NOTE 131

LOW-FREQUENCY MAGNETIC FIELD INTERACTION OF A HALF TOROID SIMULATOR WITH A PERFECTLY CONDUCTING HALF PROLATE SPHEROID

by

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June 1971

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PL/PA 5/19/97

Sensor and Simulation Notes

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Abstract

In this note we consider the low frequency interaction of a half toroid EMP simulator in the vertical position and a perfectly conducting half prolate spheroid resting on a perfectly conducting ground. In particular, we calculate the magnetostatic field on the surface of the spheroid and compare it to the case of the spheroid immersed in a homogeneous magnetic field.

PL 96-1225

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ABSTRACT

In this note we consider the low frequency interaction of a half toroid EMP simulator in the vertical position and a perfectly conducting half prolate spheroid resting on a perfectly conducting ground. In particular, we calculate the magnetostatic field on the surface of the spheroid and compare it to the case of the spheroid immersed in a homogeneous magnetic field.

I. Introduction

In this note we consider the low frequency magnetic field interaction of a half toroid EMP simulator in the vertical position with a perfectly conducting half prolate spheroid situated on the ground. The geometry of this simulator has been described in a previous note[1]. The effect of a hemispherical perfectly conducting body or a half infinite cylinder, on the low frequency magnetic field of the half toroid was investigated in notes 120 (Ref. 2) and 124 (Ref. 3) respectively.

The ground will be taken perfectly conducting under the same set of assumptions that were made before. (See for example Refs. 2, 3, 4). We can then apply image theory to reduce the problem to that of the magnetostatic interaction between a circular loop of radius a and current I and a coaxial perfectly conducting prolate spheroid. The major and minor axes of the spheroid will be denoted by 2c and 2b respectively. We will calculate the current density on the surface of the spheroid and compare it to the situation corresponding to $b/a \to 0$, $I/2a = \text{const.}$, i.e., when the spheroid is immersed in a homogeneous magnetic field $H_o = I/2a$. For a fixed finite value of the radius of the loop "a" the limiting cases $b/c \to 1$ and $b/c \to 0$ should correspond to the problems investigated in notes 120 (Ref. 2) and 124 (Ref. 3) respectively. We present plots of the normalized magnetic field $h = (2aH)/I$ where H is the magnetic field on the surface of the spheroid (equal to the current density) versus z/c with the ratios b/a and c/b as parameters. Plots are also given of $\Delta = (H - H_\infty)/H_\infty$ where H_∞ is the magnetic field on the spheroid when the incident magnetic field is homogeneous and equal to I/2a, versus z/c with b/a and c/b as parameters. Finally, we plot h versus b/a at z/c = 0 and .5 with c/b as a parameter, and the maximum value of Δ over the spheroid versus c/b with b/a as a parameter.

II. Formulation of the Problem

In this note we consider the case of a half toroid simulator in the vertical position. Our test body is a perfectly conducting prolate hemispheroid of major and minor axes 2c and 2b respectively situated on a perfectly conducting ground. Applying the method of images we can reduce our problem to that of a circular current loop of radius a engulfing a full spheroid (Fig. 1).

Spheroidal coordinates ξ, η, ϕ are best suited to the present geometry and we will characterize the circular loop by $\eta = \eta_o$ and $\xi = 0$, and the test body by $\eta = \eta_1$. The family of surfaces ξ = const are hyperboloids of revolution and are orthogonal to the family of the spheroids η = const. All surfaces are confocal with a focal length $f = (c^2 - b^2)^{\frac{1}{2}}$ (Fig. 2). The connection with the cartesian coordinates is

$$\begin{aligned} & & z &= f\xi\eta \\ -1 &\le \xi \le 1 & & \\ & & x &= f\sqrt{(\eta^2 - 1)(1 - \xi^2)}\cos\phi \\ 1 &\le \eta \le \infty & & \\ & & y &= f\sqrt{(\eta^2 - 1)(1 - \xi^2)}\sin\phi \end{aligned} \tag{1}$$

As $f \to 0$ the spheroidal coordinates reduce to spherical ones as $\xi = z/f\eta \to z/r = \cos\theta$, $\eta \to r/f$. Finally, the relationship between η_o, η_1 and a, b, c is

$$\begin{aligned} \eta_1 &= \frac{c}{\sqrt{c^2-b^2}} \\ \eta_o &= \sqrt{\frac{c^2-b^2+a^2}{c^2-b^2}} \end{aligned} \tag{2}$$

For the situation depicted in figure 1 both the incident vector potential (due to the circular current loop) and also the induced or scattered vector potential (due to the induced current on the spheroid) are ϕ-independent and have only a ϕ component. The incident vector potential A_ϕ^{inc} is (Ref. 5)

$$A_\phi^{inc} = -\frac{1}{2}\mu_o I[(\eta_o^2 - 1)(1 - \xi_o^2)]^{\frac{1}{2}} \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{2n+1}{n(n+1)} P_n^1(\xi_o)Q_n^1(\eta_o)P_n^1(\eta)P_n^1(\xi), \quad \eta < \eta_o \qquad (3)$$

where $\xi_o = 0$ and $P_n^1(\xi)(\xi \leq 1)$, $Q_n^1(\eta)(1 \leq \eta < \infty)$ are the associated Legendre functions of the first and second kind. A suitable expression for the scattered vector potential A_ϕ^{sc} is (see Appendix)

$$A_\phi^{sc} = \sum_{n=1}^{\infty} S_n P_n^1(\xi)Q_n^1(\eta), \quad \eta > \eta_1 \qquad (4)$$

To calculate S_n we must satisfy the boundary condition of a vanishing normal magnetic field on the surface of the spheroid, i.e., $H_\eta = 0$. The curl equation $\underline{H} = \mu_o^{-1}\nabla \times \underline{A}$ in spheroidal coordinates is

$$\mu_o H_\xi = \frac{1}{h_\eta h_\phi}\left[\frac{\partial(h_\phi A_\phi)}{\partial\eta} - \frac{\partial(h_\eta A_\eta)}{\partial\phi}\right]$$

$$\mu_o H_\eta = \frac{1}{h_\xi h_\phi}\left[\frac{\partial(h_\xi A_\xi)}{\partial\phi} - \frac{\partial(h_\phi A_\phi)}{\partial\xi}\right] \qquad (5)$$

$$\mu_o H_\phi = \frac{1}{h_\xi h_\eta}\left[\frac{\partial(h_\eta A_\eta)}{\partial\xi} - \frac{\partial(h_\xi A_\xi)}{\partial\eta}\right]$$

where

$$h_\xi = f\left(\frac{\eta^2-\xi^2}{1-\xi^2}\right)^{\frac{1}{2}}$$

$$h_\eta = f\left(\frac{\eta^2-\xi^2}{\eta^2-1}\right)^{\frac{1}{2}} \qquad (6)$$

$$h_\phi = f[(\eta^2 - 1)(1 - \xi^2)]^{\frac{1}{2}} = \rho$$

Recalling that $A_\xi = A_\eta = 0$ we obtain

$$\mu_o H_\xi = \frac{1}{h_\eta h_\phi} \frac{\partial}{\partial \eta} (h_\phi A_\phi)$$

$$\mu_o H_\eta = \frac{-1}{h_\xi h_\phi} \frac{\partial}{\partial \xi} (h_\phi A_\phi) \tag{7}$$

$$\mu_o H_\phi = 0$$

The boundary condition $H_\eta = 0$ can now be written as

$$\left[\frac{\partial h_\phi}{\partial \xi} A_\phi + h_\phi \frac{\partial A_\phi}{\partial \xi}\right]_{\eta=\eta_1} = 0 \tag{8}$$

If we require $A_\phi(\eta = \eta_1) = 0$, (8) is satisfied since A_ϕ is then constant on the surface of the spheroid and consequently $(\partial/\partial\xi)A_\phi = 0$. Applying

$$A_\phi = A_\phi^{inc}(\eta_1) + A_\phi^{sc}(\eta_1) = 0$$

we can find with the help of (3) and (4)

$$S_n = \frac{1}{2} \mu_o I[(\eta_o^2 - 1)(1 - \xi_o^2)]^{\frac{1}{2}} \frac{2n+1}{n^2(n+1)^2} \frac{P_n^1(0)Q_n^1(\eta_o)P_n^1(\eta_1)}{Q_n^1(\eta_1)}$$

i.e.

$$A_\phi^{sc} = \frac{1}{2} \mu_o I[(\eta_o^2 - 1)(1 - \xi_o^2)]^{\frac{1}{2}} \sum_{\substack{n=1 \\ odd}}^{\infty} \frac{2n+1}{n^2(n+1)^2} \frac{P_n^1(0)Q_n^1(\eta_o)P_n^1(\eta_1)}{Q_n^1(\eta_1)} P_n^1(\xi)Q_n^1(\eta) \tag{9}$$

In this note we are interested in the tangential magnetic field along the surface of the spheroid H_ξ. This component is equal to the current density K_ϕ.

$$\mu_o H_\xi = \frac{1}{h_\eta h_\phi}\left[\frac{\partial h_\phi}{\partial \eta} A_\phi + h_\phi \frac{\partial A_\phi}{\partial \eta}\right]_{\eta=\eta_1} \tag{10}$$

Due to the boundary condition $A_\phi(\eta_1) = 0$ (10) reduces to

$$\mu_o H_\xi = \frac{1}{h_\eta} \frac{\partial A_\phi}{\partial \eta} . \tag{11}$$

Performing the differentiation we obtain (setting $\xi_o = 0$)

$$H_\xi(\xi,\eta_1) = -\frac{1}{2}\frac{I}{f}\left(\frac{\eta_1^2-1}{\eta_1^2-\xi^2}\right)^{\frac{1}{2}} (\eta_o^2 - 1)^{\frac{1}{2}}$$

$$\times \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{2n+1}{n^2(n+1)^2} P_n^1(\xi_o) Q_n^1(\eta_o) P_n^1(\xi) \left[\frac{\partial P_n^1(\eta)}{\partial \eta} - \frac{P_n^1(\eta_1)}{Q_n^1(\eta_1)} \frac{\partial Q_n^1(\eta)}{\partial \eta}\right]_{\eta=\eta_1} \tag{12}$$

Using the Wronskian relation

$$P_n^1(\eta) \frac{d}{d\eta} Q_n^1(\eta) - Q_n^1(\eta) \frac{d}{d\eta} P_n^1(\eta) = \frac{n(n+1)}{\eta^2-1} ,$$

we can rewrite (12) as

$$H_\xi = \frac{I}{2f}\left(\frac{\eta_o^2-1}{\eta_1^2-1}\right)^{\frac{1}{2}} \frac{1}{(\eta_1^2-\xi^2)^{\frac{1}{2}}} \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{2n+1}{n(n+1)} P_n^1(0) P_n^1(\xi) \frac{Q_n^1(\eta_o)}{Q_n^1(\eta_1)} \tag{13}$$

Using (2) we can see that $(\eta_o^2 - 1)^{\frac{1}{2}}/(\eta_1^2 - 1)^{\frac{1}{2}} = a/b$. In the Appendix we show that as $b/c \to 1$, (13) reduces, as it should, to the corresponding expression for a sphere. We want to compare H_ξ given by (13) to the situation $a/b \to \infty$ and $I/2a$ = const, i.e., when the spheroid is immersed in a homogeneous magnetic field $H_\infty^{inc} = I/2a$. To find $H_\infty = H_\infty^{inc} + H_\infty^{sc}$ we consider (13) in the limit $a/b \to \infty$ with b and c finite. Recall that

$$Q_n^1(\eta) = -2^n(\eta^2 - 1)^{\frac{1}{2}} \sum_{s=0}^{\infty} \frac{(n+s)!(n+2s+1)!}{s!(2n+2s+1)!} \eta^{-n-2s-2} \tag{14}$$

As $a/b \to \infty$ then $\eta_o = (c^2-b^2+a^2)^{\frac{1}{2}}/(c^2-b^2)^{\frac{1}{2}} \to \infty$, and

$$Q_n^1(\eta_o) \xrightarrow[\eta_o\to\infty]{} -2^n \eta_o \frac{n!(n+1)!}{(2n+1)!} \eta_o^{-n-2} = -2^n \frac{n!(n+1)!}{(2n+1)!} \eta_o^{-(n+1)} . \tag{15}$$

Also notice that as $\eta_o \to \infty$

$$\frac{I}{2f}(\eta_o^2 - 1)^{\frac{1}{2}} \to \frac{I}{2f}\eta_o \to \frac{I}{2a}\eta_o^2 . \tag{16}$$

From (13), (14) and (15) we see that the term involving η_o is η_o^{-n+1} and consequently the only nonzero contribution comes from n = 1. The final result is

$$H_\xi(\eta_o \to \infty) = H_\infty = -\frac{I}{2a}\frac{1}{(\eta_1^2-\xi^2)^{\frac{1}{2}}(\eta_1^2-1)^{\frac{1}{2}}}\frac{P_1^1(\xi)}{Q_1^1(\eta_1)} \qquad (17)$$

We can arrive at (17) by solving the problem directly, and this is done in the Appendix.

In this note, the two quantities of interest are

$$h = \frac{2aH_\xi}{I}$$

$$\Delta = \frac{H_\xi - H_\infty}{H_\infty} \qquad (18)$$

We plot h and Δ versus z/c with the ratios b/a and c/b as parameters. For the case of the infinite cylinder ($c \to \infty$, b finite) we plot h and Δ versus z/a with b/a as a parameter. We show in the Appendix that as $c/b \to \infty$ then $H_\infty \to I/2a$. Thus, $h_\infty = 1$ and $\Delta = h - 1$. We also plot h versus b/a at z/c = 0 and .5 with c/b as a parameter, and the maximum value of Δ over the spheroid versus c/b with b/a as a parameter. For a given finite value of the radius of the loop "a" and as $b/c \to 1$ or $b/c \to 0$ our plots coincide with the corresponding plots considered in notes 120 (Ref. 2) and 124 (Ref. 3) respectively.

III. Numerical Calculations

The calculation of H , given by (13), involves the knowledge of $P_n^1(\xi)(\xi \leq 1)$ and $Q_n^1(\eta)(1 \leq \eta < \infty)$. From the numerical point of view it is advantageous to evaluate these quantities using the recursion formula

$$nF_{n+1}(x) = (2n + 1)xF_n(x) - (n + 1)F_{n-1}(x) \tag{19}$$

where $F_n(x)$ is either $P_n^1(\xi)$ or $Q_n^1(\eta)$.

A recursion relation like (19) is useful only when the errors that may be propagated in the evaluation process do not grow relative to the size of the wanted quantity. Such a process is called stable. For the associated Legendre polynomials an increasing n recursion process is stable; however, this is not the case with the Q_n^1's. The opposite is true, i.e., the process is stable for decreasing n. For this type of calculation the knowledge of the two starting values of the Q_n^1 is required. For a given η the Q_n^1's decrease rapidly with increasing n. This observation allows us to apply Miller's algorithm[6] which suggests that we set $Q_{N+1} = 0$ where N is the starting value for n and assign to Q_N an arbitrary small value, say M. Using the recursion formula for decreasing n we arrive at a value for Q_1^1 which depends linearly on M. Knowing the exact value for Q_1^1 we can now find the true value of any Q_n^1 by multiplying by Q/M where Q is the exact value for Q_1^1. In applying the above procedure we encountered the following difficulty. As η gets larger the Q_n^1's decrease progressively more rapidly with increasing n. Assigning to Q_N^1 the lowest possible value that the computer can register as nonzero we may, depending on η and N, arrive at an intermediate Q_n^1 with a value larger than the maximum allowable number that the computer registers as finite. Luckily we can overcome the difficulty just described, by noting that the series for H_ξ converges faster for larger η_o's and consequently for large η_o, where the difficulty is pronounced, we can only consider a small number of Q_n^1's to arrive at a finite Q_1^1.

Appendix

I. Derivation of equation (4).

The scattered vector potential $\underline{A}^{sc} = A_\phi^{sc}\hat{a}_\phi$ satisfies Laplace's equation, but A_ϕ^{sc} does not, i.e.

$$\nabla^2 A_\phi^{sc} - \frac{1}{f^2(\eta^2-1)(1-\xi^2)} A_\phi^{sc} = 0 \qquad \text{(A-1)}$$

where

$$\nabla^2 A_\phi^{sc} = \left\{\frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial A_\phi}{\partial\xi}\right] + \frac{\partial}{\partial\eta}\left[(\eta^2-1)\frac{\partial A_\phi}{\partial\eta}\right]\right\}\frac{\eta^2-\xi^2}{f^2} \qquad \text{(A-2)}$$

Consider now $u = A_\phi^{sc} e^{i\phi}$. The Laplacian of u is

$$\nabla^2 u = \frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial u}{\partial\xi}\right] + \frac{\partial}{\partial\eta}\left[(\eta^2-1)\frac{\partial u}{\partial\eta}\right] + \frac{\eta^2-\xi^2}{(1-\xi^2)(\eta^2-1)}\frac{\partial^2 u}{\partial\phi^2} \qquad \text{(A-3)}$$

In view of (A-1) and (A-2) we see that u satisfies Laplace's equation and consequently

$$u = \sum_n \sum_m [A_1 P_n^m(\xi) + A_2 Q_n^m(\xi)][A_3 P_n^m(\eta) + A_4 Q_n^m(\eta)][A_5 e^{im\phi} + A_6 e^{-im\phi}]$$

Recalling that $u = A_\phi^{sc} e^{i\phi}$, we understand that $m = 1$. Furthermore, A_ϕ^{sc} must be finite at $\xi = 1$ and at infinity $(\eta \to \infty)$; this implies that $A_2 = A_3 = 0$. Finally, in view of (3) for the incident vector potential we need only consider odd values for n.

II. Equation (13) in the limit $b/c \to 1$.

In this limit (2) gives $\eta_1 \to \infty$, $\eta_o \to \infty$ and we can use the asymptotic expression for $Q_n^1(x)$,

$$Q_n^1(x) \xrightarrow[x\to\infty]{} -\frac{n!(n+1)!}{(2n+1)!} 2^n x^{-(n+1)}$$

We also notice that

$$\xi = \frac{z}{\eta_1 f} = \frac{z}{c} = \cos\theta$$

and (13) can be written as

$$H_\xi(\eta = \eta_1) \to \frac{I}{2a}\frac{a}{f\eta_1}\frac{\eta_o}{\eta_1}\sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{2n+1}{n(n+1)} P_n^1(0)P_n^1(\cos\theta)\left(\frac{\eta_1}{\eta_o}\right)^{n+1} \qquad \text{(A-4)}$$

We have $\eta_1/\eta_o = c/(c^2 - b^2 + a^2)^{\frac{1}{2}} = c/a$, $f\eta_1 = c$ and (A-4) becomes

$$H_\xi(r = b) \to \frac{I}{2a}\sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{2n+1}{n(n+1)} P_n^1(0)P_n^1(\cos\theta)\left(\frac{b}{a}\right)^{n-1} \qquad \text{(A-5)}$$

Equation (A-5), as we expected, is equal to $-H_\theta$ which corresponds to the situation of a perfectly conducting sphere surrounded by a circular current loop.

III. A perfectly conducting prolate spheroid in a uniform magnetic field.

If the uniform magnetic field is $\underline{H}^{inc} = H_o\hat{a}_z$ then it can be derived from a scalar potential $\Phi^{inc} = -H_o z = -H_o f\xi\eta = -H_o fP_1(\xi)P_1(\eta)$. The spheroid has its major axis along the z axis and the scattered scalar potential should have the form $\Phi^{sc} = AP_1(\xi)Q_1(\eta)$. The boundary condition is $H_\eta(\eta = \eta_1) = 0$, i.e., $\partial\Phi/\partial\eta = 0$, which yields

$$A = fH_o\left[\frac{(d/d\eta)P_1}{(d/d\eta)Q_1}\right]_{\eta=\eta_1}$$

The total potential $\Phi = \Phi^{sc} + \Phi^{inc}$ is then given by

$$\Phi = -fH_oP_1(\xi)\left\{P_1(\eta) - Q_1(\eta)\left[\frac{(d/d\eta)P_1}{(d/d\eta)Q_1}\right]_{\eta=\eta_1}\right\}$$

and $H_\xi = -1/h_\xi(\partial/\partial\xi)\Phi$. If we calculate the field at $\eta = \eta_1$ we can use the Wronskian relation

$$P_1(\eta)\,\frac{dQ_1}{d\eta} - Q_1(\eta)\,\frac{dP_1(\eta)}{d\eta} = \frac{1}{1-\eta^2}$$

to finally obtain

$$H_\xi = -\,H_o\,\frac{P_1^1(\xi)}{(\eta_1^2-\xi^2)^{\frac{1}{2}}}\,\frac{1}{(\eta_1^2-1)^{\frac{1}{2}}}\,\frac{1}{Q_1^1(\eta_1)} \tag{A-6}$$

where $P_1^1(\xi) = (1 - \xi^2)^{\frac{1}{2}}(d/d\xi)P_1(\xi)$ and $Q_1^1(\eta) = (\eta^2 - 1)^{\frac{1}{2}}(d/d\eta)Q_1$. We can easily show that as $c/b \to 1$ (the spheroid becomes a sphere) (A-6) gives $H_\xi = (3/2)H_o \sin\theta$ and as $c/b \to \infty$ (either $c = \infty$, b finite which is the cylinder case or $b = 0$, i.e., no spheroid) $H_\xi = H_o$ as it should.

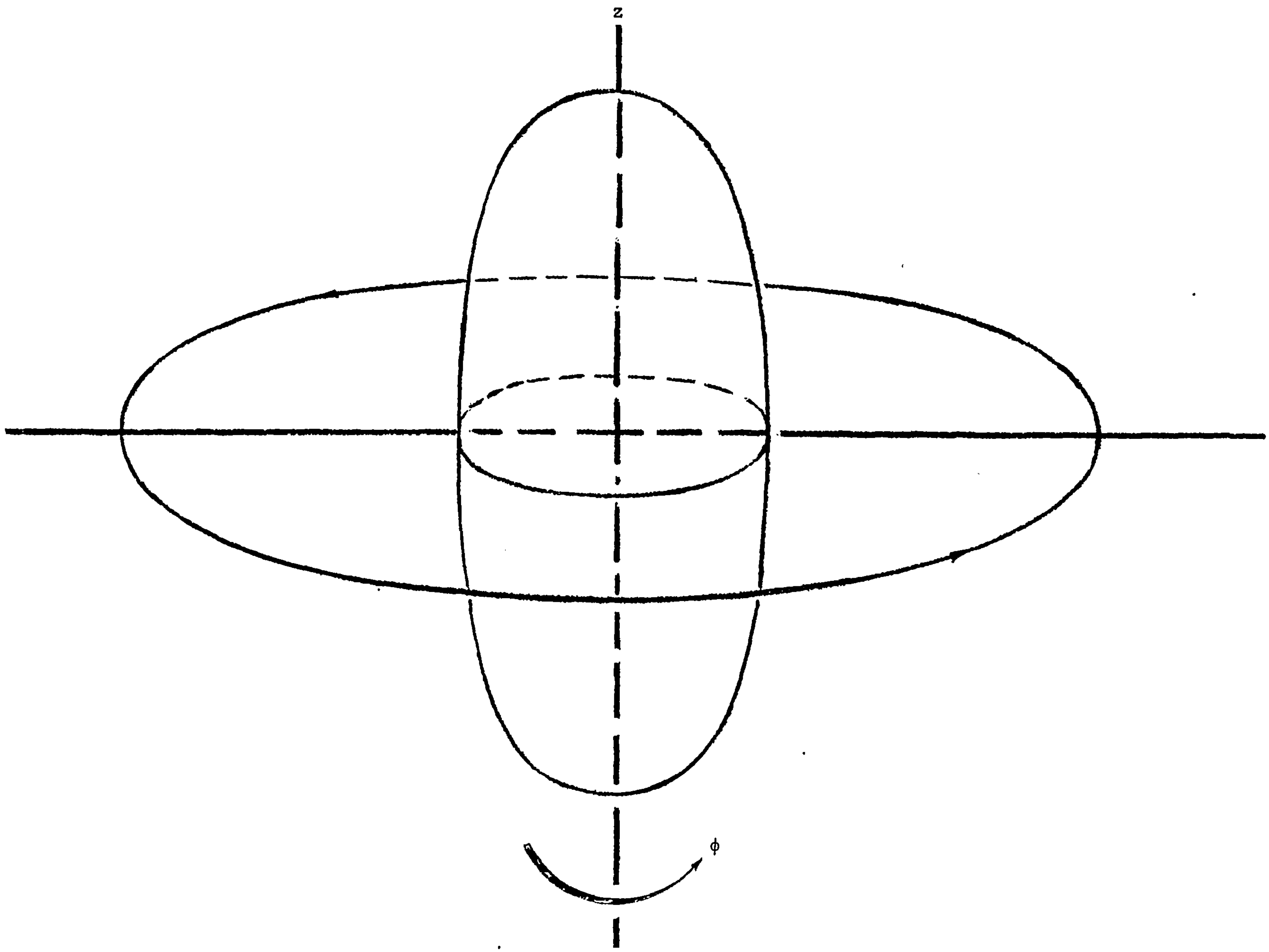


FIGURE 1. The Current Loop, Prolate Spheroid Geometry.

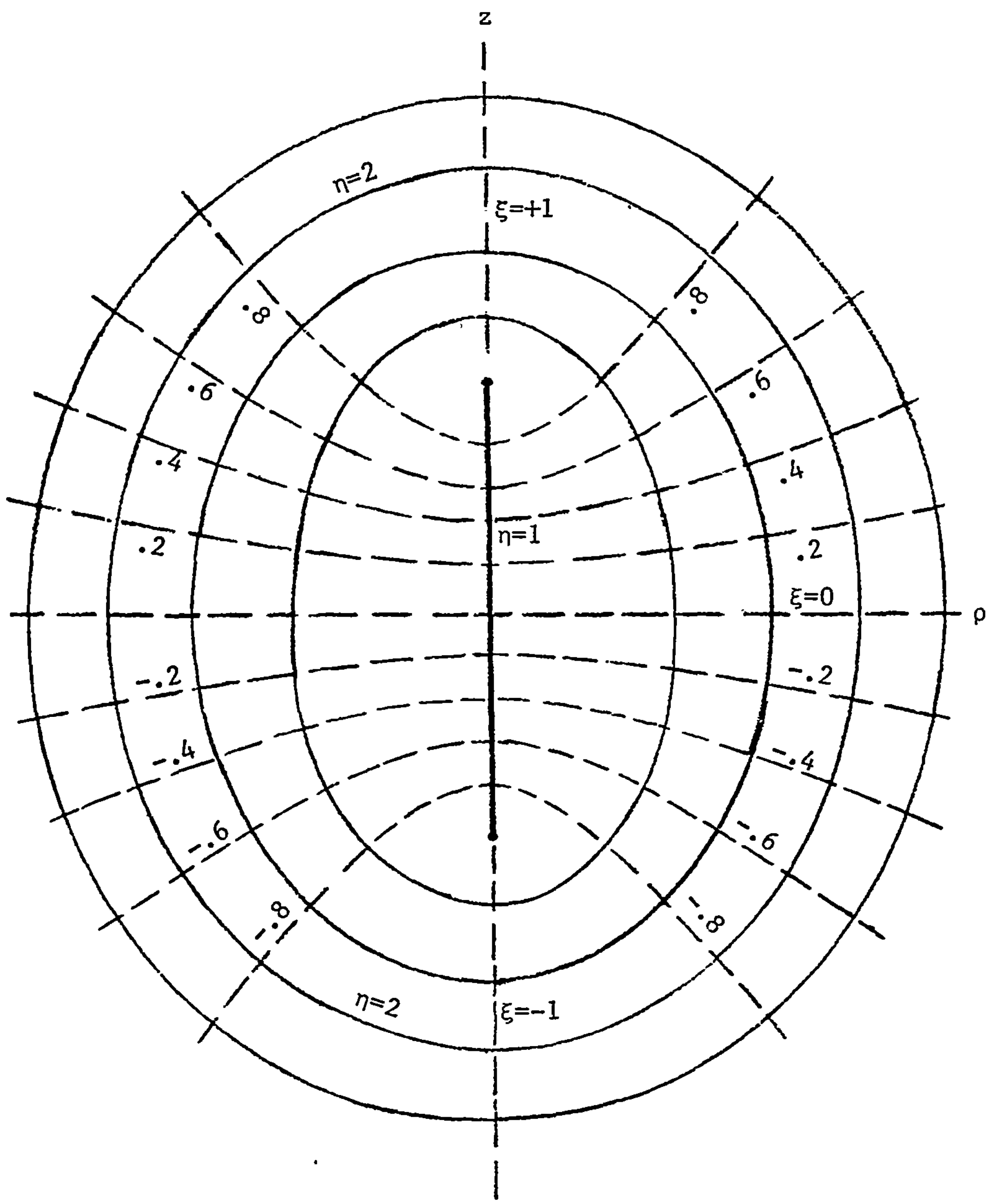
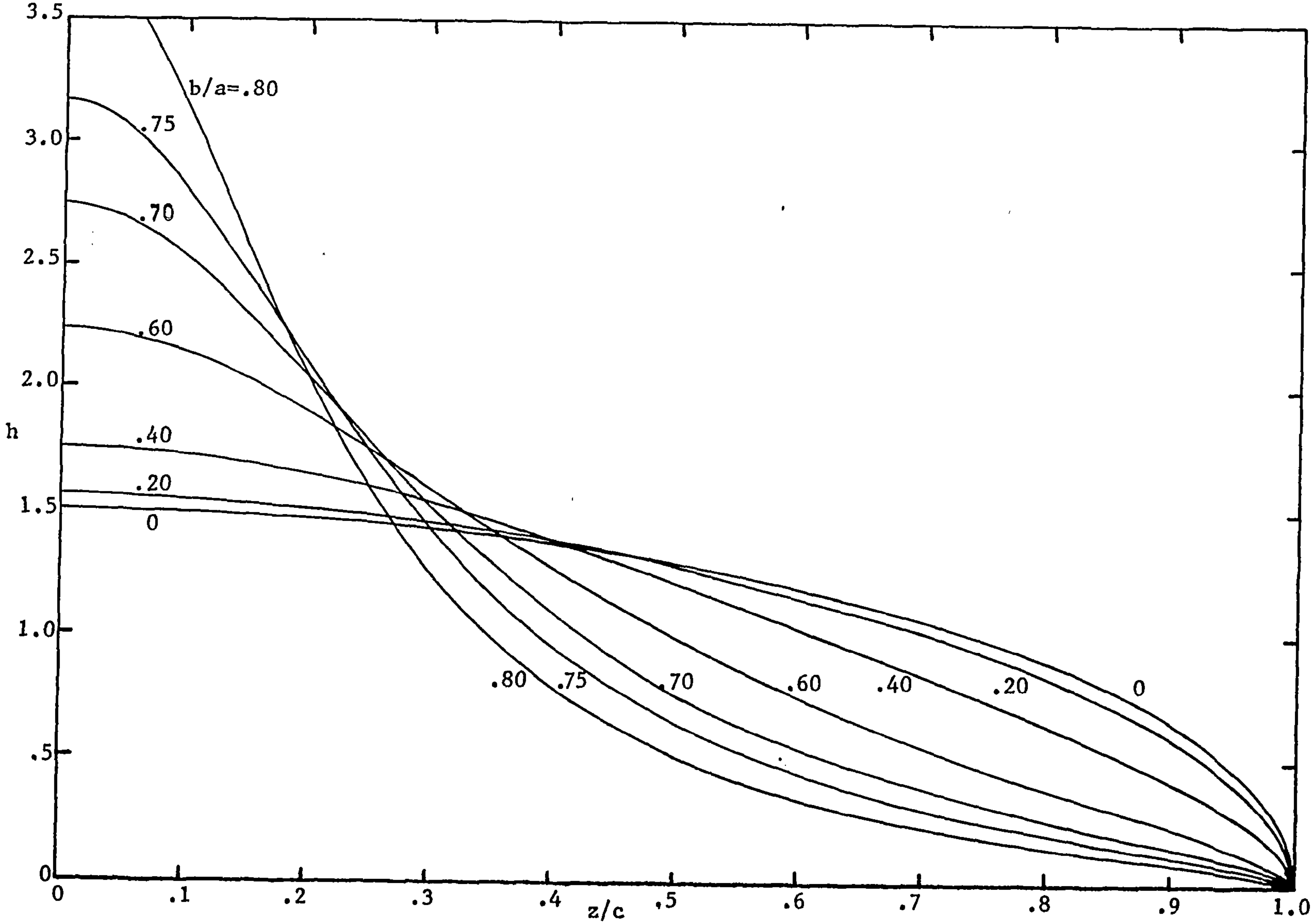


FIGURE 2. Prolate Spheroidal System of Coordinates.



3.5
3.0
2.5
2.0
1.5
1.0
.5
0
h
b/a=.80
.75
.70
.60
.40
.20
0
.80
.75
.70
.60
.40
.20
0
0
.1
.2
.3
.4
z/c
.5
.6
.7
.8
.9
1.0

FIGURE 3. c/b=1

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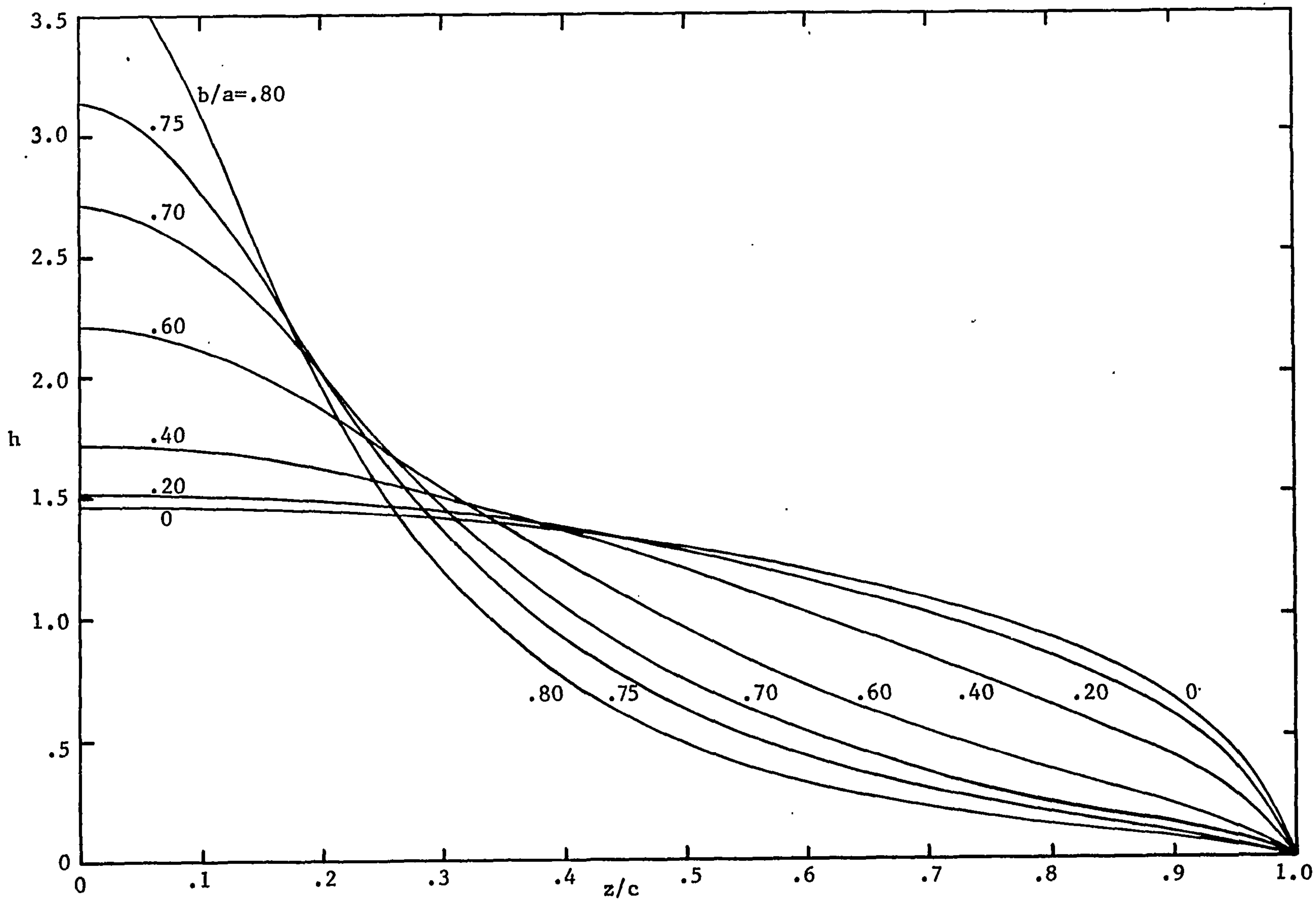

FIGURE 4. c/b=1.05

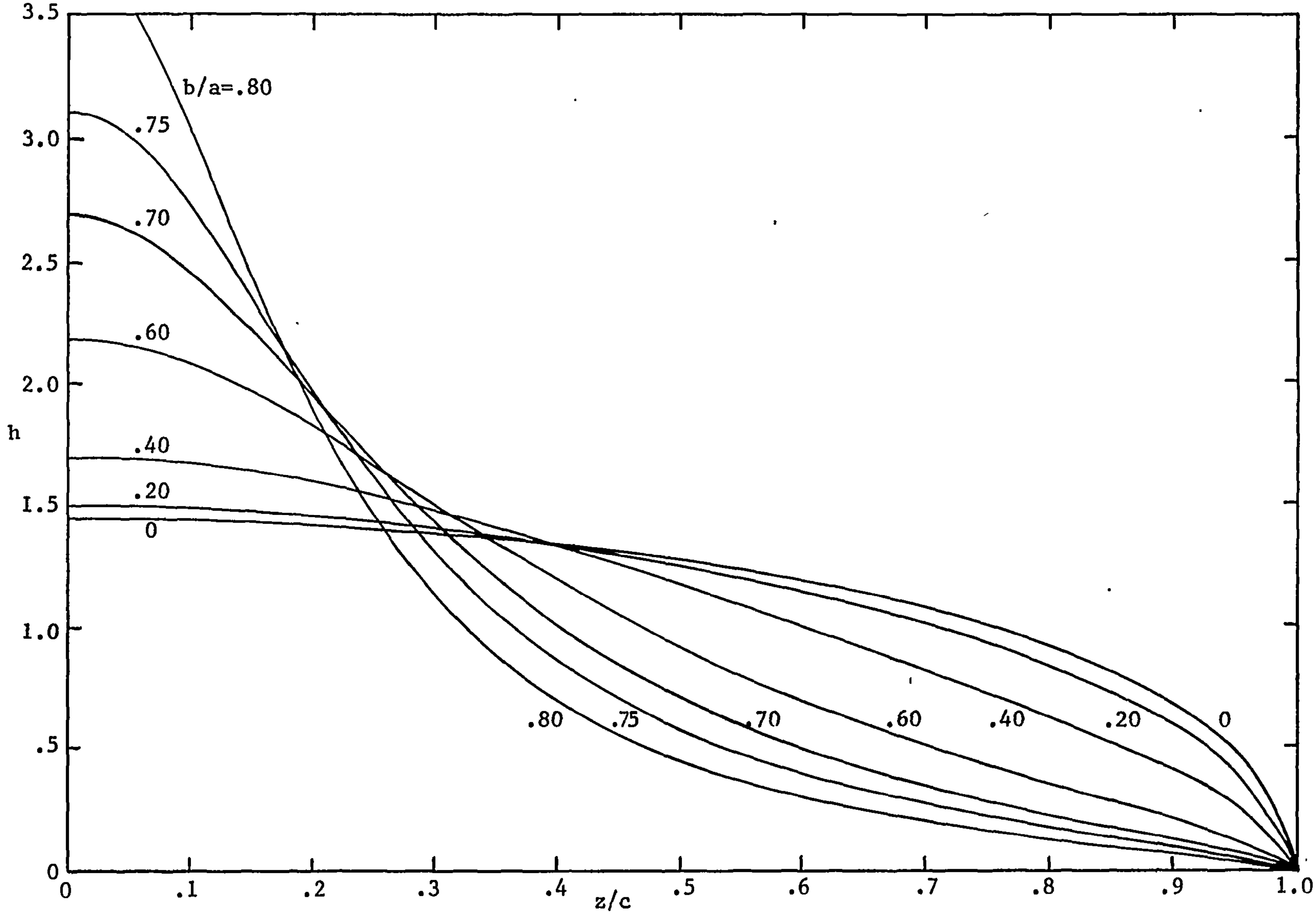

FIGURE 5. c/b=1.1

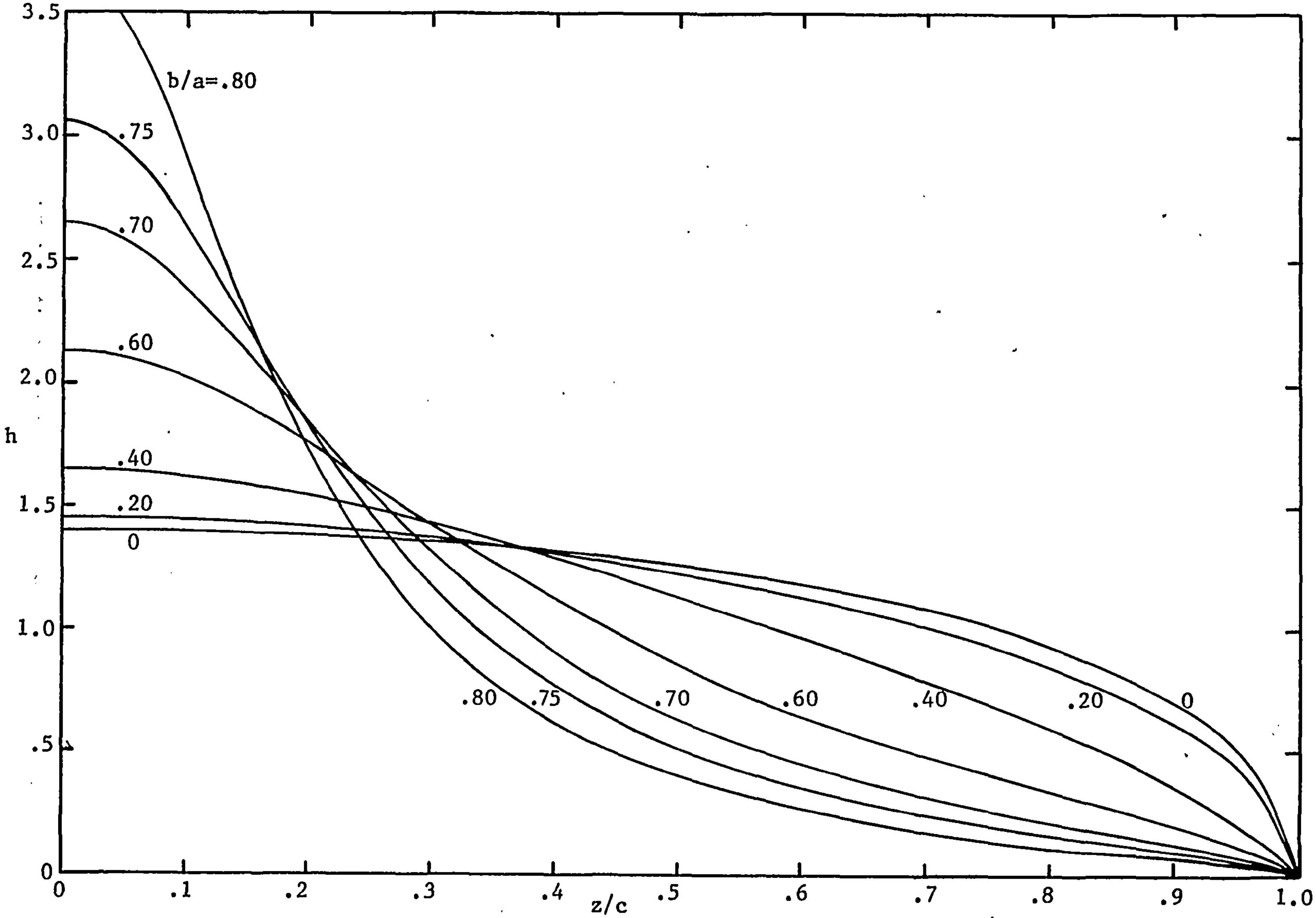

FIGURE 6. c/b=1.2

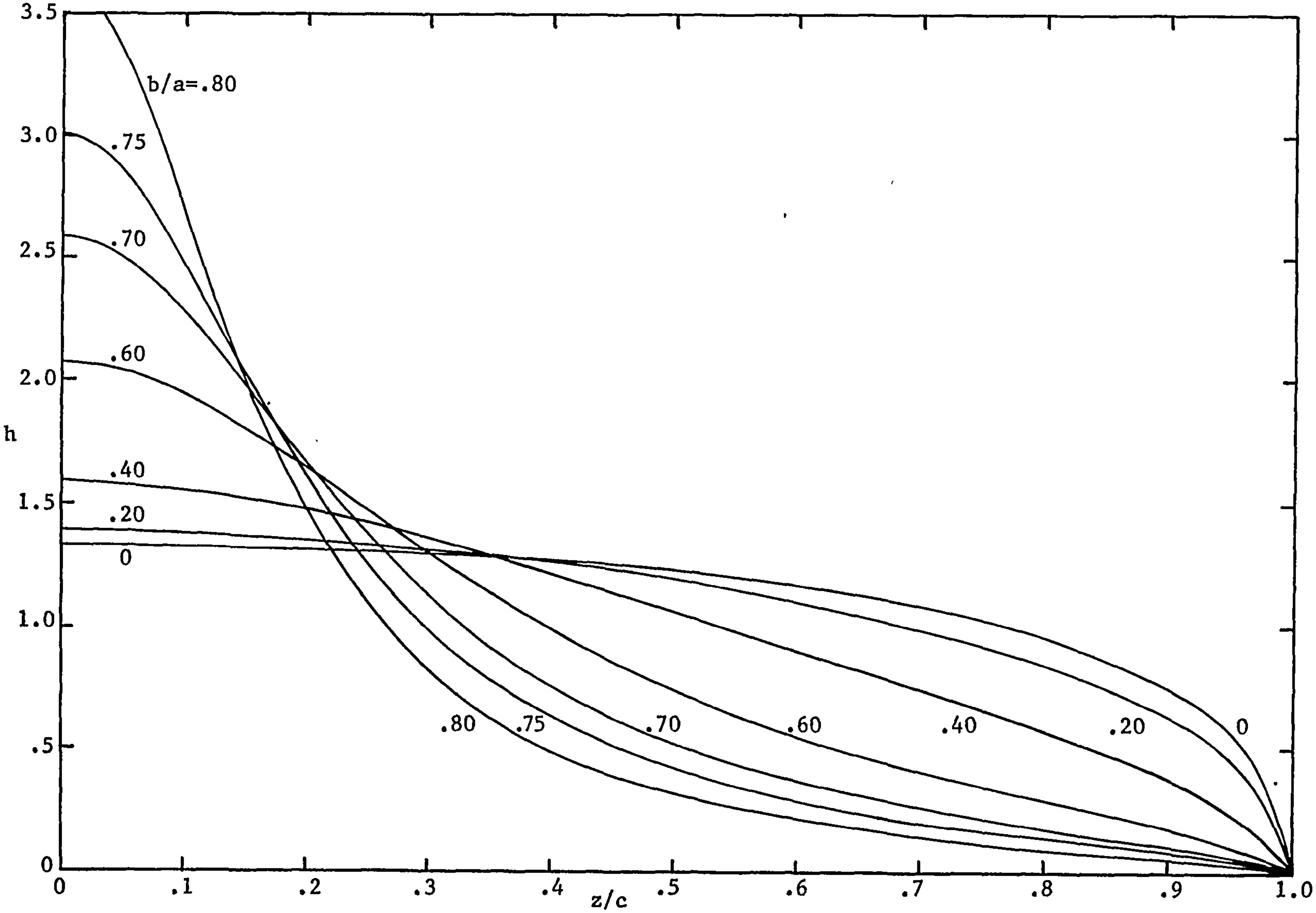

FIGURE 7. c/b=1.4

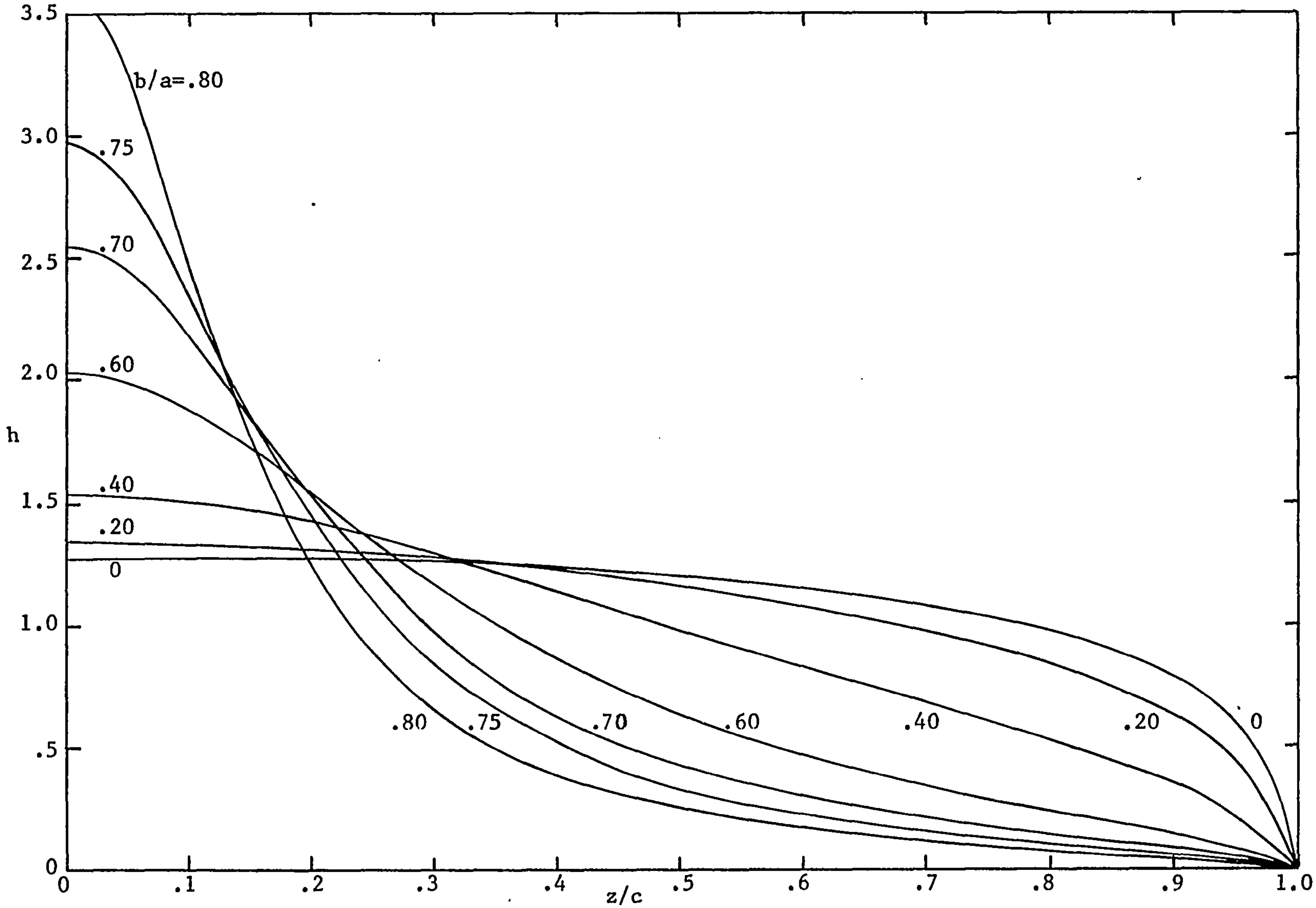

FIGURE 8. c/b=1.6

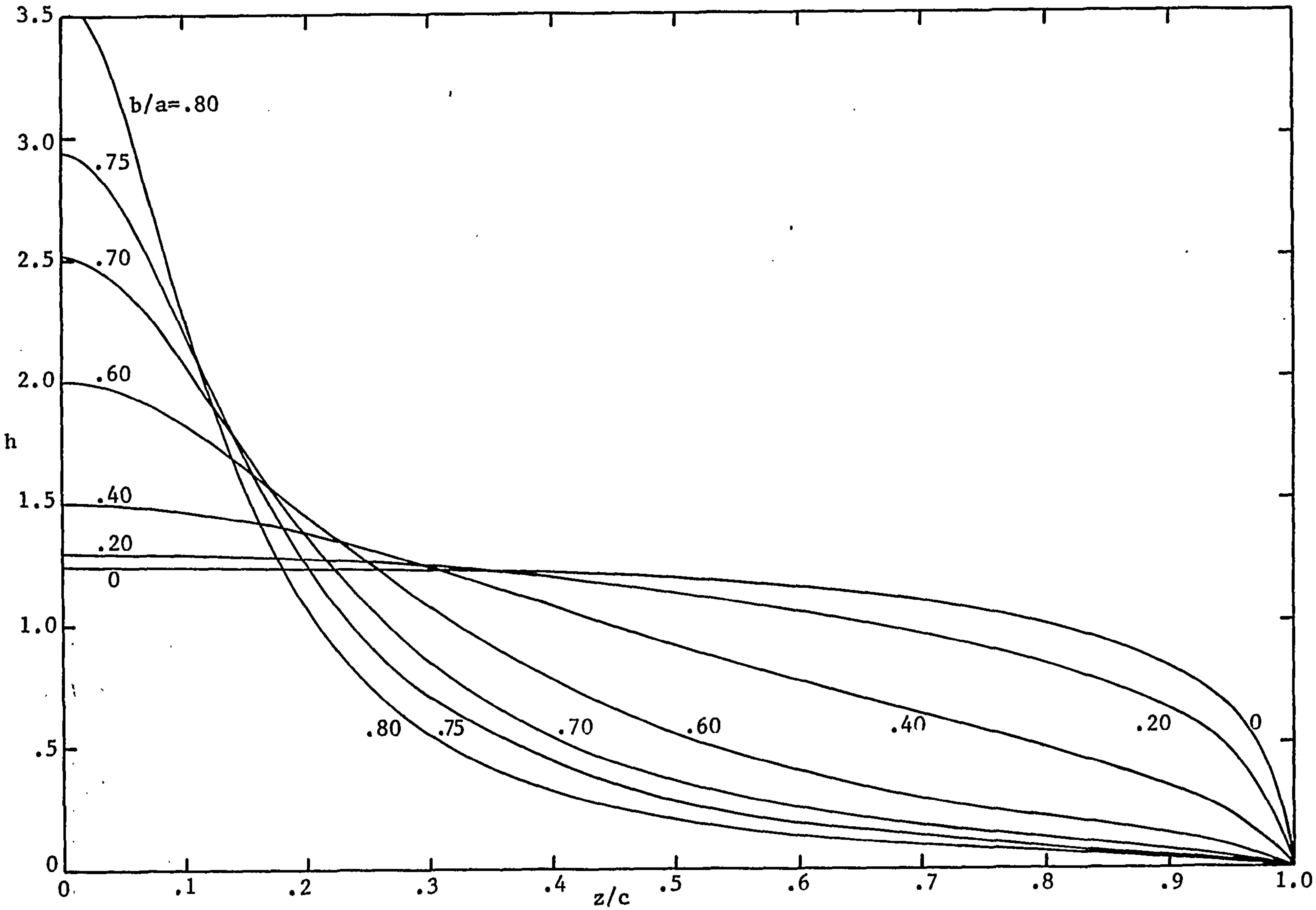

FIGURE 9. c/b=1.8

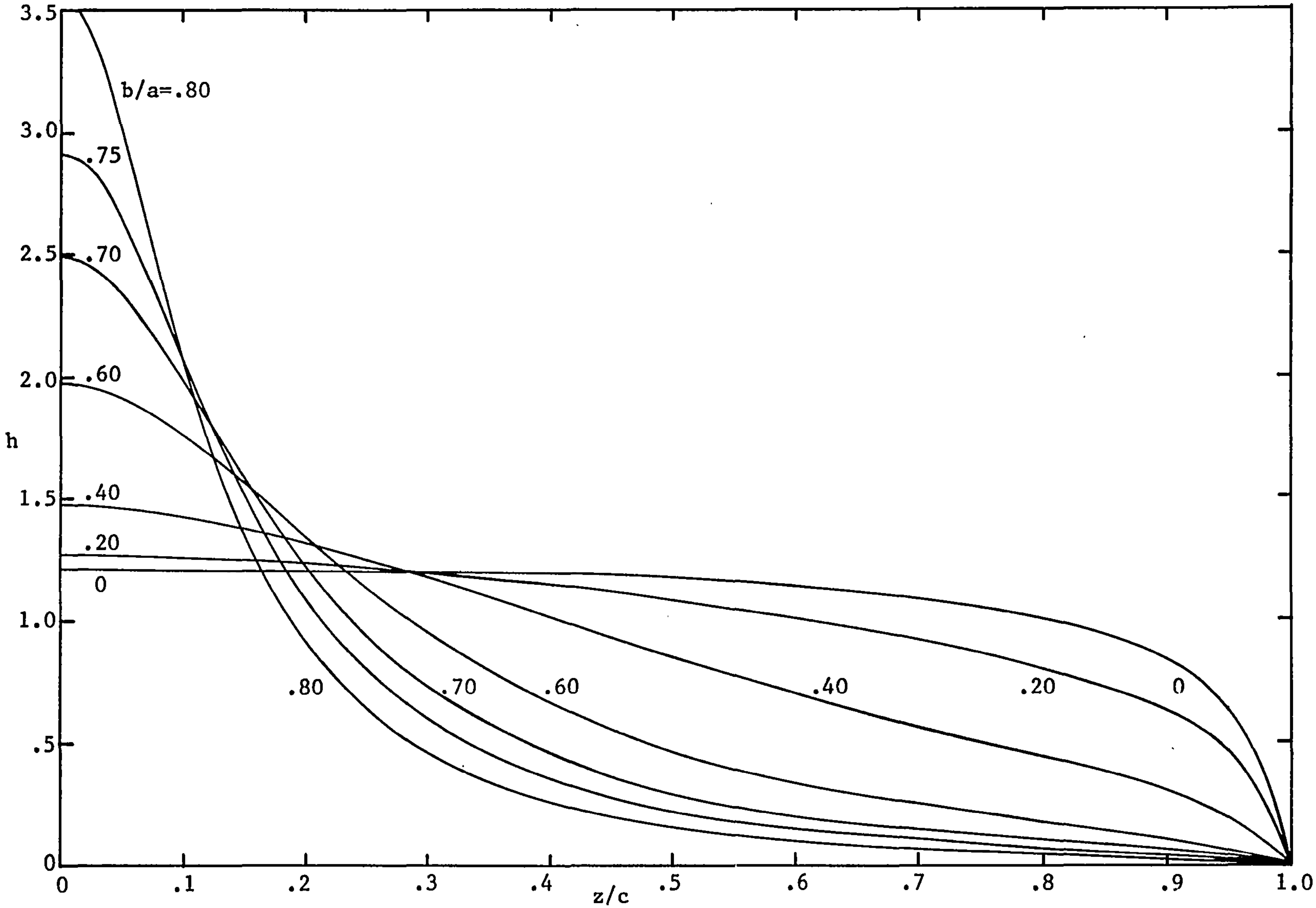

FIGURE 10. c/b=2

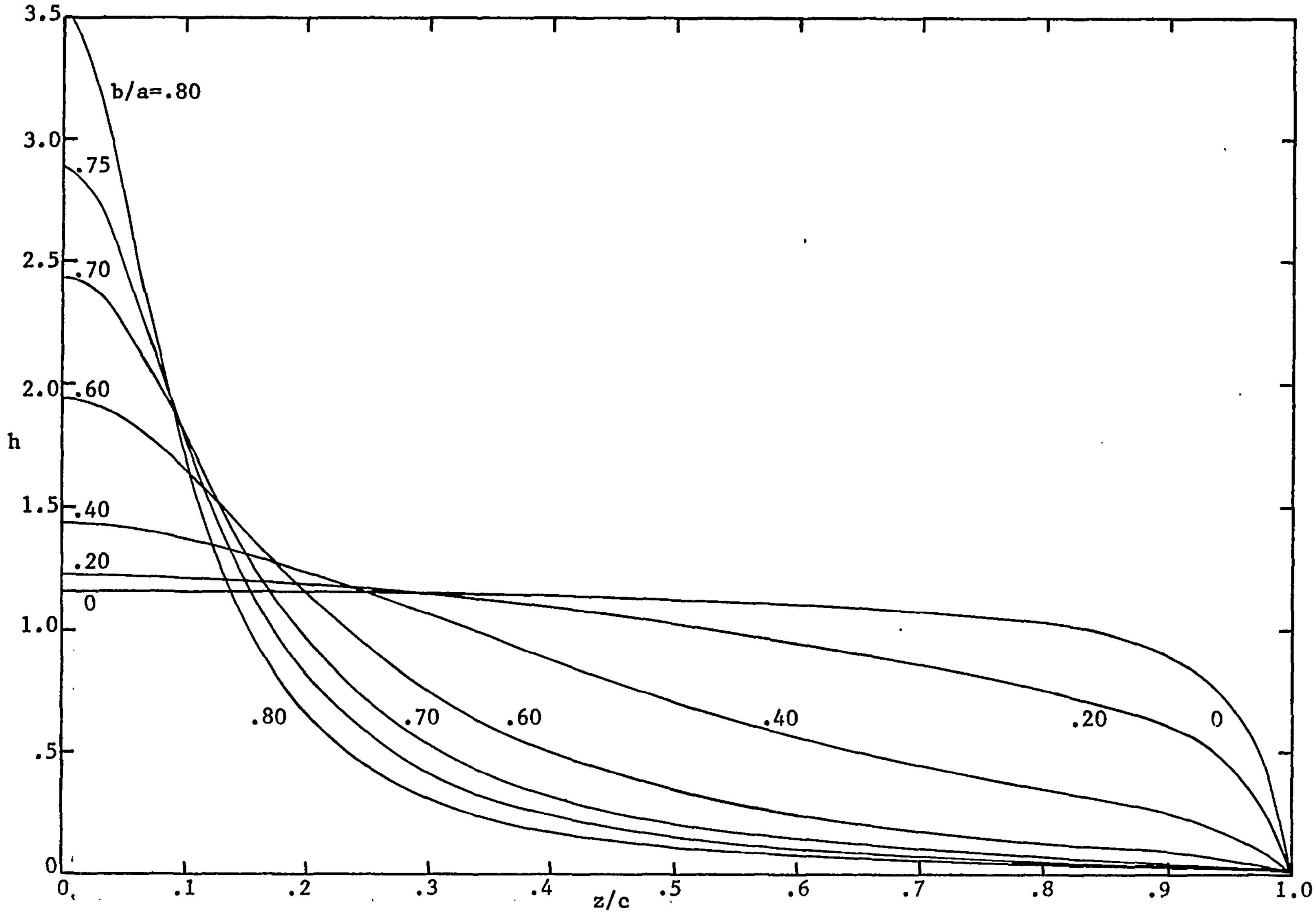

FIGURE 11. c/b=2.5

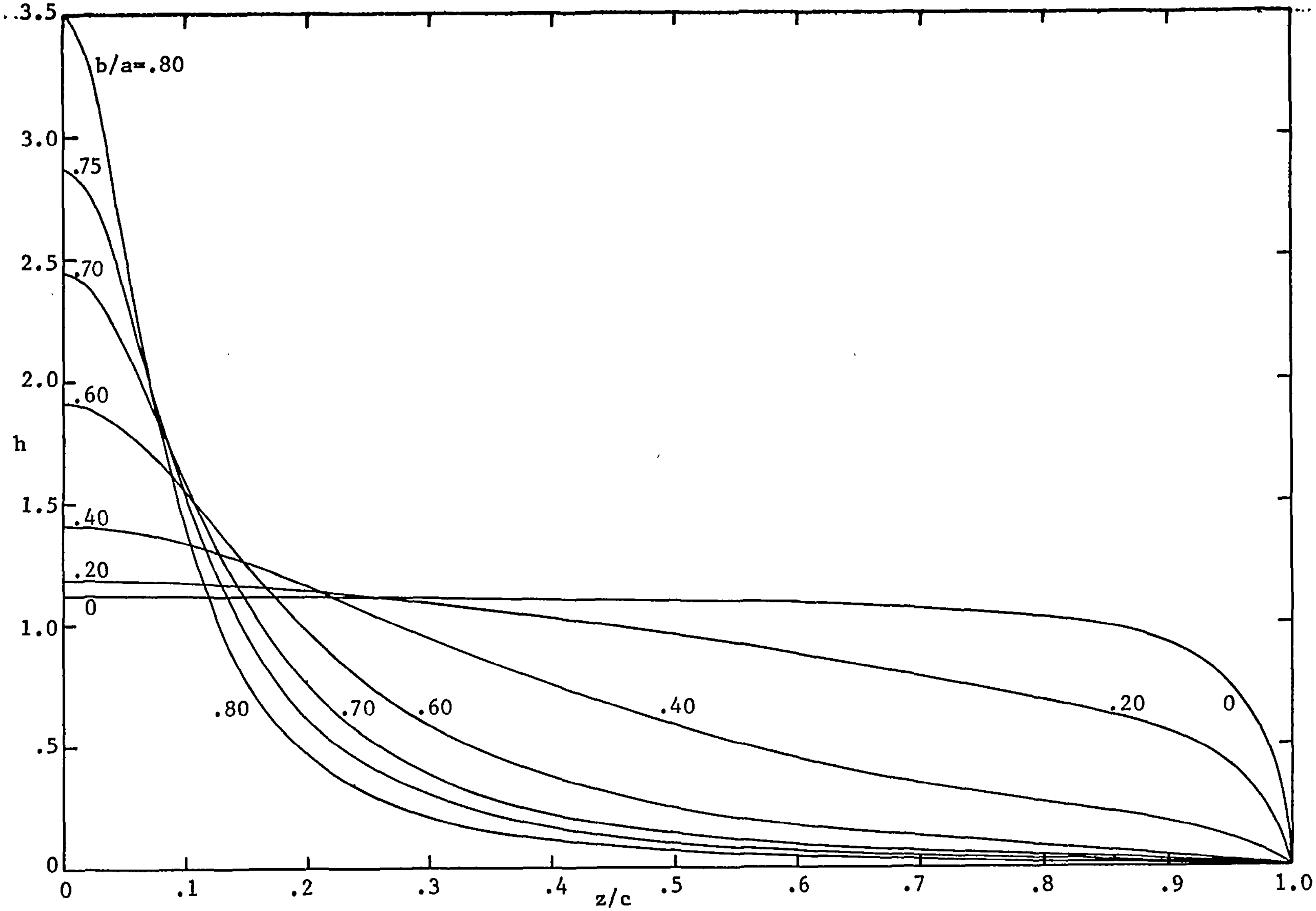

FIGURE 12. c/b=3

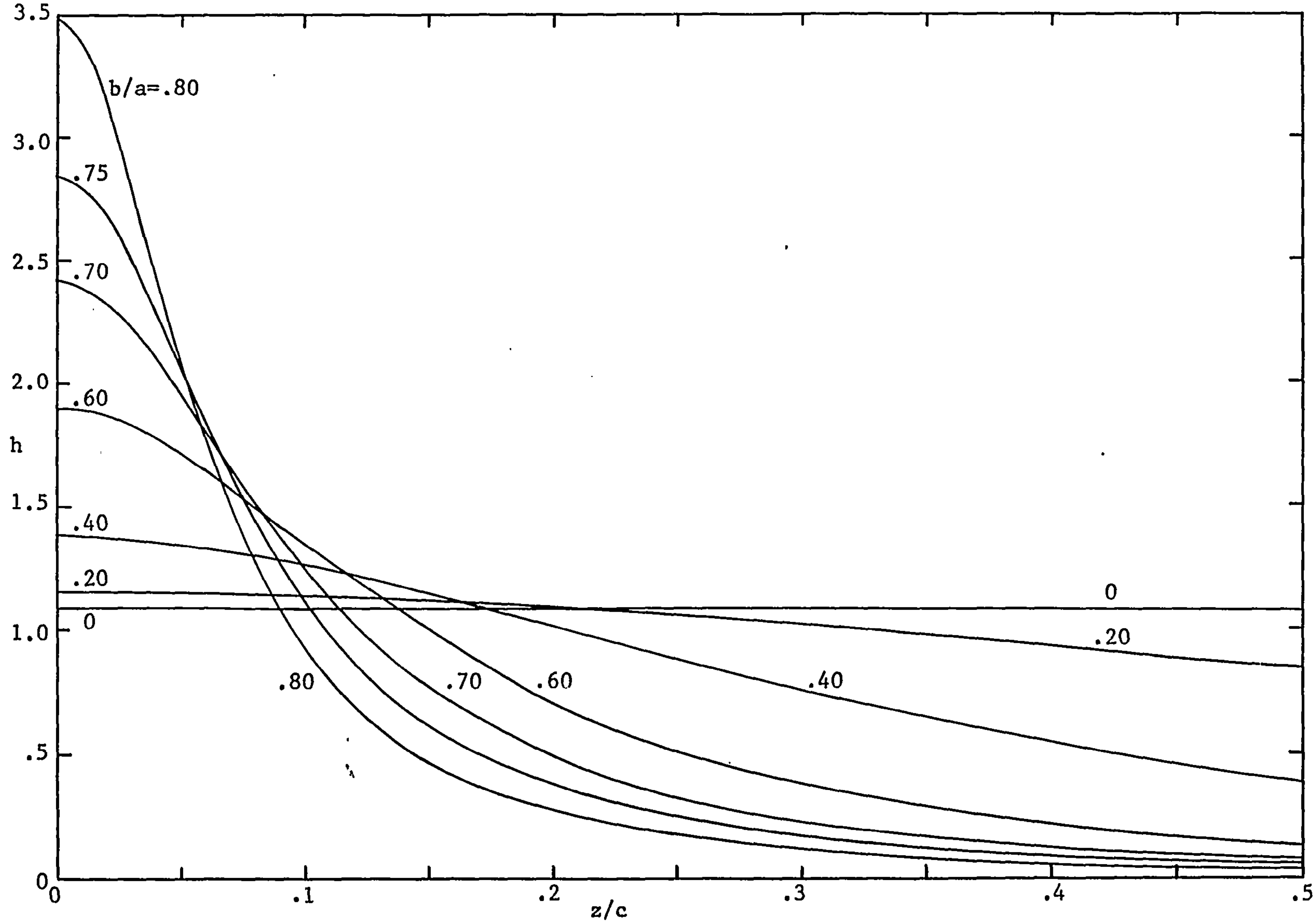

FIGURE 13. c/b=4

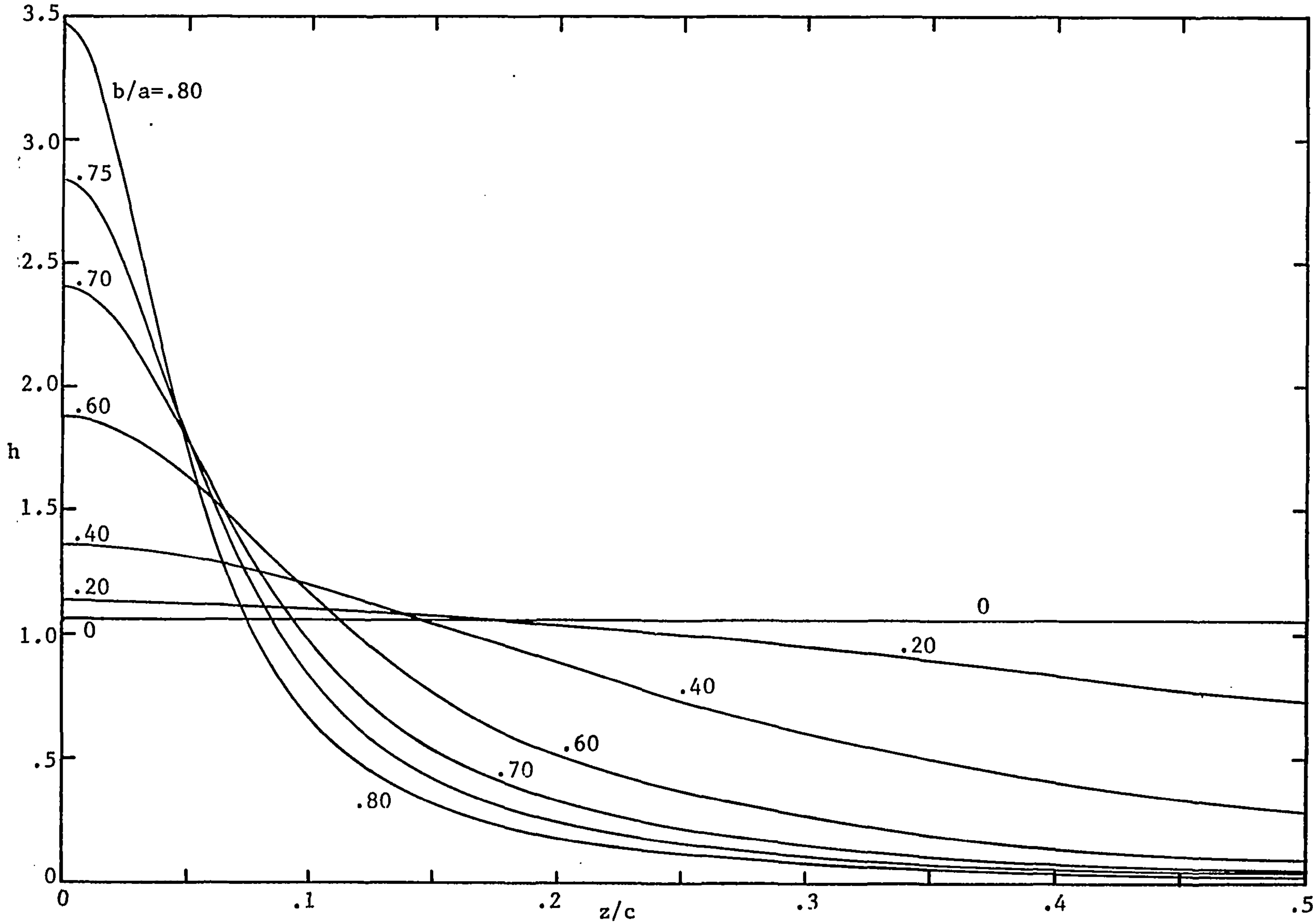

FIGURE 14. c/b=5

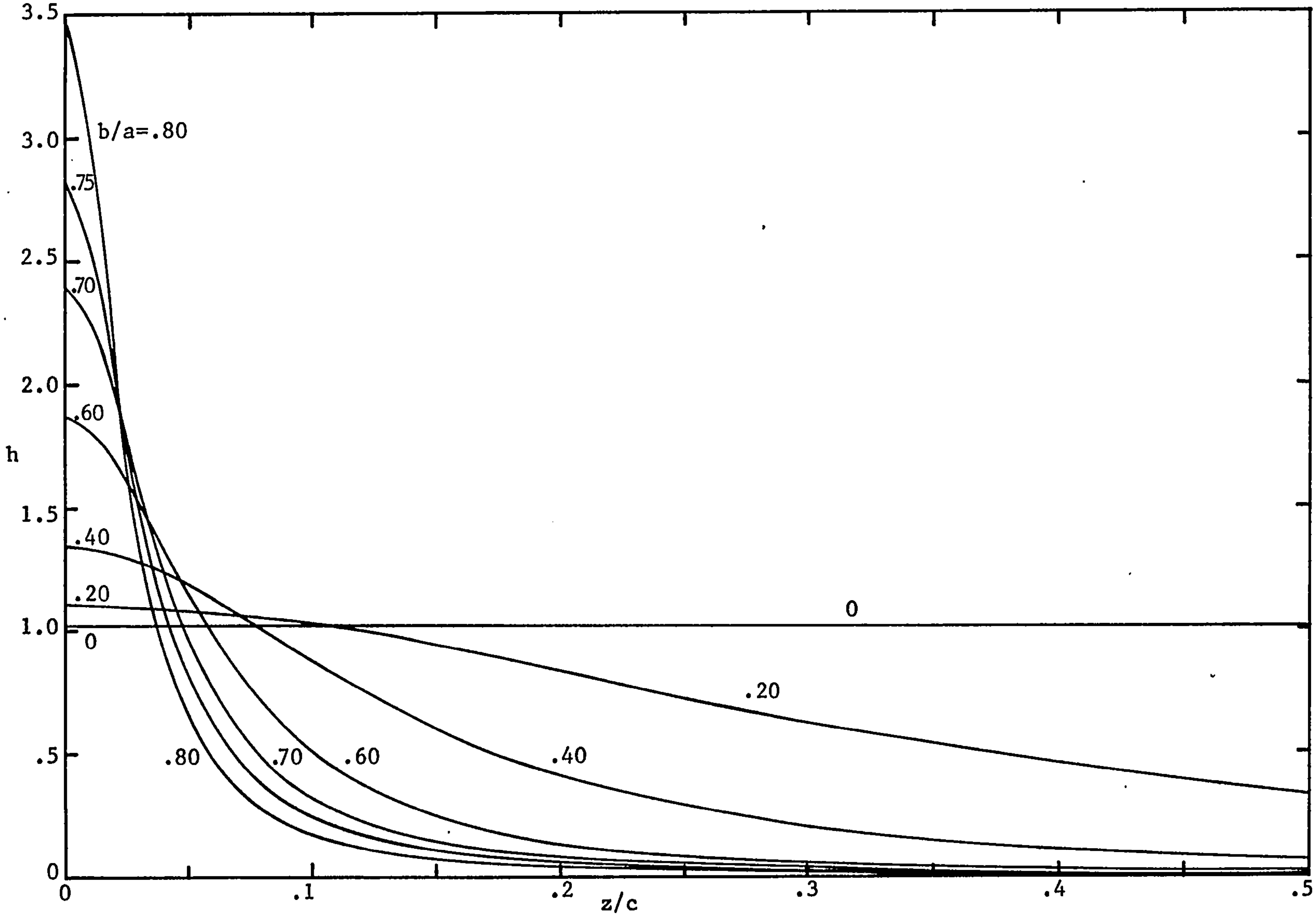

FIGURE 15. c/b=10

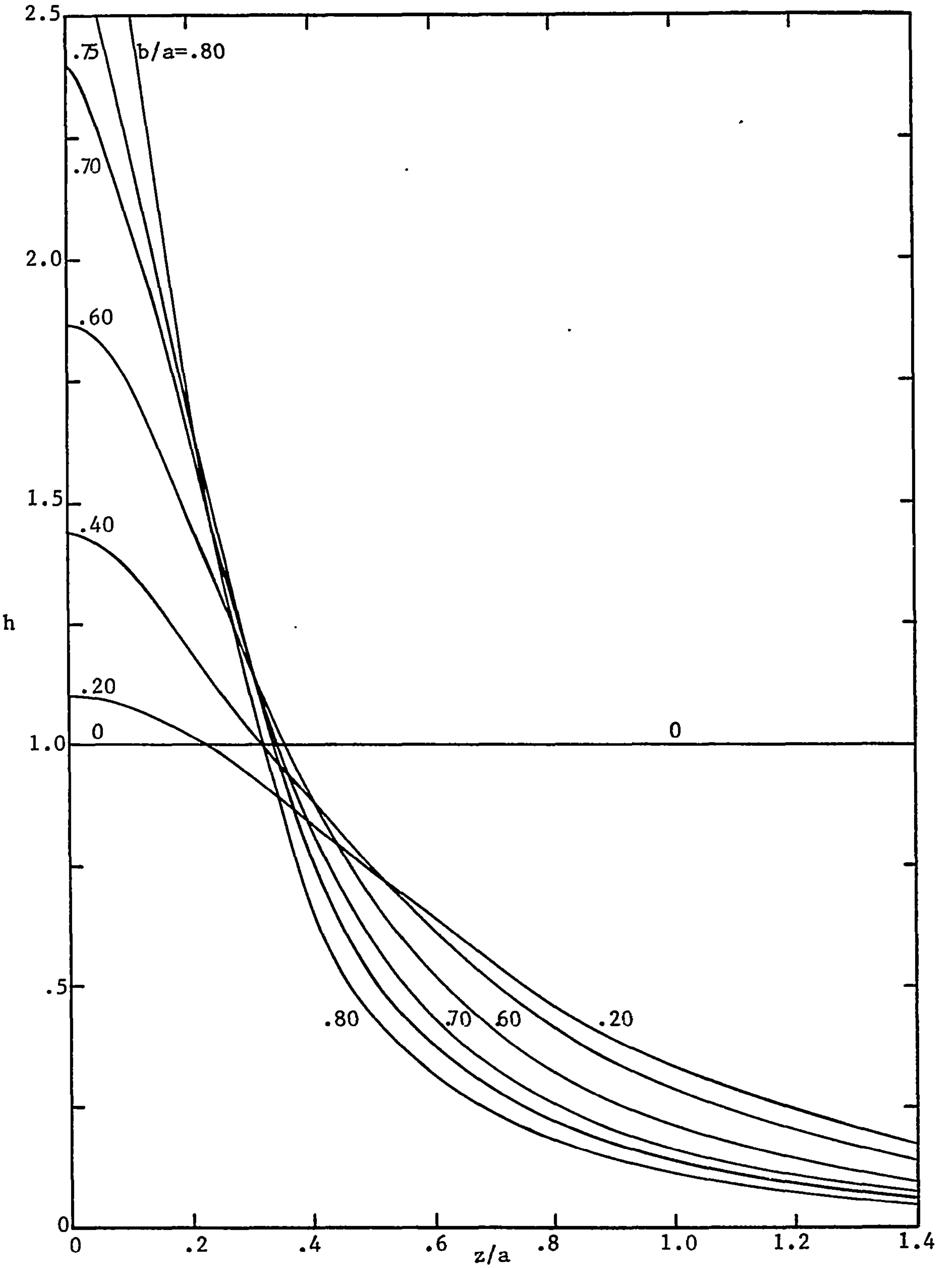

FIGURE 16. $c/b \to \infty$

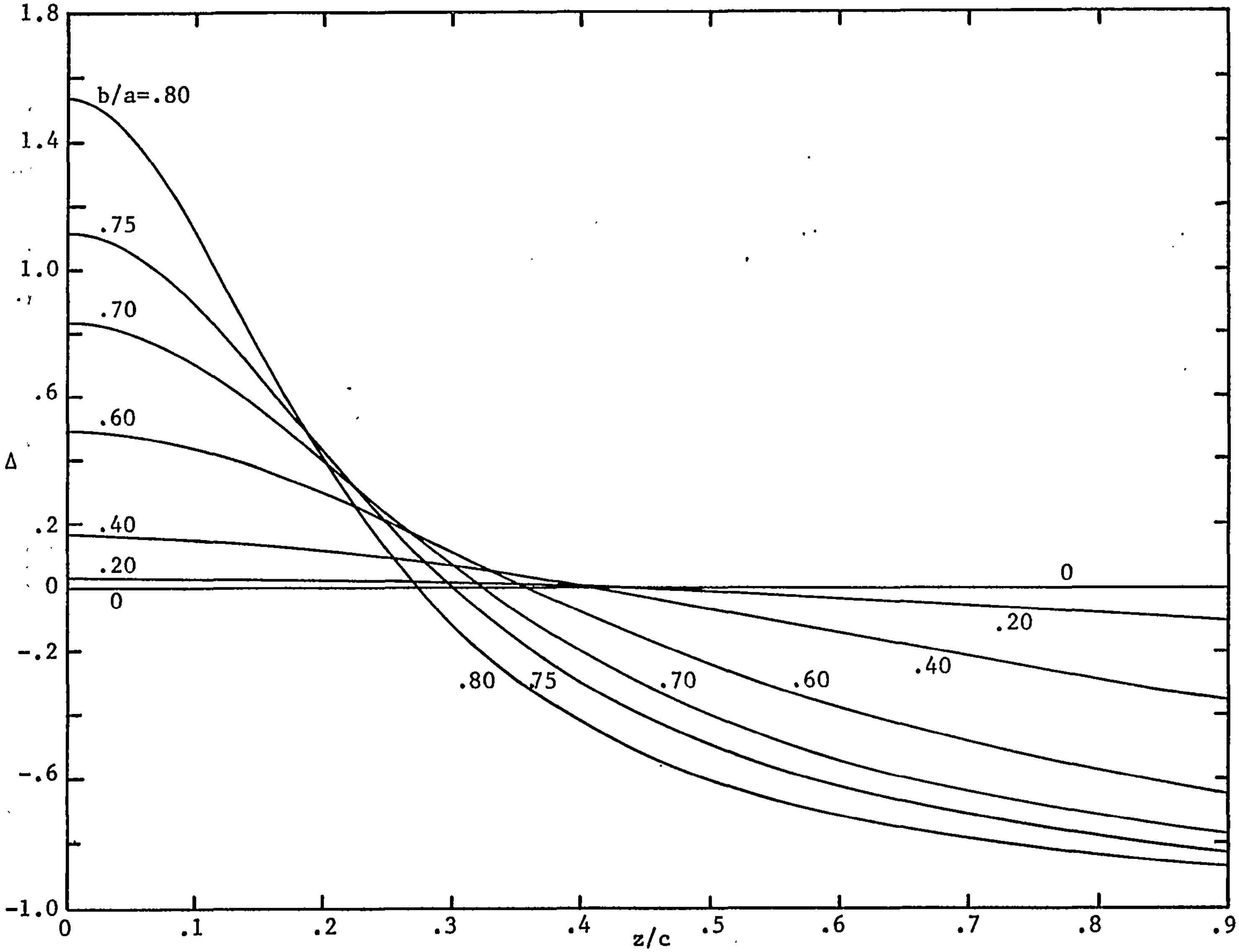

FIGURE 17. c/b=1

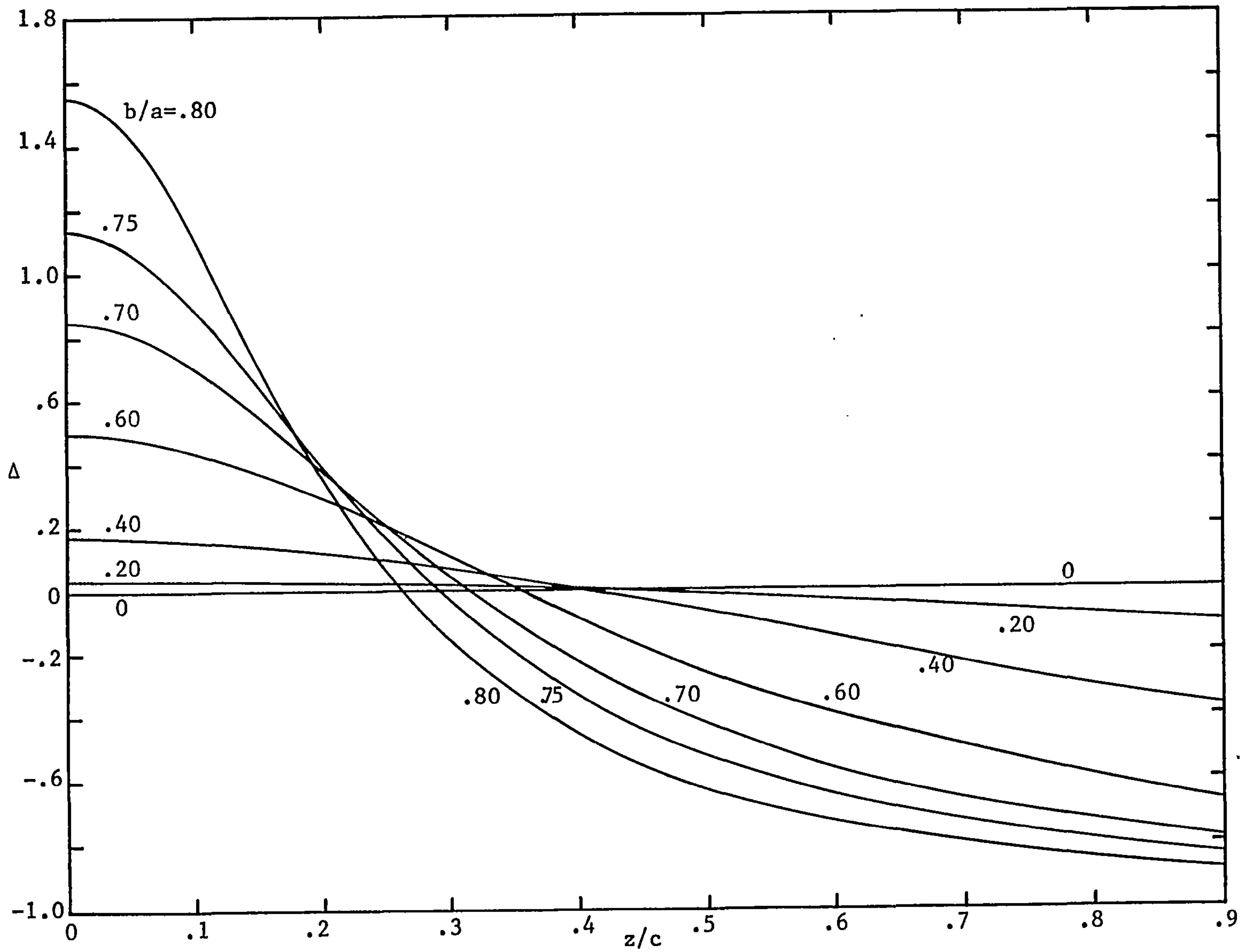

FIGURE 18. c/b=1.05

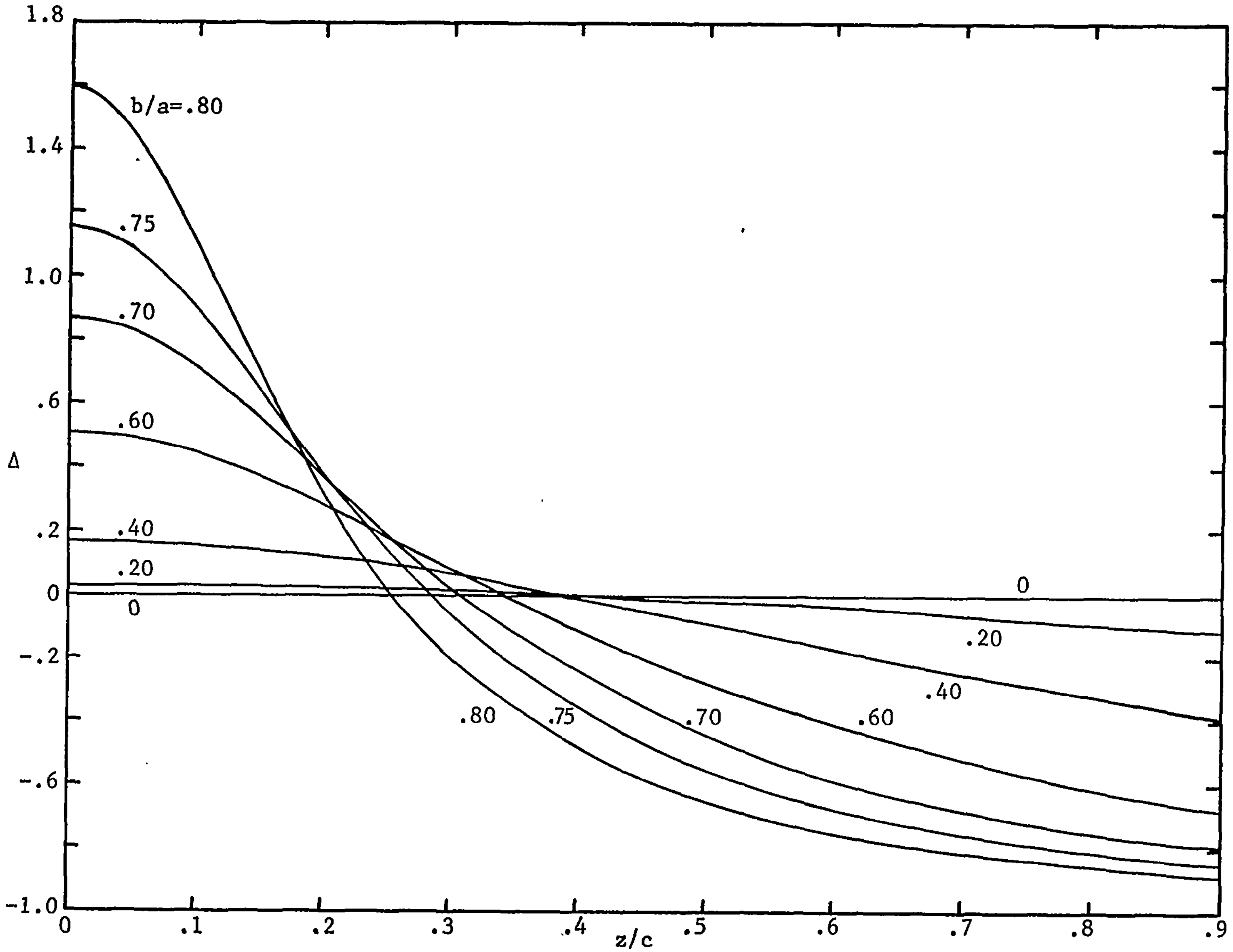

FIGURE 19. c/b=1.1

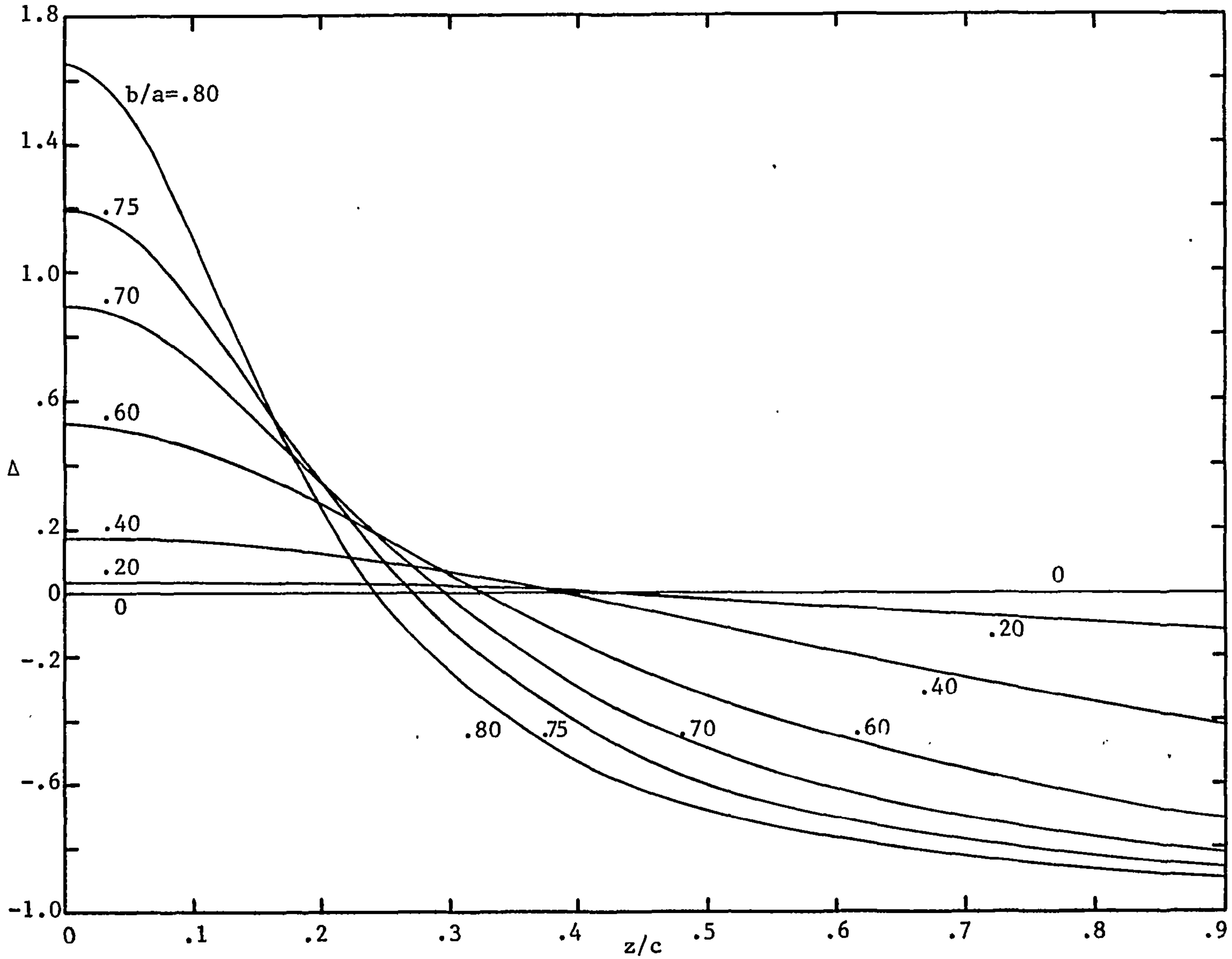

FIGURE 20. c/b=1.2

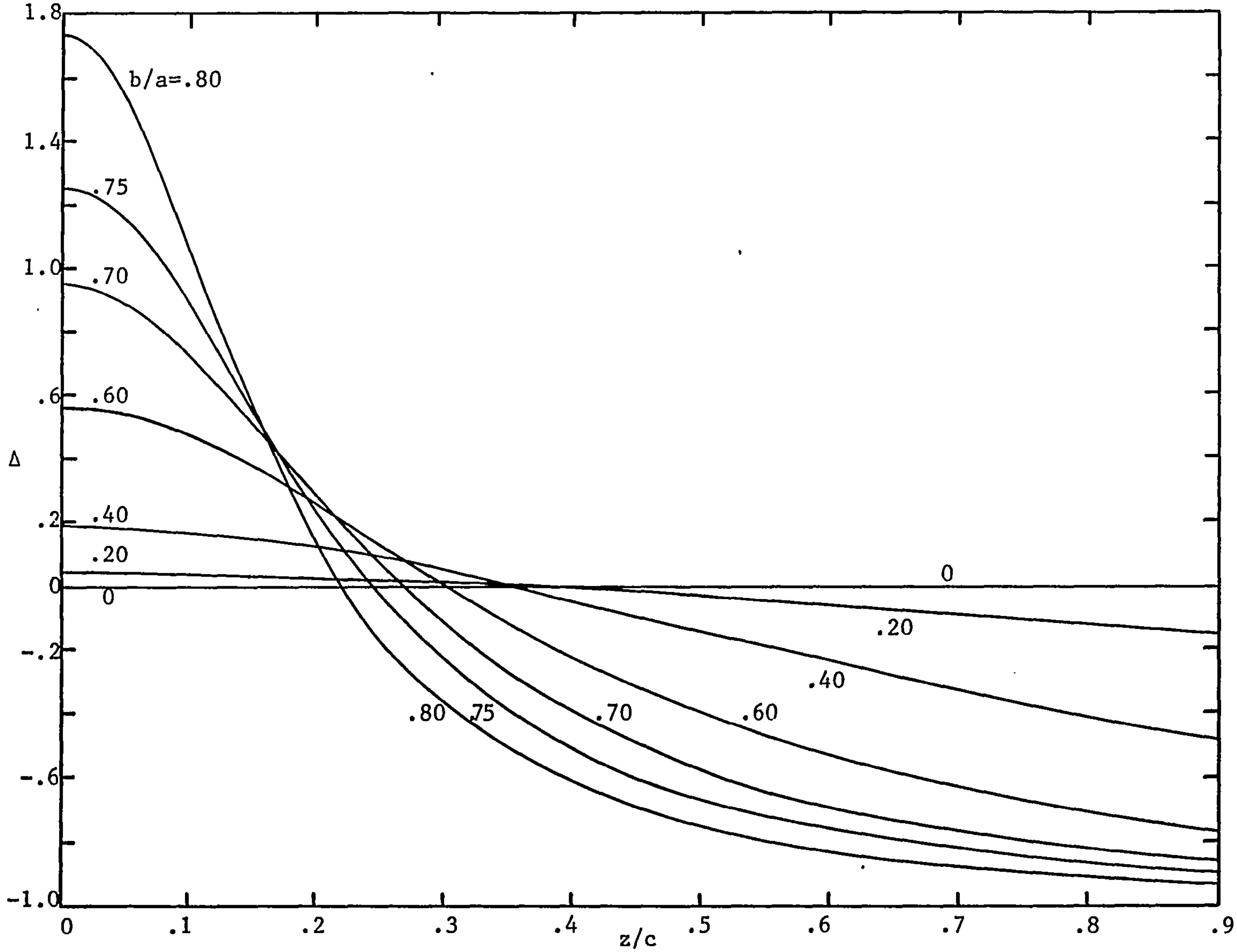

FIGURE 21. c/b=1.4

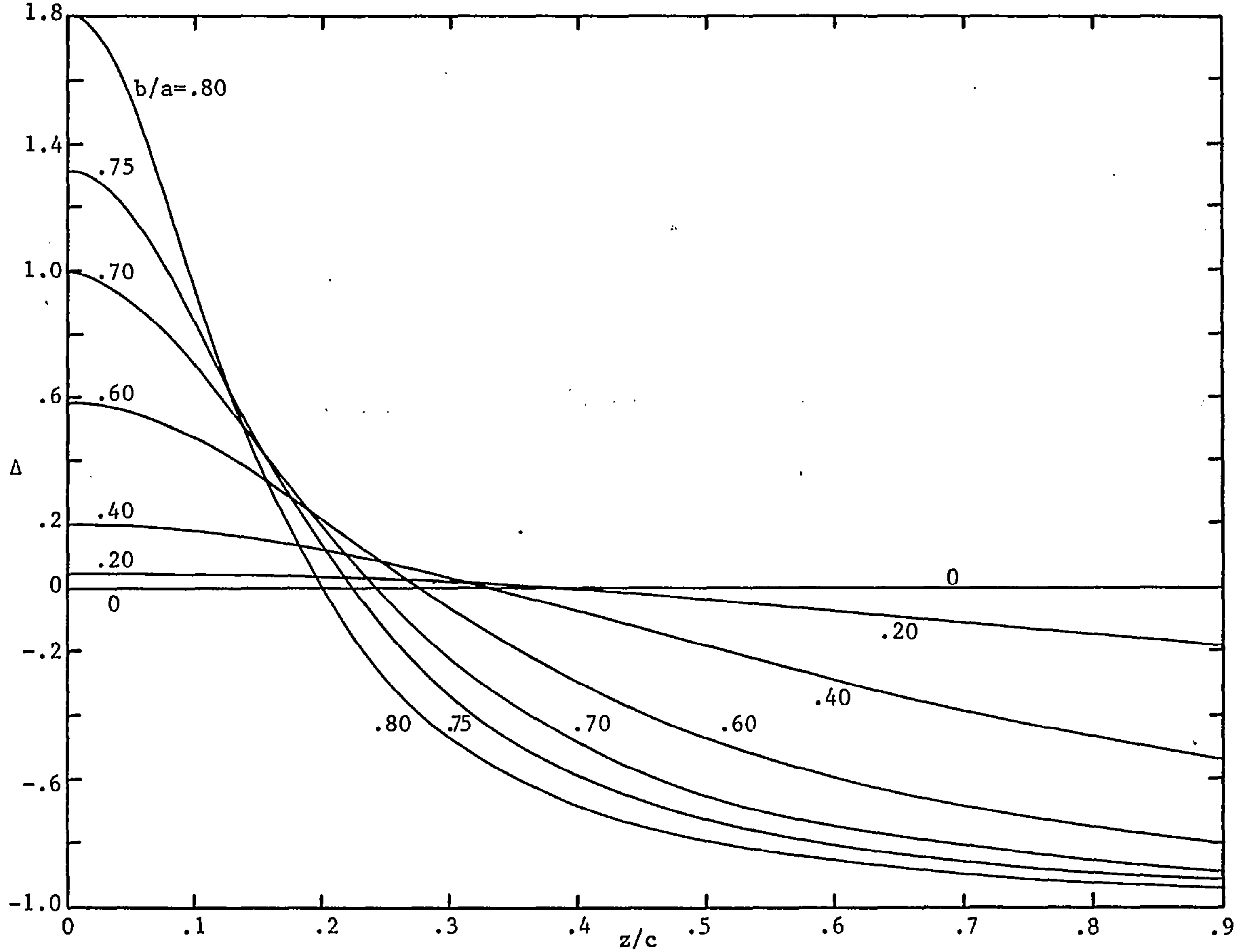

FIGURE 22. c/b=1.6

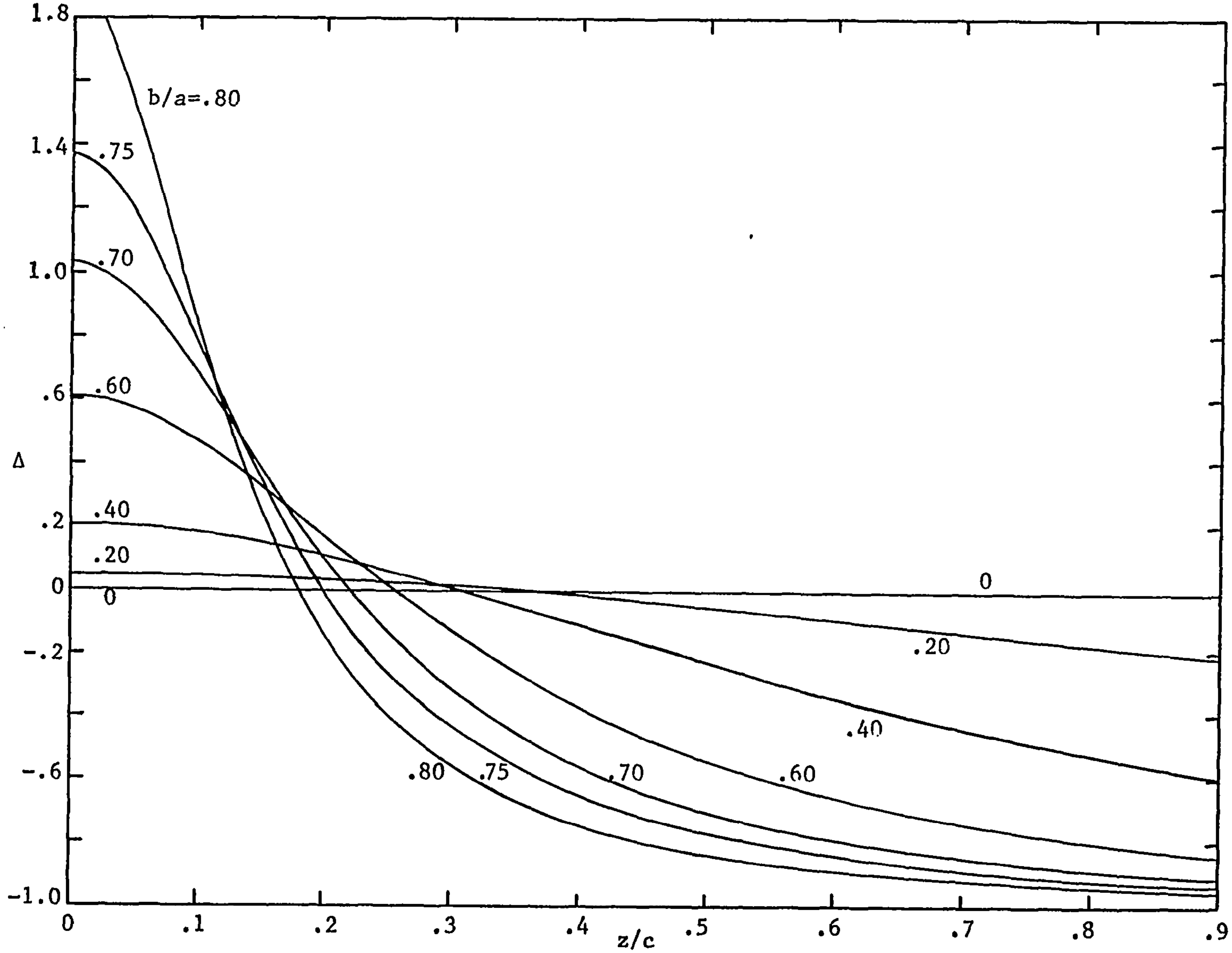

FIGURE 23. c/b=1.8

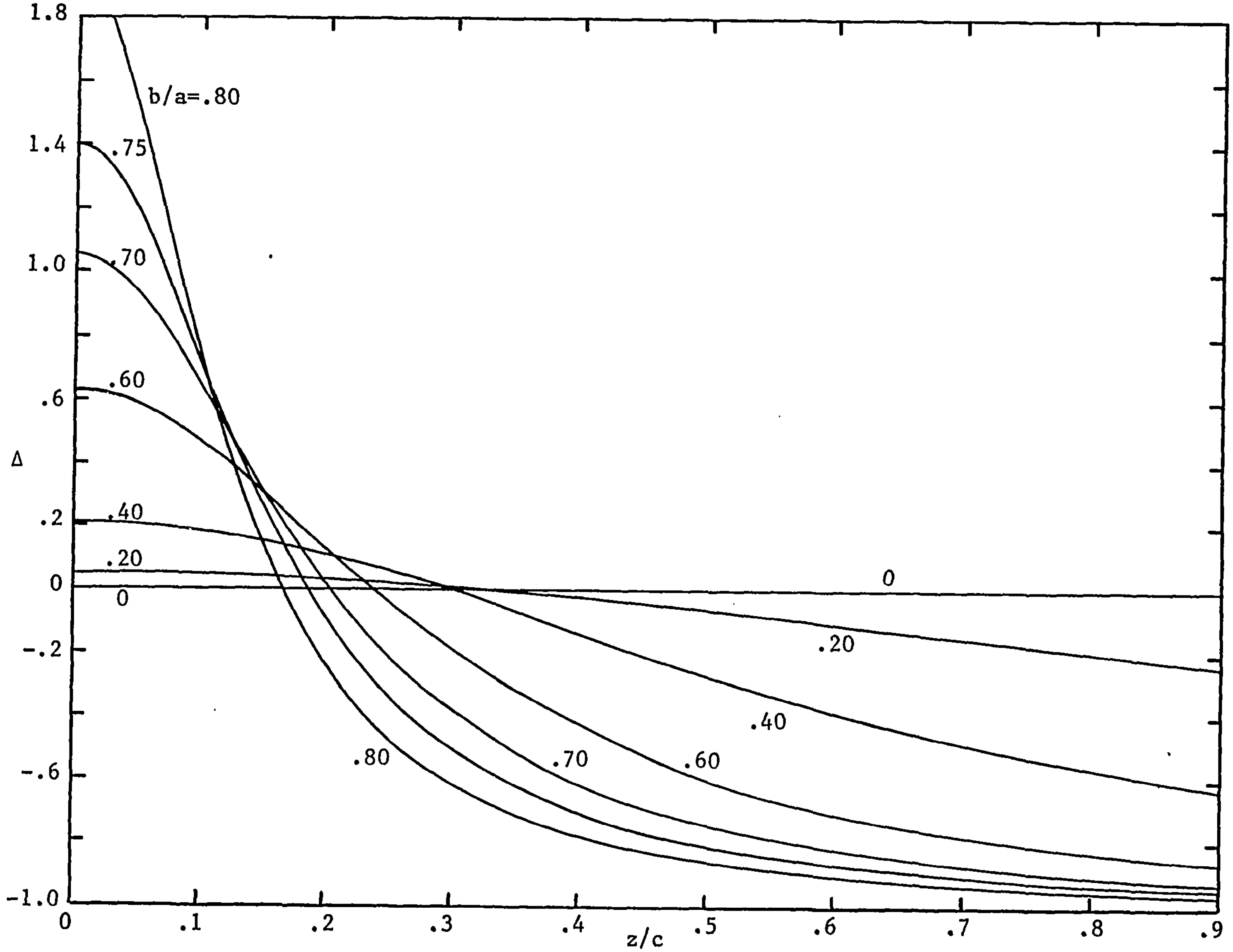

FIGURE 24. c/b=2

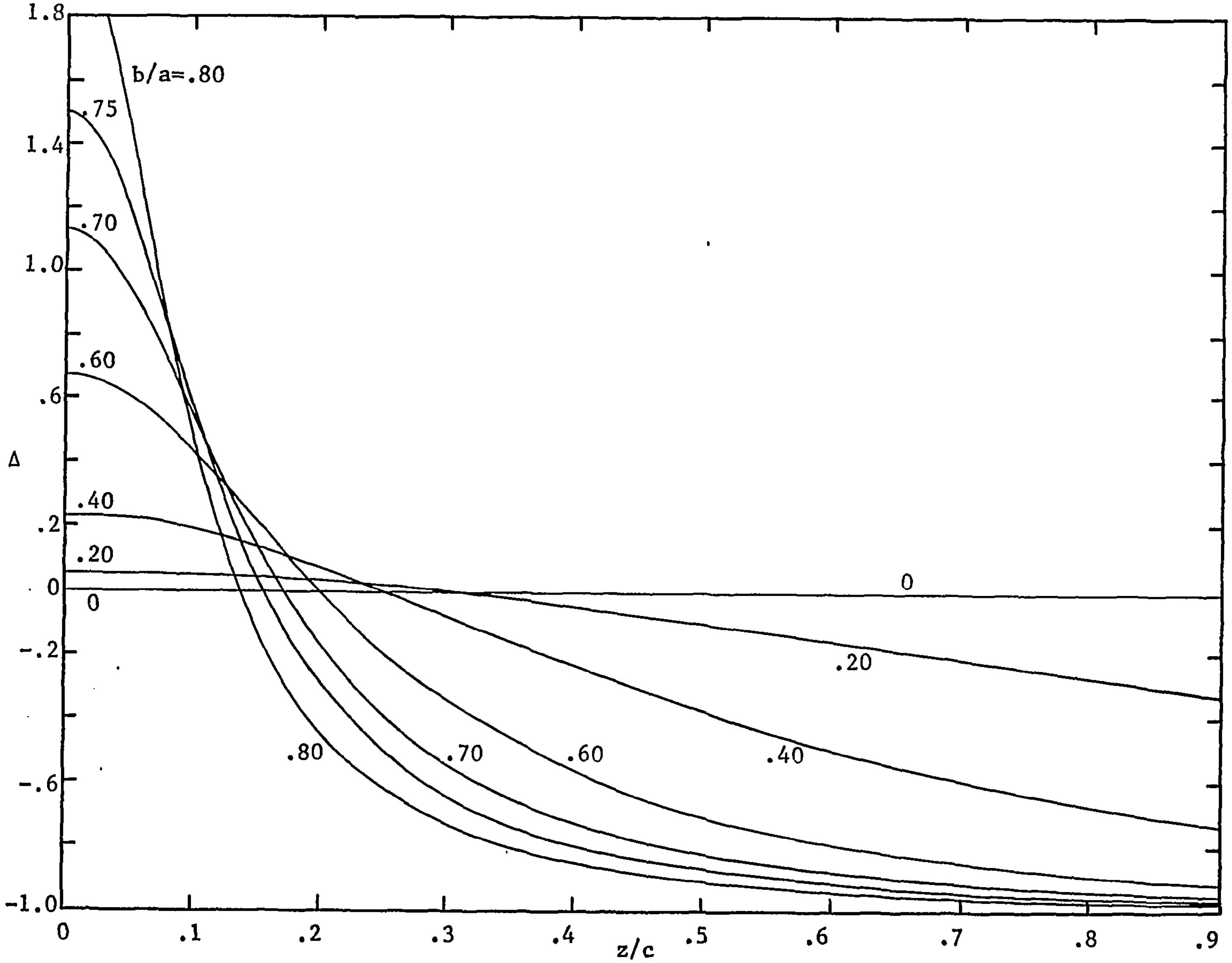

FIGURE 25. c/b=2.5

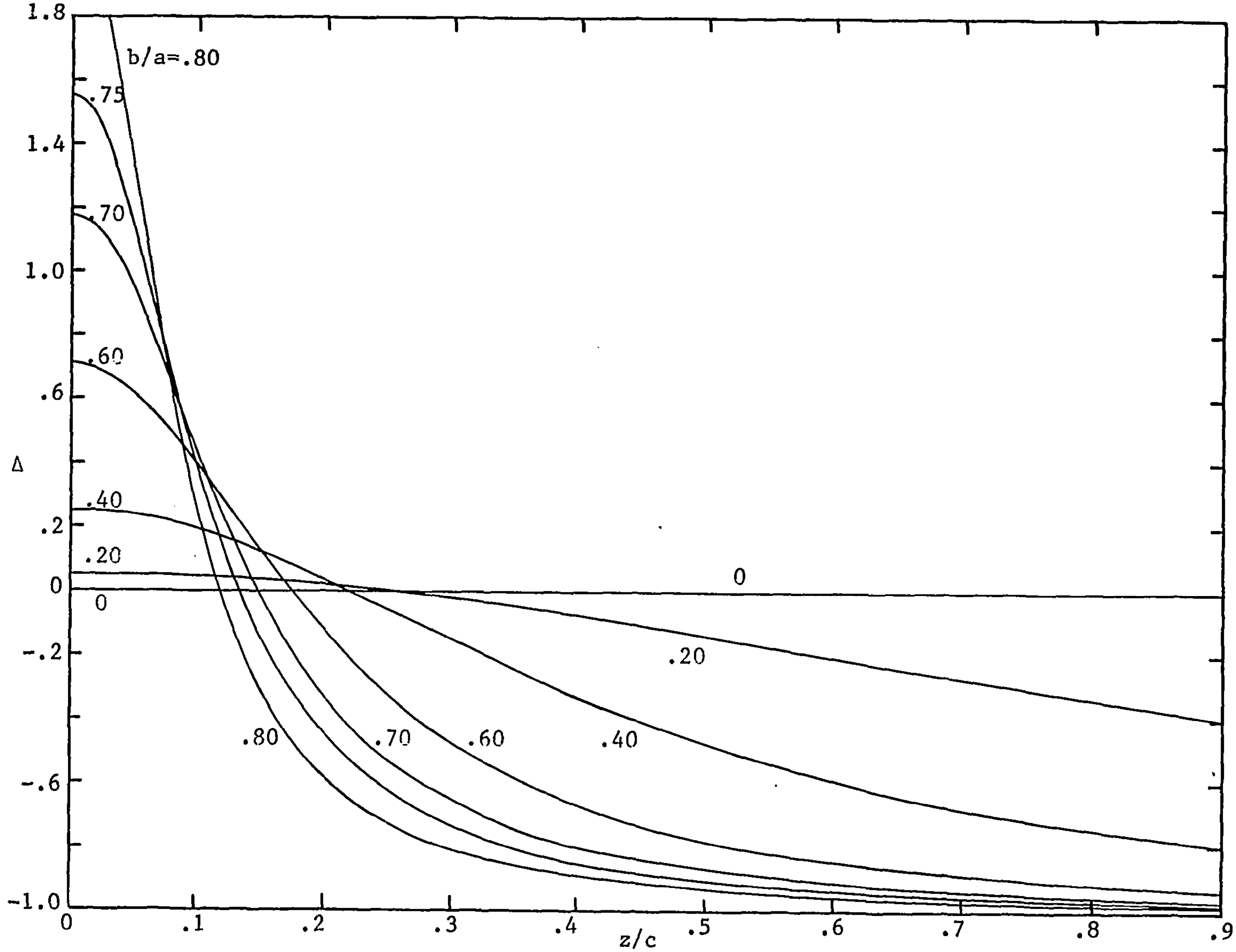

FIGURE 26. c/b=3

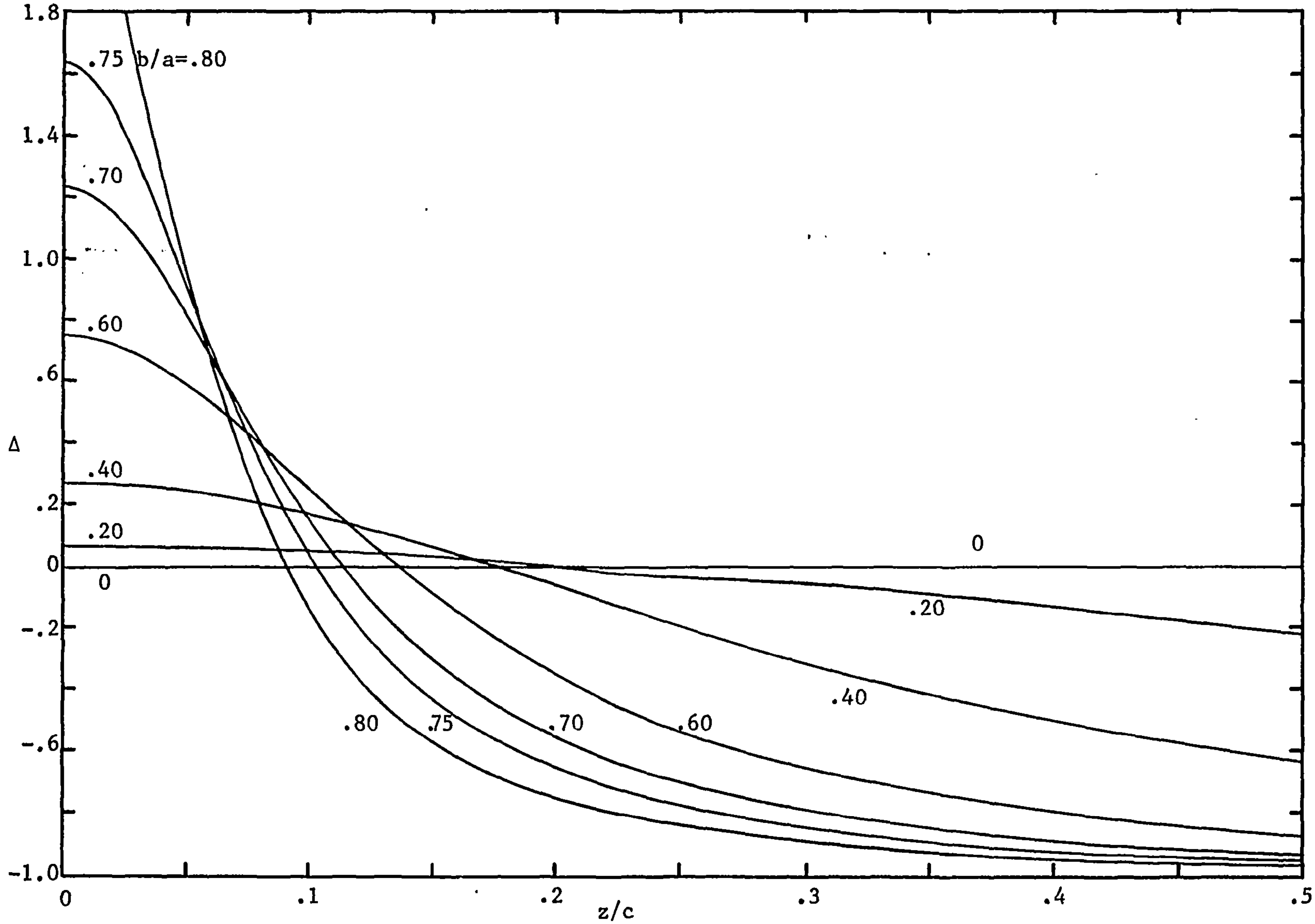

FIGURE 27. c/b=4

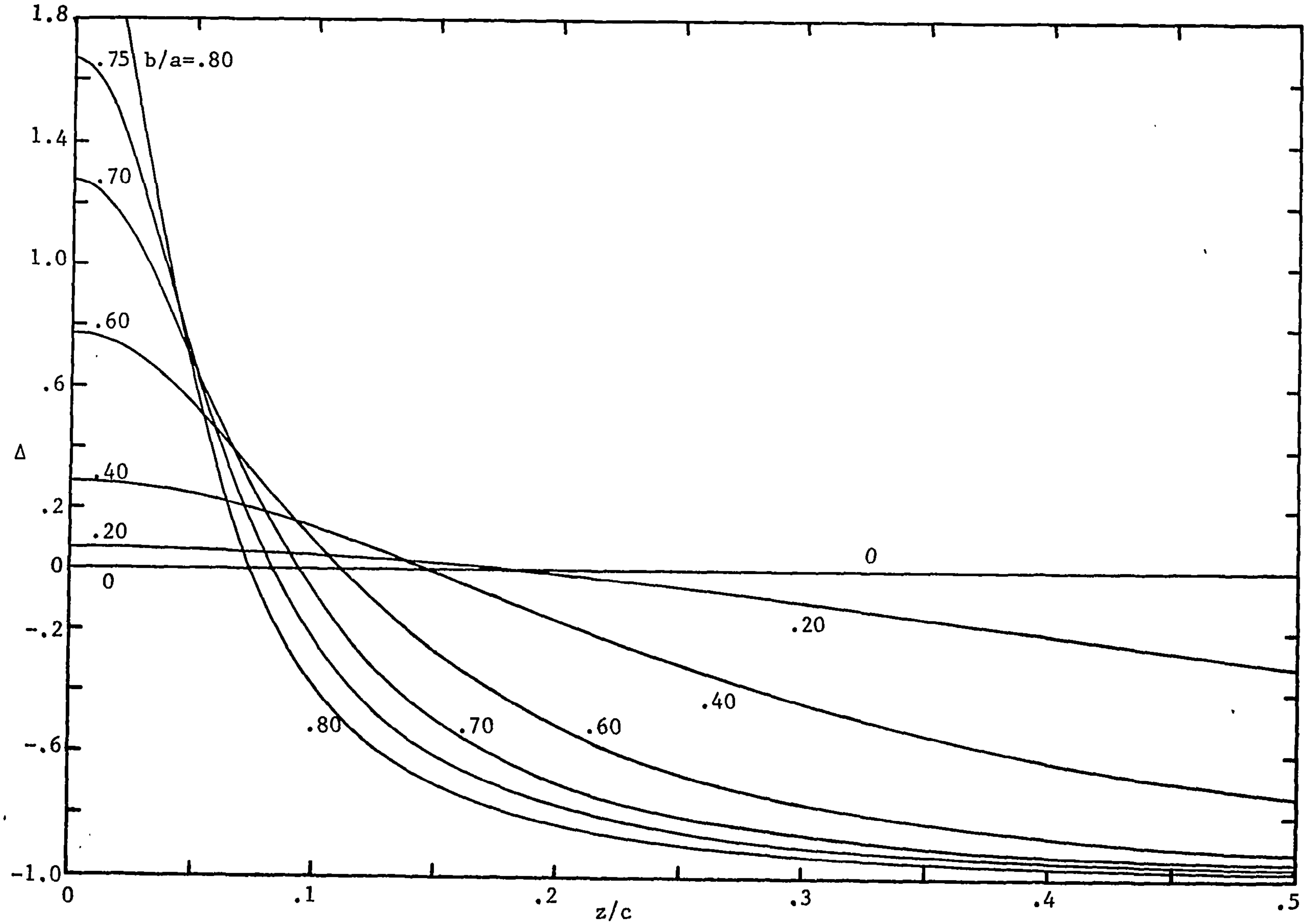

FIGURE 28. c/b=5

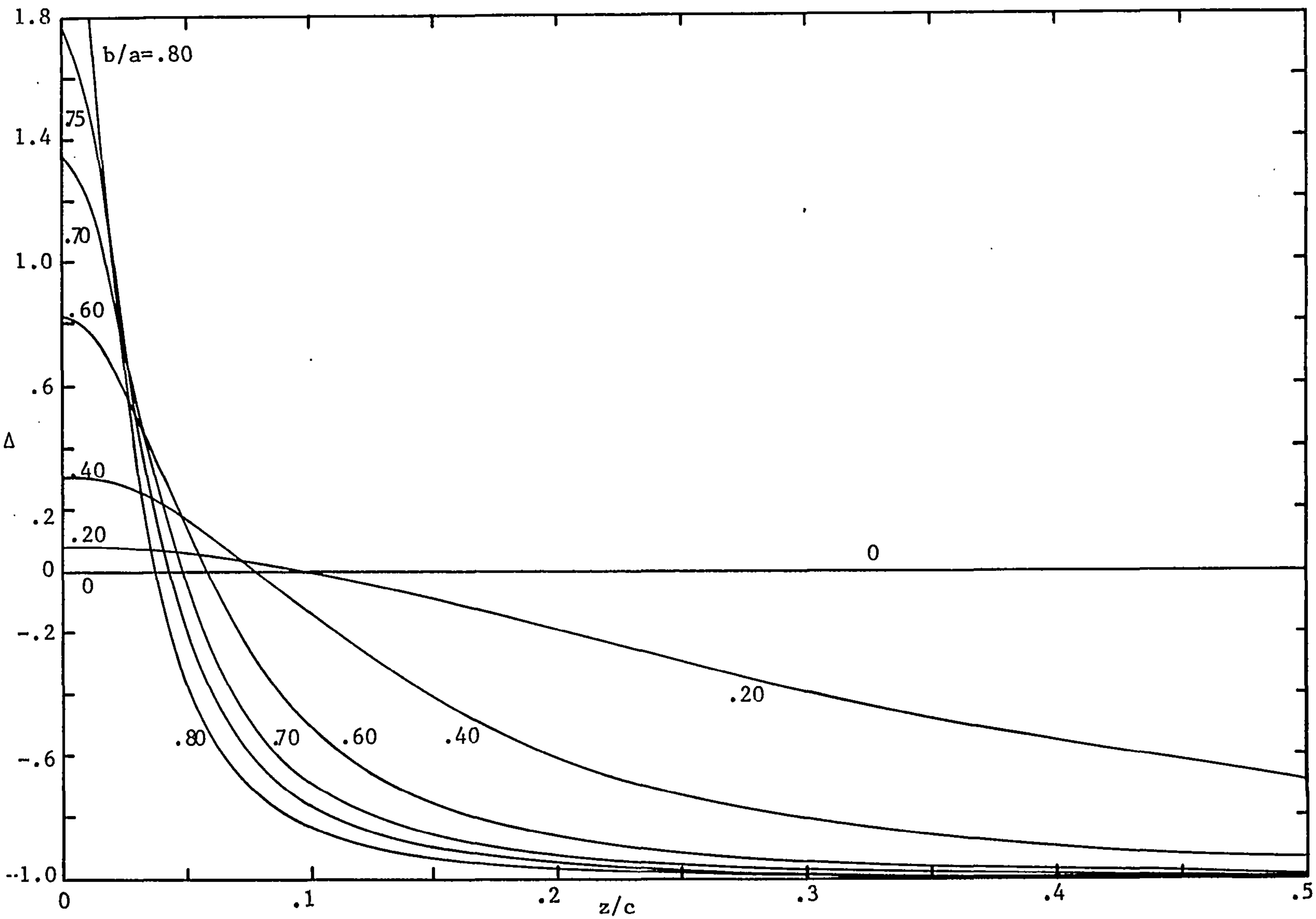

FIGURE 29. c/b=10

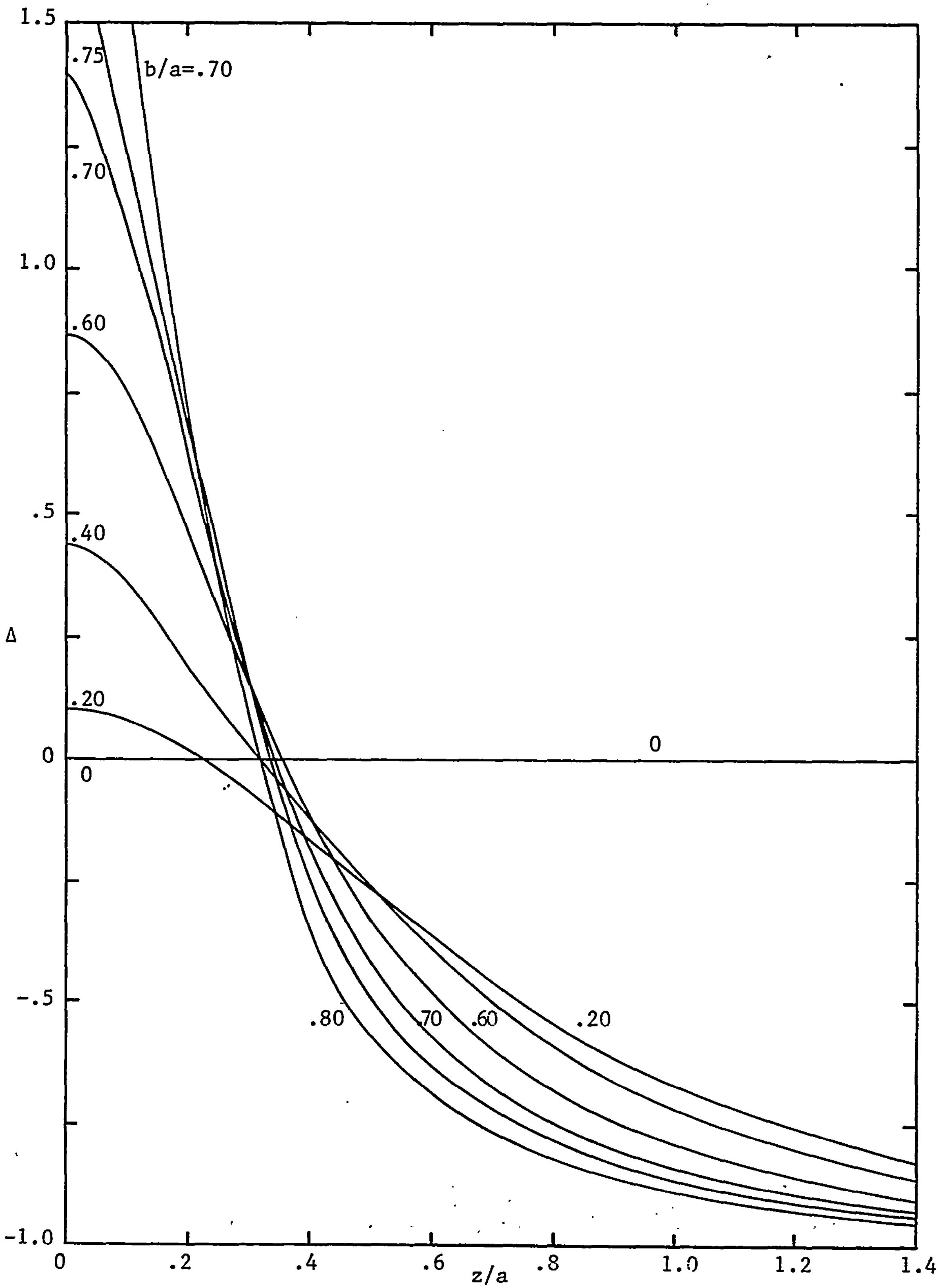

FIGURE 30. $c/b \to \infty$

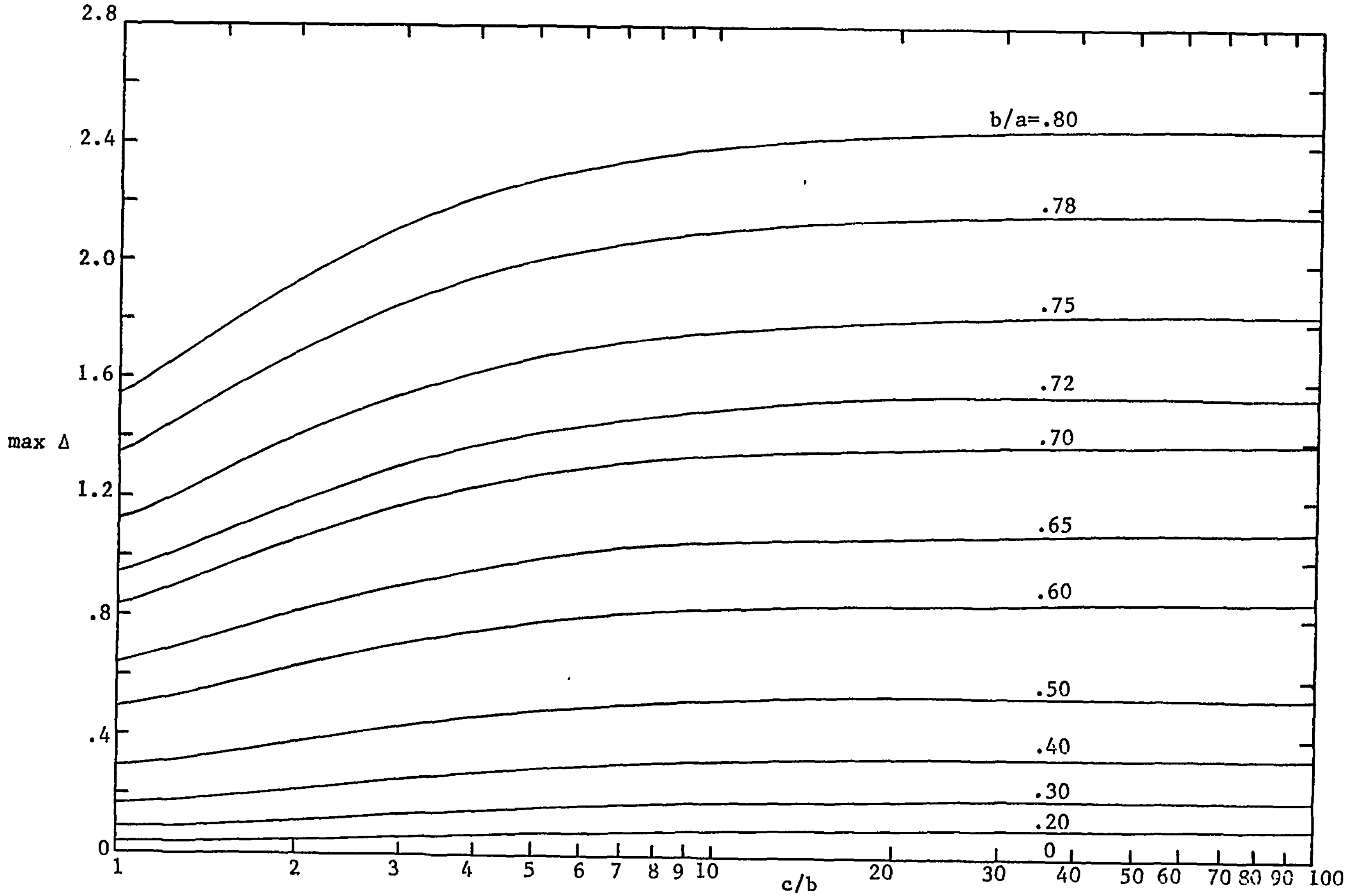

FIGURE 31. max Δ is the maximum value of Δ over the spheroid.

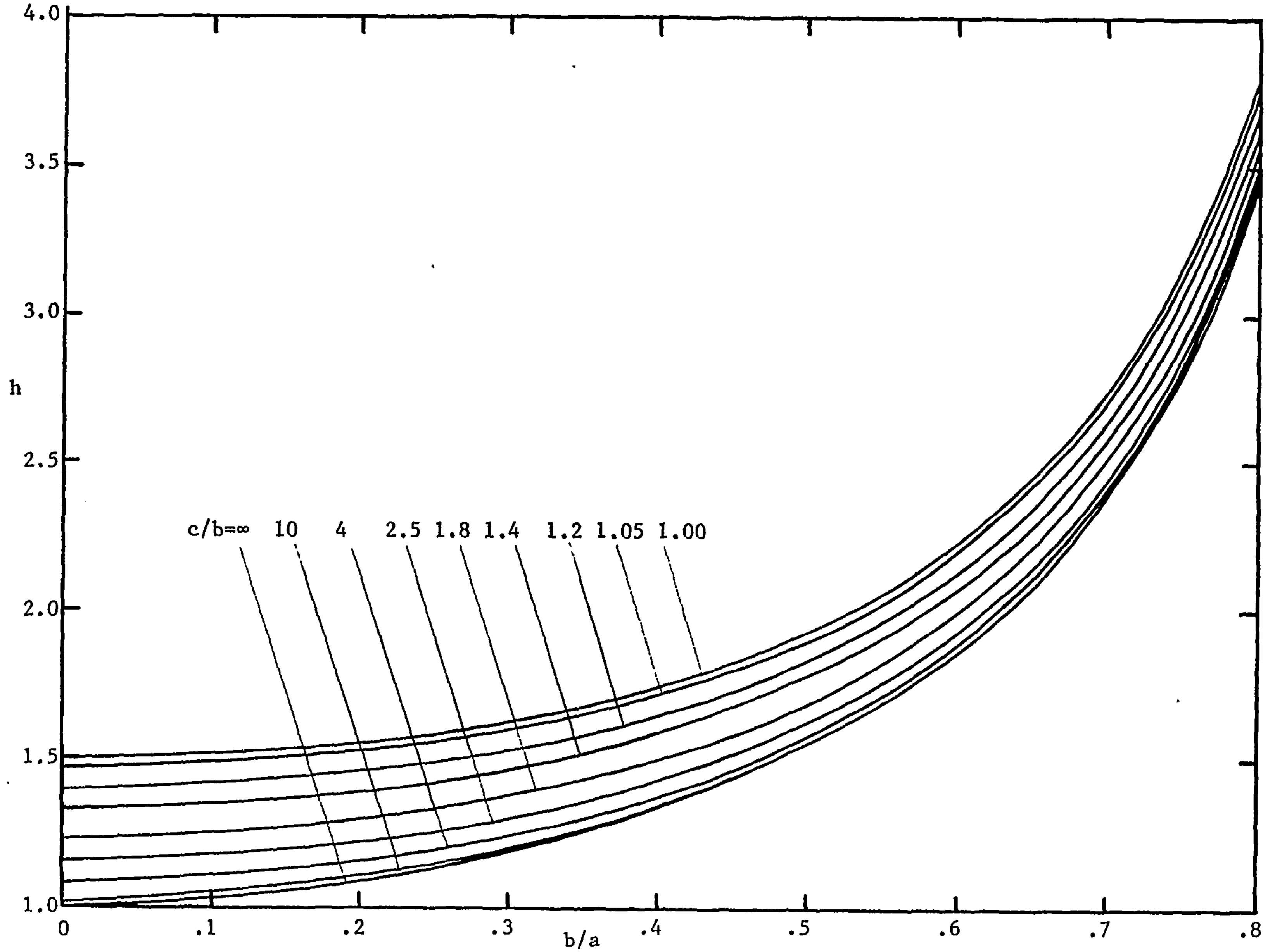

FIGURE 32. z/c=0

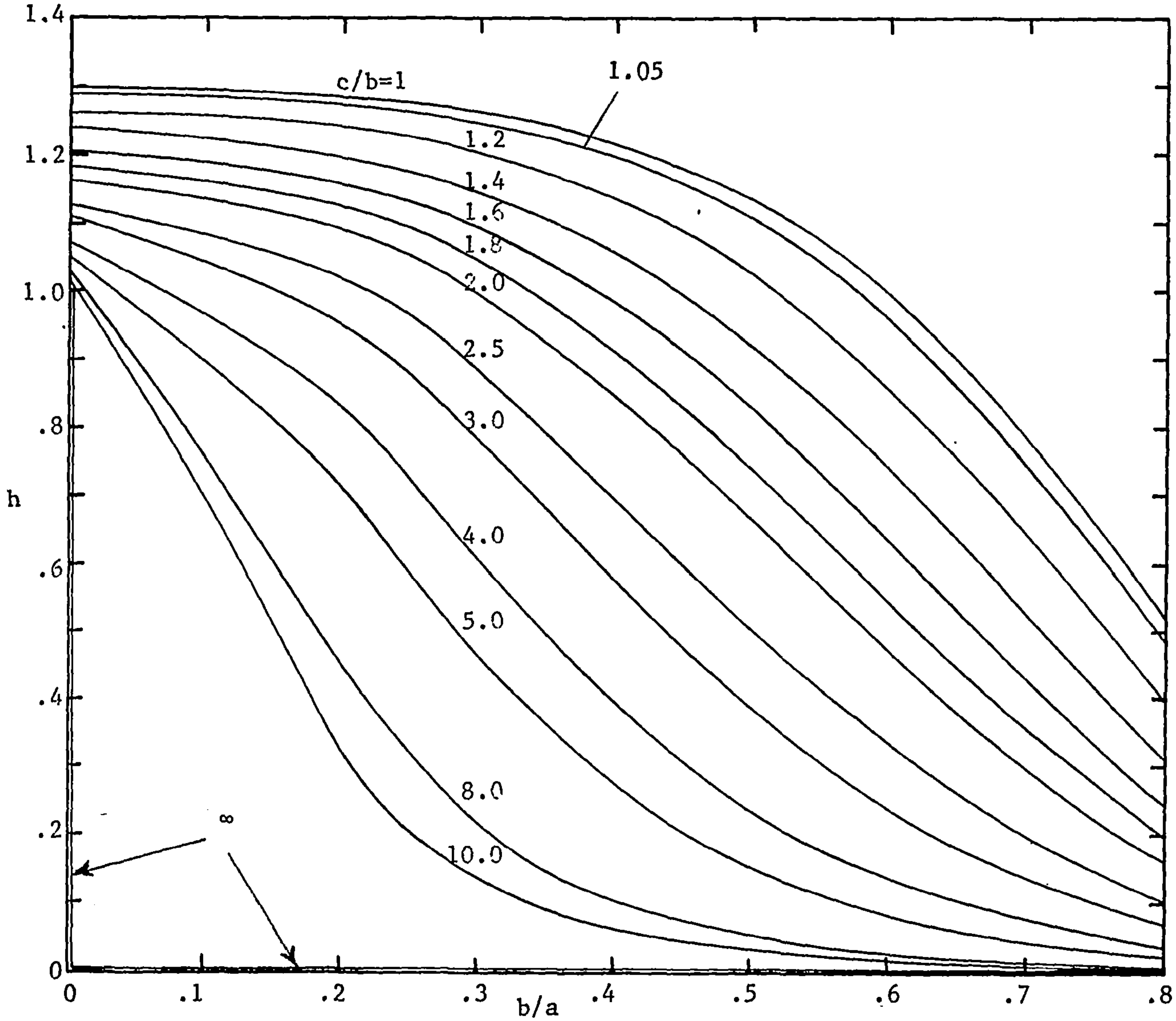

FIGURE 33. z/c=.5

Acknowledgement

The authors wish to thank Mr. R. W. Sassman for computer programming and Mrs. G. Peralta for typing the manuscript.

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