Sensor and Simulation Notes
Note 108
23 May 1970

Distributed Sources and Terminations for Launching and Terminating Plane Waves Either With or Without a Reflected Plane Wave

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Abstract

This note considers some of the general characteristics of distributed sources and terminations for producing electromagnetic fields in some limited region of space. The various examples considered include one or two uniform TEM plane waves in the volume of interest; the second wave can be the reflection of the first from a conducting dielectric medium. One can also have a nonuniform TEM plane wave guided on a cylindrical transmission line in the volume of interest. As another case of interest the volume of interest is assumed to be filled with a low conductivity medium for simulating the low conductivity portions of a nuclear source region. Various source and termination geometries are illustrated. Some of the limitations in the realization of distributed sources and terminations are discussed.

I. Introduction

In simulating the nuclear electromagnetic pulse (EMP) one general technique or class of simulators uses the basic concept of a distributed source. A distributed source produces a wave of some desired form in a volume of interest by producing an appropriate electromagnetic quantity such as the tangential electric field on some appropriate surface which forms part of the boundary surface of the volume of interest. Of course, there will be approximations involved in realizing the desired field on the source surface, such as by breaking up the surface into a number of cells, each with its own energy module, switch, distribution conductors, and/or other items depending on the specific design. Such approximations in the realization of ideal distributed sources may introduce limitations in the resulting wave, such as by increasing the rise time of the wave over the basic rise time of the energy source used. These kinds of limitations can be considered separately. The basic concept of a distributed source relies on specifying the characteristics of the source on a source plane to match the desired electromagnetic wave (provided it is a possible electromagnetic wave in the first place).

Previous notes have discussed various types of distributed sources. This technique can be used for launching TEM plane waves guided on TEM transmission lines.[1] It can be used for launching special types of waves in conducting media for simulating close-in surface burst conditions.[2] It can be used for launching outward propagating spherical waves and combined with additional antenna structures.[3] This technique applies to radiating pulses to large distances from arrays.[4] Indeed there are numerous possible applications for the distributed source technique for launching various types of electromagnetic waves into various geometries of volumes.

1. Capt Carl E. Baum, Sensor and Simulation Note 31, The Conical Transmission Line as a Wave Launcher and Terminator for a Cylindrical Transmission Line, January 1967.

2. Capt Carl E. Baum, Sensor and Simulation Note 48, The Planar, Uniform Surface Transmission Line Driven from a Sheet Source, August 1967.

3. Capt Carl E. Baum, Sensor and Simulation Note 84, The Distributed Source for Launching Spherical Waves, May 1969.

4. Capt Carl E. Baum, Sensor and Simulation Note 100, Some Characteristics of Planar Distributed Sources for Radiating Transient Pulses, March 1970.

The distributed source problem and distributed termination[5] problem can be considered (electromagnetically) from similar viewpoints. For a distributed termination one has a certain desired wave on one side of the termination; on the other side there is another wave because of the tangential electric field being continuous through the termination surface. Similarly there are waves on both sides of a source surface which specifies the tangential electric field on the surface. If the energy sources and associated conductors are localized near the source plane and do not short out the wave "behind" the source (which would greatly increase the load on a transient unipolar pulsed source) then the source provides energy to both "forward" and "back" waves from the distributed source. One of the distinctions between a distributed source and a distributed termination is that basically the former supplies energy and the latter absorbs energy. However, in both cases for some assumed ideal type of wave in the volume of interest the required source and termination impedances can be rather complex with the distributed source perhaps having some energy dissipating elements and the termination perhaps having some active elements. In a real distributed source or termination various of these elements may not be included for practical reasons. The resulting fields in the volume of interest will then not be exactly the ideal fields sought and one might then consider just how much degradation has resulted to decide whether or not it is acceptable for a particular application.

The basic approach of assuming a desired wave in a volume of interest and using this wave plus any additional waves in other volumes to determine source and/or termination characteristics on surfaces of interest has importance then as a synthesis technique. One asks "What should the source and termination characteristics be?" rather than "What do the given sources and terminations do?"; the latter question is related to compromises introduced after one has determined some "ideal" characteristics from the answer to the first question.

In this note we consider some general characteristics of a class of simulators using distributed sources and terminations. These simulators are designed to produce TEM plane waves in confined volumes. There may be just one plane wave propagating in a fixed direction, or there may be two plane waves with the second wave being a reflection of the first from a conducting dielectric half space such as ground or water; the latter case applies to a simulator for an electromagnetic plane wave striking the ground at some angle of incidence and polarization. Another

5. Capt Carl E. Baum, Sensor and Simulation Note 53, Admittance Sheets for Terminating High-Frequency Transmission Lines, April 1968.

case of interest has a plane wave propagating in a medium of low conductivity occupying the volume of interest; this case applies to a simulator which includes some of the air-conductivity related effects in simulating the EMP in the outer portions of the source region associated with a near surface or air burst. For the case of a single plane wave propagating in a single direction one might have a uniform TEM plane wave in a volume with the complete boundary surface considered as the distributed source and/or termination. Alternately one might have an inhomogeneous TEM plane wave propagating on a transmission line with perfect conductors to guide the wave, with a source surface at one end, and with a termination surface at the other end.

II. General Concept of Simulator Volume and Surface

Consider a general closed surface which we designate by A as shown in figure 1. While the volume enclosed by this surface is shown as a simply connected volume and the closed surface is shown as simply connected (topologically), more complex closed surfaces as toroids can also be considered. The important point is that we are dividing space with a closed surface A to define an inside volume called V_{in} and an outside volume V_{out}. Strictly speaking A is taken as included in neither V_{in} nor V_{out}. However one does use limits (for the fields, say) as the position tends to A from either V_{out} or V_{in}. The surface A is the source and termination surface.

In figure 1 cartesian (x, y, z), cylindrical (Ψ, ϕ, z), and spherical (r, θ, ϕ) coordinates are illustrated; these are related by[6]

$$\begin{aligned} x &= \Psi \cos(\phi) \ , \qquad y = \Psi \sin(\phi) \\ \Psi &= r \sin(\theta) \ , \qquad z = r \cos(\theta) \end{aligned} \tag{1}$$

The position vector is

$$\vec{r} \equiv r\vec{e}_r = \Psi\vec{e}_\Psi + z\vec{e}_z = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z \tag{2}$$

where $\vec{e}$ with a coordinate subscript is a unit vector associated with the coordinate. In free space we have a propagation constant

6. All units are rationalized MKSA.

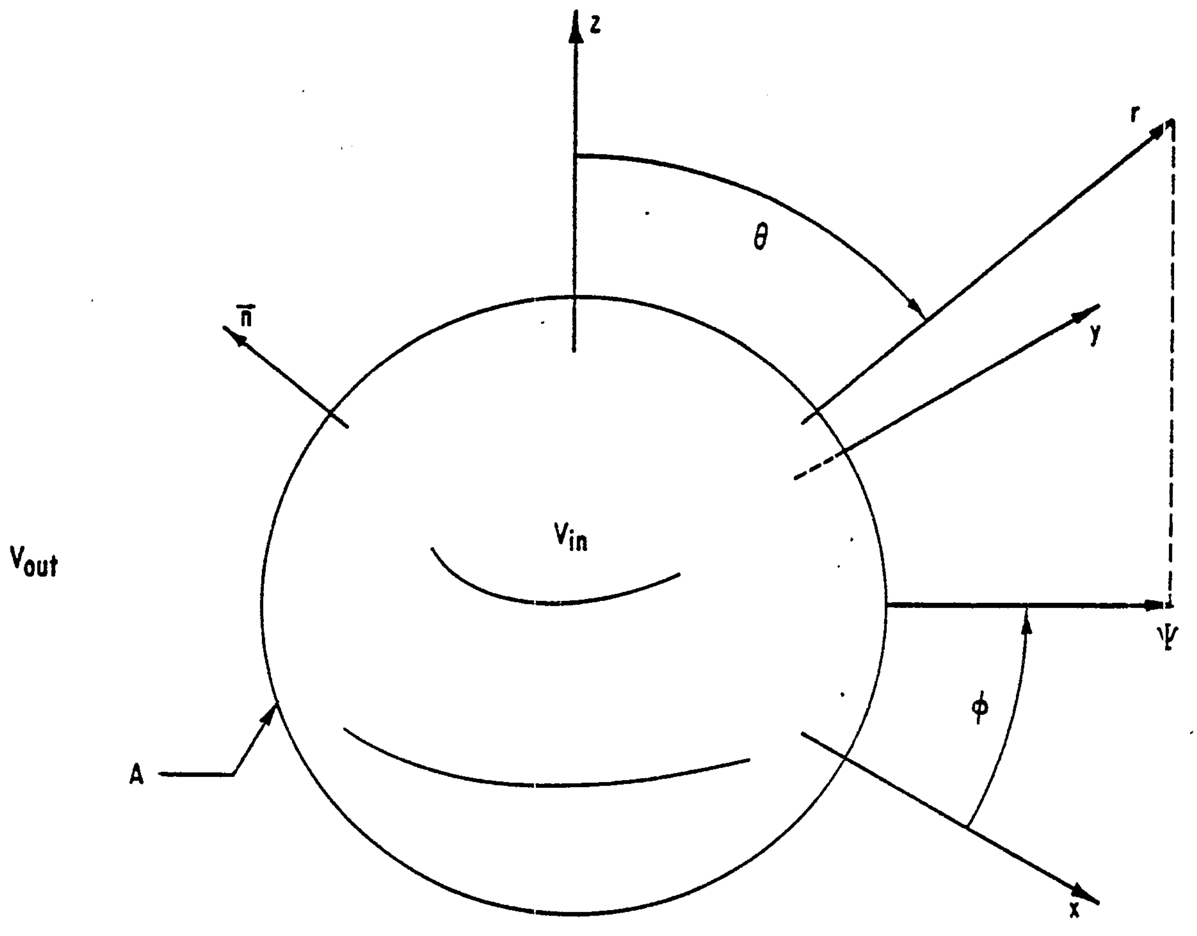


FIGURE I. SOURCE AND TERMINATION SURFACE WITH COORDINATES

$$\gamma_o = \frac{s}{c} \tag{3}$$

where s is the Laplace transform variable and the speed of light is

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \simeq 3 \times 10^8 \text{ m/s} \tag{4}$$

where μ_o and ε_o are the permeability and permittivity of free space respectively. The wave impedance of free space is

$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \simeq 377 \ \Omega \tag{5}$$

Consider a uniform TEM plane wave (labelled here as an incident wave) of the form

$$\begin{aligned} \tilde{\vec{E}}_{inc}(\vec{r}, s) &= E_o \tilde{f}(s) \vec{e}_2 \, e^{-\gamma_o \vec{e}_1 \cdot \vec{r}} \\ \tilde{\vec{H}}_{inc}(\vec{r}, s) &= \frac{E_o}{Z_o} \tilde{f}(s) \vec{e}_3 \, e^{-\gamma_o \vec{e}_1 \cdot \vec{r}} \end{aligned} \tag{6}$$

where a tilde ~ indicates the Laplace transform of the function and where E_o is a convenient constant, $\vec{e}_1$ is a unit vector specifying the direction of propagation, and $\vec{e}_2$ and $\vec{e}_3$ are unit vectors specifying the direction of the field vectors. These unit vectors are related by

$$\vec{e}_1 \times \vec{e}_2 = \vec{e}_3 \ , \qquad \vec{e}_2 \times \vec{e}_3 = \vec{e}_1 \ , \qquad \vec{e}_3 \times \vec{e}_1 = \vec{e}_2 \tag{7}$$

A tilde ~ over a quantity indicates the two sided Laplace transform with respect to time. Note that $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$ are taken independent of s. In the time domain we have

$$\vec{E}_{inc}(\vec{r}, t) = E_o f\left(t - \frac{\vec{e}_1 \cdot \vec{r}}{c}\right)\vec{e}_2$$

$$\vec{H}_{inc}(\vec{r}, t) = \frac{E_o}{Z_o} f\left(t - \frac{\vec{e}_1 \cdot \vec{r}}{c}\right)\vec{e}_3 \tag{8}$$

The function f is included to indicate the time history of some transient pulse. Let this plane wave exist in the volume V_{in}. For convenience let

$$t^* \equiv t - \frac{\vec{e}_1 \cdot \vec{r}}{c} \tag{9}$$

Note that $\vec{E}_{inc}$ and $\vec{H}_{inc}$ are only special forms for fields in V_{in}. The general case could be represented by $\vec{E}_{in}$ and $\vec{H}_{in}$.

On the surface A the electric field has a tangential component for the form of $\vec{E}_{in}$ in equations 8 given by

$$\begin{aligned}\vec{E}_A(\vec{r}_A, t) &= -[\vec{E}_{inc}(\vec{r}_A, t) \times \vec{n}(\vec{r}_A)] \times \vec{n}(\vec{r}_A) \\ &= E_o f(t_A^*)[\vec{n}(\vec{r}_A) \times \vec{e}_2] \times \vec{n}(\vec{r}_A)\end{aligned} \tag{10}$$

where $\vec{n}(\vec{r}_A)$ is the unit outward pointing normal for the closed surface A and is a function of $\vec{r}_A$. As a subscript A indicates the quantity evaluated on the surface A. In the case of $\vec{E}_A$ it indicates the component tangential to A and perpendicular to $\vec{n}$; note that $\vec{E}_A$ is a vector parallel to the surface A. We assume that $\vec{E}_A$ is continuous through the surface A. This is the appropriate assumption for cases of practical interest since we do not include the possibility of magnetic surface currents or zero impedance sources driving short circuit impedances just outside A (which would require infinite energy). One might put magnetic material outside A to raise the wave impedance but we do not consider this possibility here.

Having the tangential electric field on A specified from some desired form for $\vec{E}_{in}$, then with the radiation condition at infinity we have all one needs in principle to solve for the fields in V_{out}; we might generally call them $\vec{E}_{out}$ and $\vec{H}_{out}$. The continuity of the tangential electric field through A might then be generally expressed as

$$\vec{E}_A(\vec{r}_A, t) = -[\vec{E}_{in}(\vec{r}_A, t) \times \vec{n}(\vec{r}_A)] \times \vec{n}(\vec{r}_A)$$

$$= -[\vec{E}_{out}(\vec{r}_A, t) \times \vec{n}(\vec{r}_A)] \times \vec{n}(\vec{r}_A) \quad (11)$$

Note that these outside fields as determined by the solution of an exterior boundary value problem are much the same as the technique discussed in connection with the distributed termination in reference 5. Now $\vec{H}_{out}$ has a tangential component on A as does $\vec{H}_{in}$. The difference between these tangential magnetic fields is related to a surface current density on A as

$$\vec{J}_s(\vec{r}_A, t) = \vec{n}(\vec{r}_A) \times [\vec{H}_{out}(\vec{r}_A, t) - \vec{H}_{in}(\vec{r}_A, t)] \quad (12)$$

The relation between $\vec{E}_A$ and $\vec{J}_s$ determines whether the surface A is locally considered as a source or termination. Specifying $\vec{E}_{in}$ and $\vec{H}_{in}$, say as $\vec{E}_{inc}$ and $\vec{H}_{inc}$, then $\vec{E}_{out}$ and $\vec{H}_{out}$ can be found and then $\vec{E}_A$ and $\vec{J}_s$ can be found specifying the source and termination characteristics on A.

In considering the source and termination characteristics on A we can define a surface admittance Y_A from

$$Y_A \tilde{\vec{E}}_A(\vec{r}_A, s) \cdot \tilde{\vec{E}}_A(\vec{r}_A, s) \equiv \tilde{\vec{E}}_A(\vec{r}_A, s) \cdot \tilde{\vec{J}}_s(\vec{r}_A, s) \quad (13)$$

Note that in general $\tilde{\vec{J}}_s$ does not have to be parallel to $\tilde{\vec{E}}_A$. Thus in equation 13 Y_s is defined using the component of $\vec{J}_s$ parallel to $\vec{E}_A$. The component of $\vec{J}_s$ perpendicular to $\vec{E}_A$ corresponds to an infinite surface admittance or a short circuit; this corresponds to a zero surface impedance in this direction. Thus the surface admittance or surface impedance has directional characteristics in the general case.

Note that if $\vec{E}_{in}$ is specified as $\vec{E}_{inc}$ in equations 8, then $\vec{E}_A$ as in equation 10 has a direction which is independent of time and thus independent of frequency. This implies a Y_A associated with a fixed direction which makes the approximate realization of such a Y_A somewhat simpler to consider. On A there is a direction at each $\vec{r}_A$ which is a short circuit preventing the occurrence of any tangential electric field in this direction. This short circuit is a fixed set of conductors such as wires, one of the simpler things to realize. In some cases $\vec{J}_s$ may be parallel to $\vec{E}_A$ for all time because of the special geometry used; in such cases shorting conductors are not needed in principle but might be included anyway.

Now Y_A as defined in equation 13 is most appropriate in the case of a termination portion of A where Y_A is basically passive. For the case of $\vec{E}_{in}$ and $\vec{H}_{in}$ taken as $\vec{E}_{inc}$ and $\vec{H}_{inc}$ in equations 8 one might define A as locally a termination if $\vec{e}_1 \cdot \vec{n}$ is positive so that the Poynting vector in V_{in} given by

$$\vec{S}_{inc}(\vec{r}, t) \equiv \vec{E}_{inc} \times \vec{H}_{inc} = \frac{E_o^2}{Z_o} f^2\left(t - \frac{\vec{e}_1 \cdot \vec{r}}{c}\right)\vec{e}_1 \tag{14}$$

locally represents power flow into A. Similarly if $\vec{e}_1 \cdot \vec{n}$ is locally negative on A one might think of such a portion of A as a source. Since $\vec{e}_1$ is a fixed direction (independent of time) for this special form of $\vec{E}_{in}$ and $\vec{H}_{in}$ then this definition of termination and source is independent of frequency at each point on A. For this case such a definition would be quite appropriate. However the Poynting vector in V_{out}

$$\vec{S}_{out}(\vec{r}, t) \equiv \vec{E}_{out}(\vec{r}, t) \times \vec{H}_{out}(\vec{r}, t) \tag{15}$$

will generally vary in direction at the surface A for some particular choice of $\vec{E}_{in}$. Thus one cannot say for all frequencies that what one takes as source on A and termination on A by the above definition are really supplying or absorbing energy respectively at all times. In the frequency domain one might look at the required form of Y_A for a termination according to the above definition. As has been shown in a particular case in a previous note[7] such terminations are not always realizable with passive elements. However one can still use the solution of such problems to determine a termination using passive elements which in some sense approximates the ideal termination. Furthermore at low frequencies the termination impedance typically tends to a pure resistance because $\vec{S}_{out}$ typically becomes negligible at low frequencies if the surface A is of finite dimensions.

Those portions of A which one might call source by the above definition may have problems in realization similar to the termination problems as discussed above. Instead of considering the source as an active Y_A it is often convenient to consider it as the combination of an electric field source in series with an admittance Y_g. This electric field source is a generalization

7. R. W. Latham and K. S. H. Lee, Sensor and Simulation Note 68, Termination of Two Parallel Semi-Infinite Plates by a Matched Admittance Sheet, January 1969.

of a voltage source and has a direction parallel to the surface A. Thus a tangential electric field source can have two dimensional directional characteristics as can the surface admittance Y_g with the source; in general Y_g might even be a matrix. However for many cases of interest, including the case for a single plane wave in V_{in}, the source field has a fixed direction and the admittance needs to be considered only for this direction since it can be made infinite for the direction perpendicular to the source field on the source surface. Practically realizable field sources might be approximated as step functions associated with closing switches and source admittances might be capacitances, admittances associated with transmission lines combined with various lumped elements, etc. While there may still be required some active characteristics in Y_g to perfectly realize the ideal $\tilde{\vec{E}}_A$ and $\tilde{\vec{J}}_s$ required by the assumed $\vec{E}_{in}$ and $\vec{H}_{in}$, one may still be able to choose a passive Y_g which, together with the source tangential electric field, will give fields in V_{in} which approximate the assumed ideal $\vec{E}_{in}$ and $\vec{H}_{in}$ to some acceptable degree. Suppose the tangential source electric field on A is $\vec{E}_s$. Then including $\vec{E}_s$ equation 13 is replaced by

$$Y_s[\tilde{\vec{E}}_A(\vec{r}_A, s) - \tilde{\vec{E}}_s(\vec{r}_A, s)] \cdot \tilde{\vec{E}}_s(\vec{r}_A, s) = \tilde{\vec{E}}_s(\vec{r}_A, s) \cdot \tilde{\vec{J}}_s(\vec{r}_A, s) \qquad (16)$$

Here Y_s is defined for the direction of $\tilde{\vec{E}}_s$ and this applies to some cases of interest if the direction of $\vec{E}_s$ is time independent; the admittance normal to $\vec{E}_s$ on A can be made infinite.

While choosing $\vec{E}_{in}$ and $\vec{H}_{in}$ is useful for investigating the characteristics required of the sources and impedances on A one may want to use some discretion in the choice of $\vec{E}_{in}$ and $\vec{H}_{in}$. Even if the fields in V_{in} are chosen as a plane wave the time history $f(t^*)$ of the waveform is still left to choose. One might choose $f(t^*)$ such that simple source elements can be used to produce it. Perhaps $f(t^*)$ could be chosen as a fast rise followed by an exponential decay; such a waveform is characteristic of a charged capacitor switched into a resistive load. As an EMP simulator one may be interested in only some of the features of the waveform, such as peak amplitude, rise time, pulse width, frequency content across the spectrum of interest, etc. In this case one has some latitude in choosing practical sources and terminations which give a field distribution in space and a waveform as a function of time which reasonably approximates some simple temporal and spatial fields which are desired.

The assumption of a TEM plane wave of constant polarization in V_{in} (as in equations 8) gives a significant simplification in that $\vec{E}_A$ has a fixed direction at each position on A. However there are other forms of $\vec{E}_{in}$ and $\vec{H}_{in}$ that one might consider. Specifically one might consider the fields in V_{in} as the sum of two uniform TEM plane waves. The second TEM wave might be the reflection of the first wave from a planar boundary. Such a case of interest is encountered in simulating a plane wave incident on a ground or water surface; the reflected wave is part of the simulation problem. Considering the ground or water as a uniform conducting dielectric half space of fixed conductivity and permittivity (and permeability) independent of frequency, the reflected wave can be readily calculated by decomposing the incident and reflected waves into two orthogonal polarizations.[8] As an example of a simulator geometry for this form of $\vec{E}_{in}$ and $\vec{H}_{in}$ one might consider the prism shaped geometry shown in figure 2. One might roughly think of launching the incident plane wave from one of the sloping faces and terminating the plane wave reflected from the ground or water surface at the other sloping face. Besides these two sloping faces and the ground or water surface there are two end faces to complete A. Note, however, that fields penetrate into the ground or water medium so that the definition of a portion of the surface A here is somewhat problematical. One might replace the ground or water medium by a surface impedance at the ground or water surface but this would be an approximation; the fields actually penetrate into the lower medium and the manner in which the simulator surface joins to the lower medium has some influence on the wave reflected from the lower medium.

The inclusion of a reflected plane wave $\vec{E}_{re}$ and $\vec{H}_{re}$ gives for the fields in V_{in}

$$\vec{E}_{in} = \vec{E}_{inc} + \vec{E}_{re} , \qquad \vec{H}_{in} = \vec{H}_{inc} + \vec{H}_{re} \tag{17}$$

where $\vec{E}_{re}$ and $\vec{H}_{re}$ are perpendicular to a direction of propagation $\vec{e}_1'$ which is in general not equal to $\vec{e}_1$. While $\vec{E}_{re}$ and $\vec{H}_{re}$ are perpendicular to $\vec{e}_1'$ and form a plane wave they are not in general of the simple form as in equations 8 for the incident plane wave. Besides having a time dependence which can be different from that for the incident wave the directions of $\vec{E}_{re}$ and $\vec{H}_{re}$ may even change with time. Then at any given position of space $\vec{E}_{re}$ and $\vec{E}_{inc}$ do not in general have the same direction or the same time histories or Fourier transforms. Thus $\vec{E}_{in}$ in general has a direction which varies with time (and so with frequency) so that $\vec{E}_A$ on the source and termination surface in general varies with time (and so with frequency). This makes the appropriate source distribution somewhat more complicated.

8. Capt Carl E. Baum, EMP Theoretical Note 25, The Reflection of Pulsed Waves from the Surface of a Conducting Dielectric, February 1967.

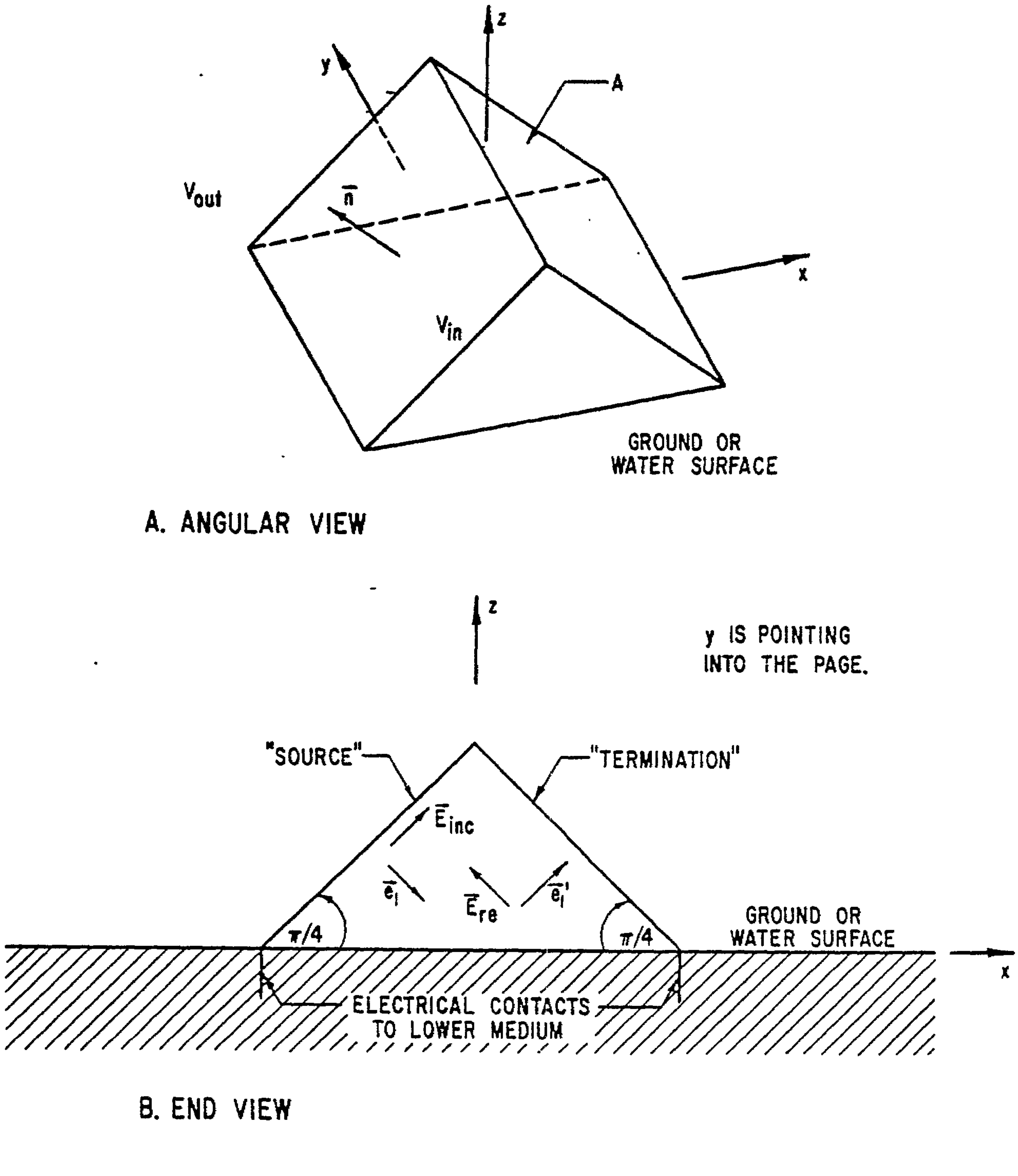


FIGURE 2. PRISM SHAPED DISTRIBUTED SOURCE AND TERMINATION GEOMETRY FOR AN INCIDENT PLANE WAVE PLUS A REFLECTED PLANE WAVE

There are special cases involving fields of the form in equations 17 for which $\vec{E}_A$ (the tangential electric field on A) has a fixed direction. Suppose, for example, that the fixed direction of $\vec{E}_{inc}$ (i.e. $\vec{e}_2$) is parallel to the ground or water surface so that $\vec{E}_{re}$ is also parallel to $\vec{e}_2$ (although of opposite direction in the time domain if f has a single polarity). Then $\vec{E}_{in}$ is parallel to $\vec{e}_2$ and so has a fixed direction. From equation 11 then at each point on A the tangential electric field $\vec{E}_A$ has a fixed direction.

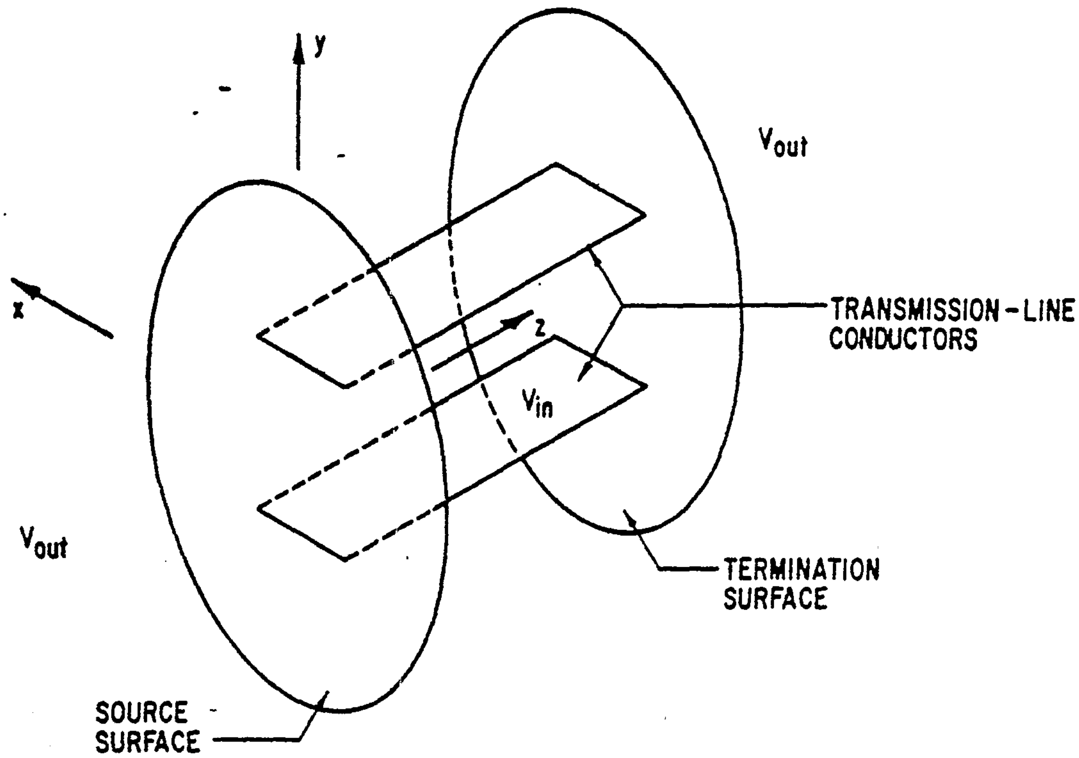
As another special case of interest let the z = 0 plane be the ground surface and let $\vec{e}_3 = \pm\vec{e}_y$ so that $\vec{e}_2$ is parallel to the x, z plane. As in figure 2 let the two sloping faces of the prism shaped surface A be perpendicular to the x, z plane. Since $\vec{H}_{inc}$ and $\vec{H}_{re}$ are both parallel to $\vec{e}_y$ then $\vec{E}_{inc}$ and $\vec{E}_{re}$ are both perpendicular to $\vec{e}_y$ and so parallel to the x, z plane. Since the sloping portions of A are parallel to $\vec{e}_y$ and perpendicular to the x, y plane, then $\vec{E}_{in}$ (which is perpendicular to $\vec{e}_y$) has a projection on the sloping surfaces with a fixed direction perpendicular to $\vec{e}_y$. Thus $\vec{E}_A$ has a fixed direction on the sloping surfaces of A; it is parallel to A and perpendicular to $\vec{e}_y$. As a further simplification the sloping surfaces might be sloped at angles of $\pi/4$ radians with respect to the z = 0 plane as illustrated in figure 2B. Furthermore $\vec{e}_1$ might be chosen perpendicular to one of these sloping surfaces so that it makes an angle of $\pi/4$ radians with respect to the z = 0 plane. Then $\vec{e}_1'$ makes an angle of $\pi/4$ radians with respect to the z = 0 plane and is perpendicular to $\vec{e}_1$. Let the second sloping surface be perpendicular to $\vec{e}_1'$. We then also have the first sloping surface parallel to $\vec{e}_1'$ and the second parallel to $\vec{e}_1$. With our choice of $\vec{e}_3 = \pm\vec{e}_y$ for this example then we have $\vec{E}_{inc}$ parallel to the first sloping surface and perpendicular to the second, while we have $\vec{E}_{re}$ parallel to the second sloping surface and perpendicular to the first. Thus for this special choice of the sloping surfaces perpendicular to each other we can in some sense launch the incident wave from one part of A and terminate the reflected wave in a separate part of A. This may simplify matters somewhat but these arguments only apply to the one direction of incidence and polarization and one set of angles for the two sloping surfaces. Of course the whole simulator could be rotated for greater flexibility. Note, however, that these considerations have not included the vertical ends of the prism or the connection of the source and termination surface to the ground or water. The considerations are then only approximate but may have some use in defining a simulator approach of this type. Even for this special case, however, one expects a detailed solution for the characteristics of A to be complex; one also expects that practical compromises will be needed in these characteristics.

In a general sense one might think of various surfaces such as hemispheres, rectangular parallelepipeds, etc. which might be joined to a ground or water surface and used as source and termination surfaces to produce fields as in equations 17 in V_{in} with no object placed in V_{in}. Of course with some system under test in V_{in} the fields are modified and one may be concerned that the presence of the system does not significantly change the simulator performance in simulating the desired plane wave incident on the ground or water surface. For various simulator geometries all the details of the source and termination surface need to be considered in detail by solving for the fields in V_{out} from boundary value problems. Furthermore sources and terminations can be considered which approximate the ideal ones in various features; such sources and terminations also require detailed boundary-value-problem solutions for accurate treatment.

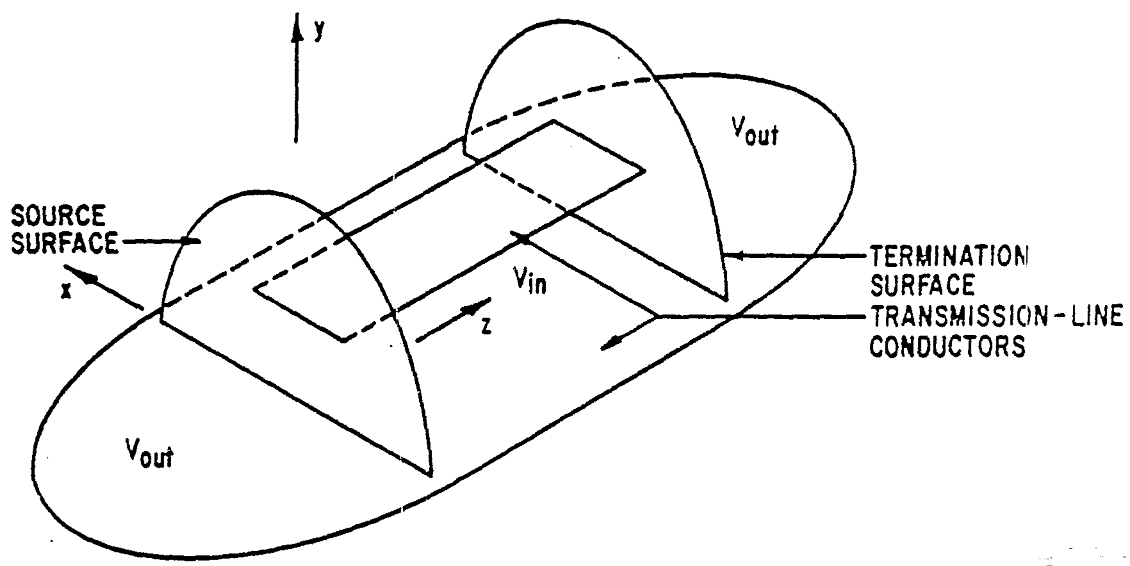
While we have considered one or two plane waves in this section for the fields in V_{in} there are many other forms of fields in V_{in} which one might consider for fields in V_{in} depending on the intended application. For example one might want to produce a transiently focused wave in V_{in} to obtain very large field strengths near some position. As long as the fields in V_{in} are possible (i.e. satisfy Maxwell's equations including nonlinear effects, etc. throughout V_{in}) then they can be used to find the required tangential fields on A. Solving for the fields in V_{out} (including the presence of the ground or water lower medium) then the source and termination characteristics required on A can be found. Based on this solution the more important characteristics on A can then be approximated with practical electrical elements.

III. TEM Transmission Line with Distributed Source and Termination

In section II we have considered some of the features of distributed sources for producing one or two uniform TEM plane waves in enclosed volumes. Another case of interest would be a nonuniform (or inhomogeneous) TEM wave in V_{in} such as can propagate on a TEM transmission line. If we include the appropriate transmission-line conductors in V_{in} then we have a simulator geometry of the type shown in figure 3. Note that even though the TEM wave is nonuniform we can still use it to approximate a uniform TEM wave over a volume of restricted dimensions (not all of V_{in}); depending on the geometry of the transmission-line conductors the corresponding TEM wave can be made to have various degrees of uniformity over restricted volumes. Here we are considering cylindrical TEM transmission lines with plane TEM waves, but the present considerations can also be applied to conical transmission lines with their corresponding spherical TEM waves.



A. SYMMETRICAL PARALLEL-PLATE TRANSMISSION LINE



B. PARALLEL-PLATE TRANSMISSION LINE WITH ONE VERY LARGE PLATE

FIGURE 3. DISTRIBUTED SOURCES AND TERMINATIONS COMBINED WITH CYLINDRICAL TEM TRANSMISSION LINES

One can have various shapes for the surface A but an interesting type of surface to consider consists in splitting A into two surfaces with basically one surface launching the nonuniform TEM wave and the second surface terminating it. In figure 3 these two surfaces are considered as planes for convenience. Note that V_{in} could be considered as infinite in extent as could each part of A. However the inhomogeneous TEM wave that we are considering is restricted to the free-space form

$$\vec{E}_{in} = [\nabla\Phi(x, y)]f\left(t - \frac{z}{c}\right)$$
$$\vec{H}_{in} = \frac{1}{Z_o}[\vec{e}_z \times \nabla\Phi(x, y)]f\left(t - \frac{z}{c}\right) \tag{18}$$

where the z direction is taken as the direction of propagation and where Φ is a potential function which is only a function of x and y, which has finite values on the transmission-line conductors, and which has a finite limit as $x^2 + y^2 \to \infty$. This potential function satisfies

$$\nabla^2\Phi(x, y) = 0 \tag{19}$$

away from the conductors. Furthermore only consider conductor geometries (which are not a function of z) such that most of the electromagnetic energy on the transmission line is confined within some finite value of $x^2 + y^2$. Then for an approximate realization of the ideal source and termination characteristics of A one might only consider portions of A close enough to the "center" of the transmission-line cross section that significant fields are present. This introduces some error in the realization of ideal sources and terminations which requires detailed boundary-value-problems for accurate determination. However one can clearly stop the extent of the source and termination surfaces somewhere such that only a certain acceptable distortion of the desired TEM wave occurs in a volume of interest near the "center" of the transmission-line cross section.

Figure 3A shows an example of a symmetrical parallel-plate transmission line in free space. The termination surface forms one boundary of part of V_{out} and the characteristics of this surface can be considered separately from those of the source surface. This type of problem is the distributed termination problem (in one form) as discussed in reference 5. The source surface also forms a boundary for a part of V_{out} and the characteristics of this distributed source can be considered (in the ideal form) independently of the termination surface.

This is one advantage in considering the surface A to be two separate surfaces, both extending to ∞. Since $\vec{E}_{in}$ and $\vec{H}_{in}$ are specified and thus have specified tangential components on A, then one can consider the ideal characteristics of the source and termination surfaces independently, each one requiring the solution of a boundary value problem in a separate portion of V_{out}. Of course in practical realizations of the source and termination surfaces the fields in V_{in} will only approximate the ideal form desired and some coupling between source and termination surfaces can occur. For example, suppose fields are launched of the form in equations 18 into V_{in} by the source surface. An imperfect termination surface will reflect a wave back toward the source surface and will interact with the source surface in some manner. Again note that practically the source and termination surfaces are of finite size so for ideal TEM waves which extend to infinity in a plane of constant z the source and termination surfaces cannot be ideal but automatically have some approximation introduced.

Figure 3B shows another case of interest involving an asymmetrical parallel-plate transmission line in which one of the conducting planes is ideally infinite in width so that we need not consider fields on the side away from the finite-width conducting sheet. Such a transmission-line configuration is convenient for setting on the ground surface. Note that the infinite-width conducting plane can even extend beyond both the source and termination surfaces infinitely far so that this geometry can be considered as an image problem and analyzed just like the geometry in figure 3A. Of course this conducting sheet is not practically infinite but can have dimensions large compared to the width of the other conducting sheet and compared to the separation between the two sheets. Also the larger conducting sheet need not extend beyond the source and termination surfaces; this just changes the boundary value problems for determining the ideal characteristics of the source and termination surfaces in that the ground characteristics or other characteristics of the medium "below" the larger conducting plate need to be included.

Since we are considering nonuniform TEM waves as in equations 18 where E_{in} has a fixed direction parallel to $\nabla\Phi(\underset{\sim}{x}, y)$ at each point in space, then the tangential electric field $\vec{E}_A$ on the source and termination surfaces has a fixed direction at each position. This direction in general varies as one moves from one position to another on these surfaces, but at least the direction is time independent. This means that on the source surface at each position electric field generators need be oriented in only one direction and conductors can be included if needed to ensure a zero tangential electric field in the orthogonal direction on the source surface. However the orientation of the generators and shorting conductors varies over the source surface in a manner to match the nonuniform TEM wave. To see how a pattern of such generators and conductors might look one can look at the potential and stream functions from the appropriate conformal transformation for the desired transmission-line geometry. The shorting conductors would be along equipotentials and the electric field generators would connect between two equipotentials, establishing the desired potential difference between the two equipotentials. For symmetrical parallel-plate transmission lines (or asymmetric ones which become symmetrical parallel-plate transmission lines when the image is included) one can see the potential and stream functions, and thus something about the source layout, in some previous notes.[9,10] Similar considerations apply to the termination surface except that instead of electric field sources one has appropriate load impedances; one can still have shorting wires along the equipotentials as before. The distribution of source admittance and termination admittance might be chosen based on the potential and stream functions except that this only includes the contribution associated with $\vec{H}_{in}$. Since $\vec{H}_{out}$ also contributes to the surface current density desired on the source and termination surfaces then some modification of the source and termination admittance distribution could be made on the basis of the solution for the fields in V_{out}. However, for convenience, one might average the effect of $\vec{H}_{out}$ over each of these surfaces to give an appropriate average correction to the admittances in each case. One might apply this average correction in the case of one or both surfaces to maintain a regularity in the elements making up the surface so as to match the pattern of the appropriate potential and stream functions characteristic of the transmission line, both in the orientation of the elements and the relative distribution of the admittance values.

Thus one can see that the fields in V_{in} can be readily generalized to include the case of nonuniform TEM waves with the accompanying transmission-line conductors. The surface A can be split into two surfaces which can be independently considered as surfaces with either basically source characteristics or basically termination characteristics. By confining the energy near the center axis of the transmission line the importance of the

9. Lt. Carl E. Baum, Sensor and Simulation Note 21, Impedances and Field Distributions for Parallel Plate Transmission Line Simulators, June 1966.

10. Terry L. Brown and Kenneth D. Granzow, Sensor and Simulation Note 52, A Parameter Study of Two Parallel Plate Transmission Line Simulators of EMP Sensor and Simulation Note 21, April 1968.

regions of space in V_{in} far away from this axis is reduced. This makes it convenient to have finite size source and termination surfaces without some of the "sides" of A to completely enclose V_{in}. Figure 3 has just shown two possible transmission-line geometries for use with distributed sources and terminations. This general technique of source and termination distribution could be applied to numerous geometries of transmission lines, not only to cylindrical transmission lines but to conical transmission lines as well.

IV. Extension to Media of Non Zero but Small Conductivity

In sections II and III we only considered TEM waves that were propagating in V_{in} in a medium of zero conductivity. However, this does not need to be the case. If one makes the medium in V_{in} conducting one can propagate an electromagnetic pulse through it but the wave characteristics will be somewhat different from those for the case of zero conductivity. One may wish to have a non zero conductivity in V_{in} in order to more adequately simulate the nuclear electromagnetic pulse as it occurs in an ionized conducting source region. This is not a completely adequate simulation of such a source region because the source currents which generate the fields throughout the nuclear source region are not present in the simulator. Furthermore in the nuclear source region the air conductivity is a function of the electric field and varies with time. However by including a constant conductivity in V_{in} for the simulator one can still simulate some additional features of the nuclear source region which are not included if the conductivity is zero. Specifically the inclusion of conductivity in V_{in} can change the impedance characteristics of various of the antennas, etc. which couple electromagnetic energy into the system under test. Also the presence of conductivity σ gives a total current density of the form

$$\vec{J}_t = \sigma\vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \tag{20}$$

The permittivity ϵ gives a displacement current density proportional to the time derivative of the electric field. At sufficiently low radian frequencies ($\omega < \sigma/\epsilon$) the conduction current $\sigma\vec{E}$ is the dominant contributor to the total current density and can then markedly increase the current picked up by certain types of electric-field antennas which might be better classed as current density sensors.[11]

11. Capt Carl E. Baum, Sensor and Simulation Note 38, Parameters for Some Electrically-Small Electromagnetic Sensors, March 1967.

One type of such a simulator has been considered in reference 2 in which the wave in the conducting medium is not TEM; this type of simulator has been considered in a later note showing various features which might be incorporated in a practical design.[12] This type of simulator can use conductivity levels which are large enough that the high frequencies rapidly attenuate in propagating through the conducting medium placed in the simulator. For such a simulator the source surface must then be placed rather close to the ground surface and the portions of the system at the ground surface or in the air; this allows the attenuation of the high frequencies, or degradation of rise time, to be minimized or reduced to some acceptable value.

In this section we consider the case that the conductivity in V_{in} is comparatively small so that the high frequencies can propagate farther. For convenience we consider TEM waves. The actual geometry of the surface A can be rather general as discussed in section II.

Let V_{in} have uniform permittivity ε, permeability μ, and conductivity σ, all independent of s. The propagation constant is

$$\gamma = [s\mu(\sigma + s\varepsilon)]^{1/2} \tag{21}$$

and the wave impedance is

$$Z = \left[\frac{s\mu}{\sigma + s\varepsilon}\right]^{1/2} \tag{22}$$

Then consider a TEM plane wave of the form

$$\begin{aligned} \tilde{\vec{E}}_{inc}(\vec{r}, s) &= E_o\tilde{f}(s)\vec{e}_2\, e^{-\gamma\vec{e}_1\cdot\vec{r}} \\ \tilde{\vec{H}}_{inc}(\vec{r}, s) &= \frac{E_o}{Z}\,\tilde{f}(s)\vec{e}_3\, e^{-\gamma\vec{e}_1\cdot\vec{r}} \end{aligned} \tag{23}$$

12. Capt Carl E. Baum, Sensor and Simulation Note 87, A Distributed-Source Conducting-Medium Simulator for Structures Near and Below the Ground Surface, July 1969.

which is the same as in equations 6 except that the new medium parameters are included. Since the medium is now dispersive the time-domain waveform cannot be written in the simple form as in equations 8.

From the propagation constant one can get an indication of some of the design problems. Considering the high-frequency propagation we have for $s \to \infty$

$$\gamma = s\sqrt{\mu\varepsilon}\left[1 + \frac{\sigma}{s\varepsilon}\right]^{1/2} = s\sqrt{\mu\varepsilon}\left[1 + \frac{\sigma}{2s} + O(s^{-2})\right]$$

$$= s\sqrt{\mu\varepsilon} + \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + O(s^{-1}) \tag{24}$$

The first term $s\sqrt{\mu\varepsilon}$ is the delay term; in propagating a distance Δ (which is the change in $\vec{e}_1 \cdot \vec{r}$) there is a time delay $\Delta\sqrt{\mu\varepsilon}$ in the leading edge of the transient waveform. One may wish to have a wave with a leading edge propagating at the speed of light because this propagation speed is characteristic of the initial fields in the nuclear source region. Then this would require μ and ε to be their minimum free space values μ_o and ε_o. Practically μ could be easily made very closely μ_o. However if some conducting dielectric is used to fill V_{in}, then ε will generally be larger than ε_o, but through careful choice of material ε should be only slightly larger than ε_o.

The second term $(\sigma/2)\sqrt{\mu/\varepsilon}$ in the expansion in equation 24 is the high frequency attenuation. In propagating a distance Δ the high frequencies in the wave are reduced in magnitude by the factor $e^{-\alpha}$ where

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}\,\Delta \tag{25}$$

which for $\mu = \mu_o$, $\varepsilon = \varepsilon_o$ is

$$\alpha = \frac{Z_o}{2}\,\sigma\Delta \tag{26}$$

and where $e^{-\alpha} < 1$. If one desires only a small attenuation so that $e^{-\alpha}$ is only slightly less than one then the attenuation factor is approximately $1 - \alpha$ which implies that one would need $\alpha << 1$. This establishes a restriction

$$\sigma << \frac{2}{\Delta}\sqrt{\frac{\varepsilon}{\mu}} \tag{27}$$

for small high-frequency attenuation; for $\mu = \mu_o$, $\varepsilon = \varepsilon_o$ this becomes

$$\sigma << \frac{2}{Z_o \Delta} \tag{28}$$

This also shows a relation between the conductivity σ and the characteristic simulator dimension Δ (for V_{in}) in the direction of propagation. For a given maximum acceptable α, as Δ is increased the maximum allowable σ is decreased. As an example suppose $\Delta \simeq 50$ m which would imply from equation 28 that σ should be small compared to 10^{-4} mho/m.

In the nuclear source region the source currents attenuate with distance because of the attenuation of the nuclear radiation. At sea level the characteristic attenuation distance for gamma rays is a few hundred meters. One might try to match the attenuation in the simulator to the attenuation of the nuclear sources. However the fields in the nuclear source attenuate with distance somewhat differently and they are not a TEM plane wave.

Define a relaxation frequency as

$$\omega_r \equiv \frac{\sigma}{\varepsilon} \tag{29}$$

and define a relaxation time as

$$t_r \equiv \frac{\varepsilon}{\sigma} \tag{30}$$

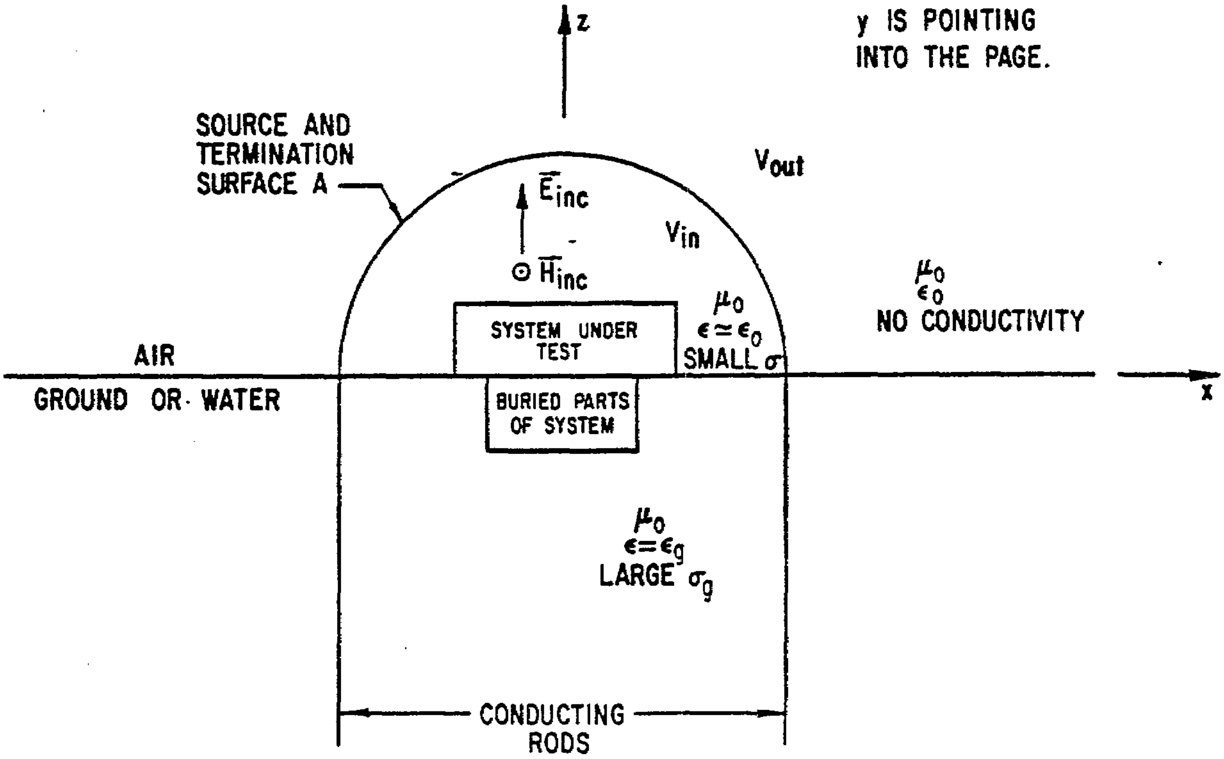
For radian frequency $\omega >> \omega_r$ then $Z \simeq \sqrt{\mu/\varepsilon}$ and for $\mu = \mu_o$, $\varepsilon = \varepsilon_o$ then $Z \simeq Z_o$. Thus for sufficiently high frequencies or sufficiently small characteristic times the electric and magnetic fields in the chosen TEM plane wave have approximately the same form. For ω of the order of or less than ω_r the TEM plane wave as in equations 23 has different characteristics for the electric and magnetic fields because of the frequency dependent characteristics of Z. For low enough frequency such that $|\gamma\Delta| << 1$(implying complex wavelengths much larger than the appropriate simulator dimensions) one might alter the source and termination characteristics of the surface A so as to give some other form to the fields, different from the form in equations 23. One could also choose some other field distribution other than a TEM plane wave around which to design such a simulator. For example, one might use a wave with curved wavefronts at early times to give some partial focusing properties to partially offset the high-frequency attenuation associated with the medium conductivity.

As an example of the use of low conductivity media in V_{in} for a distributed source and termination, consider the geometry shown in figure 4. Here the source and termination surface A is illustrated as a hemisphere and V_{in} is considered as bounded by this hemisphere plus the ground or water surface. The system under test is assumed in V_{in} on the ground or water surface or perhaps totally or partially under this surface (but at least near the ground or water surface). There is some conducting medium (a foam perhaps) used to fill V_{in} around the system under test. Let $\mu = \mu_o$, $\varepsilon \approx \varepsilon_o$, and σ be small. For convenience the TEM plane wave as in equations 23 might be assumed to propagate in the x direction with $\vec{E}$ in the z direction (up) and $\vec{H}$ in the -y direction (parallel to the ground or water surface); this case would be specified by

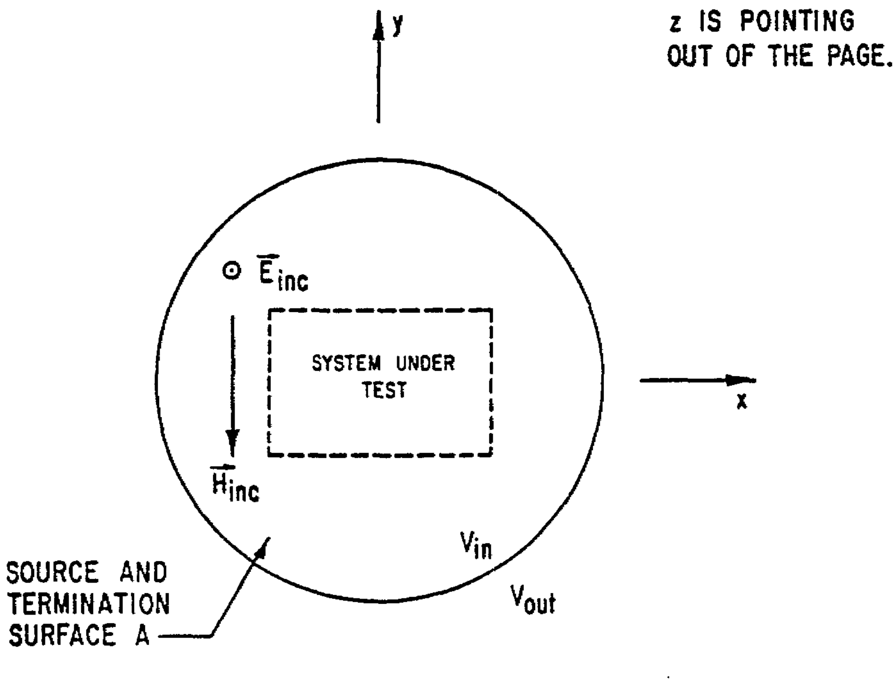
$$\vec{e}_1 = \vec{e}_x \, , \qquad \vec{e}_2 = \vec{e}_z \, , \qquad \vec{e}_3 = -\vec{e}_y \tag{31}$$

This choice gives the initial wavefront travelling at velocity $c \, \vec{e}_x$ over the ground or water surface and might be an appropriate choice for simulating the low conductivity parts of the source region for a near surface burst. Note that the ground or water conductivity is assumed large for this case so that an approximate TEM wave can propagate over it. Of course one could account for the ground or water conductivity by assuming a type of wave (such as a surface bound wave) for which the fields in the lower medium are included. Furthermore one could choose some combination of two TEM plane waves in V_{in} (as in section II) with appropriate matching at the ground or water surface.

There is still the matter of the electrical connection to the ground or water medium to be considered for our example in figure 4. This connection should be good enough to allow current to freely flow between the surface A and the lower medium so as to not significantly distort the desired fields in V_{in}. However if portions of the system under test extend significant distances into the lower medium one should consider the fields below the ground or water surface as well; one would like the fields in the lower medium to be consistent with the case of the EMP being simulated. Provided the lower medium has a high conductivity σ_g such that only sufficiently low frequency fields



A. SIDE CROSS-SECTION VIEW



B. TOP VIEW

FIGURE 4. DISTRIBUTED SOURCE AND TERMINATION BOUNDING A LOW-CONDUCTIVITY MEDIUM: EXAMPLE USING HEMISPHERICAL GEOMETRY

can penetrate to significant depths, then one can use a buried-transmission-line technique for these fields.[13] This involves placing vertical conductors into the lower medium to depths significantly beyond those of interest provided these depths are of the order of Δ or greater so that a transmission-line approximation has some validity at the low frequencies of concern. Typically the vertical conductors in the lower medium might be rods; one might connect these electrically as two groups to form a two conductor buried transmission line, or the rods could be driven as many groups tied in some appropriate fashion to the distributed source and termination on A.

While in this section we have generalized the discussion in section II to include conducting media in V_{in} the general considerations of section II still apply. Specifically the problems inherent in realizing a practical distributed source and termination must still be considered. In realizing various ideal field distributions in time and space the source and termination characteristics required may be impractical and some compromise may be necessary. Just how detrimental these compromises are requires detailed considerations of the boundary-value-problem variety. Hopefully in cases of interest such compromises as are necessary will not sacrifice much of the essential characteristics which one is trying to produce in the simulator.

V. Summary

The distributed source and termination is then a rather general technique for producing electromagnetic fields in a volume of space. There are numerous geometries of source and termination surfaces which one might use for various volumes of interest. In these volumes there could be one or two uniform TEM plane waves, or perhaps a nonuniform TEM plane wave guided on a cylindrical transmission line in the volume; these cases have been considered in this note. However, there are various other types of waves one might consider depending on the application in mind. For example one might use a distributed source surrounding V_{in} to transiently focus fields near some point in V_{in} and thereby obtain very large field strengths. Furthermore the medium filling V_{in} can have various characteristics and might even be inhomogeneous and/or anisotropic if one so desired. In this note we have considered the interesting case with $\mu = \mu_o$, $\varepsilon \approx \varepsilon_o$, and small σ because of its application for simulating the EMP in the low conductivity portions of the nuclear source region.

13. Lt. Carl E. Baum, Sensor and Simulation Note 22, A Transmission Line EMP Simulation Technique for Buried Structures, June 1966.

In designing distributed sources and terminations one first considers the form of the fields desired in the volume of interest and then determines what this implies about the source and termination characteristics. However, there may be practical limitations in realizing such ideal sources and terminations and this in general requires some compromises in the source and termination design. These compromises change the fields in V_{in}. Thus there are many detailed calculations of the boundary-value-problem type needed to quantitatively understand the impact of these compromises. Sources and terminations are in general not uniform and zero thickness sheets but are composed of arrays of elements of various shapes and electrical characteristics. Furthermore one would prefer to use comparatively simple, reliable, and inexpensive elements instead of some of the complex ones an ideal source and/or termination might imply. Thus different types of impedances and sources will be used and their impact needs to be assessed. By considering various shapes of source and termination geometries one might also see which shapes result in more desirable field distributions in space and time while using various practical source and impedance elements.