Sensor and Simulation Notes XII

A Space Charge Limited Radiation Detector

I. Introduction

In high time-resolution radiation measurements, in which there is a large amplitude narrow radiation pulse followed by a lower broader signal, one may be interested in the time history of this lower level radiation signal as well as that of the initial radiation pulse. If a linear radiation detector is being used, the signal from the initial pulse may either blur over onto the later signal because of the imperfect time resolution of the system or saturate some component of the system which does not recover until some time after the radiation reaches the level of interest.

One way to overcome this problem is to design a detector which does not convert this initial radiation pulse into an analog signal. The device should still be linear, or better, linear up to a certain radiation intensity at which point the output signal is limited. However when the radiation decreases to the level of interest, the detector must quickly recover and follow the radiation intensity for as long as it stays below this saturation level. Used properly, a space charge limited (abbreviated SCL) radiation detector should meet these requirements. The purpose of this note is to discuss a possible design of such a detector. All doses in this note are to be considered as "air equivalent" doses. The sensors may be assumed calibrated to provide this directly.

II. Space Charge Limitation

A basic mechanism which can be used for such a detector is space charge limitation (as will be shown below) which is an effect important in vacuum tube design and can be used in both SEMIRAD (devices which collect low energy secondary electrons rather than the high energy electrons) and the photodiode detectors. These devices are similar in that they both use an applied potentail to sweep low energy electrons across a vacuum. In SEMIRAD high energy charged particles (such as Compton electrons) release the low energy electrons from a surface, while in a photodiode, light quanta are normally used to release these electrons. The SEMIRAD principle will be used as the basis for the discussion in this note.

J_S, the space charge limited current density as illustrated in figure 1, is dependent on the distance between the electrodes, d, and the difference in voltage, ΔV, by the formula (assuming ΔV positive)

$$J_s = -\frac{4}{9}\epsilon_0\sqrt{2\frac{e}{m}}\,\frac{(\Delta V)^{3/2}}{d^2}\ \frac{\text{amps}}{(\text{meter})^2} \tag{1}$$

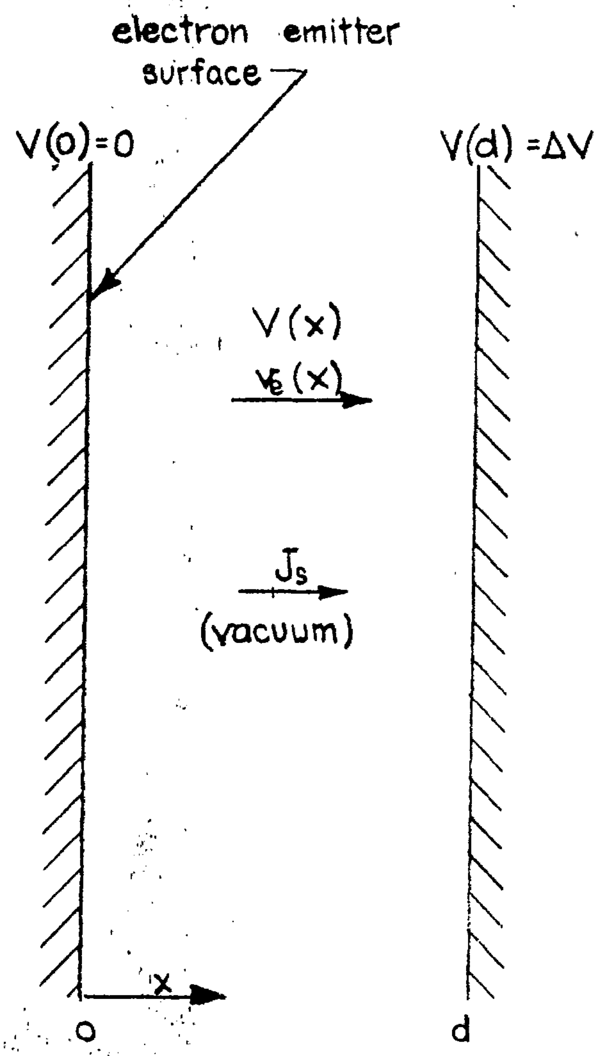


Fig. 1 Space Charge Limitation in Parallel Plate Geometry.

or

$$J_s = -2.33 \times 10^{-6} \frac{(\Delta V)^{3/2}}{d^2} \qquad \frac{\text{amps}}{(\text{meter})^2} \tag{2}$$

where all units in this note are rationalized m.k.s. unless otherwise stated. Both positive and negative ΔV can be included by using absolute magnitudes, i.e.,

$$|J_s| = 2.33 \times 10^{-6} \frac{|\Delta V|^{3/2}}{d^2} \qquad \frac{\text{amps}}{(\text{meter})^2} \tag{3}$$

The SEMIRAD device employing this principle (shown in figure 2) will have a load, Z, such as a coaxial cable on its output. Thus, ΔV will not be the applied voltage, V_1, but the applied voltage minus a saturation signal voltage, V_s. With A as the total area of negative electrode which is used as the electron emitter, V_s is given by

$$V_s = Z A J_s \tag{4}$$

Since the magnitude of V_1 must be always greater than the magnitude of V_s then

$$|\Delta V| = |V_1| - |V_s| \tag{5}$$

To obtain the saturation signal voltage one must then solve the equation

$$|V_s| + \left|\frac{V_s}{KZ}\right|^{2/3} = |V_1| \tag{6}$$

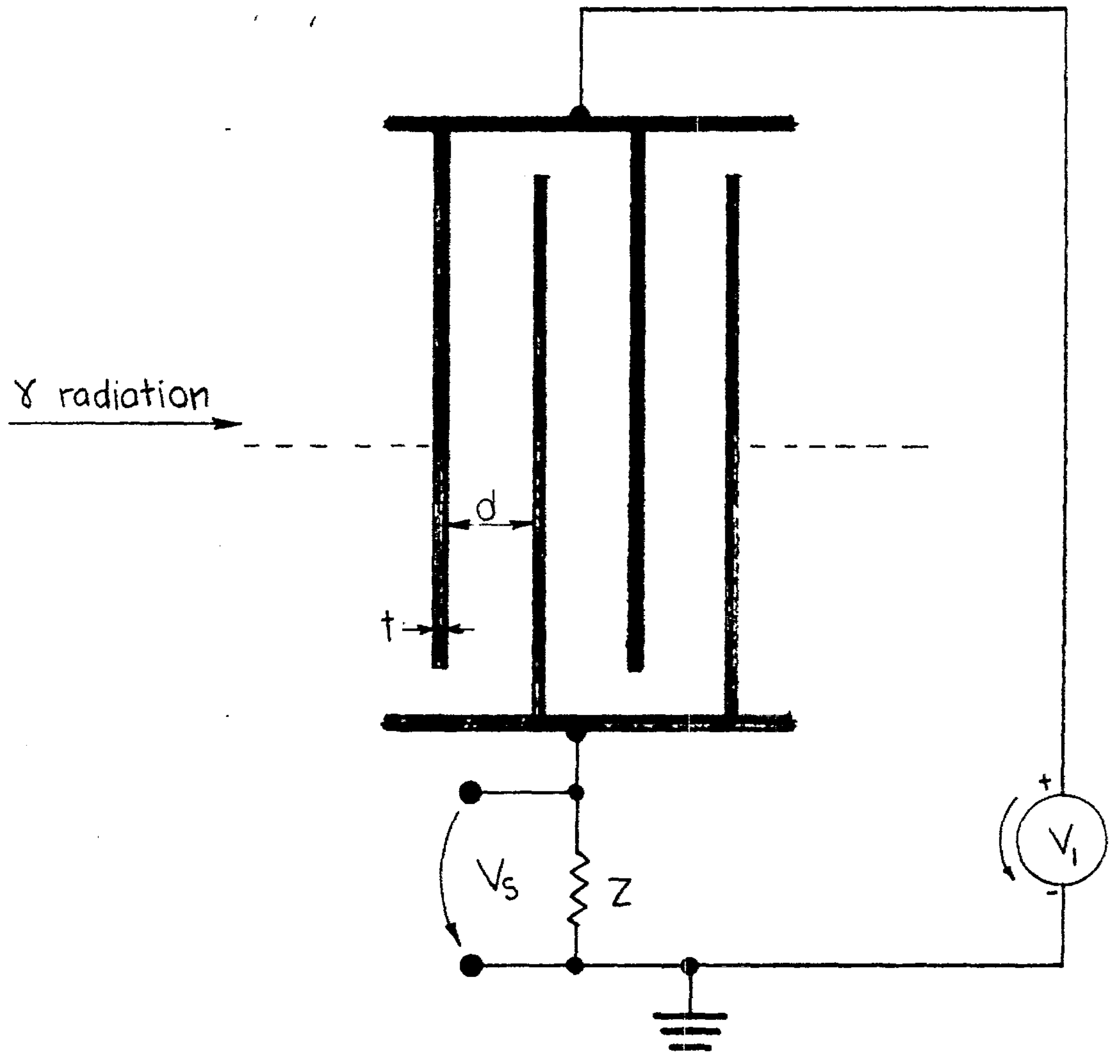
where, for parallel plate geometry

$$K = \frac{4}{9} \epsilon_0 \sqrt{2\frac{e}{m}} \frac{A}{d^2} \tag{7}$$

or

$$K = 2.33 \times 10^{-6} \frac{A}{d^2} \quad \text{ohm}^{-1}\, \text{volt}^{-1/2} \tag{8}$$

Over the appropriate range of parameters in eqn. (6) the saturation signal voltage may be limited not by the 3/2 power law, but by the first power of the applied voltage. It has been assumed in this derivation that V_1 is



A = total area of negative electrode which is used as electron emitter

Fig. 2 SEMIRAD in Parallel Plate Geometry held constant. If there is some finite source impedance in V_1 this can be included in eqn. (6) by replacing V_1 by V_1 minus some constant times V_s where this constant is determined by Z and the source impedance in V_1, (assuming this source impedance is purely resistive).

To complete the analysis for the radiation level at which the device will saturate one needs the sensitivity of the device. Assuming this SEMIRAD device is sensitive only to gamma rays, then the current per unit electron emitter area, J, is

$$|J| = Y |J_c| \tag{9}$$

where Y is the secondary electron yield per incident high energy electron (from either side of the emitter surface) and J_c is the isotropic Compton electron flux related to the isotropic gamma flux, γ, by

$$J_c \simeq -2\times 10^{-8}\,\gamma \quad \frac{\text{amps}}{\text{meter}^2} \tag{10}$$

where γ is in roentgens/sec. The current sensitivity, S_I, is

$$|S_I| \simeq 2\times 10^{-8} A\,Y \quad \frac{\text{coulombs}}{\text{roentgen}} \tag{11}$$

and the voltage sensitivity, S_V, is

$$|S_V| \simeq 2\times 10^{-8} A\,Y Z \quad \frac{\text{volt-sec}}{\text{roentgen}} \tag{12}$$

or converting roentgens to γ-Mev/cm^2.

$$|S_V| \simeq 10^{-17} A\,Y Z \quad \frac{\text{volt-sec}}{\gamma\text{-MeV/cm}^2} \tag{13}$$

where the photon energy is assumed to be about 1 Mev. Given the radiation intensity at which cut off is desired, combinations of A, d, and V_1 can be chosen to give the desired results in eqns (6), (8), (12) and (13). The secondary electron yield, Y, is typically less than 1.0 and fairly constant with respect to Compton electron energy unless special coatings for high secondary electron yield, such as those used in photodiodes, are employed.

III. Time Resolution

For an SCL detector to be useful, not only must it have a response proportional to the radiation intensity but also it must be capable of accurately following the time history of the radiation as soon as the radiation level drops below the chosen saturation level. The detector reactior time will be determined by the largest of the following characteristic times:

1. The transit time of the electrons between the electrodes.

2. The rise time corresponding to the RC time constant determined by the detector capacitance and resistive load.

3. The round trip signal transit time in the detector.

The detector must be designed so that the shortest of these time intervals is less than the time interval in which the radiation level changes significantly. The methods for determining these characteristic times are given below.

A. Electron Transit Time

If $v_e(x)$ is the electron velocity as a function of position as shown in figure 1, then the transit time, τ, is given by

$$\tau = \int_0^d \frac{dx}{v_e(x)} \tag{14}$$

where

$$v_e(x) = \sqrt{2 \frac{e}{m} V(x)} \tag{15}$$

e/m is the electron charge to mass ratio and V (x) is the potential distribution between the electrodes. For the case where there is no significant space charge in the region between the electrodes the potential distribution is given by

$$V(x) = \Delta V \frac{x}{d} \tag{16}$$

The transit time, τ_0, for this case is

$$\begin{aligned} \tau_0 &= \left(2\frac{e}{m}\Delta V\right)^{-1/2} d^{1/2} \int_0^d x^{-1/2} dx \\ &= \left(2\frac{e}{m}\Delta V\right)^{-1/2} d^{1/2} \left(2x^{1/2}\right)\Big|_0^d \end{aligned} \tag{17}$$

and thus

$$\tau_0 = 2\left(2\frac{e}{m}\Delta V\right)^{-1/2} d \tag{18}$$

or

$$\tau_0 = 3.37\times10^{-6}\,\frac{d}{\sqrt{\Delta V}}\ \text{seconds} \tag{19}$$

In the case of space charge limitation the potential distribution can be calculated from eqn. (1) by recognizing that if J_s is held constant then at any x (each of which is an equipotential) the same current relationship must hold, i.e.,

$$J_s = -\frac{4}{9}\epsilon_0\sqrt{2\frac{e}{m}}\,\frac{\left(V(x)\right)^{3/2}}{x^2} \qquad (20$$

and thus, from (1)

$$\frac{\left(V(x)\right)^{3/2}}{x^2} = \frac{\left(\Delta V\right)^{3/2}}{d^2} \tag{21}$$

or

$$V(x) = \Delta V\left(\frac{x}{d}\right)^{4/3} \tag{22}$$

The transit time, τ_s, for this case is

$$\begin{aligned}\tau_s &= \left(2\frac{e}{m}\Delta V\right)^{-1/2} d^{2/3}\int_0^d x^{-2/3}\,dx \\ &= \left(2\frac{e}{m}\Delta V\right)^{-1/2} d^{2/3}\left(3x^{1/3}\right)\Big|_0^d\end{aligned} \tag{23}$$

and thus

$$\tau_s = 3\left(2\frac{e}{m}\Delta V\right)^{-1/2} d \tag{24}$$

or

$$\tau_s = 5.06\times10^{-6}\,\frac{d}{\sqrt{\Delta V}}\ \text{seconds} \tag{25}$$

and

$$\tilde{\tau}_s = \frac{3}{2}\tilde{\tau}_0 \tag{26}$$

For our purposes there is not much difference between the two methods of calculation, but it is the case given by eqn. (25) which is important for determining recovery time from space charge limited operation. However it should be noted that in the process of recovery from space charge limitation ΔV will increase with time (ΔV is always considered postive) making the numerical evaluation of eqn. 25 only approximate. Nevertheless, this will serve for design purposes.

B. Detector Capacitance

Since the detector capacitance may combine with the load resistance to a given time constant and thus an effective rise time longer than times of interest, it is necessary to know this capacitance. Without space charge the detector capacitance, C_0 , is simply

$$C_0 = \epsilon_0 \frac{A}{d} \tag{27}$$

However, in the case of space charge limitation, the capacitance, C_s, has a somewhat more complicated form. All the stored negative charge is in transit between the electrodes and the electric field at the negative electrode is zero. All the positive charge is on the positive electrode and this charge (which must equal the the negative charge) can be calculated from the electric field, E. For this calculation the electrode at $x = d$ is considered positive (although by symmetry the result will hold for both cases). Thus from eqn. (22)

$$E(d) = -\left.\frac{\partial V}{\partial x}\right|_{x=d} = -\Delta V d^{-4/3}\left.\left(\frac{4}{3}x^{1/3}\right)\right|_{x=d} \tag{28}$$

or

$$E(d) = -\frac{4}{3}\frac{\Delta V}{d} \tag{29}$$

The total positive charge, Q, is then

$$Q = -A\epsilon_0 E(d) = \frac{4}{3}\epsilon_0 \frac{A}{d}\Delta V \tag{30}$$

and the capacitance, C_s, is

$$C_s = \frac{\partial Q}{\partial(\Delta V)} = \frac{4}{3}\epsilon_0 \frac{A}{d} \tag{31}$$

This result is dependent of ΔV and

$$C_s = \frac{4}{3} C_0 = 11.8 \frac{A}{d} \quad \text{picofarads} \tag{32}$$

so that,as with the electron transit times, there is little difference made by the space charge in the capacitance calculations.

This detector then has an effective "rise time", t_c , coming out of saturation determined by the capacitance and load impedance as

$$t_c \simeq 2ZC_s = 23.6\times10^{-3} Z\frac{A}{d} \quad \text{nanoseconds} \tag{33}$$

where Z is the load (signal cable impedance) as in figure 2. If there is any source impedance associated with V_1 this will have to be added to Z in eqn. (33) to given an accurate result.

C. Signal Transit Time

The final limitation on the response time of the detector will be the round trip transit time, $\mathcal{T}_t$, which is related to the signal path length, ℓ , from the signal outlet to the most distant point (electrically) by

$$\mathcal{T}_t = \frac{2\ell}{c} = 6.7\ell \quad \text{nanoseconds} \tag{34}$$

where c is the velocity of light and ℓ may be typically related to the electrode area as $\sqrt{A}$. The propagation velocity may also be affected to some small extent by the presence of the electrons in the vacuum but this effect is considered negligible in these calculations.

All three of these response time limitations must be considered in designing a space-charge limited detector. However, it should be noted that the resolution will not have to be near that required of a detector intended to resolve the initial pulse which a detector of this type is designed to reject.

IV. Other Effects

In an SCL detector there are additional limiting effects associated with the Compton electrons and the lower energy secondary electrons which should be mentioned. The Compton electrons have associated with them a space charge which can contribute to the space charge limitation of the detector. This effect will slightly lower the radiation level at which space charge limitation occurs and for higher radiation levels actually decrease the secondary electron current. Since one is not interested in the details of the signal from this detector during saturation (as long as it does not affect the later time signals) this effect should not be troublesome.

The SCL detector will also have a sensitivity to the Compton electron flux due to the γ-ray attenuation in the electrode connected to the signal cable. If the total array of plates (each of thickness t) presents a total thickness to the gamma flux significantly less than the γ-ray mean free path , r_γ, in the electrode material, then the fraction of the compton electron flux, f, collected in the signal electrode per unit signal electrode area is

$$f \simeq \frac{1}{2} \frac{t}{r_\gamma} \tag{35}$$

The factor of ½ results from the assumption that both sides of the signal electrode contribute to the signal electrode area as in figure 2. Referring to eqn. (9) for the sensitivity for the SEMIRAD, the magnitude of the ratio of the Compton sensitivity to the SEMIRAD sensitivity, S_{rel} , is

$$S_{rel} \simeq \frac{1}{2Y} \frac{t}{r_\gamma} \tag{36}$$

The sensitivity of the detector to the secondary electrons should be much greater than the sensitivity to the Compton electrons, but during space charge limitation, the Compton current may be orders of magnitude greater than the secondary electron current. As a design consideration then

$$\frac{t}{r_\gamma} << 2Y \tag{37}$$

For the secondary electron yield, Y, of the order of one then t/r_γ can be made extremely small.

A final effect to consider is the non-zero initial energy of the secondary electrons as they leave the emitter surface. The low energy component of the secondary electrons (which is of concern here) ranges from zero to typically 50 e.v. For ΔV's much greater than fifty volts this effect will not have much significance, but otherwise this spread in energies will have an effect in spreading out the "point" at which saturation sets in.

If one is careful these second order effects can be rendered negligible and in some cases even be made to cancel partially.

V. Sample Calculation

To illustrate how this space charge limited detector might be designed a sample calculation will be made. Let the radiation flux at saturation be $2\text{x}10^{17}$ γ-Mev/cm^2 sec (or about 10^8 roentgens/sec), the saturation signal voltage be 10 volts, the signal cable impedance be 50 ohms (the high voltage source impedance assumed zero) and the low energy secondary electron yield be 1.0. From eqn. (12) then

$$A = .1 \text{ meter}^2 \tag{38}$$

If the "risetime" from the detector capacitance in eqn. (33) is chosen to be

$$t_c = 10 \text{ ns.} \tag{39}$$

then the electrode spacing, d, is

$$d = 1.18 \text{ cm.} \tag{40}$$

From equation (34) the round trip signal transit time will be of the order of

$$\tau_t \simeq 2 \text{ ns} \tag{41}$$

depending on the details of the geometry. The saturation current is

$$J_s = 2.0 \frac{\text{amps}}{\text{meter}^2} \tag{42}$$

and from eqn. (3) then

$$\Delta V = 24 \text{ volts} \tag{43}$$

and thus from eqn. (5)

$$V_1 = 34 \text{ volts} \tag{44}$$

Finally, from eqn. (25) the electron transit time is

$$\tau_s = 12 \text{ ns}, \tag{45}$$

which is very nearly the same as obtained from eqn. (39) as a result of the detector capacitance.

It may be difficult to push some of these parameters much further. Clearly there is a premium attached to maximizing the low energy secondary electron yield because this can either increase the detector sensitivity or allow the electrode area to be decreased increasing the time resolution.

VI. Conclusion

A radiation detector designed on the space charge saturation principle should be capable of measuring the radiation level following intense radiation pulses. In the SEMIRAD it may be difficult to obtain both great sensitivity and fast time resolution. For lower radiation levels it will probably be necessary to use a fluor-photodiode system. The photodiode can also be operated as a space charge limited detector but great care will have to be given to selection of the fluor to ensure that it does not have late time decay components.

For many applications the SEMIRAD will be adequate and preferable due to its greater simplicity.

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