Sensor and Simulation Notes

Note 502

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Tuning of High-Power Antenna Resonances by Appropriately Reactive Sources

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Abstract

In designing small high-power electromagnetic radiators (of the order of a half wavelength or so in size) based on switched resonant circuits, there are questions concerning the control of the resonance frequencies. This paper explores some techniques for tuning these frequencies based on the reactive properties of the source..

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1. Introduction

Begin with some antenna with

$$\tilde{Z}_a(s) = \tilde{Y}_a^{-1}(s) \equiv \text{input impedance}$$
$$s = \Omega + j\omega = \text{Laplace-transform variable or complex frequency} \tag{1.1}$$
$$\sim \; = \text{two-sided Laplace transform}$$

Let this be driven, as in Fig. 1.1, by

$$\tilde{Z}_s(s) = \tilde{Y}_s^{-1}(s) = \text{source impedance} \tag{1.2}$$

These two impedances are connected by a closing switch which we model by

$$\tilde{V}_{sw}(s) = \frac{V_0}{s}$$
$$V_0 = \text{charge voltage before switch closure} \tag{1.3}$$

In time domain this is a step function. We should remember that the switch does not close in zero time [4], and that this limits the performance at high frequencies.

If $\tilde{Z}_s(s) = 0$ then the antenna current (at the input terminals) is just

$$\tilde{I}_a(s) = \frac{V_0}{s}\tilde{Y}_a(s) \tag{1.4}$$

This typically exhibits resonant behavior at frequencies given by

$$s_a\tilde{Z}_a(s_a) = 0 \tag{1.5}$$

Neglecting $s_a = 0$ the contribution of a pole is given in time domain by

$$I_{aa}(t) = 2V_0 \,\mathrm{Re}\left(\left[s_a \frac{d}{ds}\tilde{Z}_a(s)\bigg|_{s=s_a}\right]^{-1} e^{s_a t}u(t)\right) \tag{1.6}$$

Where the conjugate pole is now included.

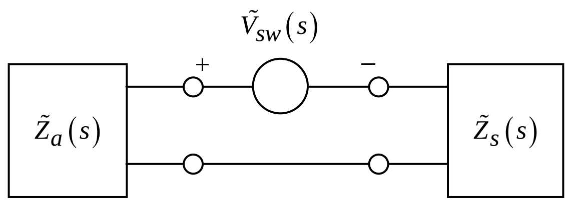

Fig. 1.1 Antenna and Source

2. General Considerations

Now consider the influence of the source impedance. This might be a simple capacitance C_s. However, at high frequencies $\tilde{Y}_s(s)$ may have more complex structure [5]. This nonzero $\tilde{Z}_s(s)$ then combines with $\tilde{z}_a(s)$ to shift the resonance frequencies.

The source impedance has resonances given by

$$\tilde{Z}_s(s_s) = 0 \tag{2.1}$$

When combined with the antenna impedance we have new natural frequencies given by

$$\tilde{Z}_a(s_m) + \tilde{Z}_s(s_m) = 0 \tag{2.2}$$

Then (1.6) is replaced for a single pole pair by

$$I_{am}(t) = 2V_0 \operatorname{Re}\left(\left[s_m \frac{d}{ds}\left[\tilde{Z}_a(s) + \tilde{Z}_s(s)\right]\Big|_{s=s_m}\right]^{-1} e^{s_m t} u(t)\right) \tag{2.3}$$

So our approach is to see how we might shift the antenna resonances in desirable directions. The factor

$$\left[s_m \frac{d}{ds}\left[\tilde{Z}_a(s) + \tilde{Z}_s(s)\right]\Big|_{s=s_m}\right]^{-1} \tag{2.4}$$

can be used (at least for high-Q resonances) as a scaling factor for the strength of the resonance.

3. Distributed Capacitive Source

For present purposes we need a model for the source impedance. Let us choose an open-circuited transmission line as indicated in Fig. 3.1. It might include a high-dielectric-constant medium with

$$\varepsilon = \varepsilon_r \varepsilon_0 \tag{2.4}$$

With a transit time t_r , the capacitance (low frequency) is just

$$\begin{aligned} C_s &= \frac{t_r}{Z_c} \\ Z_c &= \text{characteristic impedance of transmission line} \end{aligned} \tag{2.5}$$

The source impedance is then

$$\tilde{Z}_s(s) = Z_c \frac{1+e^{-2st_r}}{1-e^{-2st_r}} = Z_c \coth(st_r) \tag{2.6}$$

with open-circuit resonances at

$$\begin{aligned} \sinh\left(s_s' t_r\right) &= 0 \ , \quad \sin\left(\omega_s t_r\right) = 0 \\ \omega_s' t_r &= n\pi \ , \quad n = 0,\ 1,\ 2, \ldots \\ f_s' &= \frac{\omega_s'}{2\pi} = \frac{n}{2t_r} \end{aligned} \tag{2.7}$$

i.e., multiples of a half wavelength.

It is interesting to see at what frequencies the source has zero impedance (short-circuit resonances). These are

$$\begin{aligned} \coth\left(s_s\, t_r\right) &= 0 \ , \quad \cos\left(\omega_s\, t_r\right) = 0 \\ \omega_s\, t_r &= \frac{2n+1}{2}\pi \ , \quad n = 0,\ 1,\ 2, \ldots \\ f_s &= \frac{\omega_s}{2\pi} = \frac{2n+1}{4\, t_r} \end{aligned} \tag{2.8}$$

i.e., odd multiples of a quarter wavelength. One might choose the source then as having zero impedance at an antenna resonance so as to deliver a large voltage to the antenna.

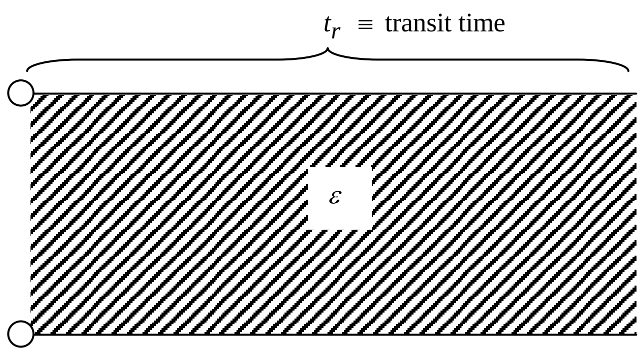

Fig. 3.1 Transmission-Line Capacitive Source

4. Combination With Magnetic Antenna

One type of electrically small antenna is a loop of some kind producing a magnetic-dipole moment. When operating in resonance condition there may be some appreciable fraction of a wavelength across the structure [1, 2]. Let us model the antenna impedance (up to first resonance of current) as

$$
\begin{aligned}
&\tilde{Z}_a(s) = \left[\frac{1}{sL_a} + sC_a\right]^{-1} \\
&L_a \equiv \text{low-frequency loop inductance} \\
&C_a \equiv \text{capacitive correction associated with leads into loop and stray capacitance of loop structure}
\end{aligned}
\tag{4.1}
$$

Note that this neglects the radiation resistance.

If the source is modeled as a simple capacitance C_s, this appears in series with $\tilde{Z}_a$ when driven by the source V_0 / s as

$$
\begin{aligned}
\tilde{Z}(s) &= \tilde{Z}_a(s) + \frac{1}{sC_g} = \left[\frac{1}{sL_a} + s\,C_a\right]^{-1} + \frac{1}{sC_s} \\
&= sL_a\left[\left[1 = s^2 L_a C_a\right]^{-1} + \frac{1}{s^2 L_a C_s}\right]
\end{aligned}
\tag{4.2}
$$

The resonance is then at

$$
\begin{aligned}
0 &= \left[1 - \omega_m^2 L_a C_a\right]^{-1} - \frac{1}{\omega_m^2 L_a C_s} \\
\omega_m^2 L_a C_s &= 1 - \omega_m^2 L_a C_s \\
\omega_m &= \left[L_a\left[C_a + C_s\right]\right]^{-1/2} = \left[L_a C_a\right]^{-1/2}\left[1 + \frac{C_s}{C_a}\right]^{-1/2}
\end{aligned}
\tag{4.3}
$$

Compared to the antenna resonance

$$
\omega_a = \left[L_a C_a\right] \tag{4.4}
$$

We see that

$$\omega_m < \omega_a \tag{4.5}$$

With equality if $C_s = 0$ (or $\tilde{Z}_s = \infty$). The effect of C_s is to *lower* the resonance frequency. Note that for infinite C_s, $\omega_m \to 0$.

At the same time the strength of the resonance is changed with the factor

$$\begin{aligned}
&\left[s_m \frac{d}{ds}\left[\tilde{Z}_a(s) + \tilde{Z}_s(s) \right]\Big|_{s=s_m} \right]^{-1} \\
&= \left[s_m \left[-\left[-\frac{1}{s_m^2 L_a} + C_a \right]\left[\frac{1}{s_m L_a} + s_m C_a \right]^{-2} - \frac{1}{s_m^2 C_s} \right]\right]^{-1} \\
&= -\frac{j}{\omega_m}\left[\left[-\frac{1}{\omega_m^2 L_a} + C_a \right]\omega_m^{-2}\left[-\frac{1}{\omega_m^2 L_a} + C_a \right]^{-2} + \frac{1}{\omega_m^2 C_s} \right]^{-1} \\
&= -\frac{j}{\omega_m}\left[\left[2C_a + C_s \right] L_a \left[C_a + C_s \right] C_s^{-2} + L_a\left[1 + \frac{C_a}{C_s} \right]\right]^{-1} \\
&= -\frac{jL_a}{\omega_m}\left[\left[1 + 2\frac{C_a}{C_s} \right]\left[1 + \frac{C_a}{C_s} \right] + 1 + \frac{C_a}{C_s} \right]^{-1} \\
&= -\frac{jL_a}{\omega_m}\frac{1}{2}\left[1 + \frac{C_a}{C_s} \right]^{-2} \\
&= -\frac{j}{2}\left[\frac{L_a}{C_a} \right]^{1/2}\left[1 + \frac{C_s}{C_a} \right]^{1/2}\left[1 + \frac{C_a}{C_s} \right]^{-2} \\
&= -\frac{j}{2}\left[\frac{L_a}{C_a} \right]^{1/2}\left[\frac{C_s}{C_a} \right]^{2}\left[1 + \frac{C_s}{C_a} \right]^{-3/2}
\end{aligned} \tag{4.6}$$

So smaller C_s *decreases* the resonant current (at the antenna port).

As $C_s \to \infty$ (zero-impedance source) this resonance has $\omega_m \to 0$, for which the antenna is zero-wavelengths long. Small C_s corresponds to a quarter wavelength. Let us consider a higher resonance corresponding to a half wavelength.

5. Transmission-Line Model of Loop and Source

Consider the case that both loop antenna and source are modeled as transmission lines as indicated in Fig. 5.1. Then we have for the antenna impedanace

$$\tilde{Z}_a(s) = Z_{ch}\frac{1-e^{-2st_1}}{1+e^{-2st_1}} = Z_{ch}\tanh(st_1) \tag{5.1}$$

For a zero-impedance source we have current resonances as

$$\begin{aligned} \sinh(s_a t_1) &= 0 \ , \ \sin(\omega_a t_1) = 0 \\ \omega_a t_1 &= n\pi \ , \ n = 0,1,2\ldots \\ f_a &= \frac{\omega_a}{2\pi} = \frac{n}{2t_1} \end{aligned} \tag{5.2}$$

which are multiples of a half wavelength.

A special simple case has

$$Z_c = Z_{ch} \tag{5.3}$$

with t_2 now the transit time of the source part. This is effectively a single transmission line of transit time, $t_1 + t_2$. With one end shorted and the other open, the first quarter-wave resonance is at

$$f_m = \frac{\omega_m}{2\pi} = \frac{1}{4[t_1+t_2]} \tag{5.4}$$

Here we see that shortening t_2 raises f_m, consistent with the previous result with lessened source capacitance. Here we can also see that as the switch approaches the right end of the transmission line, where the current in the natural mode is weakest, the strength of the resonance is also decreased.

The more general case has the resonance condition

$$Z_{ch}\frac{1-e^{-2s_m t_1}}{1+e^{-2s_m t_1}} + Z_c\frac{1+e^{-2s_m t_2}}{1-e^{-2s_m t_2}} = 0 \tag{5.5}$$

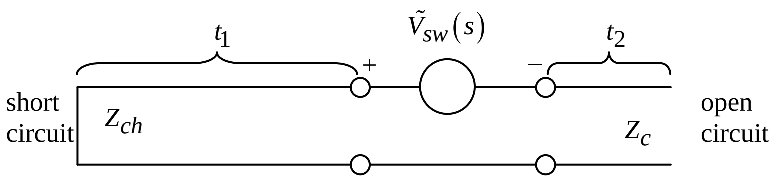

Fig. 5.1 Transmission-Line Model of Loop and Source

Note that for small Z_c we have

$$\begin{aligned} &\frac{Z_c}{Z_{ch}} \to 0 \\ &s_m \to s_a \;=\; j\omega_a \end{aligned} \tag{5.6}$$

as in (5.2).

Another special case has $t_1 = t_2$, for which we have

$$\begin{aligned} \left[1 - e^{-2 s_m t_1}\right]^2 \;+\; \frac{Z_c}{Z_{ch}}\left[1 + e^{-2 s_m t_1}\right]^2 \;&=\; 0 \\ \tanh^2\left(s_m t_1\right) \;&=\; -\frac{Z_c}{Z_{ch}} \\ \tan^2\left(\omega_m t_1\right) \;&=\; \frac{Z_c}{Z_{ch}} \\ \omega_m t_1 \;&=\; \arctan^{1/2}\left(\frac{Z_c}{Z_{ch}}\right) \end{aligned} \tag{5.7}$$

With additional solutions based on the periodicity of $\tan^2$.

The general case (5.5) is readily solved numerically for $\omega_m t_1$ or $\omega_m t_2$ as a function of Z_c / Z_{ch} and t_2 / t_1. By taking the derivative of $\tilde{Z}_a$ as in (5.1) one can also find a perturbation solution about ω_a as in Section 4.

6. Combination With Electric Antenna

Another type of electrically small antenna is an electric dipole of some kind, i.e., two separate conductors driven by some source between them, produding an electric dipole moment. Operated in resonance condition there may be some appreciable fraction of a wavelength across the structure [3]. Let us model the antenna impedance (up to first resonance of current) as

$$
\begin{aligned}
&\tilde{Z}_a(s) = \frac{1}{sC_a} + sL_a \\
&C_a \equiv \text{ low-frequency dipole capacitance} \\
&L_a \equiv \text{ inductive correction associated with leads into dipole and stray inductance of dipole structure}
\end{aligned}
\tag{6.1}
$$

Again this neglects the radiation resistance.

With the source modeled as a capacitance C_s, this appears in series with $\tilde{Z}_a$ when driven by the source V_g / s as

$$
\tilde{Z}(s) = \tilde{Z}_a(s) + \frac{1}{sC_s} = sL_a + \frac{1}{5}\left[\frac{1}{C_a} + \frac{1}{C_s}\right] \tag{6.2}
$$

The resonance is then at

$$
\begin{aligned}
0 &= \omega_m L_a - \frac{1}{\omega_m}\left[\frac{1}{C_a} + \frac{1}{C_s}\right] \\
\omega_m &= \left[\frac{1}{L_a}\left[\frac{1}{C_a} + \frac{1}{C_s}\right]\right]^{1/2}
\end{aligned}
\tag{6.3}
$$

Compared to the antenna resonance at

$$
\omega_a = \left[L_a \, C_a\right]^{-1/2} \tag{6.4}
$$

We see that

$$
\omega_m > \omega_a \tag{6.5}
$$

with equality if $C_S = \infty$ (or $\tilde{Z}_S = 0$). The effect of C_S is to *raise* the resonance frequency. For large C_S, the resonance corresponds to a quarter-wave resonance related to the source (or half wave on the two “dipole” conductors). For small C_S the result of (6.3) is unrealistic in that physically this should go to an open-circuit or half-wave resonance related to the source. For this case another model is appropriate.

The strength of the resonance is changed as

$$
\begin{aligned}
&\left[s_m \frac{d}{ds}\left[\tilde{Z}_a(s) + \tilde{Z}_S(s) \right]\Big|_{s=s_m} \right]^{-1} \\
&\quad = \left[s_m \left[L_a - \frac{1}{s_m^2}\left[\frac{1}{C_a} + \frac{1}{C_S} \right] \right] \right]^{-1} \\
&\quad = -j \left[\frac{1}{L_a}\left[\frac{1}{C_a} + \frac{1}{C_S} \right] \right]^{-1/2} \frac{1}{2L_a} \\
&\quad = -\frac{j}{2}\left[L_a \left[\frac{1}{C_a} + \frac{1}{C_S} \right] \right]^{-1/2}
\end{aligned}
\tag{6.6}
$$

So larger C_S *increases* the resonant current (at the antenna port).

7. Transmission-Line Model of Electric Antenna and Source

Model the electric antenna and source as transmission lines as indicated in Fig. 7.1. Note that this is topologically different from the loop case since both ends are open circuited. Now the antenna impedance is

$$\tilde{Z}_a(s) = Z_{ch}\frac{1+e^{-2st_1}}{1-e^{-2st_1}} = Z_{ch}\coth(st_1) \tag{7.1}$$

For a zero-impedance source we have current resonances at

$$\begin{aligned} \cosh(s_a t_1) &= 0 \ , \ \cos(\omega_a t_1) = 0 \\ \omega_a t_1 &= \frac{2n+1}{2}\pi \ , \ n = 0,1,2,\ldots \\ f_a &= \frac{\omega_a}{2\pi} = \frac{2n+1}{4t_1} \end{aligned} \tag{7.2}$$

which are odd multiples of a quarter wavelength.

For the special case of

$$Z_c = Z_{ch} \tag{7.3}$$

we have a half-wavelgneth resonant transmission line of transit time $t_1 + t_2$. This gives the lowest-order resonance at

$$f_m = \frac{\omega_m}{2\pi} = \frac{1}{2[t_1 + {}_2]} \tag{7.4}$$

For small t_2 this becomes a half wavelength on each antenna conductor. However, this also implies a small energy from the source.

The more general case has the resonance condition

$$Z_{ch}\frac{1+e^{-2s_m t_1}}{1-e^{-2s_m t_1}} + Z_c\frac{1+e^{-2s_m t_2}}{1-e^{-2s_m t_2}} = 0 \tag{7.5}$$

For small Z_c we have

$$\begin{aligned} &\frac{Z_c}{Z_{ch}} \rightarrow 0 \\ &s_m \rightarrow s_a = j\omega_a \end{aligned} \tag{7.6}$$

as in (7.2).

Another special case has $t_1 = t_2$, for which we have

$$\begin{aligned} &\sinh(2s_m t_1) + \frac{Z_c}{Z_{ch}} \sinh(2s_m t_1) = 0 \\ &\sinh(2s_m t_1) = 0 \ , \ \sinh(2\omega_m t_1) = 0 \\ &\omega_m t_1 = \frac{n\pi}{2} \ , \ n = 0,1,2,\ldots \\ &f_m = \frac{\omega_m}{2\pi} = \frac{n}{4t_1} \end{aligned} \tag{7.7}$$

The first nonzero resonance is then when each antenna conductor is a quarter-wavelength long. Note also that small Z_c means more stored energy in the source, giving a larger resonance current.

The general case (7.5) is also readily solved numerically.

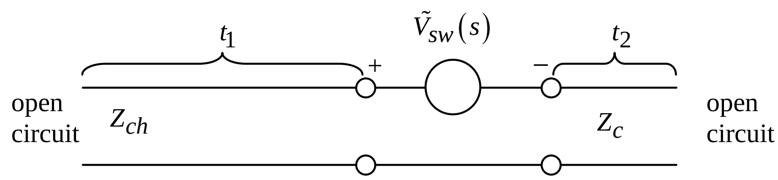

Fig. 7.1 Transmission-Line Model of Electric Antenna and Source

8. Concluding Remarks

As we can see, judicious choice of the frequency dependence of the source impedance can alter the resonance frequency and resonance strength of the antenna, whether of loop or electric-dipole type. Here we have chosen some simple forms of the source impedance for illustration. More elaborate forms can also be pursued.

References

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