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Sensor and Simulation Notes
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ABSTRACT

A theoretical study is made for obtaining the pulse radiated from an impedance loaded dipole. Numerical results are presented for an antenna that is slowly charged and suddenly shorted at the terminals. The impedance loadings are chosen as those on an existing antenna located at Sandia Laboratories, Albuquerque, New Mexico.

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INTRODUCTION

The radiation of a particular pulse shape by a dipole antenna is a difficult problem both experimentally and theoretically expecially if the pulse is very sharp. To obtain the desired pulse experimentally would require a specific voltage pulse driving the antenna. And to determine the required driving voltage pulse may be extremely difficult. However if the frequency response of the antenna is essentially flat then the radiated pulse will have essentially the same shape as the driving voltage pulse. Such an antenna was recently studied [1]. The formulation yields the current distribution under steady state conditions. The radiated fields also may be determined for steady state conditions following the procedure suggested by Harrison et al [2] . In this paper these radiated fields are superimposed to obtain the appropriate time history of the radiated fields for the specific voltage pulse excitation.

Numerical results are presented which demonstrate the solution technique. The antenna is considered to be slowly charged up to time t = o when the antenna terminals are shorted. The impedence loadings on the antenna are selected as those on the "Long-Wire" antenna at Sandia Corporation, Albuquerque, New Mexico.

ANALYSIS

Current Distribution

Consider a dipole antenna of length 2h to extend from $z = -h$ to $z = h$ (see figure 1). The antenna is driven at the center by voltage $V_o(\omega)$, ω is the radian frequency, and is symmetrically loaded with impedances Z_ℓ at points $\pm z_\ell$ along the antenna axis. For convenience, a slice generator driving mechanism is used. Then the current distribution on the antenna is obtained by solving the integral equation

$$\int_{-h}^{h} dz' \, \hat{I}(z',\omega) \, K_a(z-z') - j \frac{4\pi k}{\zeta} \sum_{\ell=1}^{N} Z_\ell \, \delta(|z|-z_\ell) \, I(z) = -j \frac{4\pi k}{\zeta} \hat{V}_o(\omega) \, \delta(z) \qquad \text{..........(1)}$$

where

$$K_a(z-z') = \left(\frac{\partial^2}{\partial z^2} + k^2\right) \left\{ \exp\left[-jk\sqrt{(z-z')^2 + a^2}\right] \Big/ \sqrt{(z-z')^2 + a^2} \right\} \qquad (2)$$

$k = \dfrac{\omega}{c}$, c is the speed of light

$\zeta \simeq 120\pi$ is the wave impedance of free space

The solution to (1) may be effected by using a finite Fourier series representation for the current distribution. It is

$$\hat{I}(z,\omega) = -j \frac{4\pi \hat{V}_o(\omega)}{\zeta} \left\{ \sum_{m=1}^{M} I_m \cos\left[\frac{(2m+1)\pi}{2} \frac{z}{h}\right] + C \sin k(h-|z|) \right\} \qquad (3)$$

The constant C is chosen to expedite the solution and the expansion coefficients are obtained by solving a system of linear equations resulting from substituting (3) into (1) [1]. To obtain the current distribution on the antenna when driven by a pulsed voltage, $\hat{V}_o(\omega)$ is taken as the Fourier transform of the voltage pulse and the resulting $\hat{I}(z,\omega)$ is interpreted as the Fourier transform of the current

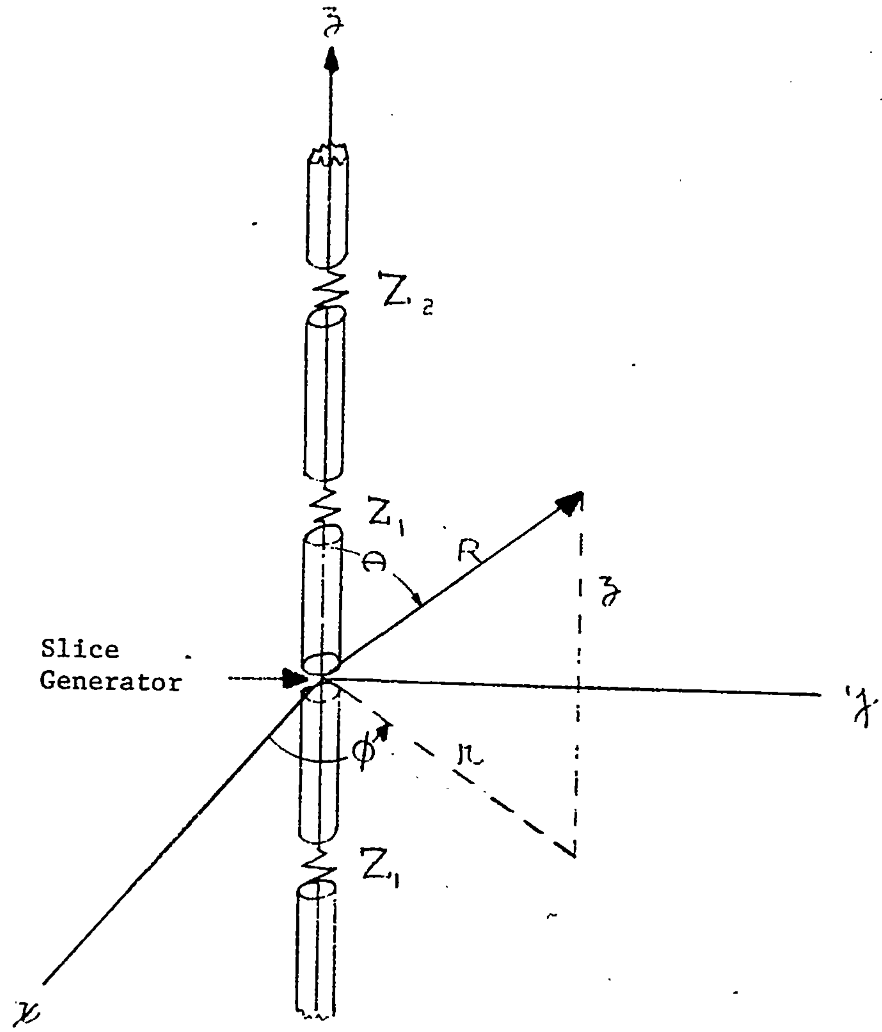

Figure 1: Impedance loaded dipole antenna with cartesian, cylindrical and spherical coordinates.

pulse. Hence the time history of the resulting current is obtained by taking the inverse Fourier transform of I (z,ω), i.e.

$$I\ (z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\ \hat{I}(z,\omega)\ e^{j\omega t} \tag{4}$$

Requiring I (z,t) to be real yields

$$\hat{I}\ (z,-\omega) = \hat{I}^*\ (z,\omega) \tag{5}$$

and using (5) in (4)

$$I\ (z,t) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} d\omega\ \mathrm{Re}\left\{I\ (z,\omega)\ e^{j\omega t}\right\} \tag{6}$$

Radiated Field Components

As derived by Harrison et al. [2], the radiated field components for a dipole antenna with current distribution I (z,ω) are

$$\hat{H}_\phi\ (r,z,\omega) = -\frac{1}{4\pi} \int_{-h}^{h} dz'\ \hat{I}\ (z',\omega)\ \frac{\partial}{\partial r}\ K\ (r,z-z') \tag{7}$$

$$\hat{E}_r\ (r,z,\omega) = -j\ \frac{\zeta}{4\pi k} \int_{-h}^{h} dz'\ \hat{I}\ (z',\omega)\ \frac{\partial^2}{\partial r \partial z}\ K\ (r,z-z') \tag{8}$$

$$\hat{E}_z\ (r,z,\omega) = -j\ \frac{\zeta}{4\pi k} \int_{-h}^{h} dz'\ \hat{I}\ (z',\omega) \left[\frac{\partial^2}{\partial z^2} + k^2\right] K\ (r,z-z') \tag{9}$$

where

$$K\ (r,z-z') = \exp\left[-jk\sqrt{(z-z')^2 + r^2}\right] / \sqrt{(z-z)^2 + r^2} \tag{10}$$

The foregoing expressions hold in both near zone and the far zone (or radiation zone) of the antenna.

In the far zone $kr >> 1$, it is easily shown that (7) reduces to

$$\hat{H}_\phi (r,z,\omega) \approx +j \frac{k \sin \theta}{4\pi} \frac{e^{-jkR}}{R} \int_{-h}^{h} dz' \hat{I}(z',\omega)e^{-jkz'\cos\theta} \tag{11}$$

where $\theta = \tan^{-1} (r/z)$

and $R = \sqrt{r^2 + z^2}$

are the usual spherical coordinates. The electric field component in the far zone is

$$\hat{E}_\theta (r,z,\omega) = \zeta \hat{H}_\phi (r,z,\omega) \tag{12}$$

It is noted that an evaluation of (7) - (9) to obtain the radiated field components requires a more accurate representation for the current distribution than is needed in (11). However it is well known that it is difficult to obtain the fields near a radiator, whereas a crude approximation of the current distribution on the radiator is sufficient to obtain reasonably accurate radiation zone field components.

Pulse Radiation

The radiated field components (7) - (9) are expressed in terms of the steady-state current distribution $\hat{I}(z,\omega)$. However the field components of a radiated pulse are obtained by an appropriate superposition of the steady-state components, i.e.

$$H_\phi (r,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \hat{H}_\phi(r,z,\omega) e^{j\omega t} \tag{13}$$

$$E_r (r,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \hat{E}_r(r,z,\omega) e^{j\omega t} \tag{14}$$

$$E_z (r,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \hat{E}_z(r,z,\omega) e^{j\omega t} \tag{15}$$

where $\hat{E}_z$, $\hat{E}_r$ and $\hat{H}_\phi$ are obtained from (7) - (9) using $\hat{I}$ (z,ω) which is obtained by solving the integral equation (1) where $\hat{V}_o(\omega)$ is the Fourier transform of the voltage pulse exciting the antenna.

Numerical Results

For convenience the antenna is considered to be charged slowly (time constant T_2). This voltage pulse expressed analytically is

$$V_o(t) = V_o e^{T_1 t} \qquad t < o$$
$$= V_o e^{-T_2 t} \qquad t > o \tag{16}$$

Then

$$\hat{V}_o(\omega) = \frac{V_o}{\sqrt{2\pi}} \quad \frac{T_1 + j\omega}{\omega^2 + T_1^{\,2}} + \frac{T_2 - j\omega}{\omega^2 + T_2^{\,2}} \tag{17}$$

and for $T_1 << 1$

$$\hat{V}_o(\omega) = \frac{V_o}{\sqrt{2\pi}} \quad \frac{T_2}{\omega^2 + T_2^{\,2}} + j\frac{T_2^{\,2}}{\omega(\omega^2 + T_2^{\,2})} \tag{18}$$

The impedance loadings of the "long-wire" antenna at Sandia Laboratory are used in the numerical computations. These are shown in Table 1.

The solution of (1) is obtained using the representation for $\hat{I}$ (z,ω) as shown in (3) where M = 15 and C is chosen as suggested in reference [1]. The result for $\hat{I}$ (z,ω) then is substituted in (7) - (9) to compute the steady-state values for the field components which are used in (13) - (15) to compute the radiated pulse. Because of smoothness of the functions that are to be integrated, it is most convenient to use the Simpson's quadrature formula.

Table 1: Impedance Loadings

Position, z_ℓ	Impedance Loading, Z_ℓ (in ohms)
0.04 h	6.0
0.08 h	9.0
0.12 h	10.5
0.16 h	12.0
0.20 h	15.0
0.24 h	21.0
0.28 h	29.0
0.32 h	32.0
0.36 h	43.0
0.40 h	44.0
0.44 h	44.0
0.48 h	49.0
0.52 h	54.0
0.56 h	71.0
0.60 h	71.0
0.64 h	71.0
0.68 h	92.0
0.72 h	100.0
0.76 h	105.0
0.80 h	120.0
0.84 h	125.0
0.88 h	150.0
0.92 h	205.0
0.96 h	250.0

But the inverse Fourier transforms (13) - (15) require a special technique. In figures (2) and (3) plots of $E_z\, e^{jkR}/kV_o(\omega)$ and $2\pi r\, H_\phi\, e^{jkR}/V_o(\omega)$, respectively, versus kh show generally slowly varying functions of frequency. Because of this, in the evaluation of the inverse Fourier transforms the ranges of integration are divided into small segments where $E_z\, e^{jkR}/kV_o(\omega)$ and $2\pi r\, H_\phi\, e^{jkR}/V_o(\omega)$ are approximated by straight lines and the integrals over the same segments than are evaluated analytically. This technioue is found to be much superior to a brute force method such as Simpson's rule. Because of the ease in obtaining the far zone field components the far electric field is investigated and is compared to the near zone predictions in the hope that the far zone approximation may be used to obtain at least qualitative predictions of the near zone fields. In figure (4) a plot of $r\, E_\theta\, e^{jkR}/V_o(\omega)$ (far zone approximation) versus kh show essentially the same result as figures (1) and (2). Hence the foregoing procedure for obtaining the inverse Fourier transform may be used.

The foregoing field components are used to obtain the time histories in figures (5) - (7). Two interesting observations are that the "rise-times" for all three components are about the same which indicated the qualitative validity of the far field approximation and the wave impedance (ratio of electric field to magnetic field) is near 300 Ω for the pulse duration. In obtaining these data, the parameters $V_o = 1$ and $h = 500$ ft. were used. Note that the time in the foregoing is actually the retarded time.

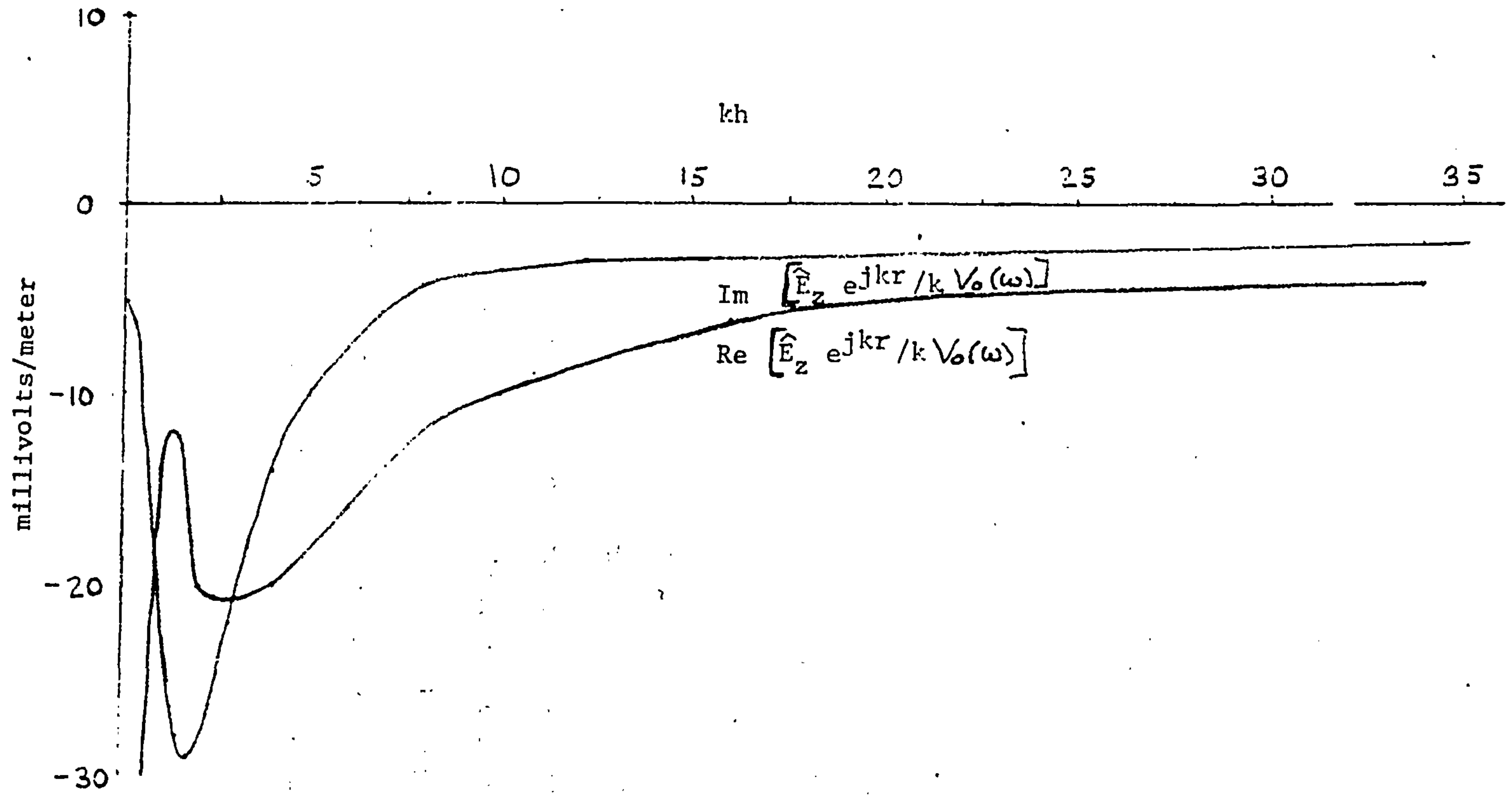

Figure 2: Real and imaginary components of the electric field versus electrical half-length of the impedance loaded dipole. Here $r = h$, $z = o$.

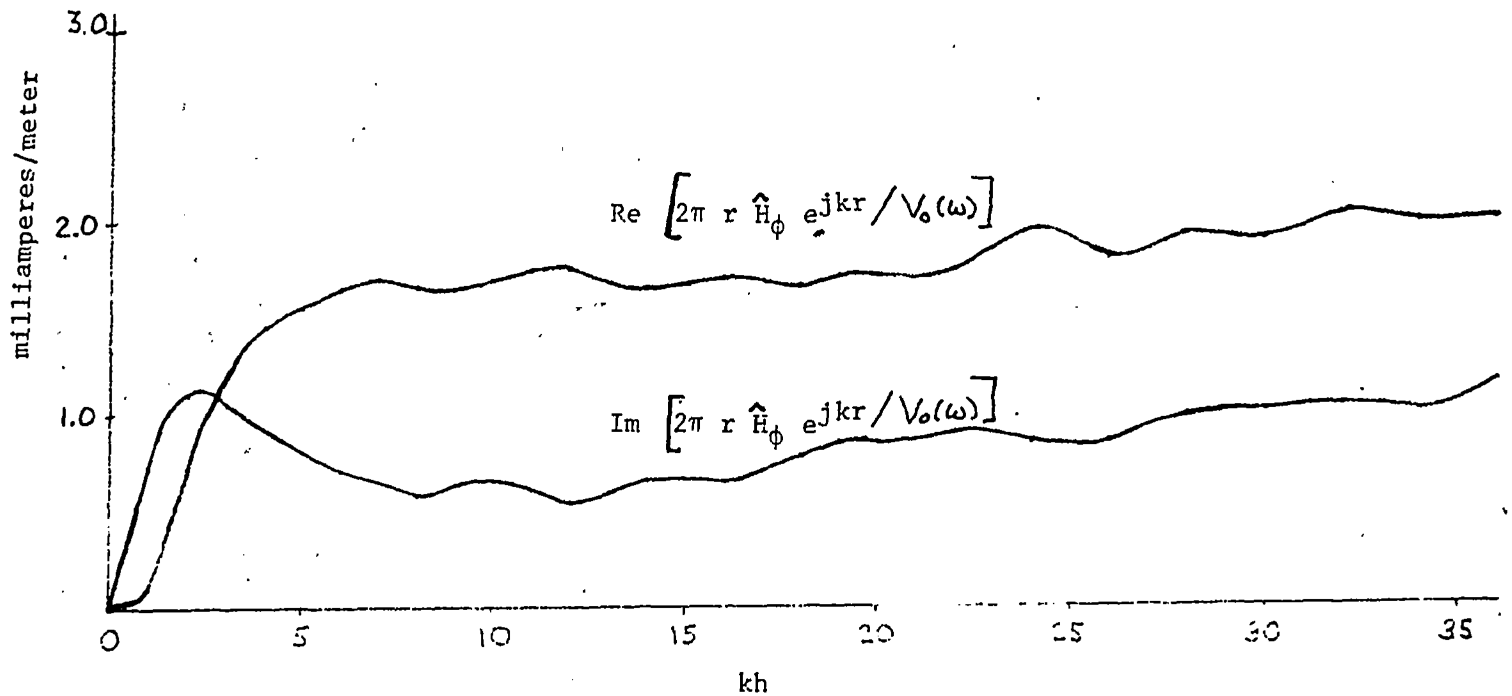

Figure 3: Real and imaginary components of the magnetic field versus electric half-length of the impedance loaded dipole. Here r = h, z = o.

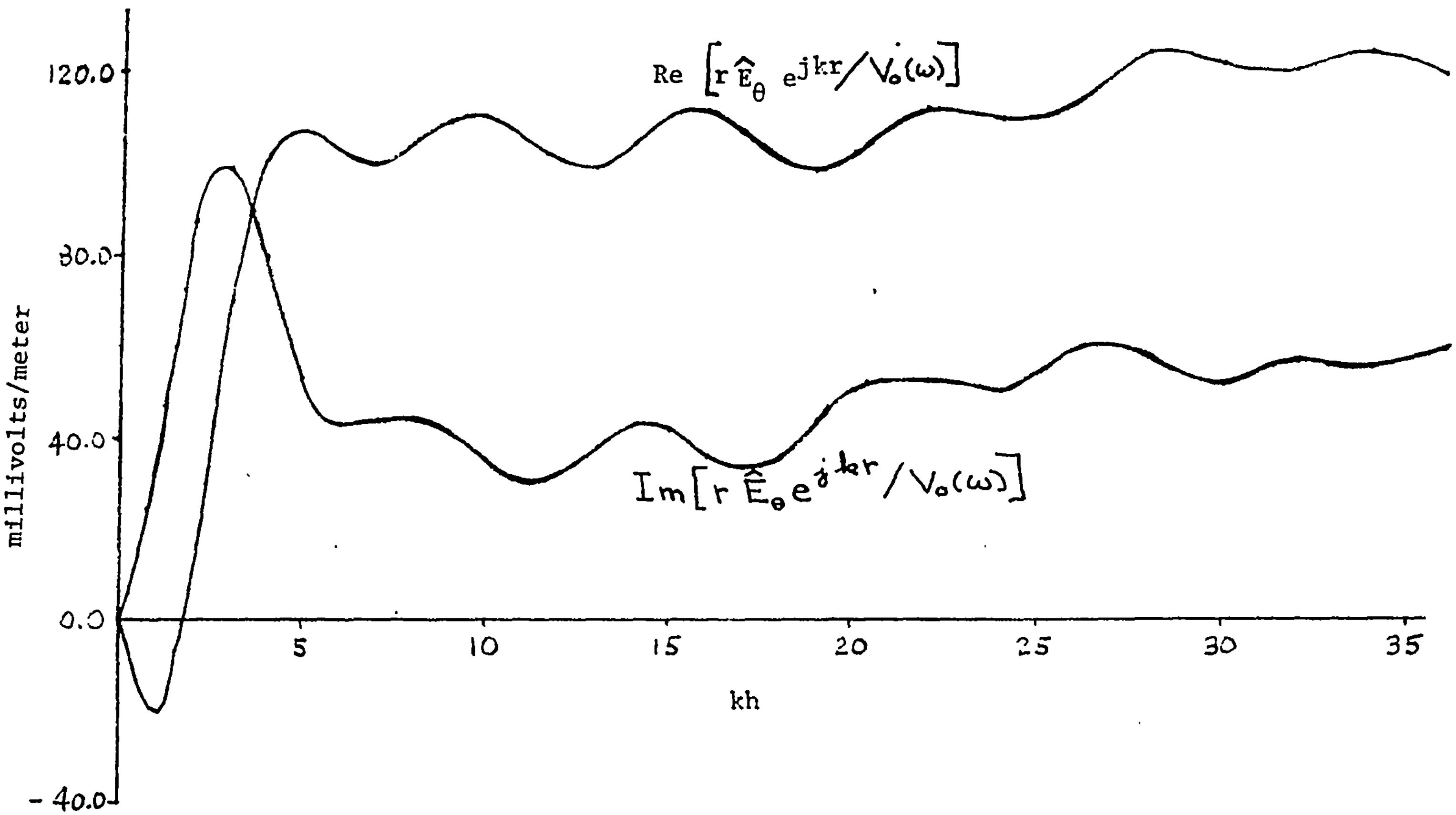

Figure 4: Real and imaginary components of the electric field (far field approximation) versus electrical half-length of the impedance loaded dipole. Here r = h, z = o.

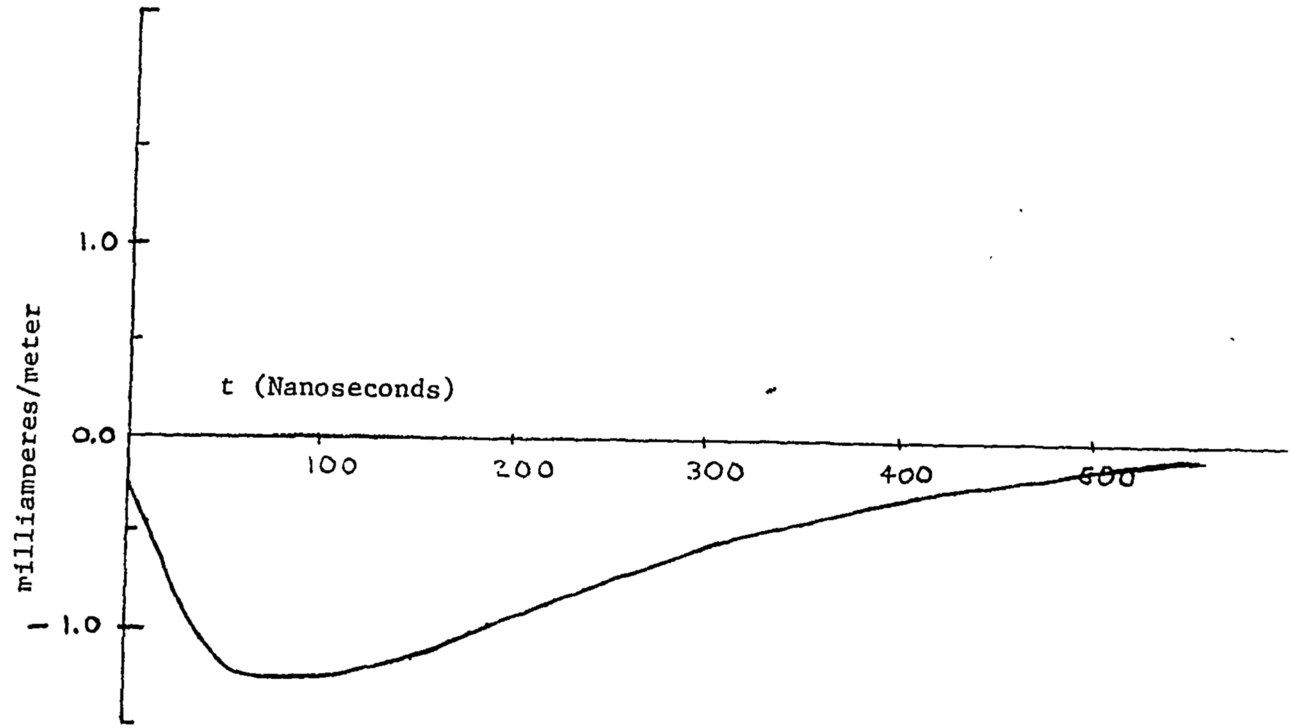

Figure 5: Time history of the magnetic field $H_\phi(r,z,t)$ for $T_2 = 2 \times 10^7$ sec^{-1}, $r = h$, $z = o$.

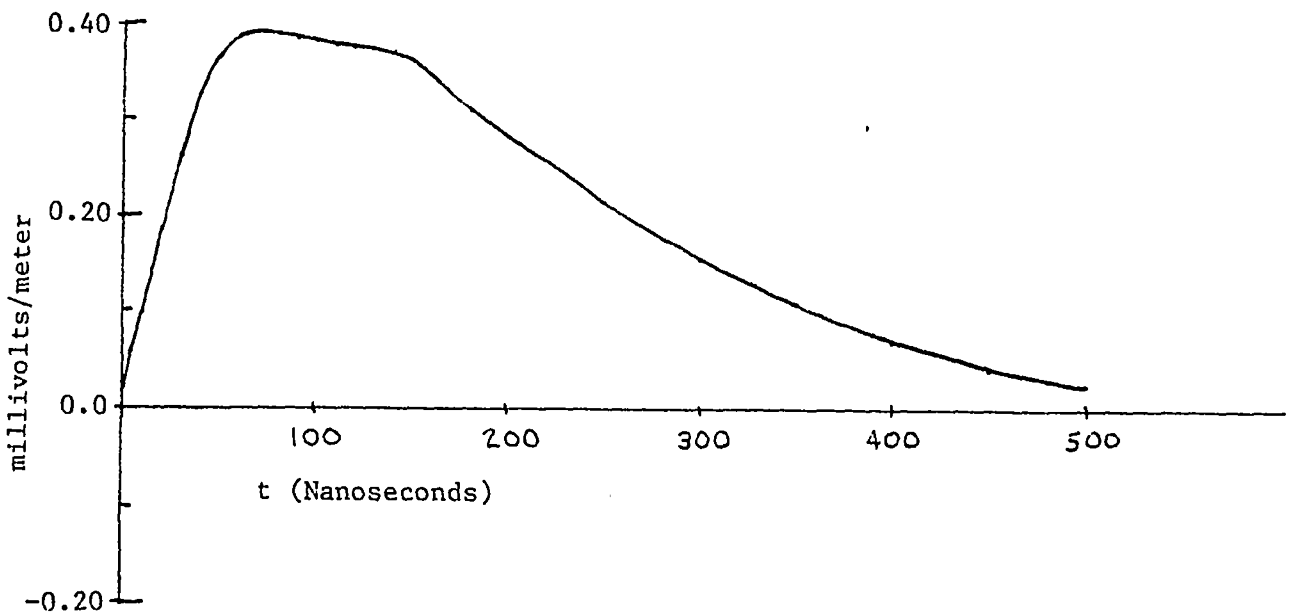

Figure 6: Time history of the electric field $E_z(r,z,t)$ for $T_2 = 2 \times 10^7$ sec^{-1}, r = h, z = o.

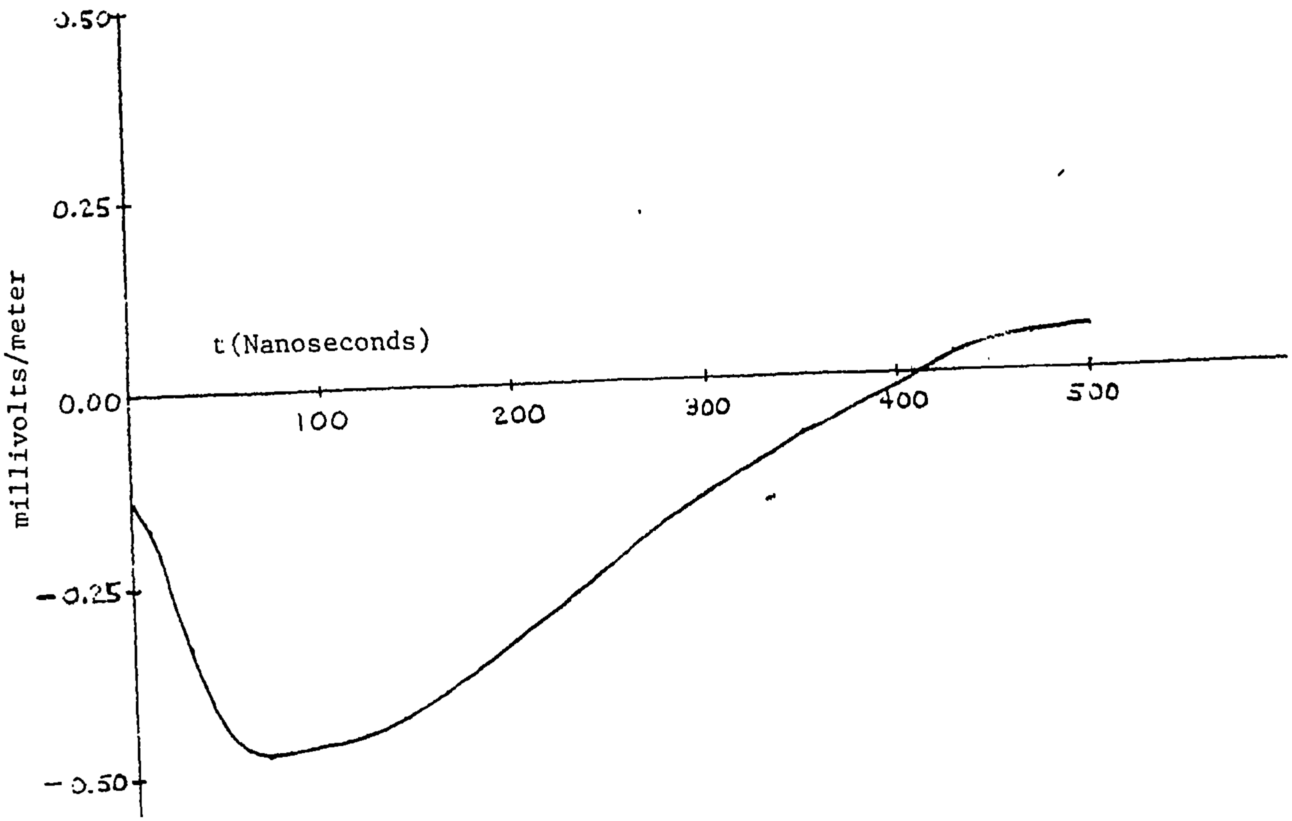

Figure 7: Time history of the far field approximation to the electric field $E_\theta(r,z,t)$ for $T_2 = 2 \times 10^7\ \text{sec}^{-1}$, $r = h$, $z = o$.

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2. C. W. Harrison, Jr., C. D. Taylor, E. A. Aronson and M. L. Houston. "An Accurate Representation of the Complete Electromagnetic Field in the Vicinity of a Base-Driven Cylindrical Monopole," Sandia Corporation Report SC-R-68-1695, January 1968.