NOTE 11

CAPACITIVE PROBE E-FIELD SENSORS

by

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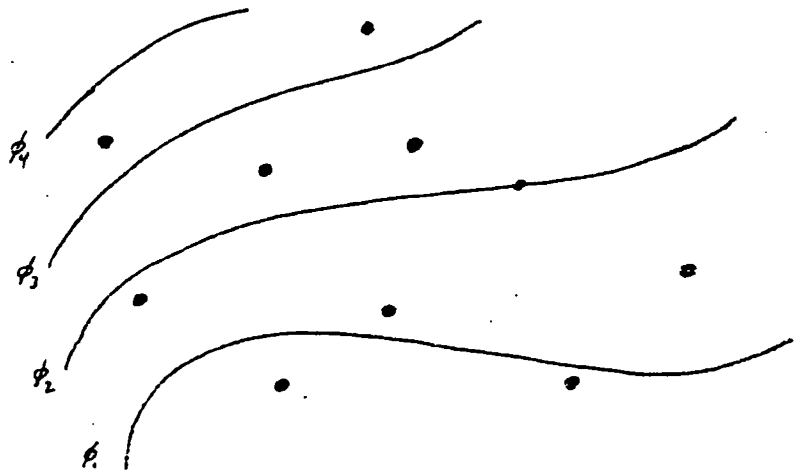
SENSOR AND SIMULATION NOTES XI

Capacitive Probe E Field Sensors

The ordinary concept of a detector sensitive to E fields but not to B or to $\nabla \times B$ is an antenna. However, the bandwidth of even the most advanced conventional antennas (e.g. 100:1 for a log-periodic) is so narrow compared to the bandwidths required for covering the signals of interest (typically 10^5:1) that such devices have very limited applicability. These bandwidths can be covered, however, by a small capacitive probe working into an essentially purely capacitive load. Such a technique was first used correctly by A. Glenn Jean of NBS on Upshot-Knothole in 1953. Such a probe in its simplest form is not suitable for use in a radiation environment because of the effects of Compton electrons, photoelectrons, secondary emission, and air conductivity. We shall restrict the present discussion to a non-radiation environment, and the effects of these other phenomena will be discussed in other S & S notes.

Consider a region of ambient electrostatic field containing an assemblage of small uncharged conducting bodies, each too small to perturb the field appreciably. Each one will acquire the potential φ_i of its local part of the field, as sketched at the right.



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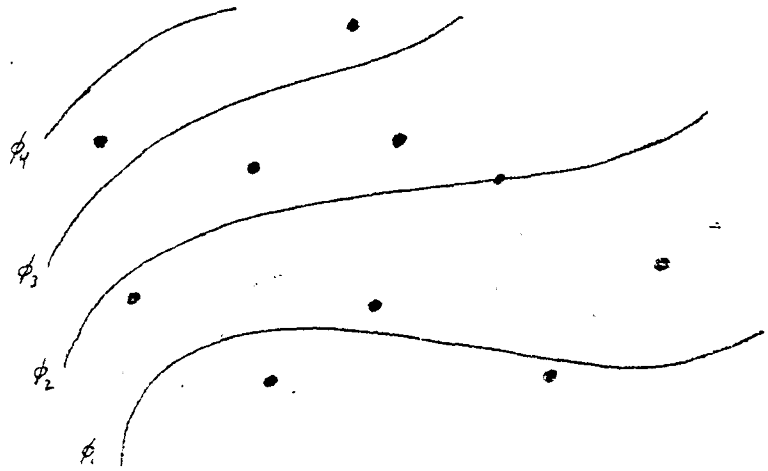


If now a number of these are to be assembled together to form a conducting probe, the differences of potential between them must cause a charge redistribution which, together with the applied field, will result in zero net field tangential to the conductor surface. The resulting composite body must be an equipotential Φ_0. This is entirely equivalent to using the principle of superposition to add the field applied to these bodies due to some remote configuration of charges to a second field arising from redistribution charges on the bodies of interest to arrive at a net field satisfying the boundary conditions. An additional condition to be met is that the net charge after redistribution must be zero since we have as yet provided no load path to ground.

Consider N particles which are to be interconnected, and assume that i-1 have now been assembled, leaving the i^{th} particle of capacitance c_i with a potential φ_i next to be connected to a main body with capacitance C_{i-1} and a potential Φ_{i-1}. For the time being we shall consider ensembles small enough so that their maximum dimensions are $\ll c\Delta t$, where Δt is the time required for any significant change in the applied field. Then the field reduces from $\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}$, where $\vec{A}$ is the vector potential, to the static approximation $\vec{E} = -\vec{\nabla}\varphi$. Under these conditions the only electrical characteristic of the particles is their capacitance (no radiation resistance or inductance), so when we connect the next particle, conservation of charge simply requires that

$$c_i \,\Delta\phi_i + C_{i-1}\,\Delta\Phi_{i-1} = 0 \qquad \text{where} \begin{cases} \Delta\phi_i = \Phi_i - \phi_i \\ \Delta\Phi_{i-1} = \Phi_i - \Phi_{i-1} \end{cases}$$

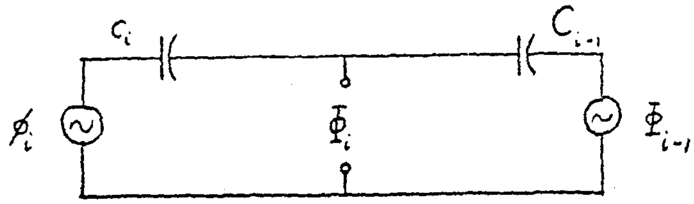
The combined body will have a total capacitance

$$C_i = \sum_{j=1}^{i} c_j = c_i + \sum_{j=1}^{i-1} c_j = c_i + C_{i-1}$$

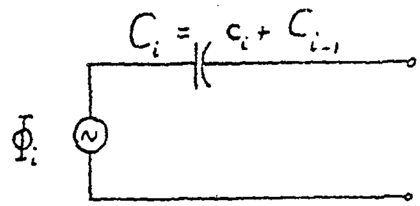
and a potential

$$\Phi_i = \phi_i + \Delta\phi_i = \Phi_{i-1} + \Delta\Phi_{i-1} = \Phi_{i-1} - \frac{c_i}{C_{i-1}}\Delta\phi_i = \frac{C_{i-1}}{C_i}\Phi_{i-1} + \frac{c_i}{C_i}\phi_i$$

The last expression above immediately implies this equivalent circuit:

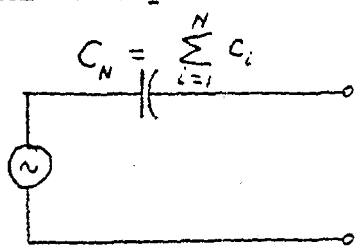


and for connecting the next particle or for driving a load, this becomes:



For the entire ensemble of N particles we have:

$$\Phi = \frac{\sum_{i=1}^{N} c_i \phi_i}{C_N}$$

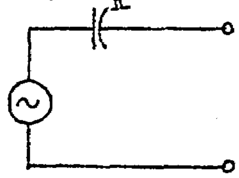


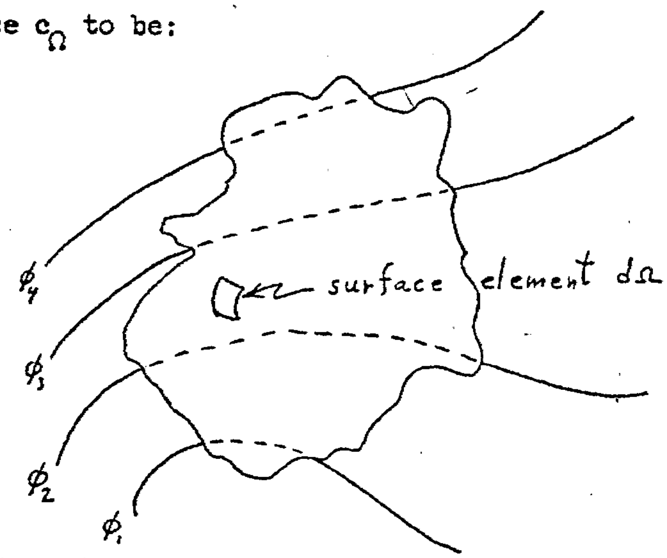
The potential contribution of each particle is thus seen to be weighted according to its capacitance because the product is a measure of the particle's ability to contribute charge, or of the charge required to change its potential from the ambient value to the final value. Thus the potential of the composite body is that of its "center of capacitance", which is somewhat analogous to a center of mass. This is the reason for the reference in standard antenna texts (e.g. Schelkunoff and Friis) to the effective height of a short antenna as the "height of its center of capacitance".

In the limit the summations above become integrals over the surface. Since the interior of a conducting object is all at the potential of its surface and since all charges appear at the surface, the behavior of a solid conductor is identical to that of a thin conducting shell. Thus we find the equivalent circuit of a surface Ω comprised of elements $d\Omega$ each at a potential φ_Ω and having a capacitance c_Ω to be:

$$C = \int_\Omega c_\Omega \, d\Omega$$

$$\Phi = \frac{\int_\Omega c_\Omega \phi_\Omega \, d\Omega}{C}$$

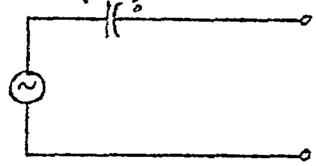




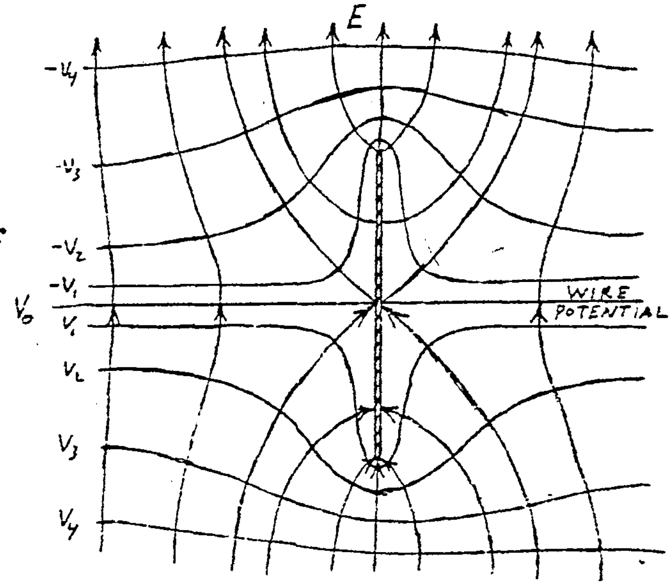
In the special case of a previously-uniform vertical field at all distances z above some reference plane, and a structure of height ℓ with uniform capacitance per unit length $\frac{\partial C}{\partial z} = c_k$, the equivalent circuit becomes

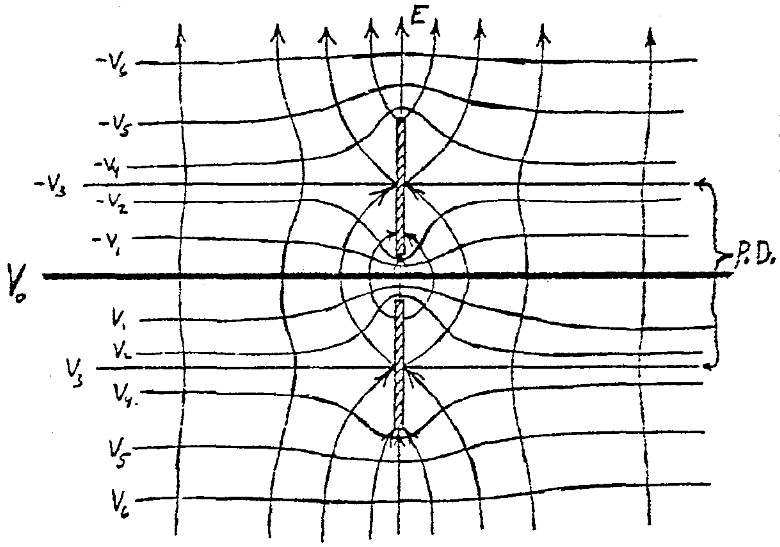
$$C_a = \int_0^\ell c_k \, dz = c_k \ell$$

$$\Phi = \frac{\int_0^\ell E z c_k \, dz}{\int_0^\ell c_k \, dz} = \frac{E \int_0^\ell z \, dz}{\int_0^\ell dz} = E \frac{\ell}{2}$$



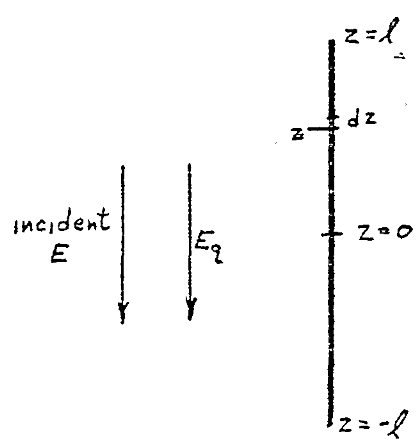
The unloaded effective height h_{eff} is the ratio of open-circuit output voltage to applied field, and is seen to be half of the physical height in this case. This is an excellent approximation for thin wires or thin cones, and is frequently used without further corrections. For more precise work, corrections are discussed later in this note. The capacitance is simply the capacitance as measured by a meter at the output terminals in the absence of applied fields.





Now let us look at a different approach. At the right are sketched the resulting field distribution after inserting a straight wire into a previously-uniform field, and of a pair of wires. The latter case is of particular interest because the equipotential V_0 can become our ground plane. First consider the single wire of length 2ℓ and a uniform ambient field E, as sketched at the lower right. The net tangential field must be zero, so the redistribution of charge must create a secondary field $E_q = -E$. (The E_q as shown must turn out to be negative.) Then we have

$E_q = -E = -\nabla V = \frac{\partial V}{\partial z}$ along the wire, where V is the potential of the charge distribution, not to be confused with the ambient potential field φ. In fact, $\varphi + V = 0$, since the wire is an equipotential whose



absolute potential V_o we shall call zero. Then

$$V = \int E_z \, dz = -\int E \, dz = -Ez + V_o = -Ez$$

For a uniform capacitance per unit length c_k, the charge per unit length q is

$$q = c_k V = -c_k E z$$

The continuity condition on the downward current i in the wire is $\frac{\partial i}{\partial z} = \frac{\partial q}{\partial t}$.

Using Laplace transform notation,

$$\frac{d\hat{I}}{dz} = \frac{d\hat{q}}{dt} = s\hat{q} = -sc_k \hat{E} z$$

$$\hat{I}(z) = -sc_k \hat{E} \int z \, dz = -sc_k \hat{E} \frac{z^2}{2} + K$$

Assume negligible end capacitance for the time being. The boundary conditions are then

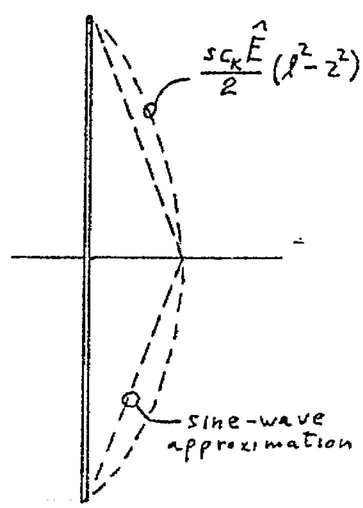
$$\hat{I}(\pm l) = 0$$

so

$$\hat{I}(z) = \frac{s c_k \hat{E}}{2} (l^2 - z^2)$$

Note that $\hat{I}$ leads $\hat{E}$ by $\pi/2$ in phase angle if s is $j\omega$.

Note that the distribution of $\hat{I}$ along the wire is parabolic, rather than the usual triangular (small-angle sine wave) distribution which is ordinarily pictured for short wires. This is because the source elements $\hat{E}dz$ are distributed all along the wire, rather than concentrated at a pair of driving terminals.



We can work the problem above without assuming $\hat{V}_0 = 0$, thusly:

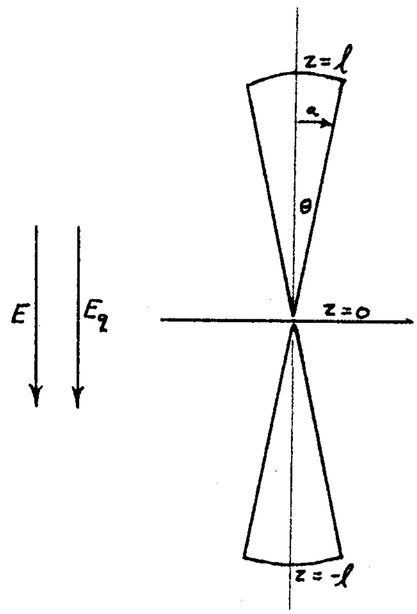
$$\frac{d\hat{I}}{dz} = s\hat{q} = -sc_k\hat{E}z + sc_k\hat{V}_0$$

$$\hat{I} = -\frac{sc_k\hat{E}}{2}z^2 + sc_k\hat{V}_0 z + K$$

$$\hat{I}(\pm\ell) = 0 = -\frac{sc_k\hat{E}}{2}\ell^2 \pm sc_k\hat{V}_0\ell + K$$

This can only be satisfied for $\hat{V}_0 = 0$, as before.

Now let us consider the split wire, of which the upper half can represent a probe above a ground plane. To get a uniform capacitance per unit length, this idealized model can be biconical. (Assume negligible end capacitance again.) The c_k of such a configuration is



$$c_k = \frac{\pi\epsilon}{\ln\cot\frac{\theta}{2}} \cong \frac{\pi\epsilon}{\ln\cot\frac{a}{2z}} \cong \frac{\pi\epsilon}{\ln\frac{2z}{a}}$$

where a is the cone radius at height z.

This time we must retain the $\hat{V}_0$ term to get the potential difference between the cone tips.

$$\hat{I} = -\frac{sc_k\hat{E}}{2}z^2 + sc_k\hat{V}_0 z + K$$

The boundary conditions are $\hat{I}(\pm\ell, 0) = 0$, so

$$\hat{I}(z>0) = \frac{sc_k\hat{E}}{2}z(\ell - z)$$

$$\hat{V}_{0_+} = \hat{E}\frac{\ell}{2}$$

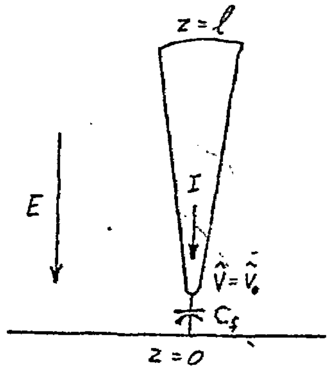
$$\hat{I}(z<0) = \frac{sc_k\hat{E}}{2}(-z)(\ell + z)$$

$$\hat{V}_{0_-} = -\hat{E}\frac{\ell}{2}$$

The open-circuit voltage difference = $\hat{V}_{o_+} - \hat{V}_{o_-} = \hat{E}\ell$.

The effective height is half of the physical height, as was expected.

Now consider a load capacitance C_f connected between the output terminal of the upper cone and ground.



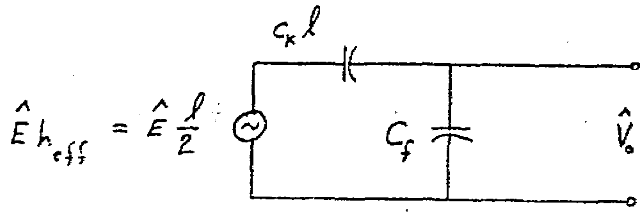
$$\hat{I}(z) = -\frac{s c_K \hat{E}}{2} z^2 + s c_K \hat{V}_o z + K$$

$$\hat{I}(o) = \hat{V}_o s C_f = K$$

$$\hat{I}(\ell) = 0 = -s\left[c_K \ell \hat{E} \frac{\ell}{2} - (c_K \ell + C_f)\hat{V}_o\right]$$

$$\hat{V}_o = \hat{E}\frac{\ell}{2}\frac{c_K \ell}{c_K \ell + C_f}$$

which gives, as expected, this equivalent circuit:



Now let us consider the more usual situation of an irregularly-shaped probe where $\frac{\partial C}{\partial z}$ is not a constant, but is an arbitrary function of z; $\frac{\partial C}{\partial z} = c(z)$:

$$\frac{d\hat{I}}{dz} = s\hat{q} = -s\,c(z)\hat{E}z + s\,c(z)\hat{V}_o$$

$$\hat{I}(z) = -s\hat{E}\int c(z)\,z\,dz + s\hat{V}_o\int c(z)\,dz + K$$

First consider an unloaded probe above a ground plane:

$$\hat{I}(o) = 0 = K$$

$$\hat{I}(\ell) = 0 = -s\left[\hat{E}\int_0^\ell c(z)\,z\,dz - \hat{V}_o\int_0^\ell c(z)\,dz\right]$$

$$\hat{V}_o = \hat{E}\frac{\int_0^\ell c(z)\,z\,dz}{\int_0^\ell c(z)\,dz} = \hat{E}\frac{\int_0^\ell c(z)\,z\,dz}{C} \equiv \hat{E}h_{eff}$$

Just as was expected, h_{eff} is the center of capacitance, since the ratio of integrals above is identical to the form for center of mass or gravity.

Now connect a load C_f at the base

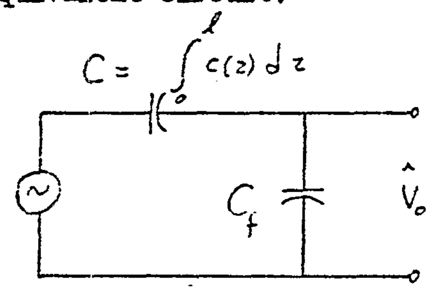
$$\hat{I}(0) = \hat{V}_o s C_f = K$$

$$\hat{I}(\ell) = 0 = -s\left[\hat{E}\int_0^{\ell} c(z)\, z\, dz - \hat{V}_o\left(C_f + C\right)\right]$$

$$\hat{V}_o = \hat{E}\,\frac{\int_0^{\ell} c(z)\, z\, dz}{C + C_f}$$

which gives us the expected equivalent circuit:

$$\hat{E}\,h_{eff} = \hat{E}\,\frac{\int_0^{\ell} c(z)\, z\, dz}{C}$$

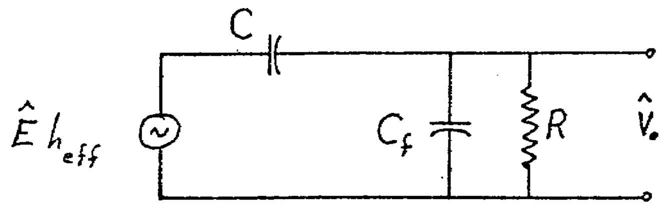


A few comments on the implications of the results above are now in order:

If we have a probe of uniform capacitance per unit length c_k (e.g., a thin cone) and change the value of this capacitance while keeping it uniform (e.g., changing the cone angle), the source impedance in the equivalent circuit is affected but the effective height is not.

If we have a non-uniform $c(z)$ and change its magnitude while holding its distribution constant with respect to z, the same situation obtains.

The paragraphs above are significant in considering the load which the probe can drive without distorting the waveform, since any physically realizable load will contain some resistance which usually can be represented thusly (we ignore for now active devices which can generate negative impedance):



From conventional circuit analysis we know that $\hat{V}_o$ will be a faithful replica of $\hat{E}$ for portions of the waveform for which $s \gg 1/R(C+C_f)$, and that V_o will be proportional to dE/dt for $s \ll 1/R(C+C_f)$.

For the cases considered below where R may be ignored, the frequency response is flat and we shall omit the circumflexes from E and V_o.

In the general case of a non-uniform $c(z)$ we must use the complete integrals above. However, this can be simplified in many practical cases where the total capacitance can be considered to be made up of a uniformly-distributed portion $c_k \ell$ plus a concentrated portion C_{z_o} localized near some particular height z_o. We then write

$$c(z) = c_k + c_{z_o}\,\delta(z-z_o)$$

where $\delta(z-z_o)$ is a delta function located at $z = z_o$, such that $\int_0^{\ell} \delta(z-z_o)dz = 1$. The total capacitance is then

$$C = \int_0^{\ell} c(z)\,dz = \int_0^{\ell} c_k\,dz + \int_0^{\ell} c_{z_o}\,\delta(z-z_o)\,dz = c_k \ell + C_{z_o}$$

and the open-circuit voltage is

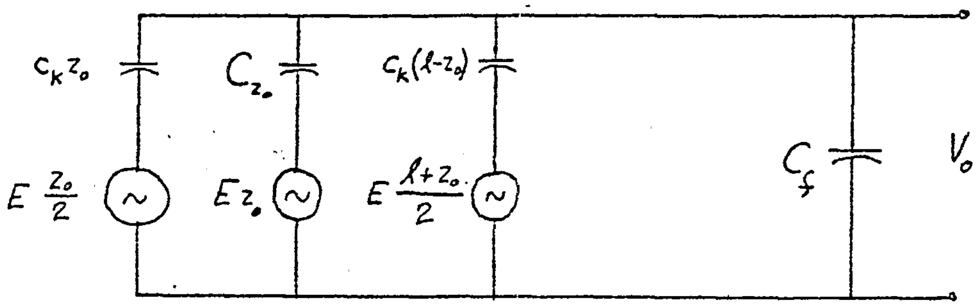
$$E h_{eff} = \frac{E}{C}\int_0^{\ell} c(z)\,z\,dz = \frac{E}{C}\int_0^{\ell} c_k\,z\,dz + \frac{E}{C}\int_0^{\ell} c_{z_o}\,z\,\delta(z-z_o)\,dz$$

For convenience we break the first integral up into two portions:

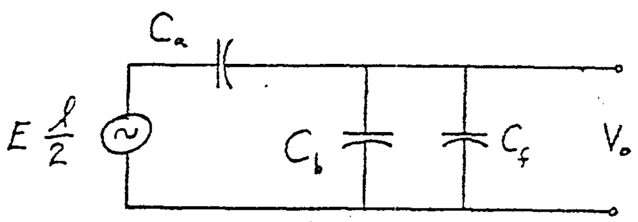
$$E h_{eff} = \frac{E}{C}\int_0^{z_0} c_K z\,dz + \frac{E}{C}\int_0^{\ell} C_{z_0}\, z\,\delta(z-z_0)\,dz + \frac{E}{C}\int_{z_0}^{\ell} c_K z\,dz$$

$$= E\,\frac{z_0}{2}\,\frac{c_K z_0}{c_K\ell + C_{z_0}} + E z_0\,\frac{C_{z_0}}{c_K\ell + C_{z_0}} + E\,\frac{\ell+z_0}{2}\,\frac{c_K(\ell-z_0)}{c_K\ell + C_{z_0}}$$

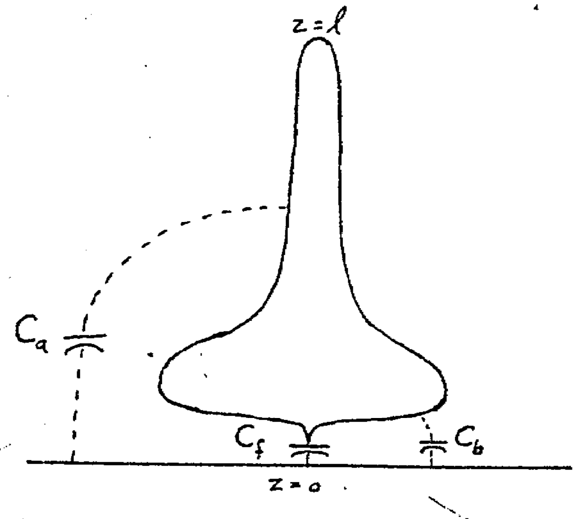
These three terms represent, respectively, the contributions of the uniform portion below the concentrated capacitance, that of the lumped capacitance, and that of the portion above. The equivalent circuit of these three with a load capacitance C_f is:

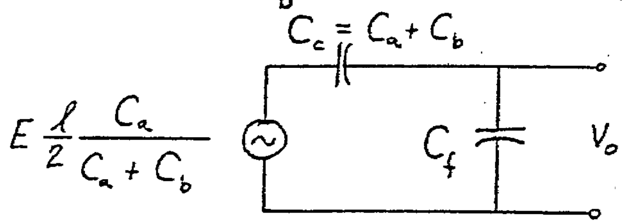


Two special cases are of particular interest. The first is where there is a concentration of capacitance near the base ($z_0 \to 0$) and C_{z_0} becomes a base capacitance C_b while $c_k\ell$ is the main antenna capacitance C_a. Then our equivalent circuit becomes:



There are two ways of looking at the effect of C_b. The first is emphasized in the equivalent circuit above, where C_b is represented as a load which attenuates the signal. C_f is a special case of C_b. The alternative way of looking at the effect of C_b is seen by redrawing the equivalent circuit:

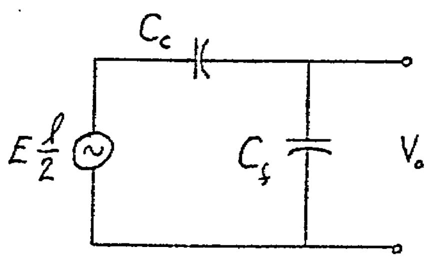




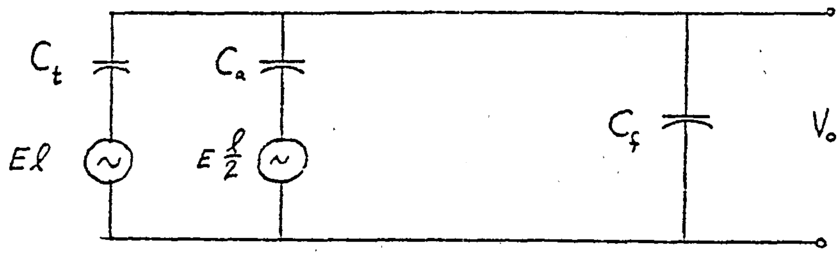
The effective height is now seen to be lowered because of the lower center of capacitance, while C_c is the measured capacitance. Either point of view results in the same output signal:

$$V_o \doteq E\frac{\ell}{2}\frac{C_a}{C_a + C_b + C_f} = E\frac{\ell}{2}\frac{C_a}{C_c + C_f}$$

A common error of experimenters in the past has been to use the equivalent circuit at the right, in which the effective height is half of the physical height, and C_c is the measured capacitance. By not recognizing that part of C_c may be C_b, the error is a factor of $C_a/(C_a+C_b)$.

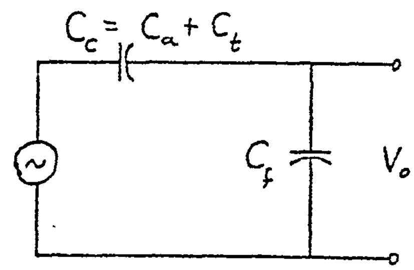


The other special case is that of a concentrated end-effect capacitance or of a top plate. Now $z_0 \to \ell$ and $C_{z_0} = C_t$:

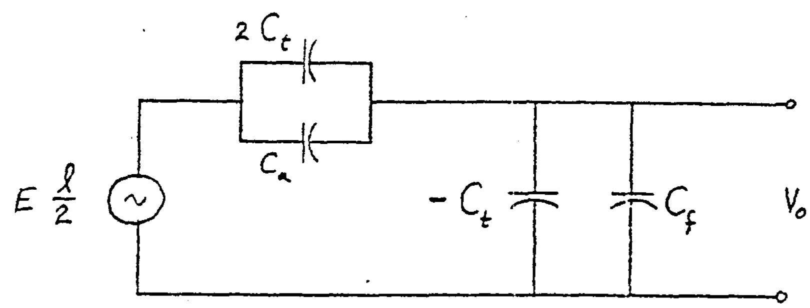


Again there are two points of view. One emphasizes the increase in effective height due to the higher center of capacitance with C_t twice as effective as C_a:

$$E\ell \frac{C_t}{C_a + C_t} + E\frac{\ell}{2}\frac{C_a}{C_a + C_t} = E\frac{\ell}{2}\frac{2C_t + C_a}{C_a + C_t}$$



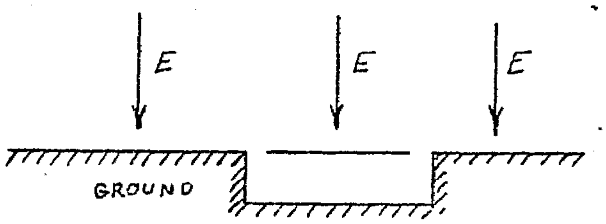
The alternative point of view considers C_t as a negative loading capacitor which increases the signal:



Another common error of experimenters is pointed up by statements made to the writer to the effect that the effective height of some probes has seemed to change as a function of the load capacitance C_f, i.e. the output of a primarily base-loaded probe has not seemed to follow the expression

$$V_o = E\frac{l}{2}\frac{C_a}{C_c + C_f} \qquad \text{as a function of } C_f.$$

The main cause of this discrepancy appears to be in the division of C_c into C_a and C_b, rather than ignoring of C_b. In particular, the non-ideal geometry of practical configurations (which arises as a result of the simultaneous needs for mechanical support and electrical insulation at the base) results in a signal contribution from metallic areas where $z_o = o$, and which thus would normally be considered as contributing only to C_b. As an extreme case, consider an insulated flat plate flush with the ground plane. The fact that this has non-zero dimensions transverse to the E field means that a signal will be picked up even though $z_o = o$. This effect may be calculated by noting that every D (=εE) line must terminate on a charge.



Thus D·A coulombs must be supplied to the plate from the surrounding ground, where A is the top area of the plate, just to keep the plate at ground potential. If a non-zero impedance connects the plate to ground, a signal will appear on the plate. Normally this impedance is the capacitance of the plate to the walls of the enclosure below it, plus its edge capacitance, and plus the load C_f. Such a plate (called a Watt plate, after B.E. Watt of LASL who suggested it) was used by Karl Theobald of LASL on Teapot in 1955. The signal from any portion of the base of a probe which resembles a plate must be added as a fourth contribution to the three-source model used earlier. In order to be sure that nothing has been left out of these models which might indeed indicate a change in effective height with load, the writer ran a controlled experiment under laboratory conditions on a probe which offered significant contributions from all four terms. Within the limits of experimental error (much smaller than any of the four effects) the behavior with varying loads was as expected, and the effective height remained constant.

Hopefully, a discussion of techniques for making these effective height and capacitance calculations and capacitance measurements can be included in an S & S note in the not-too-far distant future.

Ralph Partridge
February 10, 1965