Sensor and Simulation Notes

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Lens Design for a Prolate-Spheroidal Impulse radiating Antenna (IRA)

Serhat Altunc, Carl E. Baum, Christos G. Christodoulou and Edl Schamiloglu
University of New Mexico
Department of Electrical and Computer Engineering
Albuquerque New Mexico 87131

Abstract

In this paper, we discuss the design procedure for different types of dielectric lenses for better concentrating the fields at the second focus of a prolate-spheroidal IRA to increase the fields and decrease the spot size.

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1 Introduction

In this paper, we discuss the design procedure for different types of dielectric lenses for better concentrating the fields at the second focus of a prolate-spheroidal IRA to increase the fields and decrease the spot size. We have a very fast and intense electromagnetic pulse to illuminate the target [1] which is located at the second focal point. One of the most important problems with concentrating the fields on the target is reflection. We have to deal with this reflection because the dielectric property of the target medium and the medium through which the incident wave propagates are different. The reflection of the pulse leads to a smaller field at the second focus where our target is buried. We discuss the addition of a lens to better match the wave to the target. We can obtain larger fields and smaller spot size [2].

To obtain better concentration at the target we can use different types of lenses. The transmission coefficient from one medium to the another one is

$$T = 2\left[1+\varepsilon_{rt}^{1/2}\right]^{-1}, \tag{1}$$

where ε_{rt} is the relative permittivity of the target medium.

Suppose now that we have a lens in front of the target with relative permittivity

$$\varepsilon_{\ell} = \varepsilon_{rt}. \tag{2}$$

The fields from the reflector are transmitted with a transmission coefficient given by

$$T_0 = 2\left[1+\varepsilon_{rt}^{1/2}\right]^{-1}. \tag{3}$$

We will have a slower wave speed and an enhancement factor which is an increase in the impulse portion of the focal waveform from [2] as

$$\begin{aligned} v &= \left[\varepsilon_{r\ell}\varepsilon_0\mu_0\right]^{-1/2} = c\,\varepsilon_{r\ell}^{-1/2}, \\ F_0 &= \varepsilon_{r\ell}^{1/2} = \varepsilon_{rt}^{1/2}. \end{aligned} \tag{4}$$

Thus, for the impulse part of the field we will have a net increase of

$$F_0\, T_0 = 2\left[1 + \varepsilon_{rt}^{-1/2}\right]^{-1} \tag{5}$$

Suppose now that we have a lens in front of the target with relative permittivity

$$1 < \varepsilon_{r\ell} < \varepsilon_{rt}\ . \tag{6}$$

We will have then two transmission coefficients and the total transmission coefficient is

$$T = T_1 T_2 = 2\left[1 + \varepsilon_{rt}^{1/2}\right]^{-1} 2\left[1 + \left(\frac{\varepsilon_{rt}}{\varepsilon_{r\ell}}\right)^{1/2}\right]^{-1} = \frac{4}{\left(1 + \varepsilon_{rt}^{1/2}\right)\left(1 + \left(\frac{\varepsilon_{rt}}{\varepsilon_{r\ell}}\right)^{1/2}\right)}\ . \tag{7}$$

Finally, suppose we have a lens with a graded relative permittivity given by

$$\begin{aligned} & r_1 \geq r_\ell \geq r_2\ , \\ & \varepsilon_{r\ell}(r_1) = 1\ , \\ & \varepsilon_{r\ell}(r_2) = \varepsilon_{rt}\ . \end{aligned} \tag{8}$$

The wave propagating through this takes a similar form as that of a wave in a transmission-line transformer. The high frequency early-time transfer function can be computed for a continuous variation of ε as [2]

$$T = \left(\frac{Z_0}{Z_t}\right)^{1/2} = \varepsilon_{rt}^{-1/4}\ . \tag{9}$$

We still have the enhancement factor the transmission enhancement

$$F_0 T = \varepsilon_{rt}^{+1/2}\, \varepsilon_{rt}^{-1/4} = \varepsilon_{rt}^{+1/4}\ . \tag{10}$$

The transmission enhancement of the lens, as discussed in [3] for an exponential variation of the characteristic impedance of the transmission line (for constant wave speed) along the line, is somewhat optimal. In this paper we present different types of graded lenses for stronger focusing at the target.

The focal point is $z_0 = 37.5$ cm and the other parameters of the prolate-spheroidal IRA are defined in [4].

2 Calculating the Optimum Number of Layers for a Lens

In this section we calculate the optimum number of layers to obtain the required field at the focal point of a prolate-spheroidal IRA based on a plane-wave approximation. N layers of increasing dielectric constant lenses which have the same ratio of dielectric constant are considered for a prolate-spheroidal IRA that is based on [2]. The geometrical illustration of this design is presented in Figure 1.

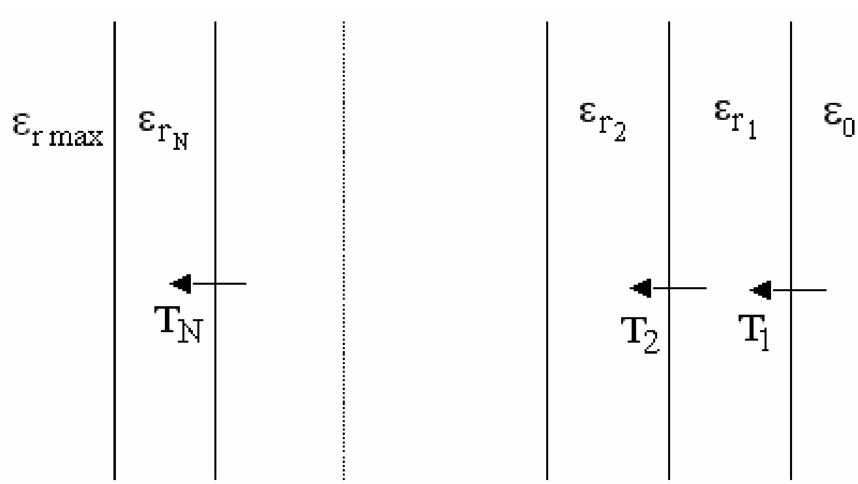


Figure 1: N layers of lens, dielectric constants and transmission coefficients.

The total transmission coefficient can be defined as

$$T_{total} = \prod_{1}^{N} T_n \ , \tag{2.1}$$

where T_n is the transmission coefficient between nth and nth+1st layer and it can be defined as

$$T_n = \frac{2Z_{n+1}}{Z_n + Z_{n+1}} = \frac{2\,\varepsilon_{r\,n+1}^{-1/2}}{\varepsilon_{r\,n}^{-1/2} + \varepsilon_{r\,n+1}^{-1/2}} \, . \tag{2.2}$$

The ratio of dielectric constant between subsequent layers are constrained to be the same,

$$\varepsilon_{\text{ratio}} = \varepsilon_{\text{r}_{\text{n+1}}} / \varepsilon_{\text{r}_{\text{n}}} \ . \tag{2.3}$$

For N layers

$$\left(\varepsilon_{r_{n+1}} / \varepsilon_{r_n}\right)^N = \varepsilon_{ratio}^N = \varepsilon_{r\,max} . \tag{2.4}$$

Substituting (4) in (2), we have

$$T_n = \frac{2\left(\varepsilon_{r\,max}^{1/N}\right)^{-1/2}}{1+\left(\varepsilon_{r\,max}^{1/N}\right)^{-1/2}} . \tag{2.5}$$

For N layers from (1)

$$T_{total} = \left(\frac{2\left(\varepsilon_{r\,max}^{1/N}\right)^{-1/2}}{1+\left(\varepsilon_{r\,max}^{1/N}\right)^{-1/2}}\right)^N . \tag{2.6}$$

If we have a continuously increasing dielectric lens we have a total transmission coefficient defined in (10) as

$$T_{total} = \varepsilon_{r\,max}^{-1/4} . \tag{2.7}$$

If we have an infinite number of layers, (2.6) approaches (2.7). We should decide how many layers will be acceptable to obtain a sufficiently close transmission coefficient to the continuously increasing dielectric lens case.

Table 1: Transmission coefficients for different N and $\varepsilon_{r\,max}$.

T_{total}	$\varepsilon_{r\,max}$					
N	**16**	**25**	**36**	**49**	**64**	**81**
2	0.4444	0.382	0.336	0.3009	0.273	0.25
3	0.4618	0.402	0.358	0.3237	0.296	0.274
4	0.471	0.413	0.37	0.3362	0.309	0.287
5	0.4766	0.419	0.377	0.344	0.318	0.296
10	0.4881	0.433	0.392	0.3605	0.335	0.314
20	0.494	0.44	0.4	0.3691	0.344	0.323
40	0.497	0.444	0.404	0.3735	0.349	0.328
50	0.4976	0.444	0.405	0.3744	0.35	0.329
100	0.4988	0.446	0.407	0.3762	0.352	0.331
$\varepsilon_{r\,max}^{-1/4}$	**0.5**	**0.447**	**0.408**	**0.378**	**0.354**	**0.333**

The number of layers depends on the sensitivity of the application accuracy. In general building more than 10 layers is not practical for manufacturing and we try to obtain the closest transmission coefficient to the continuously increasing case. From Table 1 one can see that, for 10 layers, N=10, T_{total} approaches close to the continuously increasing dielectric lens case. Even though 10 layers does not give us that much improvement if we compare it with N=2 layers, we took N=10 layers for our later calculations. One can easily decrease or increase the number of layers for specific applications. We took the maximum number of layers, which is N=10, that can be manufactured for later calculations.

3 Three Different Types of Graded Lens Design for a Prolate-Spheroidal IRA

The basic design considerations for the physical concept of three different types of increasing permittivity dielectric lens are considered. The focal point is $z_0 = 37.5$ cm and the other parameters of the prolate-spheroidal IRA are defined in (4.1). The lens is a half sphere (or half ball in mathematicians terms) and its radius is r_{max}, as shown in Figure 2.

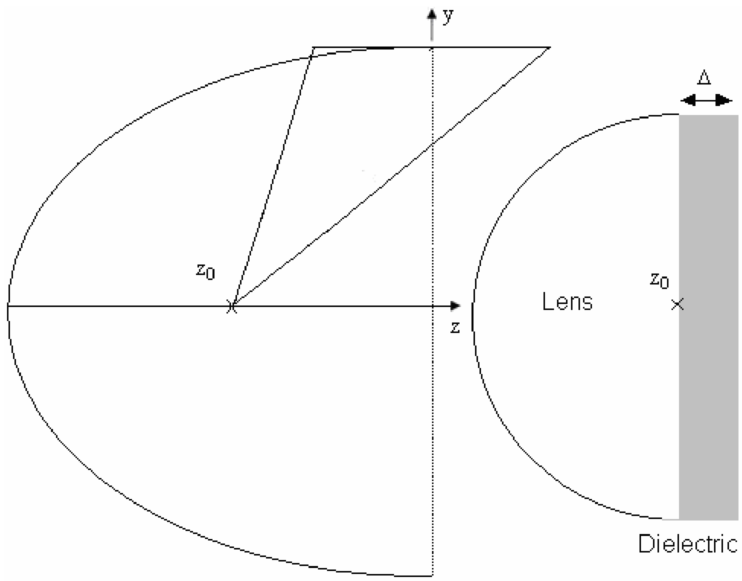


Figure 2: Addition of lens with prolate-spheroidal IRA geometry.

As discussed in [5] before, the exponential variation of the characteristic impedance of a transmission line along the line is optimal, provided that the speed of propagation is constant along the line. Some modification may be useful here since the speed varies inversely with the square root of the dielectric constant.

The lens relative permittivity is

$$\varepsilon_r(r) = \begin{cases} 1 & \text{at } r = r_{max} \\ \varepsilon_{r\,max} & \text{at } r = 0 \end{cases} \quad . \tag{3.1}$$

3.1 Exponential Variation of ε_r

One suitable form for ε_r is an exponential function as

$$\varepsilon_r(r) = e^{q(r_{max}-r)}. \tag{3.2}$$

As we know at $r = 0$ the relative permittivity is $\varepsilon_{r=}\varepsilon_{r\,max}$ so

$$\begin{aligned} \varepsilon_{r\,max} &= e^{C_1(r_{max})}, \\ C_1 &= \frac{1}{r_{max}} ln(\varepsilon_{r\,max}). \end{aligned} \tag{3.3}$$

If we substitute (3.3) in (3.2), ε_r can be found as

$$\varepsilon_r(r) = e^{ln(\varepsilon_{r\,max})(1-\frac{r}{r_{max}})}. \tag{3.4}$$

The rise time is estimated as $t_\delta = 100\,ps$ the distance corresponding to this rise time is

$$\ell_\delta = c\,t_\delta = 3\,cm, \tag{3.5}$$

in air.

The propagation distance of the wave from $r = r_{max}$ *to* $r = 0$ is

$$\begin{aligned} ct_{lens} &= c\int_0^{r_{max}} \frac{1}{v} dr = \int_0^{r_{max}} \varepsilon_r^{1/2}(r)\,dr = \int_0^{r_{max}} e^{\frac{1}{2} ln(\varepsilon_{r\,max})(1-\frac{r}{r_{max}})} dr \\ &= (\varepsilon_{r\,max}^{1/2} - 1)\frac{2\,r_{max}}{ln(\varepsilon_{r\,max})}. \end{aligned} \tag{3.6}$$

The normalized ct_{lens} is

$$\frac{ct_{lens}}{r_{max}} = (\varepsilon_{r\,max}^{1/2} - 1)\frac{2}{ln(\varepsilon_{r\,max})}. \tag{3.7}$$

The distance between the source and lens is $(0.375\,\mathrm{m} + \mathrm{r_{max}})$.

After this design procedure we designed a lens that is matched to the target dielectric $\varepsilon_{r\,max}$. The thickness of the target dielectric material should be

$$\Delta = n \frac{c\,t_\delta}{\sqrt{\varepsilon_{r\,max}}}\ ,\ n \geq 2\ , \tag{3.8}$$

in order to minimize the effect of the reflected wave on the impulse term.

3.2 Compensated Incremental Speed (CIS) form of ε_r

As we mentioned before the exponential form assumes that the propagation speed is constant. However, it is not constant we need to compensate for this assumption. Let us assume we have a plane wave problem in an inhomogeneous (isotropic) slab with $\varepsilon_r(z)$ and set the relative change in wave impedance over a transit time $\Delta\tau$

$$\frac{\Delta ln(\varepsilon_r^{-1/2})}{\Delta\tau} = C_2 \tag{3.9}$$

with the wave impedance Z_c proportional to $\varepsilon_r^{-1/2}$. The distance based on the transit time can be written as

$$c\,d\tau = \varepsilon_r^{1/2}\ dz\,. \tag{3.10}$$

For a given $\Delta\tau$ the Δz decreases as $\varepsilon_r^{-1/2}$. For a given $\Delta\tau$ the change in $ln(\varepsilon_r^{-1/2})$ is independent of z and if we substitute (3.10) in (3.9) we obtain

$$\varepsilon_r^{-1/2}\,\frac{d\,ln(\varepsilon_r^{-1/2})}{dz} = C_2\,. \tag{3.11}$$

Integrating (3.11)

$$\begin{aligned} &\int \varepsilon_r^{-1/2}\,d\,ln(\varepsilon_r^{-1/2}) = \int e^{ln(\varepsilon_r^{-1/2})}\,d\,ln(\varepsilon_r^{-1/2}) = \int C_2\,dz \\ &\varepsilon_r^{-1/2} = C_2 z + C_3\,. \end{aligned} \tag{3.12}$$

We can define ε_r from (3.12) as

$$\varepsilon_r = \left(C_2 z + C_3\right)^{-2} = \begin{cases} 1 & at\ z = z_{max} \\ \varepsilon_{r\,max} & at\ z = 0 \end{cases}, \tag{3.13}$$

such that from (3.13)

$$C_2 = \frac{1-\varepsilon_{r\,max}^{-1/2}}{z_{max}},\ C_3 = \varepsilon_{r\,max}^{-1/2}. \tag{3.14}$$

Then, if we substitute (3.14) in (3.13) we have

$$\varepsilon_r = \left((1-\varepsilon_{r\,max}^{-1/2}) \frac{z}{z_{max}} + \varepsilon_{r\,max}^{-1/2} \right)^{-2}. \tag{3.15}$$

How much time does the propagation of the wave take from r_{max} to the focal point in the lens? Substituting (3.15) in (3.7)

$$ct_{lens} = \int_0^{r_{max}} \varepsilon_r^{1/2}\, dr = \int_0^{r_{max}} \left((1-\varepsilon_{r\,max}^{1/2}) \frac{r}{r_{max}} + \varepsilon_{r\,max}^{-1/2} \right)^{-1} dr. \tag{3.16}$$

Changing the variable of the integral as $\xi = r / r_{max}$, we obtain

$$ct_{lens} = r_{max} \int_0^1 \left((1-\varepsilon_{r\,max}^{-1/2})\xi + \varepsilon_{r\,max}^{-1/2} \right)^{-1} d\xi. \tag{3.17}$$

Let us change the variable of the integral as $\varsigma = (1-\varepsilon_{r\,max}^{-1/2})\xi + \varepsilon_{r\,max}^{-1/2}$

$d\varsigma = (1-\varepsilon_{r\,max}^{-1/2})\, d\xi$ and we will have normalized

$$ct_{lens} = \frac{r_{max}}{1-\varepsilon_{r\,max}^{-1/2}} \int_{\varepsilon_{r\,max}^{-1/2}}^{1} (\zeta)^{-1}\, d\zeta = \frac{1}{2} \frac{r_{max}}{1-\varepsilon_{r\,max}^{-1/2}}\, ln(\varepsilon_{r\,max}). \tag{3.18}$$

Then, we can find the normalized ct_{lens} is

$$\frac{ct_{lens}}{r_{max}} = \frac{1}{2} \frac{1}{1-\varepsilon_{r\,max}^{-1/2}}\, ln(\varepsilon_{r\,max}). \tag{3.19}$$

3.3 Linear form of ε_r

The exponential variation and CIS form of ε_r are two different approaches having some advantages and disadvantages in terms of focusing. After these approaches we tried to use another approach, a linearly increasing form of ε_r. Let us assume we have a linear ε_r variation as

$$\varepsilon_r(r) = r/r_{max} + \varepsilon_{r\,max}\left(1 - r/r_{max}\right), \tag{3.20}$$

which satisfies (2.2), we can find the normalized propagation time of the wave from $r = r_{max}$ *to* $r = 0$ as

$$\frac{ct_{lens}}{r_{max}} = \frac{1}{r_{max}} \int_0^{r_{max}} \varepsilon_r^{1/2}(r)\,dr = \int_0^{r_{max}} \left(r/r_{max} + \varepsilon_{r\,max}\left(1 - r/r_{max}\right)\right)^{1/2} dr\,. \tag{3.21}$$

Let us change the variable of this integral as $\zeta = r/r_{max}$ we will have

$$\frac{ct_{lens}}{r_{max}} = \int_0^1 \left[\zeta + \varepsilon_{r\,max}(1-\zeta)\right]^{1/2} d\zeta\,. \tag{3.22}$$

We can also change this variable ζ as

$$\begin{aligned} \xi &= \zeta + \varepsilon_{r\,max}[1-\zeta] \\ d\xi &= d\zeta\left[1 - \varepsilon_{r\,max}\right] \end{aligned}\,. \tag{3.23}$$

Using (3.22),

$$\frac{ct_{lens}}{r_{max}} = \frac{1}{1-\varepsilon_{r\,max}} \int_{\varepsilon_{r\,max}}^1 \xi^{1/2} d\xi = \frac{2}{3}(\varepsilon_{r\,max} - 1)^{-1}(\varepsilon_{r\,max}^{3/2} - 1)\,. \tag{31}$$

3.4 Conclusion

A dielectric exponentially increasing dielectric constant, CIS, and linear increasing lens designs were discussed. One can see from Figure 3 that the wave propagates faster for the CIS form of ε_r. We can see from Figure 4 a-d) that if $\varepsilon_{r\,max}$ increases, the wave propagates slower as expected. $\varepsilon_{r\,max}$ varies from 1 to 81 (with 81 corresponding to water, which is the highest ε_r that is used in biological applications). If we increase $\varepsilon_{r\,max}$ from 36 to 81, the CIS design of ε_r has the deepest curvature. The focusing property of the lens increases from the CIS to the linear design because for the same r/r_{max} we have an increase in ε_r, we expect the lens to become more effective. Also from [2] if we increase ε_r the spot size decreases while the wave impedance decreases and the amplitude of the waveform increases by a factor of $\varepsilon_r^{1/4}$. This rough calculation has to extend out some distance from the target for effective focusing to occur and thus requires more detailed calculations.

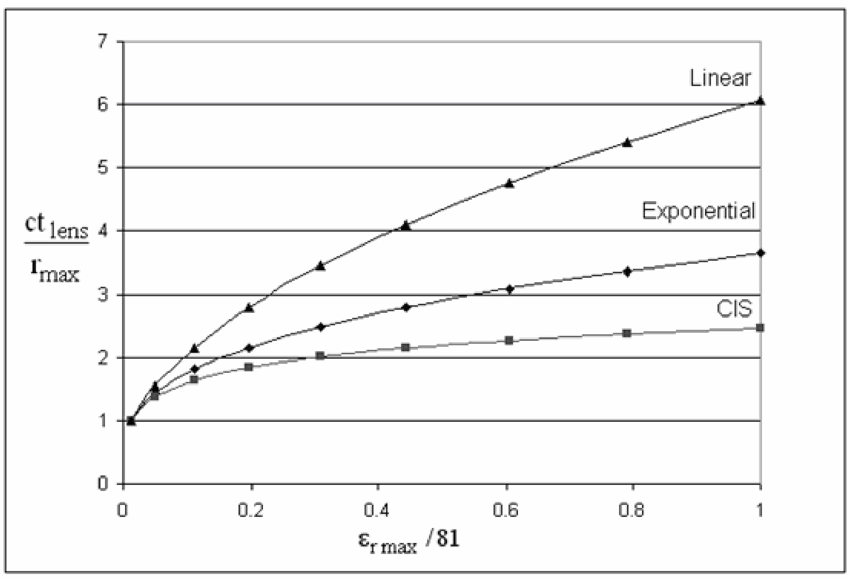


Figure 3: ct_{lens}/r_{max} for linear, exponential and CIS forms of ε_r.

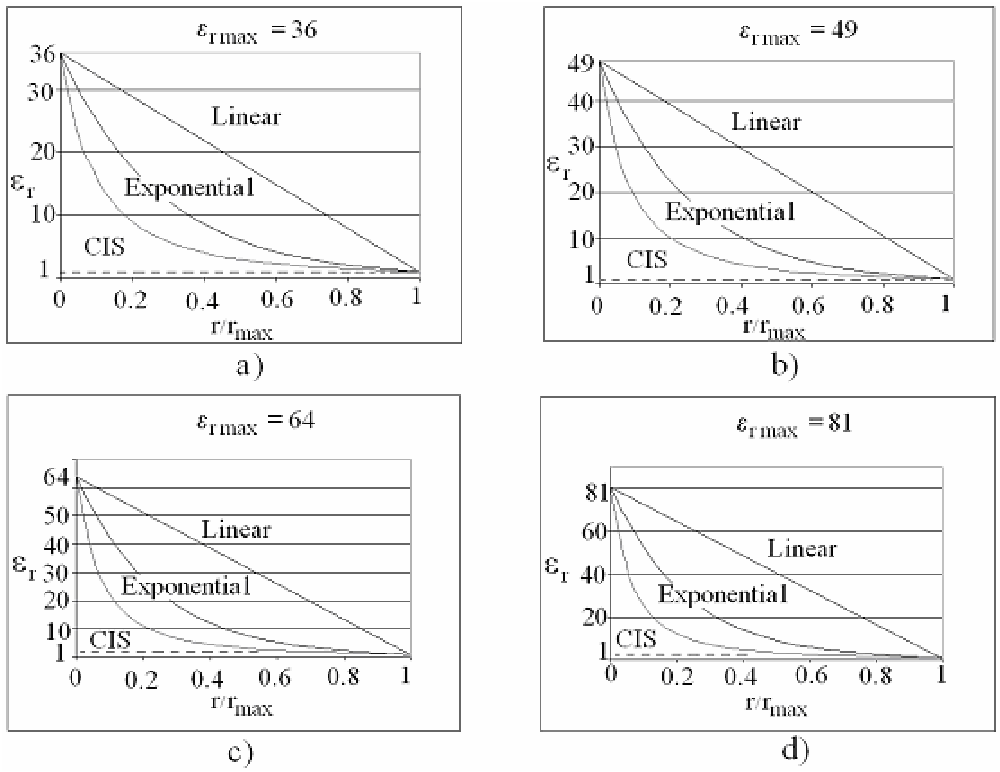


Figure 4: ε_r values for linear, exponential and CIS forms of ε_r for different $\varepsilon_{r\,max}$ with respect to r / r_{max} .

4. Spatially Limited Exponential Lens Design for Better Focusing an Impulse

A spatial limited exponential lens design is discussed and an analytical formulation has been used to examine the pulse droop in order to minimize it.

A formulation in [3] has been used to examine the pulse droop for a transmission line with an exponentially tapered impedance profile. We wish to minimize this droop, or ask how long the transmission line should be for a given droop. The exponentially tapered transmission line has an optimal transfer function in terms of early-time voltage gain and improved droop characteristics. We apply this result to an exponentially tapered dielectric constant of a focusing lens. We find the required lens dimensions for a given droop. The lens geometry and incoming spherical wave are presented in Figure 5. Our calculations are based on a one-dimensional plane-wave approximation (Figure 6). This will not directly give an estimate of spot size, only the transmission/reflection by the lens. Other considerations also apply [2].

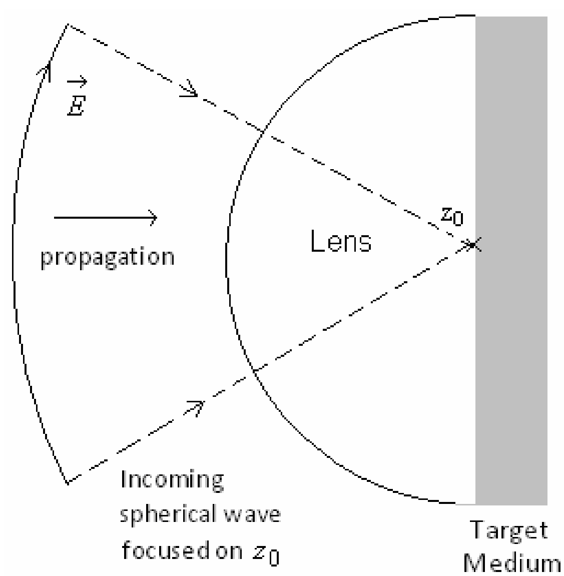


Figure 5: Lens geometry and incoming spherical wave.

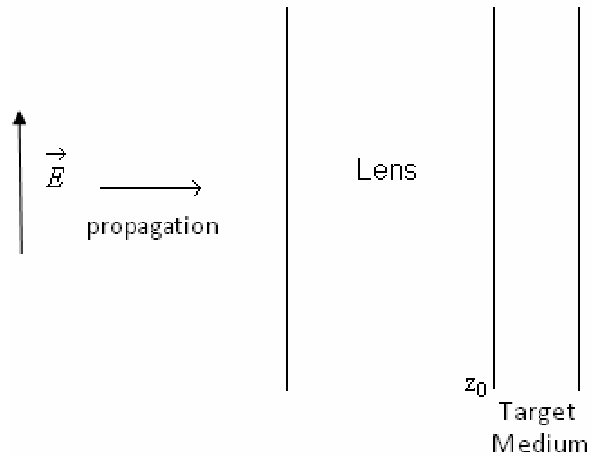


Figure 6: Equivalent plane wave geometry.

4.1 Equivalent Transmission-Line Model (One Dimensional) of Lens

As discussed in [3], the exponentially tapered lens has a minimized droop and the optimal transfer function for the case of uniform propagation speed. Here we adapt this solution to a dielectric lens, noting that the propagation speed slows as the wave propagates in higher-permittivity media. This model does not include any information about spot size.

We can define the lens wave impedance as follows: z =spatial coordinate, ζ = modified space coordinate. We have a new coordinate where the wave propagates with a constant v_1 speed and has an exponential wave impedance variation through the lens. We use a plane wave approximation and this approximation is valid up to the case when the wavelength is still small compared to the cross section of the beam

$$\frac{\zeta}{c} = \text{transit time to } z \text{ and hence } \zeta . \tag{4.1}$$

Let

$$Z(\zeta) = Z_1 \, e^{-\zeta/\zeta_0} , \tag{4.2}$$

where Z_1 is the wave impedance at the beginning of the lens; which is $Z_0 = 377\,\Omega$ in our case.

$$Z_2 = Z_1 \, e^{-\zeta_{max}/\zeta_0} , \tag{4.3}$$

where Z_2 is the wave impedance at the end of the lens.

$$Z(\zeta) = \left[\frac{\mu_0}{\varepsilon(\zeta)} \right]^{1/2} = Z_1 \, \varepsilon_r^{-1/2}(\zeta) \, . \tag{4.4}$$

The propagation speed can be defined as

$$\mathrm{v} = \frac{1}{\left[\mu_0 \, \varepsilon(\zeta) \right]^{1/2}} = \mathrm{v}_1 \, \varepsilon_r^{-1/2}(\zeta) \, , \tag{4.5}$$

where v_1 is the propagation speed before the lens, which is typically c.

The transit time through the lens can be defined as

$$t_\zeta \equiv t_z \equiv \int_0^z \mathrm{v}^{-1}(z') \, dz' = \int_0^\zeta \mathrm{v}_1^{-1} \, d\zeta = \frac{\zeta}{\mathrm{v}_1} \, . \tag{4.6}$$

Taking the derivative of both sides of (4.6), we have

$$\begin{aligned} &\frac{dt}{d\zeta} = \mathrm{v}_1^{-1} = \mathrm{v}^{-1}(\mathrm{z}) \frac{dz}{d\zeta} \, , \\ &\frac{dt}{dz} = \mathrm{v}^{-1}(\mathrm{z}) \, , \\ &\frac{d\zeta}{dz} = \frac{\mathrm{v}_1}{\mathrm{v(z)}} = \varepsilon_r^{1/2}(\zeta) = e^{\zeta_{max}/\zeta_0} \, . \end{aligned} \tag{4.7}$$

Using (4.7) to solve for the spatial coordinate z in terms of modified space coordinate ζ can be find as

$$z = \int_0^\zeta \varepsilon_r^{-1/2}(\zeta') \, d\zeta' \, . \tag{4.8}$$

From (4.2) and (4.4) we can write (4.8) as

$$z = \int_0^\zeta e^{-\zeta'/\zeta_0} \, d\zeta' = \zeta_0 \left[1 - e^{-\zeta/\zeta_0} \right] . \tag{4.9}$$

We can see from (4.9) as $\zeta \to \infty$, $z \to \zeta_0$ and this does not continue to grow. This gives us a spatially limited lens. This is convenient for purposes of implementation.

The wave propagation can be described by the source-free telegrapher equations ((2.3) in [3]). We can transform the 1D wave equation to an equivalent ζ space coordinate as

$$\frac{dE(\zeta,s)}{d\zeta} = -\frac{\mu_0}{Z_0} Z_0 e^{-\zeta/\zeta_0} \; H\,,$$
$$\frac{dH(\zeta,s)}{d\zeta} = -\frac{\varepsilon_r \varepsilon_0}{Z_0} Z_0 e^{-\zeta/\zeta_0} \; E = -\frac{\varepsilon_0}{Z_0} Z_0 e^{\zeta/\zeta_0} \; E\,. \tag{4.10}$$

4.4 Solution of the Transmission-Line Equations

We solve an equivalent problem of [3], but instead of an increase in the transmission-line impedance we have a decrease in wave impedance, but the equations in [3] still apply.
One can define the transmission coefficient for high T_h and low frequency T_ℓ as follows

$$T_h = \left[\frac{Z_2}{Z_1}\right]^{1/2} \quad , \quad T_\ell = \frac{2Z_2}{Z_1 + Z_2} \tag{4.11}$$

Calculate the difference between these two coefficients as

$$T_h - T_\ell = \frac{Z_2^{1/2}\left[Z_1 + Z_2\right] - 2Z_2 Z_1^{1/2}}{Z_1 + Z_2} = \frac{Z_2^{1/2}\left[Z_1^{1/2} - Z_2^{1/2}\right]^2}{Z_1 + Z_2} > 0 \ . \tag{4.12}$$

This is always positive except at $Z_1 = Z_2$. Thus, there is a droop (positive, i.e. a decrease) from initial to final value for both increasing and decreasing impedances. The impedance is decreasing but there is still a droop.

We can use the exact solution of the transfer function in (3.8) of [3],

$$\tilde{T} = e^{S+G}\left[\cosh\left(\left(S^2 + G^2\right)^{1/2}\right) + \frac{S}{\left(S^2 + G^2\right)^{1/2}} \sinh\left(\left(S^2 + G^2\right)^{1/2}\right)\right]^{-1} , \tag{4.13}$$

where S is the normalized complex frequency

$$S = s\, t_{\zeta\ max} = (\Omega + j\omega)\, t_{\zeta\ max}\,. \tag{4.14}$$

The high-frequency gain is defined in (3.4) of [3] as

$$g = e^G = \sqrt{Z_2 / Z_1} = \varepsilon_r^{-1/4}\,. \tag{4.15}$$

One can define the transit, normalized and droop time ((3.11) of [3]) parameters as follows

$$
\begin{aligned}
& t_{\zeta\ max} \equiv \zeta_{max} / \mathrm{v}_1 && \text{transit time through lens}, \\
& \tau \equiv t / t_{\zeta} && \text{normalized time}, \\
& \tau_d = 2\ ln^{-2}(g) && \text{normalized droop time} \\
& \quad = t_d / t_{\zeta\ max}.
\end{aligned}
\tag{4.16}
$$

t_d is the droop time, the step-response form is defined as (3.13) of [3]

$$
R(\tau) = g\left[1 + \tau / \tau_d + \mathrm{O}(\tau^2)\right] \quad as \quad \tau \to 0. \tag{4.17}
$$

4.3 Example

Now we can calculate the lens thickness for a given dielectric target permittivity $\varepsilon_{r\,max}$. Setting $t / t_d = 0.05\ \ and\ \ 0.1$, and using a $t = 100\,\mathrm{ps}$ pulse width(maximum time of interest) from (4.14)

$$
\begin{aligned}
& t_d = 2\ \mathrm{ns}\ \ and\ \ 1\mathrm{ns}, \\
& \zeta_{max} = \frac{2\ t_d}{ln^2(g)}.
\end{aligned}
\tag{4.18}
$$

From (4.3) and (4.16)

$$
\begin{aligned}
& \frac{Z_2}{Z_1} = \varepsilon_{r\ max}^{-1/2} = e^{-\zeta_{max} / \zeta_0}, \\
& \zeta_0 = \frac{c\,t_d\ ln^2(g)}{ln(\varepsilon_{r\ max})} \quad \text{(meters)}.
\end{aligned}
\tag{4.19}
$$

Substituting (4.18) in (4.9) we have

$$
z_{max} = \zeta_0\left[1 - e^{-\zeta_{max} / \zeta_0}\right] = \frac{c\,t_d\ ln^2(g)}{ln(\varepsilon_{r\ max})}\left[1 - \varepsilon_{r\ max}^{-1/2}\right] \quad \text{(meters)}. \tag{4.20}
$$

4.4 Conclusion

We might design a spatially limited exponential lens based on [3]. This lens is designed for a biological application [6]. From (51) we can find the z_{max} values for different biological tissues, which are summarized in Table 2.

Table 2 Design parameter values for different biological tissues [7,8].

		Water	Muscle	Tumor	Skin	Fat
	$\varepsilon_{r\,max}$	81	70	50.74	34.7	9.8
	g	0.33	0.34	0.37	0.41	0.56
	τ_d	1.65	1.77	2.07	2.5	6.1
t_d = 1ns	$t_{\zeta\,max}$ (ns)	0.6	0.56	0.48	0.39	0.16
	ζ_{max} (cm)	18.1	16.9	14.5	11.8	4.9
	ζ_0 (cm)	8.2	8	7.4	6.7	4.3
	z_{max}(cm)	7.3	7	6.3	5.5	2.9
t_d = 2 ns	$t_{\zeta\,max}$ (ns)	1.2	1.1	0.96	0.79	0.33
	ζ_{max} (cm)	36.2	33.8	28.9	23.6	9.8
	ζ_0 (cm)	16.5	15.9	14.7	13.3	8.6
	z_{max}(cm)	14.6	14	12.7	11	5.8

One can see from Table 2. that, if we have lower dielectric constant for target biological tissue, we need a smaller lens. This is not the only consideration. A larger dielectric constant in the lens exit results in a smaller spot size and higher fields. The smaller spot size concentrates the energy in the vicinity of the skin cancer.

One can find how ε_r changes as a function of ζ and z from (4.9) and (4.19)

$$\varepsilon_r(\zeta) = e^{2\zeta/\zeta_0}, \qquad (4.21)$$
$$\varepsilon_r(z) = \left(\frac{\zeta_0}{\zeta_0 - z}\right)^2.$$

Let us consider $t_d = 1$ ns and find $\varepsilon_r(\zeta)$ *and* $\varepsilon_r(z)$ with respect to ζ and z for different dielectric tissues. These are presented in Figures 7 and 7.8.

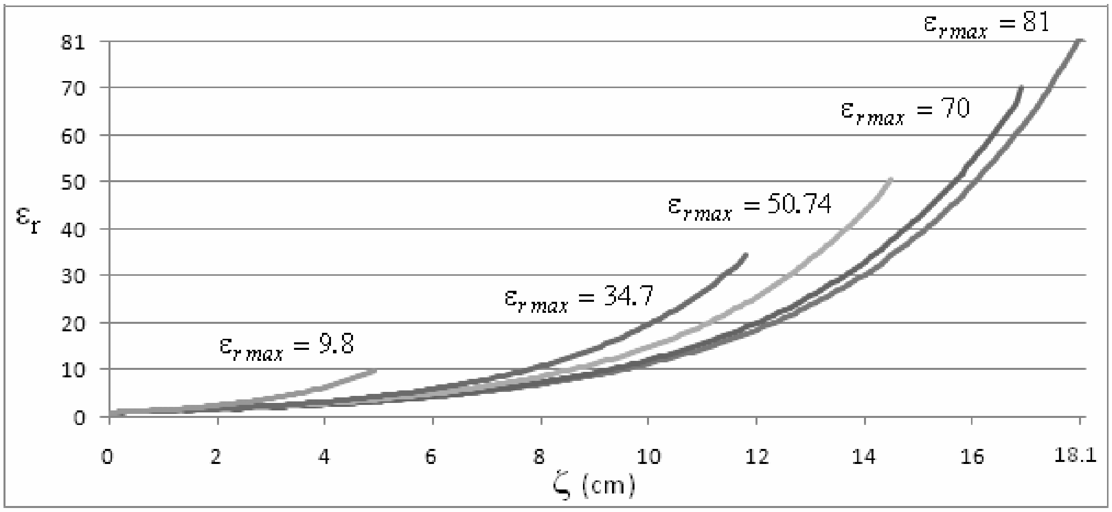


Figure 7: $\varepsilon_r(\zeta)$ values for different dielectric tissues.

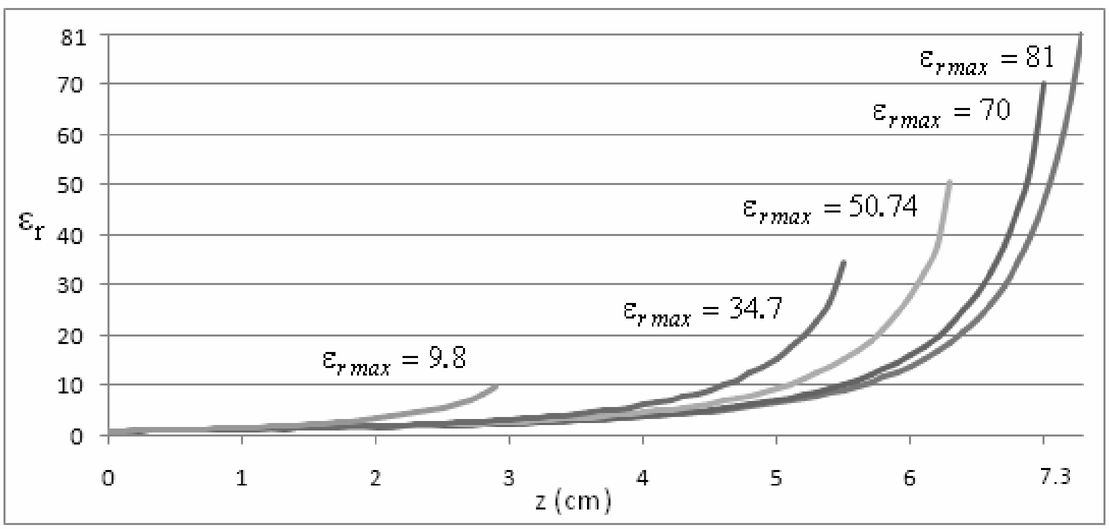


Figure 8: $\varepsilon_r(z)$ values for different dielectric tissues.

The compression of the coordinates for $t_d = 1$ ns and $\varepsilon_{r\,max} = 81$ is presented in Figure 9.

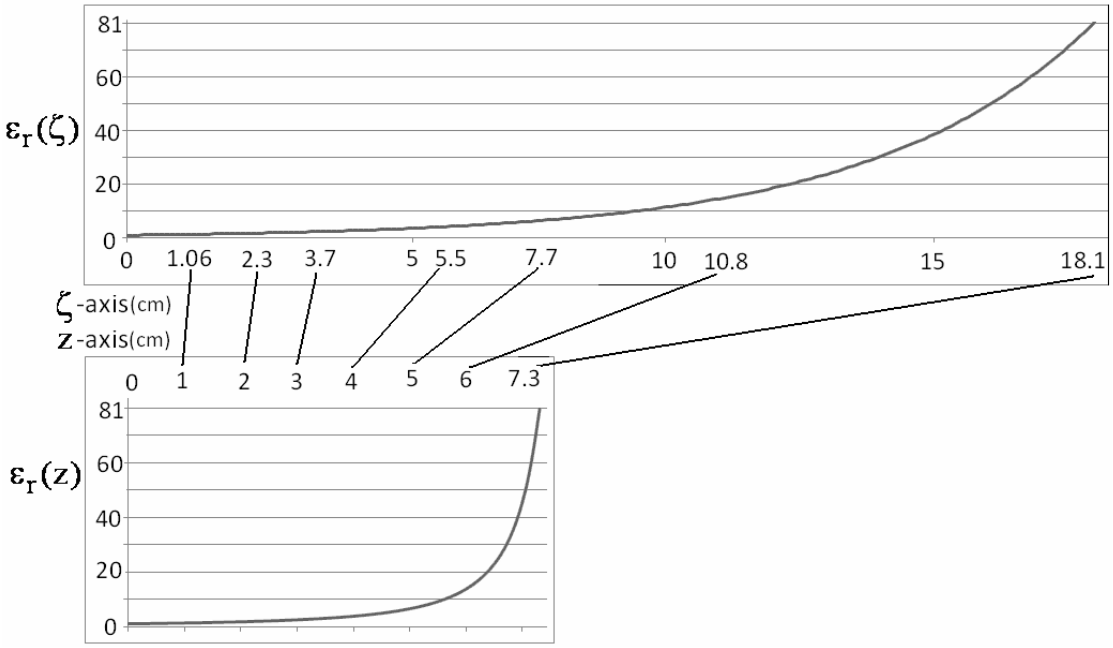


Figure 9: Compression of the coordinates for $t_d = 1$ ns and $\varepsilon_{r\ max} = 81$.

5. Lens Design for Incoming Spherical Wave

In this section an alternate lens design procedure is discussed to obtain better focusing from a prolate-spheroidal in which the lens is not a sphere. The lens design considerations are based on [9]. N layers of an increasing dielectric lens, which have the same ratio of dielectric constants between adjacent layers, are considered for a prolate-spheroidal IRA. Instead of using a half-spherical lens, a new approach is proposed for incoming spherical waves to obtain better focusing for a prolate-spheroidal IRA

5.1 Design Considerations

10 layers of increasing-dielectric-constant lens are used based on the calculations in Section 2. We use the same ratio of dielectric constant between subsequent layers as

$$\varepsilon_{ratio} = \varepsilon_{r_{n+1}} / \varepsilon_{r_n} \;,\; \left(\varepsilon_{r_{n+1}} / \varepsilon_{r_n}\right)^N = \varepsilon_{ratio}^N = \varepsilon_{r\,max} \;, \qquad (5.1)$$
$$\varepsilon_{ratio} = \varepsilon_{r\,max}^{1/N} \,.$$

We use N=10 layers and $\varepsilon_{r\max} = 81$ for the worst case scenario for biological applications. We start from free space $\varepsilon_r = 1$ and our target dielectric is $\varepsilon_{r\max} = 81$ and $\varepsilon_{ratio} = 1.55$ between subsequent layers. The first shell of the lens for incoming spherical wave is illustrated in Figure 10.

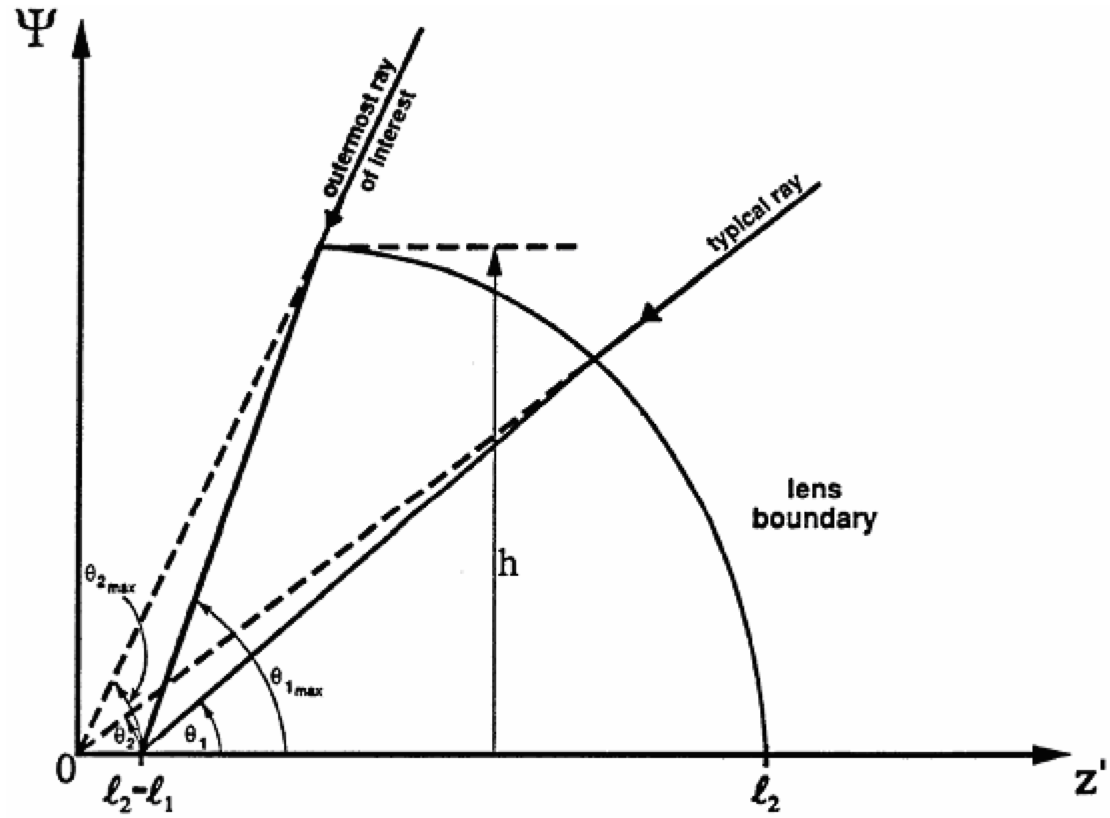


Figure 10: Lens for incoming spherical wave [9].

In Figure 10

$$\theta_{2\,max} = arctan(b / z_0)\ ,\ 0 \le \theta_2 \le \theta_{2\,max}. \tag{5.2}$$

Equation (54) represents the range of interest of angles for the incoming wave from the prolate-spheroidal IRA which has the dimensions as given in [10]. From (5.2) and (5.3) for the first shell $\theta_{2\,max} = 53.13^o$. Inside the lens the rays change their direction to the angle of θ_1 with respect to the z'-axis and $\theta_{1\,max} \le \pi / 2$ for geometrical design purposes. ℓ_1 *and* ℓ_2 are the distances on the z'-axis, h is the height of the lens. The normalized ℓ_1 *and* ℓ_2 parameters are defined from (4.7) in [9] as

$$\begin{aligned}\frac{\ell_1}{h} &= \frac{sin(\theta_{1\,max} - \theta_{2\,max}) + \varepsilon_r^{1/2}\ sin(\theta_{2\,max}) - sin(\theta_{1\,max})}{(\varepsilon_r^{1/2} - 1)\, sin(\theta_{1\,max})\, sin(\theta_{2\,max})},\\ \frac{\ell_2}{h} &= \frac{\varepsilon_r^{1/2}\left[sin(\theta_{1\,max} - \theta_{2\,max}) + sin(\theta_{2\,max})\right] - sin(\theta_{1\,max})}{(\varepsilon_r^{1/2} - 1)\, sin(\theta_{1\,max})\, sin(\theta_{2\,max})}.\end{aligned} \tag{5.3}$$

To find θ_2 as a function of θ_1 a quadratic equation in either $cos(\theta_2\,)$ *or* $sin(\theta_2\,)$ can be solved from (4.8-5.10) in [9] as

$$
\begin{aligned}
cos(\theta_2) &= \frac{AB sin^2(\theta_1) + \left| B cos(\theta_1) - A\,\varepsilon_r^{1/2} \right| \sqrt{\left[B^2 - 2AB\varepsilon_r^{1/2} cos(\theta_1) + A\varepsilon_r \right] - A^2 sin^2(\theta_1)}}{B^2 - 2AB\varepsilon_r^{1/2} cos(\theta_1) + A\varepsilon_r}, \\
sin(\theta_2) &= \frac{A(A\,\varepsilon_r^{1/2} - B cos(\theta_1)) + \left| B \right| sin(\theta_1) \sqrt{\left[B^2 - 2AB\varepsilon_r^{1/2} cos(\theta_1) + A\varepsilon_r \right] - A^2 sin^2(\theta_1)}}{B^2 - 2AB\varepsilon_r^{1/2} cos(\theta_1) + A\varepsilon_r}, \qquad (5.4) \\
A &= (\ell_2/\ell_1) - 1\ ,\ B = (\ell_2/\ell_1) - \varepsilon_r^{1/2}\,.
\end{aligned}
$$

A lens boundary curve can be defined by the coordinates of z' and Ψ as a function of θ_1 and θ_2 from (4.11) and (4.12) in [9] as

$$
\begin{aligned}
\frac{z'}{h} &= \frac{(\ell_2 - \ell_1)/h\, tan(\theta_1)}{tan(\theta_1) - tan(\theta_2)} \\
\frac{\Psi}{h} &= \frac{z}{h} tan(\theta_2) = \frac{(\ell_2 - \ell_1)/h\, tan(\theta_1) tan(\theta_2)}{tan(\theta_1) - tan(\theta_2)}.
\end{aligned}
\qquad (5.5)
$$

This approach is just for the first shell, but we can expand it to the other shells. $\varepsilon_r = \varepsilon_{ratio} = 1.55$ and we will have different $\ell_1, \ell_2, \theta_{1\,max}$ *and* $\theta_{2\,max}$ for each layer.

We can define a new coordinate system which is centered at $z = z_0$. We will call this system z' and it can be defined as

$$
z'/h = -(z - z_0)/h\,. \qquad (5.6)
$$

The IRA and lens geometry are presented in Figure 11. The angles of $\theta_{1\max}$ and $\theta_{2\max}$ are in given Figure 12

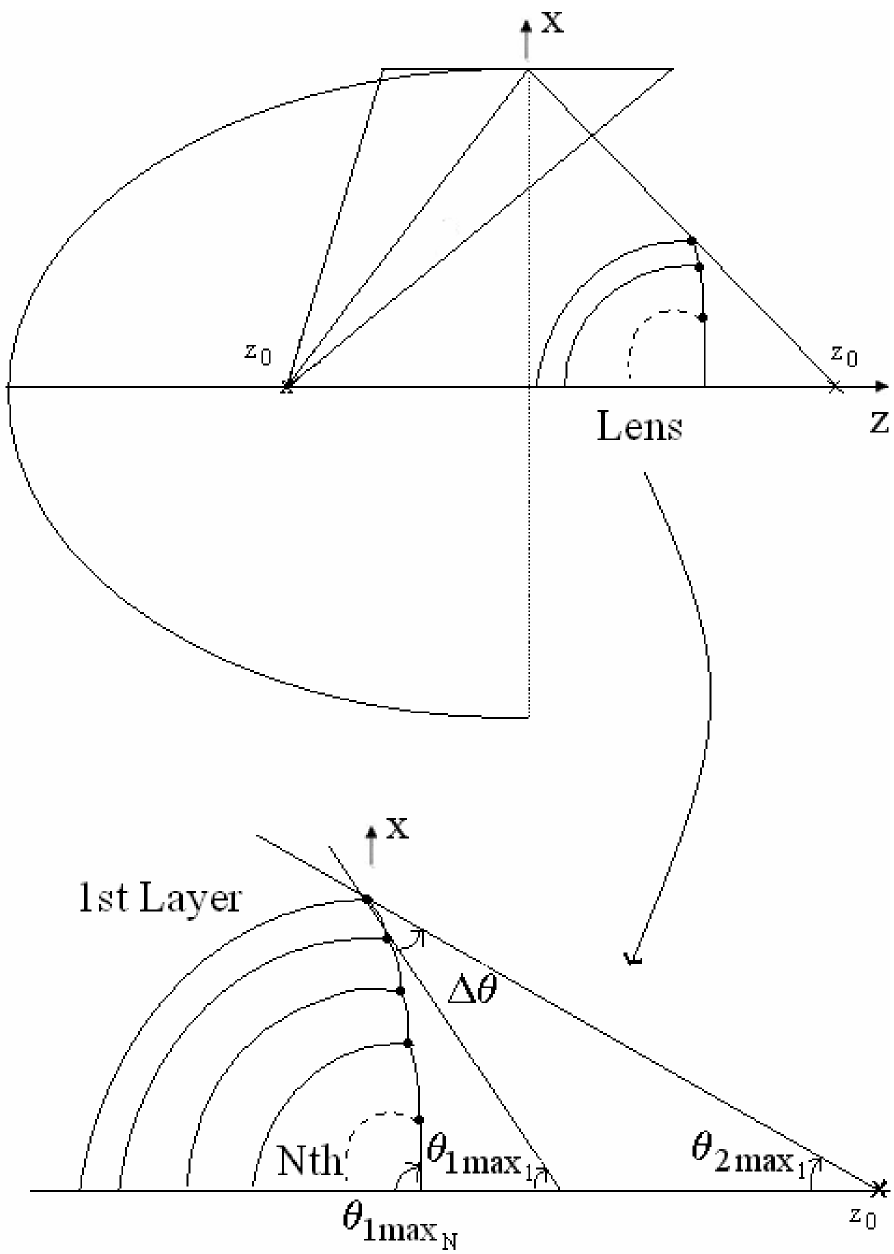


Figure 11: IRA and lens geometry.

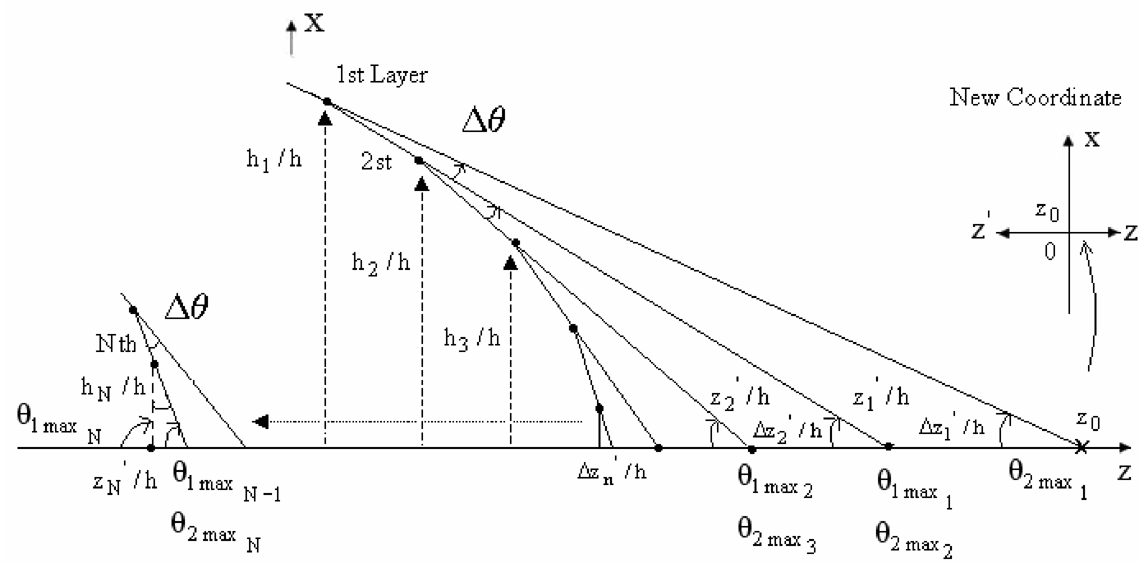


Figure 12: $\theta_{1\,max}$ and $\theta_{2\,max}$ values.

We use N=10 layers and $\Delta\theta$ is the change in the angle as one goes from one layer to the next. This is constant and is given by

$$\Delta\theta = (\theta_{1max_N} - \theta_{2max_1})/N\,. \tag{5.7}$$

We design the lens for two different $\theta_{1max_{10}}$ angles as:

$$\theta_{1max_{10}} = \begin{cases} 90^o\,(\pi/2) \\ 85^o \end{cases}. \tag{5.8}$$

For the $\pi/2$ case $\Delta\theta = 3.7^o$ and for the 85^o case $\Delta\theta = 3.2^o$. $\Delta z_n'/h$ is the normalized distance between each layer-beginning point on the z'-axis. z_n'/h is the sum of the n distances on the z'-axis which is shown in Figure 12.

5.2 Concluding Remarks for the Lens Design for Incoming Spherical Wave

We have designed a lens for incoming spherical waves to obtain better focusing from a prolate-spheroidal IRA. This design is based on the same procedure as in [9].In this design, however, just a single layer was used. We extended this design to N=10 layers. In this case we have different ℓ_1 / h ,ℓ_2 / h ,θ_{1max}, θ_{2max} , h_n / h, and $z_n^{\prime} / h$. We calculated these values for the first layer. Then we correct the values for the other layers.

First we calculate the Ψ / h and z' / h values for the first layer, then for the second layer we calculate Ψ / h and z' / h. We correct them by multiplying with $h_{corrected} = h_n / h$ value, then we add $\Delta z_n^{\prime} / h$ for each layer to find the corrected $\mathrm{z}_\mathrm{n}' / \mathrm{h}$ values at that layer.

As one can see from Figures 13 and 7.14, for $\theta_{1max_{10}} = 85^{\mathrm{o}}$ case we obtain better focusing. We call *h* the radius of the shell, it is a universal normalization parameter. But this calculation is not determining h because it is an optical calculation (infinite frequency). To determine how large *h* should be is a difficult problem. Clearly *h/c* must be much greater than the focus pulse width at the focus, and the rise-time of the incoming wave, otherwise it does not focus, $\lambda <<$ other dimensions of the lens. *h* should be smaller than the radius of the reflector as well

$$ct_\delta = 3\,cm << h < b = 50\ cm\,. \tag{5.9}$$

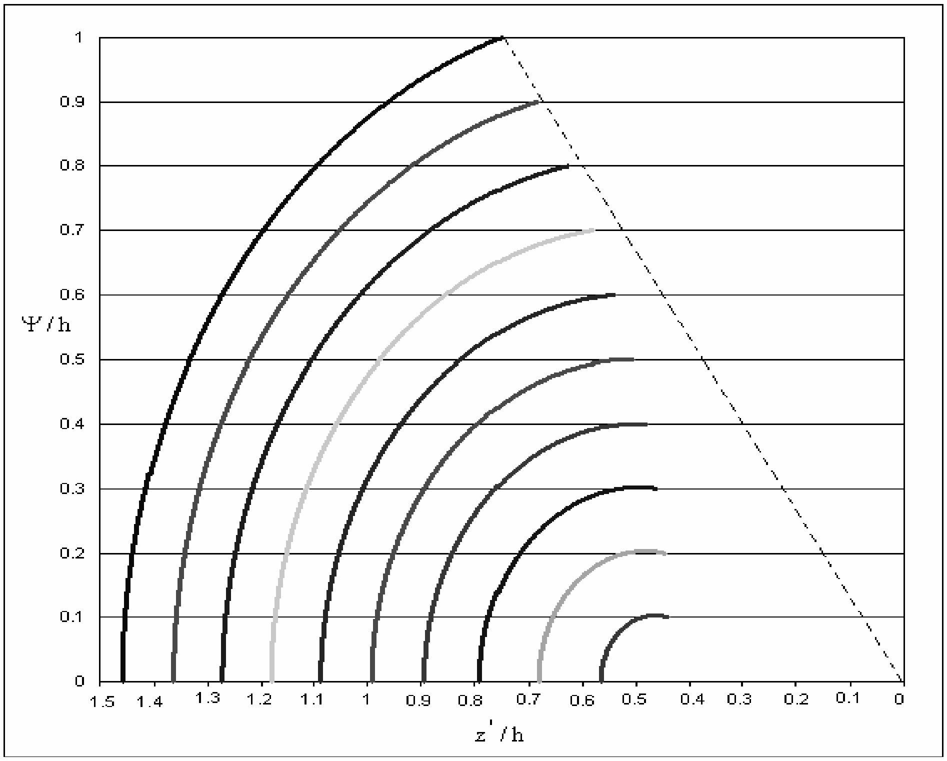


Figure 13: Ψ / h vs z' / h for $\theta_{1\max_{10}} = \pi / 2$.

Table 3: h_n / h, $\Delta z_n' / h$, z_n' / h, θ_{1max}, and θ_{2max} values for $\theta_{1max_{10}} = \pi / 2$.

Layer	h_n / h	$\Delta z_n' / h$	z_n' / h	θ_{1max}	θ_{2max}
1	1.0	0.096	0.000	0.992	0.927
2	0.9	0.079	0.096	1.056	0.992
3	0.8	0.066	0.175	1.120	1.056
4	0.7	0.054	0.241	1.185	1.120
5	0.6	0.044	0.295	1.249	1.185
6	0.5	0.035	0.339	1.313	1.249
7	0.4	0.027	0.374	1.378	1.313
8	0.3	0.020	0.401	1.442	1.378
9	0.2	0.013	0.421	1.506	1.442
10	0.1	0.006	0.434	1.571	1.506

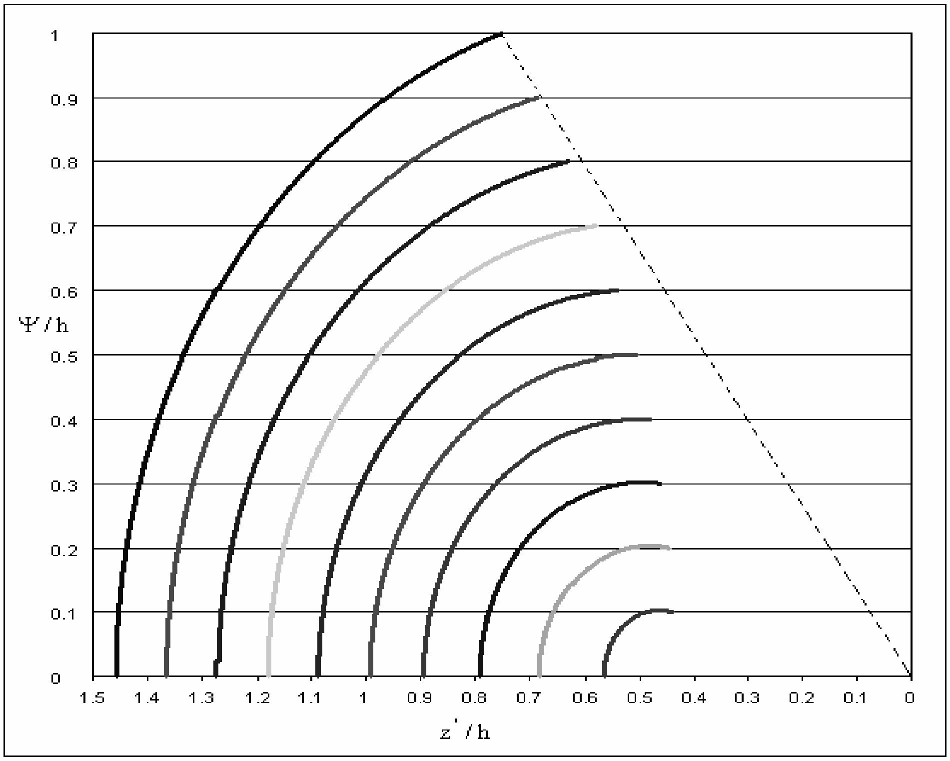


Figure 14: Ψ / h *and* z' / h for $\theta_{1\,max10} = 85^o$.

Table 4: h_n / h, $\Delta z_n' / h$, z_n' / h, $\theta_{1\,max}$, and $\theta_{2\,max}$ values for $\theta_{1\,max10} = 85^o$.

Layer	h_n / h	$\Delta z_n' / h$	z_n' / h	θ_{1max}	θ_{2max}
1	1.0	0.096	0.000	0.983	0.927
2	0.9	0.079	0.096	1.039	0.983
3	0.8	0.066	0.175	1.094	1.039
4	0.7	0.054	0.241	1.150	1.094
5	0.6	0.044	0.295	1.205	1.150
6	0.5	0.035	0.339	1.261	1.205
7	0.4	0.027	0.374	1.317	1.261
8	0.3	0.020	0.401	1.372	1.317
9	0.2	0.013	0.421	1.428	1.372
10	0.1	0.006	0.434	1.484	1.428

5.3 Lens Design for Incoming Spherical Wave for Different Biological Dielectric Tissues

Five different biological dielectric tissues are used as different target dielectrics and we try to obtain better focusing from a prolate-spheroidal IRA for an incoming spherical wave from the reflector for these tissues. This subsection is an extension of the previous one. We use 5 different target dielectric tissues comprising water, muscle, tumor, skin and fat. Ten layers of an increasing dielectric constant lens that have the same ratio of dielectric constants between adjacent layers are considered for a prolate-spheroidal IRA.We use the same ratio of dielectric constant between subsequent layers as

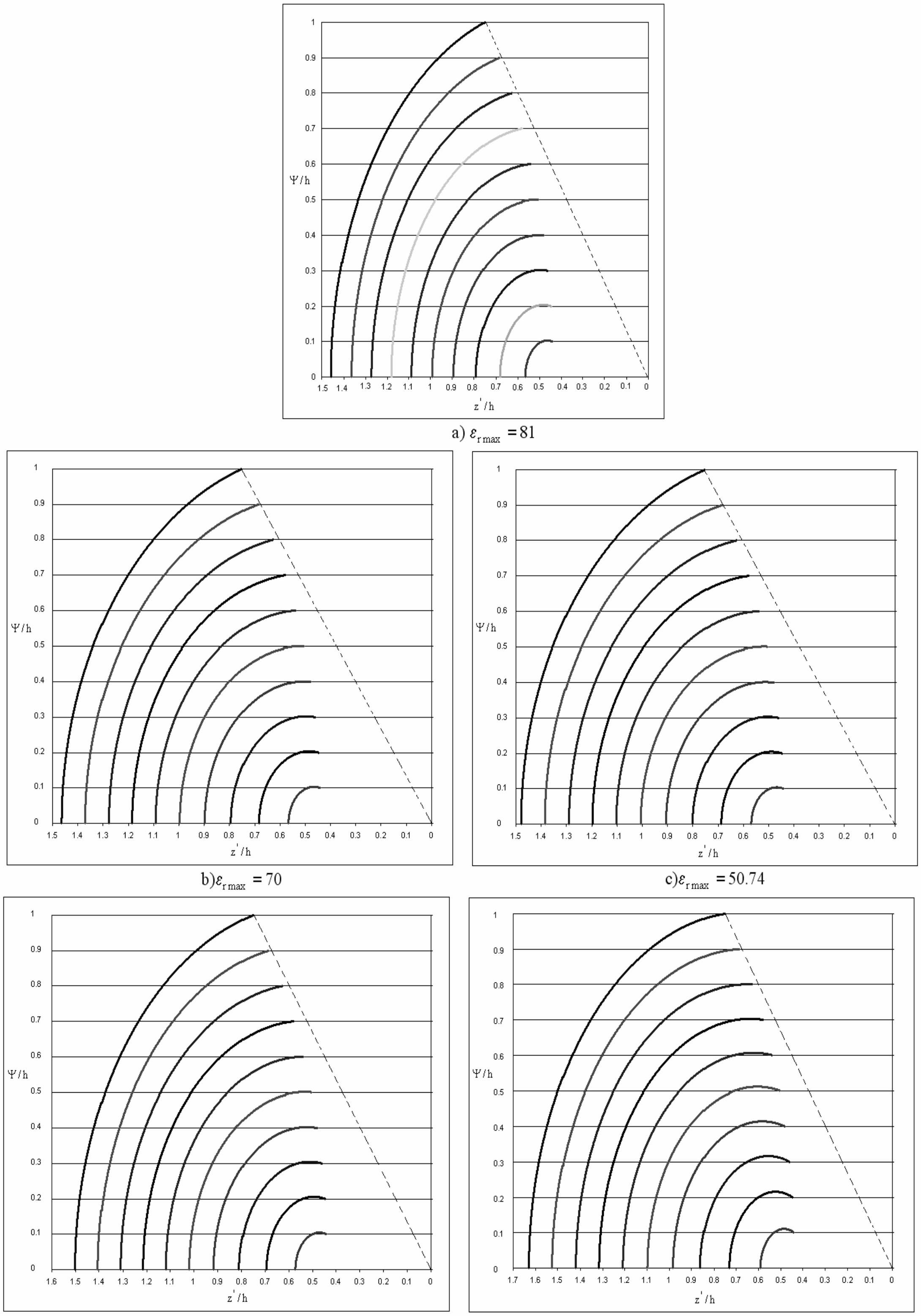
$$\varepsilon_{ratio} = \varepsilon_{r\,max}^{1/N}, \tag{5.10}$$

where $\varepsilon_{r\,max}$ and ε_{ratio} values for different human tissues are presented in Table 5.

Table 5: ε_{ratio} and $\varepsilon_{r\,max}$ values for different human tissues [7,8].

	Water	Muscle	Tumor	Skin	Fat
$\varepsilon_{r\,max}$	81	70	50.74	34.7	9.8
ε_{ratio}	1.55	1.53	1.48	1.43	1.26

A lens is designed for incoming spherical waves to obtain better focusing from a prolate-spheroidal IRA for different dielectric human tissues. We obtain better focusing for the higher dielectric lens. Ψ / h *vs* z' / h values for $\theta_{1\,max_{10}} = \pi / 2$ *and* 85^o for different $\varepsilon_{r\,max}$ are presented in Figure 3.1 and Figure 3.2. One can see from Figure 15 and Figure 16 that for smaller $\varepsilon_{r\,max}$, the first shell moves left. We have fixed the vertical (Ψ / h) axis values to increment by a uniform value of 0.1, leaving some variation (small) in the location along the horizontal coordinate.



a) $\varepsilon_{r\,max} = 81$

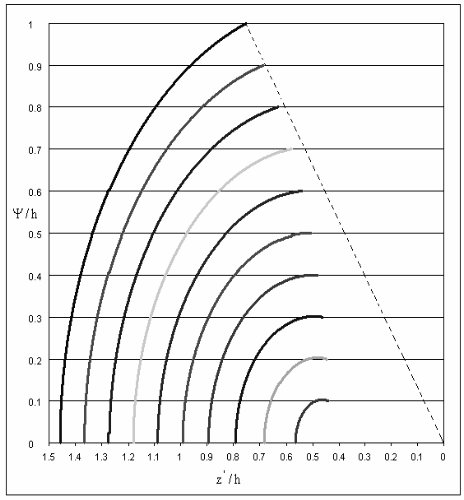
b) $\varepsilon_{r\,max} = 70$

c) $\varepsilon_{r\,max} = 50.74$

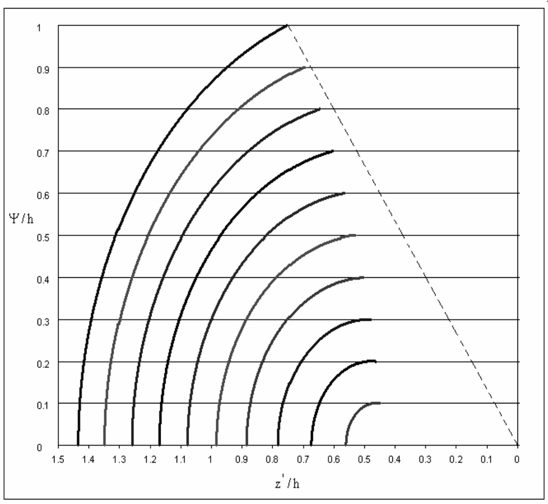
d) $\varepsilon_{r\,max} = 34.7$

e) $\varepsilon_{r\,max} = 9.8$

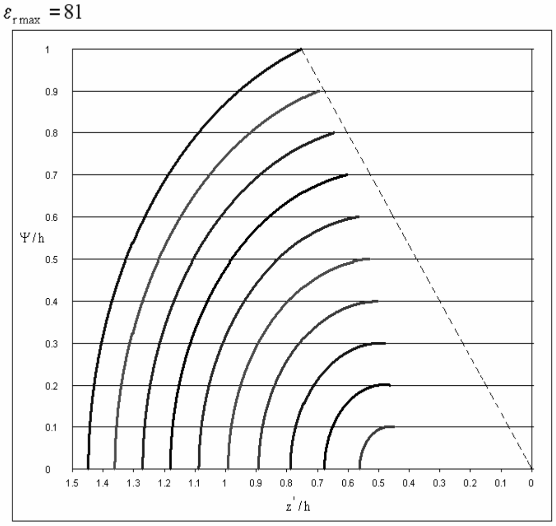
Figure 15: Ψ / h *vs* z' / h for $\theta_{1max_{10}} = 90^{o}$ and different $\varepsilon_{r\,max}$.



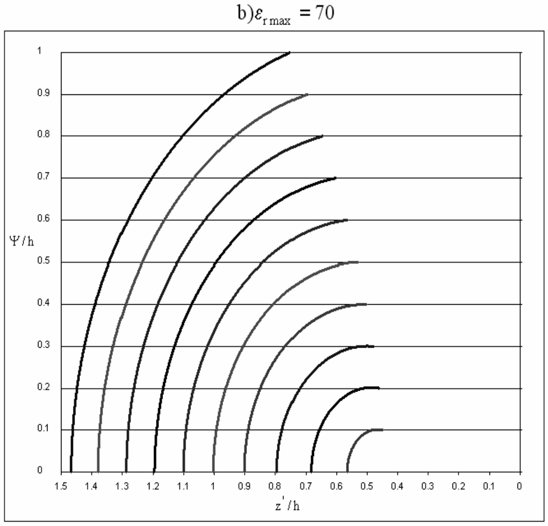
a) $\varepsilon_{r\,max} = 81$



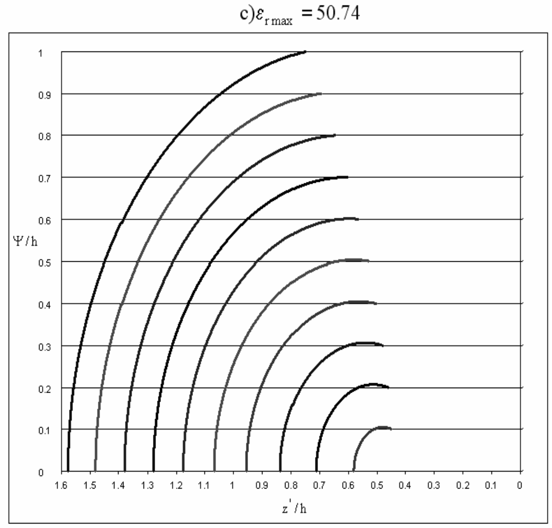
b) $\varepsilon_{r\,max} = 70$



c) $\varepsilon_{r\,max} = 50.74$



d) $\varepsilon_{r\,max} = 34.7$



e) $\varepsilon_{r\,max} = 9.8$

Figure 16: Ψ / h *vs* z' / h for $\theta_{1max_{10}} = 85^o$ and different $\varepsilon_{r\,max}$.

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