Sensor and Simulation Notes

Note 446

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Location and Orientation of Electrically Small Transmitting and Receiving Antenna Pairs with Common Linear Polarization and Beam Direction for Minimal Mutual Coupling

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Abstract

In measurement of the scattering from some target one needs transmitting and receiving antennas. For copol measurements, these can be separate antennas to minimize the direct coupling from the transmit channel into the receive channel. By appropriate positioning of the receive antenna relative to the transmit antenna one can further reduce this coupling. Based on electric- and magnetic-dipole representations, one can compute optimal receive-antenna locations. In particular for antennas comprising balanced electric and magnetic-dipoles ($\vec{p}\times\vec{m}$ antennas) one can find eight such optimal locations at the corners of a cube centered on the transmit antenna with cube faces perpendicular to the directions of the two dipole moments and the beam direction of the transmit antenna.

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1. Introduction

In measurements of electromagnetic scattering (such as by a radar) one needs to transmit (from an antenna) an incident wave which in turn scatters from the object of interest (target), the scattered field being sensed in turn via some receive antenna. The far-field amplitude decays with distance r proportional to r^{-1}. Similarly the scattered far field is proportional to r^{-1} for a total decay of the received signal (voltage) proportional to r^{-2}. If the same antenna is used for both transmission and reception (monostatic) there is then a problem in accurately measuring the scattered signal in the presence of the transmitted signal. One can use time gating to separate these if the time duration is shorter than the round-trip transit time to the target. So a short pulse in transmission offers some advantages. However, the transmitted signal should decay sufficiently rapidly in the source and antenna, noting the possibility of small reflections and resonances, which carry out in time. The decay should be sufficiently rapid such that at the round-trip time from the target the signal is negligible compared to the scattered signal in the antenna.

For a given (say linear) transmit polarization there are, in general, two scattering polarizations to measure. For two independent transmit polarizations (say horizontal, h, and vertical, v) this gies a 2 x 2 scattering dyadic $\overleftrightarrow{\Lambda}$ which one can use to identify the target if broadbnad pulses (or multiple frequencies) are used. One can use symmetry planes [6. 7. 9] to give both horizontal and vertical operation in the same antenna. For monostatic (backscatttering) crosspol measurements ($\Lambda_{b_{h,v}} = \Lambda_{b_{v,h}}$ (reciprocity)) the horizontal transmit does not couple (ideally) into the vertical receive (and conversely) as one has symmetric and the other antisymmetric fields with respect to a particular symmetry plane (there being possibly multiple symmetry planes). Practically speaking, geometric imperfections mean that the transmit signal (voltage) is reduced about two decades (depending on accuracy) in the receive channel. For copol measurements ($\Lambda_{b_{h,h}}$ and $\Lambda_{b_{v,v}}$) one does not have the advantage of the reduced coupling to the transmit signal.

One can improve the situation somewhat by having the transmit and receive channels not in the same antenna (including dual polarization), but by using physically separated transmit and receive antennas. In general, this gives a bistatic radar, but for present purposes, let us assume that the separation between the antennas is small compared to the distance to the target so that the orientations of the antennas with respect to the target are approximately the same, giving approximately a monostatic radar (quasi monostatic, quasi backscattering). One can easily correct for the different distances to the target (for staggered antennas) via the speed of light. We cn then think of an approximate backscattering dyadic $\overleftrightarrow{\Lambda}_b$ which we can regard as 2 x 2 in the h, v coordinates.

With separated antennas we can still use a common symmetry plane for symmetric/antisymmetric measurements of crosspol [5, 6]. This gives additional (perhaps decades) of isolation beyond that discussed above.

More importantly, separated antennas give isolation between copol transmit/receive antennas. This suggests the question: What should be the relative positions of the two copol antennas for minimum coupling? This paper is primarily concerned with this question.

For simplicity we assume identical transmit and receive antennas. Furthermore, we assume that the antennas are electrically small for a definitive analysis. Such antennas are characterized by their electric and magnetic-dipole moments in transmission, and reciprocity takes care of reception. The canonical cases are electric dipoles, magnetic dipoles, and combined dipoles (balanced electric and magnetic moments at right angles). Furthermore, we assume that the two antennas are identical except for a spatial translation (i.e., no rotation) so that they are both oriented the same with respect to the target.

2. Fields from Dipoles

Fundamental to the following are the fields produced by electric and magnetic dipoles. For this discussion we have the illustration in Fig. 2.1. Dipoles subscripted 1 are located at the coordinate origin $\overrightarrow{r} = \overrightarrow{0}$. For convenience we take the dipole orientations as

$$\overrightarrow{p}_1 = p_1 \overrightarrow{1}_z \; , \quad \overrightarrow{m}_1 = m_1 \overrightarrow{1}_x \tag{2.1}$$

and think of these as characterizing the transmit antenna. The receive antenna is characterized by dipole moments (in transmission) subscripted 2. They are located at some arbitrary $\overrightarrow{r} \neq \overrightarrow{0}$ but are oriented as

$$\overrightarrow{p}_2 = p_2 \overrightarrow{1}_z \; , \quad \overrightarrow{m}_2 = m_2 \overrightarrow{1}_x \tag{2.2}$$

i. e., parallel to the moments of antenna 1. Note that

$$\overrightarrow{p}_n \cdot \overrightarrow{m}_n = 0 \quad \text{for} \quad n = 1, 2 \tag{2.3}$$

so that when we come to combined dipoles the electric and magnetic moments are mutually perpendicular [1]. Note also that

$$\overrightarrow{1}_c \equiv \overrightarrow{1}_y = \overrightarrow{1}_z \times \overrightarrow{1}_x \tag{2.4}$$

is a particular direction of interest as the beam center for combined dipoles.

For the present analysis it is convenient to work in complex-frequency domain with

$$\begin{aligned} &\sim \;\equiv\; \text{two-sided Laplace transform from time t to complex frequency s} \\ &s \equiv \Omega + j \equiv \text{Laplace-transform variable or complex frequency} \end{aligned} \tag{2.5}$$

For convenience we also have

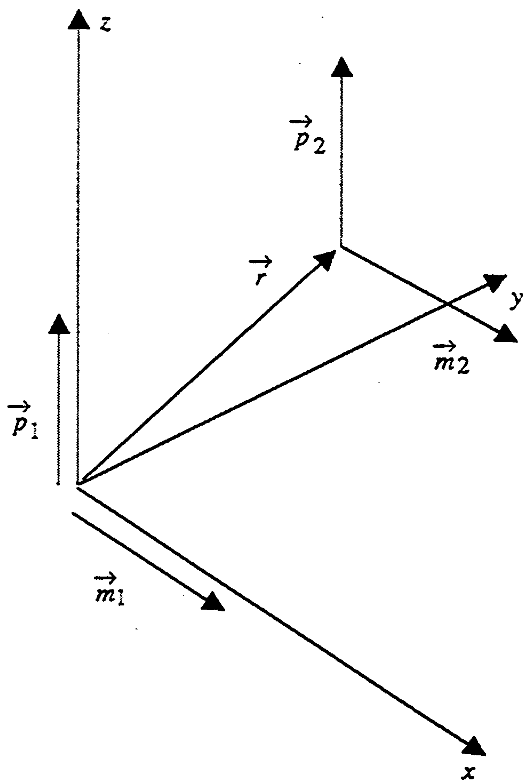

Fig. 2.1 Dipole Locations

$$\gamma = \frac{s}{c} \equiv \text{propagation constant}$$

$$c = [\mu_o \varepsilon_o]^{-\frac{1}{2}} \equiv \text{speed of light}$$

$$Z_o \equiv \left[\frac{\mu_o}{\varepsilon_o}\right]^{\frac{1}{2}} \equiv \text{wave impedance of free space} \tag{2.6}$$

$$\mu_o \equiv \text{permeability of free space}$$

$$\varepsilon_o \equiv \text{permittivity of free space}$$

We have various vectors and dyadics as

$$\begin{aligned} \overleftrightarrow{1} &\equiv \vec{1}_x \vec{1}_x + \vec{1}_y \vec{1}_y + \vec{1}_z \vec{1}_z \\ &= \vec{1}_r \vec{1}_r + \vec{1}_\theta \vec{1}_\theta + \vec{1}_\phi \vec{1}_\phi \equiv \text{identity} \\ \overleftrightarrow{1}_r &\equiv \overleftrightarrow{1} - \vec{1}_r \vec{1}_r \equiv \text{transverse (to } r\text{) identity} \end{aligned} \tag{2.7}$$

Besides Cartesian (x, y, z) coordinates we have spherical (r, θ, ϕ) coordinates. Spherical coordinates are normally defined with z as a preferred axis. Let us subscript these coordinates with z which is the preferred axis giving

$$\begin{aligned} \vec{r}_p &= x\vec{1}_x + y\vec{1}_y + z\vec{1}_z \\ &= r_p\left[\sin(\theta_z)\cos(\phi_z)\vec{1}_x + \sin(\theta_z)\sin(\theta_z)\vec{1}_y + \cos(\theta_z)\vec{1}_z\right] \end{aligned} \tag{2.8}$$

Similarly base a set of spherical coordinates on $\vec{m}_1$ (x axis) as

$$\vec{r}_m = r_m\left[-\sin(\theta_x)\cos(\phi_x)\vec{1}_z + \sin(\theta_x)\sin(\theta_x)\vec{1}_y + \cos(\theta_x)\vec{1}_x\right] \tag{2.9}$$

which is readily found by the substitutions $z \to x$, $x \to -z$. Note the commonality of

$$
\begin{aligned}
\vec{r}_p &= \vec{r}_m = \vec{r} \\
\vec{1}_{r_p} &= \vec{1}_{r_m} = \vec{1}_r \\
\overleftrightarrow{1}_{r_p} &= \overleftrightarrow{1}_{r_m} = \overleftrightarrow{1}_r
\end{aligned}
\tag{2.10}
$$

2.1 Electric-dipole fields

Following [1] we have the fields from an electric dipole $\vec{p}_1$ as

$$
\begin{aligned}
\vec{E}_p &= e^{-\gamma r}\left[-\left[\frac{1}{4\pi r^3}\frac{1}{\varepsilon_0} + \frac{sZ_o}{4\pi r^2}\right]\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right] - \frac{s^2\mu_o}{4\pi r}\overleftrightarrow{1}_r\right]\cdot\vec{p}_1 \\
\vec{H}_p &= e^{-\gamma r}\left[-\frac{s}{4\pi r^2} - \frac{s^2}{4\pi r}\frac{1}{c}\right]\vec{1}_r \times \vec{p}_1
\end{aligned}
\tag{2.11}
$$

2.2 Magnetic-dipole fields

Likewise we have the fields from a magnetic dipole $\vec{m}_1$ as

$$
\begin{aligned}
\vec{E}_m &= e^{-\gamma r}\left[\frac{s\mu_o}{4\pi r^2} + \frac{s^2\mu_o}{4\pi r}\frac{1}{c}\right]\vec{1}_r \times \vec{m}_1 \\
\vec{H}_m &= e^{-\gamma r}\left[-\left[\frac{1}{4\pi r^3} + \frac{s}{4\pi r^2}\frac{1}{c}\right]\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right] - \frac{s^2}{4\pi r}\frac{1}{c^2}\overleftrightarrow{1}_r\right]\cdot\vec{m}_1
\end{aligned}
\tag{2.12}
$$

2.3 Combined-dipole fields

For combined dipoles we use the special $\vec{p}\times\vec{m}$ form given by (2.2) together with [1]

$$
\begin{aligned}
p_n &= \frac{m_n}{c} \quad \text{for} \quad n = 1,2 \\
\vec{p}_n &= -\frac{1}{c}\vec{1}_y \times \vec{m}_n\,, \quad \vec{m}_n = c\,\vec{1}_y \times \vec{p}_n
\end{aligned}
\tag{2.13}
$$

The fields produced by antenna 1 are then

$$\vec{E} = \vec{E}_p + \vec{E}_m = e^{-\gamma r}\left[-\frac{1}{4\pi r^3}\frac{1}{\varepsilon_0}\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right]\cdot\vec{p}_1\right.$$

$$\left. - \frac{sZ_o}{4\pi r^2}\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right]\cdot\vec{p}_1 - \vec{1}_r\times\frac{\vec{m}_1}{c} - \frac{s^2\mu_o}{4\pi r}\left[\vec{1}_r\cdot\vec{p}_1 - \vec{1}_r\times\frac{\vec{m}_1}{c}\right]\right]$$

$$= e^{-\gamma r}\left[-\frac{1}{4\pi r^3}\frac{1}{\varepsilon_0}\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right]\right.$$

$$- \frac{sZ_o}{4\pi r^2}\left[\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right]\cdot\vec{p}_1 - \vec{1}_y\,\vec{1}_r + \left[\vec{1}_r\cdot\vec{1}_y\right]\overleftrightarrow{1}\right]$$

$$\left. - \frac{s^2\mu_o}{4\pi r}\left[\overleftrightarrow{1}_r - \vec{1}_y\,\vec{1}_r + \left[\vec{1}_r\cdot\vec{1}_y\right]\overleftrightarrow{1}\right]\right]\cdot\vec{p}_1$$

$$\overleftrightarrow{H} = \overleftrightarrow{H}_m + \overleftrightarrow{H}_p = e^{-\gamma r}\left[-\frac{1}{4\pi r^3}\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right]\cdot\vec{m}_1\right.$$

$$\left. - \frac{s}{4\pi r^2}\left[\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right]\cdot\frac{\vec{m}_1}{c} + \vec{1}_r\times\vec{p}_1\right] - \frac{s^2}{4\pi r}\frac{1}{c}\left[\overleftrightarrow{1}_r\cdot\frac{\vec{m}_1}{c} + \vec{1}_r\times\vec{p}_1\right]\right]$$

$$= e^{-\gamma r}\left[-\frac{1}{4\pi r^3}\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right]\right. \tag{2.14}$$

$$- \frac{s}{4\pi r^2}\frac{1}{c}\left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r - \vec{1}_y\,\vec{1}_r + \left[\vec{1}_r\cdot\vec{1}_y\right]\overleftrightarrow{1}\right]$$

$$\left. - \frac{s^2}{4\pi r}\frac{1}{c^2}\left[\overleftrightarrow{1}_r - \vec{1}_y\,\vec{1}_r + \left[\vec{1}_r\cdot\vec{1}_y\right]\overleftrightarrow{1}\right]\right]\cdot\vec{m}_1$$

(See [8] for appropriate vector/dyadic identities.) Here we see the symmetry in the expressions for $\vec{E}$ and $\vec{H}$, this being more transparent if we form $Z_0\vec{H}$ and use $\vec{m}_1/c$ to keep the units consistent with $\vec{p}_1$ (but not the orientation), and thereby make the coefficients of the various terms also agree. For example, we can see that the variation of $\vec{E}$ in the *yz* plane is the same as the variation of $\vec{H}$ in the *xy* plane.

In the $\vec{1}_y$ direction (beam center) the fields are

$$
\begin{aligned}
\vec{E} &= e^{-\gamma r}\left[-\frac{1}{4\pi r^3}\frac{1}{\varepsilon_o} - \frac{sZ_0}{2\pi r^2} - \frac{s^2\mu_o}{2\pi r}\right]\vec{1}_z\,\tilde{p}_1 \\
\vec{H} &= e^{-\gamma r}\left[-\frac{1}{4\pi r^3} - \frac{s}{2\pi r^2}\frac{1}{c} - \frac{s^2}{2\pi r}\frac{1}{c^2}\right]\vec{1}_x\,\tilde{m}_1 = \frac{1}{Z_0}\vec{1}_y \times \vec{E}
\end{aligned}
\tag{2.15}
$$

showing the TEM character of the fields, including the near-field terms. In the $-\vec{1}_y$ (back) direction the fields are

$$
\vec{E} = -\frac{e^{-\gamma r}}{4\pi r^3}\frac{\tilde{p}_1}{\varepsilon_o}\vec{1}_z \quad , \quad \vec{H} = -\frac{e^{-\gamma r}}{4\pi r^3}\tilde{m}_1\,\vec{1}_x = \frac{1}{Z_0}\vec{1}_y \times \vec{E}
\tag{2.16}
$$

Note that the fields here are also TEM, but the Poynting vector is in the $+\vec{1}_y$ direction, i.e., back toward the antenna.

Considering only the far field (the r^{-1} term) the radiated power has been shown [3, 5] to be proportional to

$$
\left[1+\cos\left(\theta_y\right)\right]^2 = \left[1 + \vec{1}_r \cdot \vec{1}_y\right]^2
\tag{2.17}
$$

where θ_y is the angle of the observer from the y axis. This is often called a cardiod pattern.

3. Far-Field Coupling Between Antennas

3.1 General considerations

As discussed in [3, 4] the far field radiated by an antenna (r^{-1} term) can be characterized by

$$\tilde{\vec{E}}_r = \frac{e^{-\gamma r}}{r} \tilde{\vec{F}}(\vec{1}_0, s) \tilde{V}_t \tag{3.1}$$

$\vec{1}_0 \equiv$ direction to observer

Here we take the form of the equations where the voltage $\tilde{V}_t$ is the "transmitted" voltage and the source impedance is assumed matched to the antenna impedance (resistive and frequency-independent in some interesting cases) [10].

As a receiver such an antenna has a response characterized by an effective height as

$$\tilde{V}_r = \tilde{\vec{h}}(\vec{1}_i, s) \cdot \tilde{\vec{E}}^{(inc)} \tag{3.2}$$

$\vec{1}_i \equiv$ direction of incidence toward antenna

where the load impedance is also assumed matched to the antenna impedance. Reciprocity relates transmission and reception as

$$\tilde{\vec{F}}(\vec{1}_0, s) = \frac{s\mu_0}{2\pi R} \tilde{\vec{h}}(-\vec{1}_0, s) \tag{3.3}$$

R = source impedance = load impedance = antenna input impedance

Note that this constraint on the impedance is not essential and will not affect our choices of antenna locations.

3.2 Electric dipoles

Considering both antennas are electric dipoles as in (2.11) we can identify from the r^{-1} term the transmission from antenna 1 via

$$\vec{1}_0 = \vec{1}_r$$

$$\vec{F}_p(\vec{1}_r,s) = -\frac{s^2\mu_o}{4\pi}\overleftrightarrow{1}_r \cdot \frac{\vec{p}_1}{\tilde{V}_t} = -\frac{s^2\mu_o}{4\pi}\overleftrightarrow{1}_r \cdot \vec{1}_z\tilde{f}_p = \frac{s^2\mu_o}{4\pi}\vec{1}_{\theta_z}\ \sin(\theta_z)\tilde{f}_p \tag{3.4}$$

$$\tilde{f}_p \equiv \frac{\tilde{p}_1}{\tilde{V}_t}$$

Here $\tilde{f}_p$ merely characterizes the linear property of the antenna in terms of electric-dipole moment per unit driving voltage. The angle θ_z is the usual spherical polar angle (i.e., θ) with respect to the electric-dipole orientation as in Fig. 2.1 and (2.8).

In reception an electric-dipole antenna at $\vec{r}$ with the same orientation as the first is characterized by

$$\vec{1}_i = -\vec{1}_r$$

$$\vec{h}_p(-\vec{1}_r,s) = \frac{2\pi R}{s\mu_o}\vec{F}(\vec{1}_r,s) = -\frac{sR}{2}\overleftrightarrow{1}_r \cdot \vec{1}_z\tilde{f}_p = \frac{sR}{2}\vec{1}_{\theta_z}\sin(\theta_z)\tilde{f}_p \tag{3.5}$$

Here the second antenna is assumed identical to the first giving the same $\tilde{f}_p$, but this is not necessary. Only the orientation is important for present purposes.

The response of antenna 2 to antenna 1 is then proportional to

$$\vec{F}_p(\vec{1}_r,s) \cdot \vec{h}_p(\vec{1}_r,s) = \frac{s^3R\mu_o}{8\pi}\sin^2(\theta_z)\tilde{f}_p^2 \tag{3.6}$$

which is zero on the z axis, i.e., at

$$\theta_z = 0,\pi \quad , \quad \vec{r} = z\vec{1}_z \quad , \quad r \neq 0 \tag{3.7}$$

So parallel electric dipoles do not couple in the far-field approximation if they are *colinear*.

3.3 Magnetic dipoles

The case of two parallel magnetic dipoles is dual to that of Section 3.2. Here we have in transmission (from (2.12))

$$\begin{aligned}
\vec{1}_0 &= \vec{1}_r \\
\vec{\tilde{F}}_m(\vec{1}_r,s) &= \frac{s^2\mu_0}{4\pi c}\vec{1}_r \times \vec{1}_x \tilde{f}_m \\
&= \frac{s^2\mu_o}{4\pi c}\left[-\sin(\theta_m)\cos(\theta_m)\vec{1}_y - \sin(\theta_m)\sin(\theta_m)\vec{1}_z\right]\tilde{f}_m \\
&= -\frac{s^2\mu_o}{4\pi c}\sin(\theta_m)\vec{1}_{\phi_m} \\
\tilde{f}_m &\equiv \frac{\tilde{m}_1}{\tilde{V}_t}
\end{aligned} \tag{3.8}$$

using the spherical coordinates in (2.9) based on the magnetic-dipole orientation.

In reception we have

$$\vec{\tilde{h}}_m(-\vec{1}_r,s) = \frac{2\pi R}{s\mu_o}\vec{\tilde{F}}_m(\vec{1}_r,s) = -\frac{sR}{2c}\sin(\theta_m)\vec{1}_{\phi_m}\tilde{f}_m \tag{3.9}$$

The response of antenna 2 to antenna 1 is then proportional to

$$\vec{\tilde{F}}_m(\vec{1}_r,s) = \vec{\tilde{h}}_m(\vec{1}_r,s) = \frac{s^3\mu_o R}{8\pi c^2}\sin^2(\theta_m)\tilde{f}_m \tag{3.10}$$

which is zero on the x axis, i.e., at

$$\theta_m = 0,\pi \quad , \quad \vec{r} = x\vec{1}_x \quad , \quad r \neq 0 \tag{3.11}$$

Parallel magnetic dipoles then do not couple in the far-field approximation if they are colinear. Note that this result is inconsistent with that for electric dipoles in (3.7) with electric dipoles on the z axis but magnetic dipoles on the x axis. With the given dipole orientations then we cannot simultaneously match both conditions.

3.4 Combined dipoles

Now combine the electric and magnetic dipoles for both antennas 1 and 2 in the balanced form discussed in Section 2.3. From (2.14) we have from the r^{-1} term the antenna transmission as

$$
\begin{aligned}
\vec{1}_0 &= \vec{1}_r \\
\vec{\tilde{F}}_c(\vec{1}_r,s) &= -\frac{s^2\mu_o}{4\pi}\left[\overleftrightarrow{1}_r - \vec{1}_y\vec{1}_r + \left[\vec{1}_r \cdot \vec{1}_y\right]\overleftrightarrow{1}\right] \cdot \vec{1}_z \tilde{f}_c \\
&= -\frac{s^2\mu_o}{4\pi}\left[\overleftrightarrow{1}_r \cdot \vec{1}_z - \vec{1}_r \times \vec{1}_x\right]\tilde{f}_c \\
\tilde{f}_c &= \frac{\tilde{p}_1}{\tilde{V}_t} = \frac{\tilde{m}_1}{c\tilde{V}_t}
\end{aligned}
\tag{3.12}
$$

From reciprocity an antenna at $\vec{r}$ with the same orientation as the first is characterized by

$$
\begin{aligned}
\vec{1}_i &= \vec{1}_r \\
\vec{\tilde{h}}_c(-\vec{1}_r,s) &= \frac{2\pi R}{s\mu_o}\vec{\tilde{F}}_c(\vec{1}_r,s) \\
&= -\frac{sR}{2}\left[\overleftrightarrow{1}_r - \vec{1}_y\vec{1}_r + \left[\vec{1}_r \cdot \vec{1}_y\right]\overleftrightarrow{1}\right] \cdot \vec{1}_z \tilde{f}_c \\
&= -\frac{sR}{2}\left[\overleftrightarrow{1}_r \cdot \vec{1}_z - \vec{1}_r \times \vec{1}_x\right]\tilde{f}_c
\end{aligned}
\tag{3.13}
$$

The response of antenna 2 to antenna 1 is then proportional to

$$
\begin{aligned}
&\vec{\tilde{F}}_c(\vec{1}_r,s) \cdot \vec{\tilde{h}}_c(\vec{1}_r,s) \\
&= \frac{s^3R}{8\pi}\tilde{f}_c^2\left[\overleftrightarrow{1}_r \cdot \vec{1}_z - \vec{1}_r \times \vec{1}_x\right] \cdot \left[\overleftrightarrow{1}_r \cdot \vec{1}_z + \vec{1}_r \times \vec{1}_x\right] \\
&= \frac{s^3R}{8\pi}\tilde{f}_c^2\left[\vec{1}_r \cdot \overleftrightarrow{1}_r \cdot \vec{1}_z - \left[\vec{1}_r \times \vec{1}_x\right] \cdot \left[\vec{1}_r \times \vec{1}_x\right]\right] \\
&= \frac{s^3R}{8\pi}\tilde{f}_c^2\left[\vec{1}_z \cdot \overleftrightarrow{1}_r \cdot \vec{1}_z - \left[\vec{1}_r \cdot \vec{1}_r\right]\left[\vec{1}_x \cdot \vec{1}_x\right] + \left[\vec{1}_r \cdot \vec{1}_x\right]\left[\vec{1}_r \cdot \vec{1}_x\right]\right] \\
&= \frac{s^3R}{8\pi}\tilde{f}_c^2\left[\vec{1}_z \cdot \overleftrightarrow{1}_r \cdot \vec{1}_z - 1 + \vec{1}_x \cdot \left[\vec{1}_r\vec{1}_r\right] \cdot \vec{1}_x\right]
\end{aligned}
$$

$$= \frac{s^3 R}{8\pi} \tilde{f}_c^2 \left[- \vec{1}_z \cdot \left[\vec{1}_r \vec{1}_r \right] \cdot \vec{1}_z + \vec{1}_x \cdot \left[\vec{1}_r \vec{1}_r \right] \cdot \vec{1}_x \right] \tag{3.14}$$

which is zero iff

$$\vec{1}_r \cdot \vec{1}_z = \pm \vec{1}_r \cdot \vec{1}_x \tag{3.15}$$

This corresponds to two planes given by

$$x = \pm z \tag{3.16}$$

So one may position antenna 2 anywhere on these two planes as in Fig. 3.1, except at $\vec{r} = \vec{0}$ (or near here due to other moments associated with real antennas). One can, in principle, place antenna 2 on the $+y$ axis, but this may not be advisable due to high frequency blockage of antenna 1. At high frequencies an impulse radiating antenna [10] has a narrow beam and antenna 2 is best not located here since the electric and magnetic response is not in general perfectly balanced in a real antenna due to various imperfections in its construction. Using reciprocity it may be similarly inadvisable to locate antenna 2 on the $-y$ axis. This still leaves many locations on the $x = \pm z$ planes.

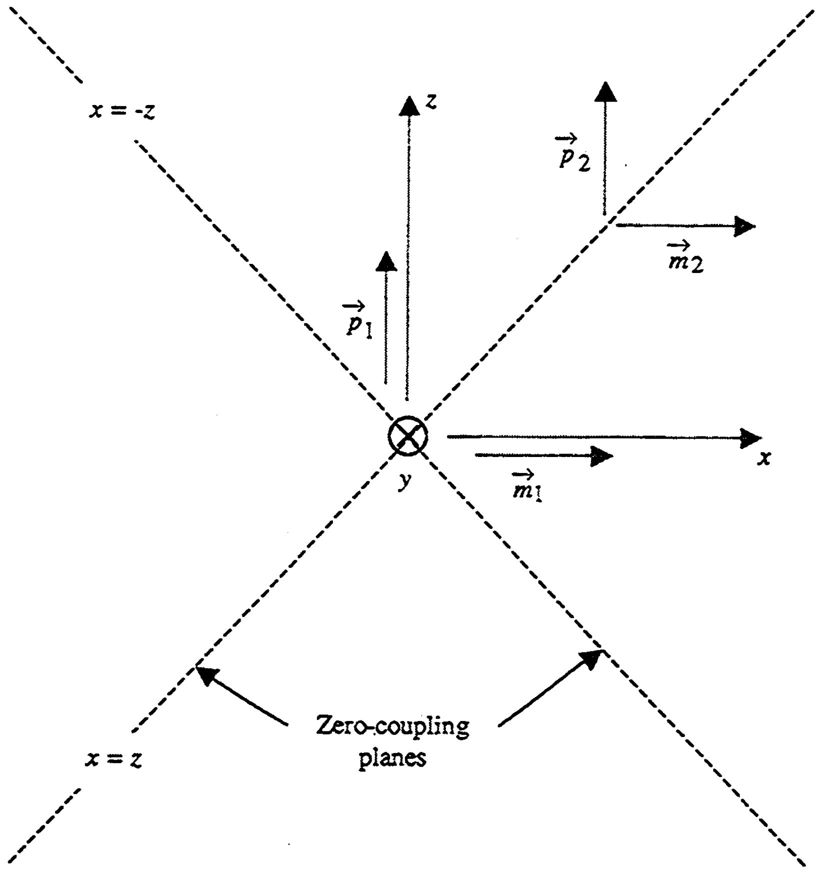

Fig. 3.1 Planes of Noncoupling Combined Dipoles in the Far-Field Approximation.

4. Near-Field Coupling Between Antennas

4.1 General considerations

As one goes down in frequencies such that the distance r between the two antennas becomes comparable to a radian wavelength, the near field terms (r^{-2} and r^{-3} in (2.10) and (2.11)) become important. Some of these have angular dependences different from those of the r^{-1} terms. Note, however, that the r^{-3} terms have the same angular dependence as the r^{-2} terms in both $\vec{E}_p$ and $\vec{H}_m$. Similarly r^{-2} and r^{-1} terms have the same angular dependence in both $\vec{H}_p$ and $\vec{E}_m$.

So let us consider the coupling of these additional terms between the two antennas. First, we have $\vec{p}_1$ to $\vec{p}_2$ and $\vec{m}_1$ to $\vec{m}_2$ coupling, followed by mixed dipole pairs, and finally by combined dipoles.

4.2 Electric dipoles

Consider first the coupling of r^{-2} and r^{-3} terms in $\vec{E}_p$, designated by superscript 2, to $\vec{p}_2$. This is proportional to

$$\vec{E}_p^{(2)} \cdot \vec{p}_2 = -e^{-\gamma r}\left[\frac{1}{4\pi r^3}\frac{1}{\varepsilon_o} + \frac{sZ_o}{4\pi r^2}\right]\vec{p}_1 \cdot \left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right] \cdot \vec{p}_2 \tag{4.1}$$

The angular part is

$$\vec{1}_z \cdot \left[\overleftrightarrow{1} - 3\,\vec{1}_r\,\vec{1}_r\right] \cdot \vec{1}_z = 1 - 3\cos^2(\theta_z) \tag{4.2}$$

which is zero at [2]

$$\begin{aligned} \cos(\theta_z) &= \pm\frac{1}{\sqrt{3}} \\ \theta_z &\simeq 54.74^\circ \quad , \quad 125.26^\circ \end{aligned} \tag{4.3}$$

This solution describes two circular cones coaxial with the z axis. For a given r this describes two circles of radius $r\sqrt{2/3}$ on planes given by

$$z = r\cos(\theta_z) = \pm\frac{r}{\sqrt{3}} \tag{4.4}$$

Note that this solution is in conflict with that for the far field for an electric-dipole pair in (3.7) for which $\theta_z = 0^\circ, 180^\circ$. However, it is *not* in conflict with the far-field solution for combined dipoles in (3.16) with $x = \pm z$. These two solutions intersect at

$$\begin{aligned} z^2 &= r^2\cos^2(\theta_z) = r^2\sin^2(\theta_z)\cos^2(\theta_z) \\ \cos(\phi_z) &= \cot(\theta_z) = \frac{1}{\sqrt{2}} \\ \phi_z &= 45^\circ, \ 135^\circ, \ 225^\circ, \ 315^\circ \end{aligned} \tag{4.5}$$

Together with the two values of z (for fixed r) this gives eight points of common solution.

4.3 Magnetic dipoles

Similarly the coupling of r^{-2} and r^{-3} terms in $\vec{H}_m$ (superscript 2) to $\vec{m}_2$ is proportional to

$$\vec{H}_m^{(2)} \cdot \vec{m}_2 = -e^{-\gamma r}\left[\frac{1}{4\pi r^3} + \frac{s}{4\pi r^2}\frac{1}{c}\right]\vec{m}_i \cdot \left[\overleftrightarrow{1} - 3\vec{1}_r\vec{1}_r\right] \cdot \vec{m}_2 \tag{4.6}$$

The angular part is

$$\vec{1}_x \cdot \left[\overleftrightarrow{1} - 3\vec{1}_r\vec{1}_r\right] \cdot \vec{1}_x = 1 - 3\cos^2(\theta_x) \tag{4.7}$$

which is zero at [2]

$$\begin{aligned} \cos(\theta_x) &= \pm\frac{1}{\sqrt{3}} \\ \theta_x &= 54.74^\circ, \ 125.26^\circ \end{aligned} \tag{4.8}$$

where now the spherical coordinates are defined with respect to the x axis as in (2.3)

This solution is just like that in Section 4.2 except that the solution circles are coaxial with the x asis. Merely replace the z subscripts with x subscripts giving

$$
\begin{aligned}
x &= r\cos(\theta_x) = \pm\frac{r}{\sqrt{3}} \\
x^2 &= r^2\cos^2(\theta_x) = r^2\sin^2(\theta_x)\cos^2(\theta_x) \\
\cos(\phi_x) &= \cot(\theta_x) = \frac{1}{\sqrt{2}} \\
\phi_x &= 45^\circ \, , \, 135^\circ \, , \, 225^\circ \, , \, 315^\circ
\end{aligned}
\tag{4.9}
$$

Noting that $\theta_z = \theta_x$ for points of common solution, this says that the eight points of common solution are common between (4.5) and (4.9).

In Fig. 2.1 we see that looking parallel to the y axis both $\vec{p}_1$ to $\vec{p}_2$ coupling and $\vec{m}_1$ to $\vec{m}_2$ coupling in the near field are zero for $\vec{r}$ on the $x = \pm z$ planes, but not on the $y = 0$ plane. Note that the direction cosines have

$$
\begin{aligned}
\cos^2(\theta_x) + \cos^2(\theta_y) + \cos^2(\theta_z) &= 1 \\
\cos(\theta_y) &= \pm\frac{1}{\sqrt{3}}
\end{aligned}
\tag{4.10}
$$

So the common solution points lie on four lines equally spaced between the coordinate axes. Said another way, the solutions lie on eight directions with the three direction cosines taking all eight choices of combinations of $\pm 1/\sqrt{3}$.

4.4 Mixed terms

Since electric dipoles produce magnetic fields, and conversely, we need to consider the response of the magnetic part of antenna 2 to $\vec{p}_1$ and the electric part of antenna 2 to $\vec{m}_1$. If we consider these couplings separately, we find that $\vec{m}_2$ needs to be on the $y = 0$ plane for zero coupling, and that $\vec{p}_2$ needs to be on the $y = 0$ plane for zero coupling. These two requirements are consistent with each other, but inconsistent with the results of Section 4.2 and 4.3 for the $\vec{p}_1$ to $\vec{p}_2$ and $\vec{m}_1$ to $\vec{m}_2$ near-field coupling.

Another approach consists of summing these mixed terms. For this purpose one needs to adjust them to allow for the balance between $\vec{p}_n$ and $\vec{m}_n/c$, and normalize the magnetic field as $Z_o\vec{H}$. So we form

$$
-Z_o\vec{H}_p \cdot \frac{\vec{m}_2}{c} + \vec{E}_m \cdot \vec{p}_2
$$

$$= e^{-\gamma r}\left[\frac{s}{4\pi r^2} + \frac{s^2}{4\pi r}\frac{1}{c}\right]\left[\vec{1}_x \cdot \left[\vec{1}_r \times \vec{1}_z\right]\tilde{p}_1 \frac{\tilde{m}_2}{c} Z_o + \vec{1}_z \cdot \left[\vec{1}_r \times \vec{1}_x\right]\tilde{p}_2\, \tilde{m}_1\, \mu_o\right] \tag{4.11}$$

Notice the minus sign in the sum; this needs some explanation. This is related to the minus sign appearing in one of the two Maxwell equations. Depending on the convention used, there is a sign reversal on the electric- or magnetic-dipole term in transmission versus reception. This can be seen in some of the previous results. For example, the far-field coupling in Section 3 can be viewed from the r^{-1} terms in (2.11) and (2.12). Observe that for an observer on the y axis $\vec{E}_p \cdot \vec{\tilde{p}}_2$ has a minus sign and $\vec{H}_m \cdot \vec{\tilde{m}}_2$ also has a minus sign. However, these two terms *subtract* to give zero far-field response of antenna 2 on the y axis (Section 3). The reader may wish to delve into this more deeply.

Returning to (4.11) we see that the angular part of the response is

$$\begin{aligned}
&\vec{1}_x \cdot \left[\vec{1}_r \times \vec{1}_z\right] \div \vec{1}_z \cdot \left[\vec{1}_r \times \vec{1}_x\right] \\
&= \vec{1}_r \cdot \left[\vec{1}_z \times \vec{1}_x\right] + \vec{1}_r \cdot \left[\vec{1}_x \times \vec{1}_z\right] = \vec{1}_x \cdot \vec{1}_y - \vec{1}_x \cdot \vec{1}_y \\
&= 0
\end{aligned} \tag{4.12}$$

i.e., the mixed terms cancel everywhere (except $\vec{r} = \vec{0}$).

5. Conclusions

The pair of balanced combined dipoles with parallel $\overrightarrow{p}_n$ and parallel $\overrightarrow{m}_n$ does not couple from antenna 1 to antenna 2 (and conversely) under the conditions:

1. Far field terms. Zero coupling occurs if antenna 2 is located on the $x = \pm z$ planes (Section 3.4, Fig. 3.1).
2. Near field terms. Zero coupling occurs for both $\overrightarrow{p}_1$ to $\overrightarrow{p}_2$ and $\overrightarrow{m}_1$ to $\overrightarrow{m}_2$ (separately, Section 4.2 and 4.3) if antenna 2 is located such that the direction cosines with respect to the three coordinate axes are any combination of $+1/\sqrt{3}$ and $-1/\sqrt{3}$, thereby defining eight directions.
3. Mixed terms. Zero coupling occurs for the sum of $\overrightarrow{p}_1$ to $\overrightarrow{m}_2$ and $\overrightarrow{m}_1$ to $\overrightarrow{p}_2$ everywhere.

These three conditions can all be satisfied using the form in condition 2 above. For a given r this gives eight points at the intersections of the $x=\pm z$, $y=\pm x$, and $y=\pm z$ planes (excepting $\overrightarrow{r} = \overrightarrow{0}$), or for general r, four lines of intersection. For a given r the eight points are the corners of a cube centered at the origin with each face perpendicular to a coordinate axis. The mutual relation of these points is found in the O (octahedral) symmetry group.

Of course, various imperfections in the antennas mean that the coupling is not truly zero. This points out the desirability of closely matching p and m/c and making these at right angles in the antenna design. Likewise accurate relative positioning and orientation of the two antennas is required.

At high-frequencies for which the radian wavelength is less than the antenna dimensions, the antennas may not be characterized by only electric and magnetic dipoles. For example, impulse radiating antennas (IRAs) are designed to be highly directive at such frequencies [10]. Fortunately condition 2 above places the antennas such that neither is in the main beam of the other at such high frequencies. Nevertheless, there may be some residual coupling here from the higher-order multipoles. Further studies and perhaps use of absorbers between the two antennas may help. There is also the spacing r between the two antennas that one can choose. Larger r gives smaller coupling (especially for the near-field terms). However, r should be small compared to the distance to the target and large r may present mechanical difficulties.

Note that various other conductors such as coaxial cables connected to the antennas may increase the coupling. Use of chokes (inductance) to suppress currents on such conductors may help. Appropriate positioning of such conductors may also help. Replication of antenna 2 to occupy two or more of the eight optimal positions (and summing the signals) can add more symmetry to the ensemble and perhaps cancel some of the unwanted coupling. So the present paper is a first step in realizing practical designs.

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