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A Fast Transmission-Line Voltage Divider With Large Signal Reduction

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Abstract

In monitoring a high-voltage waveform one may use a resistive or capacitive divider. For large divider ratios (to bring the signal to relatively low levels for recording) one is concerned with questions of accuracy and bandwidth. This paper describes a kind of transmission-line divider suitable for fast, high-amplitude pulses.

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1. Introduction

One is sometimes faced with the problem of measuring fast voltages in the 100 ps range with amplitudes of hundreds of kV. This amplitude needs to be reduced to the kV range for typical signal cables, and to the volt range for recording instruments (oscilloscopes, transient digitizers). At the same time one needs to maintain very faithful high-frequency response so as to accurately record the wave shape with characteristic times t_c of the order of 100 ps, or even less.

There are various traditional approaches to this problem. One can install a D-dot sensor in the outer coaxial conductor. This can be very fast and can achieve large signal reduction. However, it is a time-derivative sensor which some may find inconvenient due to a requirement to integrate the output. Depending on how far out one goes in time, there can also be integration errors. This can be modified to give a capacitive divider, but this can introduce bandwidth problems based on the sensor size (capacitance) and the integrating resistor.

One can also use a resistive divider on a transverse plane (transverse to the direction of propagation), connecting the center conductor to a connector on the outer conductor [14]. In this case one is concerned with the voltage and frequency-response characteristics of the resistors. One can minimize the effect of stray capacitance of the resistor string by grading the resistance per unit length so as to match the potential distribution of the TEM mode [5, 6].

In this paper we explore another approach to this problem, one that we might term a transmission-line divider. In this case a conductor, parallel to the coax axis, is placed very near inside the outer coax conductor. A small piece of the TEM wave passing over it is sensed to give the desired voltage division. Depending on how one connects to this one can also have a directional coupler which can separate the TEM waves propagating in two (opposite) directions [10-12]. This type is especially appropriate for the case that the pulse width is shorter than the round-trip transit time on the transmission-line sensor [8]. For the case that the pulse width is shorter than the transit times across (transverse) the coax, we will also need to take particular care to maintain the field distribution in the coaxial TEM mode.

2. Coaxial Transmission-Line Divider

Consider, as in Fig. 2.1, that we have a coaxial transmission line with

$$
\begin{aligned}
&\Psi_0 \equiv \text{ outer radius (inside)} \\
&\Psi_1 \equiv \text{ inner radius} \\
&Z_w = \left[\frac{\mu}{\varepsilon}\right]^{1/2} = \text{ wave impedance of medium} \\
&\text{v} = [\mu\varepsilon]^{-1/2} = \text{ wave speed of medium} \\
&Z_c = f_g Z_w = \text{ characteristic impedance of coax} \\
&f_g = \frac{1}{2\pi}\ell n\left(\frac{\Psi_0}{\Psi_1}\right)
\end{aligned}
\tag{2.1}
$$

The electric field of the TEM mode takes the form (positive in the radial direction $\vec{1}_\Psi$)

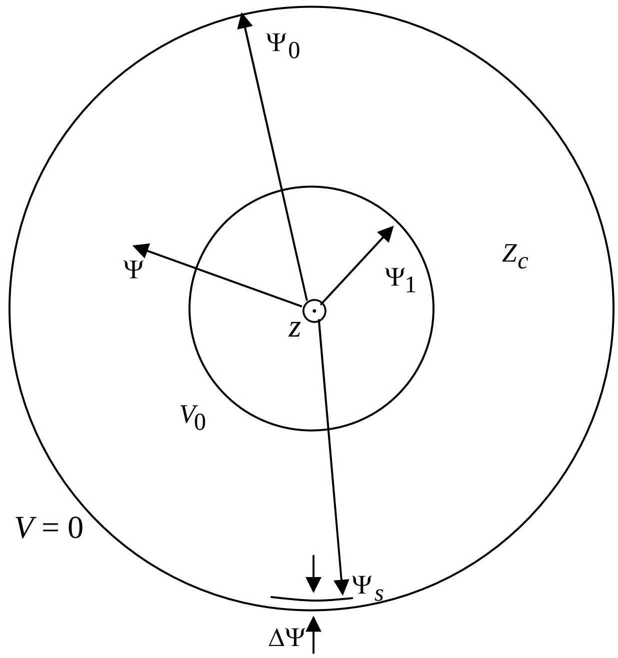
$$
\begin{aligned}
&E = V_0\ \ell n^{-1}\left(\frac{\Psi_0}{\Psi_1}\right)\Psi^{-1} \\
&V_0 = \text{ potential of center conductor (zero potential on outer conductor)}
\end{aligned}
\tag{2.2}
$$

Near the outer conductor at

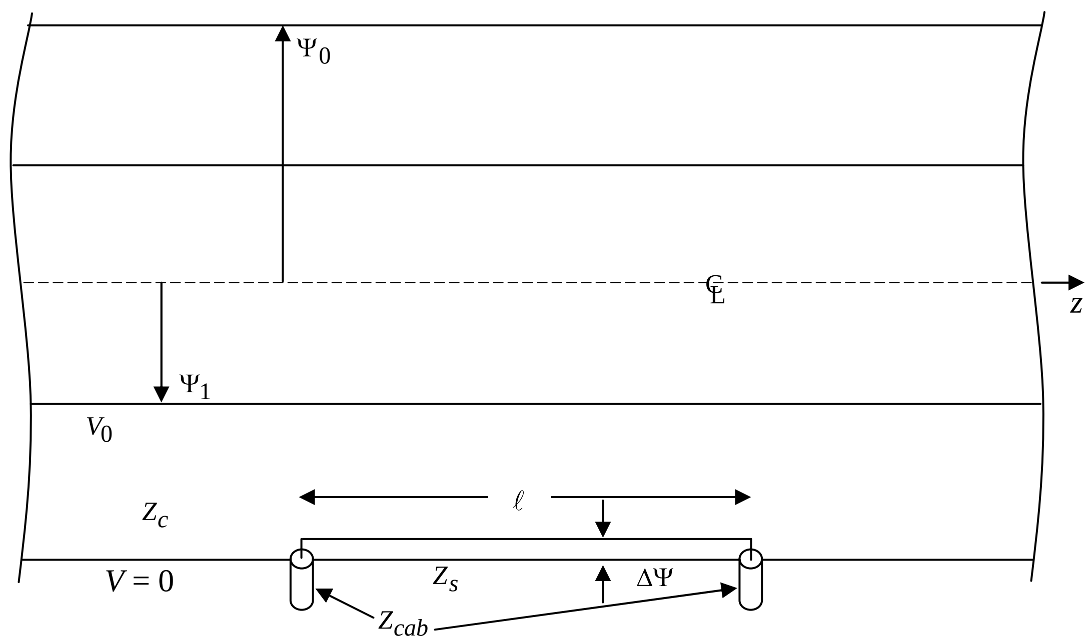
$$
\begin{aligned}
&\Psi_s = \Psi_0 - \Delta\Psi \equiv \text{ sensor conductor radius} \\
&\Delta\Psi << \Psi_0 - \Psi_1
\end{aligned}
\tag{2.3}
$$

we place a thin conductor (ideally << Ψ_s) on a surface of constant radius Ψ_s. Before connecting signal cables to this note that the TEM wave passes over this without disturbance giving an open-circuit voltage

$$
\begin{aligned}
V_{oc} &= \int_{\Psi_s}^{\Psi_0} E d\Psi = V_0 \ell n^{-1}\left(\frac{\Psi_0}{\Psi_1}\right)\ell n\left(\frac{\Psi_0}{\Psi_s}\right) \\
&= V_0 \ell n^{-1}\left(\frac{\Psi_0}{\Psi_1}\right)\ell n\left(1+\frac{\Delta\Psi}{\Psi_s}\right) \\
&= V_0 \ell n^{-1}\left(\frac{\Psi_0}{\Psi_1}\right)\left[\frac{\Delta\Psi}{\Psi_s} + O\left(\left[\frac{\Delta\Psi}{\Psi_0}\right]^2\right)\right]
\end{aligned}
\tag{2.4}
$$



A. Cross-section view



B. Transverse view

Fig. 2.1 Coaxial Transmission-Line Divider

For small $\Psi_0^{-1}\Delta\Psi$ we have

$$\frac{V_{oc}}{V_0} \simeq \ell n^{-1}\left(\frac{\Psi_0}{\Psi_1}\right)\frac{\Delta\Psi}{\Psi_0} \tag{2.5}$$

which can be made quite small for large Ψ_0. (It can be made yet smaller by recessing the sensor conductor in a depression in the outer coax conductor.)

If the sensor conductor has length

$$\begin{aligned} \ell &= \mathrm{v}t_r \\ t_r &= \text{transit time along conductor} \end{aligned} \tag{2.6}$$

we can think of this type of sensor itself as a transmission line with

$$Z_s \equiv \text{characteristic impedance of sensor transmission line} \tag{2.7}$$

This raises various possibilities.

As a first possibility let us connect a signal cable through the outer coaxial shield via some large resistance R with

$$\mathrm{R} \gg Z_s \tag{2.8}$$

Then this acts like a high-frequency capacitive divider, provided R is of sufficiently low capacitance and is graded between Ψ_s and Ψ_0 approximately uniformly (to avoid stray capacitances [5, 6]). Note that R connects to a coaxial cable. This gives a divider ratio

$$\begin{aligned} T_V &= \frac{Z_{cab}}{\mathrm{R} + Z_{cab}} \\ Z_{cab} &= \text{sensor cable characteristic impedance} \end{aligned} \tag{2.9}$$

There is also a long-time decay constant

$$\begin{aligned} \tau &= \left[\mathrm{R} + Z_{cab}\right] C_s \\ C_s &= \frac{t_r}{Z_s} \end{aligned} \tag{2.10}$$

which should be large compared to the pulse width

$$t_p \equiv \text{pulse width} \tag{2.11}$$

A second possibility is to use this sensor to measure only for a time $2t_r$ (the round-trip transit time) with

$$t_p < 2t_r \tag{2.12}$$

and with the sensor cable connected to the end of the sensor transmission line first contacted by the TEM wave on the large coax. This acts like previously discussed limited-time electric sensors [8, 15, 16]. This signal can be taken by connecting a resistor R, or in an interesting case for high speed

$$\begin{aligned} \mathrm{R} &= 0 \quad , \quad Z_{cab} = Z_s \\ T_{\mathrm{v}} &= \frac{1}{2} \end{aligned} \tag{2.13}$$

In this case there is a single reflection from the far end of the sensor transmission line with terminates without reflection at the near end.

A third possibility is to operate this sensor as a directional coupler [12, 13]. In this case, as illustrated in Fig. 2.1B, two sensor cables are connected to the sensor transmission line, one at each end terminating the transmission line in its characteristic impedance. Enforcing the condition in (2.13), this gives two limited-time measurements, one for each of the two directions of propagation on the large coax.

With a large coax (to achieve a large divider ratio) the fast times t_c in the pulse to be measured may be less than Ψ_0 / v. This raises the possibility of overmoding the coax with nonTEM modes entering. The sensor with small $\Delta\Psi$ should not give much of a problem in this regard.

One approach to aleviating this problem is to make the coaxial geometry conical so that the radius Ψ_0 expands slowly from the coaxial source of the pulse to be measured. This can lead to a rather long structure for large Ψ_0. There is also the problem of the dielectric medium to be used. For small coaxes (signal-source end), solid dielectrics like polyethylene are typically used. If one wishes to use a gas (such as air or SF_6) in the large-cross-section region there is the problem of the interface of the two media.

3. Thin Lens

In order to shorten the divider, one can have a short conical section connected to a cylindrical section. This would lead to the significant introduction of higher-order modes for the case of fast pulses. This can be remedied, at least in a major part, by the introduction of a lens at the junction of the conical and cylindrical sections as in Fig. 3.1. A first constraint is to make the conical-transmission-line impedance [3]

$$\begin{aligned} Z_{con} &= Z_w f_g \\ f_g &= \frac{1}{2\pi} \ell n \left(\frac{\tan\left(\frac{\theta_0}{2}\right)}{\tan\left(\frac{\theta_1}{2}\right)} \right) \end{aligned} \tag{3.1}$$

equal to Z_c, the coax characteristic impedance as in (2.1).

Next we need to add delay to the wave for small θ (and small Ψ) to match the transit time in going from a spherical wavefront to a planar one as discussed in [2]. A convenient reference surface for the matching is found as in [1] by matching ray paths for the TEM modes on both sides of the lens (Fig. 3.2). This gives an approximate continuity of TEM potential on both sides of the lens. As derived in [1], there is a transition paraboloid given by

$$\begin{aligned} &\left[\frac{\Psi}{2\, z_{ref}} \right] = 1 - \frac{z_t}{z_{ref}} \\ &z_t \equiv \text{coordinate of transition paraboloid} \\ &z_{ref} \equiv \text{intersection of transition paraboloid with the } z \text{ axis} \end{aligned} \tag{3.2}$$

The lens thickness d is such as to delay the wave an additional amount approximately (for unchanged μ)

$$\begin{aligned} &\mathrm{v} t_d = \left[\varepsilon_r^{1/2} - 1 \right] d \\ &\varepsilon_r \equiv \text{relative dielectric constant of lens medium} \end{aligned} \tag{3.3}$$

With zero lens thicknes at $\theta = \theta_0$, we can find the required extra delay (for small θ_0) as

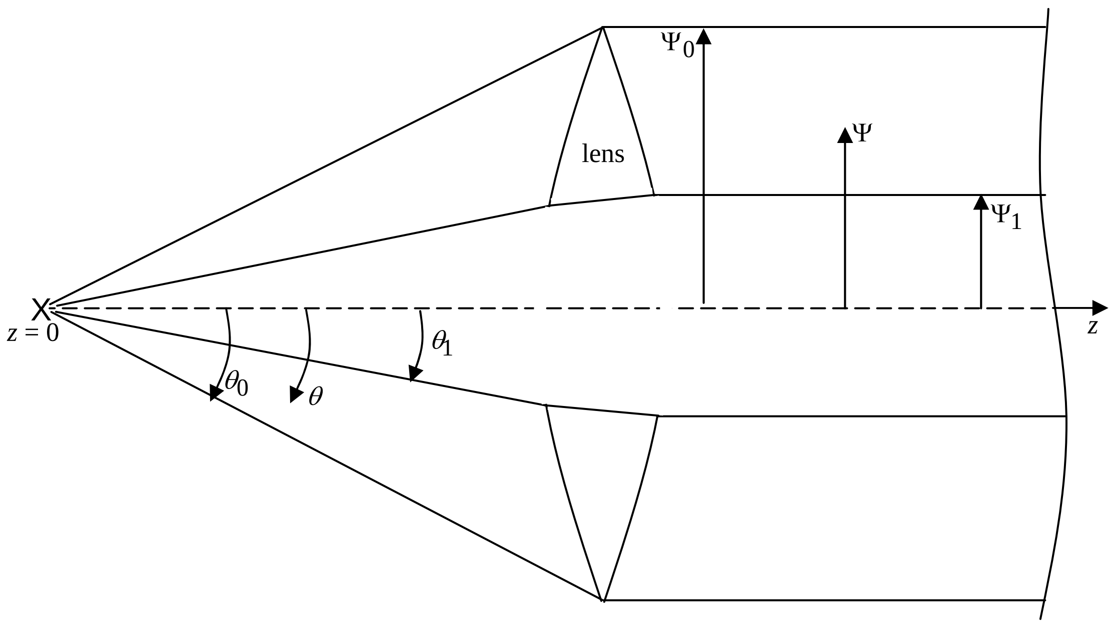


Fig. 3.1 Lens at Junction of Conical and Cylindrical Sections

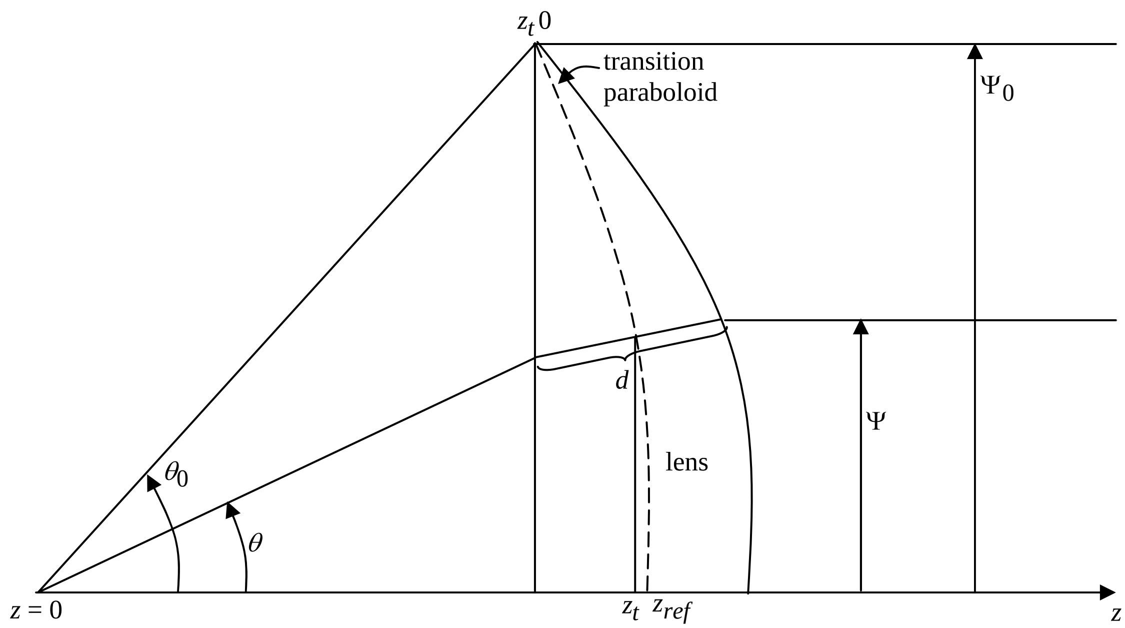


Fig. 3.2 Lens Geometry

$$
\begin{aligned}
\mathrm{v}t_d &= \left[\left[\Psi_0^2 + z_{t_0}^2\right]^{1/2} + z_{ref} \;\; - z_{t0}\right] \\
&\quad -\left[\left[\Psi^2 + z_t^2\right]^{1/2} + z_{ref} \;\; - z_t\right] \\
&= \frac{1}{2}\left[\frac{\Psi_0^2}{z_{t_0}} - \frac{\Psi^2}{z_t}\right] + \; z_t \;\; \mathrm{O}\left(\left[\frac{\Psi}{\mathrm{z_t}}\right]^4\right) \\
&\simeq \frac{1}{2\, z_{ref}}\left[\Psi_0^2 \;=\; \Psi^2\right] \\
z_{t0} &= z_t \;\; \text{at} \;\; \Psi \;=\; \Psi_0
\end{aligned}
\tag{3.4}
$$

How thick one makes this lens then depends on how small one makes θ_0 (making a long conical section) and how large an ε_r one uses. We can note that for our high-voltage applications one often uses polyethelene (equivalent to transformer oil) with

$$
\varepsilon_r \;\simeq\; 2.25 \;\; , \;\; \varepsilon_r^{1/2} \;\simeq\; 1.5 \tag{3.5}
$$

This can imply a fairly thick lens, but its transmission coefficient through the two lens surfaces (normal incidence) is

$$
T \;=\; \frac{2\varepsilon_r^{-1/2}}{1+\varepsilon_r^{-1/2}}\,\frac{2}{1+\varepsilon_r^{-1/2}} \;=\; \frac{4\varepsilon_r^{1/2}}{\left[1+\varepsilon_r^{1/2}\right]^2} \;\simeq\; 0.96 \tag{3.6}
$$

Which is quite good. Also significant are the multiple reflections between the two lens surfaces which are small for small ε_r .

4. Prolate-Spheroidal Lens

Another kind of lens, appropriate to this problem, involves but a single lens surface with a "thick" lens. As indicated in Fig. 4.1 this has a prolate spheroidal boundary for launching a spherical wave from a medium of relative dielectric constant ε_r into a plane wave in a medium of relative dielectric constant 1.0. This is covered in detail in [7]. So we need not repeat the details here.

This type of lens has a useful advantage for high-voltage applications. For places where the electric field is large due to small spacing between conductors we can have polyethylene or transformer oil. Where the spacing is large the electric field is much lower allowing gas (air, SF_6) insulation.

As shown in [7] the transmission-line impedances are matched, the ray transit times are matched, and the transmission coefficient of the TEM mode is nearly unity.

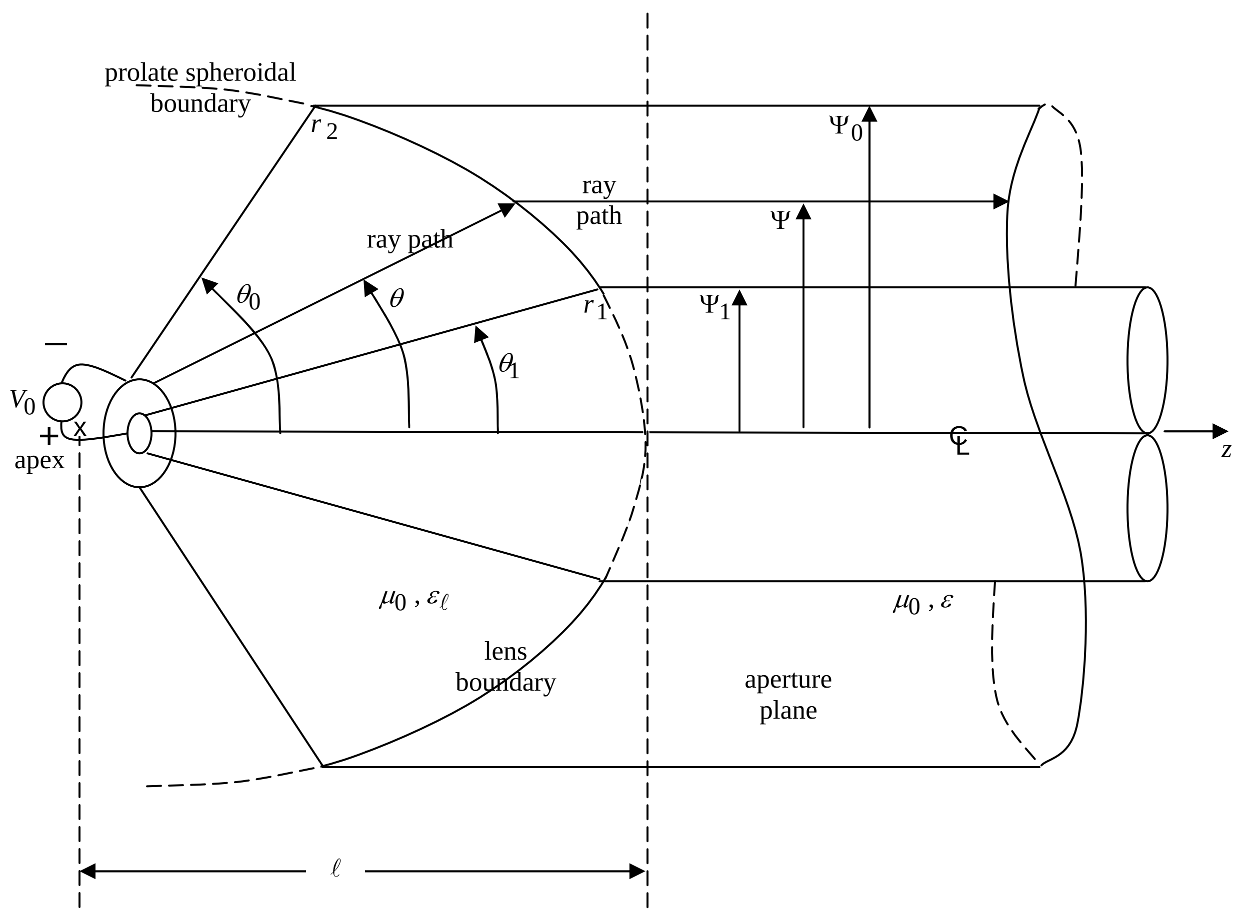


Fig. 4.1 Prolate Spheroidal Lens with Circular Conical Transmission Line Feeding Circular Coax

5. Termination of the Coaxial Transmission Line

With a high-voltage wave propagating over the transmission-line sensor we need to terminate this wave in some fashion. Depending on application, the return of a reflected wave back over the sensor to the source of the signal being measured may or may not pose a significant problem.

For a resistive termination one needs to consider the energy to be absorbed. There is also the issue of inductance between resistor strings. An approximate solution of the distributed terminator has been found [4, 9].

An alternate approach uses a smaller termination by focusing the TEM wave on the coax down into a smaller coaxial region where the terminator is placed. This is accomplished by reversing the procedure in Sections 3 and 4. One turns the problem around (by reciprocity) by sending the coaxial TEM wave into a lens which converts this to a TEM wave on a conical transmission line. At a suitably small cross-section dimension the conical structure can be connected to a coaxial one for further disposition of the signal, including termination.

6. Concluding Remarks

It would appear that it is possible to design a transmission-line voltage divider for large signal reduction, faithful pulse response, and a fast response time. This requires a careful consideration of the electromagnetic wave transport in the sensor structure.

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