System Design and Assessment Notes

Note 23

A Brief Presentation and Discussion of the Algorithm Used to Determine EC-135 EMP Margin Reliability-Confidence

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Abstract

In the assessment of EMP vulnerability of the EC-135 conducted by Rockwell Autonetics under contract to the Air Force Weapons Laboratory, not only was the EMP reliability of that system estimated but also the confidence level which should be assigned to that reliability estimate. This note presents and discusses the algorithm which was used to determine confidence level as a function of the amount of data available to support the reliability estimate.

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Introduction

In a reliability assessment program it is necessary to estimate the possible error in the margin estimation process. The reason for this is, if that error is greater than the nominal margin estimated then the conclusion that the system is hard remains in doubt. (That is if the nominal margin is positive. If it is negative, then the conclusion that the system is soft similarly remains in doubt.)

Boeing has recently begun referring to this possible error in margin estimation as "Data Quality". (Since the quality of the data is decreasing as the possible error increases, perhaps it would be more accurate to refer to this possible error instead as "data unquality".) Regardless of the labels used, it is an inescapable fact that possible error in the data must always be itself estimated using a finite amount of less than perfect data. Because the amount of error data is finite, the confidence in the error estimate is less than unity. In fact, this confidence is rigorously calculable as a function of how many observations (and other pieces of supplementary information) are available to support the total error estimate. When this confidence is calculated, it is in practice often found to be sufficiently less than unity (typically .9 or less) to require that the confidence level be advertised along with the total error estimate.

This was in fact done on the EC-135 EMP assessment program. The purpose of this note is to provide a brief presentation and discussion of the algorithm used to compute the EMP reliability-confidence in that program.

An algorithm for determining reliability-confidence

1. List all n independent, additive, zero mean margin error sources. (If there exist significant error sources not on this list, STOP. In this case it would be necessary to devise a different algorithm.)
2. For each margin error source i:
   a. Determine the empirical histogram (i.e., sample density) of errors.
   b. Obtain estimate s_i of standard deviation σ_i at confidence level C_i. Begin by GOF (goodness of fit) testing the histogram for normality. Then:
      α. If clearly passes test, compute

$$s_i = \frac{\sqrt{\sum_{j=1}^{N_i} (x_{ij}-\bar{x}_i)^2}}{X_{N_i-1;1-C_i}}$$

      and then go to step 2.b.γ.

      β. If GOF test equivocal or results in rejection, then use the extreme value statistic for s_i. Compute nonparametric confidence

$$C_i(.68) = 1 - .68^{L_i+1}$$

      (where L_i was the total number of observed sample values in the histogram of margin error source i) that probability is at least .68 that the true absolute value of the error from that source is less than or equal to s_i.

      γ. Compute $\hat{V}_i = s_i^2$.

3. Compute $\hat{V} = \sum^{n} \hat{V}_i$.
4. Compute $s = \sqrt{\hat{V}}$.
5. Assume n sufficiently large so Central Limit Theorem makes total (sum) error sufficiently approximately normal.
6. Accept s as upper bound on standard deviation σ for this total error distribution.

(Continued)

7. Compute margin lower bound $m_R \triangleq m_{.5} - zs$, where $m_{.5}$ is the measured (i.e., "nominal") value of margin and z is an (invertible) function of reliability R given by the standard normal (one-tailed) "z table".
8. Assign to m_R the confidence level $C \triangleq \min\{C_i\}$. (This relies upon Reference [R4].)

9. To obtain system RC (reliability-confidence), for each mission determine specific functional relationship (e.g., specific series-parallel relationship) between subsystems, and then perform the appropriate probability-confidence computations.

Notes on the algorithm as applied in the EC-135 program

These notes are numbered to correspond to the algorithm step to which each refers.

1. For EC-135 $n = 7$, with the seven significant error sources (independent and additive in dB) being:
   a. external H fields,
   b. analytic cable currents,
   c. impedance ratio calculations,
   d. measurement error,
   e. simulation error,
   f. power on-off, and
   g. threshold calculations.
2. The equation for nonparametric confidence given in 2.b.β. is derived in Reference [R1], where it appears as equation (10). (Alternatively, a close approximation to this equation appears as equation (13.6.4) on p. 219 of Reference [R3].)

   In the EC-135 program the sample size was sufficiently large for each error source listed above to permit $C_i \geq .9$ for all $i \in \{1,\ldots,7\}$.

3 to 7. The procedure given says to compute an estimate s of the standard deviation σ of the total error, and then multiply by z to capture R or more of the population. It is equivalent to estimate zs_i (instead of just s_i) for each individual error source in step 2.b., and then replace steps 2.b.γ. through 4. by:

2.b.γ'. Compute $z^2\hat{V}_i = z^2 s_i^2 = (zs_i)^2$.

3'. Compute $z^2\hat{V} = z^2\Sigma\hat{V}_i = \Sigma(z^2\hat{V}_i)$.

4'. Compute $zs = z\sqrt{\hat{V}} = \sqrt{z^2\hat{V}}$.

The result of this version of step 4. is then available to plug into step 7. directly (still subject to steps 5. and 6., of course).

The basic algorithm, therefore, is referred to as "the RSS (root sum square) procedure", since the zs_i are squared in step 2.b.γ'., summed in step 3'., and rooted in step 4'. The same RSS procedure is used in the unprimed steps 2.b.γ., 3., and 4., the only difference being that in the unprimed steps $z = 1$ (until step 7.; so that the capture probability is 68%, until step 7.).

7. In the EC-135 program the reliability (minimum capture probability) value investigated was $R = .95$ (one sided).

As a result of this value for R, the value of z used in step 7. (or in step 2.b.) was $z = 1.64$.

At this point a further refinement becomes possible in the algorithm. In step 2.b.α. the quantity zs_i (instead of just s_i) can be estimated to give a high confidence $C_i = .9$ estimate for 90% (two sided) capture probability ($R = .9$); this zs_i is then available to be plugged into the primed steps described in the note immediately above. So far the procedure is exactly the same as described in the above note. The further refinement has to do with how the 90% confidence 90% reliability value is obtained in step 2.b.β. In the EC-135 program it turned out that, whenever it was necessary to resort to that nonparametric step, the size L_i of the sample from which the extreme value statistic was drawn was always large enough so that the computation $C_i(.9) = 1-(.9)^{L_i+1}$ yielded a confidence C_i of .9 or greater. Therefore in such cases it was possible to use the extreme value statistic unchanged for the 90% confidence 90% reliability value of zs_i (instead of calling the extreme value statistic just s_i, and having to multiply it by $z = 1.64$).

Incorporating this refinement, the primed portion of the algorithm then becomes:

2.b.α'. If clearly passes GOF test, compute s_i as in the unprimed step 2.b.α., multiply by $z = 1.64$, and then go to step 2.b.γ.

2.b.β'. Otherwise, use the extreme value statistic for zs_i. Compute nonparametric confidence

$$C_i(.9) = 1 - .9^{L_i+1}$$

that probability is at least .9 that the absolute value of the error from that source is less than or equal to zs_i.

2.b.γ'. Compute $z^2\hat{V}_i = (zs_i)^2$.

3'. Compute $z^2\hat{V} = \Sigma(z^2\hat{V}_i)$.

4'. Compute $zs = \sqrt{z^2\hat{V}}$.

(Continued)

In the EC-135 program it is the primed version of these steps in the basic RSS algorithm which the contractor has chosen to use for the assessment and which therefore appears in the final documentation.

8. As a result of the last sentence in the note above on algorithm step 2, in the EC-135 program the overall confidence level for each subsystem was $C = .9$.

9. For example, cf. Reference [R2]. It may be possible to approximate the RC of a series path with the dominantly lowest RC in that path, if there is such a dominant path element; similarly the RC of a path of parallel elements can be approximated by a dominantly largest parallel element RC.

Weaknesses in the algorithm

Following is a brief presentation and discussion of three weaknesses in the algorithm, for which it would be desirable to supply advances in theory for future applications of the algorithm to system assessments.

A. Estimation of σ_i using algorithm step 2.b.β.

At step 2.b.γ. of the algorithm $\hat{V}_i$ is taken as an estimate of the variance V_i of error source i regardless of whether the s_i used to compute $\hat{V}_i$ in step 2.b.γ. was calculated according to the procedure in step 2.b.α. or according to that in step 2.b.β. There is no difficulty if s_i came from step 2.b.α., since s_i is then really a lower bound estimator (if $C_i > .5$) of σ_i. However, if the error distribution does not appear to be normal, then the classical interval estimate of step 2.b.α. cannot be used since it assumes normality (although robustness may provide some protection if it is carelessly used despite the invalidity of this assumption). In that case we should therefore fall back on the procedure for generating s_i described in step 2.b.β., and using the s_i generated by that step as an estimate of σ_i implicitly relies upon the following assumption:

> 2.b.β assumption: The error distribution of error source i has the property that one standard deviation σ_i "captures" approximately 68% of error population i.

The normal distribution of course satisfies this assumption. Unfortunately, however, it is in theory possible for the error source being treated by step 2.b.β. to have an error distribution which does not satisfy the assumption. As a simple counterexample, consider a distribution for which σ_i is infinite, such as the Cauchy distribution; in such a case σ_i would capture 100% (instead of just 68%) of the population.

In the EC-135 program we nonetheless did use this implicit 2.b.β assumption, and then relied upon two additional facts to guard the program from any significant underestimation of σ by s in algorithm step 5. These two facts were:

a. Only one of the seven error sources in the EC-135 program *required* step 2.b.β. to estimate s_i (although, as pointed out in the discussion of weakness B., below, the computations of that step were done even when not required, as a precaution, in several cases in addition to the required computations); for the other six sources the GOF test in step 2.b. was passed. It is therefore beleved that any small errors in s_i for that one source would not significantly affect the value of s computed in step 5 using all sources.

b. The sample size for the single error histogram which required step 2.b.β. was sufficiently large so that, for $C_i = .9$, the extreme value statistic had a minimum capture fraction (probability) appreciably greater than .68. This, together with the moderate kurtosis of the normal distribution, makes us feel the estimator s_i for that error source was more likely to be an overestimate than an underestimate if the assumption was not satisfied in that case.

B. Mixing kinds of confidence in step 8.

Assuming Reference [R4] guarantees nondecreasing confidence when individual errors are combined (which it does not; cf. weakness C., below), it is still desirable that the term "confidence" be used in the same sense for all C_i and for C. However, if some C_i come from algorithm step 2.b.α. and others from step 2.b.β., then this is not the case: the former were in fact classical confidences while the latter were Bayesian. Consequently it would be good to generate an extension of Reference [R4] which generalizes its conclusions to kinds of mathematical confidence which do not depend upon normality.

To guard against any damage to the EC-135 reliability-confidence assessment conclusions from this weakness in the algorithm, the following extra precaution was taken in the application of the algorithm in that program. For each error source for which there was concern that C_i might be inappropriate for use in step 8., C_i was computed using *both* steps 2.b.α. and 2.b.β. to confirm that the s_i used in step 2.b.γ. warranted a confidence of at least .9 *regardless* of which kind of confidence was used.

C. Incompleteness of the demonstration in Reference [R4].

We relied upon [R4] to justify the nondecreasing confidence C generated by algorithm step 8. However, not only does [R4] restrict itself to the normal case, it does not even provide a general proof for that case. Rather, it concludes by supplying tabular results for a few specific normal distribution *examples*. Now, we believe Reference [R4] is acceptable as far as it goes and given the time and program pressure in which it was produced. But it needs to be generalized, possibly first to a proof covering the general normal case and then to providing a statement of the minimum amount of supplementary information which must be available to justify rigorously the assumption of nondecreasing confidence.

As Reference [R4] points out, it is in theory possible to postulate error distributions for which the combined confidence will be *less* than the individual C_i, possibly as low as $\prod_{i=1}^{n} C_i$. This fact is recognized in Reference [R2], and was deferred to in the RFP for the E-3A EMP assessment. However, in the EC-135 program we nonetheless decided to assume nondecreasing confidence as given in algorithm step 8., and relied upon two additional facts to guard the program from any significant overestimation of C in that step. These two facts were:

a. Almost all of the individual error distributions in the EC-135 program passed the normal GOF test in algorithm step 2.b. Therefore we felt that even if Reference [R4] could be generalized only as far as the family of normal distributions (which we suspect is a plausible conjecture), nonetheless the small fraction of error sources in this program which were non-normal would not be sufficient to significantly depress C.

b. After examining the development of Reference [R4] carefully, we conjecture that it may be possible to generalize the conclusion of nondecreasing confidence to (at least) the family of unimodal densities. If this conjecture is true, then the fact that *all* error sources in the EC-135 program gave strong evidence of being unimodal (even if not normal) means that probably C was not overestimated by even a small amount.

Conclusion

The margin reliability-confidence algorithm used in the EC-135 EMP assessment program could be improved by additional research on its theoretical foundations. However, it is reasonable to assume that the data produced and extra precautions used by this particular program protect the conclusions of that program from any serious deterioration in credibility arising from the theoretical weaknesses in the algorithm discussed above.

Footnote
On applying the algorithm in other programs;
or,
Other issues not addressed herein

In actual practice step 1 of the algorithm can be difficult to execute. For example, engineering judgement is necessary to determine when data from different sources can and cannot responsibly be combined to form a single union sample for input to step 2.a. As another example, in the EC-135 program it was decided that the effective independence of the error sources should be checked by performing a Monte Carlo calculation to verify the RSS procedure's output zs. (This was done, and the corroboration obtained. Cf. Reference [R5].)

In addition, the "STOP" in step 1 may entail a great deal of soul searching. If it is decided that the conditions stated in step 1 are not nearly enough satisfied, then a program would have to begin to wrestle with the difficulties implicit in the blithe statement, "In this case it would be necessary to devise a different algorithm." For example, there may be an error source which is not considered sufficiently additive; e.g., threshold may be deemed to be too strongly a function of frequency, as opposed to being simply a function of time domain peak amplitude (as was considered adequate in the EC-135 program). As another example, if nonlinearities in coupling are considered to be very important then margin is not simply the sum of threshold and (minus) the linearly corrected and extrapolated simulated current.

Finally, other steps in the algorithm also could require considerably greater effort in another program than was the case with EC-135. For example, "the appropriate probability-confidence computations" in step 9 could be very complicated if "the specific functional relationship between subsystems" in the new program could not be well approximated by a simple series-parallel network.

References

[R1] Chris Ashley, *Confidence and Reliability in an Infinite Population.* EMP Probability and Statistics Note 2; 7 October 1971.

[R2] Chris Ashley, *Confidence in System Reliability.* EMP Probability and Statistics Note 6; 8 February 1973.

[R3] Irwin Guttman and S.S. Wilks, *Introductory Engineering Statistics.* John Wiley & Sons, Inc.; copyright 1965.

[R4] James V. Locasso, *The Confidence in Combinations of Imperfectly Known Variances.* EMP Probability and Statistics Note 9; 8 July 1977.

[R5] James V. Locasso, C.F. Juster, W.H. Cordova, J.F. Wagner, and B.A. Daiken, *EC-135 EMP Assessment Program Error Analysis.* Air Force Weapons Laboratory TR-77-254, April 1978.