{x_{rel}}\right|$ for d/b = 5.0.

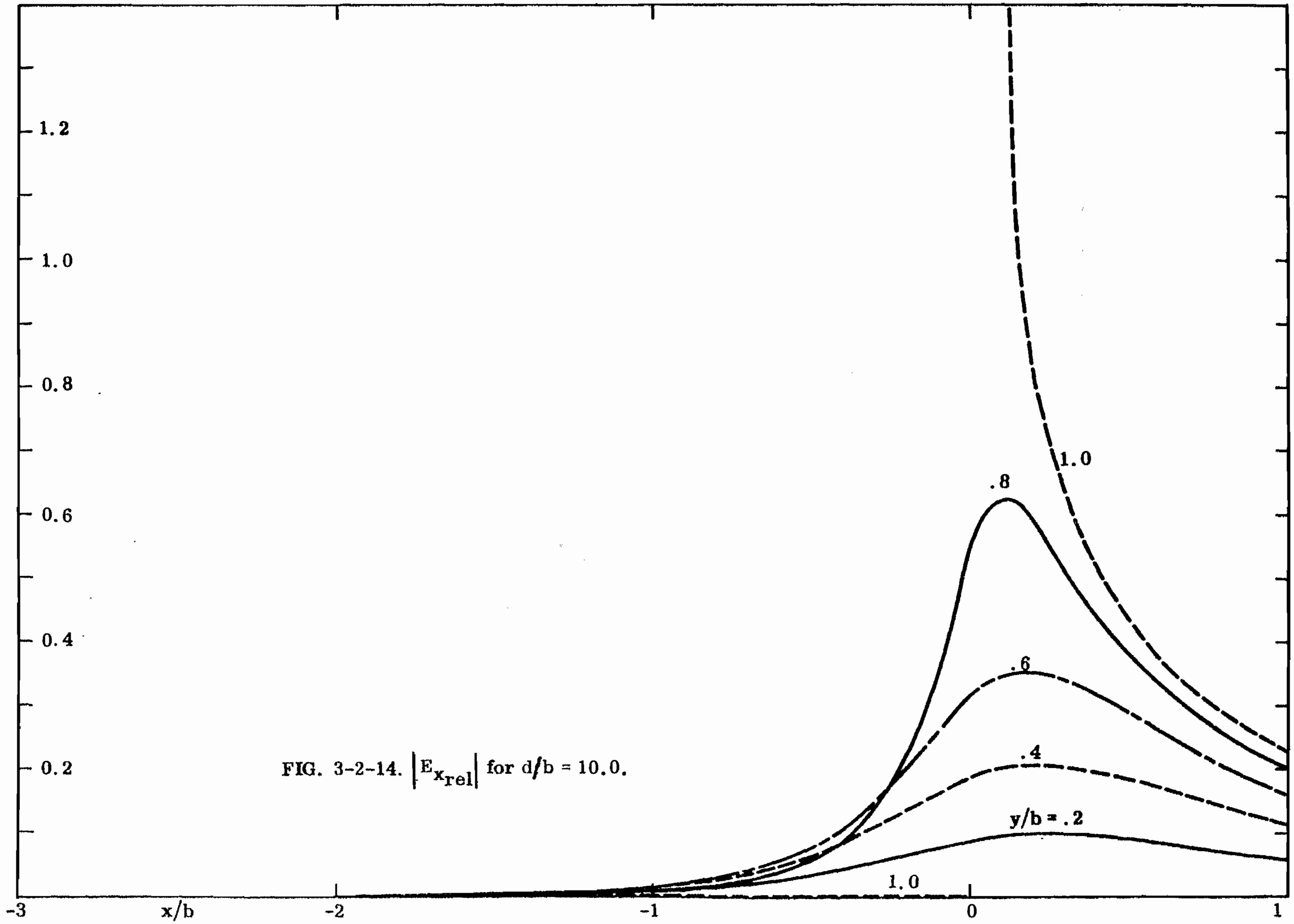

FIG. 3-2-14. $\left|E_{x_{rel}}\right|$ for d/b = 10.0.

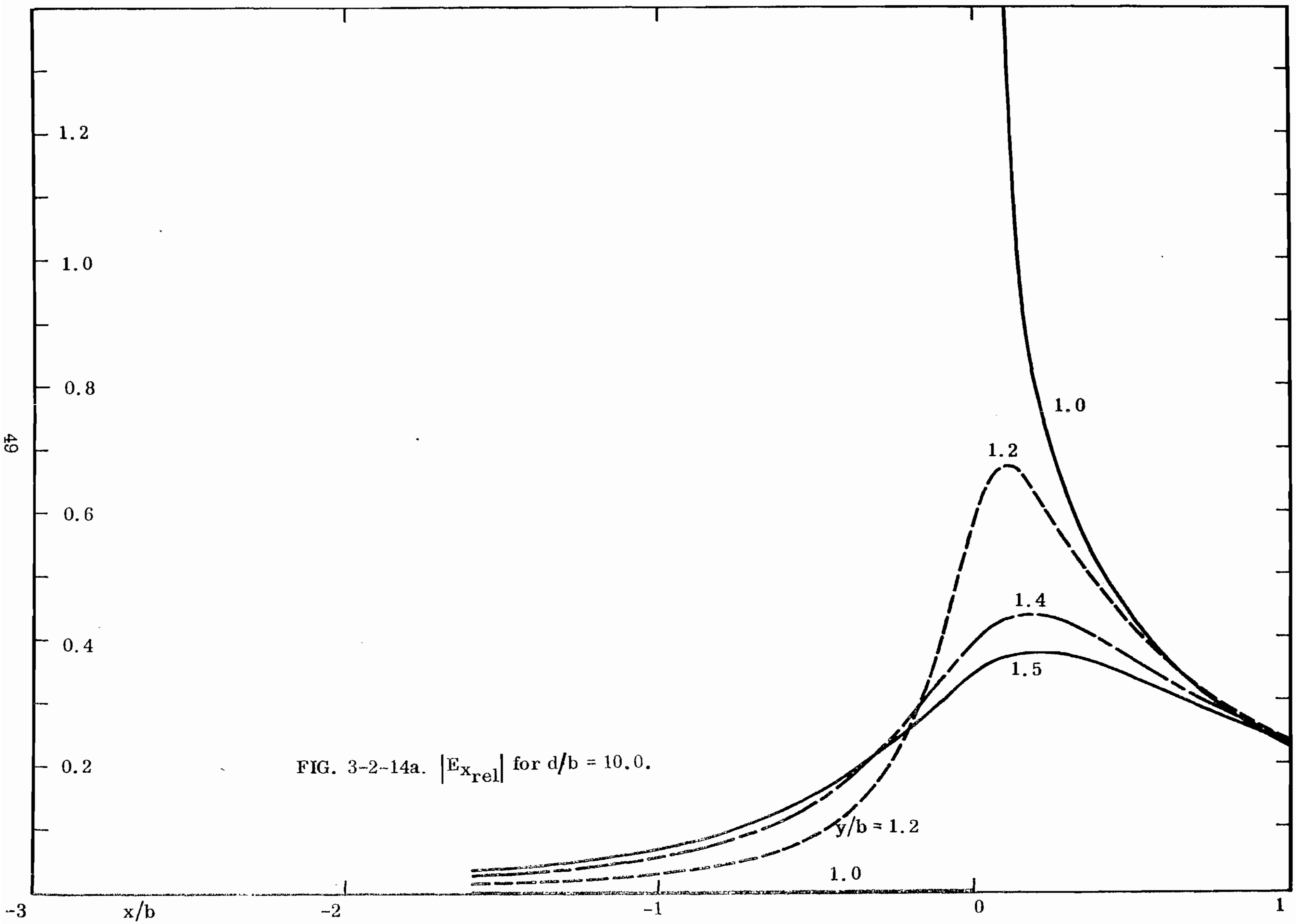

FIG. 3-2-14a. $|E_{x_{rel}}|$ for d/b = 10.0.

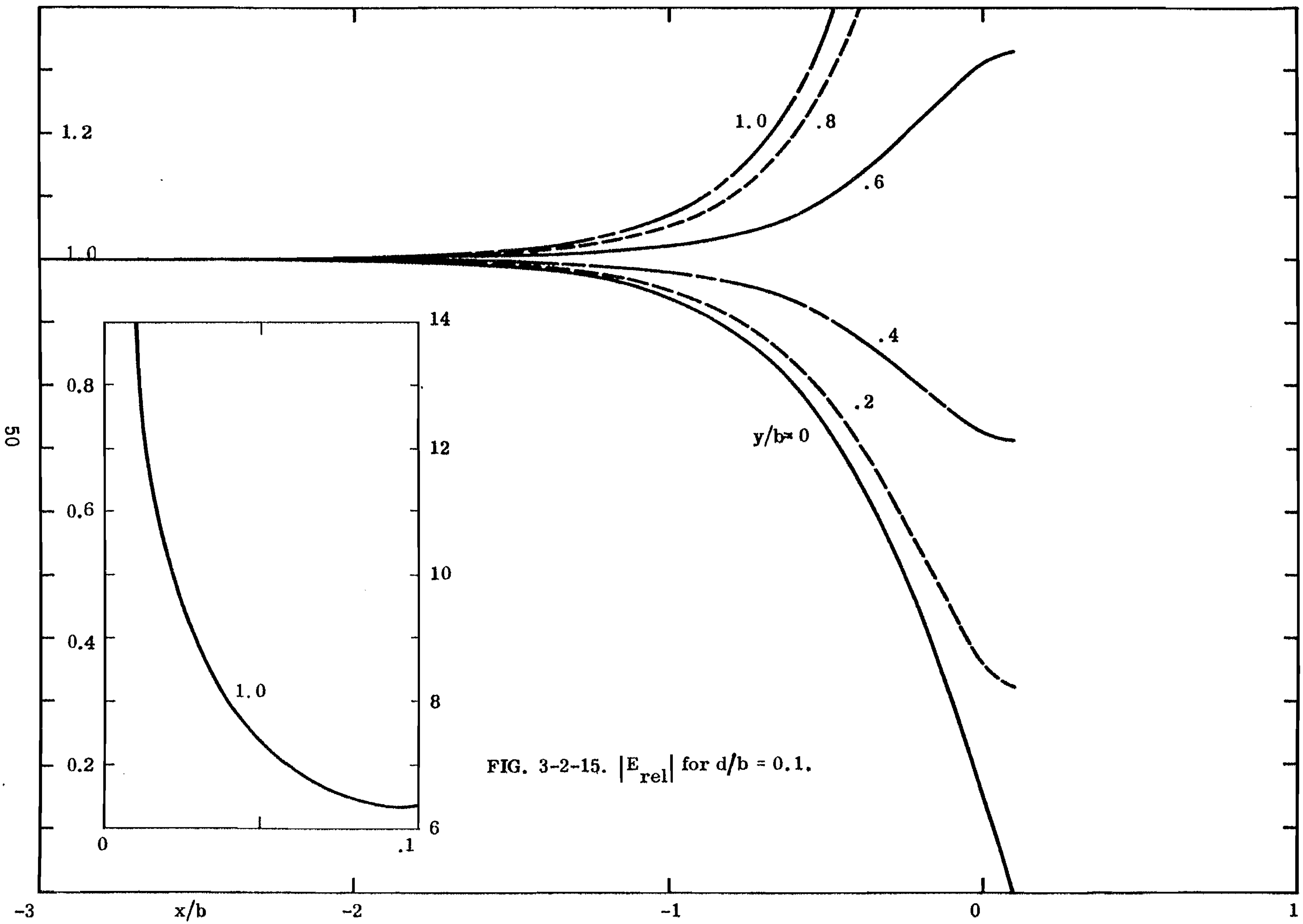

FIG. 3-2-15. $|E_{rel}|$ for $d/b = 0.1$.

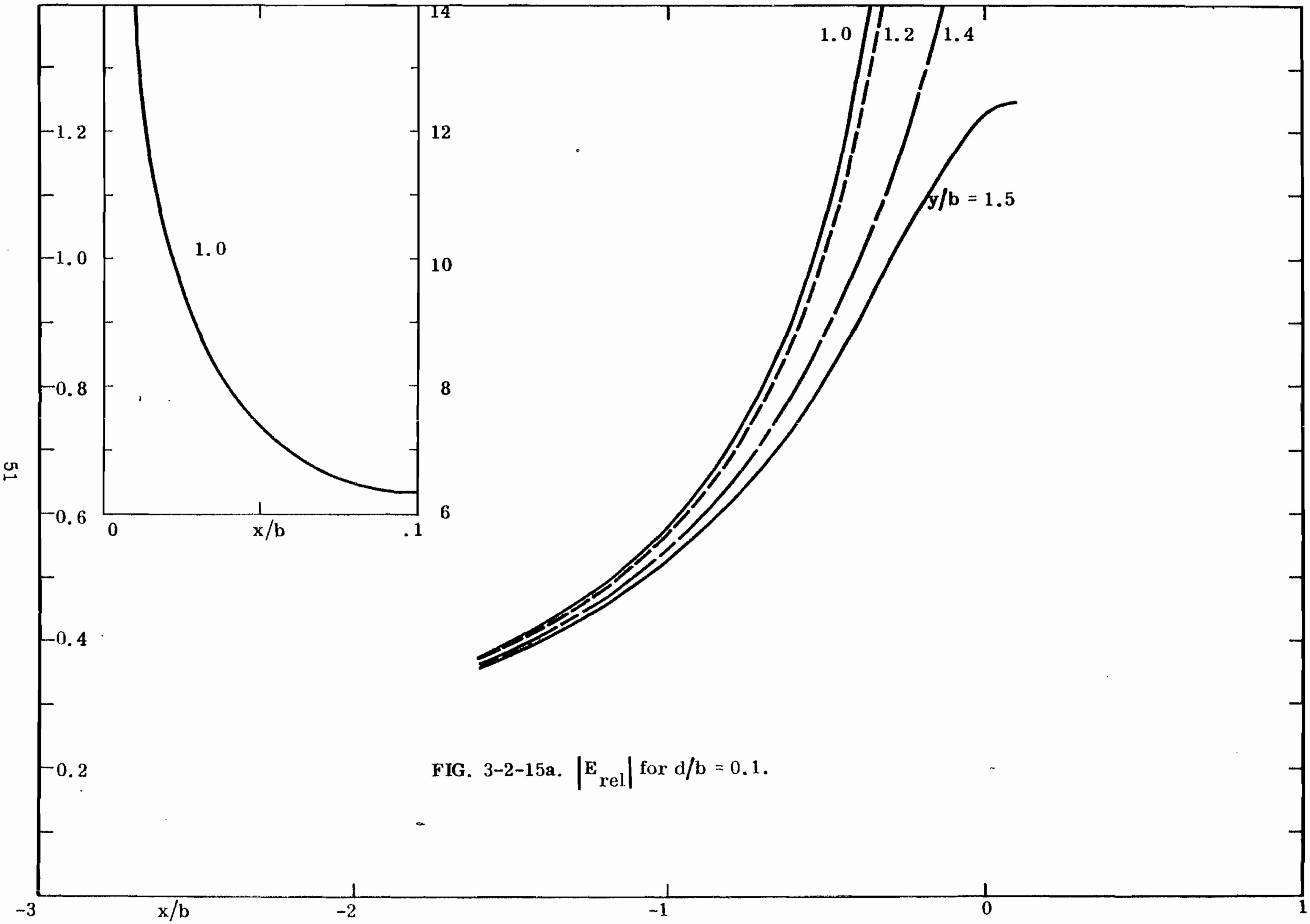

FIG. 3-2-15a. $|E_{rel}|$ for $d/b = 0.1$.

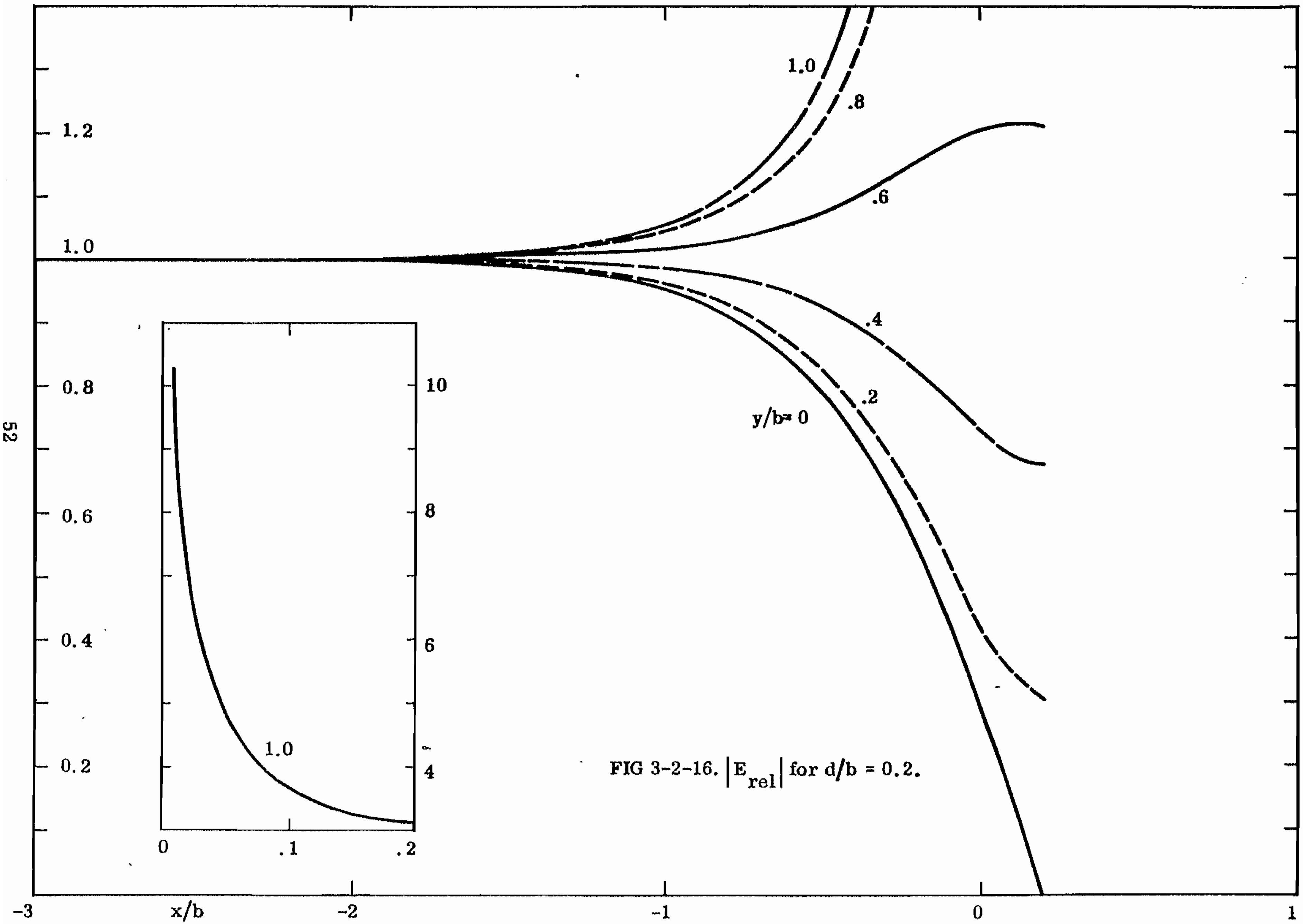

FIG 3-2-16. $|E_{rel}|$ for d/b = 0.2.

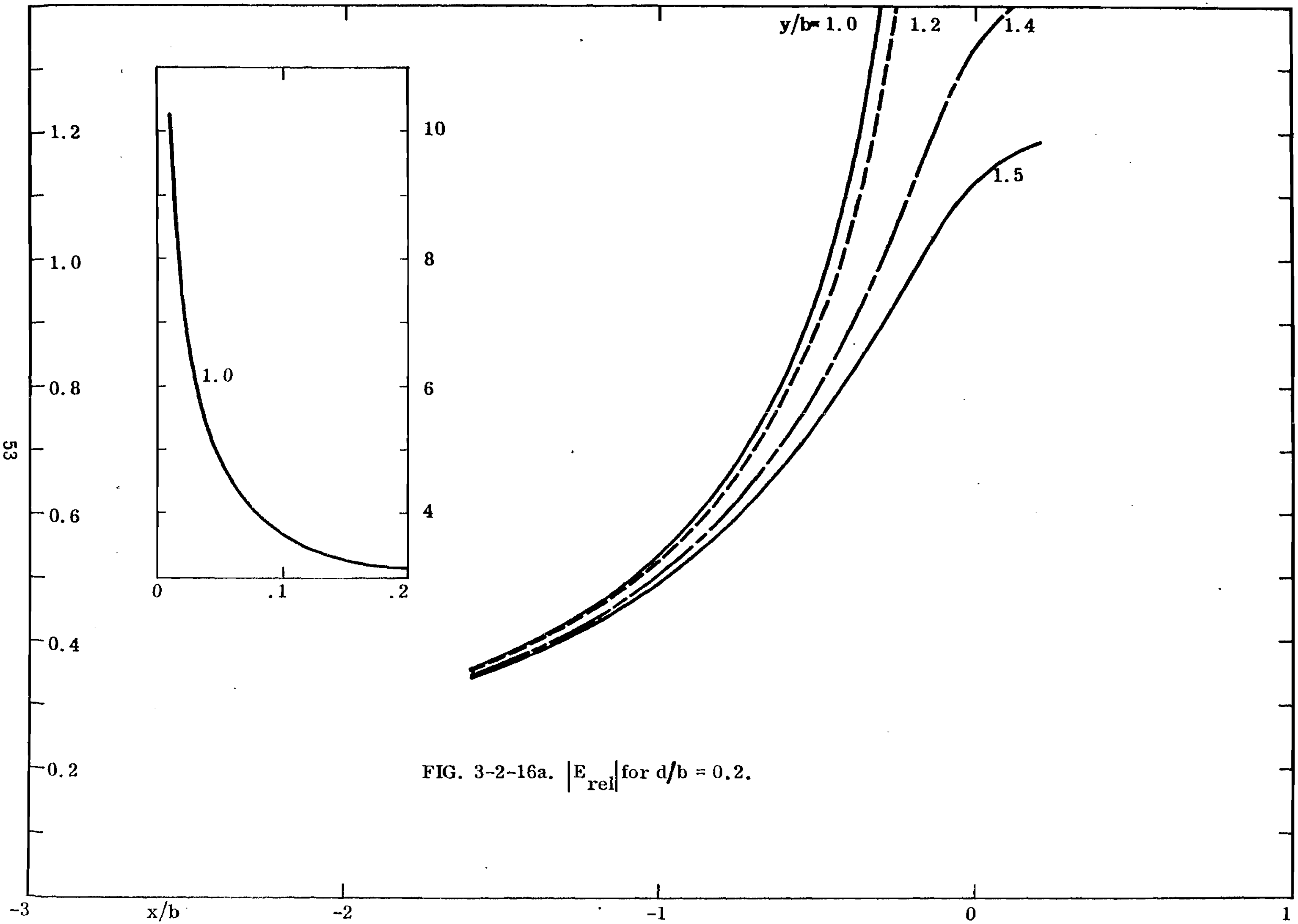

FIG. 3-2-16a. $|E_{rel}|$ for $d/b = 0.2$.

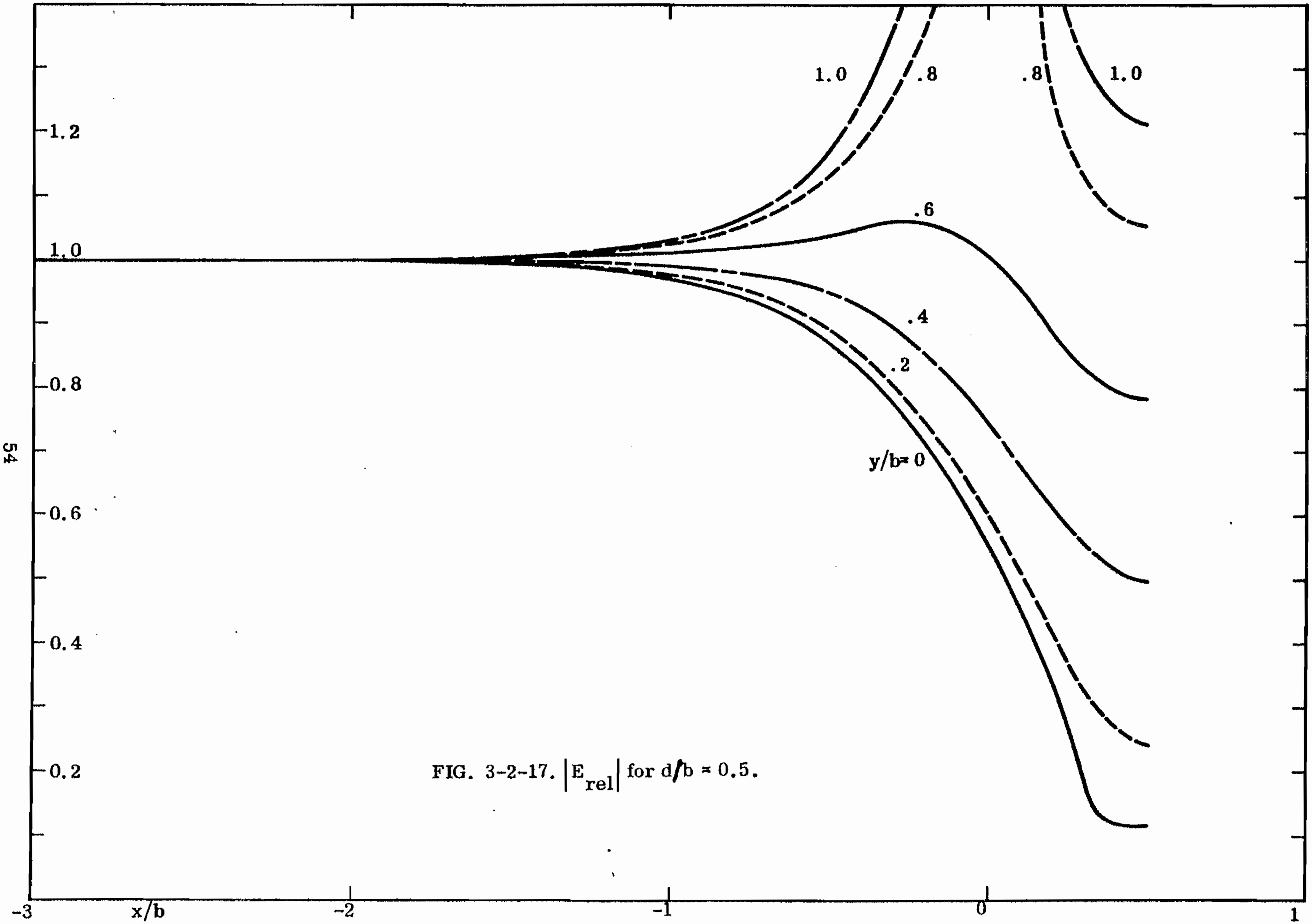

FIG. 3-2-17. $|E_{rel}|$ for $d/b = 0.5$.

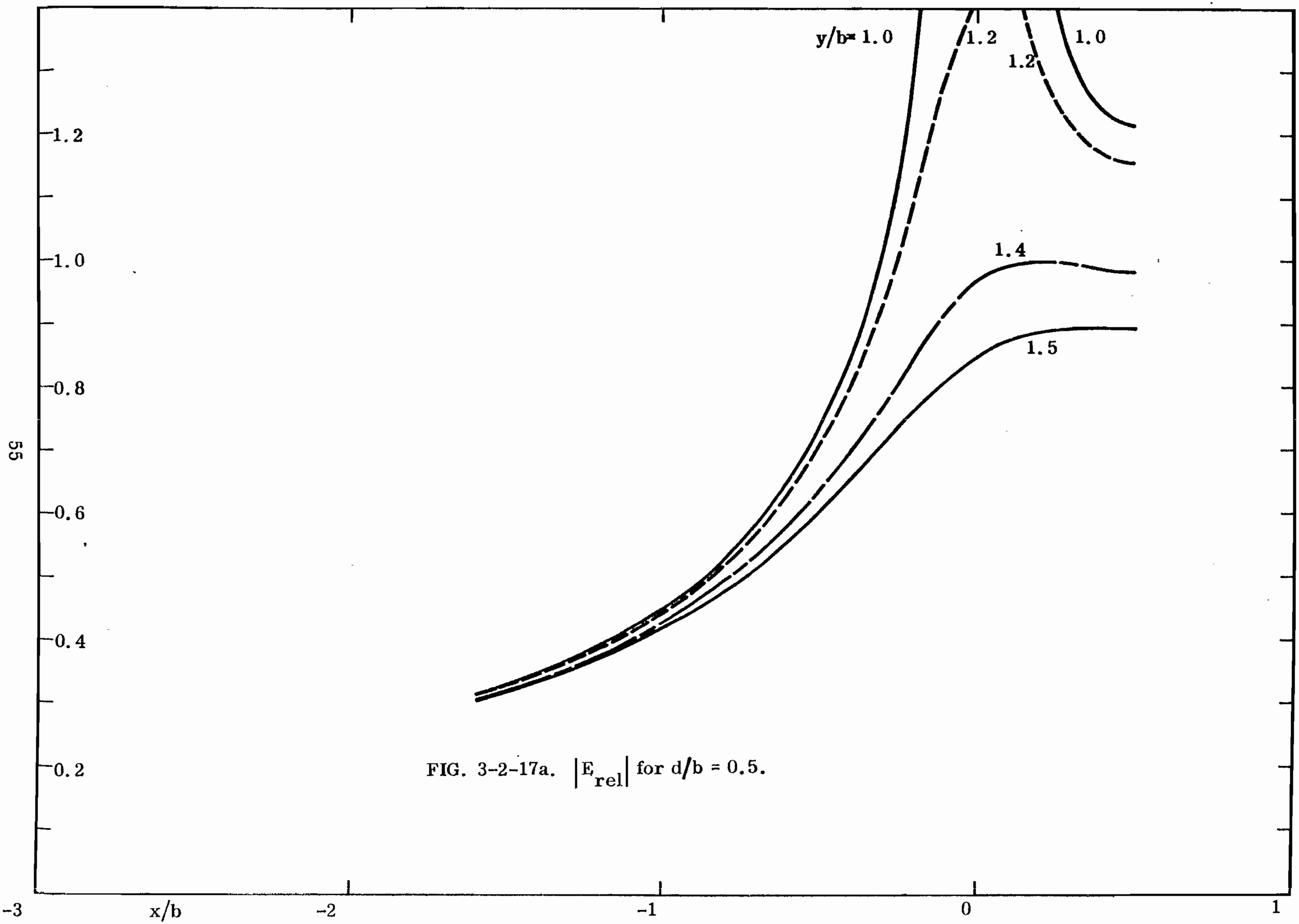

FIG. 3-2-17a. $|E_{rel}|$ for $d/b = 0.5$.

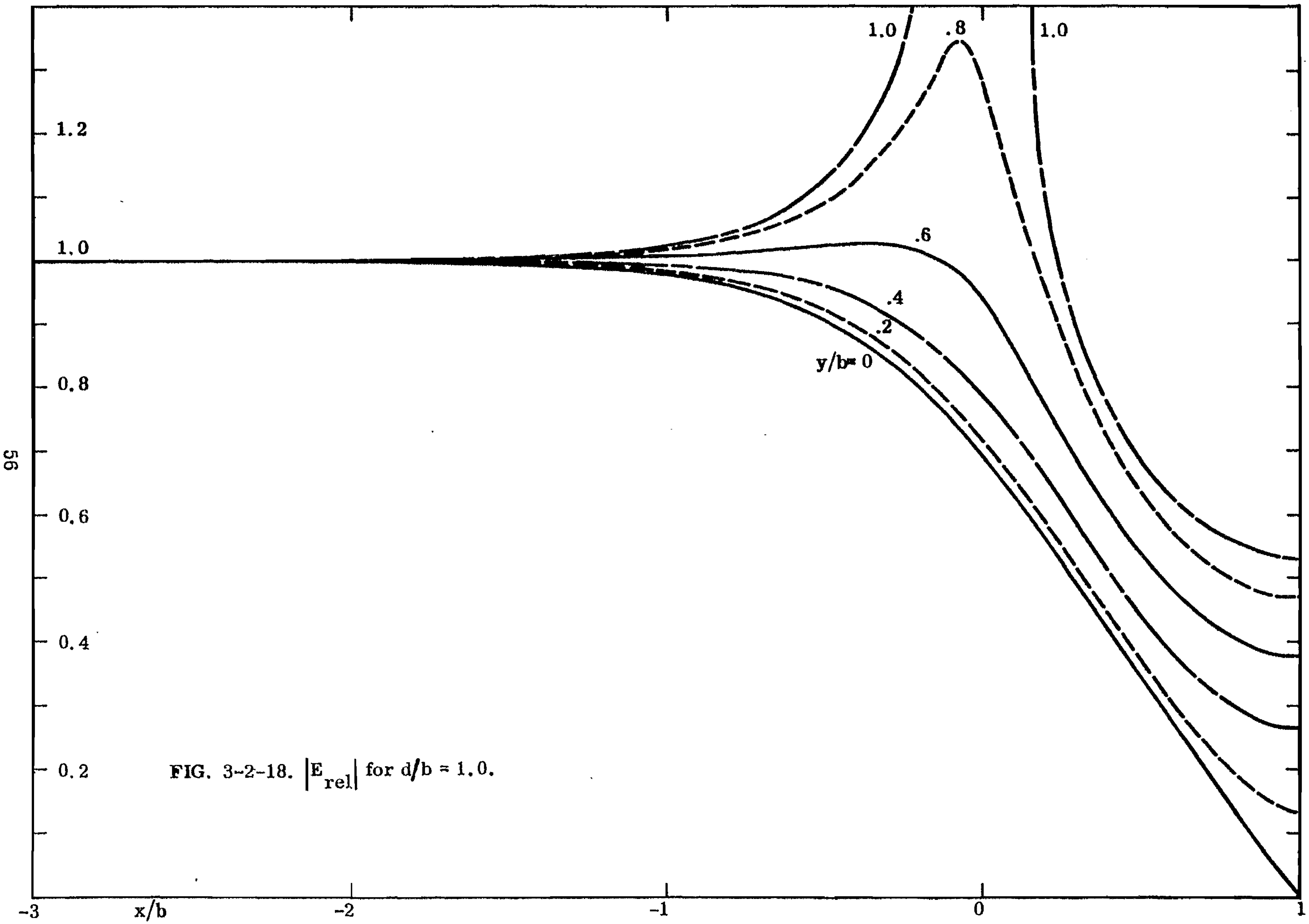

FIG. 3-2-18. $|E_{rel}|$ for $d/b = 1.0$.

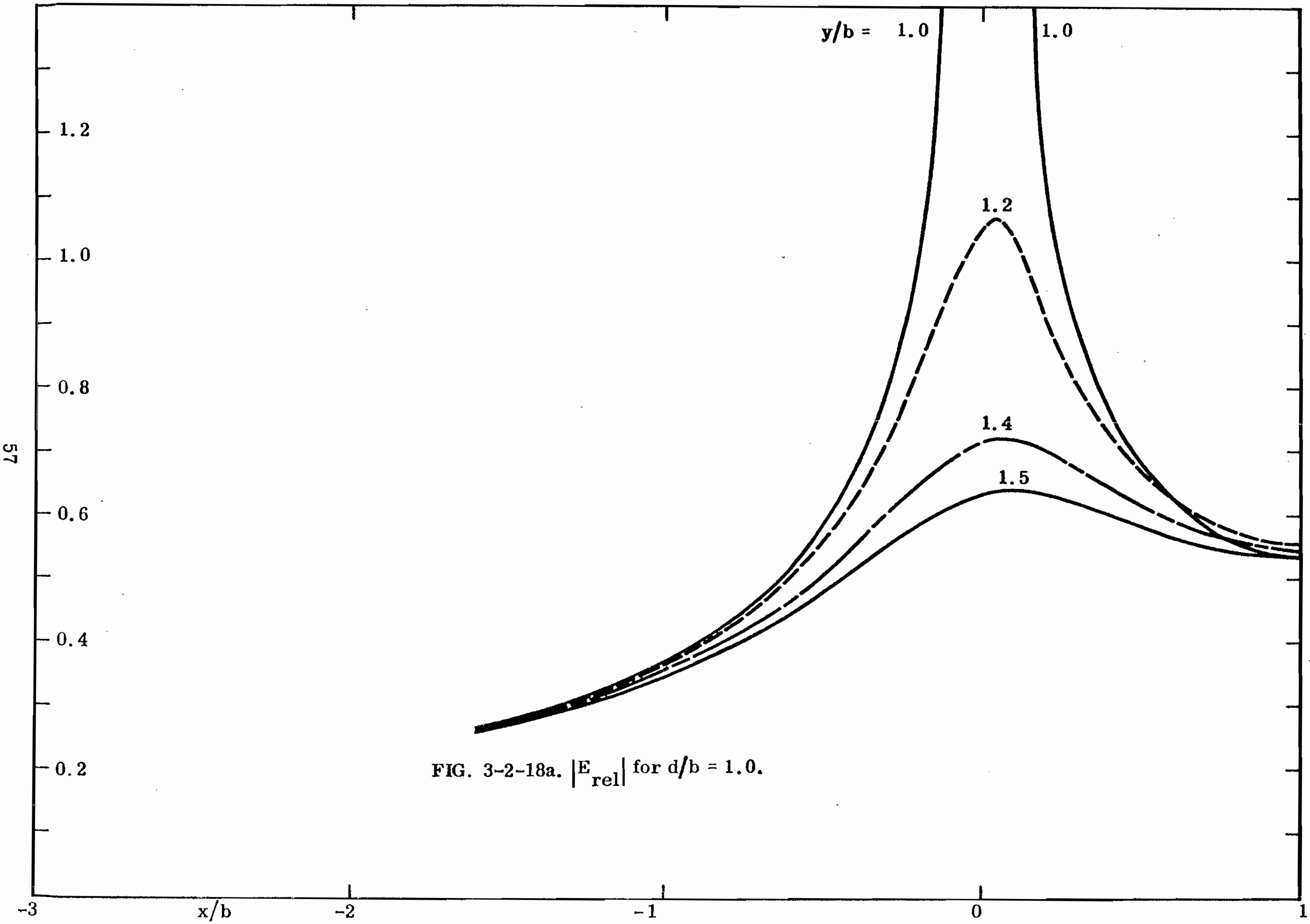

FIG. 3-2-18a. $|E_{rel}|$ for d/b = 1.0.

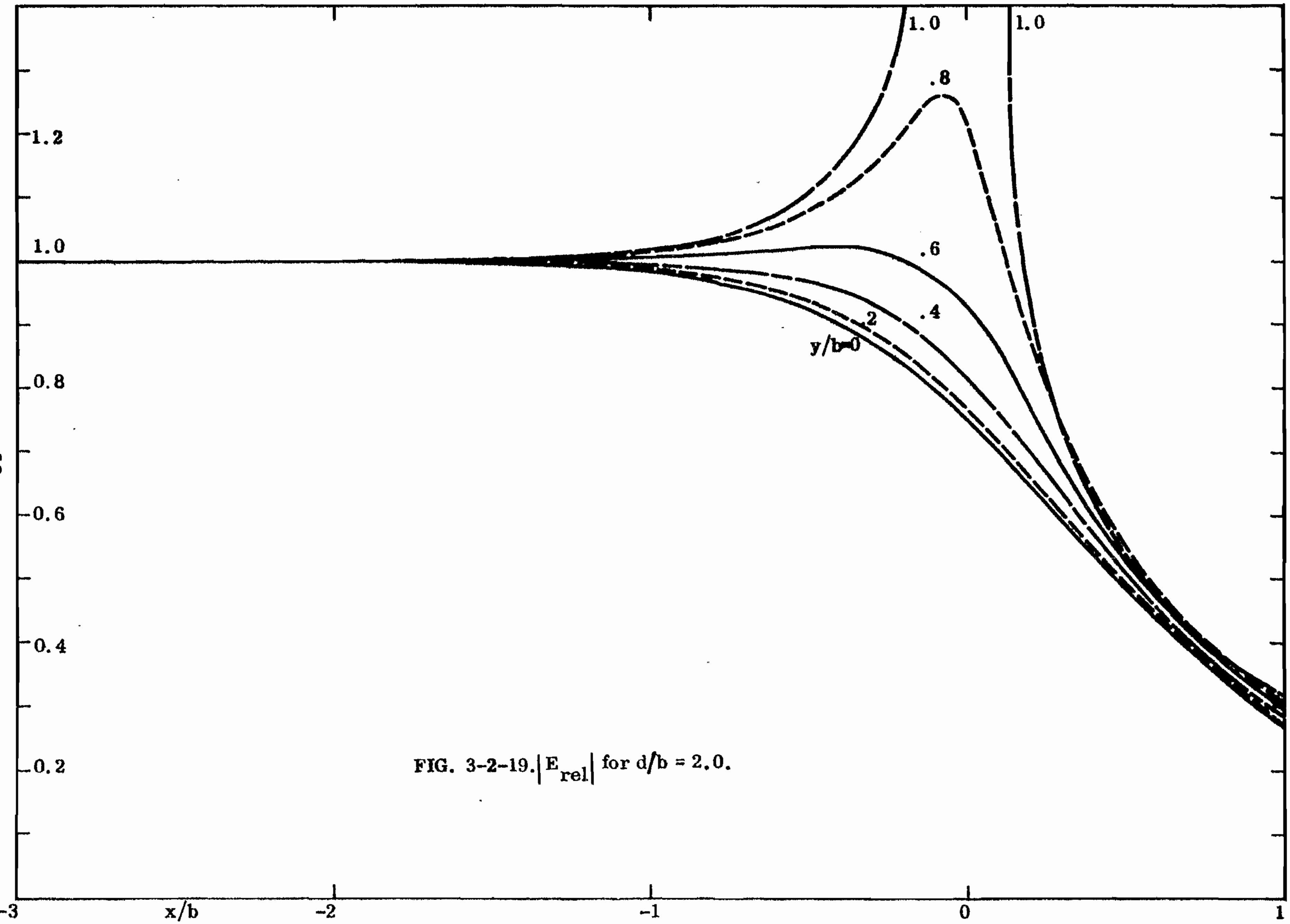

FIG. 3-2-19. $|E_{rel}|$ for $d/b = 2.0$.

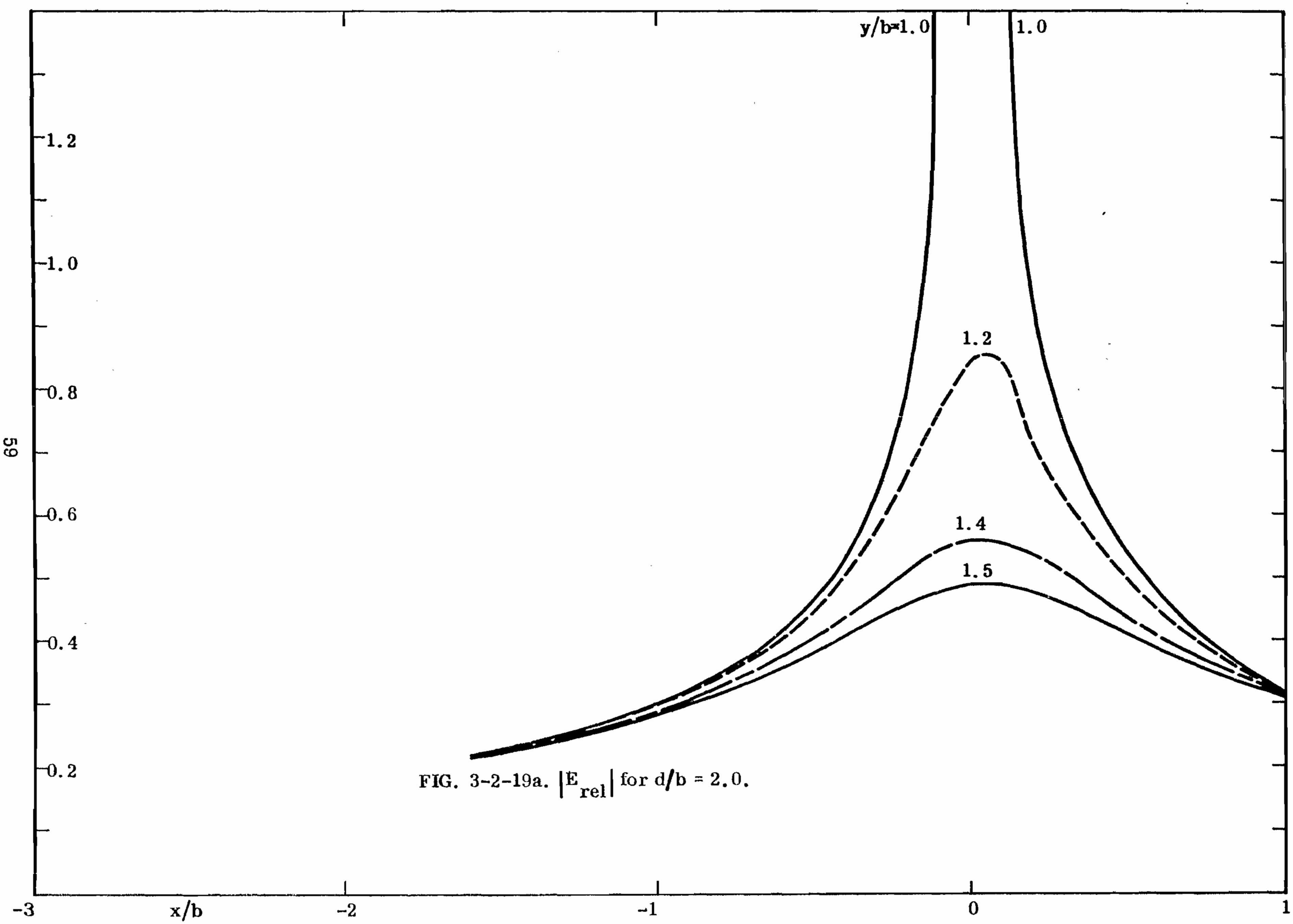

FIG. 3-2-19a. $|E_{rel}|$ for $d/b = 2.0$.

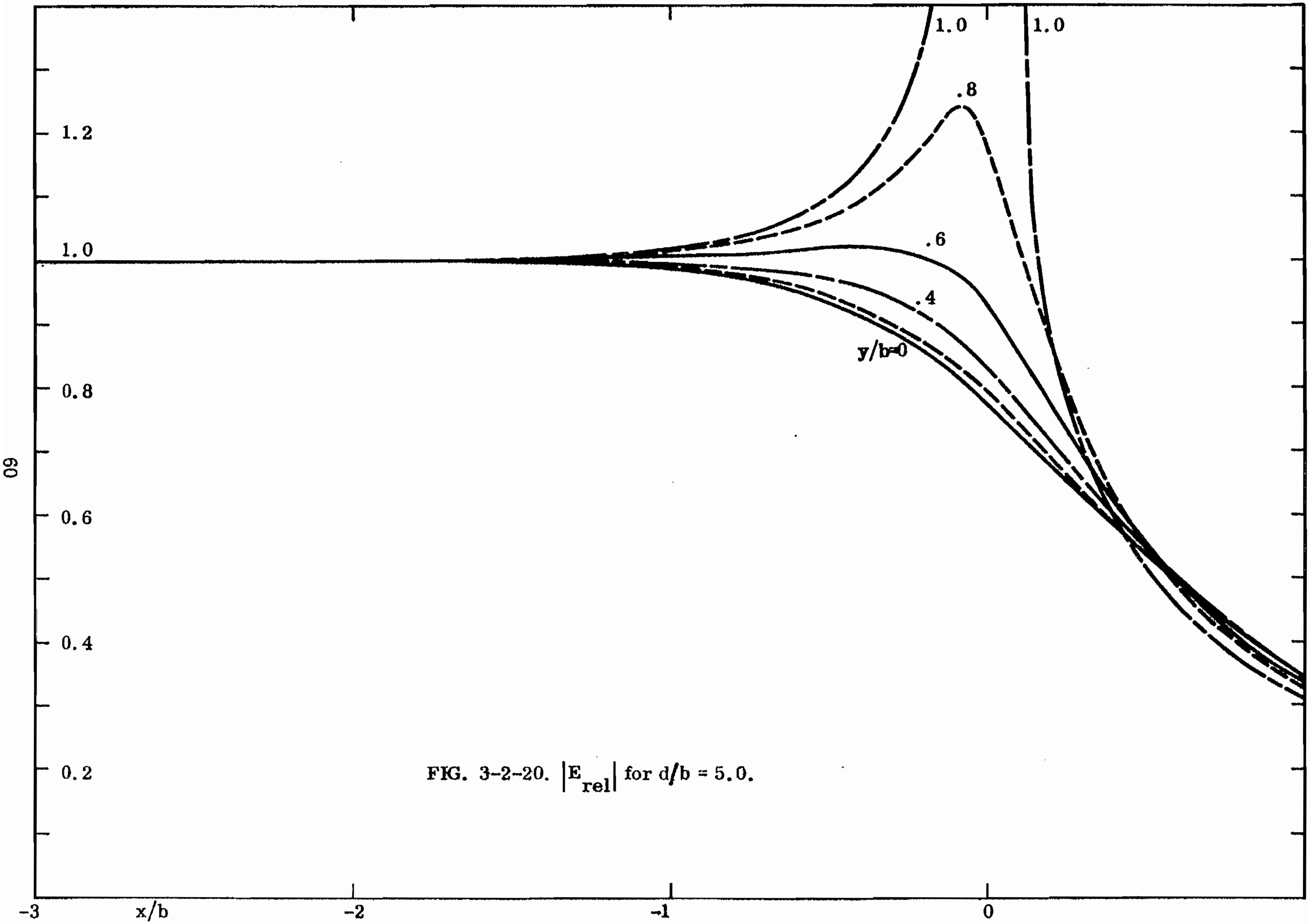

FIG. 3-2-20. $|E_{rel}|$ for $d/b = 5.0$.

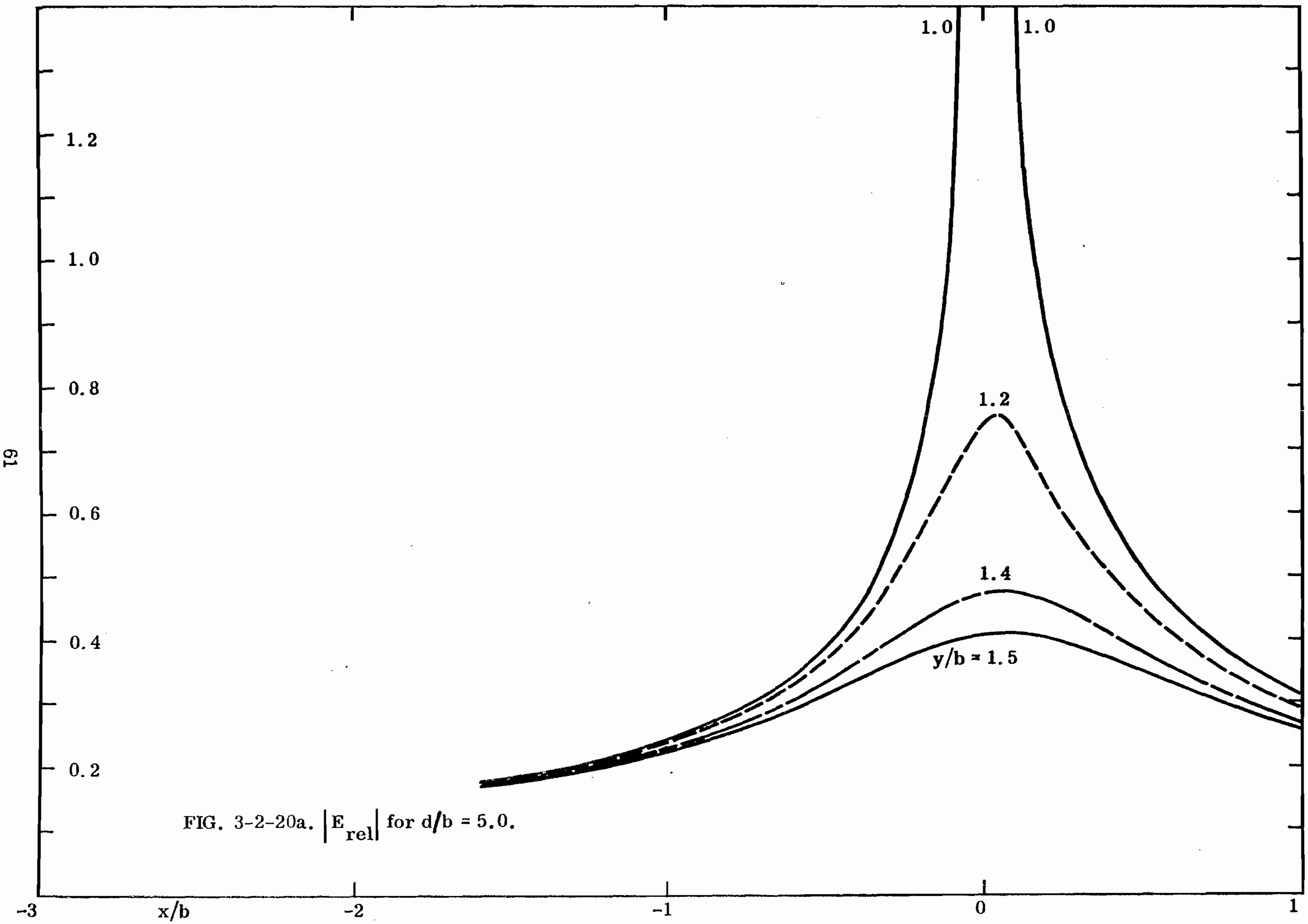

FIG. 3-2-20a. $|E_{rel}|$ for d/b = 5.0.

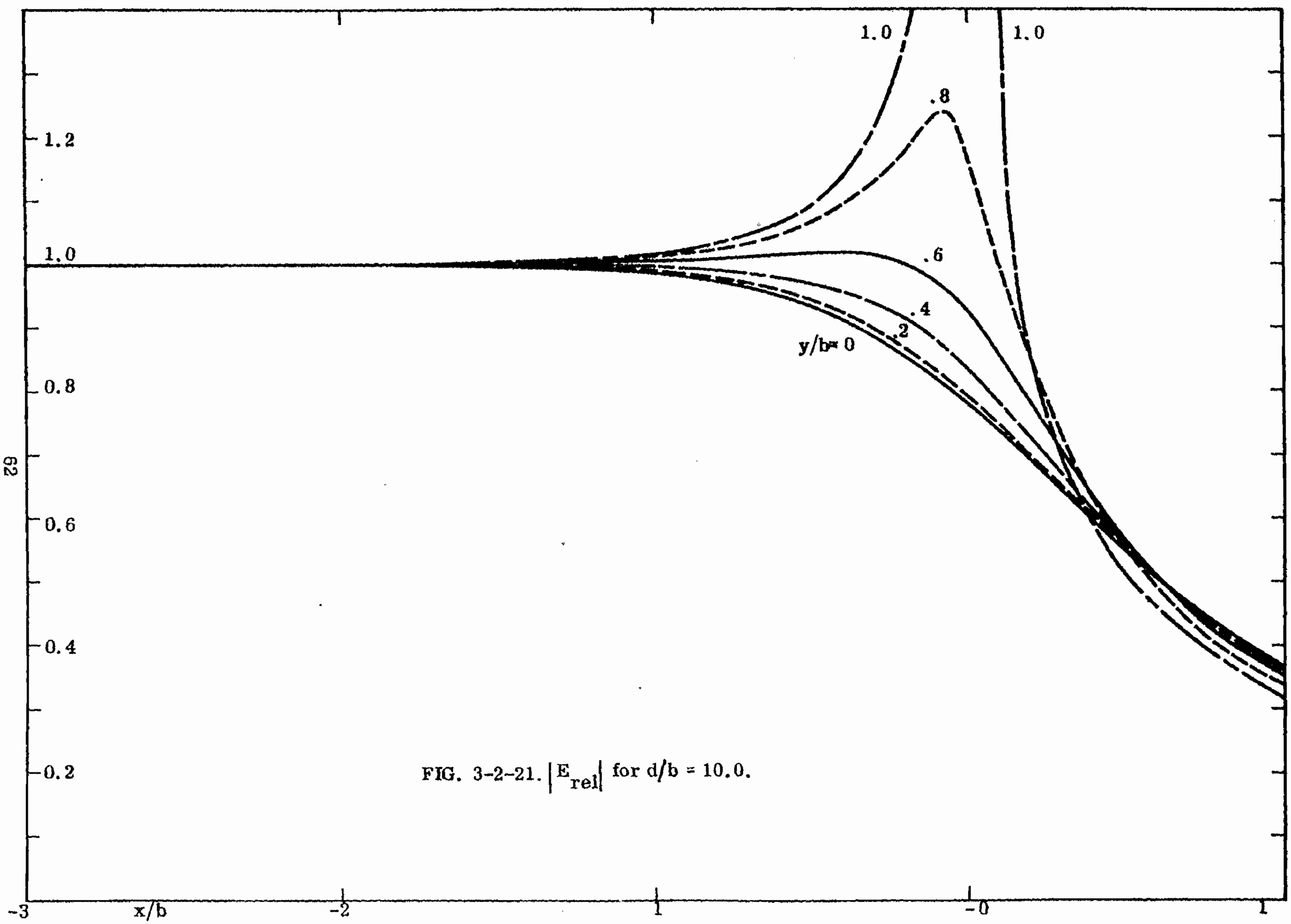

FIG. 3-2-21. $|E_{rel}|$ for $d/b = 10.0$.

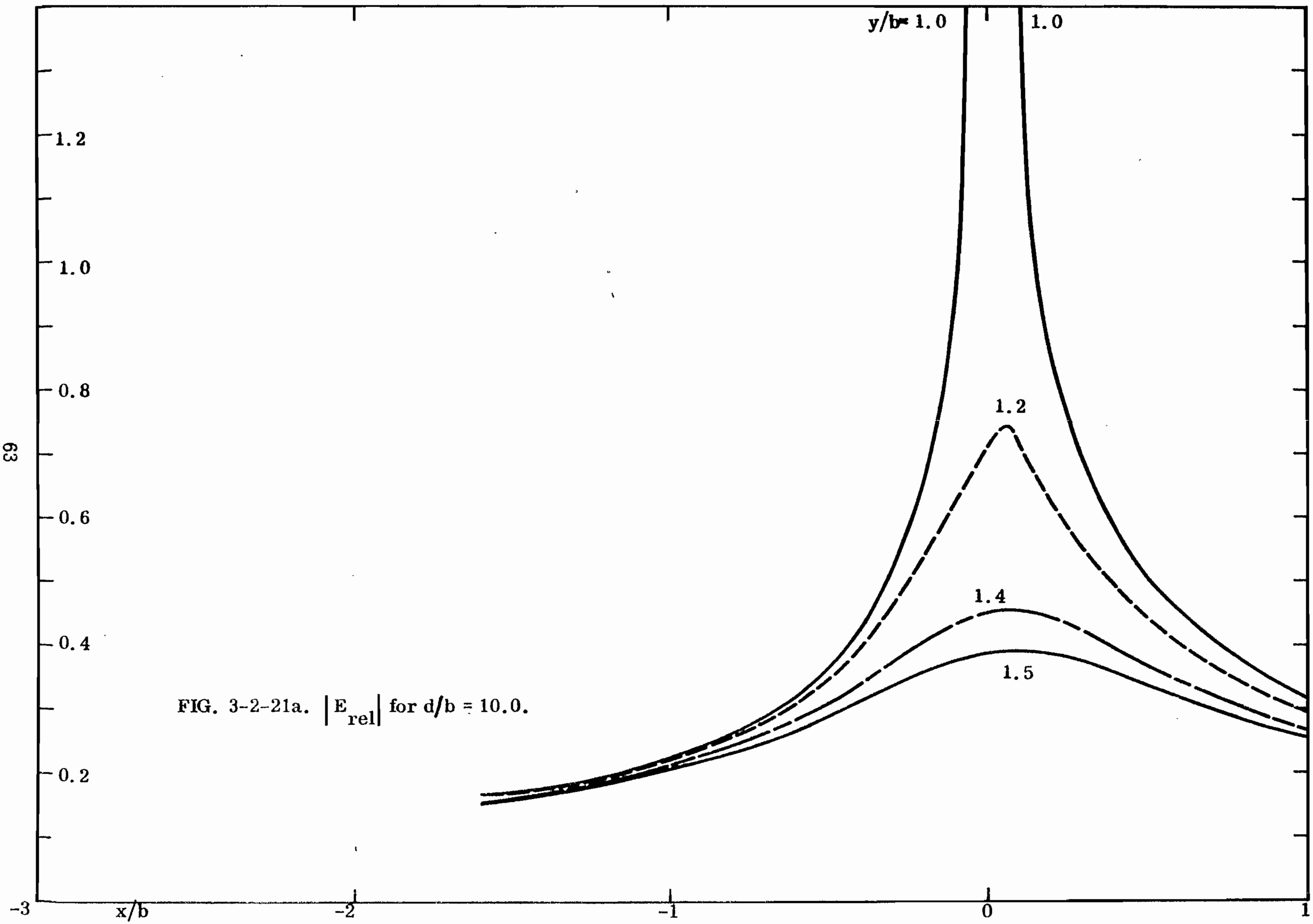

FIG. 3-2-21a. $|E_{rel}|$ for $d/b = 10.0$.

The graphs for the relative electric field intensities of a semi-infinite parallel plate transmission line show that, as a whole, the effect of the presence of a perfectly conducting ground on the field intensity is significant, as far as in the region between the upper plate and the center plane is concerned, only when the separation of the ground is less than one-half the separation distance of the parallel plates (i.e. $d/b < 1.0$).

So far we have shown the variations of the relative electric field intensities for various ground proximities as a function of x/b with y/b fixed. In Figs. 3-2-22 through 3-2-30, we present the variations of relative electric field intensities as a function of y/b with x/b as a fixed parameter. The incremental steps of x/b=0.5 are taken and only the cases of d/b=0.1, 0.2 and 0.5 are considered in view of the observation in the preceding paragraph.

TABLE III-5: Relative Electric Field Intensities Along x/b Plates

Field Component	Parameters x/b	d/b	Figure Nos.
$\lvert E_{y\,rel}\rvert$	$-1.5 \leq x/b \leq 0.1$	0.1	3-2-22
	$-1.5 \leq x/b \leq 0.2$	0.2	3-2-23
	$-1.5 \leq x/b \leq 0.5$	0.5	3-2-24
$\lvert E_{x\,rel}\rvert$	$-1.5 \leq x/b \leq 0.1$	0.1	3-2-25
	$-1.5 \leq x/b \leq 0.2$	0.2	3-2-26
	$-1.5 \leq x/b \leq 0.5$	0.5	3-2-27
$\lvert E_{rel}\rvert$	$-1.5 \leq x/b \leq 0.1$	0.1	3-2-28
	$-1.5 \leq x/b \leq 0.2$	0.2	3-2-29
	$-1.5 \leq x/b \leq 0.5$	0.5	3-2-30

For completeness and reference, the electric field intensities of a semi-infinite parallel plate transmission line without ground are given in Figs. 3-2-31 through 3-2-33-a.

TABLE III-6: Relative Electric Field Intensities Without Ground

Field Component	Parameters y/b	Figure Nos.
$\lvert E_{y\,rel}\rvert$	$0 \leq y/b \leq 1.0$	3-2-31
	$1.0 < y/b \leq 1.5$	3-2-31-a
$\lvert E_{x\,rel}\rvert$	$0 \leq y/b \leq 1.0$	3-2-32
	$1.0 < y/b \leq 1.5$	3-2-32-a
$\lvert E_{rel}\rvert$	$0 \leq y/b \leq 1.0$	3-2-33
	$1.0 < y/b \leq 1.5$	3-2-33-a

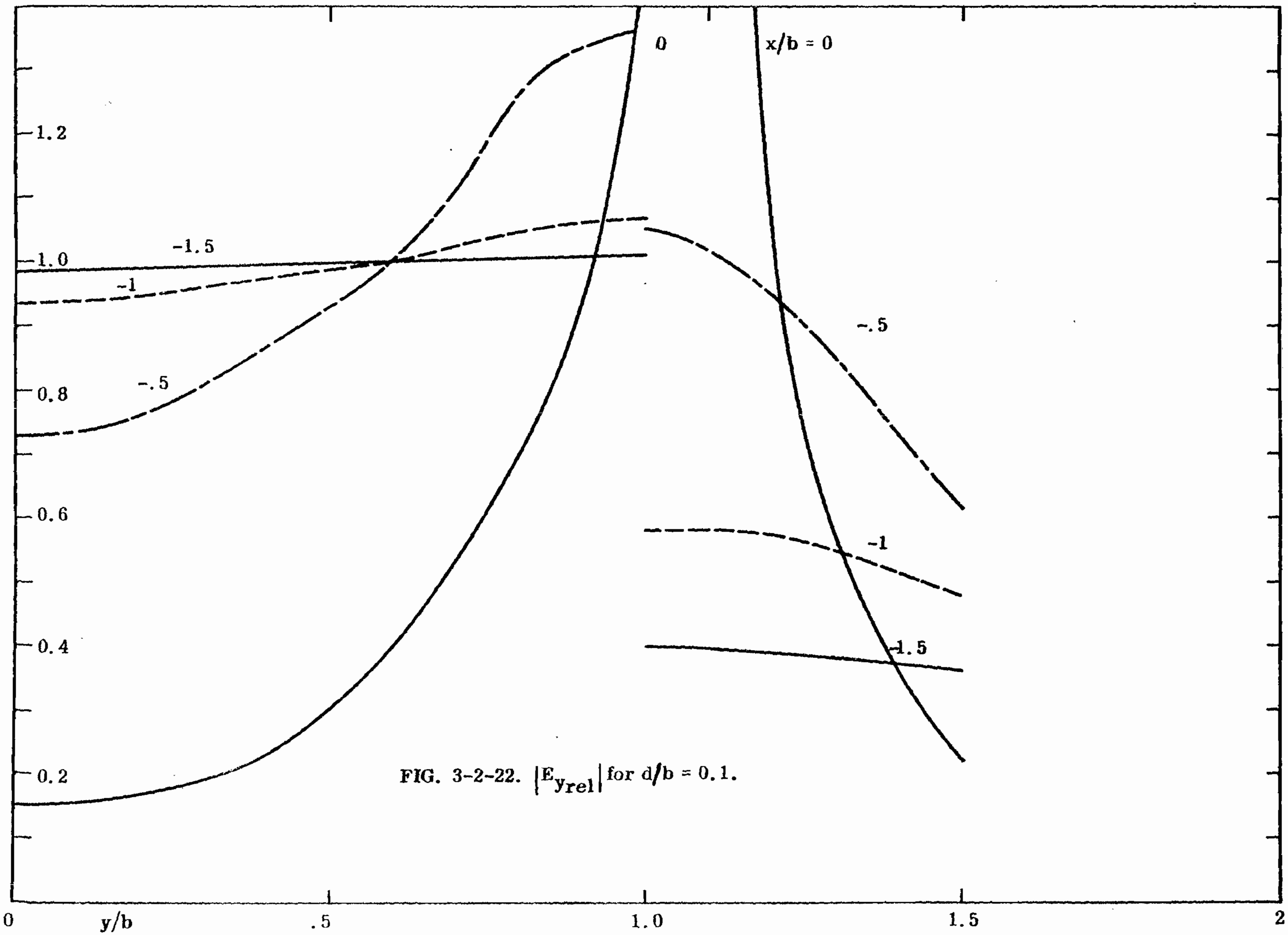

FIG. 3-2-22. $|E_{y_{rel}}|$ for d/b = 0.1.

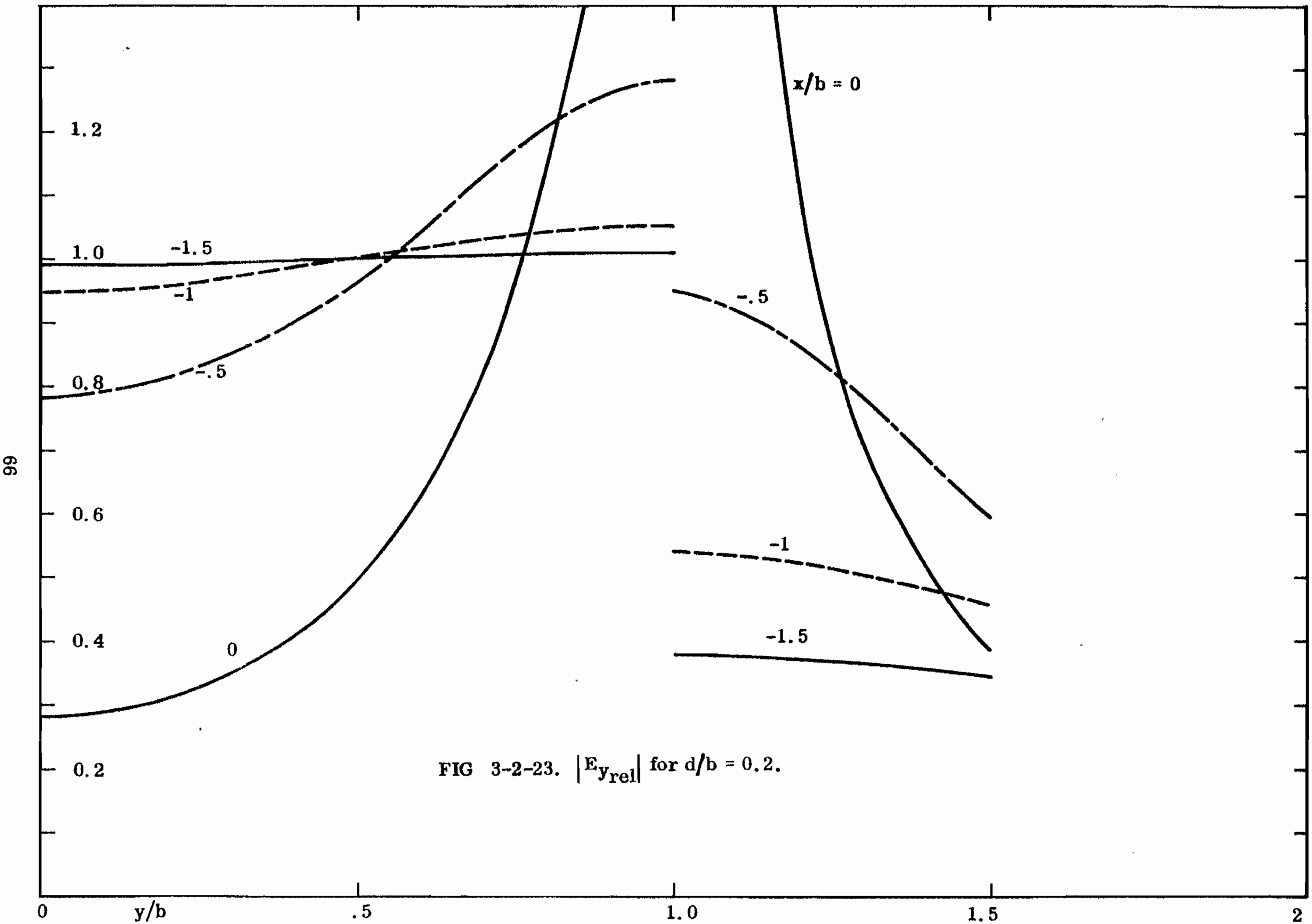

FIG 3-2-23. $|E_{y_{rel}}|$ for $d/b = 0.2$.

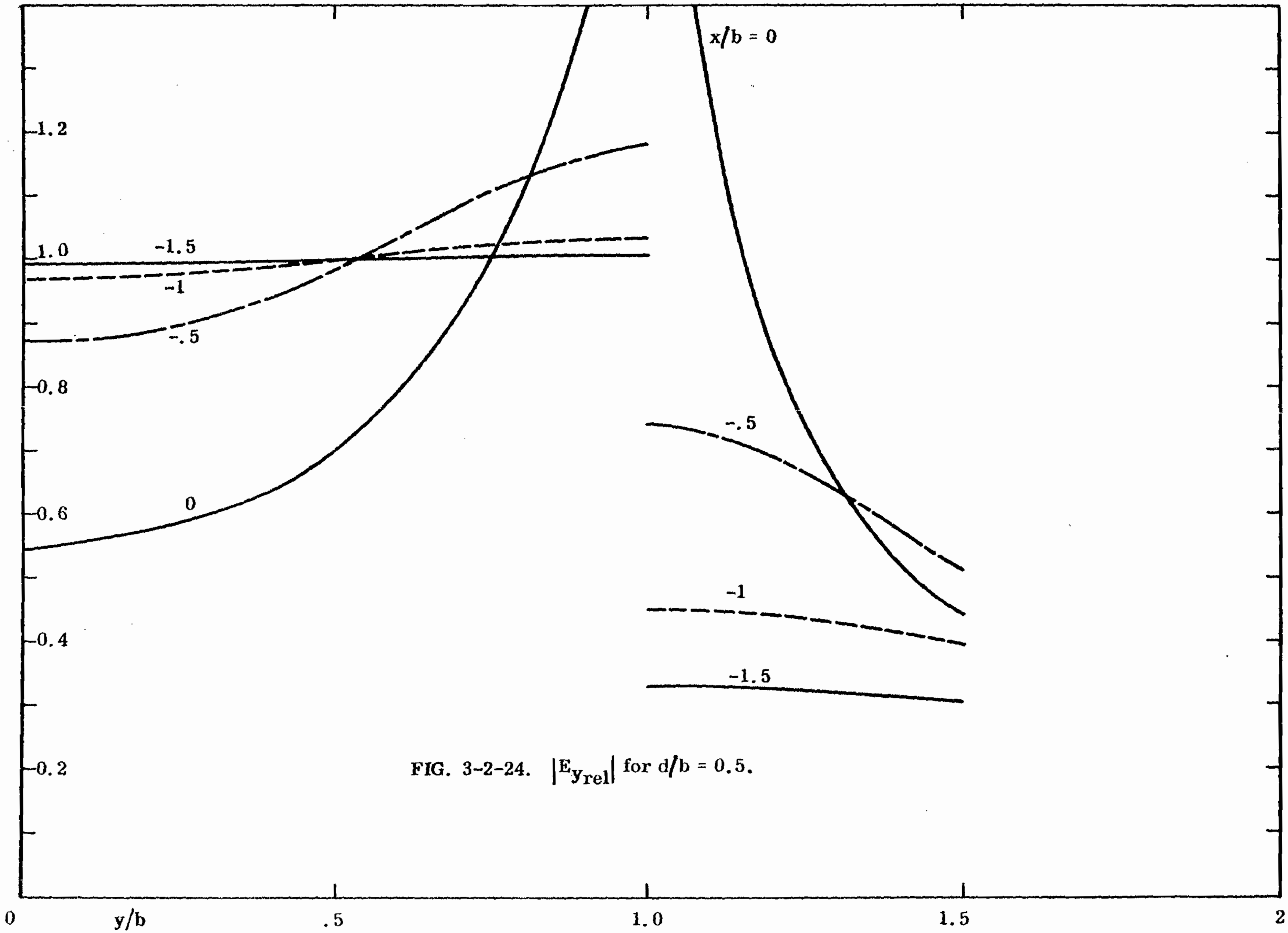

FIG. 3-2-24. $|E_{y_{rel}}|$ for d/b = 0.5.

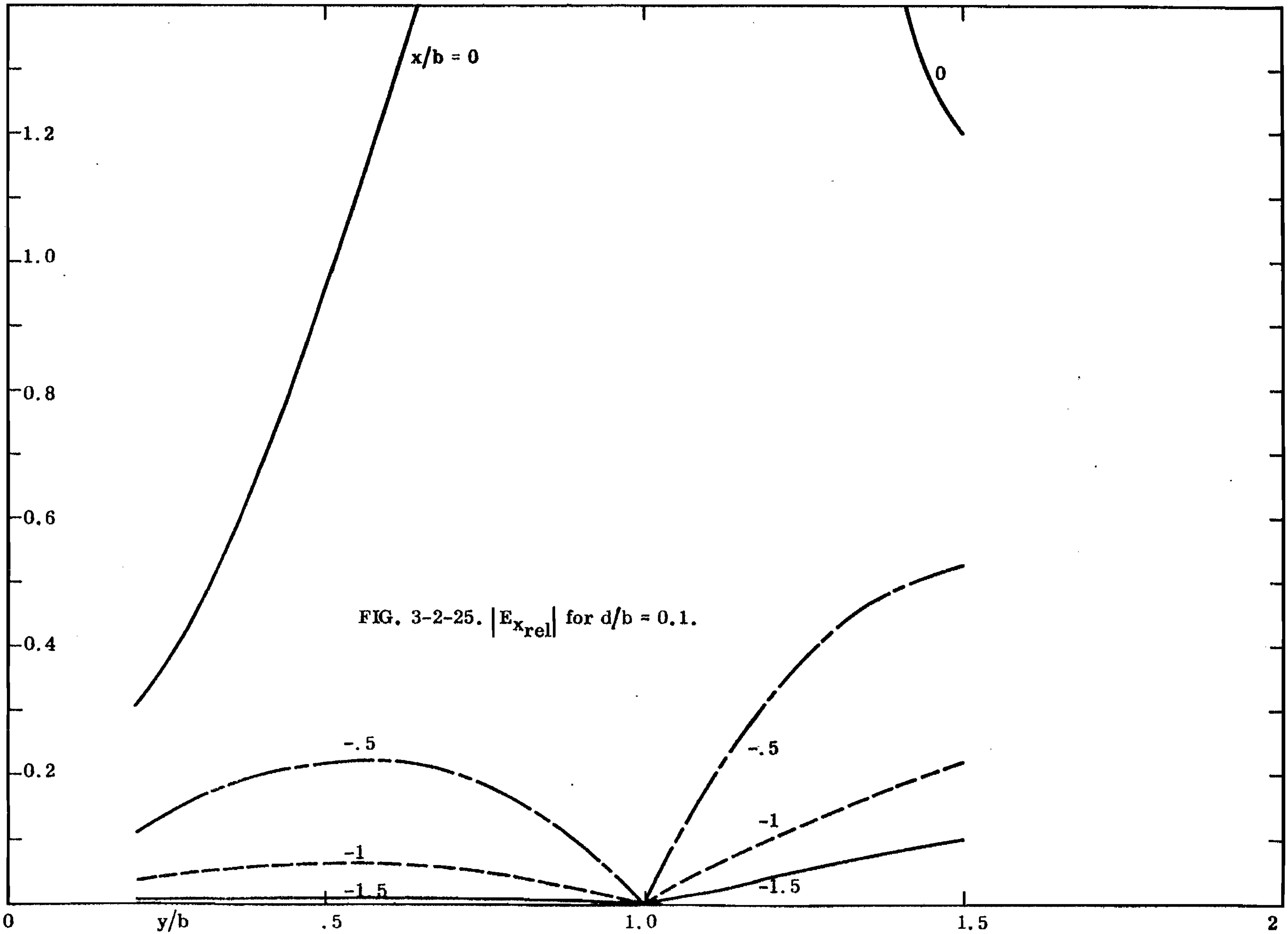

FIG. 3-2-25. $|E_{x_{rel}}|$ for $d/b = 0.1$.

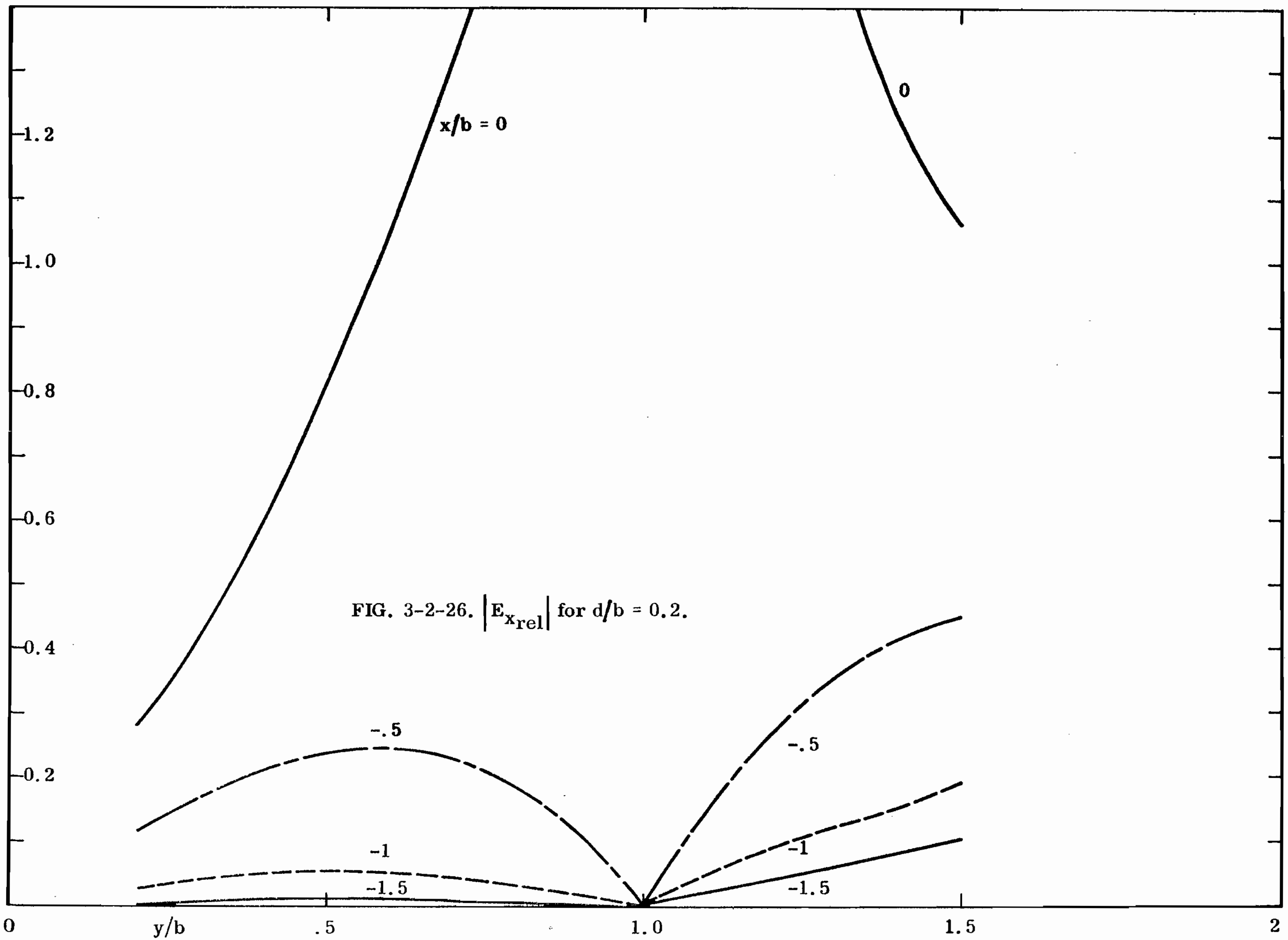

FIG. 3-2-26. $\left|E_{x_{rel}}\right|$ for d/b = 0.2.

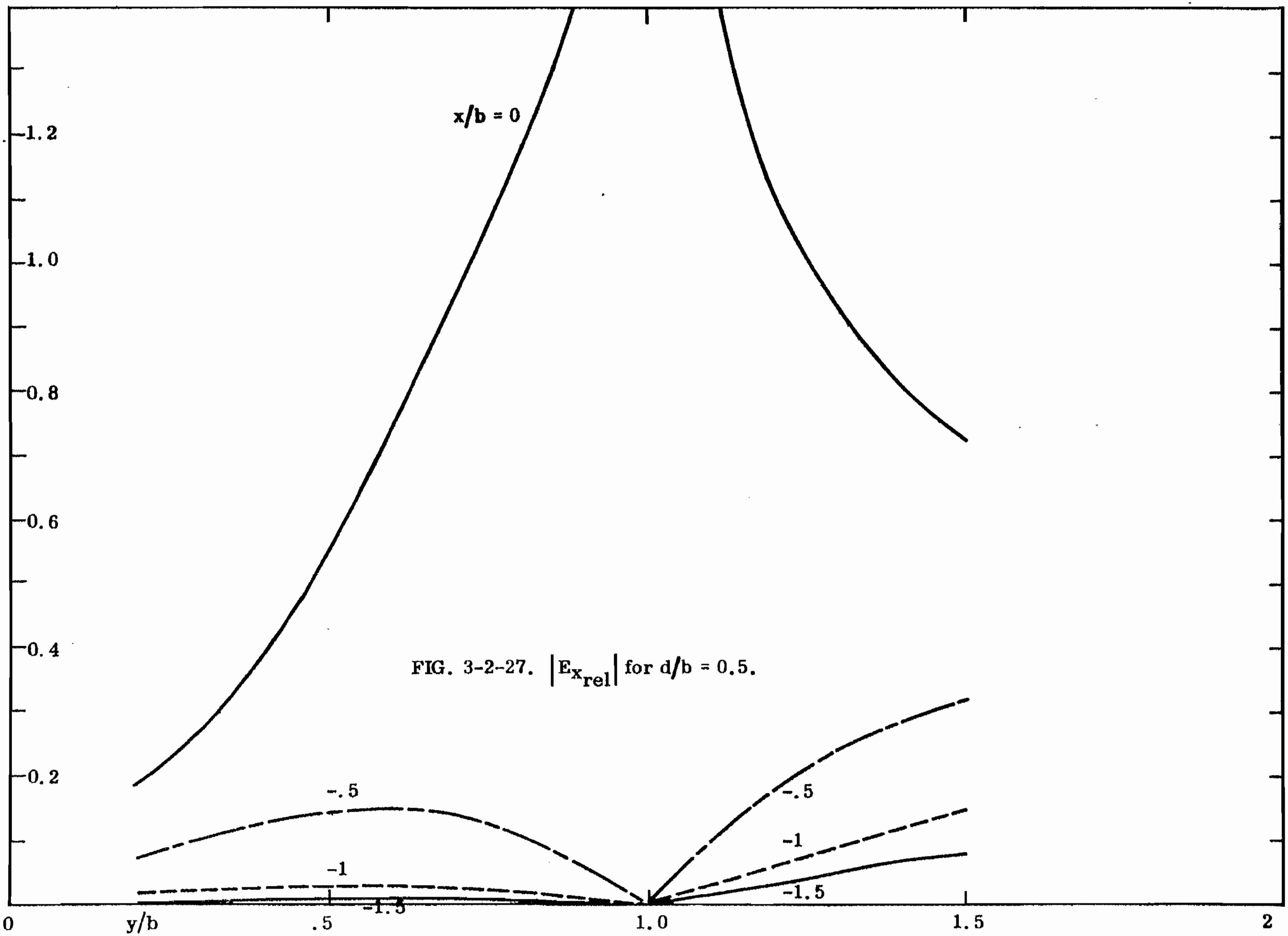

FIG. 3-2-27. $\left|E_{x_{rel}}\right|$ for d/b = 0.5.

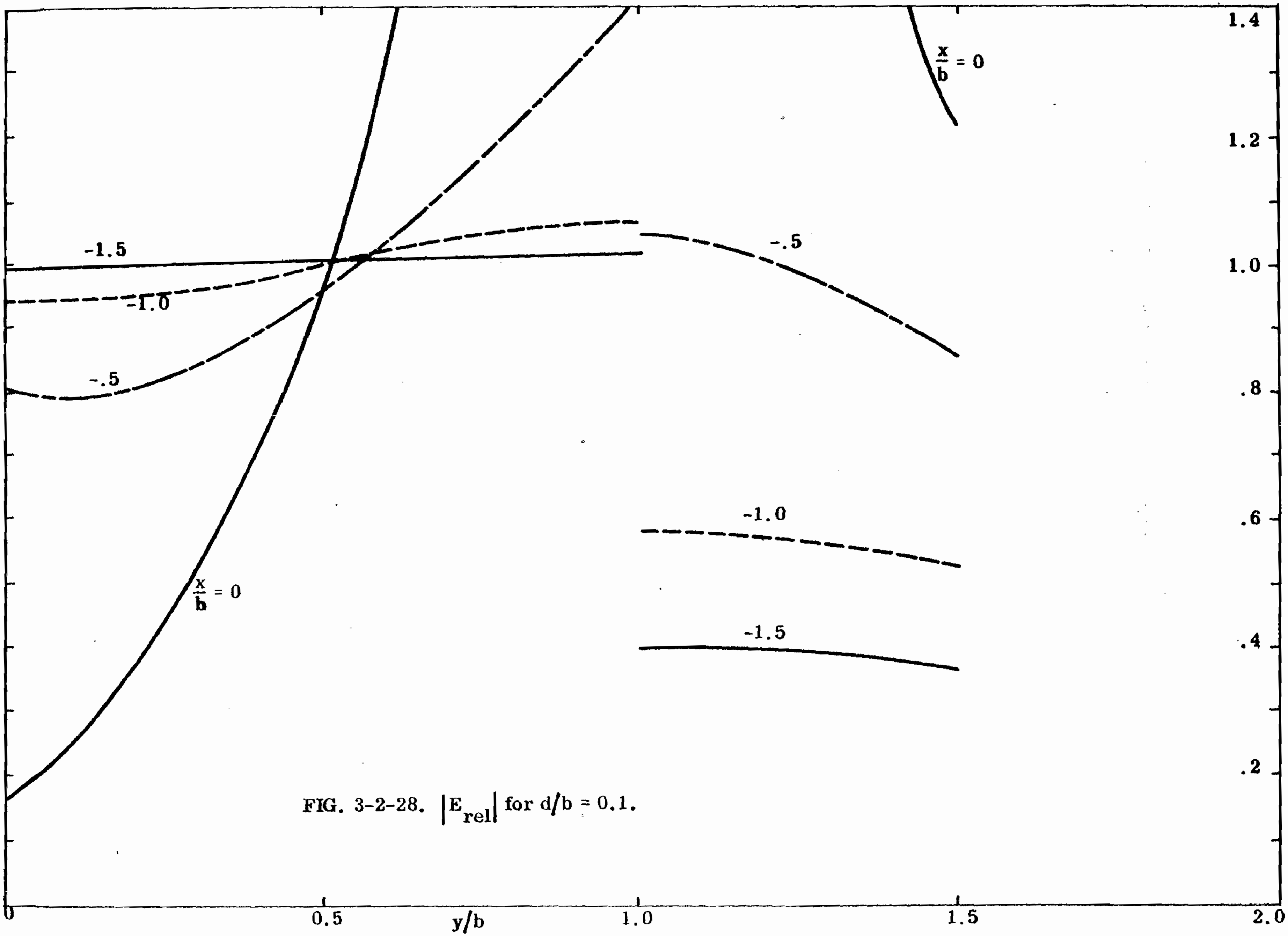

FIG. 3-2-28. $|E_{rel}|$ for $d/b = 0.1$.

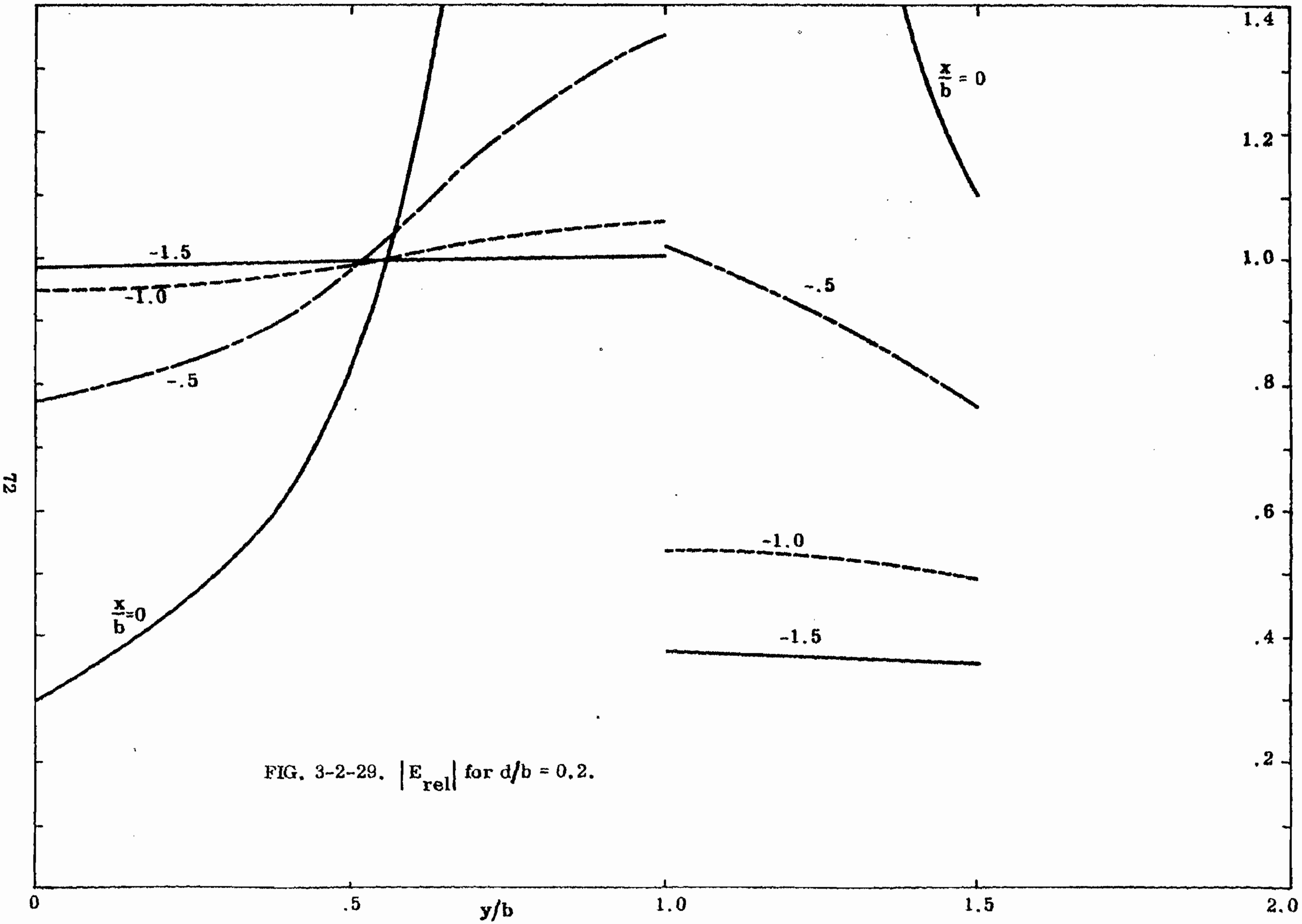

FIG. 3-2-29. $|E_{rel}|$ for $d/b = 0.2$.

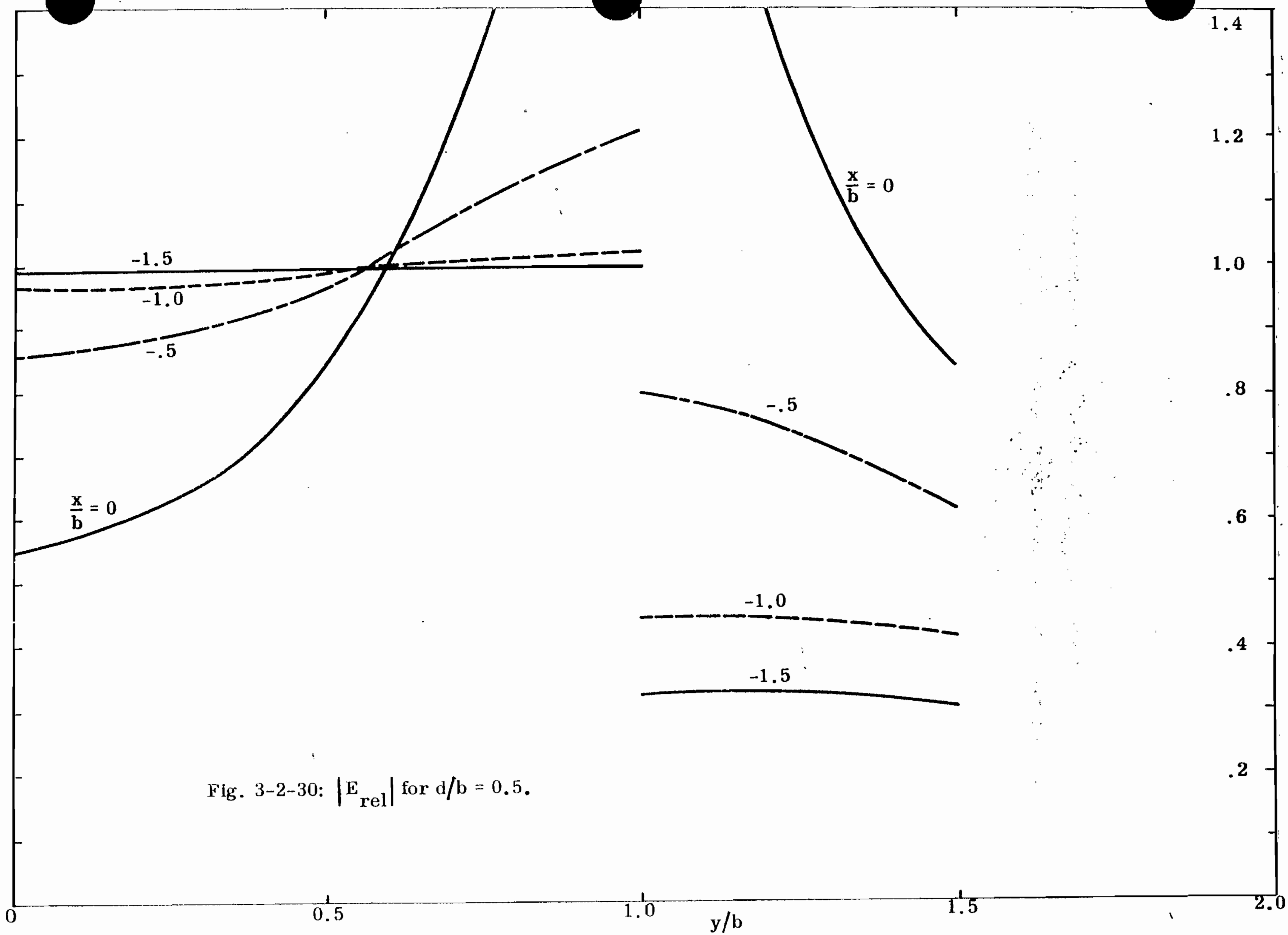

Fig. 3-2-30: $|E_{rel}|$ for d/b = 0.5.

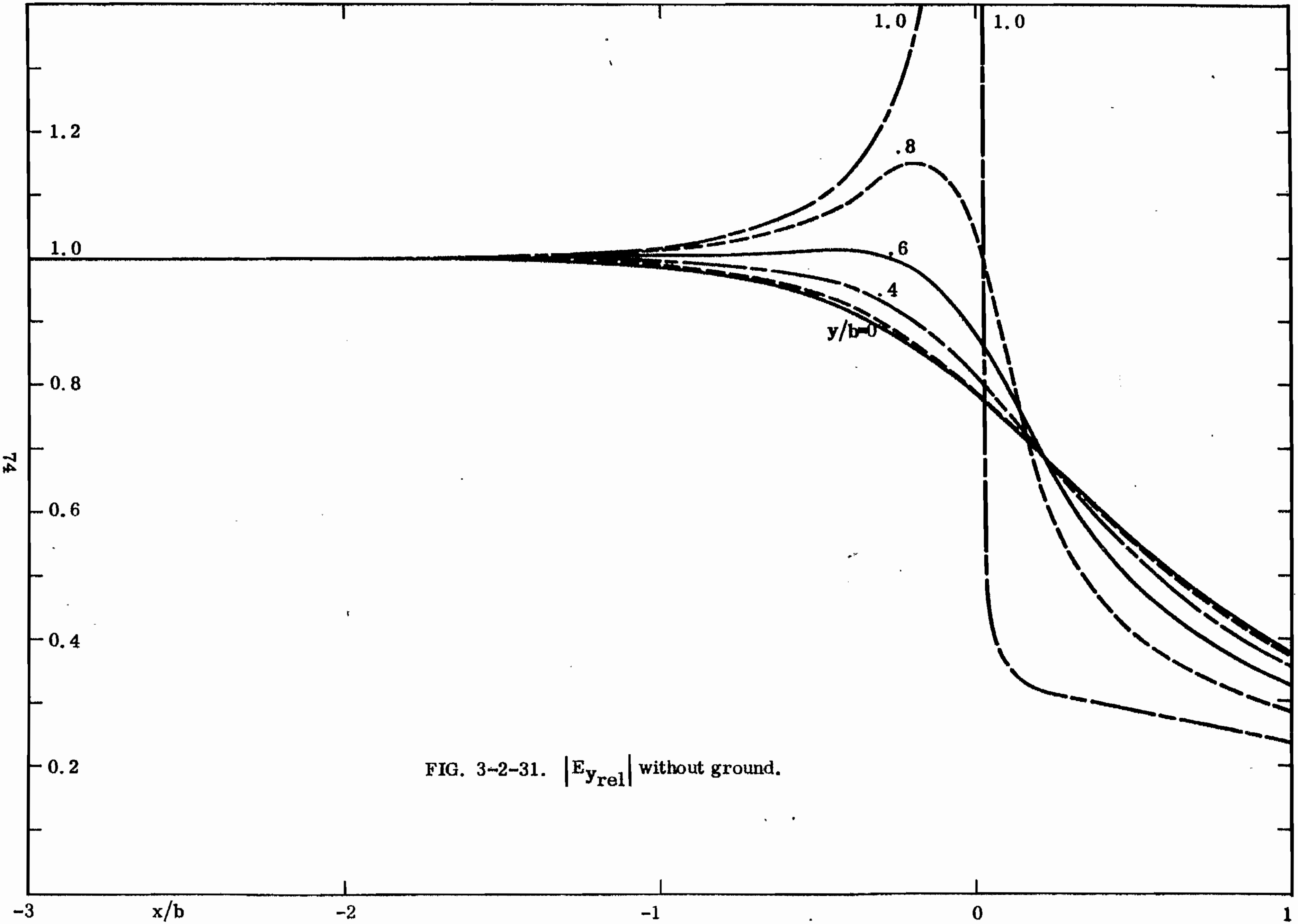

FIG. 3-2-31. $\left|E_{y_{rel}}\right|$ without ground.

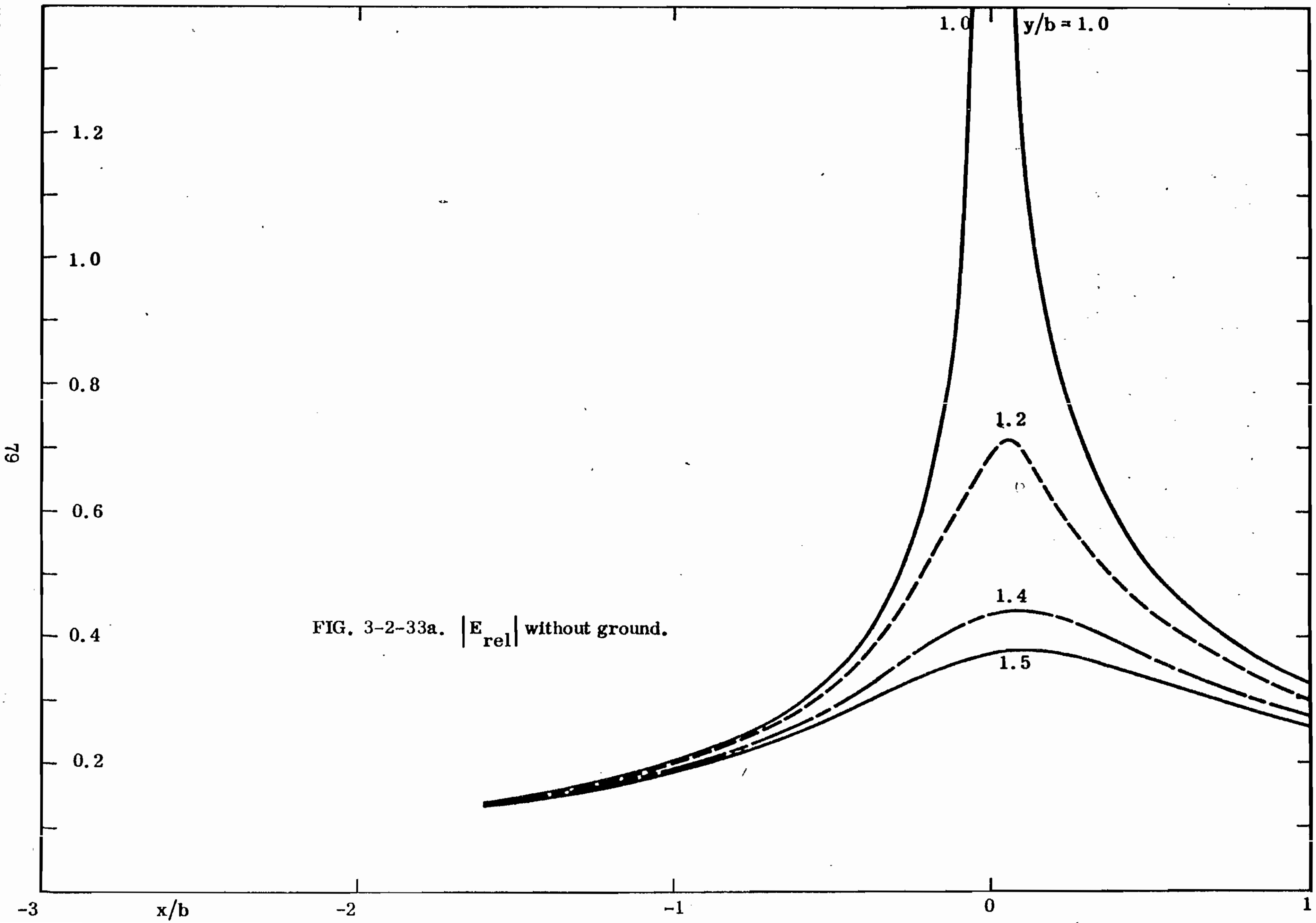

FIG. 3-2-33a. $|E_{rel}|$ without ground.

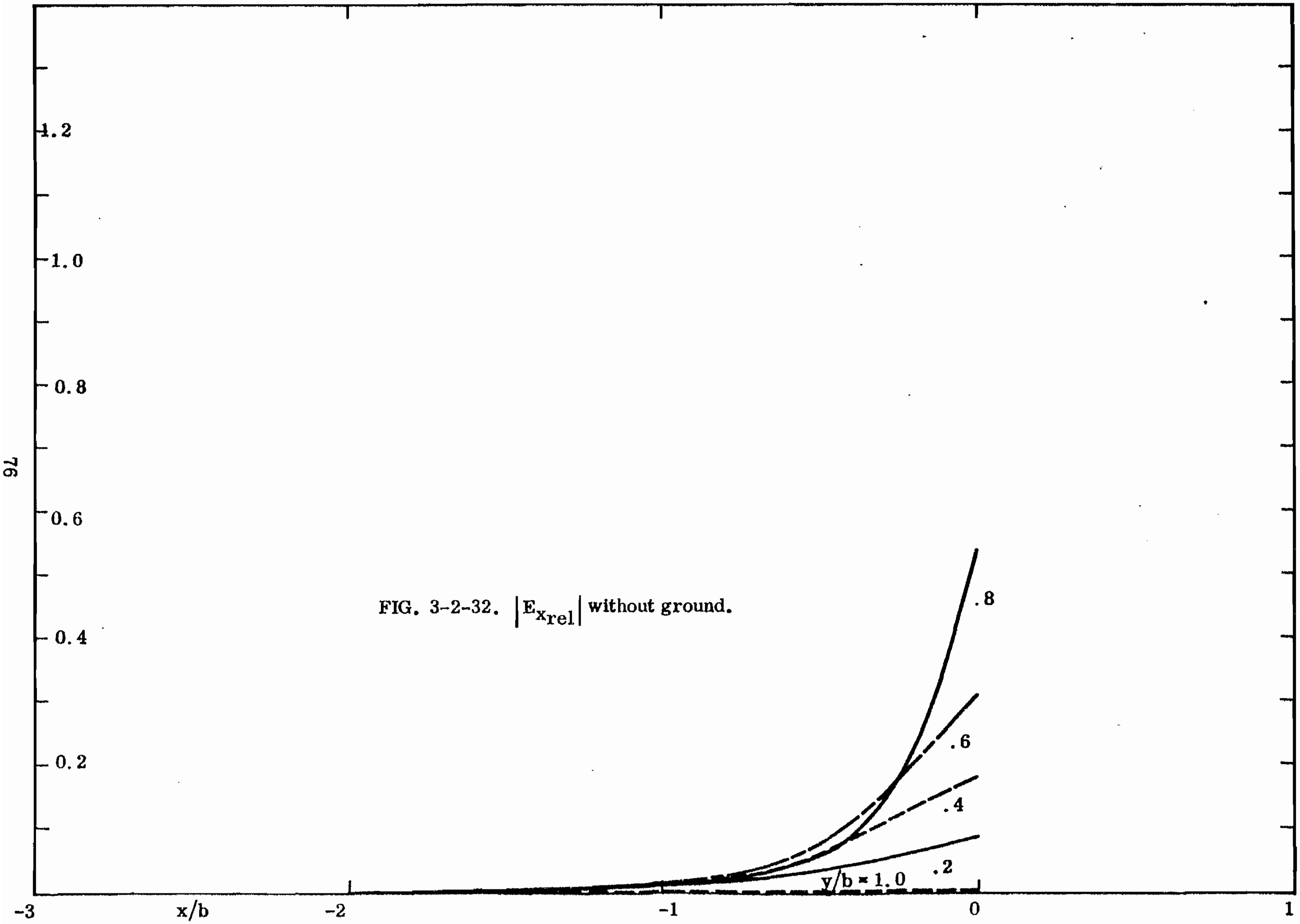

FIG. 3-2-32. $|E_{x_{rel}}|$ without ground.

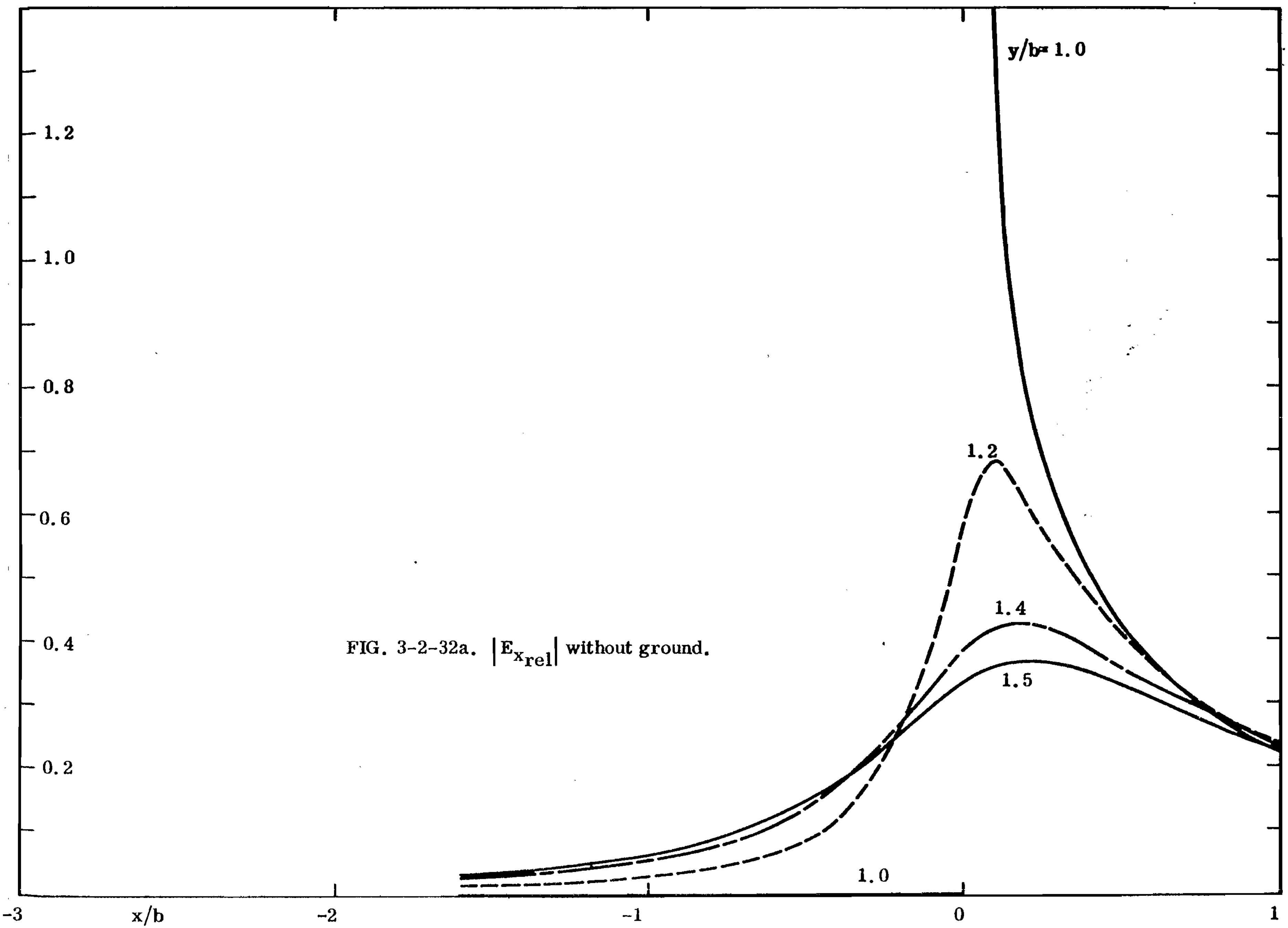

FIG. 3-2-32a. $|E_{x_{rel}}|$ without ground.

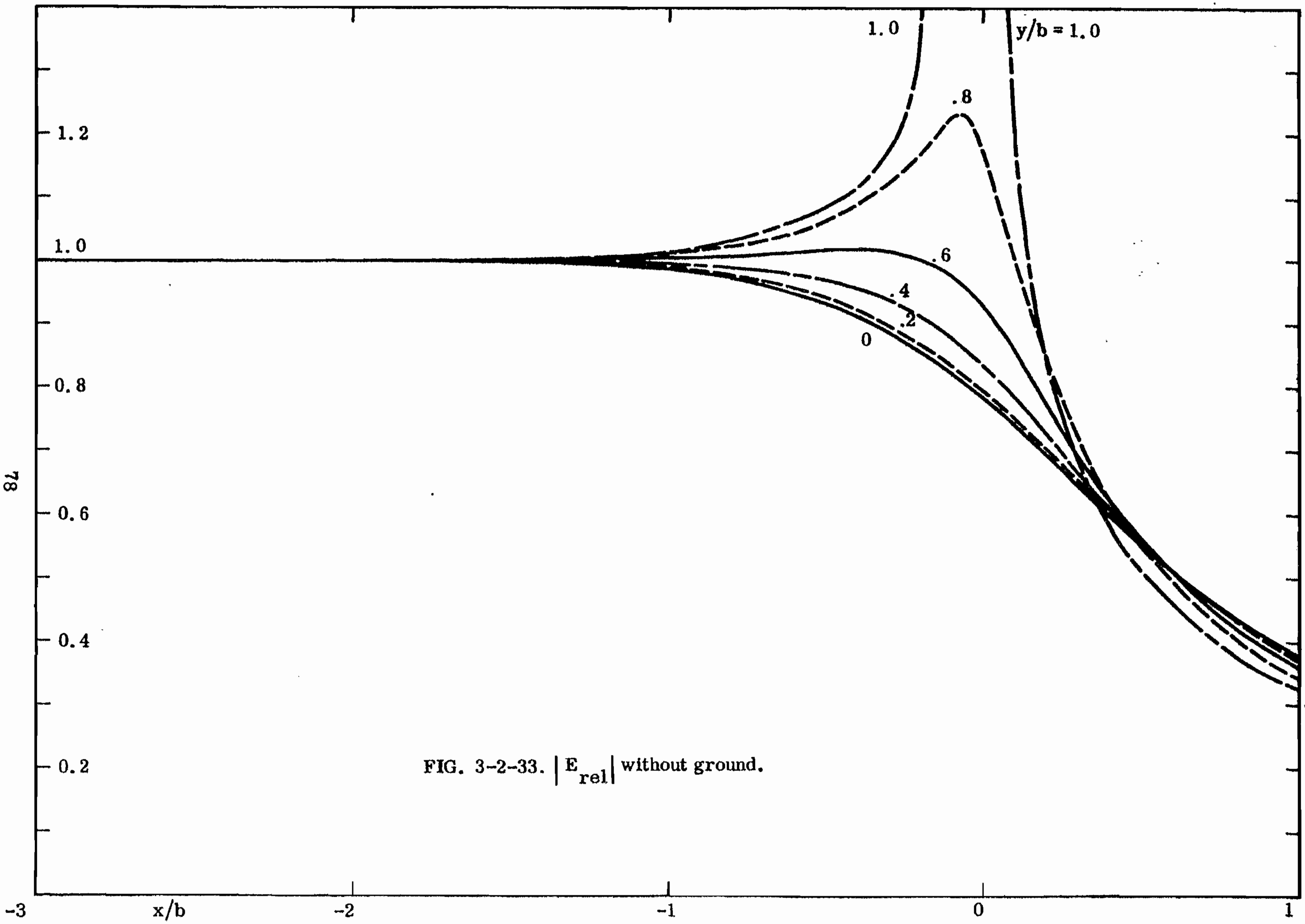

FIG. 3-2-33. $|E_{rel}|$ without ground.

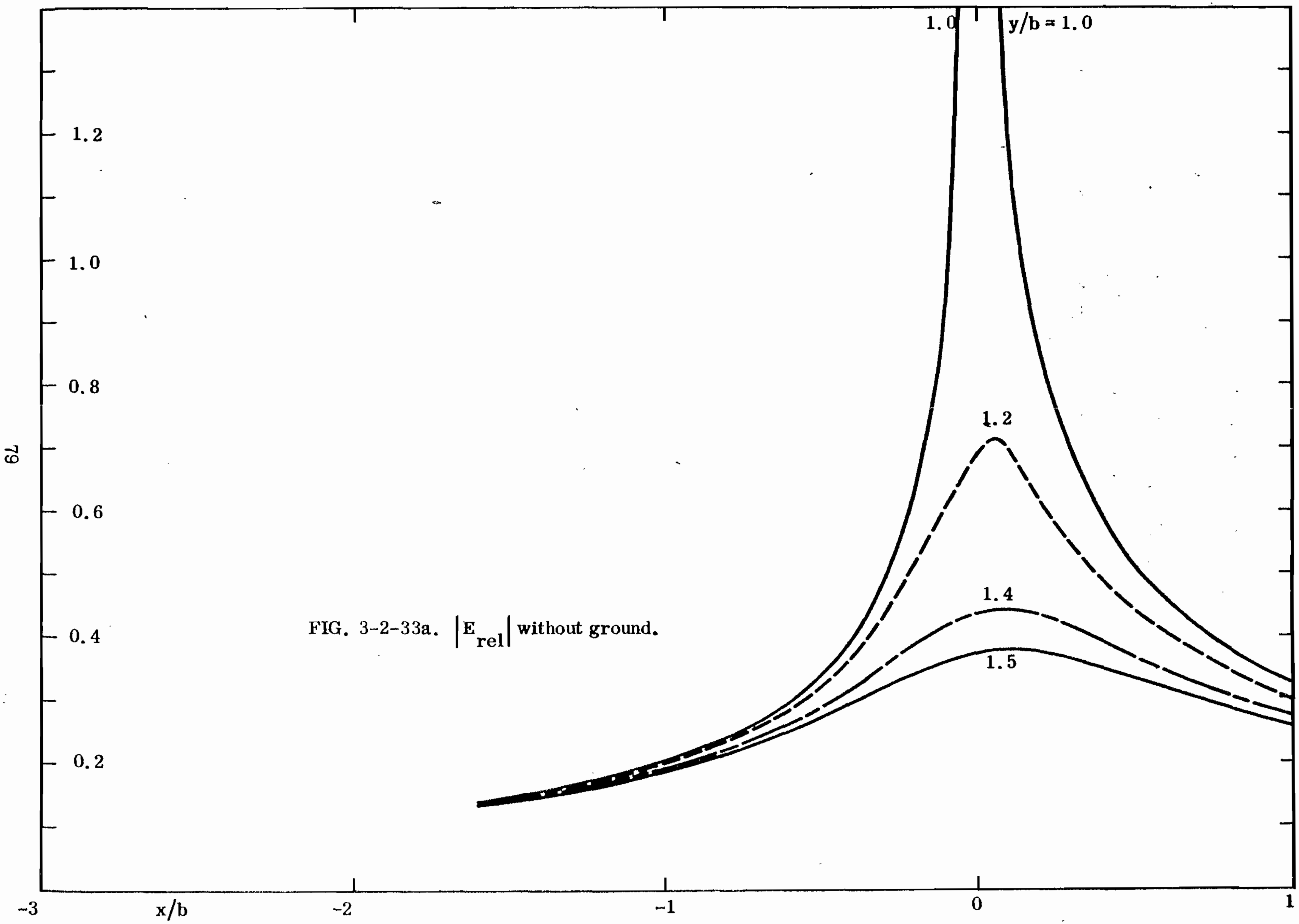

FIG. 3-2-33a. $|E_{rel}|$ without ground.

3.3 Contour Plots for Deviations of Relative Electric Field Intensities

In this section the effect of the ground on the electric field intensities of the transmission line is considered in terms of the deviation of the electric field intensity. The deviation is measured in two different ways:

i) deviation of the field intensity due to the ground from that without the ground,

ii) deviation from the uniform field.

Let the field intensity at a point in the transmission line without the ground be denoted by $E_{o_{rel}}$. For the case of (i) we define

$$|\Delta E_{rel}| = |E_{rel} - E_{o_{rel}}|;$$

for the case of (ii),

$$|\delta E_{rel}| = |E_{rel} + 1|,$$

where the uniform field intensity is given by $E_{uniform} = E_{y_{uniform}} = -1.$

The deviations for the case (i) are shown in Figs. 3-3-1 through 3-3-21 and for the case (ii) in Figs. 3-3-22 through 3-3-35.

TABLE III-7: Deviation of Relative Electric Field Intensity of Transmission Line From That Without The Ground.

Field Component	Parameters: Coordinates	Parameters: d/b	Figure Nos.
$\lvert\Delta E_{y_{rel}}\rvert$	z-plane	0.1	3-3-1
		0.2	3-3-2
		0.5	3-3-3
		1.0	3-3-4
		2.0	3-3-5
		5.0	3-3-6
		10.0	3-3-7
$\lvert\Delta E_{x_{rel}}\rvert$		0.1	3-3-8
		0.2	3-3-9
		0.5	3-3-10
		1.0	3-3-11
		2.0	3-3-12
		5.0	3-3-13
		10.0	3-3-14
$\lvert\Delta E_{rel}\rvert$		0.1	3-3-15
		0.2	3-3-16
		0.5	3-3-17
		1.0	3-3-18
		2.0	3-3-19
		5.0	3-3-20
		10.0	3-3-21

TABLE III-8: Deviation of Relative Electric Field Intensity From Uniform Field

Field Component	Parameters: Coordinates	Parameters: d/b	Figure Nos.
$\lvert \delta E_{y_{rel}} \rvert$	z-plane	0.1	3-3-22
		0.2	3-3-23
		0.5	3-3-24
		1.0	3-3-25
		2.0	3-3-26
		5.0	3-3-27
		10.0	3-3-28
$\lvert \delta E_{rel} \rvert$		0.1	3-3-29
		0.2	3-3-30
		0.5	3-3-31
		1.0	3-3-32
		2.0	3-3-33
		5.0	3-3-34
		10.0	3-3-35

In the above table, $\lvert \delta E_{x_{rel}} \rvert$ does not appear for the reason that the x-component of the uniform field is zero, hence, $\lvert \delta E_{x_{rel}} \rvert$ is the same as $\lvert E_{x_{rel}} \rvert$.

From the contour plots, it is observed that, as a whole, the presence of a perfectly conducting plane ground causes the field strength to be enhanced in a neighborhood of the upper edge and to be weakened in a region where the center plane intersects the ground plane. The ground effect is negligible in the region $-1.5 \lesssim x/b$, $0 \leq y/b \leq 1.0$.

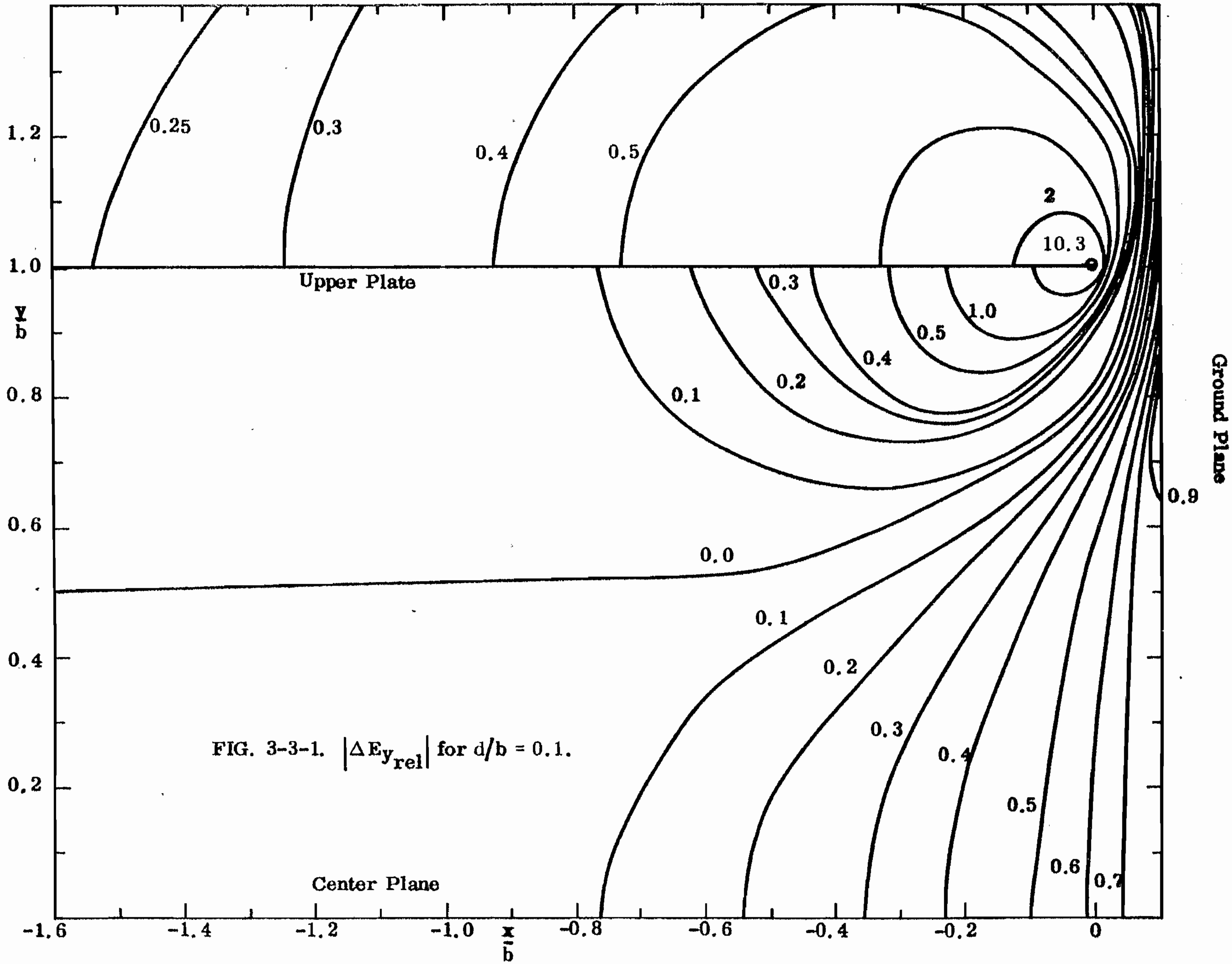

FIG. 3-3-1. $\left|\Delta E_{y_{rel}}\right|$ for d/b = 0.1.

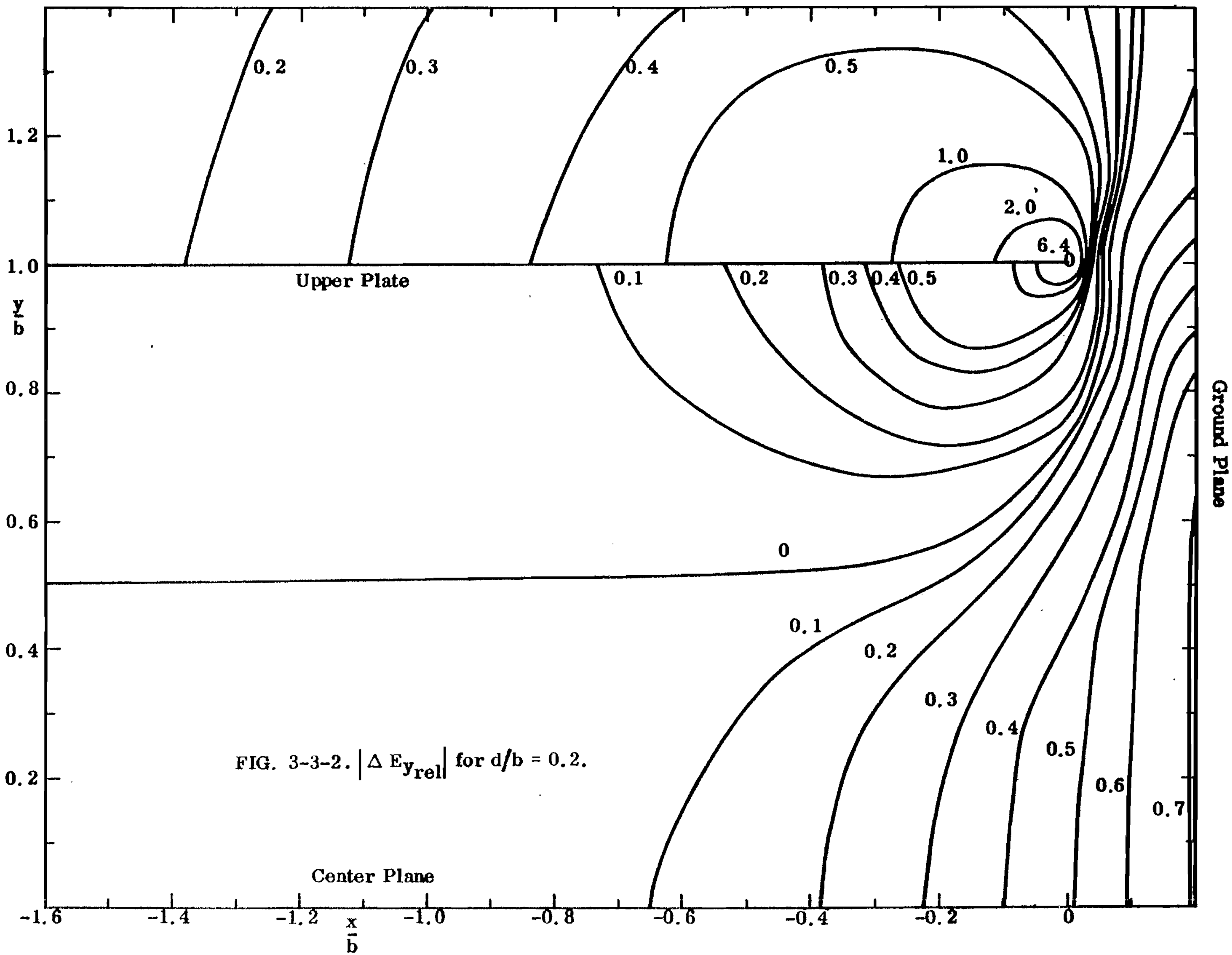

FIG. 3-3-2. $|\Delta E_{y_{rel}}|$ for d/b = 0.2.

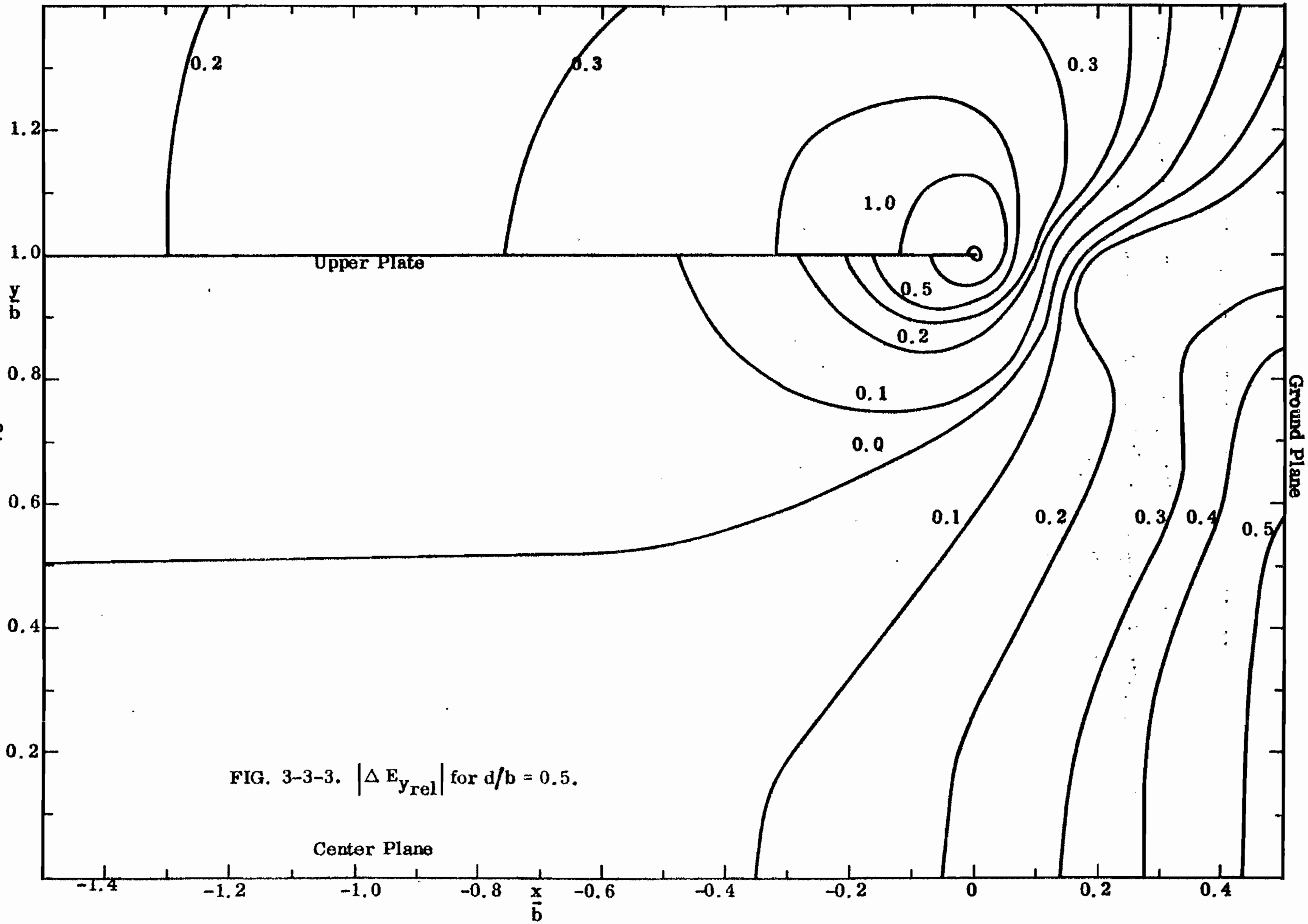

FIG. 3-3-3. $\left|\Delta E_{y_{rel}}\right|$ for $d/b = 0.5$.

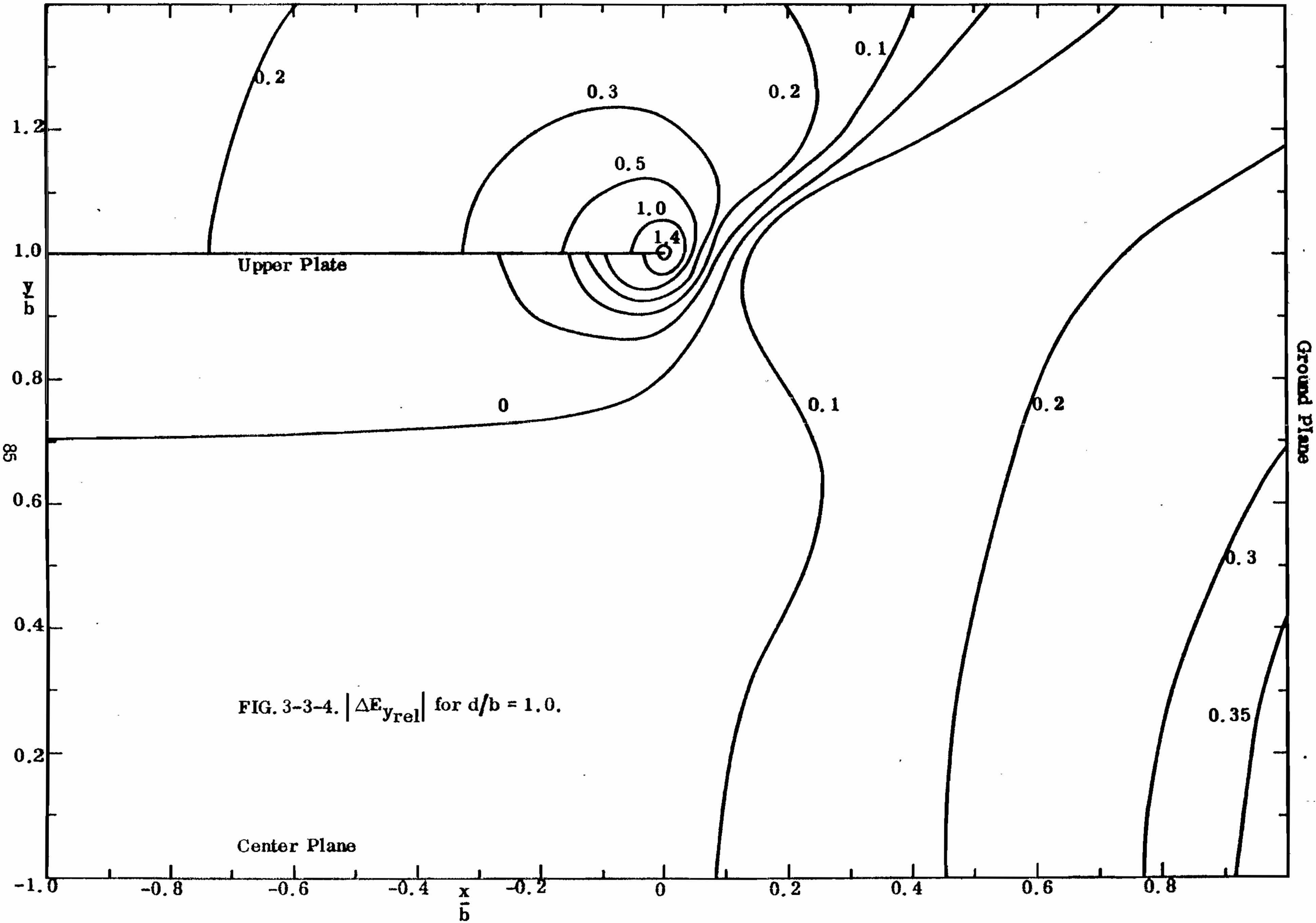

FIG. 3-3-4. $\left|\Delta E_{y_{rel}}\right|$ for d/b = 1.0.

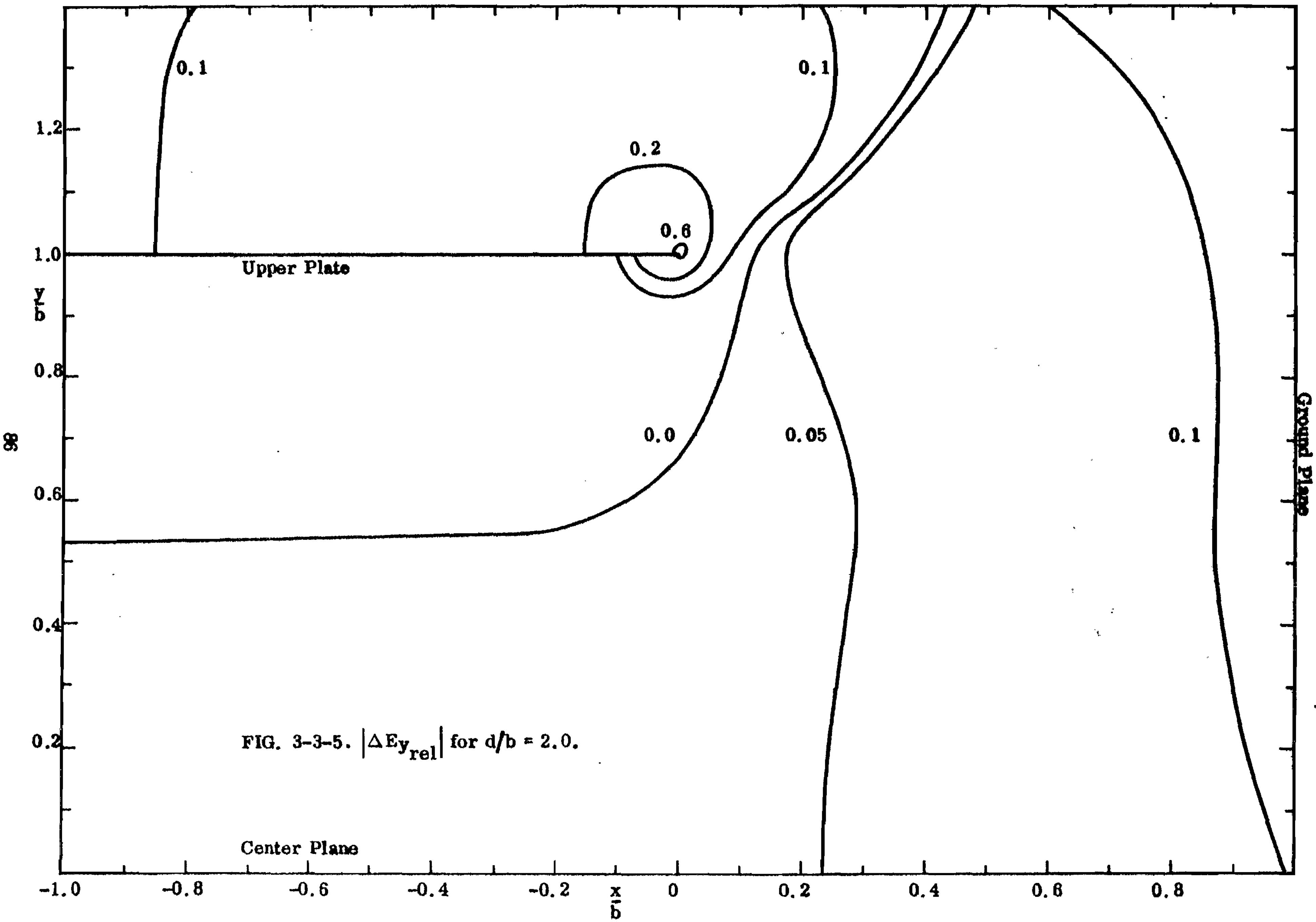

FIG. 3-3-5. $|\Delta Ey_{rel}|$ for d/b = 2.0.

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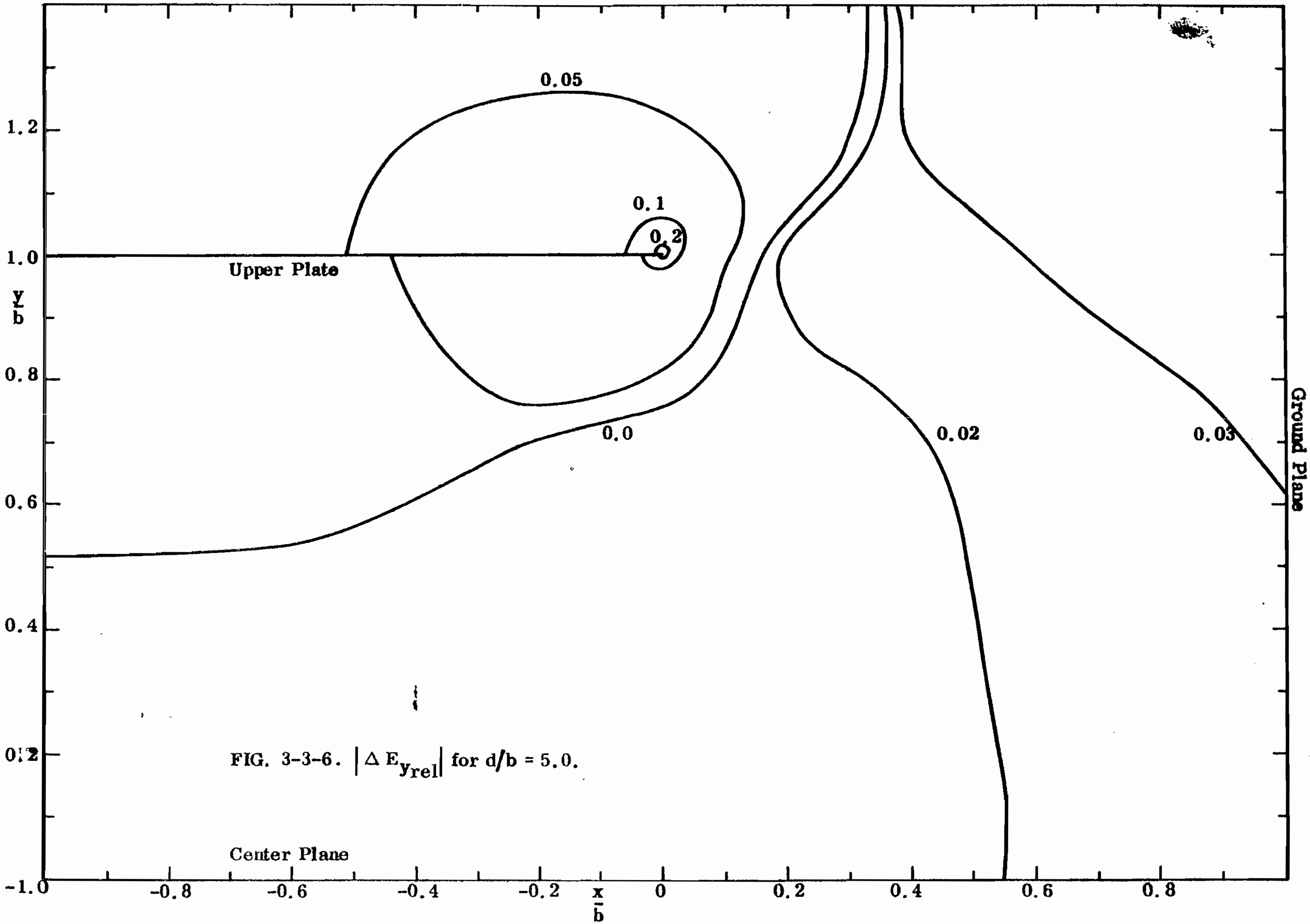

FIG. 3-3-6. $|\Delta E_{y_{rel}}|$ for $d/b = 5.0$.

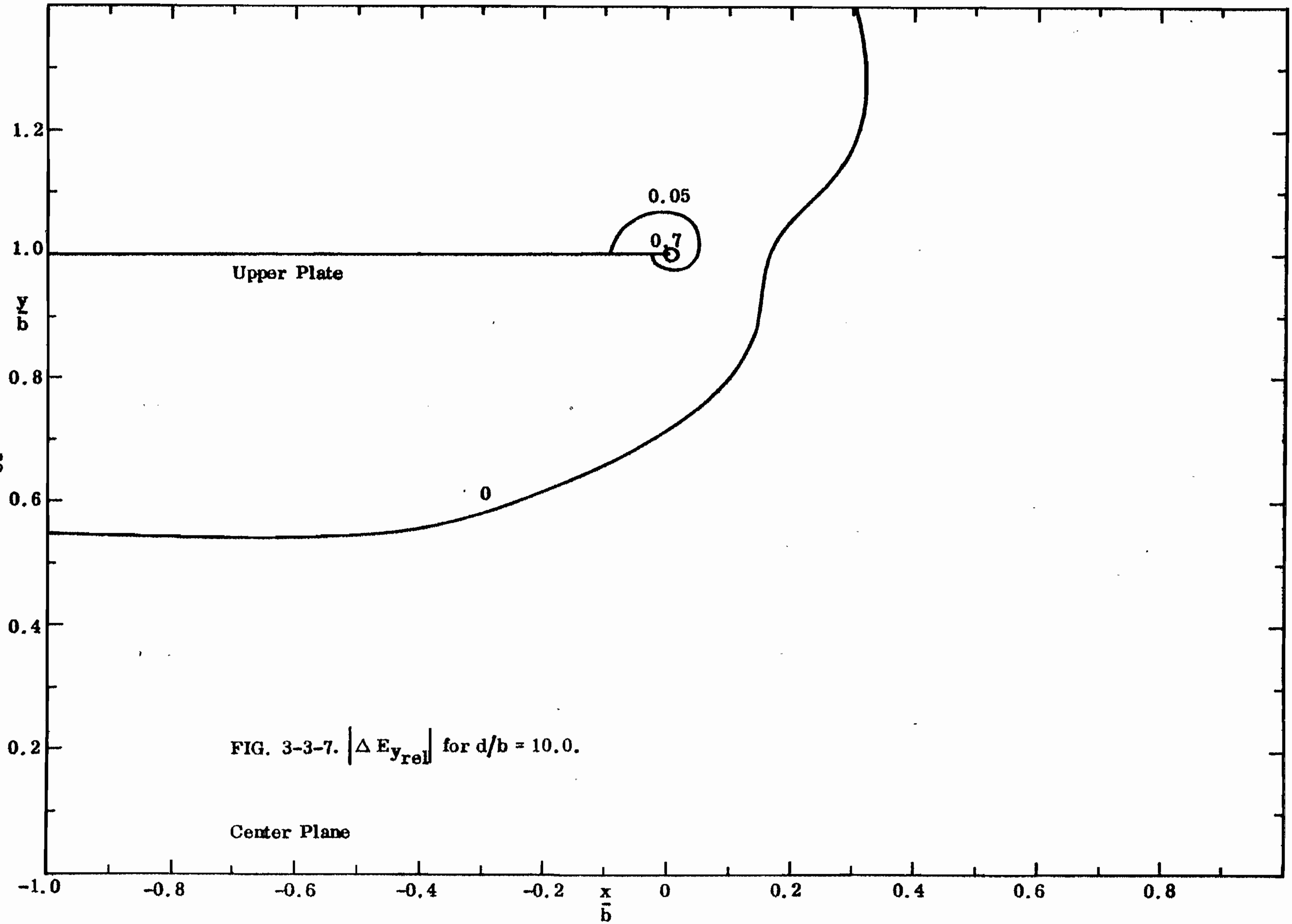

FIG. 3-3-7. $|\Delta E_{y_{rel}}|$ for $d/b = 10.0$.

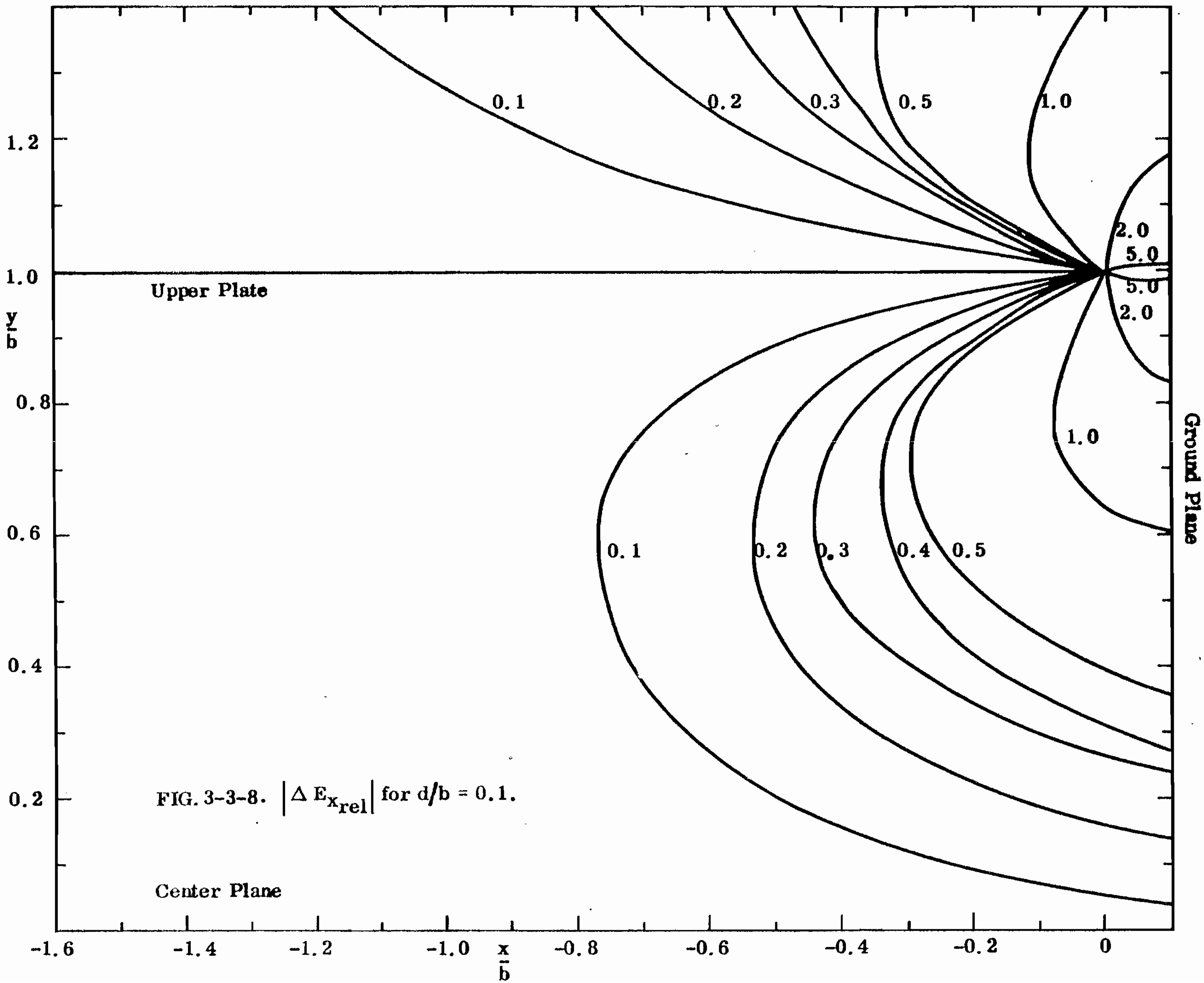

FIG. 3-3-8. $\left|\Delta E_{x_{rel}}\right|$ for $d/b = 0.1$.

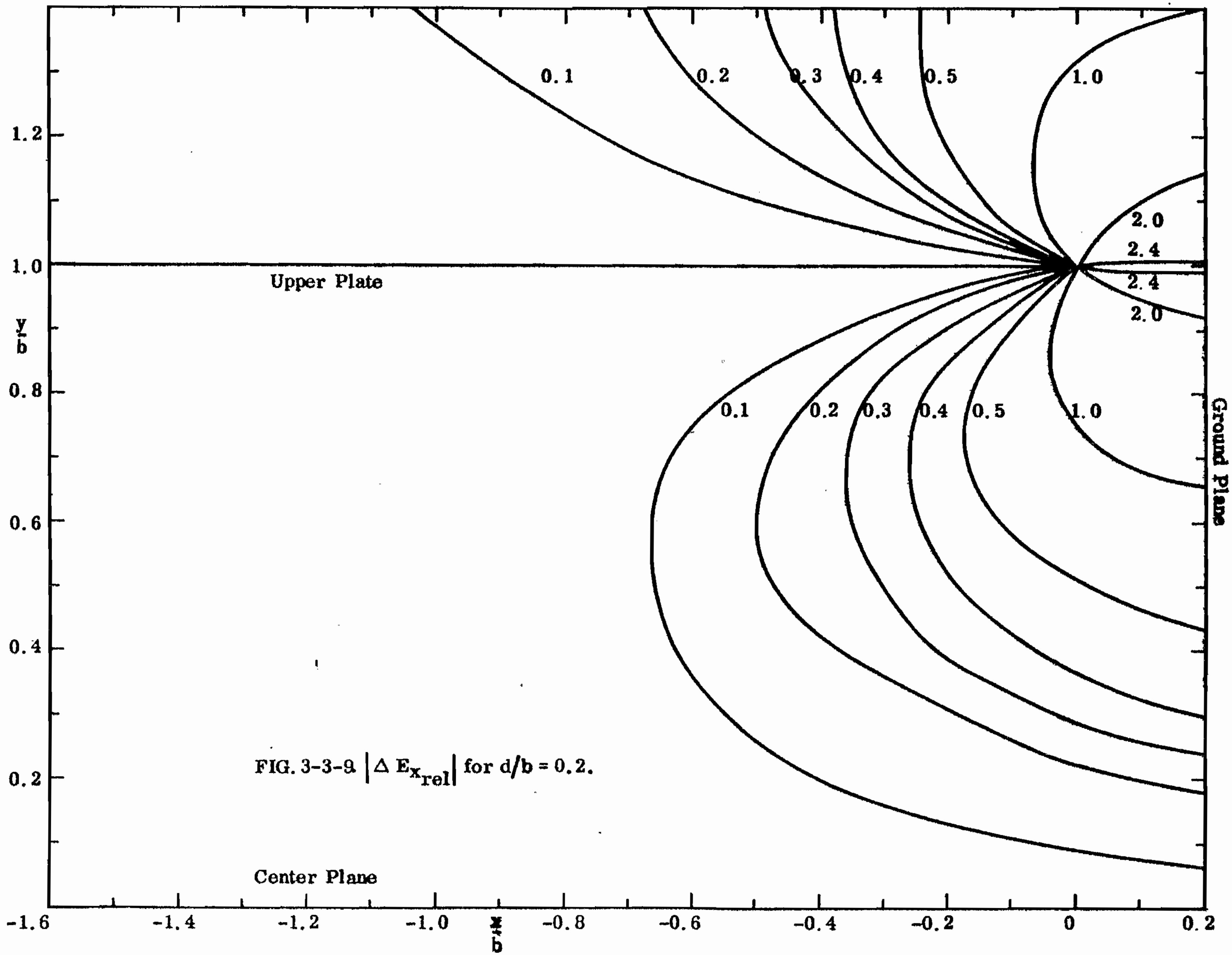

FIG. 3-3-9 $\left|\Delta E_{x_{rel}}\right|$ for d/b = 0.2.

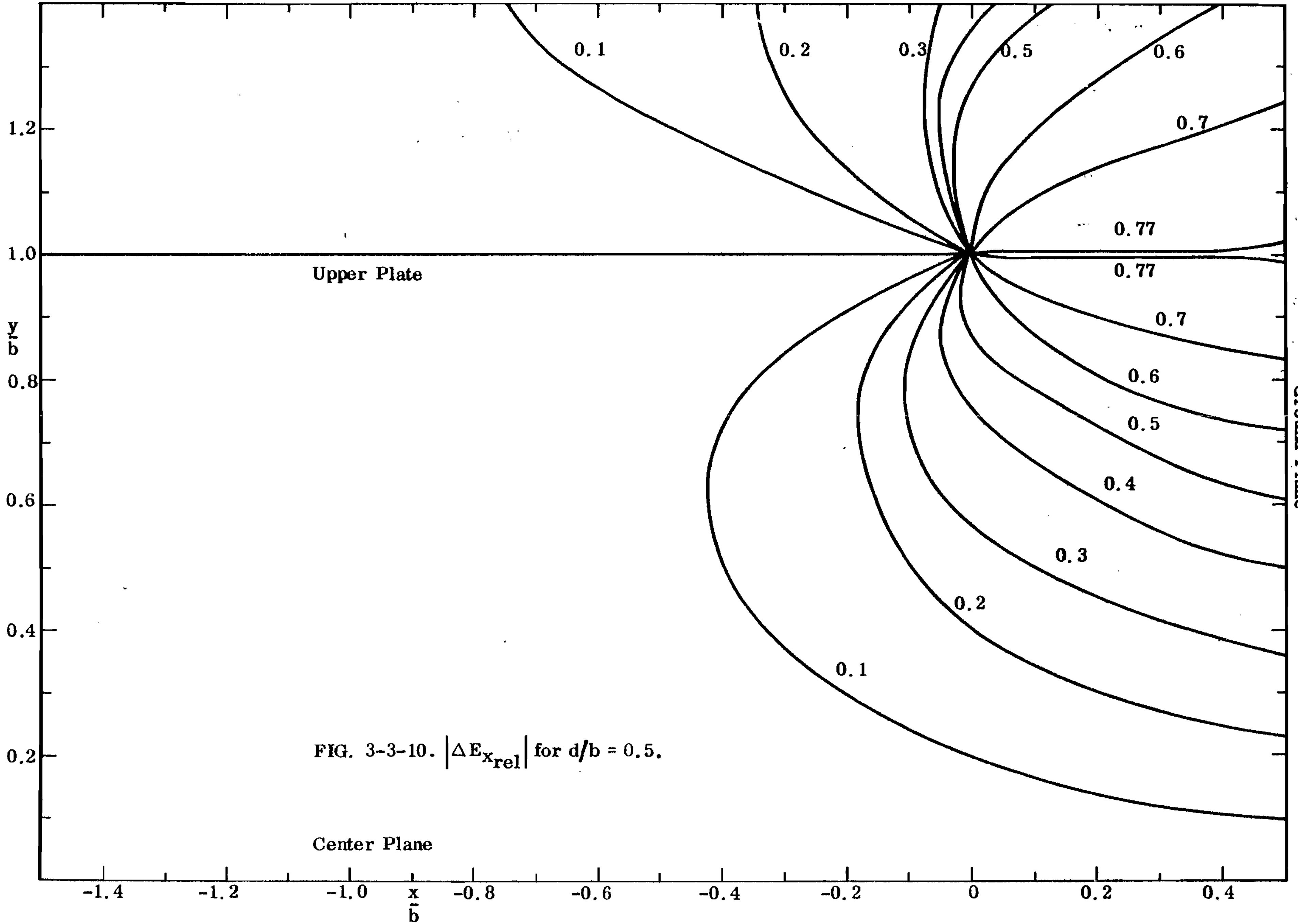

FIG. 3-3-10. $\left|\Delta E_{x_{rel}}\right|$ for $d/b = 0.5$.

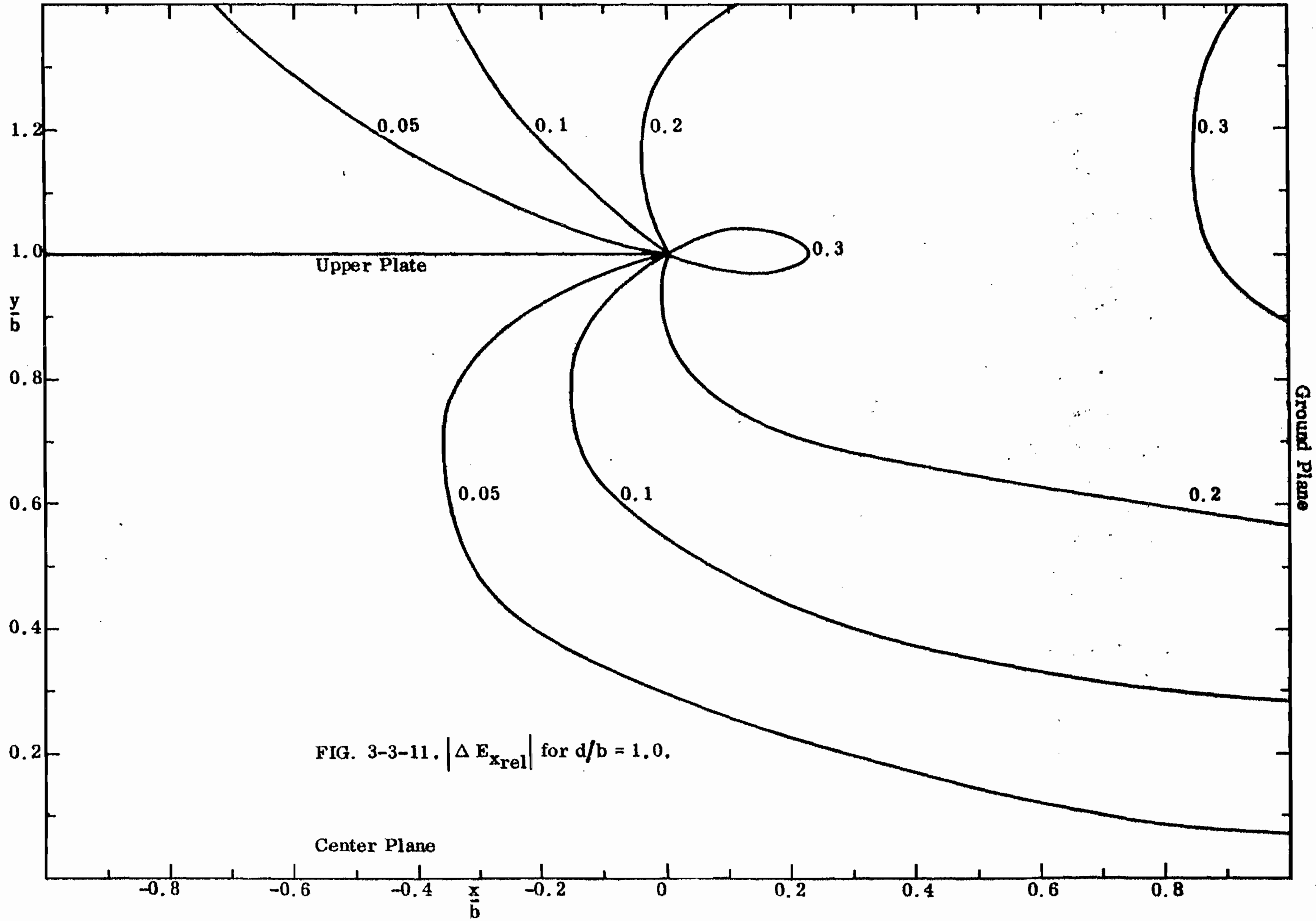

FIG. 3-3-11. $\left|\Delta E_{x_{rel}}\right|$ for d/b = 1.0.

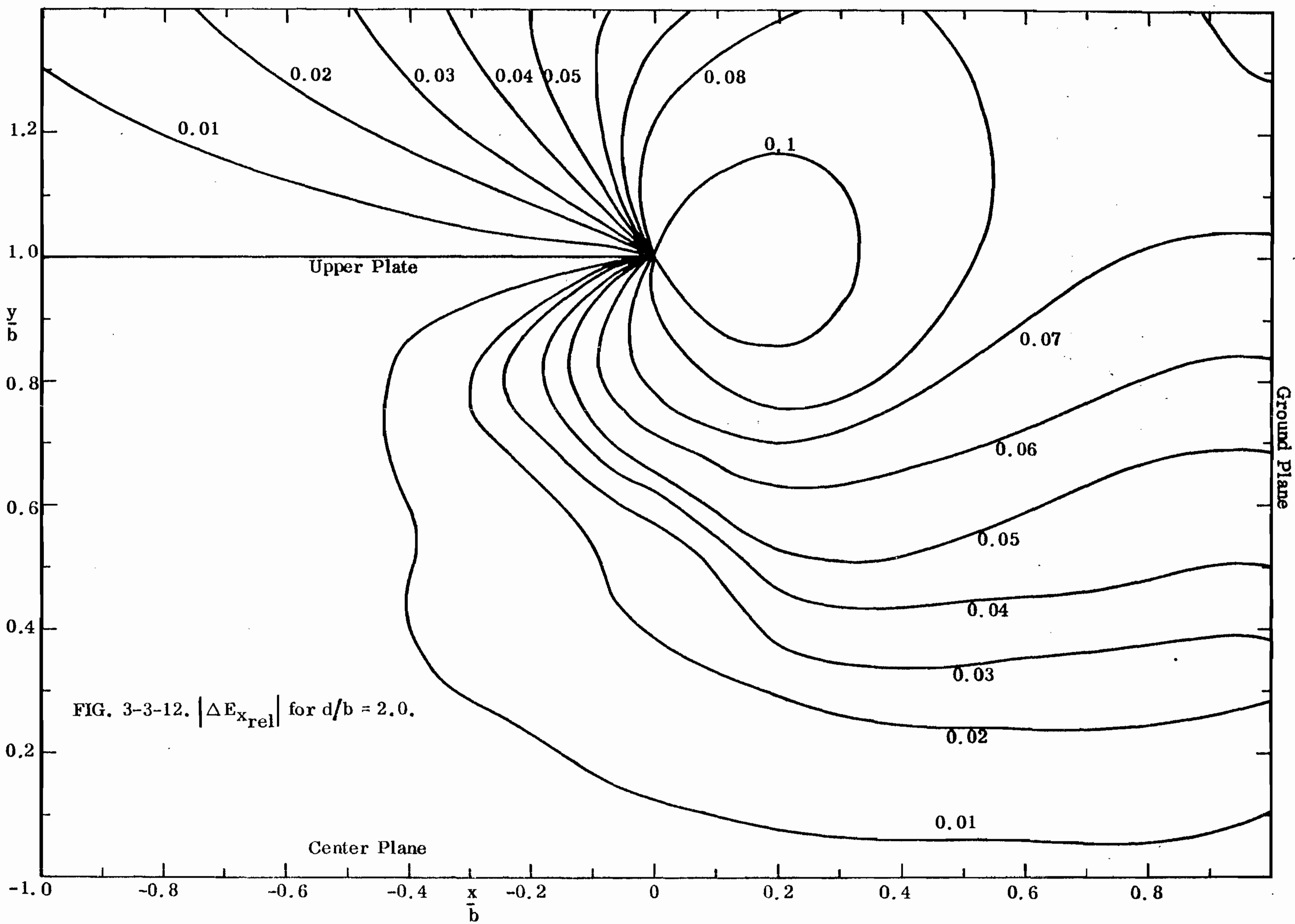

FIG. 3-3-12. $\left|\Delta E_{x_{rel}}\right|$ for $d/b = 2.0$.

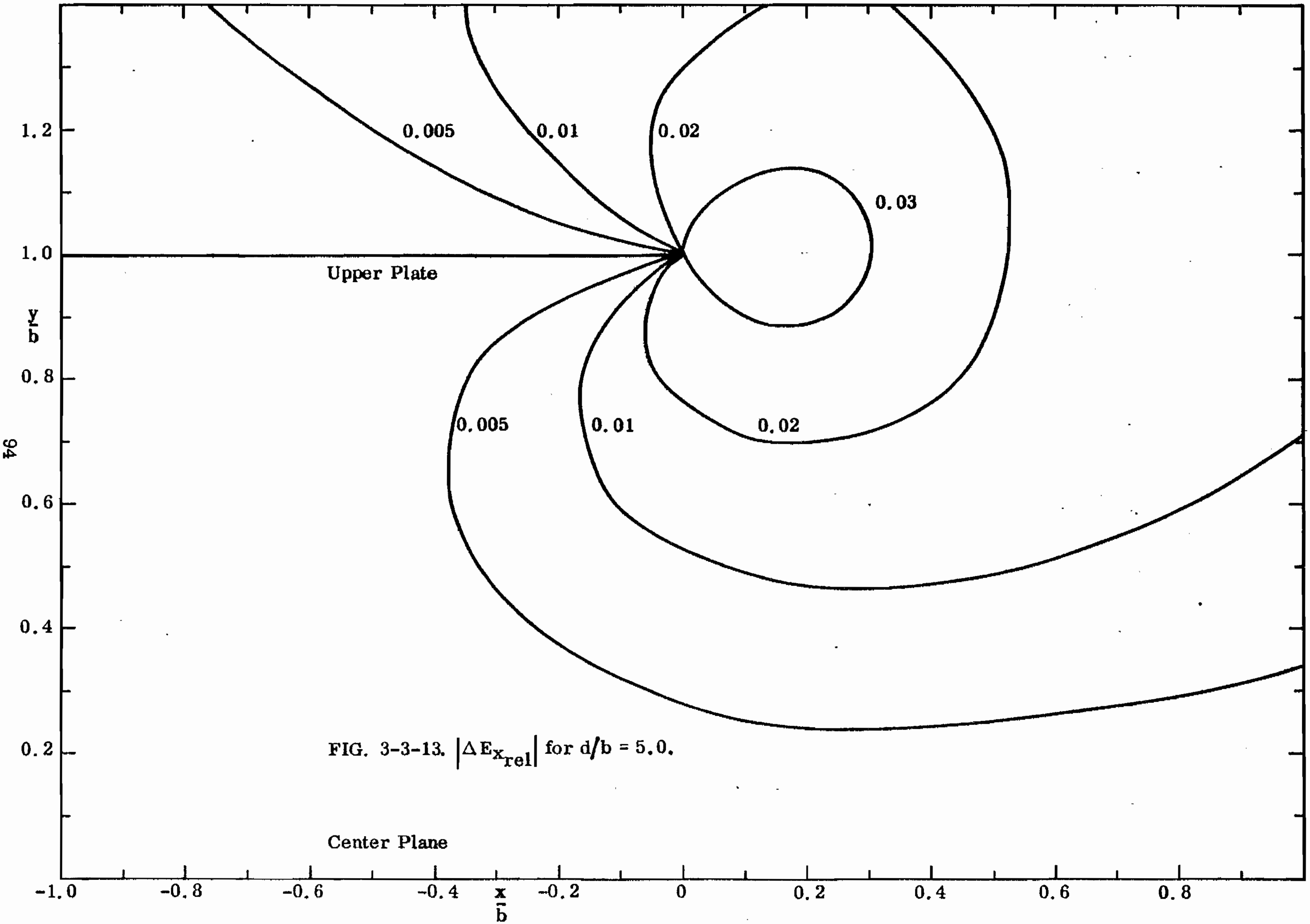

FIG. 3-3-13. $\left|\Delta E_{x_{rel}}\right|$ for d/b = 5.0.

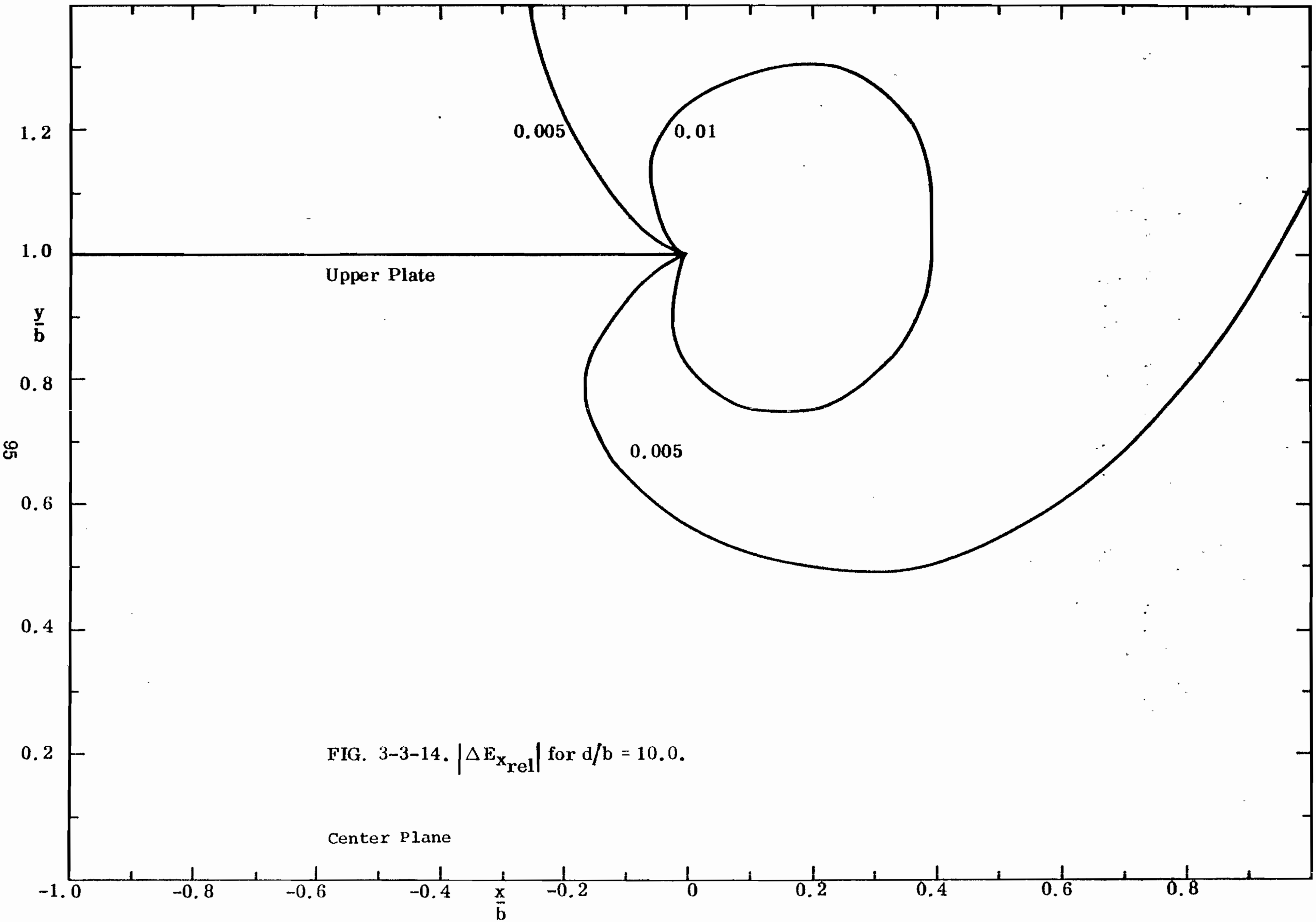

FIG. 3-3-14. $|\Delta E_{x_{rel}}|$ for $d/b = 10.0$.

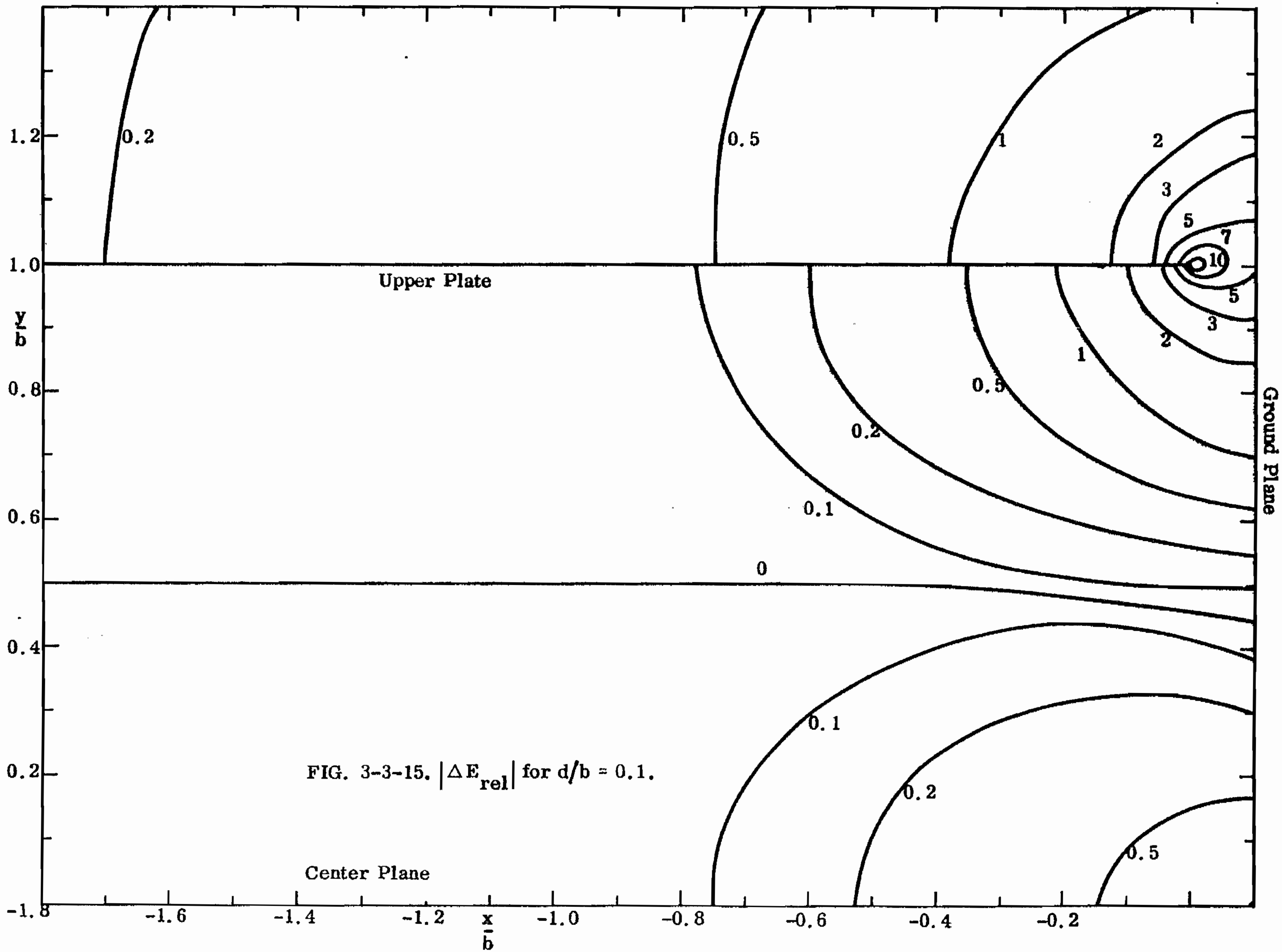

FIG. 3-3-15. $|\Delta E_{rel}|$ for $d/b = 0.1$.

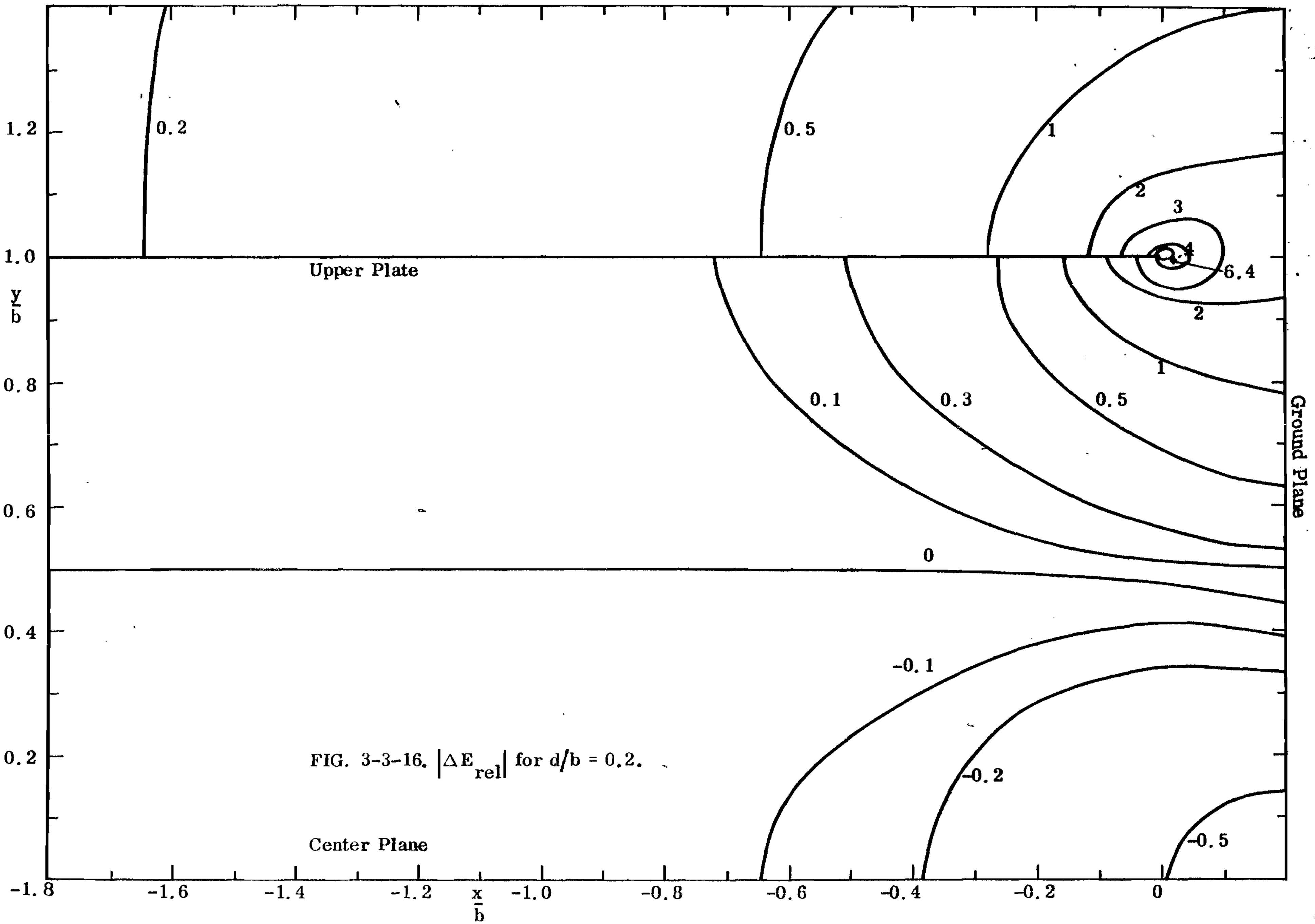

FIG. 3-3-16. $|\Delta E_{rel}|$ for d/b = 0.2.

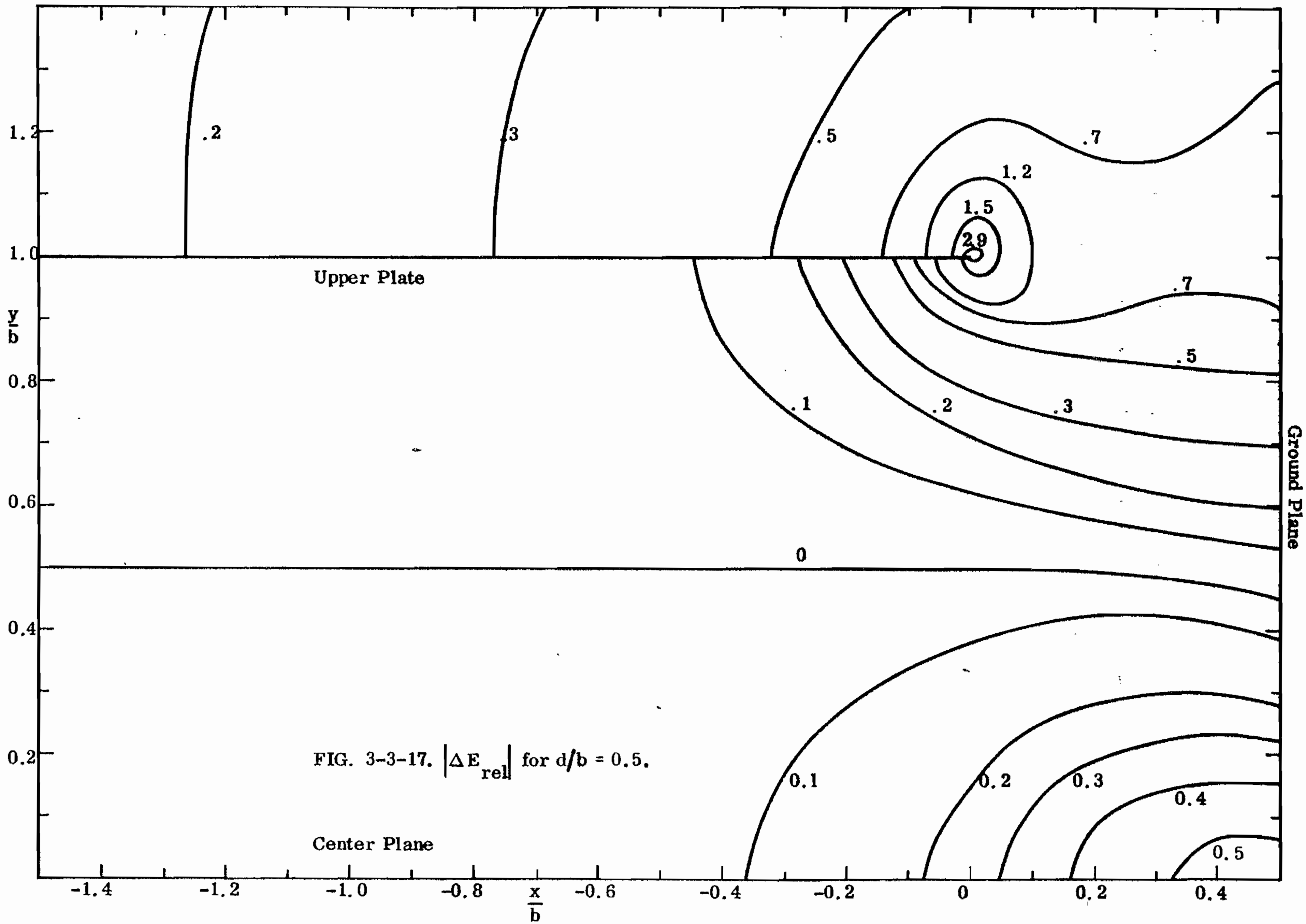

FIG. 3-3-17. $|\Delta E_{rel}|$ for d/b = 0.5.

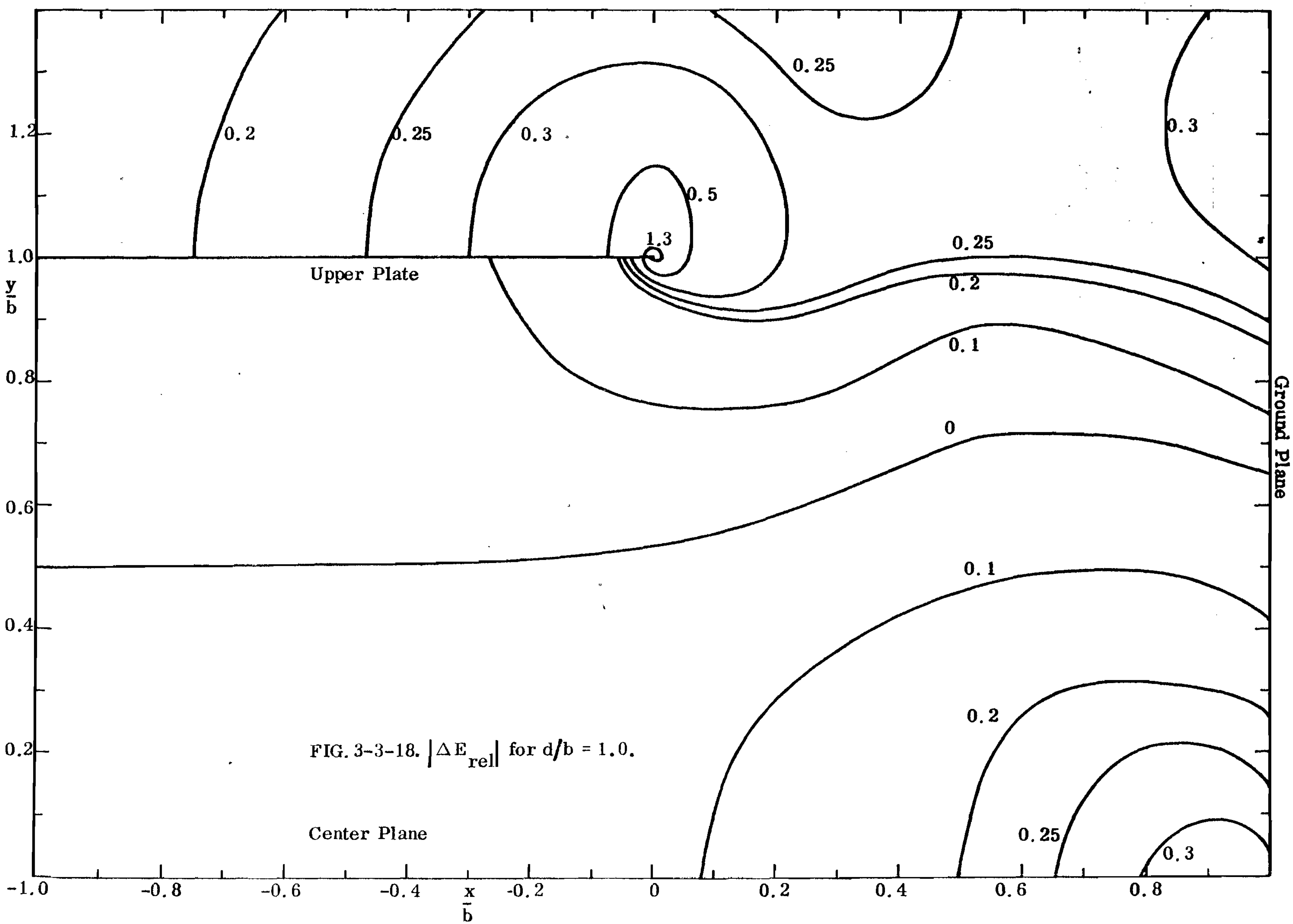

FIG. 3-3-18. $|\Delta E_{rel}|$ for $d/b = 1.0$.

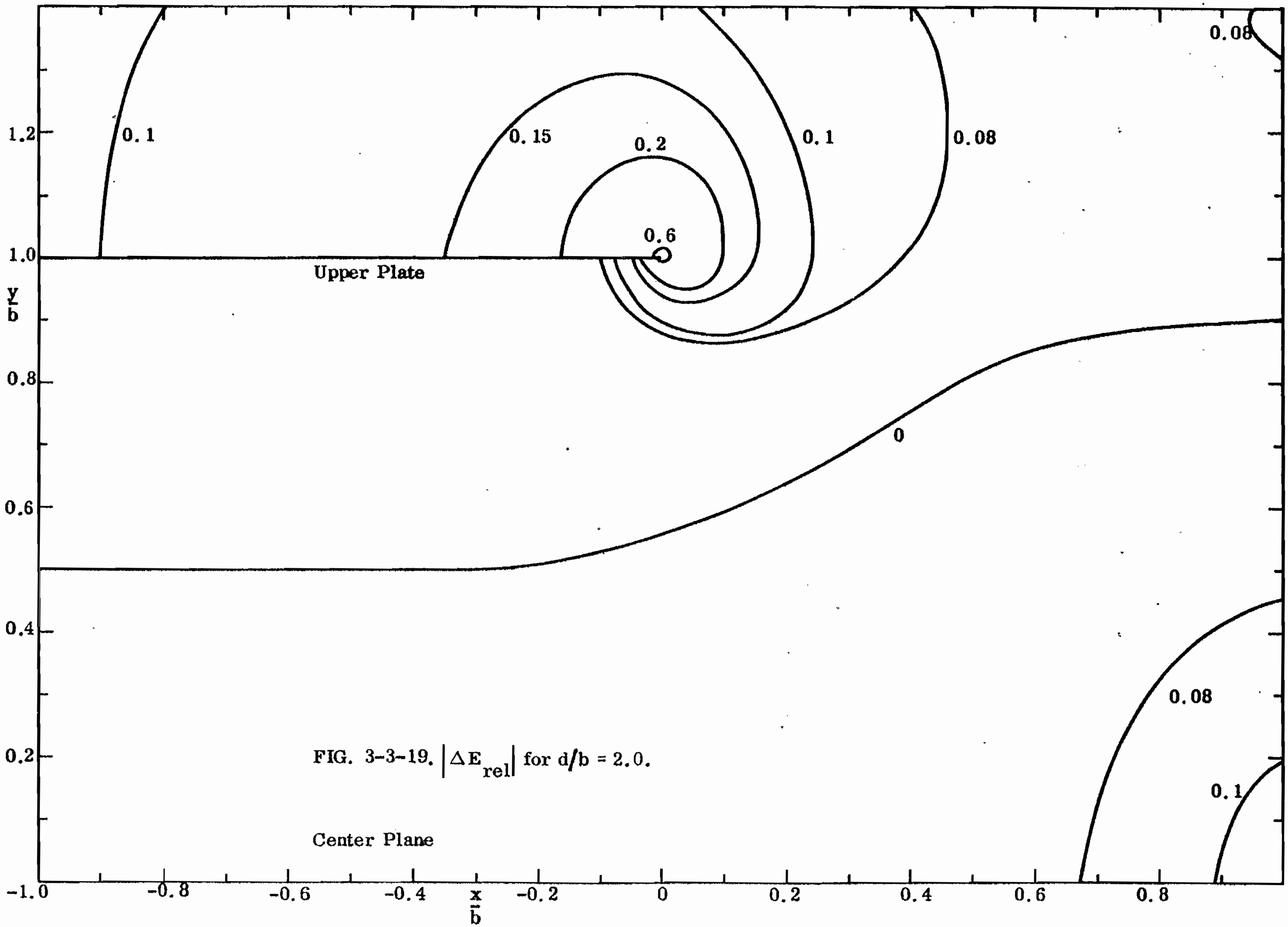

FIG. 3-3-19. $|\Delta E_{rel}|$ for d/b = 2.0.

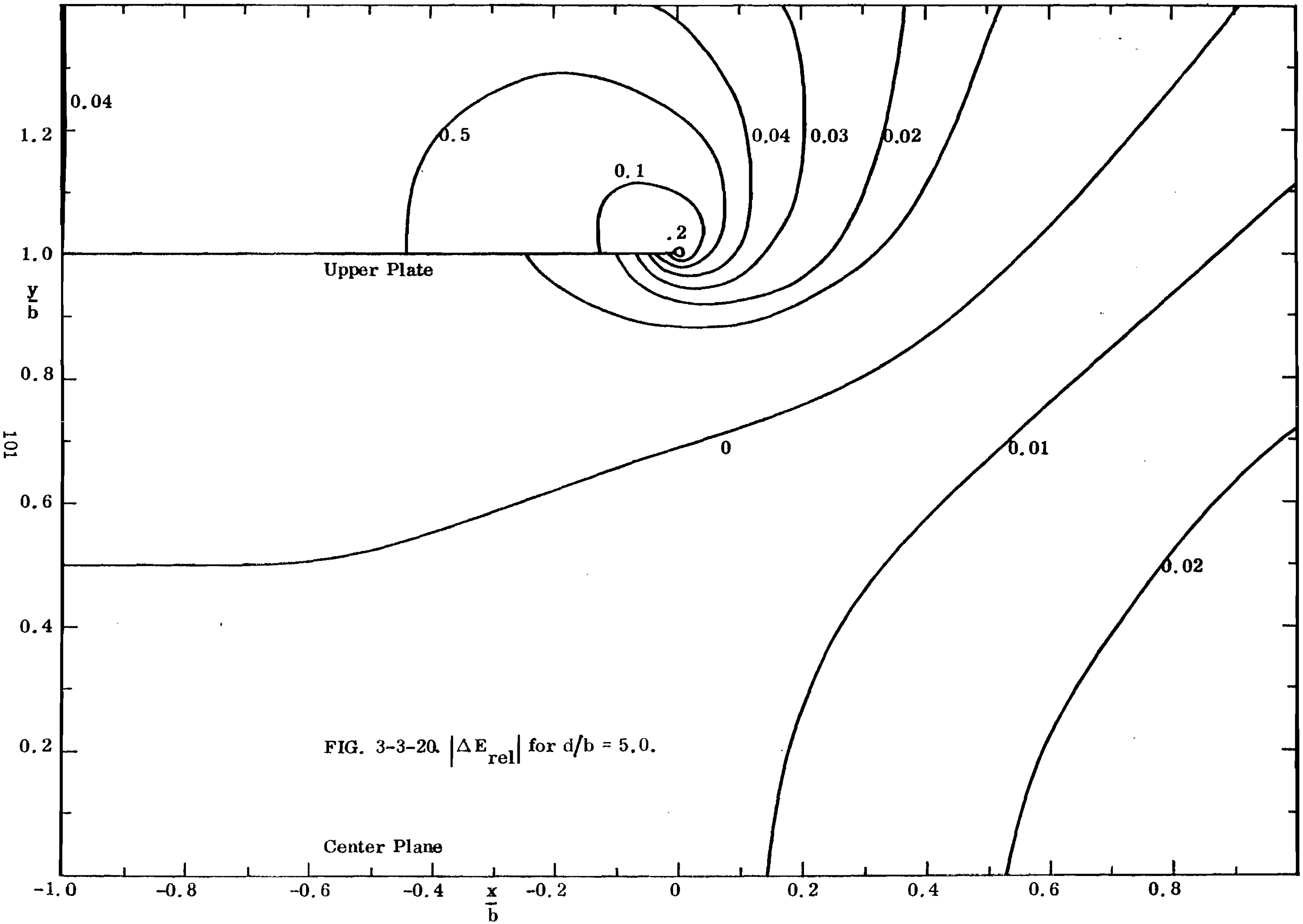

FIG. 3-3-20. $|\Delta E_{rel}|$ for d/b = 5.0.

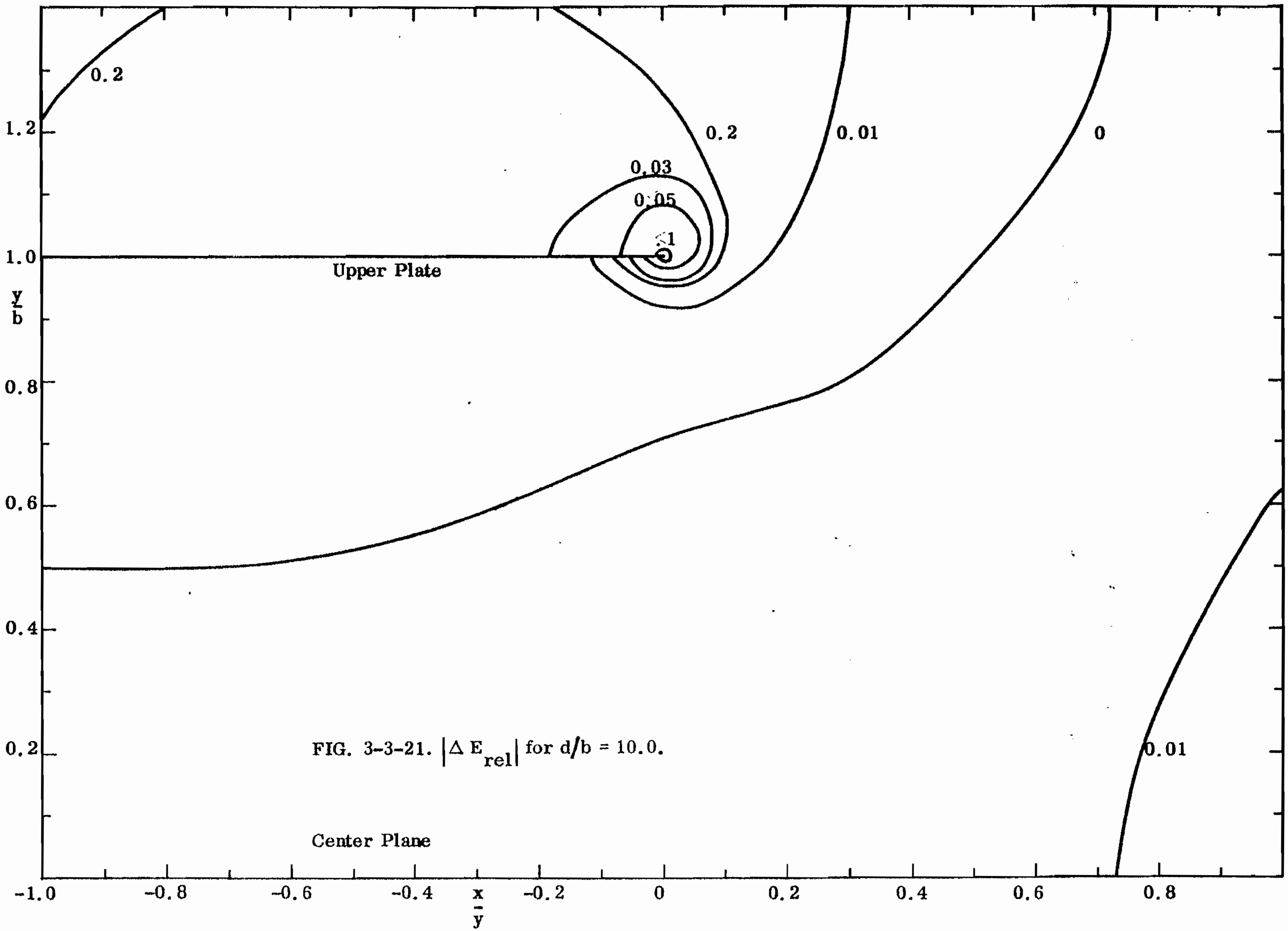

FIG. 3-3-21. $|\Delta E_{rel}|$ for $d/b = 10.0$.

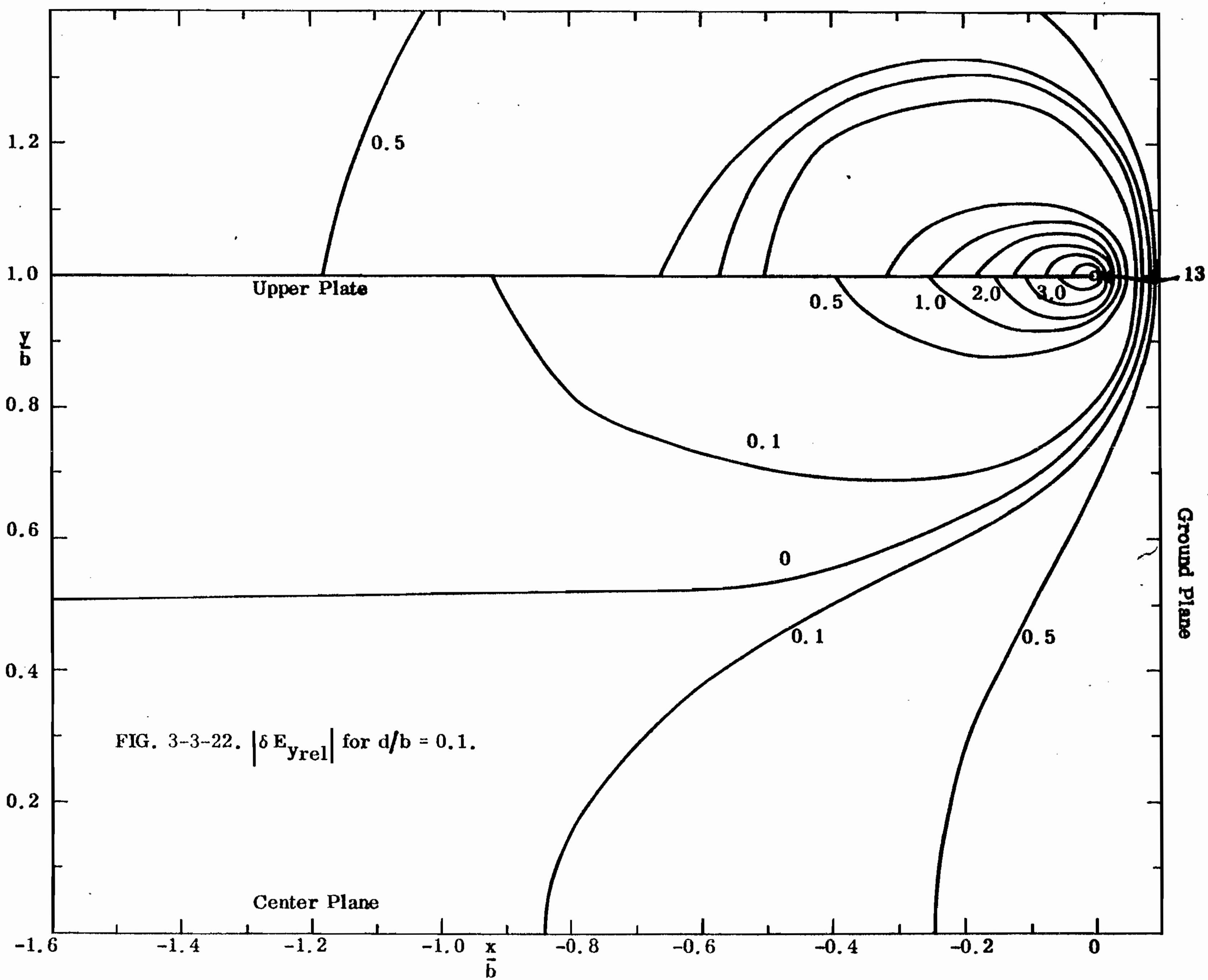

FIG. 3-3-22. $|\delta E_{y\,rel}|$ for d/b = 0.1.

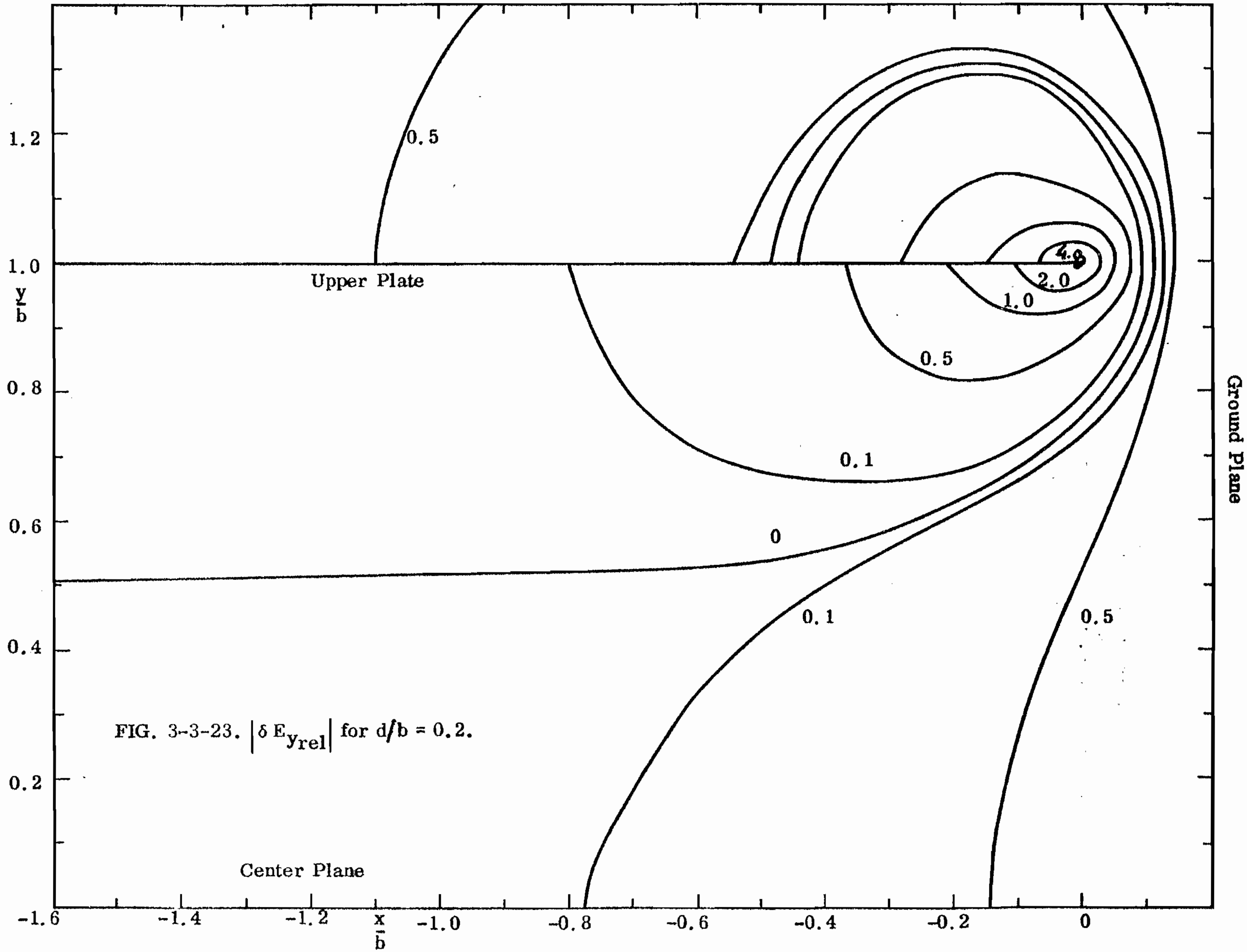

FIG. 3-3-23. $|\delta E_{y_{rel}}|$ for $d/b = 0.2$.

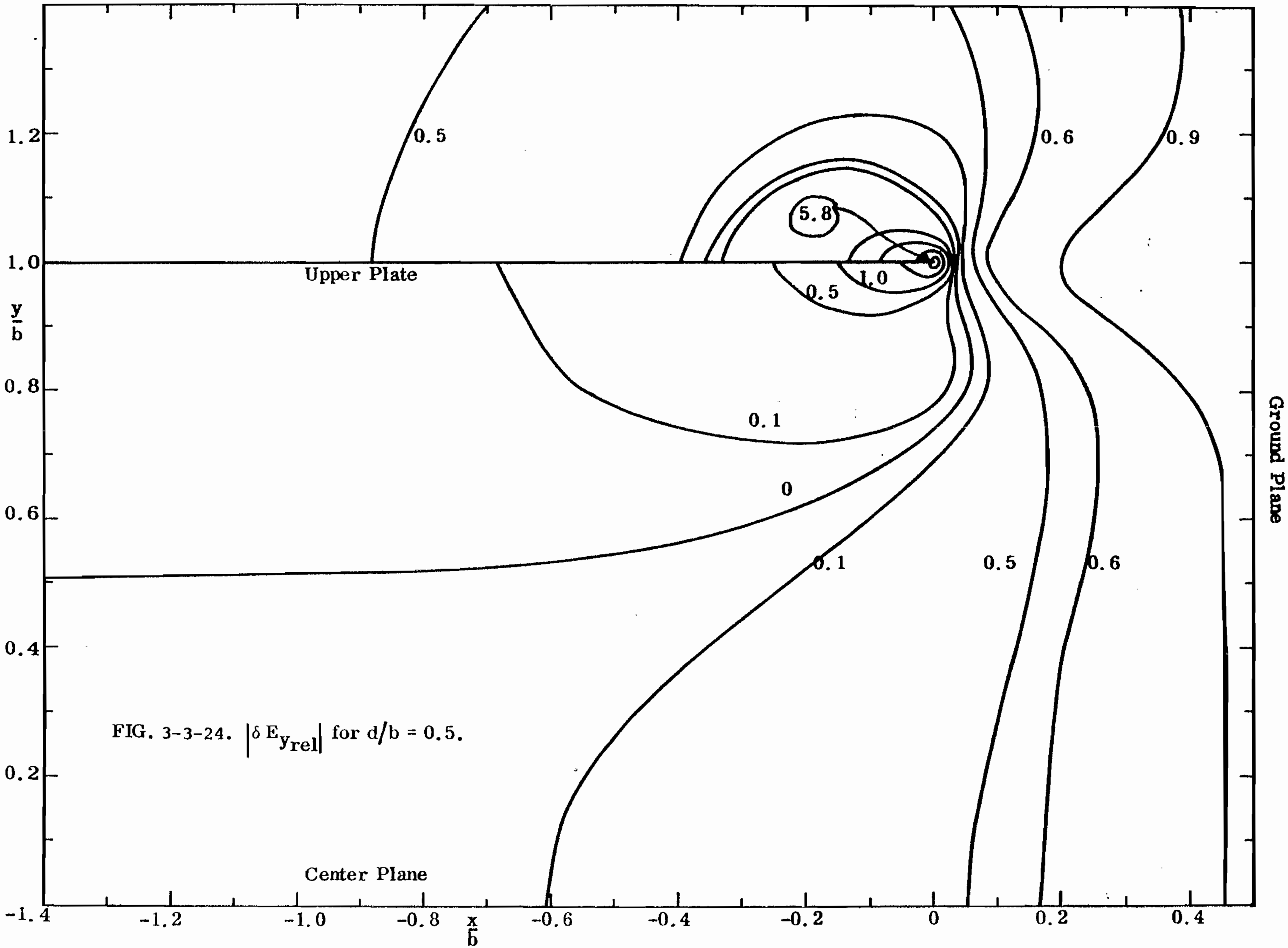

FIG. 3-3-24. $|\delta E_{y_{rel}}|$ for d/b = 0.5.

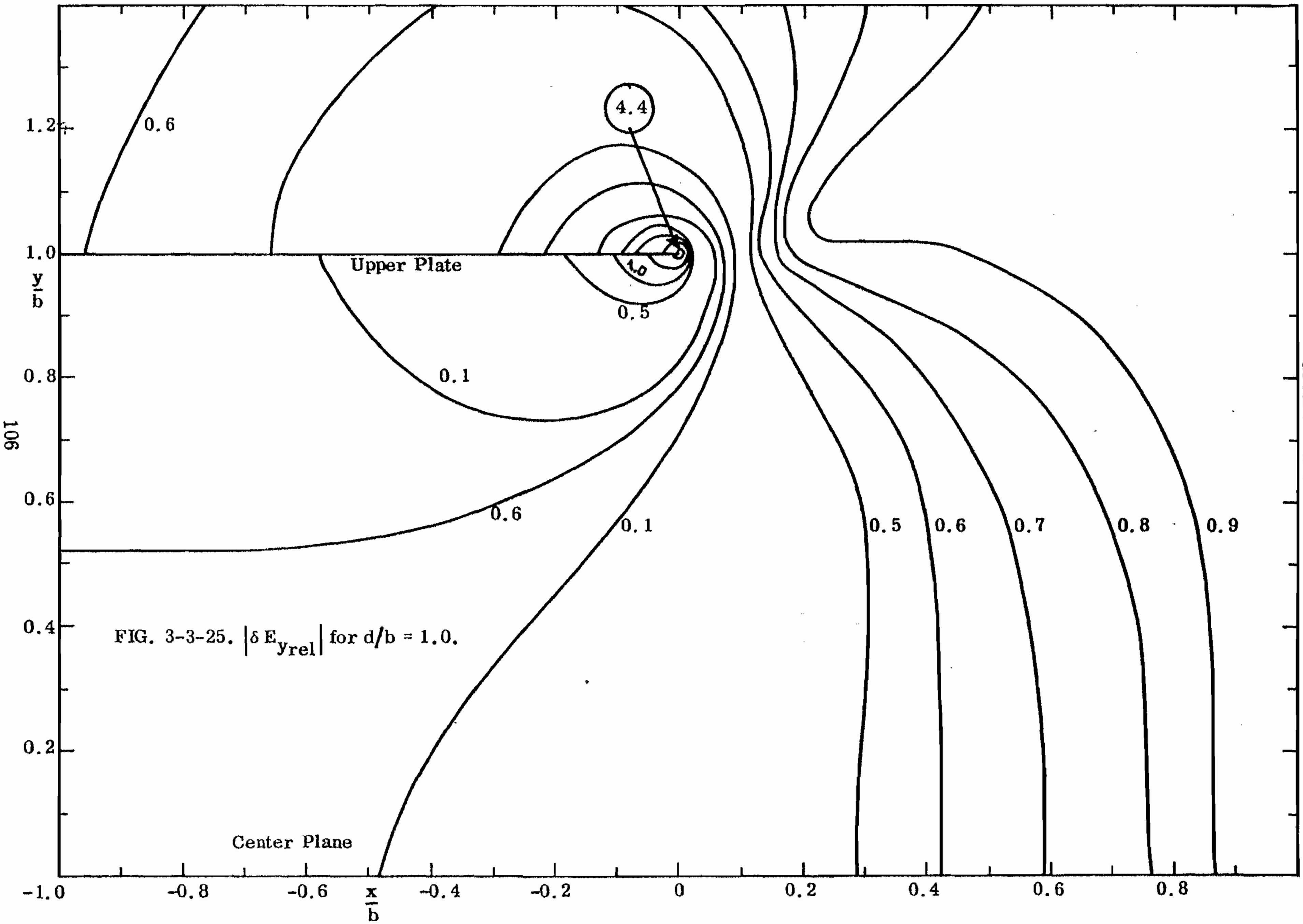

FIG. 3-3-25. $\left|\delta E_{y_{rel}}\right|$ for $d/b = 1.0$.

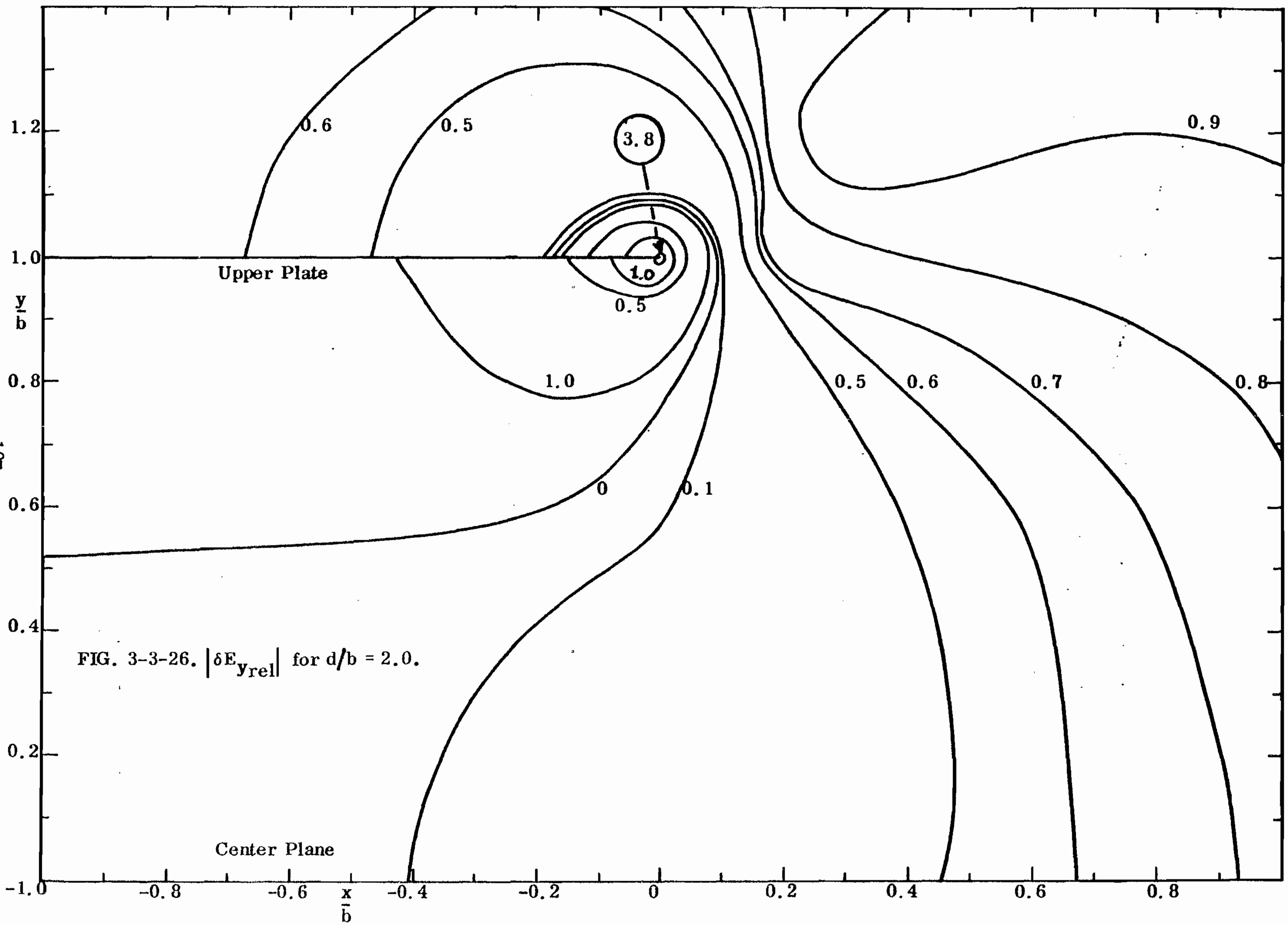

FIG. 3-3-26. $|\delta E_{y_{rel}}|$ for $d/b = 2.0$.

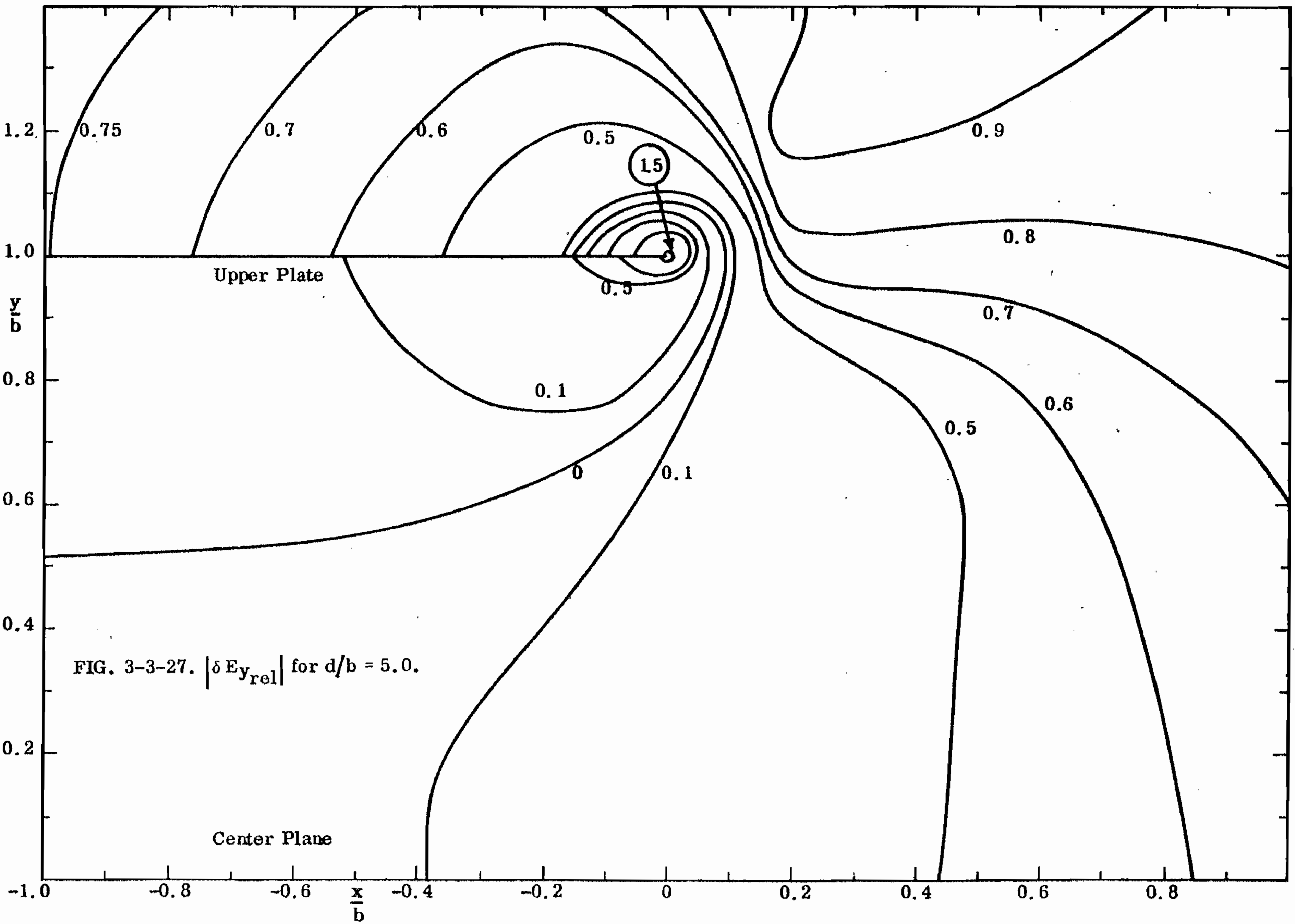

FIG. 3-3-27. $\left|\delta E_{y_{rel}}\right|$ for $d/b = 5.0$.

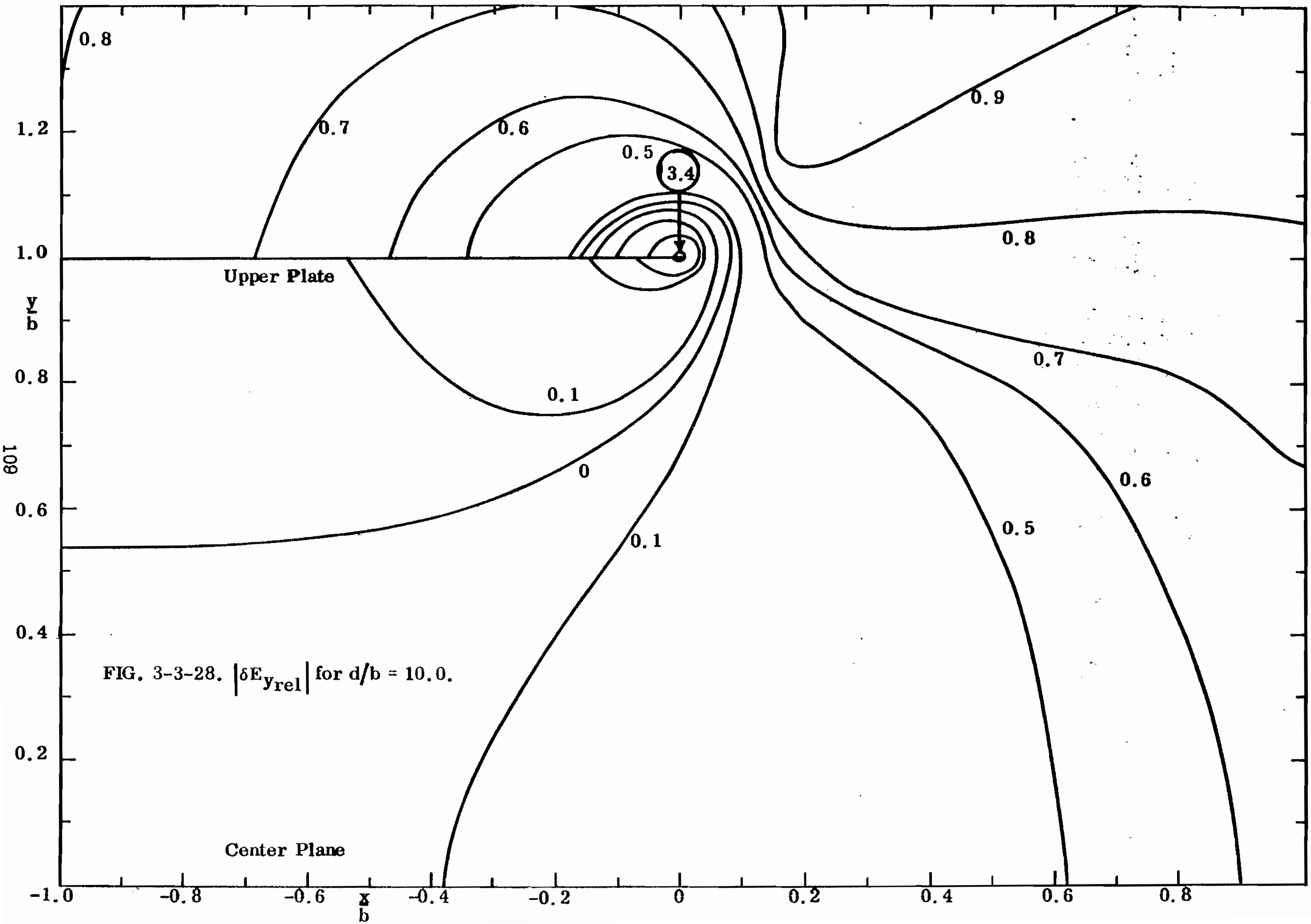

FIG. 3-3-28. $|\delta E_{y_{rel}}|$ for d/b = 10.0.

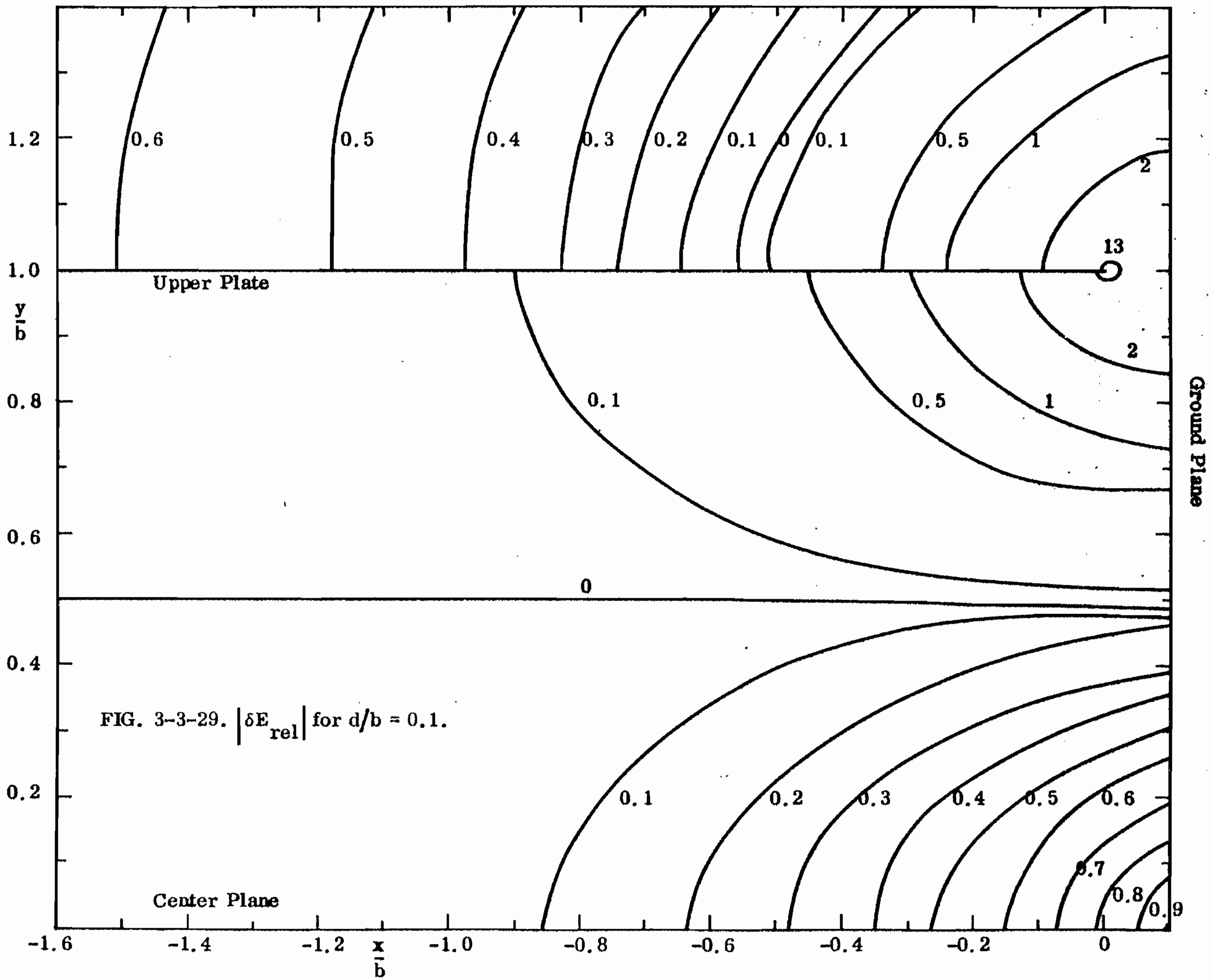

FIG. 3-3-29. $|\delta E_{rel}|$ for $d/b = 0.1$.

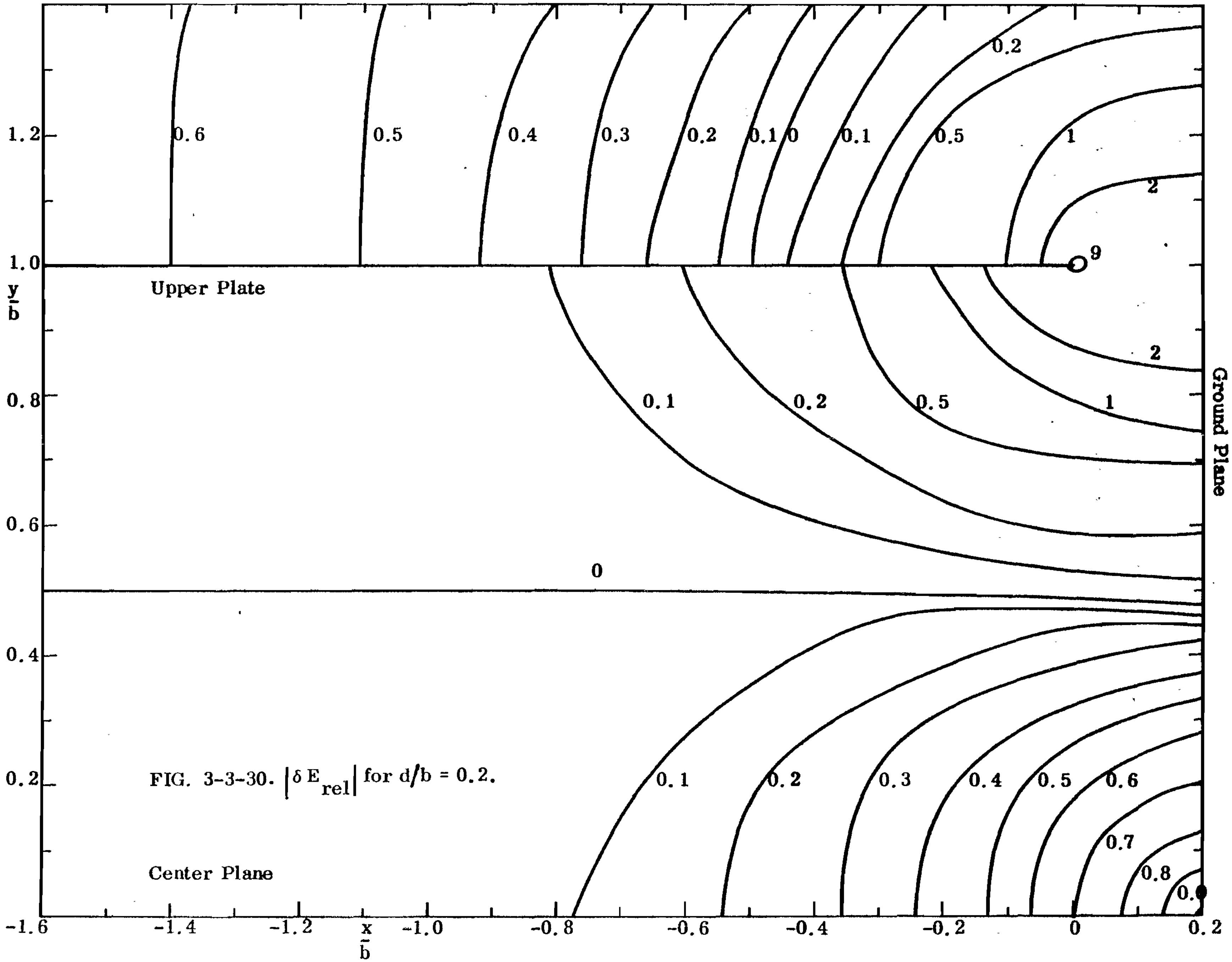

FIG. 3-3-30. $\left|\delta E_{rel}\right|$ for $d/b = 0.2$.

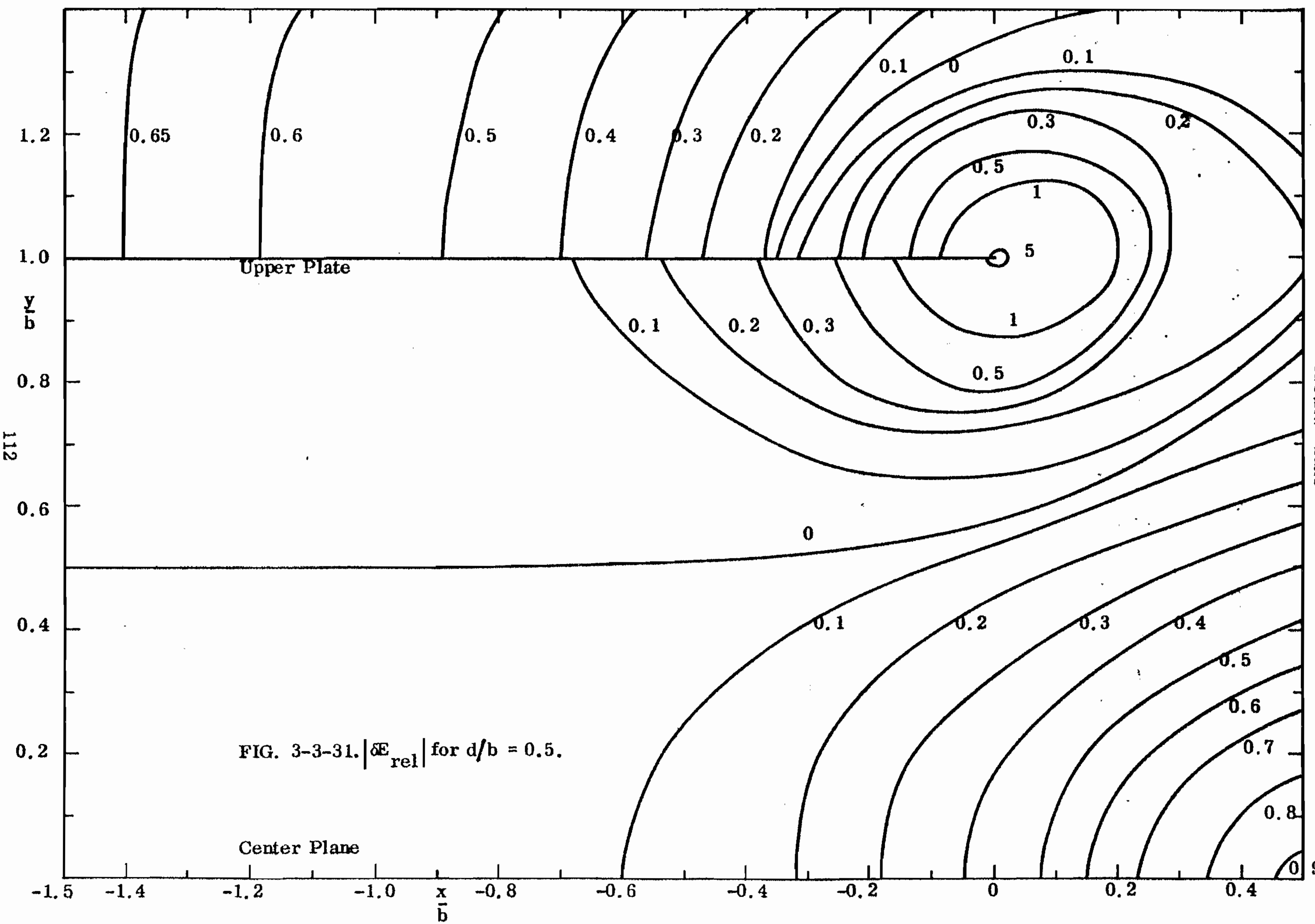

FIG. 3-3-31. $|\delta E_{rel}|$ for $d/b = 0.5$.

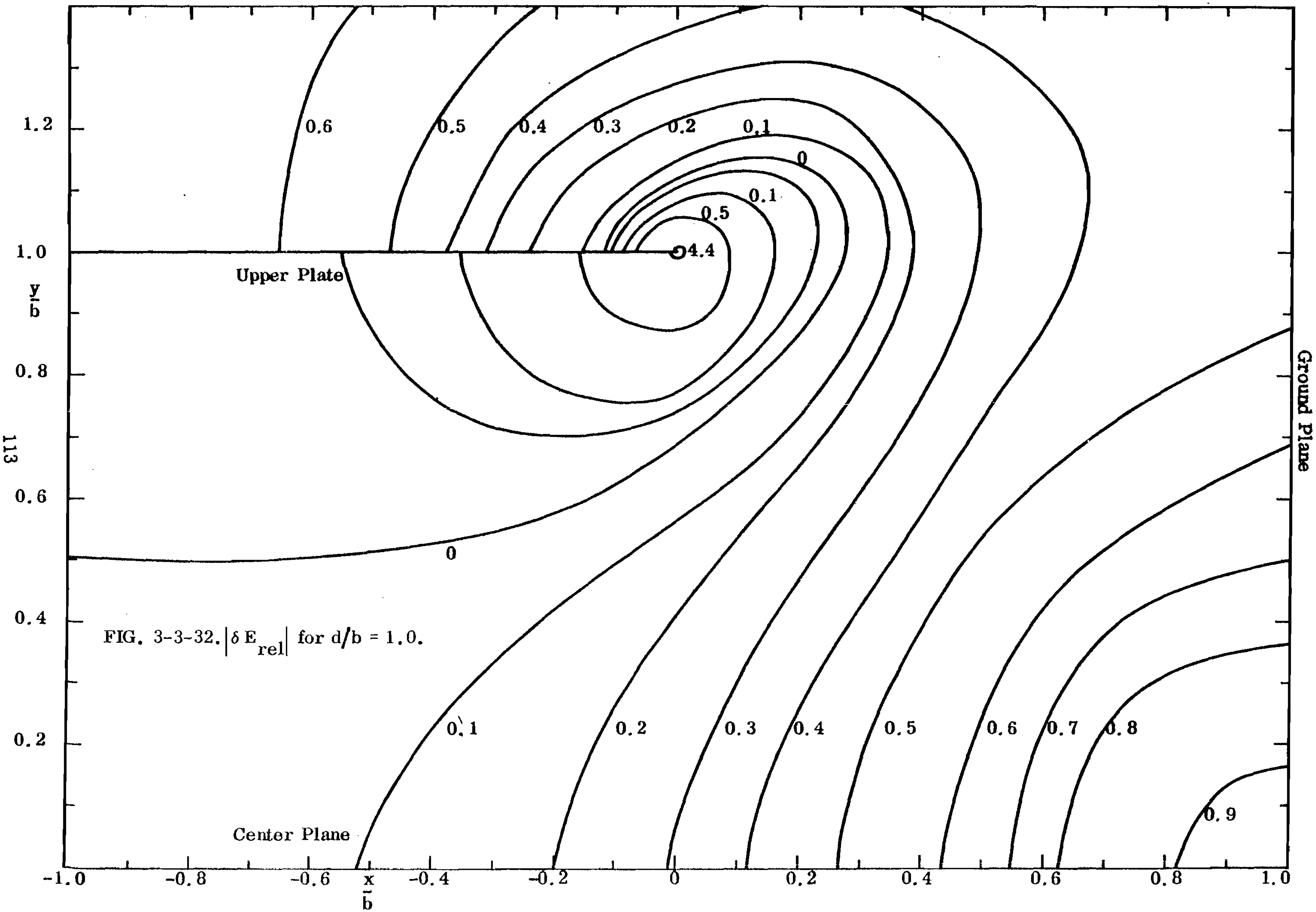

FIG. 3-3-32. $|\delta E_{rel}|$ for $d/b = 1.0$.

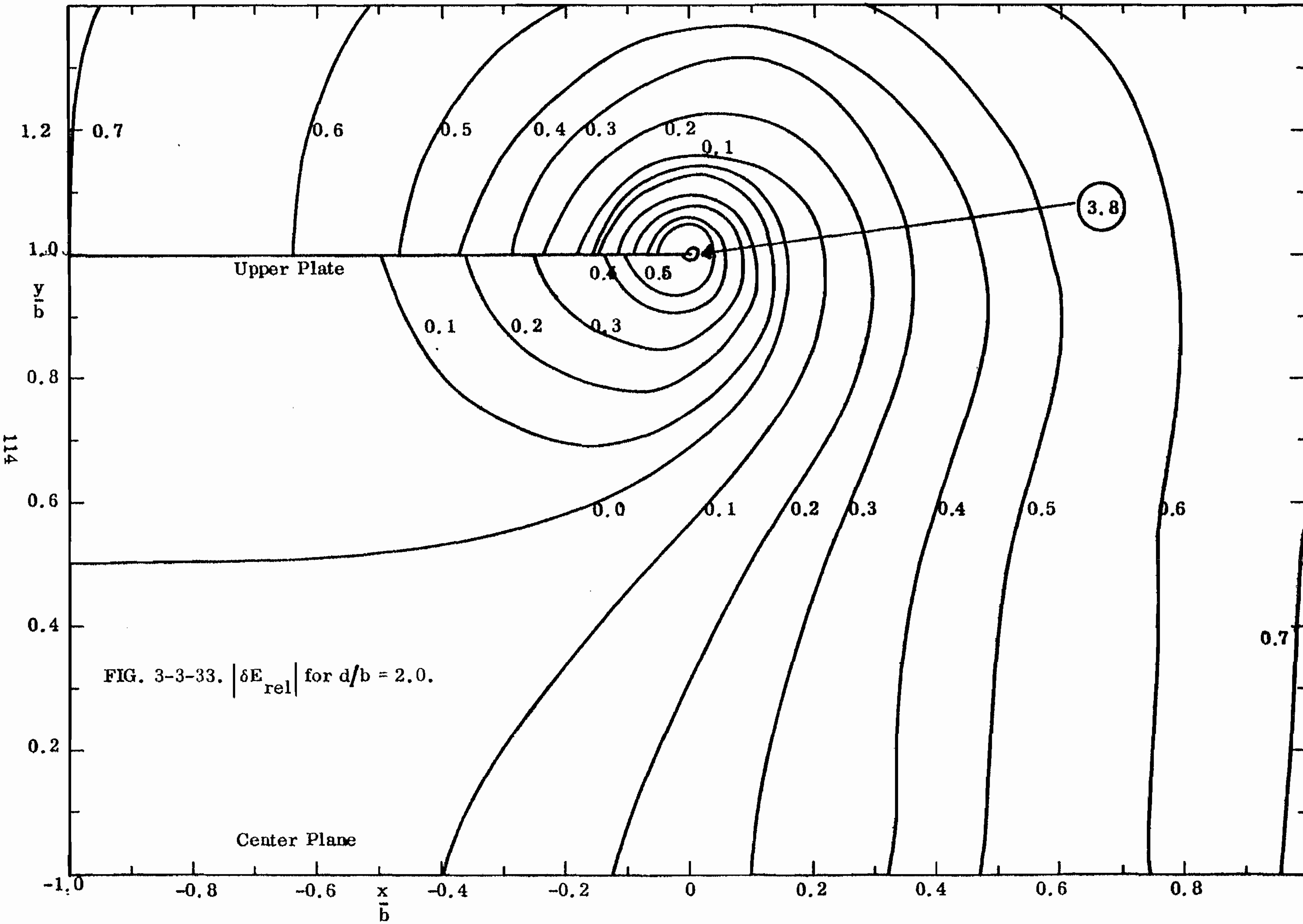

FIG. 3-3-33. $|\delta E_{rel}|$ for $d/b = 2.0$.

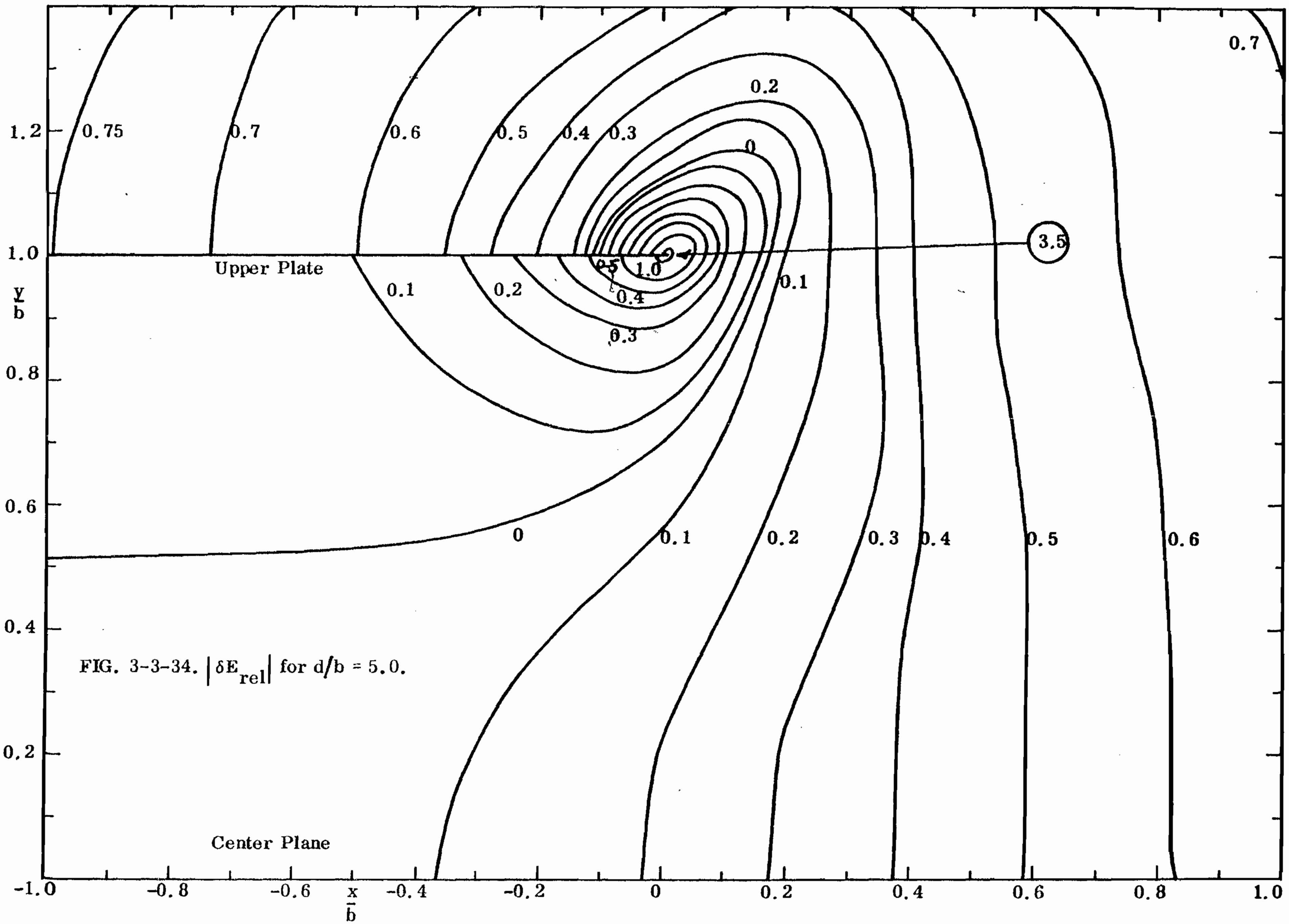

FIG. 3-3-34. $|\delta E_{rel}|$ for $d/b = 5.0$.

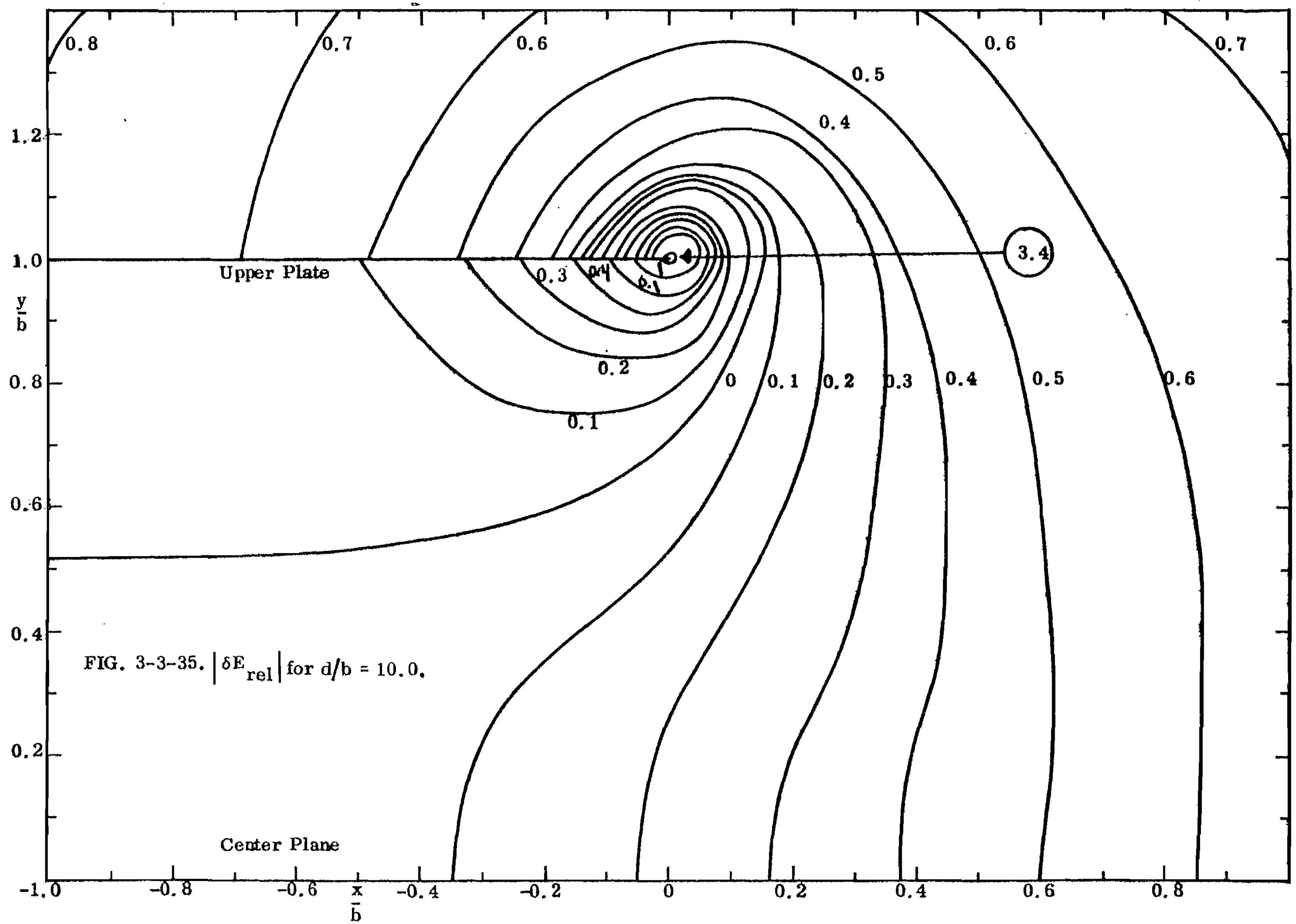

FIG. 3-3-35. $|\delta E_{rel}|$ for d/b = 10.0.

4. Conclusions

The primary objective of this work is to investigate the effect of the ground proximity on the field distribution of a semi-infinite parallel plate transmission line. Assuming that the ground is a perfectly conducting plane, an exact electrostatic solution is obtained by conformal transformation for such a transmission line system. Based on the results shown in Sections 2 and 3, the following observations are made for field lines and electric field intensities.

(a) For $d/b \gtrsim 2.0$, the presence of the perfectly conducting plane ground causes only a small modification of the fields in regions outside the plates, while the modification of the fields in the region between the plates is negligible.

(b) For all cases of d/b considered in this report, the relative electric field is nearly uniform in the region between the plates when $x/b \lesssim -1.5$.

(c) The presence of a perfectly conducting plane ground causes the electric field intensity to increase near the upper plate and to decrease on the center plane.

(d) The electric charges on the two parallel plates are increased due to the presence of the ground and hence, in general, the capacitance per unit length is increased.

(e) The increase in electric charge on the upper plate is more prominent on the outer surface than on the inner surface.

5. Acknowledgment

The authors wish to thank Captain Carl E. Baum of the Weapons Laboratory, Kirtland Air Force Base for valuable suggestions in the course of this work.

6. References

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2. Brown, T.L. and K.D.Granzow (April 1968) SENSOR AND SIMULATION NOTE 52, "A Parameter Study of Two Parallel Plate Transmission Line Simulators of EMP Sensor and Simulation Note 21."

3. Baum, Carl E. (April 1969) SENSOR AND SIMULATION NOTE 82, "Some Considerations Concerning a Horizontally Polarized Transmission Line Simulator."

4. Baum, Carl E. (July 1970) SENSOR AND SIMULATION NOTE 112, "Low Frequency Magnetic Field Distribution for a Simulator with the Geometry of a Half Toroid Joined to the Surface of a Medium with Infinite Conductivity."

APPENDIX — The Conformal Transformation

Our aim is to find the distribution of electric field and magnetic field lines of a semi-infinite parallel plate transmission line system which is placed at some arbitrary distance above the perfectly conducting plane ground as illustrated in Fig. A-1.

Because of the symmetry in the system about the center plane, it is sufficient to consider only the region in the upper- half-plane, $y \geq 0$, $x \leq d$. This region can be transformed by Schwartz-Christoffel transformation, onto the upper half of the t-plane with the line segments A-P-Q-C-D transformed into the real axis of the t-plane. The configuration of the line segments so transformed in the t-plane is shown in Fig. A-2, where A', P', Q', C' and D' are the t-plane images of the corresponding points A, P, Q, C and D in the z-plane .

The t-plane configuration is then transformed onto the w-plane by

$$w = \mathrm{Ln}\, t \tag{A. 1}$$

where

$$w = u(\text{flux}) + i\, v\ (\text{potential}) .$$

The coordinates of the corresponding points in each plane effected by the successive transformations are tabulated below. The point t_o in the t-plane corresponds to the point C in the z-plane (Fig. A-1), the coordinates of which represent the separation distance between the transmission line system and the plane ground.

Points	z-plane	t-plane	w-plane
A	$-\infty + ib$	$-\infty + io$	$\infty + i\pi$
P	$0 + ib$	$-1 + io$	$0 + i\pi$
Q	$-\infty$	$0 + io$	$-\infty$
C	$d + io$	$t_o + io$	$\mathrm{Ln}t_o + io$
D	$0 + i\infty$	$\infty + io$	$\infty + io$

The Schwartz-Christoffel transform of the configuration of the semi-infinite parallel plate transmission line system including the plane ground (Fig. A-1) is represented by the differential equation

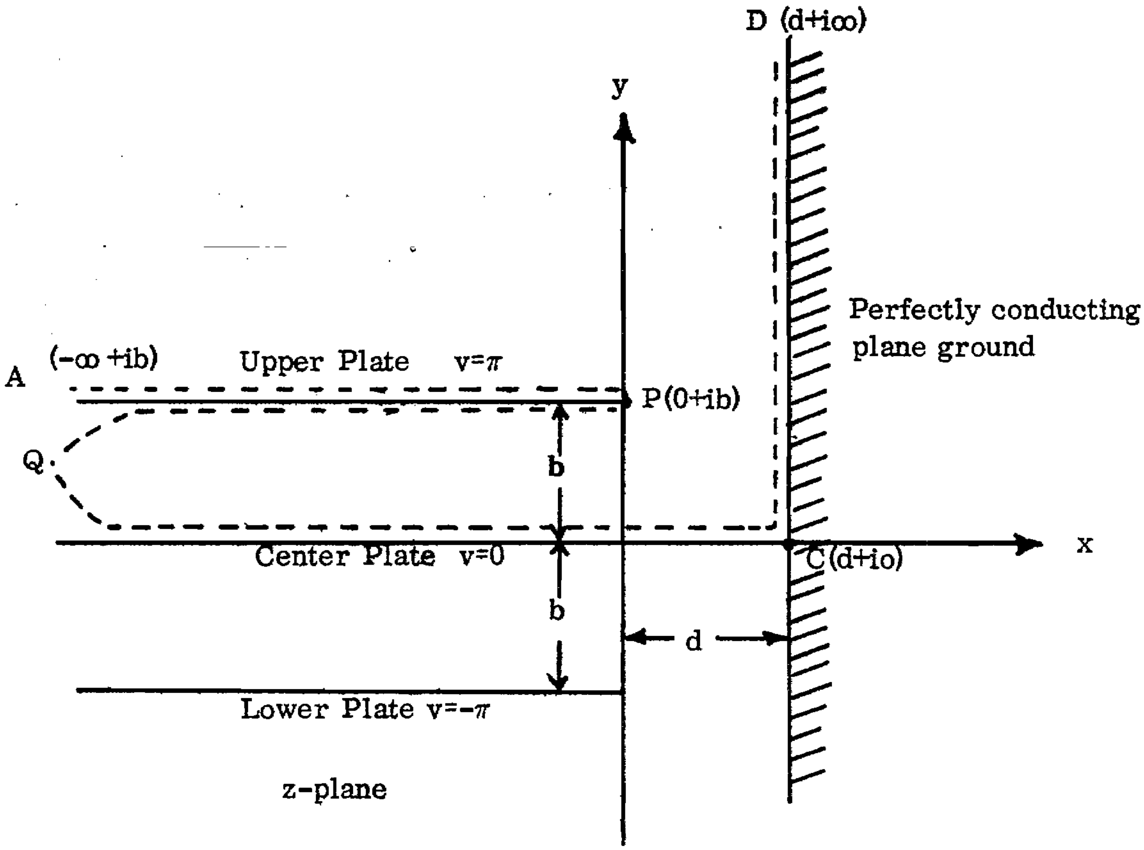

FIG. A-1: A semi-infinite parallel plate transmission line system in the z-plane in the presence of the perfectly conducting ground.

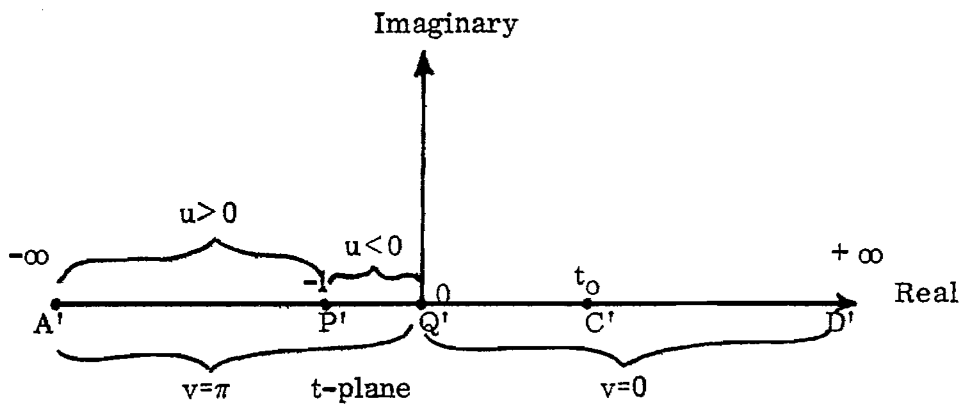

FIG. A-2: The t-plane configuration of the line segments A-P-Q-C-D in z-plane.

$$\frac{dz}{dt} = A\left[\frac{t+1}{t}\,\frac{1}{\sqrt{t-t_o}}\right] = A\left[\frac{1}{\sqrt{t-t_o}} + \frac{1}{t\sqrt{t-t_o}}\right] , \tag{A.2}$$

where the constant A is yet to be determined. Integrating (A.2) and making use of (A.1), one finds

$$z = C_1\left[\frac{w}{2} - \sqrt{t_o}\sqrt{t_o - \exp(w)} - \mathrm{Ln}\left(\sqrt{t_o} + \sqrt{t_o - \exp(w)}\right)\right] + C_2 \tag{A.3}$$

where the real constants C_1, C_2 and t_o are to be determined by use of the coordinates of the points A, C and P indicated in Fig. A-1 as follows:

a) Determination of C_2

The coordinates of the point C is given by

$$z \doteq d + io \text{ in the z-plane}$$

and

$$w = \mathrm{Ln}t_o + io \text{ in the w-plane.}$$

Upon substitution of these into (A.3), one obtains

$$C_2 = d . \tag{A.4}$$

b) Determination of C_1

The coordinates of the point A are given by

$$z = x + ib \text{ in the z-plane } (ix \rightarrow -\infty)$$

and

$$w = u + i\pi \text{ in the w-plane } (iu \rightarrow +\infty) .$$

The use of these in (A.3) yields

$$x + ib = C_1\left[\frac{u+i\pi}{2} - \sqrt{t_o}\sqrt{t_o + \exp(u)} - \mathrm{Ln}\left(\sqrt{t_o} + \sqrt{t_o + \exp(u)}\right)\right] + d.$$

Equating the real and imaginary parts on both sides, one finds

$$C_1 = \frac{2b}{\pi} . \tag{A.5}$$

c) Determination of t_o.

The coordinates of the point P is given by

$z = 0 + ib$ in the z-plane

and

$w = o + i\pi$ in the w-plane.

In this case, (A.3) becomes

$$0 + ib = \frac{2b}{\pi}\left[\frac{i\pi}{2} - \sqrt{t_o}\sqrt{t_o+1} - \mathrm{Ln}\,(\sqrt{t_o} + \sqrt{t_o+1})\right] + d.$$

Whence, one finds

$$\frac{d}{b} = \frac{2}{\pi}\left[\sqrt{t_o}\sqrt{t_o+1} + \mathrm{Ln}\,(\sqrt{t_o} + \sqrt{t_o+1})\right]. \qquad \text{(A. 6)}$$

Thus, t_o is implicitly given by the relation (A. 6).

Substituting (A. 4), (A. 5) and (A. 6) into (A. 3), one finally obtains $z = x + iy$:

$$z = \frac{2b}{\pi}\left[\frac{w}{2} + \sqrt{t_o(t_o+1)} - \sqrt{t_o(t_o - \exp(w))} - \mathrm{Ln}\,\frac{\sqrt{t_o} + \sqrt{t_o - \exp(w)}}{\sqrt{t_o} + \sqrt{t_o+1}}\right]. \qquad \text{(A. 7)}$$

Noting that $t = e^w$, (A. 7) can be alternatively expressed in the form:

$$z = \frac{2b}{\pi}\left[\frac{w}{2} + \sqrt{e^{u_o}(e^{u_o}+1)} - \sqrt{e^{u_o}(e^{u_o} - e^w)} - \mathrm{Ln}\,\frac{\sqrt{e^{u_o}} + \sqrt{e^{u_o} - e^w}}{\sqrt{e^{u_o}} + \sqrt{e^{u_o} + 1}}\right]. \qquad \text{(A. 8)}$$

Equation (A. 8) was used in the computer program for evaluation of the field line distributions.