Sensor and Simulation Notes

Note 483

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Modes of a Double Baffled, Cylindrical, Coaxial Waveguide

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ABSTRACT

There is considerable interest in antenna and transmission line structures that are conformal to curved and cylindrical surfaces. The double-baffled, coaxial transmission line is defined by inner and outer radii, and an arc length. It is conformal to curved surfaces, particularly structures cylindrical in nature. In this note we derive the TE and TM, axially propagating modes of a double-baffled, coaxial transmission line. First, the characteristic equations that define the cut off frequencies of each mode are derived, then the electric fields are explicitly expressed. Finally, an example double-baffled, coaxial transmission line geometry is defined for which the lowest TE and TM mode cutoff frequencies are computed and graphs of the normalized field components are presented.

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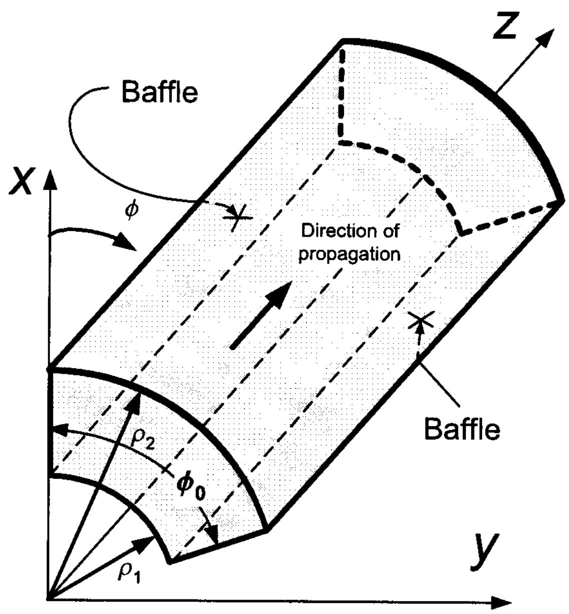
1. Introduction

The double-baffled, coaxial transmission line is defined by inner and outer radii, and an arc length, and can be conformal to curved surfaces and cylindrical structures. This note describes the propagating modes of a coaxial waveguide transmission line with two baffles, with propagation assumed in the z-direction. First, the characteristic equations that define the cut off frequencies of each mode are derived, then the electric fields are explicitly expressed. Finally, an example geometry is defined for which the lowest TE and TM mode cutoff frequencies are computed and graphs of the normalized field components are presented.

2. Geometry

The geometry of the double baffled, coaxial waveguide transmission line is shown in Figure 1. Note that the arc between the baffles has an angular extension of $\varphi = \varphi_0$.



(a)

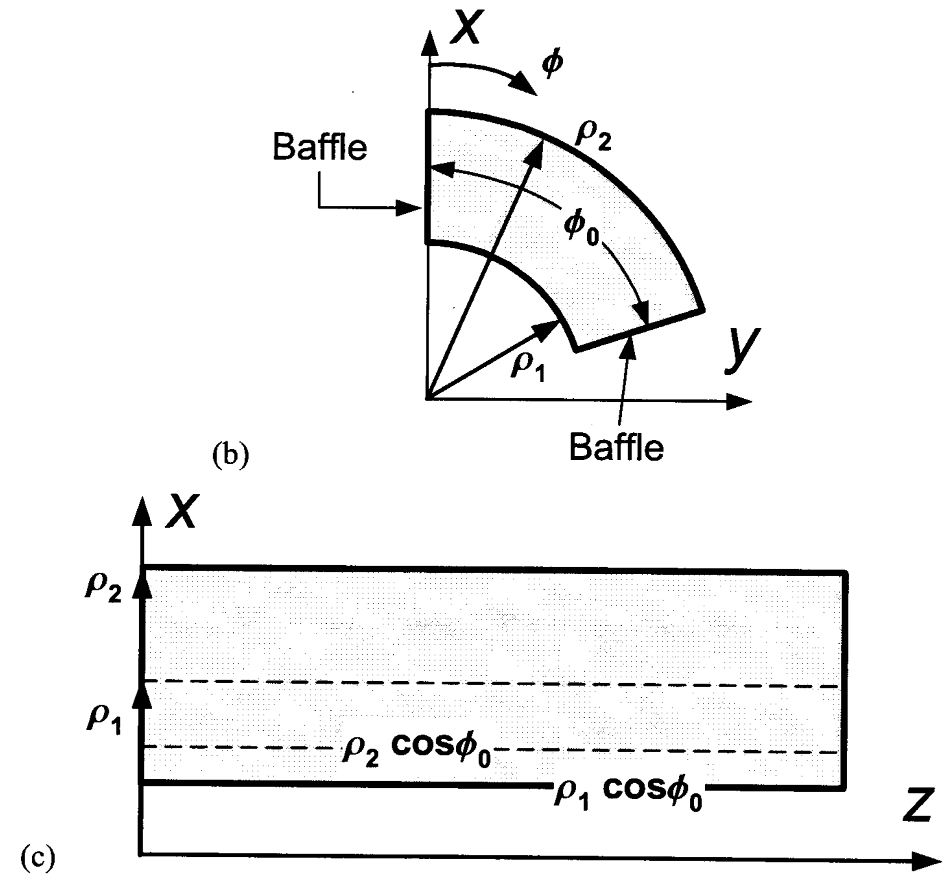


Figure 1. **The geometry of the double baffled cylindrical coaxial waveguide: (a) 3-D perspective drawing; (b) plane view o the xy-plane; and (c) plane view of the xz-plane.**

3. Wave Equation

The natural coordinate system for the coaxial waveguide transmission line with two baffles is the cylindrical coordinate system. The scalar Helmholtz wave equation in cylindrical coordinates is

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi(\rho,\varphi,z)}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\psi(\rho,\varphi,z)}{\partial\varphi^2}+\frac{\partial^2\psi(\rho,\varphi,z)}{\partial z^2}+k^2\psi(\rho,\varphi,z)=0 \quad (1)$$

Using standard separation of variable techniques the wave equation can be written as

$$\rho\frac{d}{d\rho}\left(\rho\frac{dR(\rho)}{d\rho}\right)+\left[\left(k_\rho\rho\right)^2-n^2\right]R(\rho)=0 \quad (2a)$$

$$\frac{d^2}{d\varphi^2}\Phi(\varphi)+n^2\Phi(\varphi)=0 \tag{2b}$$

$$\frac{d^2}{dz^2}Z(z)+k_z^2Z(z)=0 \tag{2c}$$

where: $\psi(\rho,\varphi,z)=R(\rho)\Phi(\varphi)Z(z)$, and $k_\rho^2+k_z^2=k^2$.

4. Boundary Conditions

The boundary conditions for the coaxial waveguide transmission line with two baffles are:

$$E_\rho=0 \text{ for } \varphi=0\text{, and } \varphi=\varphi_0 \tag{3a}$$

$$E_\varphi=0 \text{ for } \rho=\rho_1\text{, and } \rho=\rho_2 \tag{3b}$$

$$E_z=0 \text{ for } \rho=\rho_1\text{, and } \rho=\rho_2\text{, and } \varphi=0\text{, and } \varphi=\varphi_0. \tag{3c}$$

5. Solution of the Separated Wave Equation

The $\Phi(\varphi)$ and $Z(z)$ equations are harmonic equations with harmonic functions as solutions; these will be denoted $h(n\varphi)$ and $h(k_z z)$.

The equation in $R(\rho)$ is a Bessel equation, and has Bessel function solutions:

$J_n(k_\rho\rho)=$ the Bessel function of the first kind of order n
$N_n(k_\rho\rho)=$ the Bessel function of the second kind of order n
$H_n^{(1)}(k_\rho\rho)=$ the Hankel function of the first kind of order n
$H_n^{(2)}(k_\rho\rho)=$ the Hankel function of the second kind of order n

Let the function $B_n(k_\rho\rho)$ represent the linearly independent combination of two of the above. Then, the general solution to the scalar Helmholtz wave equation is:

$$\psi_{k_\rho,n,k_z}=B_n(k_\rho\rho)h(n\varphi)h(k_z z) \tag{4}$$

6. TE_z and TM_z Field Components

The electric and magnetic field components can be written in terms of fields that are TE_z and TM_z.

6.1 TM_z Field Components

The TM_z field components are found by letting $\mathbf{A} = \mathbf{u}_z\psi$, where $\mathbf{A} =$ the magnetic vector potential, and $\mathbf{u}_z =$ unit vector in the z-direction. Then

$$\mathbf{E} = -j\omega\mathbf{A} + \frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{A}), \tag{5a}$$

$$\text{and } \mathbf{H} = \frac{1}{\mu}\nabla\times\mathbf{A}. \tag{5b}$$

When expanded in cylindrical coordinates these equations become:

$$E_\rho = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2\psi}{\partial\rho\,\partial z} \tag{6a}$$

$$E_\varphi = \frac{1}{j\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial^2\psi}{\partial\varphi\,\partial z} \tag{6b}$$

$$E_z = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial z^2} + k^2\right)\psi \tag{6c}$$

$$H_\rho = \frac{1}{\mu}\frac{1}{\rho}\frac{\partial\psi}{\partial\varphi} \tag{6d}$$

$$H_\varphi = -\frac{1}{\mu}\frac{\partial\psi}{\partial\rho} \tag{6e}$$

$$H_z = 0 \tag{6f}$$

6.2 TE_z Field Components

The TE_z field components are found by letting $\mathbf{F} = \mathbf{u}_z\psi$, where $\mathbf{F} =$ the electric vector potential, and $\mathbf{u}_z =$ unit vector in the z-direction. Then

$$\mathbf{E} = -\frac{1}{\varepsilon}\nabla\times\mathbf{F}, \tag{7a}$$

$$\text{and } \mathbf{H} = -j\omega\mathbf{F} + \frac{1}{j\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{F}). \tag{7b}$$

When expanded in cylindrical coordinates these TE_z field equations become:

$$E_\rho = -\frac{1}{\varepsilon}\frac{1}{\rho}\frac{\partial\psi}{\partial\varphi} \tag{8a}$$

$$E_\varphi = \frac{1}{\varepsilon}\frac{\partial\psi}{\partial\rho} \tag{8b}$$

$$H_\rho = \frac{1}{j\omega\mu\varepsilon}\frac{\partial^2\psi}{\partial\rho\,\partial z} \tag{8d}$$

$$H_\varphi = \frac{1}{j\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial^2\psi}{\partial\varphi\,\partial z} \tag{8e}$$

$E_z = 0$ (8c) $H_z = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial z^2} + k^2\right)\psi$ (8f)

7. Solution of the Separated Wave Equation Subject to the Boundary Conditions of the Generalized Geometry

Propagating waves in the z-direction in the double baffled coaxial waveguide give rise to

$$h(k_z z) = e^{-jk_z z} \tag{9}$$

The harmonic function $h(n\varphi)$ can be written as

$$h(n\varphi) = a_n \sin(n\varphi) + b_n \cos(n\varphi) \tag{10}$$

Note that n is not necessarily an integer. The scalar wave function is then

$$\psi_{k_\rho,n,k_z} = B_n(k_\rho \rho)h(n\varphi)e^{-jk_z z} \tag{11}$$

subject to the boundary conditions. The solutions for the TE_z and TM_z modes in the guide are as follows.

7.1 TM_z Field Components

The TM_z electric field components in terms of the wave function are

$$E_\rho = \frac{(-jk_z)(k_\rho)}{j\omega\mu\varepsilon} B_n'(k_\rho \rho)h(n\varphi)e^{-jk_z z} \tag{12a}$$

$$E_\varphi = \frac{(-jk_z)(n)}{j\omega\mu\varepsilon}\frac{1}{\rho} B_n(k_\rho \rho)h'(n\varphi)e^{-jk_z z} \tag{12b}$$

$$E_z = \frac{1}{j\omega\mu\varepsilon}\left(k^2 - k_z^2\right) B_n(k_\rho \rho)h(n\varphi)e^{-jk_z z} \tag{12c}$$

Since

$E_z = 0$ for $\rho = \rho_1$, and $\rho = \rho_2$, and $\varphi = 0$, and $\varphi = \varphi_0$;

then

$$h(n\varphi)\,|_{\varphi=0,\varphi_0} = (a_n \sin(n\varphi) + b_n \cos(n\varphi))\,|_{\varphi=0,\varphi_0} = 0$$

is satisfied if: $a_n = 1$, $b_n = 0$, $n = \frac{m\pi}{\varphi_0}$, and $m = 1,2,3,\ldots.$ (13)

Note that $B_n'(k_\rho\rho) = \dfrac{d}{d(k_\rho\rho)} B_n(k_\rho\rho)$. The general Bessel function, $B_n(k_\rho\rho)$, also satisfies the boundary conditions if

$$B_n(k_\rho\rho)\,|_{\rho=\rho_1,\rho_2} = 0$$

Let $B_n(k_\rho\rho) = a_n J_n(k_\rho\rho) + b_n N_n(k_\rho\rho)$, then $a_n J_n(k_\rho\rho) + b_n N_n(k_\rho\rho)\,|_{\rho=\rho_1,\rho_2} = 0$. And,

$$a_n J_n(k_\rho\rho_1) + b_n N_n(k_\rho\rho_1) = 0$$
$$a_n J_n(k_\rho\rho_2) + b_n N_n(k_\rho\rho_2) = 0$$

Solving the first equation for a_n:

$$a_n = b_n \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)} \tag{14}$$

Substitution into the second equation yields:

$$a_n J_n(k_\rho\rho_2) + b_n N_n(k_\rho\rho_2) = -b_n \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)} J_n(k_\rho\rho_2) + a_n N_n(k_\rho\rho_2) = 0$$

Or,

$$b_n \left(N_n(k_\rho\rho_2) - \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)} J_n(k_\rho\rho_2) \right) = 0$$

For specific values of n, ρ_1 and ρ_2, the values of k_ρ that solve

$$\frac{N_n(k_\rho\rho_2)}{J_n(k_\rho\rho_2)} = \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)} \tag{15}$$

are the sought after mode numbers that are true for any non-zero value of b_n. Hence,

$b_n = 1$ and $a_n = -\dfrac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)}$. Finally, the scalar wave function for the TM$_z$ modes is:

$\psi_{k_\rho,n,k_z} = \left[N_n(k_\rho\rho) - \dfrac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)} J_n(k_\rho\rho) \right] \sin(n\varphi) e^{-jk_z z}$, for $n = \dfrac{m\pi}{\varphi_0}$, $m = 1, 2, 3, \ldots$, and $k_\rho^2 + k_z^2 = k^2$.

The convention for the zeros of the Characteristic Equation is $p = p_1, p_2, p_3, \ldots$, where the p_1 is the first zero solution, p_2 is the second solution (with increasing numerical value, and so forth). The TM_z field components are then found explicitly as:

$$E_\rho = \frac{-k_\rho k_z}{\omega\mu\varepsilon}\sin(n\varphi)\left[N_n'(k_\rho\rho) - \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)}J_n'(k_\rho\rho)\right]e^{-jk_z z} \tag{16a}$$

$$E_\varphi = -\frac{k_z n}{\omega\mu\varepsilon}\frac{1}{\rho}\left[N_n(k_\rho\rho) - \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)}J_n(k_\rho\rho)\right]\cos(n\varphi)e^{-jk_z z} \tag{16b}$$

$$E_z = \frac{k^2 - k_z^2}{j\omega\mu\varepsilon}\left[N_n(k_\rho\rho) - \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)}J_n(k_\rho\rho)\right]\sin(n\varphi)e^{-jk_z z} \tag{16c}$$

$$H_\rho = \frac{1}{\mu}\frac{n}{\rho}\left[N_n(k_\rho\rho) - \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)}J_n(k_\rho\rho)\right]\cos(n\varphi)e^{-jk_z z} \tag{16d}$$

$$H_\varphi = -\frac{k_\rho}{\mu}\left[N_n'(k_\rho\rho) - \frac{N_n(k_\rho\rho_1)}{J_n(k_\rho\rho_1)}J_n'(k_\rho\rho)\right]\sin(n\varphi)e^{-jk_z z} \tag{16e}$$

$$H_z = 0 \tag{16f}$$

7.2 TE_z Field Components

The TE_z electric field components in terms of the wave function are:

$$E_\rho = -\frac{1}{\varepsilon}\frac{1}{\rho}B_n(k_\rho\rho)\frac{d}{d\varphi}h(n\varphi)e^{-jk_z z} \tag{17a}$$

$$E_\varphi = \frac{1}{\varepsilon}\frac{d}{d\rho}B_n(k_\rho\rho)h(n\varphi)e^{-jk_z z} \tag{17b}$$

$$E_z = 0 \tag{17c}$$

Since

$E_\rho = 0$ for $\varphi = 0$, and $\varphi = \varphi_0$

$E_\varphi = 0$ for $\rho = \rho_1$, and $\rho = \rho_2$

then

$$\frac{d}{d\varphi}h(n\varphi)\,|_{\varphi=0,\varphi_0} = \frac{d}{d\varphi}(a_n \sin(n\varphi) + b_n \cos(n\varphi))\,|_{\varphi=0,\varphi_0}$$
$$= n(a_n \cos(n\varphi) - b_n \sin(n\varphi))\,|_{\varphi=0,\varphi_0} = 0$$

is satisfied if: $a_n = 0$, $b_n = 1$, $n = \dfrac{m\pi}{\varphi_0}$, and $m = 1,2,3,\ldots$ (18)

The general Bessel function, $B_n(k_\rho \rho)$, also satisfies the boundary conditions if

$$\frac{d}{d\rho}B_n(k_\rho \rho)\,|_{\rho=\rho_1,\rho_2} = 0$$

Let $B_n(k_\rho \rho) = a_n J_n(k_\rho \rho) + b_n N_n(k_\rho \rho)$, then $\frac{d}{d\rho}\{a_n J_n(k_\rho \rho) + b_n N_n(k_\rho \rho)\}\,|_{\rho=\rho_1,\rho_2} = 0$. And,

$$a_n J_n'(k_\rho \rho_1) + b_n N_n'(k_\rho \rho_1) = 0$$
$$a_n J_n'(k_\rho \rho_2) + b_n N_n'(k_\rho \rho_2) = 0$$

Solving the first equation for a_n: $\quad a_n = -b_n \dfrac{N_n'(k_\rho \rho_1)}{J_n'(k_\rho \rho_1)}$ (19)

Substitution into the second equation yields:

$$a_n J_n'(k_\rho \rho_2) + b_n N_n'(k_\rho \rho_2) = -b_n \frac{N_n'(k_\rho \rho_1)}{J_n'(k_\rho \rho_1)} J_n'(k_\rho \rho_2) + b_n N_n'(k_\rho \rho_2) = 0$$

Or,

$$b_n \left(N_n'(k_\rho \rho_2) - \frac{N_n'(k_\rho \rho_1)}{J_n'(k_\rho \rho_1)} J_n'(k_\rho \rho_2) \right) = 0$$

For specific values of n, ρ_1 and ρ_2, the values of k_ρ that solve

$$\frac{N_n'(k_\rho \rho_2)}{J_n'(k_\rho \rho_2)} = \frac{N_n'(k_\rho \rho_1)}{J_n'(k_\rho \rho_1)} \tag{20}$$

are the sought after mode numbers that are true for any non-zero value of b_n. Hence,

$b_n = 1$ and $a_n = -\dfrac{N_n'(k_\rho \rho_1)}{J_n'(k_\rho \rho_1)}$.

Finally, the scalar wave function for the TE$_z$ modes is:

$$\psi_{k_\rho,n,k_z} = \left[N_n(k_\rho \rho) - \frac{N'_n(k_\rho \rho_1)}{J'_n(k_\rho \rho_1)} J_n(k_\rho \rho) \right] \cos(n\varphi) e^{-jk_z z}$$, for $n = \frac{m\pi}{\varphi_0}$, $m = 1,2,3,\ldots$, and $k_\rho^2 + k_z^2 = k^2$.

Again, the convention for the zeros of the TE Characteristic Equation is $p = p_1, p_2, p_3, \ldots$, where the p_1 is the first solution, p_2 is the second solution (with increasing numerical value, and so forth. The TE$_z$ field components are then found explicitly as:

$$E_\rho = \frac{n}{\varepsilon} \frac{1}{\rho} \left[N_n(k_\rho \rho) - \frac{N'_n(k_\rho \rho_1)}{J'_n(k_\rho \rho_1)} J_n(k_\rho \rho) \right] \sin(n\varphi) e^{-jk_z z} \tag{21a}$$

$$E_\varphi = \frac{k_\rho}{\varepsilon} \left[N'_n(k_\rho \rho) - \frac{N'_n(k_\rho \rho_1)}{J'_n(k_\rho \rho_1)} J'_n(k_\rho \rho) \right] \cos(n\varphi) e^{-jk_z z} \tag{21b}$$

$$E_z = 0 \tag{21c}$$

$$H_\rho = \frac{-k_z k_\rho}{\omega\mu\varepsilon} \left[N'_n(k_\rho \rho) - \frac{N'_n(k_\rho \rho_1)}{J'_n(k_\rho \rho_1)} J'_n(k_\rho \rho) \right] \cos(n\varphi) e^{-jk_z z} \tag{21d}$$

$$H_\varphi = \frac{k_z}{\omega\mu\varepsilon} \frac{n}{\rho} \left[N_n(k_\rho \rho) - \frac{N'_n(k_\rho \rho_1)}{J'_n(k_\rho \rho_1)} J_n(k_\rho \rho) \right] \sin(n\varphi) e^{-jk_z z} \tag{21e}$$

$$H_z = \frac{k_\rho^2}{j\omega\mu\varepsilon} \left[N_n(k_\rho \rho) - \frac{N'_n(k_\rho \rho_1)}{J'_n(k_\rho \rho_1)} J_n(k_\rho \rho) \right] \cos(n\varphi) e^{-jk_z z} \tag{21f}$$

Note that the Characteristic Equations for the TE and TM mode are similar, but not exact, to the forms given in [Ref. 2]. The reason for the differences is at this time unknown.

8. EXAMPLE

Determine the first few cutoff frequencies of the TE and TM modes of a Double Baffled, Cylindrical, Coaxial Waveguide defined by the parameters: $\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6\,in = 0.1524\,m$ and $\varphi_0 = 2\pi/3$.

The cutoff frequencies for the first few the TE and TM modes have been computed and are presented in Table 1 and Table 2 below.

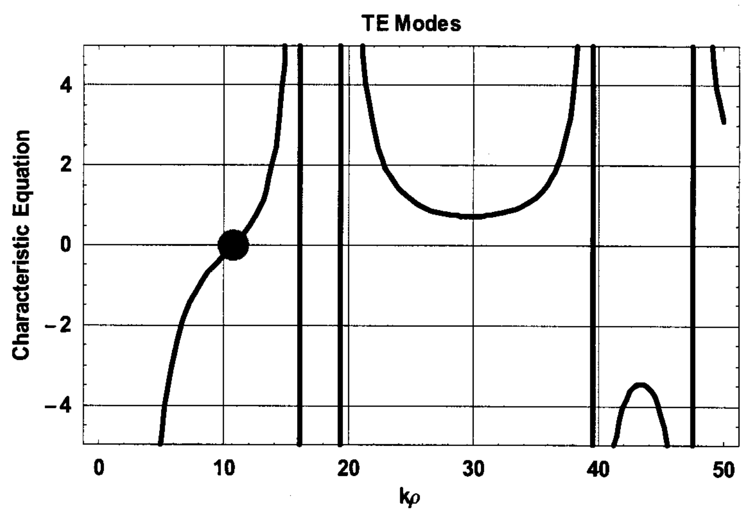
Table 1. Cutoff frequencies (GHz) of the TE modes of a double baffled, coaxial waveguide ($\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6\,in = 0.1524\,m$ and $\varphi_0 = 2\pi/3$).

m / p	1	2	3
1	0.513006	5.93146	11.8179
2	1.02589	5.99859	11.8516
3	1.53851	6.10889	11.9074

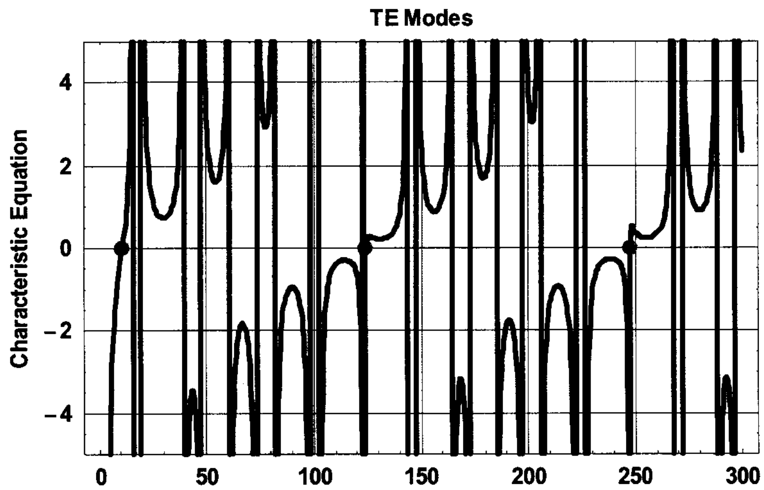
Table 2. Cutoff frequencies (GHz) of the TM modes of a double baffled, coaxial waveguide ($\rho_1 = 5\,in = 0.127\,m, \rho_2 = 6\,in = 0.1524\,m$ and $\varphi_0 = 2\pi/3$).

m / p	1	2	3
1	5.92129	11.8129	17.7111
2	5.98762	11.8464	17.7335
3	6.09656	11.9021	17.7707

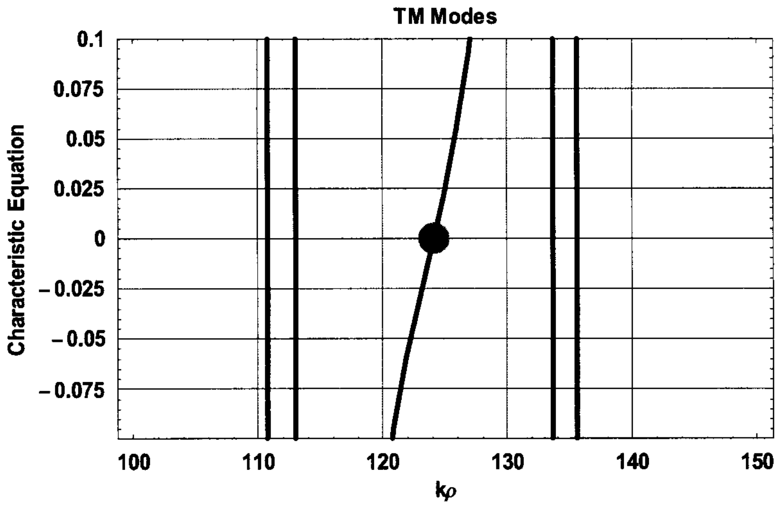
The characteristic equations for the TE_{11} and TM_{11} cases are plotted in Figure 2a and c. The characteristic equation for the TE_{1p} mode is plotted in Figure 2b showing the first 3 roots that characterize the first 3 modes. Note that the radial gap between the conductors is 1-inch, about ½ of the freespace wavelength of the 1st TM cutoff frequency. And the median arc length between the inner and outer radii is 11.52 inches, about ½ of the freespace wavelength of the 1st TE cutoff frequency.



(a)



(b)



(b)

Figure 2. **Plots of the Characteristic Equation for the double baffled, coaxial waveguide transmission line (n = 1): (a) TE_{11} mode; (b) TE_{1p} , for $p = 1,2,3$ modes; and (c) TM_{11} mode.**

Plot the electric field components over a cross section of the guide at a frequency that is 1.2 times the cutoff frequency of the lowest mode.

For $m = 1$, $n = \dfrac{m\pi}{\varphi_0} = \dfrac{\pi}{2\pi/3} = 1.5$. For the TM_{11} mode: $f_c = 5.921\,\text{GHz}$.

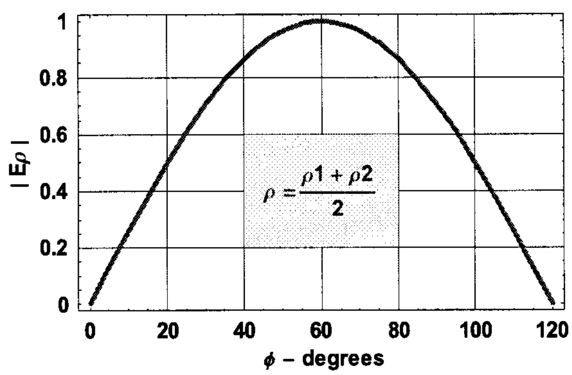
For $m = 1$, $n = \dfrac{m\pi}{\varphi_0} = \dfrac{\pi}{2\pi/3} = 1.5$. For the TE_{11} mode: $f_c = 513.00\,\text{MHz}$.

Then, compute the field distributions at $f = 1.2 \times f_c = 1.2 \times 513.00 = 615.6$ MHz and $f = 1.2 \times f_c = 1.2 \times 5.921 = 7.105$ GHz.

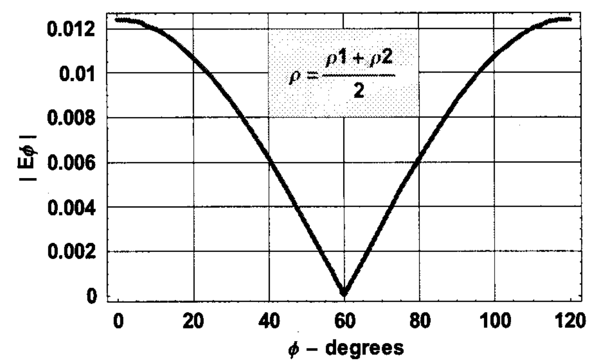
The wavelength at the operating frequency of the guide for the TE_{11} mode, $f = 615.6$ MHz, in the axial direction of the guide, λ_g, is defined as $\lambda_g = \frac{2\pi}{k_z} = \frac{2\pi}{7.13159} = 0.881036$ meters, where $k_z = \sqrt{k^2 - k_\rho^2}$. The wavelength at the operating frequency of the guide for the TM_{11} mode, $f = 7.105$ GHz, in the axial direction of the guide, λ_g, is $\lambda_g = \frac{2\pi}{k_z} = \frac{2\pi}{82.2978} = 0.076347$ meters.

Normalized distributions of the electric field components of the TE_{11} mode of the double baffled coaxial waveguide for $\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6.0\,in = 0.1524\,m$, $\varphi_0 = 2\pi/3$, and $f = 615.6$ MHz are shown in Figure 3(a) $E_\rho((\rho_1+\rho_2)/2,\varphi)$; (b) $E_\varphi((\rho_1+\rho_2)/2,\varphi)$, and $E_z = 0$. Normalized distributions of the magnetic field components are shown in Figure 4. Since $\mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}$, we can plot the current density on the interior surfaces of the waveguide. Shown in Figure 5 is a vector plot of the current density of the TE_{11} mode at $f = 615.6$ MHz on the $\rho = \rho_2$, $0 < \varphi < \varphi_0 = 2\pi/3$ surface.

Normalized distributions of the electric field components of the TM_{11} mode of the double baffled coaxial waveguide for $\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6.0\,in = 0.1524\,m$, $\varphi_0 = 2\pi/3$, and $f = 7.105$ GHz are shown in Figure 6(a) $E_\rho((\rho_1+\rho_2)/2,\varphi)$; (b) $E_\varphi((\rho_1+\rho_2)/2,\varphi)$; and (c) $E_z((\rho_1+\rho_2)/2,\varphi)$.



(a)



(b)

Figure 3. **Normalized distributions of the electric field components of the TE_{11} mode of the double baffled coaxial waveguide ($\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6.0\,in = 0.1524\,m$, $\varphi_0 = 2\pi/3$, and $f = 615.6$ MHz): (a) $E_\rho((\rho_1+\rho_2)/2,\varphi)$; and (b) $E_\varphi((\rho_1+\rho_2)/2,\varphi)$. $E_z = 0$.**

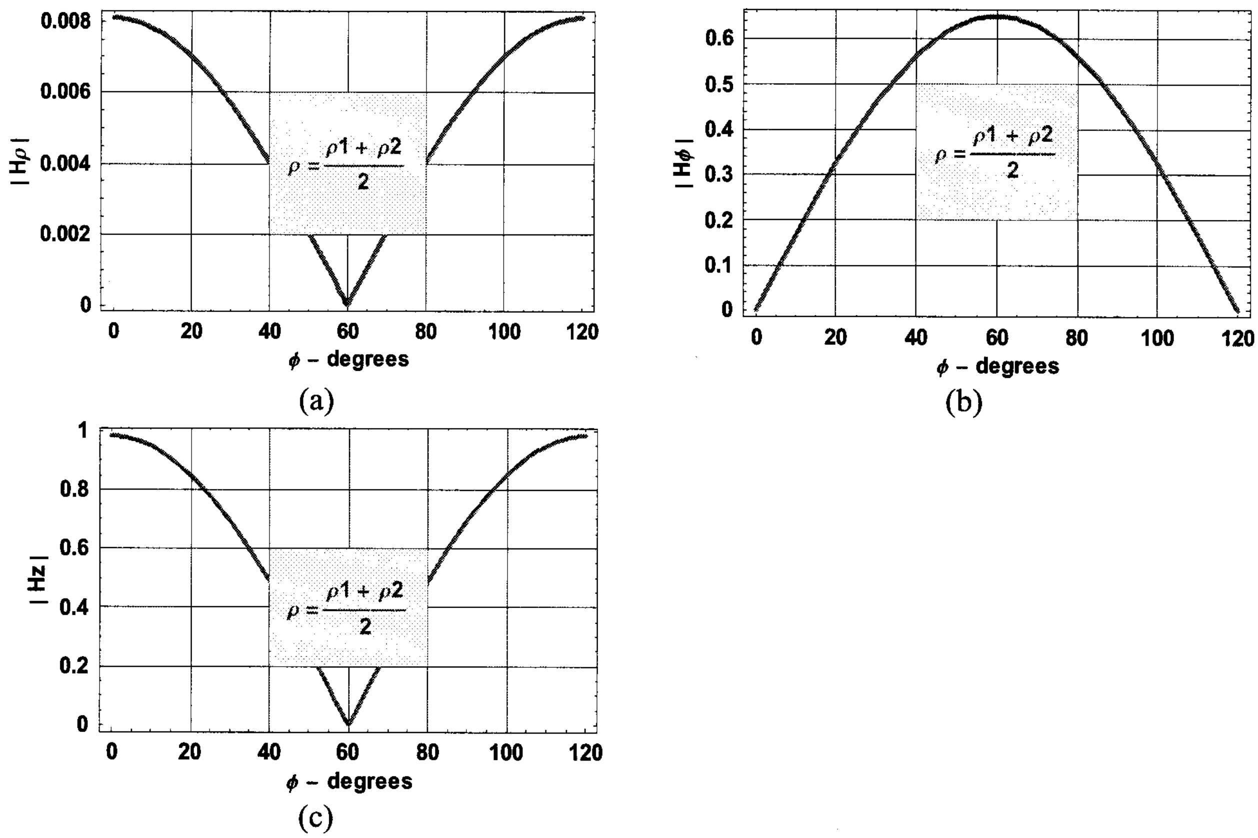


Figure 4. **Normalized distributions of the magnetic field components of the TE$_{11}$ mode of the double baffled coaxial waveguide ($\rho_1 = 5in$, $\rho_2 = 6.0in$, $\varphi_0 = 2\pi/3$, and $f = 615.6$ MHz): (a) H_ρ; (b) H_φ; and (c) H_z.**

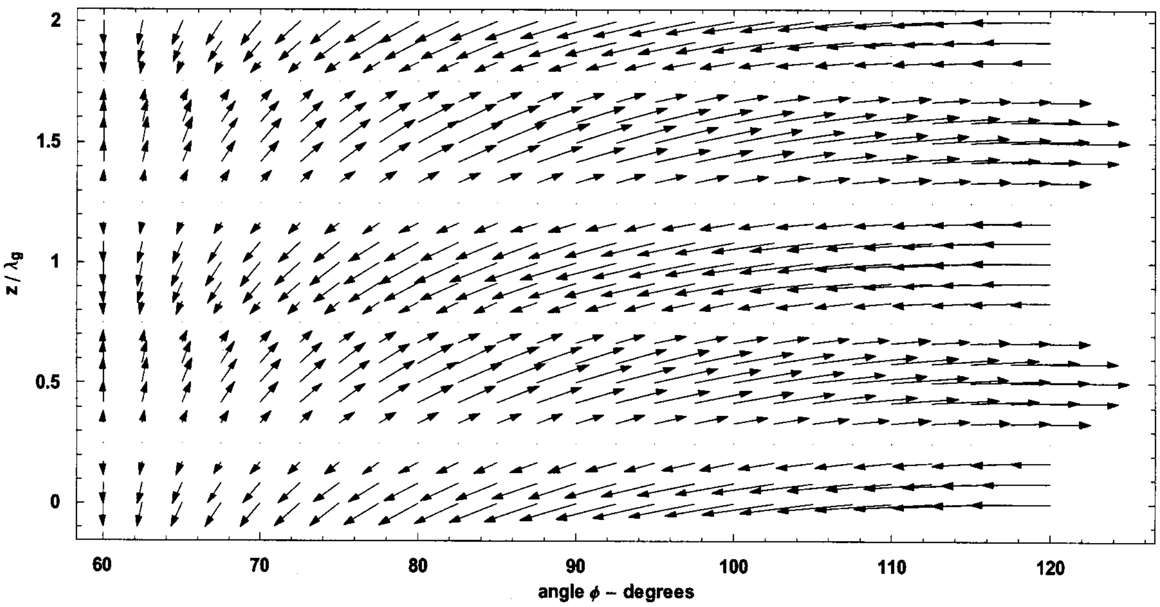


Figure 5. **Vector plot of the current density of the TE$_{11}$ mode at $f = 615.6$ MHz on the $\rho = \rho_2$, $0 < \varphi < \varphi_0 = 2\pi/3$ surface.**

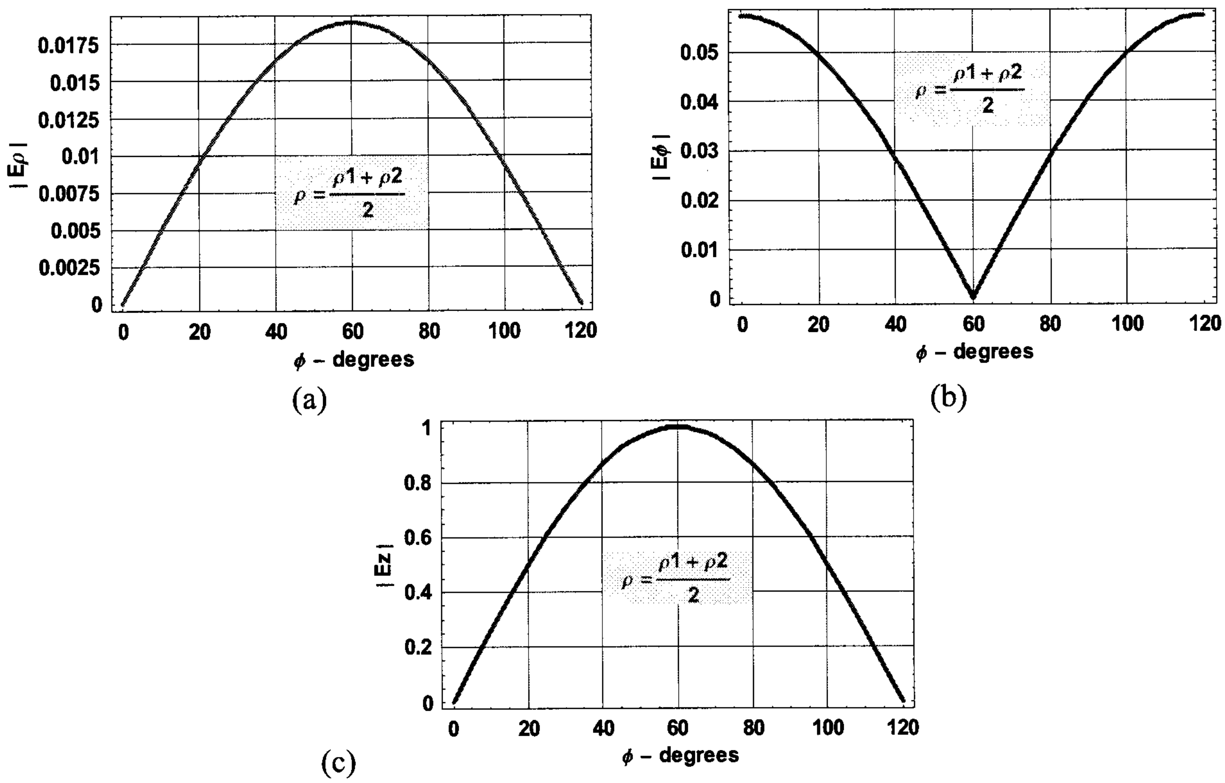


Figure 6. **Normalized distributions of the electric field components of the TM$_{11}$ mode of the double baffled coaxial waveguide ($\rho_1 = 5\,in$, $\rho_2 = 6\,in$, $\varphi_0 = 2\pi/3$, and $f = 7.105$ GHz): (a) E_ρ; (b) E_φ; and (c) E_z.**

Contour and 3D projection graphs of the normalized distributions of the electric field components of the TM11 modes of the double baffled coaxial waveguide for $\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6.0\,in = 0.1524\,m$, $\varphi_0 = 2\pi/3$, and $f = 7.105$ GHz) are shown in Figure 7. Likewise, plots of the normalized distributions of the electric field components of the TE$_{11}$ mode of the double baffled coaxial waveguide are shown in Figure 8 for $f = 615.6$ MHz

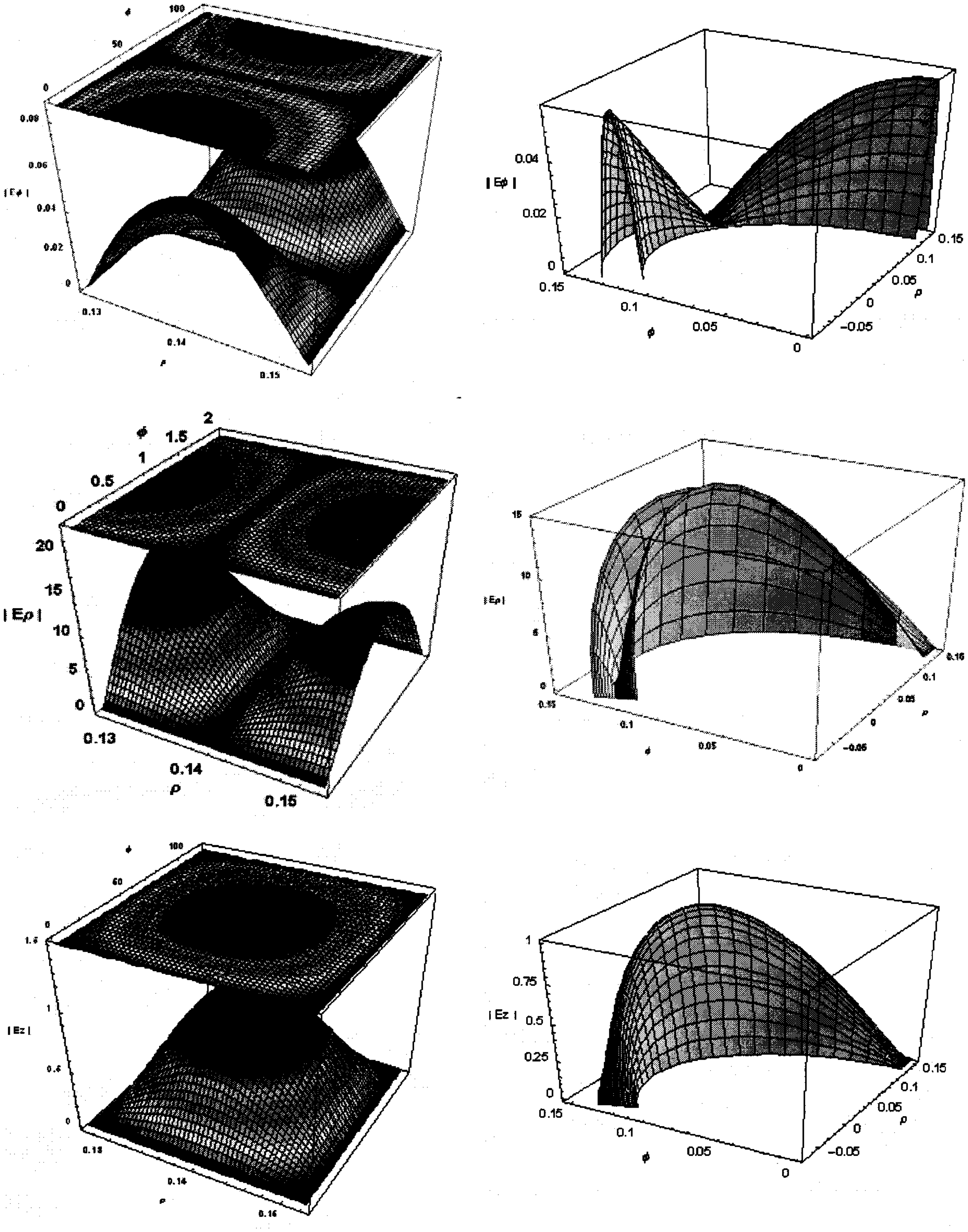
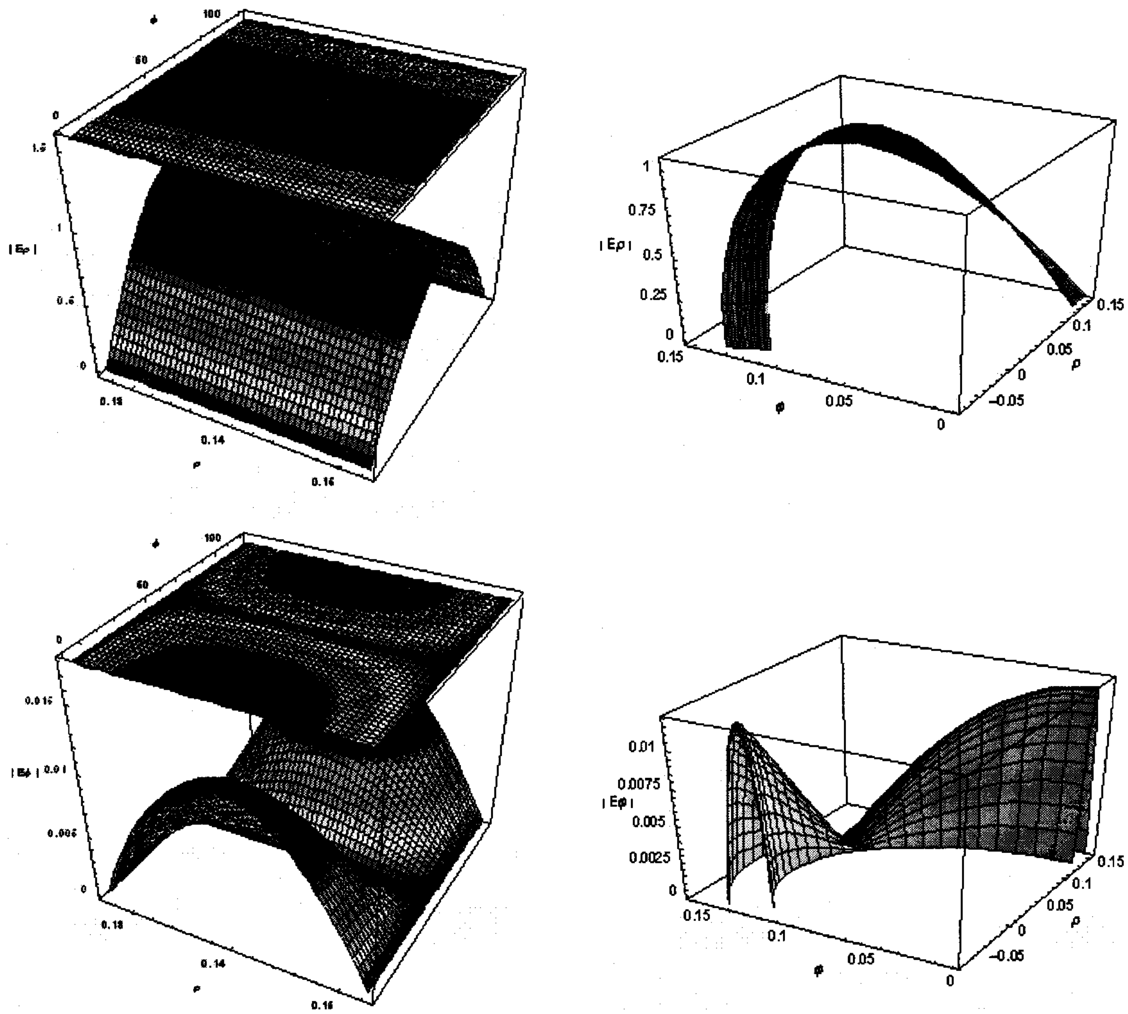


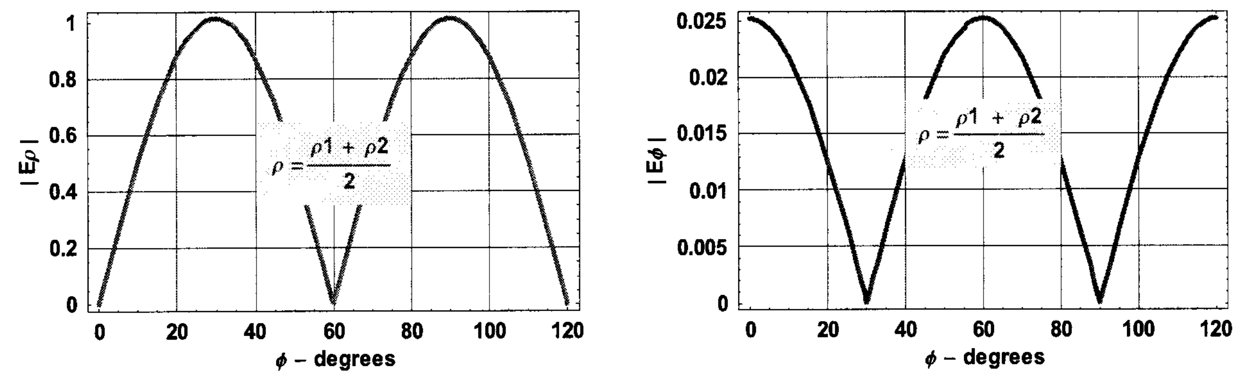
Figure 7. **Normalized distributions of electric field components of TM_{11} mode of double baffled coaxial waveguide ($\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6.0\,in = 0.1524\,m$, $\varphi_0 = 2\pi/3$, and $f = 7.105$ GHz): (a) E_ρ; (b) E_φ; and (c) E_z.**



Ez = 0 Ez = 0

Figure 8. **Normalized distributions of electric field components of TE$_{11}$ mode of double baffled coaxial waveguide ($\rho_1 = 5\,in = 0.127\,m$, $\rho_2 = 6.0\,in = 0.1524\,m$, $\varphi_0 = 2\pi/3$, and $f = 0.6156$ GHz): (a) E_ρ; (b) E_φ; and (c) E_z.**

Finally, the form of the next higher order mode is of interest. Referring to Table 1, the next propagating mode is the TE$_{21}$ mode, with a cutoff frequency of $f_c = 1.02589$ GHz. The non-zero electric fields at $\rho = \dfrac{\rho_1 + \rho_2}{2}$, as a function of φ are plotted in Figure 9.



(a) (b)

Figure 9. Normalized distributions of the electric field components of the TE_{21} mode of the double baffled coaxial waveguide ($\rho_1 = 5\,in$, $\rho_2 = 6.0\,in$, $\varphi_0 = 2\pi/3$, and $f = 1.3$ MHz): (a) E_ρ; and (b) E_φ. $E_z = 0$.

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