Sensor and Simulation Notes

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Coupled Transmission Lines as a Time-Domain Directional Coupler

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Abstract

A finite-length, two-wire-plus-reference, uniform, lossless transmission line can be used as a directional coupler. In particular it can sample a waveform without distortion for a time window of the two-way transit time on the transmission line. The fully symmetric case gives results in agreement with the literature. Solutions for more general cases are also found.

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1. Introduction

Directional couplers are a well-established microwave subject, various bibliographies and texts being available [4, 7, 11, 12, 14, 15]. Here our attention is limited to TEM transmission-line couplers for their potential time-domain application. This is further restricted to *uniform* two-conductor plus reference transmission lines of finite length because, as we shall see, the coupler samples the waveform of interest over a time window of twice the transit time in the coupler without distorting the waveform. This can be compared to another type of directional coupler which senses the time derivative of the waveform [2]. These kinds of directional couplers have application to various measurement situations, including measuring the returning transient signal in a radar antenna which is also used for transmission of the radar pulse.

The reader may consult some relevant references for related previous results [5, 6, 8, 9, 10, 13]. Our general derivation covers these with some extension. In particular the principal result is reinterpreted in time domain. Note that some authors refer to this type of device as a contra-directional coupler [6].

2. Two-Conductor-Plus Reference Uniform Transmission-Line Section

As indicated schematically in Fig. 2.1, we consider a two-wire transmission line with a reference conductor (often a shield) of length ℓ. The wires are labelled 1 and 2 which apply to the voltages and currents propagating on the transmission line. This is a 4-port network, ports 1 and 2 at the left end ($z = 0$) and ports 3 and 4 at the right end ($z = \ell$). Our interest lies in the scattering parameters $\tilde{S}_{n,m}(s)$ for waves entering the mth port and exiting the nth port. These are initially general source- and termination-impedance matrices (reciprocal). There are source voltages at the left end (with $V_2^{(s)}$ eventually being set to zero).

Looking ahead, left/right reflection symmetry (z changing to $\ell - z$), including identical source and termination impedances will allow one to interchange port numbers as

$$1 \leftrightarrow 3 \quad , \quad 2 \leftrightarrow 4 \tag{2.1}$$

in all the $\tilde{S}_{n,m}$, thereby reducing the number of elements in $(\tilde{S}_{n,m})$ to be calculated from 16 to 8. Furthermore, by making the two wires identical with symmetrical positions in the transmission line, and constraining similar symmetry in the source and impedance matrices, we will be left with only 4 scattering-matrix elements to compute due to the interchange

$$1 \leftrightarrow 2 \quad , \quad 3 \leftrightarrow 4 \tag{2.2}$$

associated with this second symmetry. Initially, however, the derivation is more general.

The telegrapher equations for the assumed uniform and lossless transmission line are

$$\begin{aligned} \frac{d}{dz}\left(\tilde{V}_n(z,s)\right) &= -\left(\tilde{Z}'_{n,m}(s)\right) \cdot \left(\tilde{I}_n(z,s)\right) \\ \frac{d}{dz}\left(\tilde{I}_n(z,s)\right) &= -\left(\tilde{Y}'_{n,m}(s)\right) \cdot \left(\tilde{V}_n(z,s)\right) \end{aligned} \tag{2.3}$$

Note that sources are not included in this case. Also note that the I_n are positive to the right (increasing z). We also need

$$\left(\tilde{Z}'_{n,m}(s)\right) = s\mu\left(f_{g_{n,m}}\right) \equiv \text{impedance-per-unit-length matrix}$$

$$\left(\tilde{Y}'_{n,m}(s)\right) = s\varepsilon\left(f_{g_{n,m}}\right)^{-1} \equiv \text{admittance-per-unit-length matrix}$$

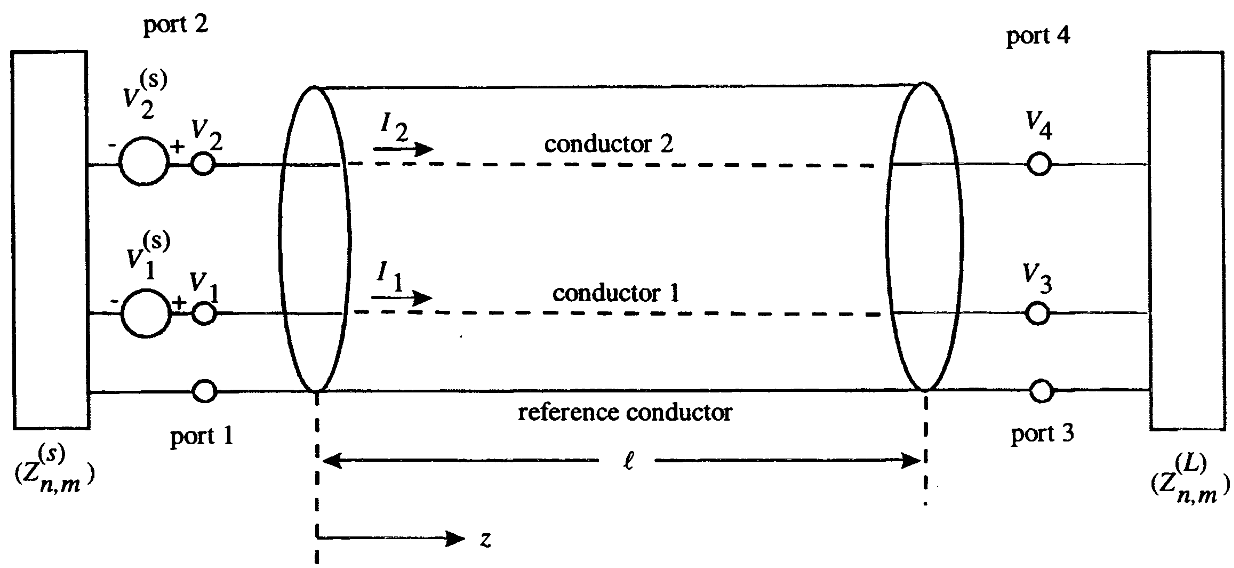

Fig. 2.1 Two-Wire Transmission-Line Directional Coupler

$$\left(f_{g_{n,m}}\right) = \left(f_{g_{n,m}}\right)^T \equiv \text{ geometric-impedance-factor matrix (real)}$$

$$\left(Z_{c_{n,m}}\right) = Z_w\left(f_{g_{n,m}}\right) = \left(Y_{c_{n,m}}\right)^{-1} \text{ characteristic impedance matrix}$$

$$\left(\tilde{\gamma}_{c_{n,m}}\right) = \gamma\left(1_{n,m}\right) \equiv \text{ propagation matrix}$$

$$\gamma = \frac{s}{\text{v}} \equiv \text{ propagation constant}$$

$$Z_w = \left[\frac{\mu}{\varepsilon}\right]^{\frac{1}{2}} \equiv \text{ wave impedance of medium containing the conductors}$$

$$\text{v} = \left[\mu\,\varepsilon\right]^{-\frac{1}{2}} \equiv \text{ propagation speed in this medium}$$

μ ≡ medium permeability (uniform, isotropic)

ε ≡ medium permittivity (uniform, isotropic) (2.4)

~ ≡ two-sided Laplace transform over time t

$s = \Omega + j\omega$ ≡ Laplace-transform variable or complex frequency

$$t_\ell = \frac{\ell}{\text{v}} \equiv \text{ transit time from one end to the other}$$

The telegrapher equations can be combined into a single supervector/supermatrix equation as

$$\begin{aligned}\frac{d}{dz}\begin{pmatrix}\left(\tilde{V}_n(z,s)\right)\\ \left(Z_{c_{n,m}}\right)\cdot\left(\tilde{I}_n(z,s)\right)\end{pmatrix} &= -\begin{pmatrix}\left(0_{n,m}\right) & \left(\tilde{Z}'_{n,m}(s)\right)\cdot\left(Y_{c_{n,m}}\right)\\ \left(Z_{c_{n,m}}\right)\cdot\left(\tilde{Y}'_{n,m}(s)\right) & \left(0_{n,m}\right)\end{pmatrix}\odot\begin{pmatrix}\left(\tilde{V}_n(z,s)\right)\\ \left(Z_{c_{n,m}}\right)\cdot\left(\tilde{I}_n(z,s)\right)\end{pmatrix}\\ &= -\gamma\begin{pmatrix}\left(0_{n,m}\right) & \left(1_{n,m}\right)\\ \left(1_{n,m}\right) & \left(0_{n,m}\right)\end{pmatrix}\odot\begin{pmatrix}\left(\tilde{V}_n(z,s)\right)\\ \left(Z_{c_{n,m}}\right)\cdot\left(\tilde{I}_n(z,s)\right)\end{pmatrix}\end{aligned} \tag{2.5}$$

3. Solution of Transmission-Line Equations

3.1. Matrizant

The solution of (2.5) is via the matrizant differential equation

$$\frac{d}{dz}\left(\left(\tilde{\Xi}_{n,m}(z,z_0;s)\right)_{u,\mathrm{v}}\right) = -\gamma\begin{pmatrix}(0_{n,m}) & (1_{n,m})\\(1_{n,m}) & (0_{n,m})\end{pmatrix} \odot \left(\left(\tilde{\Xi}_{n,m}(z,z_0;s)\right)_{u,\mathrm{v}}\right)$$
$$\left(\left(\tilde{\Xi}(z_0,z_0;s)\right)_{u,\mathrm{v}}\right) = \left(\left(1_{n,m}\right)_{u,\mathrm{v}}\right) \equiv \text{boundary condition} \qquad (3.1)$$

Here u and v range over 1, 2 (as do n,m for our two-wire case). The solution is obtained via the product integral (a simple one in this case of a uniform transmission line) as

$$\left(\left(\tilde{\Xi}_{n,m}(z,z_0;s)\right)_{u,\mathrm{v}}\right) = \prod_{z_0}^{z} e^{-\gamma\begin{pmatrix}(0_{n,m}) & (1_{n,m})\\(1_{n,m}) & (0_{n,m})\end{pmatrix}dz'}$$
$$= e^{-\gamma\int_{z_0}^{z}\begin{pmatrix}(0_{n,m}) & (1_{n,m})\\(1_{n,m}) & (0_{n,m})\end{pmatrix}dz'}$$
$$= e^{-\begin{pmatrix}(0_{n,m}) & (1_{n,m})\\(1_{n,m}) & (0_{n,m})\end{pmatrix}\gamma[z-z_0]} \qquad (3.2)$$

Using the direct product ⊗ [1, 3] we have

$$\begin{pmatrix}(0_{n,m}) & (1_{n,m})\\(1_{n,m}) & (0_{n,m})\end{pmatrix} = (1_{n,m}) \otimes \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$
$$e^{-(1_{n,m})\otimes\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\gamma[z-z_0]} = (1_{n,m}) \otimes e^{-\gamma[z-z_0]\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}}$$
$$= (1_{n,m}) \otimes \left[\cosh(\gamma[z-z_0])\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \sinh(\gamma[z-z_0])\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\right] \qquad (3.3)$$
$$= \begin{pmatrix}\cosh(\gamma[z-z_0])\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} & -\sinh(\gamma[z-z_0])\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\\ -\sinh(\gamma[z-z_0])\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} & \cosh(\gamma[z-z_0])\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\end{pmatrix}$$

from which we readily see a determinant of +1 and a trace of $4\cosh(\gamma[z-z_0])$.

From the matrizant we readily construct the solution of (2.5) as

$$\begin{pmatrix} \left(\tilde{V}_n(z,s)\right) \\ \left(Z_{c_{n,m}}\right) \cdot \left(\tilde{I}_n(z,s)\right) \end{pmatrix} = \left(\left(\tilde{\Xi}_{n,m}(z,z_0;s)\right)_{u,v}\right) \odot \begin{pmatrix} \left(\tilde{V}_n(z_0,s)\right) \\ \left(Z_{c_{n,m}}\right) \cdot \left(\tilde{I}_n(z_0,s)\right) \end{pmatrix} \tag{3.4}$$

where z_0 can be chosen at our convenience. If (2.5) were to include distributed sources then (3.4) is easily modified to accommodate such sources. However, the above is adequate for present purposes.

3.2 Boundary condition at z = ℓ: input impedance

At the right end the termination impedance gives

$$\left(\tilde{V}_n(\ell,s)\right) = \begin{pmatrix} \tilde{V}_3(s) \\ \tilde{V}_4(s) \end{pmatrix} = \left(Z_{n,m}^{(L)}\right) \cdot \left(\tilde{I}_n(\ell,s)\right) \quad , \quad \left(Y_{n,m}^{(L)}\right) \equiv \left(Z_{n,m}^{(L)}\right)^{-1} \tag{3.5}$$

From (3.4) we have

$$\begin{pmatrix} \left(\tilde{V}_n(\ell,s)\right) \\ \left(Z_{c_{n,m}}\right) \cdot \left(\tilde{I}_n(\ell,s)\right) \end{pmatrix} = \begin{pmatrix} \cosh(\gamma\ell)\left(\tilde{V}_n(0,s)\right) - \sinh(\gamma\ell)\left(Z_{c_{n,m}}\right) \cdot \left(\tilde{I}_n(0,s)\right) \\ -\sinh(\gamma\ell)\left(\tilde{V}_n(0,s)\right) + \cosh(\gamma\ell)\left(Z_{c_{n,m}}\right) \cdot \left(\tilde{I}_n(0,s)\right) \end{pmatrix} \tag{3.6}$$

The input impedance (left end) is defined ∋

$$\left(\tilde{V}_n(0,s)\right) = \left(\tilde{Z}_{n,m}^{(in)}(s)\right) \cdot \left(\tilde{I}_n(0,s)\right) \quad , \quad \left(\tilde{Y}_{n,m}^{(in)}(s)\right) = \left(\tilde{Z}_{n,m}^{(in)}(s)\right)^{-1} \tag{3.7}$$

for all choices of voltage or current vector.

Rearranging terms gives

$$\begin{aligned} \left(\tilde{Z}_{n,m}^{(in)}(s)\right) &= \left[\left(1_{n,m}\right)\cosh(\gamma\ell) + \left(Z_{n,m}^{(L)}\right) \cdot \left(Y_{c_{n,m}}\right)\sinh(\gamma\ell)\right]^{-1} \\ &\quad \cdot \left[\left(1_{n,m}\right)\sinh(\gamma\ell) + \left(Z_{n,m}^{(L)}\right) \cdot \left(Y_{c_{n,m}}\right)\cosh(\gamma\ell)\right] \cdot \left(Z_{c_{n,m}}\right) \\ &= \left(\tilde{Y}_{n,m}^{(in)}(s)\right)^{-1} \end{aligned} \tag{3.8}$$

Note that if $\left(Z_{n,m}^{(L)}\right) = \left(Z_{c_{n,m}}\right)$ then $\left(\tilde{Z}_{n,m}^{(in)}\right) = \left(Z_{c_{n,m}}\right)$ as expected.

3.3 Voltage vector at $z = 0$

With the source vector at the left end we include the source impedance matrix via

$$\begin{aligned}\left(\tilde{V}_n(0,s)\right) - \left(\tilde{V}_n^{(s)}(s)\right) &= -\left(Z_{n,m}^{(s)}\right) \cdot \left(\tilde{I}_n(0,s)\right) \\ &= -\left(Z_{n,m}^{(s)}\right) \cdot \left(\tilde{Y}_{n,m}^{(in)}(s)\right) \bullet \left(\tilde{V}_n^{(s)}(0,s)\right)\end{aligned} \tag{3.9}$$

noting the current convention as positive out of the source. Rearranging we have

$$\begin{aligned}\left(\tilde{V}_n(0,s)\right) &= \left[\left(1_{n,m}\right) + \left(Z_{n,m}^{(s)}\right) \bullet \left(\tilde{Y}^{(in)}(s)\right)\right]^{-1} \bullet \left(\tilde{V}_n^{(s)}(0)\right) \\ &= \left[\left(Z_{c_{n,m}}\right) \bullet \left(Y_{n,m}^{(s)}\right) + \left(Z_{c_{n,m}}\right) \bullet \left(\tilde{Y}_{n,m}^{(in)}(s)\right)\right]^{-1} \bullet \left(Z_{c_{n,m}}\right) \bullet \left(Y_{n,m}^{(s)}\right) \bullet \left(\tilde{V}_n^{(s)}(s)\right) \\ &= \left[\left[\left(Z_{c_{n,m}}\right) \bullet \left(Y_{n,m}^{(s)}\right) + \left(Z_{n,m}^{(L)}\right) \bullet \left(Y_{c_{n,m}}\right)\right]\sinh(\gamma\ell) + \left[\left(1_{n,m}\right) + \left(Z_{n,m}^{(L)}\right) \bullet \left(Y_{n,m}^{(s)}\right)\right]\cosh(\gamma\ell)\right]^{-1} \\ &\quad \bullet \left[\left(1_{n,m}\right)\sinh(\gamma\ell) + \left(Z_{n,m}^{(L)}\right) \bullet \left(Y_{c_{n,m}}\right)\cosh(\gamma\ell)\right] \bullet \left(Z_{c_{n,m}}\right) \bullet \left(Y_{n,m}^{(s)}\right) \bullet \left(\tilde{V}_n^{(s)}(s)\right) \\ &= \left[\left[\left(Z_{c_{n,m}}\right) \bullet \left(Y_{n,m}^{(s)}\right) + \left(Z_{n,m}^{(L)}\right) \bullet \left(Y_{c_{n,m}}\right)\right]\sinh(\gamma\ell) + \left[\left(1_{n,m}\right) + \left(Z_{n,m}^{(L)}\right) \bullet \left(Y_{n,m}^{(s)}\right)\right]\cosh(\gamma\ell)\right]^{-1} \\ &\quad \bullet \left[\left(Z_{c_{n,m}}\right) \bullet \left(Y_{n,m}^{(s)}\right)\sinh(\gamma\ell) + \left(Z_{n,m}^{(L)}\right) \bullet \left(Y_{n,m}^{(s)}\right)\cosh(\gamma\ell)\right] \bullet \left(\tilde{V}_n^{(s)}(s)\right)\end{aligned} \tag{3.10}$$

3.4 Voltage vector at $z = \ell$

From (3.6) and (3.7) we have

$$\begin{aligned}&\begin{pmatrix}\left(\tilde{V}_n(0,s)\right) \\ \left(Z_{c_{n,m}}\right) \bullet \left(\tilde{I}_n(\ell,s)\right)\end{pmatrix} \\ &= \begin{pmatrix}\left(1_{n,m}\right)\cosh(\gamma\ell) & -\left(1_{n,m}\right)\sinh(\gamma\ell) \\ -\left(1_{n,m}\right)\sinh(\gamma\ell) & \left(1_{n,m}\right)\cosh(\gamma\ell)\end{pmatrix} \odot \begin{pmatrix}\left(\tilde{V}_n(0,s)\right) \\ \left(Z_{c_{n,m}}\right) \bullet \left(\tilde{Y}_{n,m}^{(in)}(s)\right) \bullet \left(\tilde{V}_n(0,s)\right)\end{pmatrix}\end{aligned} \tag{3.11}$$

With (3.10), and separating out the equation for $(\tilde{V}_n)$, we have

$$\begin{aligned}
\left(\tilde{V}_n(\ell,s)\right) &= \left[\left(1_{n,m}\right)\cosh(\gamma\ell) - \left(Z_{c_{n,m}}\right) \bullet \left(\tilde{Y}^{(in)}_{n,m}(s)\right)\sinh(\gamma\ell)\right] \\
&\quad \bullet \left[\left(1_{n,m}\right) + \left(Z^{(s)}_{n,m}\right) \bullet \left(\tilde{Y}^{(in)}_{n,m}(s)\right)\right]^{-1} \bullet \left(\tilde{V}^{(s)}_n(s)\right) \\
&= \left[\left(1_{n,m}\right)\sinh(\gamma\ell) + \left(Z^{(L)}_{n,m}\right) \bullet \left(Y_{c_{n,m}}\right)\cosh(\gamma\ell)\right]^{-1} \bullet \left(Z^{(L)}_{n,m}\right) \bullet \left(Y_{c_{n,m}}\right) \\
&\quad \bullet \left[\left(1_{n,m}\right) + \left(Z^{(s)}_{n,m}\right) \bullet \left(\tilde{Y}^{(in)}_{n,m}(s)\right)\right]^{-1} \bullet \left(\tilde{V}^{(s)}_n(s)\right) \\
&= \left[\left(Z_{c_{n,m}}\right) \bullet \left(Y^{(L)}_{n,m}\right)\sinh(\gamma\ell) + \left(1_{n,m}\right)\cosh(\gamma\ell)\right]^{-1} \\
&\quad \bullet \Bigg[\left[\left(Z_{c_{n,m}}\right) \bullet \left(Y^{(s)}_{n,m}\right) + \left(Z^{(L)}_{n,m}\right) \bullet \left(Y^{(s)}_{c_{n,m}}\right)\right]\sinh(\gamma\ell) \\
&\qquad + \left[\left(1_{n,m}\right) + \left(Z^{(L)}_{n,m}\right) \bullet \left(Y^{(s)}_{n,m}\right)\right]\cosh(\gamma\ell)\Bigg]^{-1} \\
&\quad \bullet \left[\left(Z_{c_{n,m}}\right) \bullet \left(Y^{(s)}_{n,m}\right)\sinh(\gamma\ell) + \left(Z^{(L)}_{n,m}\right) \bullet \left(Y^{(s)}_{n,m}\right)\cosh(\gamma\ell)\right] \bullet \left(\tilde{V}^{(s)}_n(s)\right)
\end{aligned} \tag{3.12}$$

4. Special Cases

In (3.10) and (3.12) we have the general solution from which the various scattering parameters can be obtained. Looking at these formulae we see three matrices $(Z_{c_{n,m}})$, $(Z_{n,m}^{(s)})$, $(Z_{n,m}^{(L)})$, and their inverses that appear in various combinations. While these formulae are complicated by the fact that the various matrices do not in general commute, and that these matrices are symmetric (reciprocity) but the products are not in general symmetric, special choices can simplify these formulae considerably.

4.1 Equal source- and load-impedance matrices

A first simplification occurs if we choose

$$\left(Z_{n,m}^{(s)}\right) = \left(Z_{n,m}^{L}\right) \tag{4.1}$$

thereby imposing left/right symmetry (making $z = \ell/2$ a reflection symmetry plane). This also corresponds to the symmetry on interchange of port labels as in (2.1). Then define

$$\left(X_{n,m}\right) \equiv \left(Z_{c_{n,m}}\right) \cdot \left(Y_{n,m}^{(s)}\right) \tag{4.2}$$

and we find that only this matrix and the identity are needed.

At the left end (ports 1 and 2) we now have

$$\begin{aligned}\left(\tilde{V}_n(0,s)\right) = &\left[\left[\left(X_{n,m}\right) + \left(X_{n,m}\right)^{-1}\right]\sinh(\gamma\ell) + 2\left(1_{n,m}\right)\cosh(\gamma\ell)\right]^{-1} \\ &\cdot \left[\left(X_{n,m}\right)\sinh(\gamma\ell) + \left(1_{n,m}\right)\cosh(\gamma\ell)\right] \cdot \left(\tilde{V}_n^{(s)}(s)\right)\end{aligned} \tag{4.3}$$

At the right end (ports 3 and 4) we now have

$$
\begin{aligned}
\left(\tilde{V}_n(\ell,s)\right) &= \left[\left(X_{n,m}\right)\sinh(\gamma\ell) + \left(1_{n,m}\right)\cosh(\gamma\ell)\right]^{-1} \\
&\quad \bullet \left[\left[\left(X_{n,m}\right) + \left(X_{n,m}\right)^{-1}\right]\sinh(\gamma\ell) + 2\left(1_{n,m}\right)\cosh(\gamma\ell)\right]^{-1} \\
&\quad \bullet \left[\left(X_{n,m}\right)\sinh(\gamma\ell) + \left(1_{n,m}\right)\cosh(\gamma\ell)\right] \bullet \left(\tilde{V}_n^{(s)}(s)\right) \\
&= \left[\left[\left(X_{n,m}\right) + \left(X_{n,m}\right)^{-1}\right]\sinh(\gamma\ell) + 2\left(1_{n,m}\right)\cosh(\gamma\ell)\right]^{-1} \bullet \left(\tilde{V}_n^{(s)}(s)\right)
\end{aligned}
\tag{4.4}
$$

Since only $(X_{n,m})$, its inverse, and the identity matrix are involved, and they all commute, then the above dot products of bracketed terms can be arranged in *any order.* (All these matrices can also be expanded in terms of the same eigenvectors.) This allows the reduction of the right-end voltages in (4.4) to a single bracketed matrix combination.

Since our motivation is for a directional coupler, let us try to make $\tilde{V}_3(\ell,s) = 0$ under the excitation condition from $\tilde{V}_1(0,s)$ with $\tilde{V}_2(0,s) = 0$. This gives

$$
\left[\left[\left(X_{n,m}\right) + \left(X_{n,m}\right)^{-1}\right]\sinh(\gamma\ell) + 2\left(1_{n,m}\right)\cosh(\gamma\ell)\right] \bullet \begin{pmatrix} \tilde{V}_3(\ell,s) \\ 0 \end{pmatrix} = \begin{pmatrix} \tilde{V}_1(0,s) \\ 0 \end{pmatrix}
\tag{4.5}
$$

which implies

$$
\begin{aligned}
0 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bullet \left[\left[\left(X_{n,m}\right) + \left(X_{n,m}\right)^{-1}\right]\sinh(\gamma\ell) + 2\left(1_{n,m}\right)\cosh(\gamma\ell)\right] \bullet \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
0 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bullet \left[\left(X_{n,m}\right) + \left(X_{n,m}\right)^{-1}\right] \bullet \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= X_{2,1} - \frac{1}{\det\left(\left(X_{n,m}\right)\right)} X_{1,2} \\
X_{1,2} &= \det\left(\left(X_{n,m}\right)\right) X_{2,1} = \det\left(\left(Z_{c_{n,m}}\right)\right) \det\left(\left(Y_{n,m}^{(s)}\right)\right) X_{2,1}
\end{aligned}
\tag{4.6}
$$

Now $(X_{n,m})$ is not in general a symmetric matrix, but it is positive definite as the product of positive-definite matrices in (4.2) implying $\det((X_{n,m})) > 0$. Provided $X_{1,2}$ and $X_{2,1}$ are of the same sign (4.6) can be satisfied by scaling $(X_{n,m})$ (multiplying by a positive constant). This appears to be a more general solution than previously reported.

4.2 Source- and load-impedance matrices as diagonal matrices

Now let the source- and load-admittance matrices be diagonal, corresponding to coaxial cables of constant resistive characteristic impedances, as

$$\left(Y_{n,m}^{(s)}\right) = \left(Y_{n,m}^{(L)}\right) = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} = \begin{pmatrix} R_1^{-1} & 0 \\ 0 & R_2^{-1} \end{pmatrix} \tag{4.7}$$

Then we have (noting symmetric $(Z_{n,m})$)

$$\begin{aligned} \left(X_{n,m}\right) &= \begin{pmatrix} Z_{1,1} & Z_{1,2} \\ Z_{2,1} & Z_{2,2} \end{pmatrix} \cdot \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \\ &= \begin{pmatrix} G_1 Z_{1,1} & G_2 Z_{1,2} \\ G_1 Z_{2,1} & G_2 Z_{2,2} \end{pmatrix} = \begin{pmatrix} G_1 Z_{1,1} & G_2 Z_{1,2} \\ G_1 Z_{1,2} & G_2 Z_{2,2} \end{pmatrix} \\ \det\left(X_{n,m}\right) &= G_1 G_2 \det\left(\left(Z_{n,m}\right)\right) = G_1 G_2 \left[Z_{1,1} Z_{2,2} - Z_{1,2}^2 \right] \end{aligned} \tag{4.8}$$

Our directional-coupler criterion in (4.6) then becomes

$$G_1 Z_{1,2} G_1 G_2 \left[Z_{1,1} Z_{2,2} - Z_{1,2}^2 \right] = G_2 Z_{1,2} \quad , \quad G_1^2 \left[Z_{1,1} Z_{2,2} - Z_{1,2}^2 \right] = 1 \tag{4.9}$$

or

$$\det\left(\left(Z_{n,m}\right)\right) = G_1^{-2} = R_1^2 \tag{4.10}$$

Note that G_2 (or R_2, the source and load resistance for wire 2) does not enter into the directional-coupler criterion. (Very interesting!)

Note that this result includes the special case that $R_1 = R_2 = R$, corresponding to identical cable impedances R connected to each of the four ports.

4.3 Fully symmetric case

Now we come to the classical case of symmetry in $(Z_{n,m})$ on interchange of wires 1 and 2. This symmetry is also a symmetry on interchange of port labels in (2.2). Then we have

$$
\begin{aligned}
Z_{1,1} &= Z_{2,2} \\
\left(Z_{n,m}\right) &= \begin{pmatrix} Z_{1,1} & Z_{1,2} \\ Z_{1,2} & Z_{1,1} \end{pmatrix} \\
\det\left(\left(Z_{n,m}\right)\right) &= Z_{1,1}^2 - Z_{1,2}^2
\end{aligned}
\tag{4.11}
$$

Furthermore, make all four cable characteristic impedances be the same, i.e.,

$$
R \equiv R_1 = R_2 \quad , \quad G \equiv G_1 = G_2 \tag{4.12}
$$

This now makes

$$
\left(X_{n,m}\right) = G\left(Z_{n,m}\right) = \begin{pmatrix} G\,Z_{1,1} & G\,Z_{1,2} \\ G\,Z_{1,2} & G\,Z_{1,1} \end{pmatrix} = \begin{pmatrix} X_{1,1} & X_{1,2} \\ X_{1,2} & X_{1,1} \end{pmatrix} \tag{4.13}
$$

Our directional-coupler criterion is now

$$
\det\left(\left(Z_{n,m}\right)\right) = Z_{1,1}^2 - Z_{1,2}^2 = R^2 \quad , \quad \det\left(\left(X_{n,m}\right)\right) = X_{1,1}^2 - X_{1,2}^2 = 1 \tag{4.14}
$$

This is an agreement with previous results cited in Section 1.

5. Diagonalization of Symmetric $(X_{n,m})$

For the fully symmetric case in (4.13) we have equations for eigenvalues χ_n as

$$\det\left(\left(X_{n,m}\right)\right) = 1 = \chi_1\,\chi_2 \;,\quad tr\left(\left(X_{n,m}\right)\right) = 2\,X_{1,1} = \chi_1 + \chi_2$$
$$\chi_\beta^2 - 2\,X_{1,1}\chi_\beta + 1 = 0 \;,\quad \chi_{\substack{1\\2}} = X_{1,1} \pm \left[X_{1,1}^2 - 1\right]^{\frac{1}{2}} = X_{1,1} \pm X_{1,2} \tag{5.1}$$

For eigenvectors we have

$$\begin{aligned}
&\left(X_{n,m}\right) \cdot \left(x_n\right)_\beta = \chi_\beta \left(x_n\right)_\beta \\
&X_{1,1}\,x_{1;\beta} + X_{1,2}\,x_{2;\beta} = \left[X_{1,1} \pm X_{1,2}\right] x_{1;\beta} \\
&X_{1,2}\,x_{1;\beta} + X_{1,1}\,x_{2;\beta} = \left[X_{1,1} \pm X_{1,2}\right] x_{2;\beta} \\
&X_{1,2}\,x_{2;\beta} = \pm X_{1,2}\,x_{1;\beta} \\
&X_{1,2}\,x_{1;\beta} = \pm X_{1,2}\,x_{2;\beta} \\
&x_{2;\beta} = \pm x_{1;\beta} \\
&\left(x_n\right)_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \;, \left(x_n\right)_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \\
&\left(x_n\right)_{\beta_1} \cdot \left(x_n\right)_{\beta_2} = 1_{\beta_1,\beta_2} \quad \text{(biorthornormal)}
\end{aligned} \tag{5.2}$$

This is in agreement with previous results with symmetric and antisymmetric modes (eigenvectors) (or called even and odd in some literature). We then have the dyadic forms for the symmetric matrices as

$$\begin{aligned}
\left(X_{n,m}\right) &= \sum_{\beta=1}^{2} \chi_\beta \left(x_n\right)_\beta \left(x_n\right)_\beta \\
&= \left[X_{1,1} + X_{1,2}\right]\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} + \left[X_{1,1} - X_{1,2}\right]\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} \\
\left(X_{n,m}\right)^{-1} &= \begin{pmatrix} X_{1,1} & -X_{1,2} \\ -X_{1,2} & X_{1,1} \end{pmatrix} \\
&= \left[X_{2,1} + X_{1,2}\right]^{-1}\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} + \left[X_{1,1} - X_{1,2}\right]^{-1}\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} \\
\left(1_{n,m}\right) = \left(X_{n,m}\right)^0 &= \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}
\end{aligned} \tag{5.3}$$

The reader should note that these results are readily extendable to the more general cases previously discussed (Section 4). The eigenvectors and eigenvectors (left and right side for nonsymmetric matrices) are of more complicated, but still analytically computable, forms.

6. Scattering-Matrix Elements

From (3.10) and (3.12) we have the general solution for the port voltages. With equal and diagonal source and load impedances (4.7) we can determine incident voltages at the ports from which the scattering-matrix elements can be determined. Further restricting to the fully symmetric case (although more general cases can also be computed from the foregoing) we have for our computations

$$\begin{aligned}\tilde{V}_1^{(inc)}(s) &= \frac{1}{2}\tilde{V}_1^{(s)}(s) = 0\\ \tilde{V}_2^{(inc)}(s) &= \frac{1}{2}\tilde{V}_2^{(s)}(s) = 0\end{aligned} \tag{6.1}$$

The factor of 1/2 accounts for a reflection of the incident voltage of +1 if the port is open circuited.

We already have from zero voltage at port 3, including symmetry

$$\tilde{S}_{3,1}(s) = \tilde{S}_{1,3}(s) = \tilde{S}_{2,4}(s) = \tilde{S}_{4,2}(s) = 0 \tag{6.2}$$

From Section 5 and (4.3) we have

$$\begin{aligned}&\left(\tilde{V}_n(0,s)\right)\\ &= \left[\left[\left[X_{1,1}+X_{1,2}+\left[X_{1,1}+X_{1,2}\right]^{-1}\right]\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\right.\\ &\quad\left.+\left[\left[X_{1,1}-X_{1,2}+\left[X_{1,1}-X_{1,2}\right]^{-1}\right]\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\right]^{-1}\\ &\bullet\left[\left[\left[X_{1,1}+X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\right.\\ &\quad\left.+\left[\left[X_{1,1}-X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\right]\bullet\begin{pmatrix}2\tilde{V}_1^{(inc)}(s)\\0\end{pmatrix}\\ &= \left[\left[\left[X_{1,1}+X_{1,2}+\left[X_{1,1}+X_{1,2}\right]^{-1}\right]\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]^{-1}\right.\\ &\quad\left[\left[X_{1,1}+X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\\ &\quad+\left[\left[X_{1,1}-X_{1,2}+\left[X_{1,1}-X_{1,2}\right]^{-1}\right]\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]^{-1}\\ &\quad\left.\left[\left[X_{1,1}-X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\right]\bullet\begin{pmatrix}1\\0\end{pmatrix}2\tilde{V}_1^{(inc)}(s)\end{aligned}$$

$$
\begin{aligned}
&= \Big[\left[2X_{1,1}\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]^{-1}\left[\left[X_{1,1}+X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\begin{pmatrix}1\\1\end{pmatrix} \\
&\quad + \left[2X_{1,1}\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]^{-1}\left[\left[X_{1,1}-X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\begin{pmatrix}1\\-1\end{pmatrix}\Big]\tilde{V}_1^{(inc)}(s) \\
&= \left[2X_{1,1}\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]^{-1} \\
&\quad \Big[\left[\left[X_{1,1}+X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\begin{pmatrix}1\\1\end{pmatrix} + \left[\left[X_{1,1}-X_{1,2}\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]\begin{pmatrix}1\\-1\end{pmatrix}\Big]\tilde{V}_1^{(inc)}(s)
\end{aligned}
\tag{6.3}
$$

Here we have used (from the determinant (4.14))

$$
\left[X_{1,1} \pm X_{1,2}\right]^{-1} = X_{1,1} \mp X_{1,2} \tag{6.4}
$$

From this we have

$$
\tilde{S}_{1,1}(s) = \tilde{S}_{2,2}(s) = \tilde{S}_{3,3}(s) = \tilde{S}_{4,4}(s) \equiv \frac{\tilde{V}_1(0,s)}{\tilde{V}_1^{(s)}(s)} - 1 = 0 \tag{6.5}
$$

which is a simple and pleasing result. There is *no reflection* back toward the source. By symmetry, this applies to signals in any port. We also have

$$
= \left[X_{1,1}\sinh(\gamma\ell) + \cosh(\gamma\ell)\right]^{-1} X_{1,2}\sinh(\gamma\ell) \tag{6.6}
$$

Observe for the case of small coupling (small $X_{1,2}$)

$$
\begin{aligned}
\tilde{S}_{2,1}(s) &= \left[\left[1+X_{1,2}^2\right]^{\frac{1}{2}}\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]^{-1} X_{1,2}\sinh(\gamma\ell) \\
&= \left[\left[1+\frac{1}{2}X_{1,2}^2+\mathrm{O}\left(X_{1,2}^4\right)\right]\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]^{-1} X_{1,2}\sinh(\gamma\ell) \\
&= \left[e^{-\gamma\ell}+\sinh(\gamma\ell)\mathrm{O}\left(X_{1,2}\right)^2\right]X_{1,2}\sinh(\gamma\ell) \\
&= e^{-\gamma\ell}X_{1,2}\sinh(\gamma\ell)\left[1+\mathrm{O}\left(X_{1,2}\right)\right] \\
&= \frac{1-e^{-2\gamma\ell}}{2}X_{1,2}\left[1+O\left(X_{1,2}\right)\right] \text{ as } X_{1,2}\to 0
\end{aligned}
\tag{6.7}
$$

This shows a separation of the initial response of $X_{1,2}/2$, followed at a time of $2t_\ell$ by the first reflection. More precisely the early-time response is given by writing

$$\tilde{S}_{2,1}(s) = \left[\left[1+X_{1,1}\right]\frac{e^{\gamma\ell}}{2}+\left[1-X_{1,1}\right]\frac{e^{-\gamma\ell}}{2}\right]^{-1} X_{1,2}\frac{e^{\gamma\ell}e^{-\gamma\ell}}{2} \tag{6.8}$$

from which we find

$$S_{2,1}(t) = \frac{X_{1,2}}{1+X_{1,1}}\delta(t) \quad \text{for } t<2t_\ell \tag{6.9}$$

Here the delta function is used showing that $S_{2,1}$ replicates the incident voltage for a time $2t_\ell$ with an amplitude coefficient $X_{1,2}/[1+X_{1,1}]^{-1}$. Said another way

$$V_2(0,t) = \frac{X_{1,2}}{1+X_{1,1}}V_1^{(inc)}(t) \text{ for } t<2t_\ell \text{ with } V_1^{(inc)}(t) \text{ for } t<0 \tag{6.10}$$

For low frequencies we have

$$\tilde{S}_{2,1}(s) = X_{1,2}\gamma\ell\left[1+O(\gamma\ell)\right] \text{ as } s\to 0 \tag{6.11}$$

These are the directional-coupler properties as they also represent $\tilde{S}_{4,3}$ which gives the signal out of port 4 from that into port 3.

From Section 5 and (4.4) we have

$$\begin{aligned}
&\left(\tilde{V}_n(\ell,s)\right) \\
&= \Bigg[\left[\left[X_{1,1}+X_{1,2}+\left[X_{1,1}+X_{1,2}\right]^{-1}\right]\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \\
&\quad +\left[\left[X_{1,1}-X_{1,2}+\left[X_{1,1}-X_{1,2}\right]^{-1}\right]\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\Bigg]^{-1}\bullet\begin{pmatrix}2\tilde{V}_1^{(inc)}(s)\\0\end{pmatrix} \\
&= \Bigg[\left[2X_{1,1}\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \\
&\quad + \left[2X_{1,1}\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]\frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\Bigg]^{-1}\bullet\begin{pmatrix}1\\0\end{pmatrix}2\tilde{V}_1^{(inc)}(s)
\end{aligned}$$

$$
\begin{aligned}
&= \left[2X_{1,1}\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]^{-1}\left[\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\right] \bullet \begin{pmatrix}1\\0\end{pmatrix} 2\tilde{V}_1^{(inc)}(s) \\
&= \left[2X_{1,1}\sinh(\gamma\ell)+2\cosh(\gamma\ell)\right]^{-1}\left[\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}1\\-1\end{pmatrix}\right]\tilde{V}_1^{(inc)}(s) \\
&= \left[X_{1,1}\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]^{-1}\begin{pmatrix}1\\0\end{pmatrix}\tilde{V}_1^{(inc)}(s)
\end{aligned}
\tag{6.12}
$$

One of the results ($\tilde{S}_{3,1}$) has already been exhibited in (5.2). The other has

$$
\tilde{S}_{3,1} = \tilde{S}_{1,3} = \tilde{S}_{4,2} = \tilde{S}_{2,4} = \frac{\tilde{V}_1(\ell,s)}{\tilde{V}_1^{(inc)}(s)} = \left[X_{1,1}\sinh(\gamma\ell)+\cosh(\gamma\ell)\right]^{-1} \tag{6.13}
$$

This is the direct signal through the coupler. Writing this as

$$
\begin{aligned}
\tilde{S}_{3,1} &= \left[\left[1+X_{1,1}\right]\frac{e^{\gamma\ell}}{2} + \left[1+X_{1,1}\right]\frac{e^{-\gamma\ell}}{2}\right]^{-1} \\
&= \frac{2e^{-\gamma\ell}}{1+X_{1,1}}\left[1+\left[1-X_{1,1}\right]^2 e^{-2\gamma\ell}\right]^{-1}
\end{aligned}
\tag{6.14}
$$

we then find

$$
\begin{aligned}
S_{2,1}(t) &= \frac{2}{1+X_{1,1}}\delta(t-t_\ell) \quad \text{for } t<3t_\ell \\
\frac{2}{1+X_{1,1}} &= 2\left[1+\left[1+X_{1,2}^2\right]^{\frac{1}{2}}\right]^{-1} = 2\left[2+\frac{1}{2}X_{1,2}^2+O\left(X_{1,2}^4\right)\right]^{-1} \\
&= 1-\frac{1}{4}X_{1,2}^2 + O\left(X_{1,2}^4\right) \quad \text{as } X_{1,2}<0<1 \\
&< 1
\end{aligned}
\tag{6.15}
$$

So the direct signal on wire 1 is reduced at early times $t_\ell < t < 3t_\ell$. For late time we have the low-frequency form as

$$
\tilde{S}_{3,1}(s) = 1 - X_{1,1}\frac{\gamma\ell}{2} + O\left((\gamma\ell)^2\right) \quad \text{as } \gamma\ell \to 0 \tag{6.16}
$$

which goes to 1 showing no attenuation of "slow" pulses.

7. Concluding Remarks

It is then clear that a traditional type of transmission-line directional coupler can be made to operate for temporal waveforms as well. There is a time-window of width $2t_\ell$ during which the coupled waveform is the same as the incident waveform times a constant. This requires that, in the simplest operation, $2t_\ell$ be longer than the time duration of the pulse of interest. One can extend this to longer times by appropriate data processing, noting the more complete description of the coupler scattering-matrix elements.

Our general approach to the theory has revealed various cases of potential interest. The fully symmetric case (symmetry between wires 1 and 2 as well as source and load impedances) with identical resistive impedances (cable characteristic impedances) on all four ports gives rather simple final answers. There are, however, more general cases that still lead to zero transmission from port 1 to port 4 (the directional-coupler criterion) as discussed in Section 4. These may deserve further consideration.

Noting that a transmission-line model is used for the coupler, there are some errors in modelling a real such device. In particular, at frequencies high enough that radian wavelengths are not large compared to the cross-section dimensions, a full wave analysis may be required. Near the ports the abrupt changes in the cross-section geometry may make evanescent modes significant there. (Details, details!)

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