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A Modified Description of Early Time High-altitude Electromagnetic Pulse Waveform (E1)

Yan-zhao XIE

School of Electrical Engineering, Xi'an Jiaotong University
710049, Xi'an, Shaanxi Province, China
Yanzhao.xie@gmail.com

Abstract— The mathematical description of High-altitude Electromagnetic Pulse (HEMP) waveform (E1) in the standard of IEC 61000-2-9 is the difference of double exponentials (DEXP). However, in this description there is a discontinuity of the first derivative at the initial time of waveform. Another analytic waveform description, the quotient of double exponentials (QEXP) has the advantage of all time derivatives continuous, but an additional time shift parameter must be used since the amplitude is nonzero for negative time. To address these problems, this paper offers a modified description, namely *p*-power of double exponentials (PEXP). The parameter values in PEXP description corresponding to IEC 61000-2-9 have been fitted by means of varied power of double exponentials. The analysis results demonstrate that the PEXP description has two advantages of the continuous derivative at initial time of waveform and no need of the time shift factor.

Key Words— HEMP Waveform Description, Discontinuity, Difference of Double Exponentials (DEXP), Quotient of Double Exponentials (QEXP), *p*-power of Double Exponentials (PEXP)

1 INTRODUCTION

IEC 61000-2-9 standard was firstly appeared in [1] and issued in 1996, in which the HEMP waveform was described by the mathematical description of the difference of double exponentials[1]. However, this description has the discontinuity of the first derivative at the initial time of waveform. Another representative description is the quotient of double exponentials[1-4], which can solve the discontinuity problem, but it needs the time shift factor since its amplitude is not zero for finite negative time.

In this paper, a modified description of *p*-power of double exponentials, labelled as PEXP, is proposed to describe HEMP waveform. The influence of power index p to the initial time of waveform was analysed. In order to measure the closeness between different descriptions, the respective frequency spectrums based on these three descriptions were calculated and compared. In the end, the parameter values of modified HEMP waveform description corresponding to IEC 61000-2-9 are presented under two specific values of p=10 and 20.

2 THE EXISTING HEMP WAVEFORM DESCRIPTIONS AND THEIR DRAWBACKS

2.1 Difference of double exponentials (DEXP)

The commonly used term HEMP generally means the early-time portion ($E1$) of high-altitude electromagnetic pulse. The most important temporal parameters of HEMP shape are rise time t_r (10%-90%), pulse duration t_{FWHM} (50%-50%) and peak amplitude of electric-field E_0. Very popular mathematical form to describe early-time HEMP pulse shape is the difference of double exponentials with the unit step function $u(t-t_0)$[1-3]

$$E(t) = kE_0\left(e^{-(t-t_0)/\tau_f} - e^{-(t-t_0)/\tau_r}\right)u(t-t_0) \tag{1}$$

Or its equivalent form

$$E(t) = kE_0\left(e^{-\beta(t-t_0)} - e^{-\alpha(t-t_0)}\right)u(t-t_0) \tag{2}$$

where E_0 is the peak value of the waveform, generally equals to 50kV/m, k is unitary factor, t_0 is the initial time of waveform. The parameters rise time t_r, the pulse duration t_{FWHM} and E can be determined via the mathematical pulse parameters α, β and E_0 in (2). In some engineering cases, t_r and t_{FWHM} could be roughly approximated by $t_r \approx \tau_r$ and $t_{FWHM} \approx \tau_f$, only when α is much larger than β. On this account the mathematical description of (2) is much more popularly used than (1)[5].

The form of (2) is simple, time-integrable and has been widely used in IEC 61000-2-9 and Bell laboratory waveform descriptions. It turns on at $t=t_0+$ with a finite slope, reaches a peak value in a few nanoseconds, and decays slowly to zero. However, this description has a discontinuous slope, the electric-field derivative at $t=t_0$ is

$$\left.\frac{\mathrm{d}E(t)}{\mathrm{d}t}\right|_{t=t_0} = \left.kE_0\left(-\beta e^{-\beta(t-t_0)} + \alpha e^{-\alpha(t-t_0)}\right)\right|_{t=t_0} = kE_0(-\beta+\alpha) \tag{3}$$

In comparison, when $t<t_0$ the electric-field derivative is zero. The discontinuity occurs at $t=t_0$.

One knows that the mechanisms of HEMP generation do not allow the electric-field change abruptly. So DEXP of (2) is not consistent with natural physical mechanism of HEMP, leading to computational difficulties for some cases.

Moreover, since the term of radiation field of antenna is the derivative of current, the mathematical description of (2) is also not suitable for the calculation of the antenna radiation if (2) was chose as the description of current exciting source.

For the HEMP waveform defined by IEC 61000-2-9 standard [6], the pulse parameters in (2) are k=1.3, $\alpha=6\times10^{8}$, $\beta=4\times10^{7}$, t_0=0. Fig.1 shows the IEC 61000-2-9 waveform (blue dash line) and Fig.2 shows its respective derivative (blue dash line).

2.2 Quotient of double exponentials (QEXP)

In order to overcome the discontinuity problem, another canonical waveform coming into common usage is the quotient or reciprocal of the sum of two exponentials[1-3], which is given by

$$E(t) = kE_0 \frac{1}{e^{-\alpha(t-t_0)} + e^{\beta(t-t_0)}} \tag{4}$$

where t_0 is a time shift factor, k is unitary factor, E_0 is peak value of the waveform. It was found that the derivative of function (4) is continuous at t=0, while its amplitude is not zero for finite negative time as shown in Fig.1 and Fig.2 (solid line). Thus the time shift factor t_0 must be used to shift the waveform in the positive t direction so that the amplitude is near zero at t=0. The description of (4) with parameter values k=1.114， E_0=50kV/m， $\alpha=1.6\times10^{9}$， $\beta=3.7\times10^{7}$, t_0=0[1], have almost the same temporal parameters as in the IEC 61000-2-9 waveform. The waveform and its derivative are shown in Fig.1 and Fig.2 (solid line), respectively. The advantage of this expression is that it has continuous time derivative of all orders for all times. The disadvantage of this expression is it extends to t=-infinity. It could be further concluded that the value of time shift factor t_0 is strongly dependent with HEMP waveform standard specified. The corresponding time shift factor in Eq.(4) would be varied accordingly depending on different HEMP waveform standards, such as IEC 61000-2-9, Bell laboratory and other waveform standards, which is inconvenient for usage.

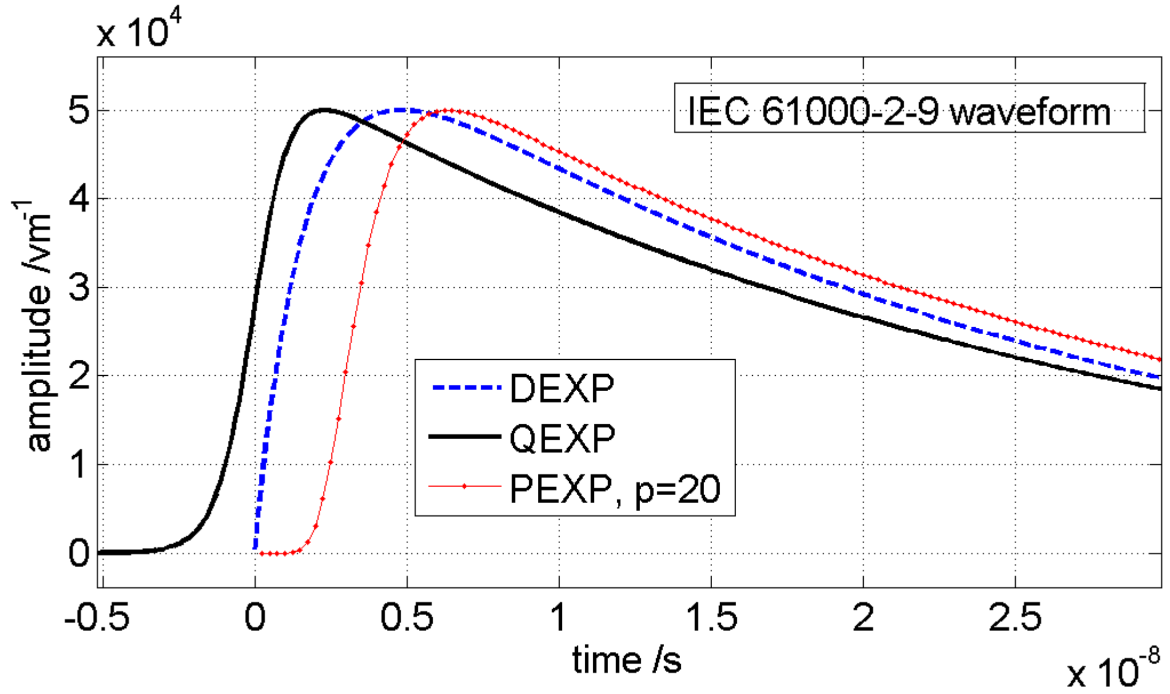

Fig.1 Temporal waveforms of three HEMP descriptions

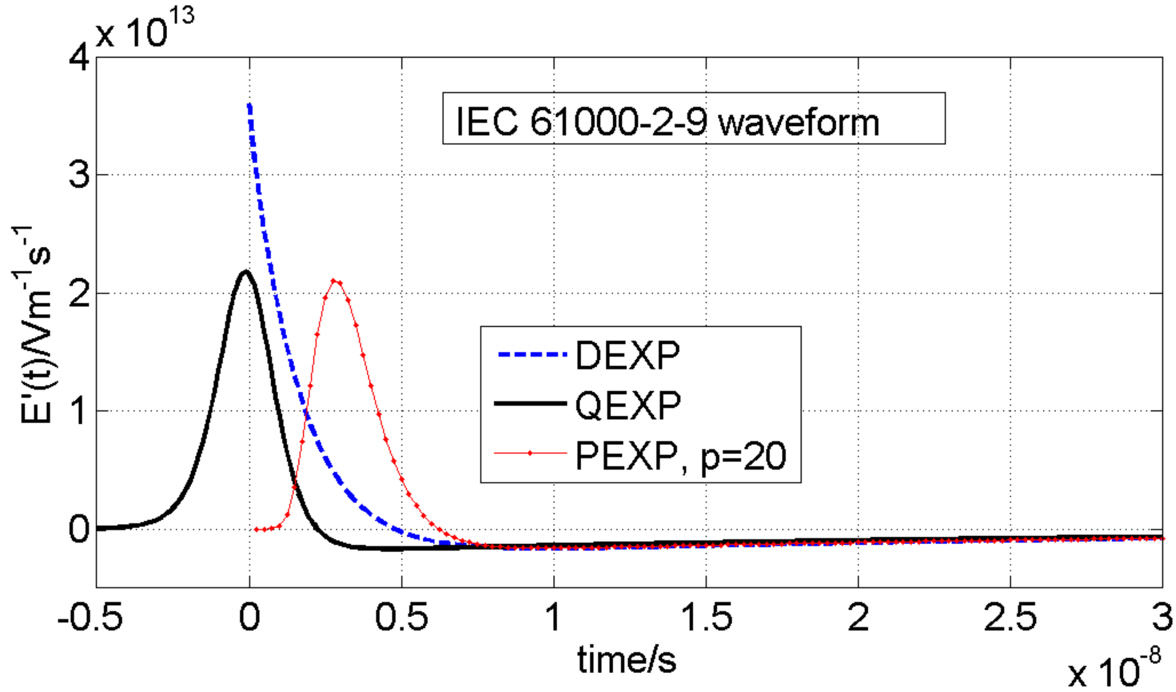

Fig.2 Derivatives of three HEMP descriptions

3 A MODIFIED HEMP WAVEFORM DESCRIPTION

3.1 Proposition of *p*-power of double exponentials

Set t_0=0 and $u(t-t_0)$=1, the description of Eq.(2) could be rewritten as

$$E(t) = kE_0\left(e^{-\beta' t} - e^{-\alpha' t}\right) = kE_0\left(1 - e^{-(\alpha' - \beta')t}\right)e^{-\beta' t} \tag{5}$$

If the term in the bracket of Eq.(5) is taken to the power index of p (p is integer), it results in

$$E(t) = kE_0\left(1 - e^{-(\alpha' - \beta')t}\right)^p e^{-\beta' t} \tag{6}$$

which can be further simplified if ($\alpha' + \beta'$) is denoted by α and β' is denoted by β.

$$E(t) = kE_0\left(1 - e^{-\alpha t}\right)^p e^{-\beta t} \tag{7}$$

Herein, the description of Eq.(7) is named as p-power of double exponentials (abbreviated as PEXP, with respect to DEXP and QEXP) which can be conceptually compared to other descriptions clearly. Eq.(7) was ever adopted as the description of lightning discharge current waveform[7] and the base functions to represent the ESD waveform[8]. The author introduced it to describe different HEMP waveform standards in 2007.

Eq.(7) is time-integrable and the expression of its time integral is given by:

$$\int E(t)\mathrm{d}t = kE_0 \sum_{n=0}^{p} \frac{(-1)^{n+1} p!}{n!(p-n)!} \frac{1}{(\alpha n + \beta)} e^{-(\alpha n + \beta)t} \tag{8}$$

Eq.(7) is also time-differentiable and the expression of its first time derivative is given by:

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = kE_0\left(1 - e^{-\alpha t}\right)^{p-1} e^{-\beta t}\left[(p\alpha + \beta)e^{-\alpha t} - \beta\right] \tag{9}$$

It is clear that, when $p \geq 2$, Eq.(9) equals to zero at t=0. This indicates that the continuity condition of fist derivative could be satisfied.

3.2 The influence of parameter p to the initial time of waveform

Note that $\alpha t > 0$， e=2.718⋯， we have

$$0 < e^{-\alpha t} < 1 \tag{10}$$

$$0 < \left(1 - e^{-\alpha t}\right) < 1 \tag{11}$$

It is clear that with the increase of p, the term $(1-e^{-\alpha t})^p$ in Eq.(7) tends to be zero and the whole wave shape would be postponed accordingly. This feature could be further illustrated in the plots of DEXP and PEXP with varied values of p as shown in Fig.3. Taking the advantage of this property, we can easily adjust the initial time of pulse as needed.

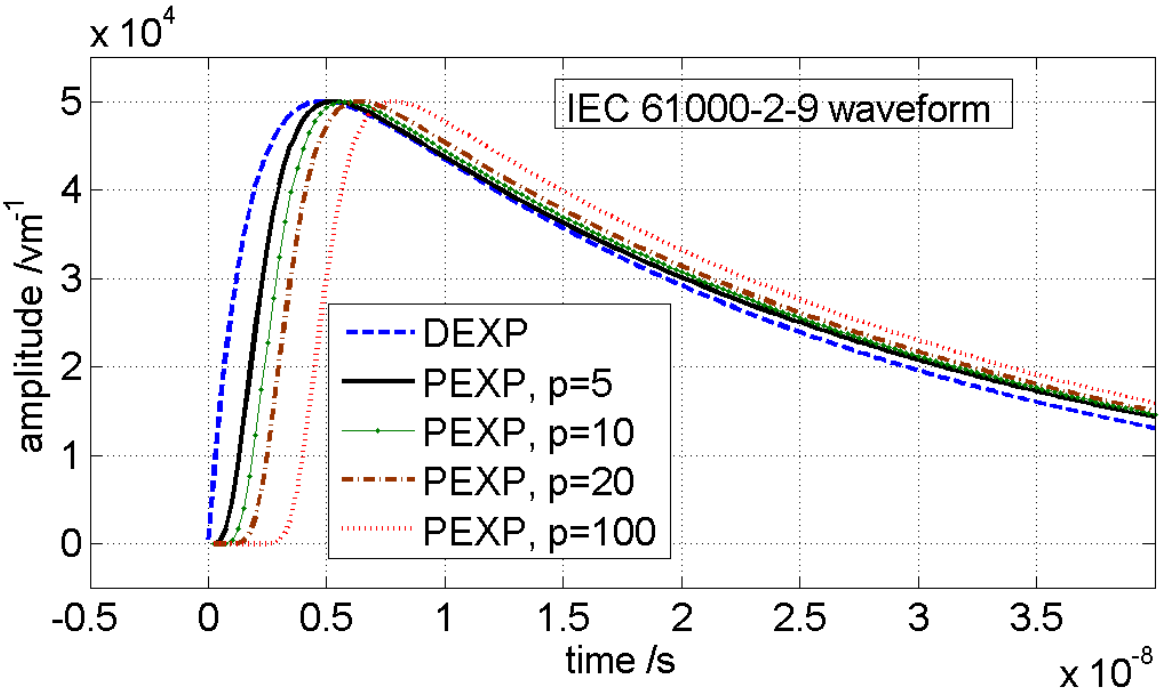

Fig.3 The influence of the value of p to pulse shape

3.3 Parameter values determination using iterative fitting method

The commonly used parameters rise time t_r and pulse length t_{fwhm} are of great interests for engineering use, which could be selected as mainly objectives for parameter optimization.

Once the value of p was specified in advance, the goal of parameter optimization is to find the adjustable parameter values of α and β in Eq.(7) that most closely match t_r and t_{fwhm} which are deduced from IEC 61000-2-9 waveform.

To perform optimization process, firstly one needs to define the tolerances of rise time and pulse length between IEC 61000-2-9 and the proposed description in Eq.(7). It is possible to set both tolerances of rise time and pulse length lower than 0.01ns.

In Eq.(7), one could empirically suppose that the rise time and pulse length are mainly influenced by the parameter α and β, respectively. Thus, the parameters determination could be implemented through a simple iterative fitting approach as follows.

1 Set the initial values of p, α, β, $\Delta\alpha$, $\Delta\beta$, *error tolerance*, t_r, t_{fwhm}, t_{r_new}, t_{fwhm_new}, etc.
2 Do the following steps:

```
while | tr - tr_new |>error tolerance or | tfwhm - tfwhm_new |>error tolerance
    Dertermine the value of k in Eq.(7) by normalization.
    Calculate of tr_new and tfwhm_new.
        if tr < tr_new
            Δα=Δα/2
            α=α-Δα
        else
            α=α+Δα
        end
        if tfwhm < tfwhm_new
            β=β-Δβ
        else
            Δβ=Δβ/2
            β=β+Δβ
        end
end
```

Based on Eq.(7) with p=20, the plots of temporal waveform and its first derivative of IEC 61000-2-9 waveform are shown in Fig.1 and Fig.2 (dotted line), respectively.

TABLE 1
PARAMETER VALUES IN DEXP AND PEXP CORRESPONDING TO IIEC 61000-2-9 STANDARD

	DEXP	PEXP, p=10	PEXP, p=20
α/s^{-1}	6.0×10^8	9.63×10^8	9.81×10^8
β/s^{-1}	4.0×10^7	3.69×10^7	3.66×10^7
k	1.30	1.286	1.313
t_r/ns	2.47	2.46	2.48
t_{FWHM}/ns	22.98	23.00	23.12

The temporal parameters of t_r, t_{FWHM} and pulse function parameters of k, α, β, which based on PEXP description with p=10 and 20, are fitted and presented in TABLE 1 and compared with the DEXP description. It is found that the temporal parameter, such as the peak value E_0, rise time t_r and pulse width t_{FWHM}, almost have the same values.

In addition to t_r and t_{FWHM}, the area under the temporal curve, corresponding to the total energy flux included in the waveform, would be an alternative objective for optimization as well. However, the tolerance requirement of these three objectives could not be satisfied simultaneously, except for t_r, normally one can only choose either the t_{FWHM} or the total energy flux, because of the intrinsic characteristics of the PEXP expression. In other words, the total energy flux would have relatively big fitted error compared to t_r and t_{FWHM}.

The measure of the total energy flux j is defined by

$$j = \frac{\int_0^\infty E^2(t)\mathrm{d}t}{\eta} = \eta\int_0^\infty H^2(t)\mathrm{d}t$$

where η is the wave impedance of free space. For the parameters listed in TABLE 1, the corresponding values of j are 0.1144, 0.1064 and 0.1139 J/m^2, respectively.

In a similar way, some other HEMP waveform standards, such as Bell laboratory and 1976 publication[9,10], could be represented and fitted by PEXP description. In [11], the author presented the fitted values of these two other waveform standards based on PEXP description.

3.4 The spectrum comparison of the three HEMP descriptions

In terms of the Fourier transformation equation

$$\tilde{E}(\mathrm{j}\omega) = \int_{-\infty}^{\infty} E(t)e^{-\mathrm{j}\omega t}\mathrm{d}t \tag{12}$$

The above-mentioned three descriptions can be transformed to frequency domain since all of them are integrable. The respective frequency spectrum and spectral magnitude are given in TABLE 2.

TABLE 2
THE EXPRESSIONS OF FREQUENCY SPECTRUM AND SPECTRAL MAGNITUDE

Descriptions	Items	Expressions
DEXP	Frequency spectrum	$\tilde{E}(\mathrm{j}\omega) = \frac{E_0 k(\alpha-\beta)}{(\mathrm{j}\omega+\beta)(\mathrm{j}\omega+\alpha)}e^{-\mathrm{j}\omega t_0}$
	Spectral magnitude	$\lvert\tilde{E}(\mathrm{j}\omega)\rvert = E_0 k\frac{(\alpha-\beta)\sqrt{\alpha^2\beta^2+\alpha^2\omega^2+\beta^2\omega^2+\omega^4}}{(\alpha\beta-\omega^2)^2+(\alpha\omega+\beta\omega)^2}$
QEXP	Frequency spectrum	$\tilde{E}(\mathrm{j}\omega) = E_0 k\frac{\pi}{\alpha+\beta}\csc\left(\pi\frac{\mathrm{j}\omega+\beta}{\alpha+\beta}\right)e^{-\mathrm{j}\omega t_0}$
	Spectral magnitude	$\lvert\tilde{E}(\mathrm{j}\omega)\rvert = E_0 k\frac{\pi}{\alpha+\beta}\left[\sinh^2\left(\frac{\pi\omega}{\alpha+\beta}\right)+\sin^2\left(\frac{\pi\beta}{\alpha+\beta}\right)\right]^{-1/2}$
PEXP	Frequency spectrum	$\tilde{E}(\mathrm{j}\omega) = E_0 k\sum_{n=0}^{p}\left(\frac{(-1)^n p!}{n!(p-n)!}\frac{1}{n\alpha+\beta+\mathrm{j}\omega}\right)$
	Spectral magnitude	$\lvert\tilde{E}(\mathrm{j}\omega)\rvert = E_0 k\sqrt{\left[\sum_{n=0}^{p}\left(\frac{(-1)^n p!}{n!(p-n)!}\frac{n\alpha+\beta}{(n\alpha+\beta)^2+\omega^2}\right)\right]^2+\left[\sum_{n=0}^{p}\left(\frac{(-1)^n p!}{n!(p-n)!}\frac{\omega}{(n\alpha+\beta)^2+\omega^2}\right)\right]^2}$

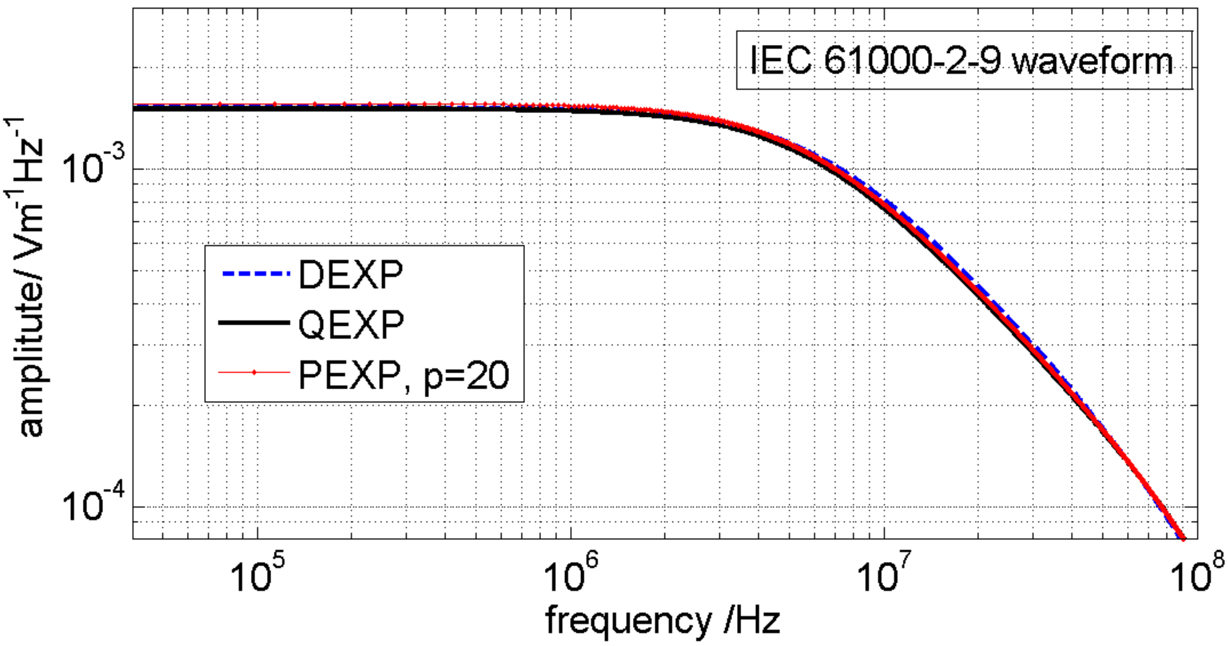

Fig.4 Spectral magnitude of the waveforms in Fig.1

Fig.4 shows the respective spectral magnitude of the three HEMP descriptions. Over the whole frequency range there is no significant differences between three descriptions. If we enlarge this figure, it can be found that the DEXP description introduces artificial high frequency content due to the discontinuous first derivative at t=0.

5 CONCLUSIONS

Considering the drawbacks existed in the previous HEMP waveform descriptions for IEC 61000-2-9 standard, this paper presents a modified description to solve the problems of the first time derivative discontinuity at the initial time of waveform in DEXP and the need of time shift factor in QEXP.

The presented PEXP description conforms to the physical requirement of electric-field and its first derivative at the initial time of waveform. On the other hand, the PEXP description of HEMP waveform has the advantage of time-integrable, time-differentiable and without need of time shift factor for all HEMP waveform standards.

The frequency spectrum analysis of IEC 61000-2-9 shows that there is no significant difference between the three HEMP waveform descriptions. The parameter values based on PEXP description are calculated and presented in this paper. The PEXP mathematical description is convenient for usage especially for numerical computation, and could be a quite good alternative for HEMP waveform. In addition to HEMP waveform, PEXP also could be a prospective description of HPEM waveform, such as UWB waveform, etc.

6 APPRECIATION

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