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RELIABILITY THEORY AND APPLICATIONS TO ELECTROMAGNETIC PULSE TESTING

by

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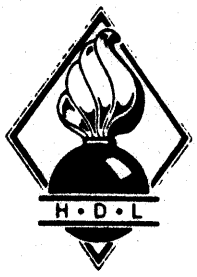

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ABSTRACT

Methods for calculating the reliability of EMP-perturbed systems are presented and discussed. Formulas for the reliability of components subjected to random stresses are derived with the aid of the theory of discrete Markov processes. Techniques for evaluating the formulas are illustrated by examples. The reliability of systems subjected to certain types of failure and repair is computed. Techniques for solving the resulting differential equations are derived. The theory of structure reliability in terms of component reliability is extended to include continuous Markov processes which arise in the consideration of electromagnetic-pulse (EMP) perturbations of a complex structure. Classical methods for evaluating reliability functions for communications systems are presented and their applicability to EMP problems considered. The work was sponsored by the joint DCA/DNA Program for EMP testing (PREMPT).

CONTENTS

1. INTRODUCTION

These notes were prepared for a series of talks on reliability theory, given as part of a "stochastic processes workshop" at the Electromagnetic Effects Laboratory (EMEL) of Harry Diamond Laboratories. One programmatic goal of EMEL is to determine the reliability of communication nodes of the Defense Communication System when subjected to electromagnetic pulses (EMP) arising from high-altitude nuclear explosions. The talks have been directed toward solving some problems that arise in this field, while introducing the basic concepts in reliability theory. Since most participants of this workshop had postgraduate training in mathematics and physics, no attempt was made to keep the discussions at a low level of sophistication. If any technique appeared to be useful, it was used without presenting any background material.

The first problem concerns a possible approach to calculating the degradation in reliability of an electrical component when it is subjected to random electrical stresses such as might arise in an EMP attack.

The second problem treats a communication node as a set of well-defined subsystems subject to periodic failure and repair. This leads to an enumeration of the possible states of the system and consideration of a set of linear differential equations describing transformations among these states.

The third problem deals with the relationship between the reliability of the components of a structure and that of the structure itself. This discussion is mostly concerned with the formal theory of the subject but it leads quite naturally to the study of the reliability of large, extended networks and some of the real problems associated with any meaningful calculation.

2. RELIABILITY OF AN ELECTRICAL COMPONENT SUBJECTED TO RANDOM ELECTRICAL STRESSES

This section defines a model that will allow the calculation of the reliability of an electrical component of a structure subjected to a series of EMP events. We assume that the EMP events arrive randomly at the structure at a given average rate. The effect of the EMP is to produce an electrical surge at some or all of the inputs to the component. We further assume that we can characterize the component by a finite number of failure probability levels, to each of which we can assign a failure rate. These failure levels could represent the different electronic states of the component (as in a digital logic circuit), some of which would have different susceptibilities to failure during an electrical surge. Since the failure rates depend on the strength of an EMP event, they should be continuous functions for each electronic state. The calculation is greatly simplified if the continuous function is replaced by some average value. The continuous distribution is discussed in section 4.8. By suitably defining transition rates between levels and the probabilities associated with the occurrence of the internal states, we then provide methods for calculating, as a function of time, the average probability for the component to be operative.

2.1 Derivation of the Cumulative Distribution for the Time to Failure

To obtain the probability function, we must define various quantities. Let

P_1 = the probability that no failure of a component occurred prior to or at time t

P_2 = the probability that failure of a component occurs between times t and $t + dt$

P_{21} = the conditional probability that a failure of a component occurs between t and $t + dt$, given that no failure occurs prior to or at time t

P_{12} = the conditional probability that no failure of a component has occurred prior to or at time t, given that a failure has occurred between t and $t + dt$.

It is clear that $P_{12} = 1$ since we do not admit processes such as repair and return to service.

Noting that we have two dependent events, "1" and "2," the conditional and unconditional probabilities are related by (Bayes' theorem)

$$P_{21} \times P_1 = P_{12} \times P_2 . \tag{1}$$

Let $F(t, \gamma)$ be the probability of failure occurring prior to or at time t if the failure rate is γ. Then

$$P_1 = 1 - F(t, \gamma). \tag{2}$$

P_2 is given by the probability density function $f(t, \gamma)$ (associated with the cumulative distribution function $F(t, \gamma)$) times the increment of time dt

$$P_2 = f(t, \gamma)dt = \frac{dF(t, \gamma)}{dt}\, dt. \tag{3}$$

P_{21} is given by the product of the failure rate $\gamma(t)$ (sometimes called the hazard rate) and the interval dt

$$P_{21} = \gamma(t)dt. \tag{4}$$

From equations (1) through (4)

$$\gamma(t)dt = \frac{f(t, \gamma)dt}{[1 - F(t, \gamma)]} . \tag{5}$$

Integrating equation (5) and solving for $F(t, \gamma)$

$$F(t, \gamma) = 1 - \exp\left[-\int_0^t \gamma(t')dt'\right]. \tag{6}$$

If we have a constant failure rate $\gamma(t) = \gamma_0$, we obtain the familiar exponential time-to-failure law,

$$F(t, \gamma_0) = 1 - \exp(-\gamma_0 t). \tag{7}$$

By suitably choosing γ, other familiar distributions can be obtained. Some are tabulated in references 1 and 2.

2.2 Definition of the Averaged Probability Function

By suitably classifying the types of electrical stresses and internal states of a component, a set $|\gamma|$ of failure rates can be defined $|\gamma| = |\gamma_1, \gamma_2, \ldots, \gamma_q|$. For each pair of failure rates, a transition probability must be defined to describe the physical processes under consideration. For every failure rate, it is necessary to specify the a priori probabilities that it occurs. All these quantities are defined precisely in the following.

From our previous discussion it is clear that we need to evaluate the average of

$$\exp\left[-\int_0^t \gamma(t')dt'\right]$$

under the assumption that $\gamma(t)$ randomly assumes values in the finite set of failure rates $|\gamma_1, \gamma_2, \ldots, \gamma_q|$. Denoting the averaging process by $\langle\ \rangle$ and defining $p(t)$ as the averaged probability,

$$p(t) = \langle \exp\left[-\int_0^t \gamma(t')dt'\right] \rangle. \tag{8}$$

The average will be evaluated by assuming that $\gamma(t)$ is a stationary Markov function.

2.3 Properties of Homogeneous Markov Functions

A homogeneous random function $\gamma(t)$ is a Markov function if its probability of having value γ_2 at instant t, given that it had value γ_1 Δt seconds earlier, is independent of all values that the function might have had prior to the instant $t - \Delta t$. In addition this probability depends only on the time difference between the two events. Denote this probability by $W(\gamma_1, \gamma_2, \Delta t)$ and hypothesize that for small Δt

$$W(\gamma_1, \gamma_2, \Delta t) = \delta_{\gamma_1\gamma_2} + \pi(\gamma_1, \gamma_2)\Delta t, \tag{9}$$

$$W(\gamma_1, \gamma_2, \Delta t) = \delta_{\gamma_1\gamma_2}[1 - \Omega(\gamma_1)\Delta t] + \Omega(\gamma_1)P(\gamma_1, \gamma_2)\Delta t, \tag{10}$$

where $\Omega(\gamma_1)\Delta t$ is the total probability of going from value γ_2 to a different value during time Δt, and $P(\gamma_1, \gamma_2)$ is the relative probability that the final value is γ_2. Given the condition that

$$\sum_{k=1}^{q} W(\gamma_i, \gamma_k, \Delta t) = 1,$$

the following identities are apparent from equations (9) and (10):

$$\pi(\gamma_i, \gamma_j) = -\delta_{\gamma_i\gamma_j}\Omega(\gamma_i) + \Omega(\gamma_i)P(\gamma_i, \gamma_j),$$

$$P(\gamma_j, \gamma_j) = 0,$$

$$\sum_{k=1}^{q} P(\gamma_j, \gamma_k) = 1, \tag{11}$$

$$\sum_{k=1}^{q} \pi(\gamma_j, \gamma_k) = 0.$$

2.4 Evaluation of $\langle \exp\left[-\int_0^t \gamma(t')dt'\right]\rangle$

We now return to our evaluation of

$$\langle \exp\left[-\int_0^t \gamma(t')dt'\right]\rangle.$$

Divide the integral into n equal intervals and let $\gamma_{i_1}, \gamma_{i_2}, \ldots, \gamma_{i_n}$ be the values of the failure rates at the different instants, $t_1, t_2, \ldots, t_n$, where $t_j = jt/n$. Thus

$$\exp\left[-\int_0^t \gamma(t')dt'\right] = \lim_{n\to\infty} \exp\left[-\frac{t}{n}(\gamma_{i_1} + \gamma_{i_2} + \cdots + \gamma_{i_n})\right]. \tag{12}$$

Let $P(\gamma_{i_1}, t_1, \gamma_{i_2}, t_2, \ldots, \gamma_{i_n}, t_n)$ be the probability that the failure rate is γ_{i_1} at t_1, γ_{i_2} at t_2, ..., γ_{i_n} at t_n. The average (eq. 8) is obtained by summing the product of equation (12) and the probability P over all possible combinations of the γ's. Thus

$$p(t) = \lim_{n\to\infty} \sum_{i_1, i_2, \ldots, i_n=1}^{q} P(\gamma_{i_1}, t_1, \gamma_{i_2}, t_2, \ldots, \gamma_{i_n}, t_n) \exp\left[-\frac{t}{n}(\gamma_{i_1} + \gamma_{i_2} + \cdots + \gamma_{i_n})\right]. \tag{13}$$

Since the process is a stationary Markov process, the probability $P(\gamma_{i_1}, t_1, \gamma_{i_2}, t_2, \ldots, \gamma_{i_n}, t_n)$ can be written as a product of the probabilities $W(\gamma_{i_j}, \gamma_{i_{j+1}}, t/n)$, thus:

$$p(t) = \lim_{n\to\infty} \sum_{i_1, i_2, \ldots, i_n=1}^{q} W(\gamma_{i_1}) \cdot W(\gamma_{i_1}, \gamma_{i_2}, \tfrac{t}{n}) \cdots W(\gamma_{i_{n-1}}, \gamma_{i_n}, \tfrac{t}{n}) \exp\left[-\frac{t}{n}(\gamma_{i_1} + \gamma_{i_2} + \cdots + \gamma_{i_n})\right], \tag{14}$$

$$p(t) = \lim_{n\to\infty} \sum_{i_1, i_2, \ldots i_n=1}^{q} W(\gamma_{i_1}) \prod_{m=2}^{n} \left[\exp\left(-\frac{t}{n}\gamma_{i_{m-1}}\right) W\left(\gamma_{i_{m-1}}, \gamma_{i_m}, \frac{t}{n}\right)\right] \exp\left(-\frac{t}{n}\gamma_{i_n}\right). \quad (15)$$

$W(\gamma_{i_{m-1}}, \gamma_{i_m}, t/n)$ is the element of a $q \times q$ matrix represented by $\underset{\sim}{W}(t/n)$, $\exp[-(t/n)\gamma_{i_m}]$ is an element of a $q \times q$ diagonal matrix $\exp[-(t/n)\gamma]$ whose diagonal values are $\exp[-(t/n)\gamma_1]$, $\exp[-(t/n)\gamma_2], \ldots, \exp[-(t/n)\gamma_q]$. $W(\gamma_i)$ is the i^{th} component of a row vector of dimension q, specifically $\vec{W} = [W(\gamma_1), W(\gamma_2), \ldots, W(\gamma_q)]$. $W(\gamma_k)$ is the a priori probability of finding the system in the failure-rate state γ_k. As $n \to \infty, (t/n) \to 0$, and $\exp[-(t/n)\gamma_{i_n}] \to [\vec{1}]_{i_n}$, where $\vec{1}$ is a column vector with ones for each of its elements. Equation (15) is clearly in the form of a matrix product. Thus

$$p(t) = \lim_{n\to\infty} \vec{W} \cdot \left[\exp\left(-\frac{t}{n}\underset{\sim}{\gamma}\right)\underset{\sim}{W}\left(\frac{t}{n}\right)\right]^{n-1} \cdot \vec{1}. \quad (16)$$

Since t/n is small, in the limit $n \to \infty$,

$$\exp\left(-\frac{t}{n}\underset{\sim}{\gamma}\right)\underset{\sim}{W}\left(\frac{t}{n}\right) \sim \left(1 - \frac{t}{n}\underset{\sim}{\gamma}\right)\left(1 + \underset{\sim}{\pi}\frac{t}{n}\right) = 1 + \frac{t}{n}\left(-\underset{\sim}{\gamma} + \underset{\sim}{\pi}\right). \quad (17)$$

Therefore

$$\lim_{n\to\infty}\left[\exp\left(-\frac{t}{n}\underset{\sim}{\gamma}\right)\underset{\sim}{W}\left(\frac{t}{n}\right)\right]^{n-1} = \lim_{n\to\infty}\left[1 + \frac{t}{n}\left(-\underset{\sim}{\gamma} + \underset{\sim}{\pi}\right)\right]^{n-1} = \exp\left[t\left(-\underset{\sim}{\gamma} + \underset{\sim}{\pi}\right)\right]. \quad (18)$$

From equations (13) through (18)

$$p(t) = \vec{W} \cdot \exp\left[t\left(-\underset{\sim}{\gamma} + \underset{\sim}{\pi}\right)\right] \cdot \vec{1}. \quad (19)$$

Note that $p(t)$ is the average probability of no failure prior to and including time t. The quantity $p(t)$ is also defined to be the reliability of the component at time t.

The evaluation of $p(t)$ in any specific case is somewhat complicated by the term $\exp\{\underset{\sim}{A}t\}$ where $\underset{\sim}{A} = -\underset{\sim}{\gamma} + \underset{\sim}{\pi}$ is a square matrix of order q. By definition

$$\exp(\underset{\sim}{A}t) = \sum_{n=0}^{\infty} \frac{(\underset{\sim}{A}t)^n}{n!}, \quad (20)$$

so to determine $p(t)$ we must evaluate and sum a series of matrices. This is tedious at best, and impossible in most cases. The calculation can be simplified by evaluating the Laplace transform of $p(t)$, denoted by $p(s)$ where

$$p(s) = \vec{W} \cdot \left[\int_0^{\infty} \exp(-s\underset{\sim}{I}t) \exp(\underset{\sim}{A}t)dt\right] \cdot \vec{1}, \quad (21)$$

$$p(s) = \vec{W} \cdot (s\underset{\sim}{I} - \underset{\sim}{A})^{-1} \cdot \vec{1}. \tag{22}$$

Here $\underset{\sim}{I}$ is the $q \times q$ identity matrix.

After matrix inversion, we obtain $p(t)$ by inversion of the Laplace transform, usually by using a table of transforms.

2.5 Exact Solution of a Two-state Homogeneous Markov System

We now examine a tractable case of a homogeneous two-state system, where γ_1 is the failure rate in the normal operating state and γ_2 the rate in the EMP-excited state:

$$\underset{\sim}{\gamma} = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}. \tag{23}$$

The vector of a priori probabilities is of the form $W_1 = (P_1, P_2)$ where we expect that $P_1 >> P_2$. From the definition of the transition matrix π,

$$\pi(\gamma_\alpha, \gamma_\beta) = -\delta_{\gamma_\alpha \gamma_\beta} \Omega(\gamma_\alpha) + \Omega(\gamma_\alpha) P(\gamma_\alpha, \gamma_\beta), \tag{24}$$

we have

$$\underset{\sim}{\pi} = \begin{pmatrix} -\Omega_1, & \Omega_1 \\ \Omega_2, & -\Omega_2 \end{pmatrix}. \tag{25}$$

Here Ω_1 is the rate at which transitions take place from the normal state to the excited state and Ω_2 is the rate at which transitions take place from the excited state to the normal state.

The matrix $s\underset{\sim}{I} - \underset{\sim}{A}$ then becomes

$$s\underset{\sim}{I} - \underset{\sim}{A} = \begin{bmatrix} s + \gamma_1 + \Omega_1, & -\Omega_1 \\ -\Omega_2, & s + \gamma_2 + \Omega_2 \end{bmatrix}. \tag{26}$$

The inverse of the matrix $s\underset{\sim}{I} - \underset{\sim}{A}$ is

$$(s\underset{\sim}{I} - \underset{\sim}{A})^{-1} = \{1/[(s + \gamma_1 + \Omega_1)(s + \gamma_2 + \Omega_2) - \Omega_1 \Omega_2]\} \begin{bmatrix} s + \gamma_2 + \Omega_2, & \Omega_1 \\ \Omega_2, & s + \gamma_1 + \Omega_1 \end{bmatrix}. \tag{27}$$

The Laplace transform of $p(t)$ is

$$p(s) = (P_1, P_2)(s\underset{\sim}{I} - \underset{\sim}{A})^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{P_1(s + \gamma_2 + \Omega_1 + \Omega_2) + P_2(s + \gamma_1 + \Omega_1 + \Omega_2)}{(s + \gamma_1 + \Omega_1)(s + \gamma_2 + \Omega_2) - \Omega_1 \Omega_2}$$

$$\equiv (\lambda s + \mu)/(s + \alpha)(s + \beta). \tag{28}$$

From a table of Laplace transforms we can now find $p(t)$

$$p(t) = \frac{\alpha\lambda - \mu}{\alpha - \beta} \exp(-\alpha t) + \frac{\beta\lambda - \mu}{\beta - \alpha} \exp(-\beta t) \tag{29}$$

where

$$\begin{matrix}\alpha \\ \beta\end{matrix} = (\gamma_1 + \Omega_1 + \gamma_2 + \Omega_2)/2 \pm \sqrt{(\gamma_1 + \Omega_1 - \gamma_2 - \Omega_2)^2 + 4\Omega_1\Omega_2}\Big/2$$

$$\lambda = 1$$

$$\mu = \Omega_1 + \Omega_2 + P_1\gamma_2 + P_2\gamma_1.$$

2.5.1 Some Limiting Cases of a Two-state System

One simple example has as conditions

$\gamma_1 \to 0$, which implies that the apparatus does not fail in its normal operating state,

$\Omega_1 = \Omega_2 = \Omega$, the transition rates between states are equal, and

$P_1 \sim 1$. For notational convenience let $\gamma = \gamma_2$.

Then

$$\begin{matrix}\alpha \\ \beta\end{matrix} = (\gamma + 2\Omega)/2 \pm \sqrt{\gamma^2 + 4\Omega^2}\Big/2$$

$$\mu = \gamma + 2\Omega,\ \alpha - \beta = \sqrt{\gamma^2 + 4\Omega^2} \tag{30}$$

$$\begin{matrix}\alpha - \mu \\ \beta - \mu\end{matrix} = -(\gamma + 2\Omega)/2 \pm \sqrt{\gamma^2 + 4\Omega^2}\Big/2.$$

This yields, after some algebraic manipulation for the probability of no failure, the expression

$$p(t) = \frac{\exp\left[\frac{-(\gamma + 2\Omega)t}{2}\right]}{\sqrt{\gamma^2 + 4\Omega^2}} (\gamma + 2\Omega) \sinh t\sqrt{\gamma^2 + 4\Omega^2} + \sqrt{\gamma^2 + 4\Omega^2} \cosh t\sqrt{\gamma^2 + 4\Omega^2}. \tag{31}$$

As a second example we choose the following conditions:

$\gamma_1 \to 0$, indicating that the component does not normally break,

$\Omega_2 \to 0$, implying that once the component appears in the excited state it does not return to the ground state, and

$P_1 \sim 1$. For notational convenience let $\gamma = \gamma_2$, $\Omega = \Omega_1$.

From equation (29)

$$\alpha = \Omega,\ \beta = \gamma,\ \alpha - \beta = \Omega - \gamma$$

$$\mu = \Omega + \gamma,\ \lambda = 1 \tag{32}$$

$$\alpha - \mu = -\gamma,\ \beta - \mu = -\Omega.$$

This yields, for the probability of no failure,

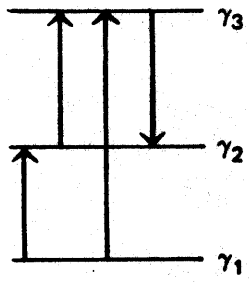
$$p(t) = \frac{\gamma}{\gamma - \Omega} \exp(-\Omega t) - \frac{\Omega}{\gamma - \Omega} \exp(-\gamma t). \tag{33}$$

It can easily be shown that $p(t)$ satisfies a fundamental property of probabilities, that is, $0 \leq p(t) \leq 1$.

2.6 Exact Solution for a Three-state System

The two-state system with homogeneous Markov chains describing the transformations is of course the simplest possible case; one of our examples, equation (38), can easily be derived from a differential equation approach, to be discussed shortly.

Before leaving this section, note that many "failure rate levels" may exist in an EMP-stressed system. Another possibility might be illustrated in the "failure rate diagram" shown below:



γ_1 might correspond to the normal operating failure rate, γ_3 to a transient, high-risk state occurring only during EMP perturbations, and γ_2 to a failure rate state obtained by weakening the component during EMP attack. The possible transitions are indicated by arrows on the diagram. It is possible to write down $p(s)$ directly from our definitions.

$$\begin{aligned}
&\pi_{11} = -\Omega_1,\ \pi_{12} = \Omega_1 P_{12},\ \pi_{13} = \Omega_1 P_{13},\\
&P_{12} + P_{13} = 1,\ \text{define } p = P_{12},\\
&\pi_{21} = 0,\ \pi_{22} = -\Omega_2,\ \pi_{23} = \Omega_2 P_{23},\\
&P_{23} = 1,\\
&\pi_{31} = 0,\ \pi_{32} = \Omega_3 P_{32},\ \pi_{33} = -\Omega_3,\\
&P_{32} = 1.
\end{aligned} \tag{34}$$

The π matrix then becomes

$$\underset{\sim}{\pi} = \begin{bmatrix} -\Omega_1, & \Omega_1 p, & \Omega_1(1-p) \\ 0, & -\Omega_2, & \Omega_2 \\ 0, & \Omega_3, & -\Omega_3 \end{bmatrix}. \tag{35}$$

The $\underset{\sim}{\gamma}$ matrix and the $\vec{W}$ vector have the form

$$\underset{\sim}{\gamma} = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{bmatrix}, \vec{W} = (1, 0, 0). \tag{36}$$

From equation (22), the Laplace transform of the reliability is

$$p(s) = \vec{W} \cdot \begin{bmatrix} s + \gamma_1 + \Omega_1, & -\Omega_1 p, & -\Omega_1(1-p) \\ 0, & s + \gamma_2 + \Omega_2, & -\Omega_2 \\ 0, & -\Omega_3, & s + \gamma_3 + \Omega_3 \end{bmatrix}^{-1} \cdot \vec{1}. \tag{37}$$

By carrying out the inversion and the inner products indicated, $p(s)$ can be seen to have the form

$$p(s) = \frac{\lambda_1 s^2 + \lambda_2 s + \lambda_3}{(s + \alpha_1)(s + \alpha_2)(s + \alpha_3)}. \tag{38}$$

Here the λ_i and $\alpha_i > 0$, $i = 1, 3$ depend on the γ's, Ω's, and a priori probabilities. The inverse transform yields the following expression for $p(t)$:

$$\begin{aligned} p(t) = {} & \frac{\lambda_1(-\alpha_1)^2 + \lambda_2(-\alpha_1) + \lambda_3}{(\alpha_2 - \alpha_1)(\alpha_3 - \alpha_1)} \cdot \exp(-\alpha_1 t) \\ & + \frac{\lambda_1(-\alpha_2)^2 + \lambda_2(-\alpha_2) + \lambda_3}{(\alpha_3 - \alpha_2)(\alpha_1 - \alpha_2)} \cdot \exp(-\alpha_2 t) \\ & + \frac{\lambda_1(-\alpha_3)^2 + \lambda_2(-\alpha_3) + \lambda_3}{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3)} \cdot \exp(-\alpha_3 t). \end{aligned} \tag{39}$$

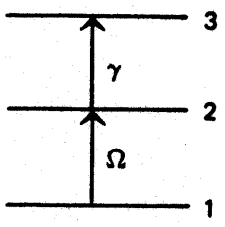
The solution for the n-state case is readily obtained in a similar manner. It has the general form

$$p(t) = \sum_{i=1}^{n} W_i \exp(-\lambda_i t).$$

3. METHODS FOR CALCULATING THE RELIABILITY OF SYSTEMS SUBJECT TO FAILURE AND REPAIR

3.1 Introduction

As an introduction to another method of calculating reliabilities we will examine a three-state system. (This corresponds to the example that leads to equation (33).)



(failure state of the system)

(state created as a result of stress)

(normal operating state).

In state 1, the component does not fail; in state 2, which is being fed from state 1 at rate Ω, the component fails at rate γ into state 3, which we call the failure state. If P_1, P_2, and P_3 are the probabilities of occupation of states, then clearly we have the following differential equations:

$$\begin{aligned} \frac{dP_1(t)}{dt} &= -\Omega P_1(t), \quad P_1(0) = 1 \\ \frac{dP_2(t)}{dt} &= -\gamma P_2(t) + \Omega P_1(t), \quad P_2(0) = 0 \\ \frac{dP_3(t)}{dt} &= \gamma P_2(t), \quad P_3(0) = 0. \end{aligned} \tag{40}$$

It is a straightforward task to solve the above system of equations subject to the listed initial conditions. The solution is

$$\begin{aligned} P_1(t) &= \exp(-\Omega t) \\ P_2(t) &= \frac{\Omega}{\gamma - \Omega}\left[\exp(-\Omega t) - \exp(-\gamma t)\right] \\ P_3(t) &= 1 + \frac{\Omega}{\gamma - \Omega}\exp(-\gamma t) - \frac{\gamma}{\gamma - \Omega}\exp(-\Omega t). \end{aligned} \tag{41}$$

The reliability is given by the sum of the probabilities of the nonfailure states

$$p(t) = P_1(t) + P_2(t) = 1 - P_3(t), \tag{42}$$

$$p(t) = \frac{\gamma}{\gamma - \Omega}\exp(-\Omega t) - \frac{\Omega}{\gamma - \Omega}\exp(-\gamma t), \tag{43}$$

which agrees with equation (33).

3.2 Formal Discussion of Markov Processes

Now a more general method can be discussed, which allows the determination of the probability of occupying a given state of the system once the failure rates and the repair and/or replacement rates of the components are known. First a formal discussion of Markov processes is needed.

A continuous parameter stochastic process $[X(t), t \geq 0]$ is said to be a Markov process if, for any set of n time points $t_1, t_2, \ldots, t_n$ with

$$[X(t_1), X(t_2), \ldots, X(t_n) : t_1 < t_2 < \cdots < t_n], \tag{44}$$

The conditional distribution $X(t_n)$ for given values of $X(t_1), \ldots, (t_{n-1})$, depends only on $X(t_{n-1})$. Thus we may write

$$\begin{aligned} P[X(t_n) &= x_n | X(t_1) = x_1, \ldots, X(t_{n-1}) = x_{n-1}] \\ &= P[X(t_n) = x_n | X(t_{n-1}) = x_{n-1}]. \end{aligned} \tag{45}$$

The sequence $[X(t_j)]$ is called a chain.

To specify the probability law of a continuous parameter Markov chain, it suffices to state for all times $t \geq s \geq 0$, and states j and k, the probability function,

$$P_k(t) = P[X(t) = k], \tag{46}$$

and the conditional probability function (sometimes called the transition probability function)

$$P_{jk}(s, t) = P[X(t) = k | X(s) = j]. \tag{47}$$

The chain is homogeneous if we impose the following condition:

$$P_{jk}(t) = P[X(t + u) = k | X(u) = j], \text{ for any } u \geq 0. \tag{48}$$

3.3 Derivation of the Chapman-Kolmogorov Equation

The transition probabilities satisfy a fundamental relation called the Chapman-Kolmogorov equation. For all times $t \geq u \geq s \geq 0$ we must have

$$P[X(t) = k | X(s) = j] = \sum_i P[X(t) = k | X(u) = i, X(s) = j] \times P[X(u) = i | X(s) = j], \tag{49}$$

$$P_{jk}(s, t) = \sum_i P_{ji}(s, u) P_{ik}(u, t). \tag{50}$$

If a transition probability matrix $\underset{\sim}{P} = (P_{jk})$ is defined, this can be written as

$$\underset{\sim}{P}(s, t) = \underset{\sim}{P}(s, u)\, \underset{\sim}{P}(u, t). \tag{51}$$

3.4 Derivation of the Forward Differential Equation for the Markov Chain

To obtain the transition probabilities one usually obtains differential equations for them; one does, however, have to make some assumptions.

Assume that

$$\lim_{h\to 0} \frac{1}{h}[1 - P_{jj}(t, t+h)] = q_j(t). \tag{52}$$

The quantity $[1 - P_{jj}(t, t+h)]$ is the probability of transition from a state j to some other state during the time interval h, and $q_j(t)$ is sometimes called the intensity of passage. Similarly, assume that

$$\lim_{h\to 0} \frac{P_{jk}(t, t+h)}{h} = q_{jk}(t), \quad j \neq k. \tag{53}$$

The quantity $q_{jk}(t)$ is called the intensity of transition. It is also written as $q_j(t)\rho_{jk}(t)$. The quantities q_j and q_{jk} correspond to the $\underset{\sim}{\Omega}$ and $\underset{\sim}{\pi}$ transition rates introduced earlier. Clearly, if the chain is homogeneous, the intensities are too. Manipulating equation (50) to take advantage of the assumptions,

$$P_{jk}(\tau, t+h) = \sum_{\nu} P_{j\nu}(\tau, t)P_{\nu k}(t, t+h), \tag{54}$$

$$P_{jk}(\tau, t+h) = P_{jk}(\tau, t)P_{kk}(t, t+h) + \sum_{\nu\neq k} P_{j\nu}(\tau, t)P_{\nu k}(t, t+h). \tag{55}$$

Utilizing equations (52) and (53)

$$P_{jk}(\tau, t+h) = P_{jk}(\tau, t)[1 - hq_k(t)] + \sum_{\nu\neq k} P_{j\nu}(\tau, t)hq_{\nu k}(t). \tag{56}$$

Rearranging and taking the limit $h \to 0$

$$\lim_{h\to 0} \frac{1}{h}[P_{jk}(\tau, t+h) - P_{jk}(\tau, t)] = -q_k(t)P_{jk}(\tau, t) + \sum_{\nu\neq k} P_{j\nu}(\tau, t)q_{\nu k}(t). \tag{57}$$

In the limit

$$\frac{\partial}{\partial t}P_{jk}(\tau, t) = -P_{jk}(\tau, t)q_k(t) + \sum_{\nu\neq k} P_{j\nu}(\tau, t)q_{\nu k}(t). \tag{58}$$

This is the so-called forward differential equation for the Markov chain. The backward differential equation treats τ as the variable. It is not needed for present purposes, but is listed for completeness.

$$\frac{\partial}{\partial t}P_{jk}(\tau, t) = q_j(\tau)P_{jk}(\tau, t) - \sum_{\nu\neq k} q_{j\nu}(\tau)P_{\nu k}(\tau, t). \tag{59}$$

If the chain is homogeneous, the intensities are independent of t and the transition probabilities do not depend on t and τ but only on $t - \tau = t'$. Thus equation (58) can be written as

$$\frac{d}{dt}P_{jk}(t) = -P_{jk}(t)q_k + \sum_{v\neq k} P_{jv}(t)q_{vk}. \tag{60}$$

Suppose now we are not interested in the initial state j. We then can average over all initial states and define the probability of occupation of the state k, $P_k(t)$ as

$$P_k(t) = \sum_j a_j P_{jk}(t). \tag{61}$$

Thus

$$\frac{d}{dt}P_k(t) = -P_k(t)q_k + \sum_{v\neq k} P_v(t)q_{vk}. \tag{62}$$

Equations (58), (60), and (62) all have the following form:

$$\frac{d}{dt}\underset{\sim}{U}(t) = \underset{\sim}{H}(t)\underset{\sim}{U}(t). \tag{63}$$

For example, if U and H are given by the following matrices,

$$\underset{\sim}{U} = \begin{bmatrix} P_{11}, P_{21}, \ldots, P_{l1} \\ P_{12}, P_{22} \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ P_{1l} \quad \cdot \quad \cdot \quad \cdot \quad P_{ll} \end{bmatrix}, \quad \underset{\sim}{H} = \begin{bmatrix} -q_1, & +q_{21}, \ldots, q_{l1} \\ +q_{12}, & -q_2, \\ \cdot \quad & \cdot \quad & \cdot \\ \cdot \quad & \cdot \\ \cdot \quad & \cdot \quad & \cdot \\ q_{1l} \quad & \cdot \quad & \cdot \quad & -q_l \end{bmatrix}, \tag{64}$$

equation (58) or (60) follows. If $\underset{\sim}{U}$ is a vector given by

$$\vec{U} = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_l \end{bmatrix}, \tag{65}$$

equation (62) follows.

3.5 Solutions of the Forward Differential Equation

3.5.1 Equivalent Integral Equation and Some Special Case Solutions

Equation (63) is equivalent to the integral equation:

$$\underset{\sim}{U}(t) = \underset{\sim}{U}(0) + \int_0^t \underset{\sim}{H}(t')\underset{\sim}{U}(t')dt'. \tag{66}$$

By an iterative procedure the following equation is obtained:

$$U(t) = \sum_{n=0}^{\infty} \int_0^t dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n \underset{\sim}{H}(t_1)\underset{\sim}{H}(t_2) \cdots \underset{\sim}{H}(t_n)\underset{\sim}{U}(0). \tag{67}$$

This is a solution to our equation, though not in a very usable form.

3.5.1.1 Time-independent Case

If H is independent of time as for the homogeneous case,

$$\underset{\sim}{U}(t) = \sum_{n=0}^{\infty} \underset{\sim}{H}^n t^n / n! \underset{\sim}{U}(0) = \exp(\underset{\sim}{H}t)\underset{\sim}{U}(0). \tag{68}$$

As in our earlier work, we can introduce the Laplace transform of $\underset{\sim}{U}$

$$\underset{\sim}{U}(s) = \int_0^{\infty} \exp(-s\underset{\sim}{I}t) \exp(\underset{\sim}{H}t)\underset{\sim}{U}(0)dt, \tag{69}$$

which has the solution

$$\underset{\sim}{U}(s) = (s\underset{\sim}{I} - \underset{\sim}{H})^{-1}\underset{\sim}{U}(0). \tag{70}$$

After evaluating $U(s)$, $U(t)$ can be found from a table of Laplace transforms.

3.5.1.2 Use of the Feynman Calculus for Solving the Time-independent Case

An equivalent approach to solving equation (68) is to separate the operator $\underset{\sim}{H}$ into two operators $\underset{\sim}{\phi}$ and $\underset{\sim}{\psi}$ and use the techniques of Feynman's operator calculus (ref. 3 and 4), which disentangles them and thus makes computations considerably simpler. A convenient choice for the two operators would be to let $\underset{\sim}{\phi}$ be a diagonal matrix whose elements are the intensity of passage q, introduced in equation (52),

$$\underset{\sim}{\phi} = \begin{bmatrix} q_1 & & & \\ & q_2 & & \\ & & \ddots & \\ & & & q_l \end{bmatrix}, \tag{71}$$

and let $\underset{\sim}{\psi}$ be the matrix consisting of the intensities of transition, q_{jk}, introduced in equation (53),

$$\underset{\sim}{\psi} = -\begin{bmatrix} 0 & q_{21} & \cdots & q_{l1} \\ q_{12} & 0 & & \\ \cdot & & \cdot & \\ \cdot & & & \cdot \\ \cdot & & & \\ q_{1l} & \cdots & & 0 \end{bmatrix}. \tag{72}$$

Equation (63) then can be written

$$\frac{d}{dt}\underset{\sim}{U}(t) = -(\underset{\sim}{\phi} + \underset{\sim}{\psi})U(t). \tag{73}$$

Transform $\underset{\sim}{U}(t)$ into $\underset{\sim}{V}(t)$, where $\underset{\sim}{V}(t) = \exp(\underset{\sim}{\psi}t)U(t)$ and $\underset{\sim}{V}(0) = \underset{\sim}{U}(0)$. The operator $\underset{\sim}{V}(t)$ then satisfies the following differential equation:

$$\frac{d}{dt}\underset{\sim}{V}(t) = -\underset{\sim}{\bar{\psi}}(t)\underset{\sim}{V}(t) \tag{74}$$

where

$$\underset{\sim}{\bar{\psi}}(t) = \exp(\underset{\sim}{\phi}t)\underset{\sim}{\psi}\exp(-\underset{\sim}{\phi}t). \tag{75}$$

The solution to equation (74) is obtained by iteration of its equivalent integral equation, thus:

$$\underset{\sim}{V}(t) = \sum_{n=0}^{\infty}(-)^n \int_0^t dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n \underset{\sim}{\bar{\psi}}(t_1)\underset{\sim}{\bar{\psi}}(t_2) \times \cdots \times \underset{\sim}{\bar{\psi}}(t_n)\underset{\sim}{V}(0). \tag{76}$$

Thus we may also express the solution to equation (73) as

$$\underset{\sim}{U}(t) = \exp(-\underset{\sim}{\phi}t)\sum_{n=0}^{\infty}(-)^n \int_0^t dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n \underset{\sim}{\bar{\psi}}(t_1)\underset{\sim}{\bar{\psi}}(t_2)\ldots\underset{\sim}{\bar{\psi}}(t_n)\underset{\sim}{U}(0). \tag{77}$$

The Laplace transform $\underset{\sim}{U}(s)$ is, as before,

$$\underset{\sim}{U}(s) = \int_0^{\infty} \exp(-s\underset{\sim}{I}t)\underset{\sim}{U}(t)dt. \tag{78}$$

Define

$$\underset{\sim}{A}(t_n - t_{n'}) = \exp[-\underset{\sim}{\phi}(t_n - t_{n'})], \tag{79}$$

$$\underset{\sim}{B}(t_n - t_{n'}) = \underset{\sim}{\psi}\underset{\sim}{A}(t_n - t_{n'}), \tag{80}$$

and their Laplace transforms

$$\underset{\sim}{A}(s) = \int_0^\infty \exp(-s\underset{\sim}{I}t)\underset{\sim}{A}(t)dt = (s\underset{\sim}{I} + \underset{\sim}{\phi})^{-1}$$

$$\underset{\sim}{B}(s) = \underset{\sim}{\psi}\underset{\sim}{A}(s). \tag{81}$$

The first few terms of equation (77) look like

$$\underset{\sim}{U}(t) = \Big\{\underset{\sim}{A}(t) - \int_0^t \underset{\sim}{A}(t - t_1)\underset{\sim}{\psi}\underset{\sim}{A}(t_1)dt_1$$

$$+ \int_0^t dt_1 \int_0^{t_1} dt_2 \underset{\sim}{A}(t - t_1)\underset{\sim}{\psi}\underset{\sim}{A}(t_1 - t_2)\underset{\sim}{\psi}\underset{\sim}{A}(t_2) + \cdots\Big\}\underset{\sim}{U}(0). \tag{82}$$

By using the convolution theorem of Laplace transform theory, equations (78) through (82) can be combined to yield

$$\underset{\sim}{U}(s) = \underset{\sim}{A}(s)[\underset{\sim}{I} - \underset{\sim}{B}(s) + \underset{\sim}{B}(s)^2 + \cdots]\underset{\sim}{U}(0), \tag{83}$$

$$\underset{\sim}{U}(s) = \underset{\sim}{A}(s)[\underset{\sim}{I} + \underset{\sim}{B}(s)]^{-1}\underset{\sim}{U}(0). \tag{84}$$

If numerical difficulties arise in inverting equation (70), equation (84) represents another solution that may be easier to obtain. This is especially true if B can be considered small compared to I; then $(\underset{\sim}{I} + \underset{\sim}{B})^{-1}$ may be expanded and only the first few terms kept. (Note that $A(s)$, defined by equation (81), is a diagonal matrix; its k^{th} element is just $1/(s + \phi_k)$. In this case no complicated inversion of matrices need be carried out.

3.5.1.3 A Solution for a Certain Class of Time-dependent Equations

It is well known that if $\underset{\sim}{H}$ depends on the time t and if

$$[\underset{\sim}{H}(t), \underset{\sim}{H}(t')] = 0 \tag{85}$$

then a solution to equation (63) is

$$\underset{\sim}{U}(t) = \exp\left[\int_0^t \underset{\sim}{H}(t')dt'\right]\underset{\sim}{U}(0). \tag{86}$$

It is not known whether any useful, practical system will have associated operators satisfying equation (85). Most time-dependent cases are very complex.

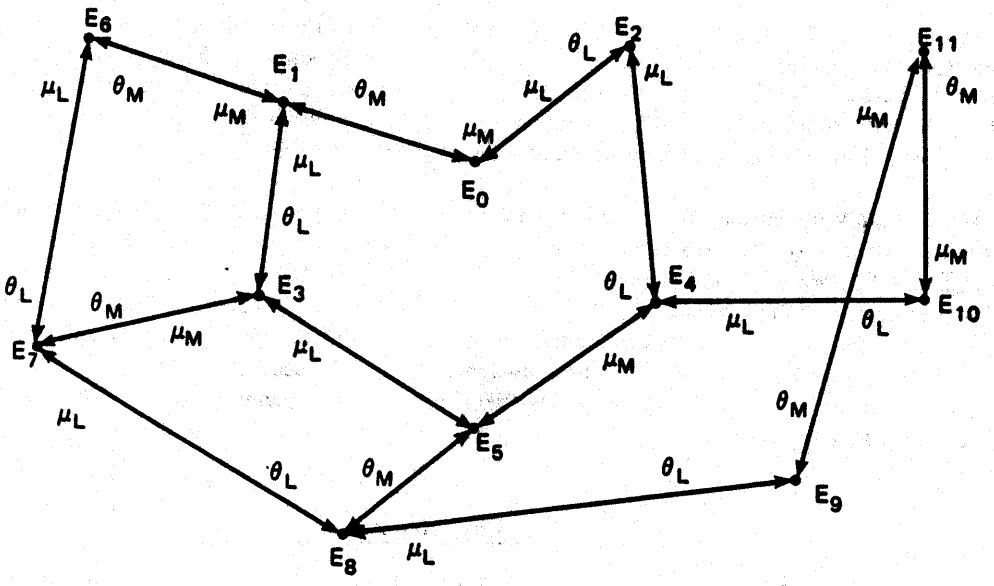
3.6 Characterization of a Subsystem of an AECo Switch in a Reliability Model

In most cases of practical interest (such as an EMP-stressed AECo switch) one usually limits the number of states so calculations can be easily performed. A finer structure in the calculations can always be obtained by further classification of the original set of states. The procedure to be followed in obtaining the reliability function for a system is to calculate the probabilities of finding the system in non-failure states, add those probabilities, and thus form the reliability function.

We will apply the formalism in a simple manner to a subsystem of an AECo switch. Let us, for the moment, consider only five subsystems (ref. 5): the three logic systems (A, B, and C) and the two memories (X and Y). We assume that the switch is in some kind of EMP environment, such as would result from a real or simulated nuclear attack. This causes the breakdown (at some rate) of the five subsystems, which are repaired at a different rate. Repair can also mean, in this context, just the resetting of the logic system, which may have been switched into an unacceptable logic state. This type of calculation is obviously included under the title "reliability models" including repair. We can clearly identify 12 states, as listed below:

State	Characterization
E_0	nothing failed
E_1	1 memory failed 0 logic system failed
E_2	0 " " 1 " " "
E_3	1 " " 1 " " "
E_4	0 " " 2 " " "
E_5	1 " " 2 " " "
E_6	2 " " 0 " " "
E_7	2 " " 1 " " "
E_8	2 " " 2 " " "
E_9	2 " " 3 " " "
E_{10}	0 " " 3 " " "
E_{11}	1 " " 3 " " "

We might want to restrict ourselves to fixing or breaking only one item at a time. Thus we let θ_L and θ_M correspond to the rate at which the logic system and memory, respectively, break, and μ_L and μ_M to their rates of repair. The diagram shown below is called the "state space of the system." The possible transitions, assuming single unit breakdown and repair, are indicated by arrows linking the states.



State Space of System

The differential equations of the system are:

$$
\begin{aligned}
P_0' &= -(\theta_L + \theta_M)P_0 + \mu_L P_2 + \mu_M P_1 \\
P_1' &= -(\theta_L + \mu_M + \theta_M)P_1 + \theta_M P_0 + \mu_L P_3 + \mu_M P_6 \\
P_2' &= -(\theta_L + \mu_L)P_2 + \theta_L P_0 + \mu_L P_4 \\
P_3' &= -(\mu_L + \theta_L + \theta_M)P_3 + \theta_L P_1 + \mu_L P_5 + \mu_M P_7 \\
P_4' &= -(\theta_L + \theta_M + \mu_L)P_4 + \theta_L P_2 + \mu_M P_5 + \mu_L P_{10} \\
P_5' &= -(\mu_M + \mu_L + \theta_M)P_5 + \theta_L P_3 + \theta_M P_4 + \mu_M P_8 \\
P_6' &= -(\mu_M + \theta_L)P_6 + \theta_M P_1 + \mu_L P_7 \\
P_7' &= -(\mu_L + \mu_M + \theta_L)P_7 + \theta_L P_6 + \theta_M P_3 + \mu_L P_8 \\
P_8' &= -(\mu_M + \theta_L + \mu_L)P_8 + \theta_M P_5 + \theta_L P_7 + \mu_L P_9 \\
P_9' &= -(\mu_M + \mu_L)P_9 + \theta_L P_8 + \theta_M P_{11} \\
P_{10}' &= -(\mu_L + \theta_M)P_{10} + \theta_L P_4 + \mu_M P_{11} \\
P_{11}' &= -(\mu_M + \theta_M)P_{11} + \mu_M P_9 + \theta_M P_{10}.
\end{aligned}
\tag{87}
$$

We could pick for the initial condition (see equations (62) and (65))

$$P_0 = (1,0,0,0,0,0,0,0,0,0,0,0)$$

After the differential equation is solved, the reliability may be calculated as the sum of the probabilities of the favorable states (those for which the system works). In this case

$$R(t) = \sum_{i=0}^{5} P_i(t). \tag{88}$$

Using a computer, there is no difficulty in solving large systems of linear differential equations, but extreme difficulties can arise in the interpretation of the parameters θ and μ. As used here, they are clearly average values and do not take into account the intensity of the EMP's or their frequency. In any simulation of a real communications node, the θ's and μ's must be given some sort of time dependence (to reflect the realities of a nuclear attack). Their magnitudes clearly can only be obtained from experimental work.

4. STRUCTURE RELIABILITY IN TERMS OF COMPONENT RELIABILITY

4.1 Definition of the Structure Function with Some Elementary Examples

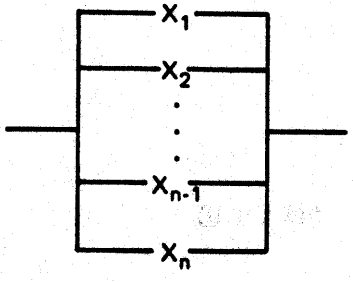
Suppose that a structure is made up of n components. For each i^{th} component we define a two-value variable x_i; $x_i = 1$ implies that the component is functional, and $x_i = 0$ implies that the component is not functional. From the n variables x_i we define a structure vector $\vec{x} = (x_1, x_2, \ldots, x_n)$. Note that there are 2^n possible vectors for an n component structure. These 2^n vectors naturally divide into two distinct classes: those for which the structure works and those for which it fails. It is natural, then, to define a two-valued function of the vector $\vec{x}$, called the structure function $\Phi(\vec{x})$ such that,

$\Phi(\vec{x}) = 1 \rightarrow$ structure works, so $\vec{x}$ is a path for the structure

$\Phi(\vec{x}) = 0 \rightarrow$ structure fails, so $\vec{x}$ is a cut for the structure.

For simple structures it is a straightforward task to write down the structure function, as the following examples illustrate.

For a parallel structure of n components, the only possible mode of failure is when all components fail.

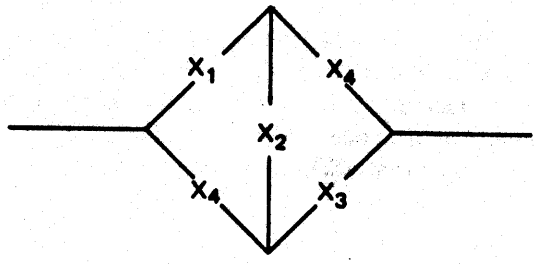


$$\Phi_p(\vec{x}) = 1 - \prod_{i=1}^{n} (1 - x_i). \tag{89}$$

For a series structure of n components the only possible mode of operation is when all components are operable,

$$\text{—}\,x_1\,\text{—}\,x_2\,\text{—}\cdots\text{—}\,x_n\,\text{—} \qquad \Phi_s(\vec{x}) = \prod_{i=1}^{n} x_i. \tag{90}$$

By judiciously combining series and/or parallel subsystems, it is possible to write down the structure function for any system. For example, it can be verified that the structure function for



is

$$\begin{aligned}\Psi(\vec{x}) = {} & x_2[1-(1-x_1)(1-x_4)][1-(1-x_3)(1-x_4)] \\ & + (1-x_2)[1-(1-x_3x_4)(1-x_1x_4)].\end{aligned} \tag{91}$$

4.2 Combination of Structures

Suppose now that there is an n-component structure described by $\Phi(\vec{x})$, $\vec{x} = (x_1, x_2, \ldots, x_n)$. We note the following identity:

$$\Phi(\vec{x}) = x_n\Phi(\vec{x}', 1) + (1-x_n)\Phi(\vec{x}', 0) \tag{92}$$

where $\vec{x}' = (x_1, x_2, \ldots, x_{n-1})$. By applying the same manipulation to $\Phi(\vec{x}', 1)$ and $\Phi(\vec{x}', 0)$, we find

$$\Phi(\vec{x}', 1) = x_{n-1}\Phi(\vec{x}'', 1, 1) + (1-x_{n-1})\Phi(\vec{x}'', 0, 1) \tag{93}$$

and

$$\Phi(\vec{x}', 0) = x_{n-1}\Phi(\vec{x}'', 1, 0) + (1-x_{n-1})\Phi(\vec{x}'', 0, 0) \tag{94}$$

where $\vec{x}'' = (x_1, x_2, \ldots, x_{n-2})$.
Thus

$$\begin{aligned}\Phi(\vec{x}) = {} & x_nx_{n-1}\Phi(\vec{x}'', 1, 1) + x_n(1-x_{n-1})\Phi(\vec{x}'', 0, 1) \\ & + (1-x_n)x_{n-1}\Phi(\vec{x}'', 1, 0) + (1-x_n)(1-x_n)\Phi(\vec{x}'', 0, 0).\end{aligned} \tag{95}$$

By continuing the process we find

$$\Phi(\vec{x}) = \sum_{\vec{y}} \prod_{j=1}^{n} x_j^{y_j} (1 - x_j)^{1-y_j} \Phi(\vec{y}) \tag{96}$$

where $\vec{y}$ ranges over all 2^n vectors of length n, and $\Phi(\vec{y}) = 1$ if $\vec{y}$ is a path and $\Phi(\vec{y}) = 0$ if $\vec{y}$ is a cut.

4.3 Composition of Structures

Another well-known process of building complex structures is the modular approach. One conceives an n-component structure and then for some or all of the n components an m-component structure (module) is substituted.

If γ is a structure of order n and Ψ_i is a structure of order k_i, $i = 1, \ldots, n$, the composition of $\Psi_1(\vec{x}), \Psi_2(\vec{y}), \ldots, \Psi_n(\vec{z})$ into γ is defined as χ, where

$$\chi(\vec{x}, \vec{y}, \ldots, \vec{z}) = \gamma[\Psi_1(\vec{x}), \Psi_2(\vec{y}), \ldots, \Psi_n(\vec{z})]. \tag{97}$$

The order of χ is $\sum_{i=1}^{n} k_i$.

4.4 Definition of the Reliability Function for a Structure of n Identical, Independent Components

Let $\Phi(\vec{x})$ be a structure function of order n. Assume that its n components are independent random variables all having the same probability distributions, $P(x)$,

$$P(x_i = 1) = p, P(x_i = 0) = 1 - p, i = 1, \ldots, n. \tag{98}$$

The reliability function for the structure Φ, denoted by $R_\Phi(p)$, is defined as

$$R_\Phi(p) = P[\Phi(\vec{x}) = 1] = E[\Phi(x)]. \tag{99}$$

Here $E[\,]$ means to take the expectation value of the quantity enclosed in the parentheses.

As a specific example consider a two-component series structure so that $\Phi(x_1, x_2) = x_1 x_2$. We have

$$R_\Phi(p_1, p_2) = \sum_{x_1, x_2=0}^{1} x_1 x_2 p(x_1, x_2). \tag{100}$$

Since the components are independent and identical $p(x_1, x_2) = p(x_1) \cdot p(x_2)$, thus

$$R_\Phi(p) = \sum_{x_1=0}^{1} x_1 p(x_1) \sum_{x_2=0}^{1} x_2 p(x_2) = p^2. \tag{101}$$

For a general n-component structure we have from equations (96) and (99) that

$$R_\Phi(p) = E\left[\sum_{\vec{y}} \prod_{j=1}^{n} x_j^{y_j} (1 - x_j)^{1-y_j} \Phi(\vec{y})\right], \tag{102}$$

$$R_\Phi(p) = \sum_{\vec{y}} E\left[\prod_{j=1}^{n} x_j^{y_j} (1 - x_j)^{1-y_j}\right] \Phi(\vec{y}). \tag{103}$$

If $\vec{y}$ is a path for Φ then $\Phi(\vec{y}) = 1$ otherwise $\Phi(\vec{y}) = 0$, thus we need only calculate the expectation values for the paths of the structure. If $\vec{y}$ is any structure vector then the size of $\vec{y}$ is defined by

$$s(\vec{y}) = \sum_{i=1}^{n} y_i. \tag{104}$$

The "size" is numerically equal to the number of ones in the vector or simply to the number of working components in the structure. If a path $\vec{y}$ has size j then there are exactly j ones and $n-j$ zeros. If the components are also independent and identical, then

$$E\left[\prod_{j=1}^{n} x_j^{y_j} (1 - x_j)^{1-y_j}\right] = p^j (1 - p)^{n-j}. \tag{105}$$

If there are A_j paths of size j, then clearly

$$R_\Phi(p) = \sum_{j=0}^{n} A_j p^j (1 - p)^{n-j}. \tag{106}$$

4.5 Reliability of Composite Structures

Let Φ be a structure of order n and $\Psi(\vec{y})$, $\vec{y} = (y_1, \ldots, y_n)$, a structure of order m. The composition of ψ into Φ is χ.

$$\chi(\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_n) = \Phi[\Psi(\vec{y}_1), \Psi(\vec{y}_2), \ldots, \Psi(\vec{y}_n)]. \tag{107}$$

If the y_i, $i = 1, 2, \ldots, n \times m$ are independent random variables with the same probability distribution, then

$$R_\chi(p) = R_\Phi[E\{\Psi\}] = R_\Phi[R_\Psi(p)]. \tag{108}$$

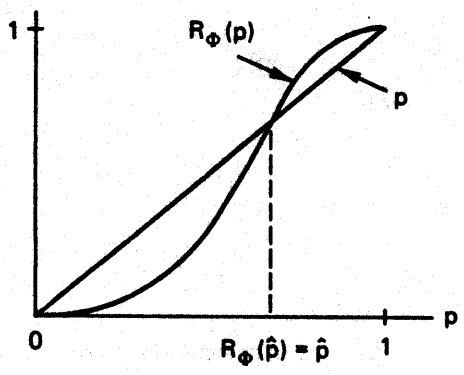
It is of some interest to examine what happens to the structure reliability under repeated compositions of the structure into itself. Define the iterated composition of Φ as

$$
\begin{aligned}
R_1(p) &= R_\Phi(p) \\
R_2(p) &= R_\Phi[R_1(p)] \\
&\vdots \\
R_k(p) &= R_\Phi[R_{k-1}(p)].
\end{aligned}
\tag{109}
$$

Assume that the structure Φ has the following properties:

$$
\begin{aligned}
&R_\Phi(0) = 0 \\
&R_\Phi(1) = 1 \\
&\frac{dR_\Phi(p)}{dp} > 0 \quad 0 \leq p \leq 1 \\
&R_\Phi(p) < p \qquad p \text{ in some neighborhood of } 0 \\
&R_\Phi(p) > p \qquad p \text{ in some neighborhood of } 1 \\
&R(p) = p \qquad \text{has only 1 root in the interval } (0, 1).
\end{aligned}
\tag{110}
$$

The structures for which the above conditions hold are referred to in the literature as "S-shaped structures," since the conditions force the reliability function to have the form displayed in the diagram below.



If $\hat{p}$ is the root of the equation $R_\Phi(p) = p$, then by applying the conditions of equation (110) the iterated composition of Φ must satisfy

$$
\begin{aligned}
&R_1(p) > R_2(p) > \cdots > R_k(p) \to 0 \quad \text{if} \quad p < \hat{p} \\
&R_1(p) < R_2(p) < \cdots < R_k(p) \to 1 \quad \text{if} \quad p > \hat{p}.
\end{aligned}
\tag{111}
$$

This demonstrates that one can obtain a structure that works perfectly or not at all, depending on the reliability of the components.

4.6 Structure Failure Rate in Terms of the Component Failure Rate

The component failure rate $\gamma(t)$ is the probability that a component of age t will fail in the interval $(t, t+dt)$,

$$\gamma(t) = \frac{-1}{\bar{F}} \frac{d\bar{F}}{dt}, \tag{112}$$

where $\bar{F}(t)$ is the probability of no failure at time t.

If we have a structure of n identical independent components described by the structure function Φ, then the reliability function, $R_\Phi(p) = R_\Phi[\bar{F}(t)]$, gives the probability of no failure of the structure at time t. The structure failure rate, denoted by $\Delta(t)$, is defined as

$$\Delta(t) = \frac{-1}{R_\Phi[\bar{F}(t)]} \frac{d}{dt} R_\Phi[\bar{F}(t)]. \tag{113}$$

Equation (113) can be rewritten as

$$\Delta(t) = \frac{-1}{R_\Phi(p)} \left.\frac{dR_\Phi(p)}{dp}\right|_{p=\bar{F}(t)} \cdot \frac{d\bar{F}(t)}{dt}. \tag{114}$$

With the help of equation (112) we have

$$\Delta(t) = \frac{+1}{R_\Phi(p)} \left.\frac{d}{dp} R_\Phi(p)\right|_{p=\bar{F}(t)} [-p\gamma(t)]. \tag{115}$$

Thus we finally obtain

$$\frac{\Delta(t)}{\gamma(t)} = p \left.\frac{R'_\Phi(p)}{R_\Phi(p)}\right|_{p=\bar{F}(t)}. \tag{116}$$

It is apparent that equation (116) will allow us to calculate the structure failure rate, provided we can calculate the structure function Φ.

4.7 Structure Failure Rate for Structures of Nonidentical Components

We now consider the case of a structure made up of n independent components in which the i^{th} component has probability $p_i(t) = \bar{F}_i(t)$ of not failing at time t, $i = 1, \ldots, n$. If $R[\vec{p}(t)]$ is the structure reliability function, where $\vec{p}(t) = [p_1(t), p_2(t), \ldots, p_n(t)]$, then the structure failure rate $\Delta(t)$ is

$$\Delta(t) = \frac{-1}{R[\vec{p}(t)]} \frac{dR[\vec{p}(t)]}{dt}$$

$$= \frac{-1}{R[\vec{p}(t)]} \sum_{i=1}^{n} \frac{\partial R(\vec{p})}{\partial p_i} \frac{dp_i}{dt} \tag{117}$$

which can be written

$$\Delta(t) = \sum_{i=1}^{n} \frac{p_i(t)}{R(\vec{p})} \frac{\partial R(\vec{p})}{\partial p_i} \left[\frac{-1}{p_i(t)} \frac{d}{dt} p_i(t) \right]. \tag{118}$$

From equation (112),

$$\Delta(t) = \sum_{i=1}^{n} \frac{p_i}{R(\vec{p})} \frac{\partial R(\vec{p})}{\partial p_i} \gamma_i(t). \tag{119}$$

If we define vectors $\vec{\gamma} = [\gamma_1(t), \gamma_2(t), \ldots, \gamma_n(t)]$ and $\vec{\beta} = [1/R(\vec{p})] \cdot (p_1 \partial R/\partial p_1, p_2 \partial R/\partial p_2, \ldots, p_n \partial R/\partial p_n)$, then equation (119) can be written as the inner product of two vectors

$$\Delta(t) = \vec{\beta} \cdot \vec{\gamma}. \tag{120}$$

The significance of equation (120) lies in the fact that $\vec{\beta}$ is a vector which depends only on the structure while $\vec{\gamma}$ depends only on the failure rate.

4.8 Determination of the Reliability Function for a Structure in an EMP Environment

Suppose now that we have a structure Φ, consisting of many components which are subjected to random EMP stresses. The problem is to find a method which would allow us to calculate the average reliability of the structure in much the same manner as in section 2.4. By integrating equation (113) and solving for $R_\Phi(t)$, we find that

$$R_\Phi(t) = \exp\left[- \int_0^t \Delta(t')dt' \right]. \tag{121}$$

Equation (121) is formally identical with equation (8). Assuming that the components have a discrete spectrum of failure rates that are known, equation (120) gives us a prescription for determining the discrete spectrum of Δ for the structure Φ. Once again we note that the set of failure rates for a component could be due to the component existing in a set of electronic states each of which allows a slightly different and larger susceptibility to failure when EMP-induced energy is coupled into the component. For a very complex structure (for example, an AUTOVON switching center), the number of values of Δ may be extremely large and their spacing sufficiently small that we could consider Δ to be essentially a continuous function. We shall devote a short amount of space to indicating some of the

difficulties encountered in evaluating the average of R_Φ given by equation (121) when Δ is a continuous function.

To find a solution, we need to backtrack momentarily and discuss another technique for the evaluation of average reliabilities as given by equation (19) in the discrete case. The basic idea is to make a transformation that will diagonalize the matrix $-\underset{\sim}{\gamma}+\underset{\sim}{\pi}$. The transformation $\underset{\sim}{T}$ is defined by

$$\underset{\sim}{T}^{-1}(-\underset{\sim}{\gamma}+\underset{\sim}{\pi})\underset{\sim}{T} = \underset{\sim}{\lambda}. \tag{122}$$

Here $\underset{\sim}{\lambda}$ is a diagonal matrix whose elements are the eigenvalues λ_i, $i = 1, \ldots, q$ of $-\underset{\sim}{\gamma}+\underset{\sim}{\pi}$. Equation (19) can be manipulated into a form that allows us to use equation (122).

$$p(t) = \vec{W}\cdot\underset{\sim}{T}\underset{\sim}{T}^{-1} \exp\left[t(-\underset{\sim}{\gamma}+\underset{\sim}{\pi})\right]\underset{\sim}{T}\underset{\sim}{T}^{-1}\cdot\vec{1} \tag{123}$$

$$= \vec{W}\cdot\underset{\sim}{T} \exp\left[t\underset{\sim}{T}^{-1}(-\underset{\sim}{\gamma}+\underset{\sim}{\pi})T\right]T^{-1}\cdot\vec{1} \tag{124}$$

$$= \vec{W}\cdot\underset{\sim}{T} \exp\left[\underset{\sim}{\lambda}t\right]\underset{\sim}{T}^{-1}\cdot\vec{1} \tag{125}$$

$$= \sum_{j,i,k} W(\gamma_j)\underset{\sim}{T}_{ji} \exp\left[\lambda_i t\right]\underset{\sim}{T}^{-1}_{ik} \tag{126}$$

$$= \sum_i \left[\sum_{j,k} W(\gamma_j)\underset{\sim}{T}_{ji}T^{-1}_{ik}\right] \exp\left(\lambda_i t\right) \tag{127}$$

$$= \sum \omega_i \exp\left(\lambda_i t\right) \tag{128}$$

Thus the average reliability function has been decomposed into a sum of exponentials of scalars multiplied by a weighting factor ω_i. Note that for some simple cases this same form was obtained by using Laplace transform theory (eq. 39).

We now return to the consideration of the continuous case. The matrix analogous to $\underset{\sim}{\gamma}$ is now $\underset{\sim}{\Delta}$, where

$$\underset{\sim}{\Delta}(\Delta_1, \Delta_2) = \Delta_1\delta(\Delta_1 - \Delta_2). \tag{129}$$

Assume that the failure rates Δ are distributed according to a normalized probability function $P(\Delta)$. $P(\Delta)$ is the generalization to the continuous case of the vector of a priori probabilities. The $\underset{\sim}{\pi}$ matrix is given by

$$\underset{\sim}{\pi}(\Delta_1, \Delta_2) = -\Omega(\Delta_1)\delta(\Delta_1 - \Delta_2) + \Omega(\Delta_1)P(\Delta_1, \Delta_2). \tag{130}$$

$\Omega(\Delta)$ and $P(\Delta_1, \Delta_2)$ have their usual meaning but are now continuous functions.

It is well known that the columns of the matrix $\underset{\sim}{T}$ in equation (122) are the eigenvectors $\vec{\phi}_i$ of $-\underset{\sim}{\gamma}+\underset{\sim}{\pi}$ associated with eigenvalue λ_i. Thus to determine $\underset{\sim}{T}$, one first must solve a matrix eigenvalue problem for the discrete case.

$$(-\underset{\sim}{\gamma} + \underset{\sim}{\pi})\vec{\phi} = \lambda\vec{\phi} \tag{131}$$

In the continuous case it is necessary to solve an integral eigenvalue equation

$$\int d\Delta_2 [-\underset{\sim}{\Delta}(\Delta_1, \Delta_2) + \underset{\sim}{\pi}(\Delta_1\Delta_2)]\, \phi(\Delta_2) = \lambda\phi(\Delta_1). \tag{132}$$

Substituting from equations (129) and (130) into equation (132),

$$\Omega(\Delta_1)\int d\Delta_2 P(\Delta_1, \Delta_2)\phi(\Delta_2) = [\lambda + \Delta_1 + \Omega(\Delta_1)]\, \phi(\Delta_1). \tag{133}$$

If a solution to equation (133) exists (a serious difficulty), then a formula analogous to equation (128) can be written down. Further simplification and manipulation of equation (133) is not very rewarding. If the functions Δ, $\Omega(\Delta)$, $P(\Delta)$, and $P(\Delta_1\Delta_2)$ are actually determined from some experiment, then the best approach would be a numerical solution of the integral equation.

5. METHODS FOR CALCULATING THE RELIABILITY OF A COMPLEX STRUCTURE

As we have seen, it is possible to determine the system reliability in terms of the component reliability for any system. The method we have used is to list each of the possible states of the system, determine which of the states are paths for the system, calculate the probability that each path is functional, and finally add all the probabilities together and obtain the system reliability. Although this approach appears quite simple in principle, its applicability is very limited because of the extremely large number of states possible for any sizable system. For example, a 20-component system has over a million states. Most analysts, therefore, resort to approximation schemes (ref. 6 through 8) which alleviate to a considerable extent the computational difficulties.

5.1 Minimal Paths and Cuts

In section 4.1 we introduced the idea of paths and cuts for a structure function $\Phi(\vec{x})$. If $\Phi(\vec{x}) = 1$, then $\vec{x}$ is a path for the structure, while if $\Phi(\vec{x}) = 0$, then $\vec{x}$ is a cut for the structure. The structure vectors $\vec{x}$ are either cuts or paths, but not both, so that the vectors are naturally divided into two classes, the path set and the cut set. If there are p paths and c cuts in an n-component structure, then $p + c = 2^n$. In most cases both p and c are large numbers of the order 2^{n-1}. To make the problem more tractable, the notion of a minimal path set and a minimal cut set can be introduced.

A minimal path $\vec{Z}$ of a structure Φ is a path such that

$$\Phi(\vec{Z}) = 1$$
$$\Phi(\vec{x}) = 0 \quad \forall\, \vec{x} < \vec{Z}. \tag{134}$$

The notation $\vec{x} < \vec{Z}$ means $x_i \leq Z_i$ for all $i = 1, \ldots, n$; and $x_j < Z_j$ for some j (at least one). The set A of all minimal paths is called the minimal path set. Physically the elements of $\vec{Z}$ that are 1 (working

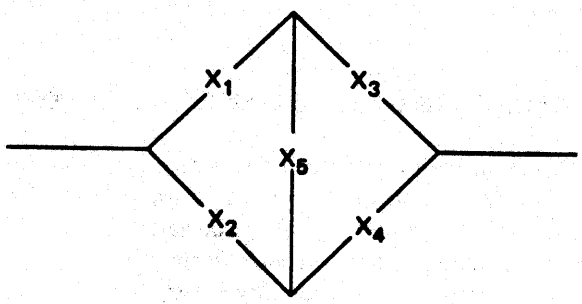
components) correspond to a smallest set of components that allows the system to function. A representation of the structure could be made by imagining the working components of a minimal path to act in series and the set of all such series structures to be acting in parallel. For each of these series structures, associate a series structure function α. If there are r minimal paths, then there are r structure functions α_j.

$$\alpha_j = \prod_{i \in A_j} Z \quad j = 1, \ldots, r \tag{135}$$

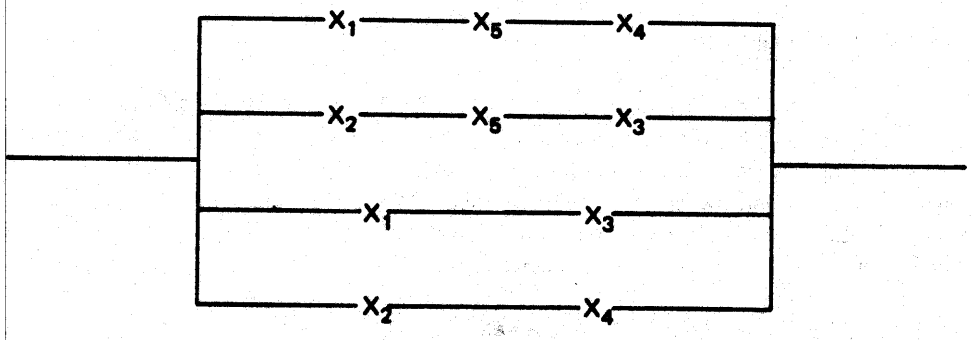
where the product is over the working components of the j^{th} minimal path. The representation of the structure constructed from the minimum path set is denoted by Φ_u,

$$\Phi_u = 1 - \prod_{j=1}^{r} (1 - \alpha_j) \tag{136}$$

As an example, consider the bridge structure diagrammed below.



The bridge is represented by the following structure,



The series structure functions $\alpha_j, j = 1, \ldots, 4$ are

$$\begin{aligned} \alpha_1 &= x_1 x_5 x_4 & \alpha_3 &= x_1 x_3 \\ \alpha_2 &= x_2 x_5 x_3 & \alpha_4 &= x_2 x_4 . \end{aligned} \tag{137}$$

The structure representation Φ_u is

$$\Phi_u = 1 - (1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)(1 - \alpha_4). \tag{138}$$

Since the same component appears in more than one path for this representation, replications of the components must always be forced to fail or function simultaneously.

A minimal cut $\vec{y}$ of a structure Φ is a cut such that

$$\Phi(\vec{y}) = 0$$
$$\Phi(\vec{x}) = 1 \quad \forall\ \vec{x} > \vec{y}. \tag{139}$$

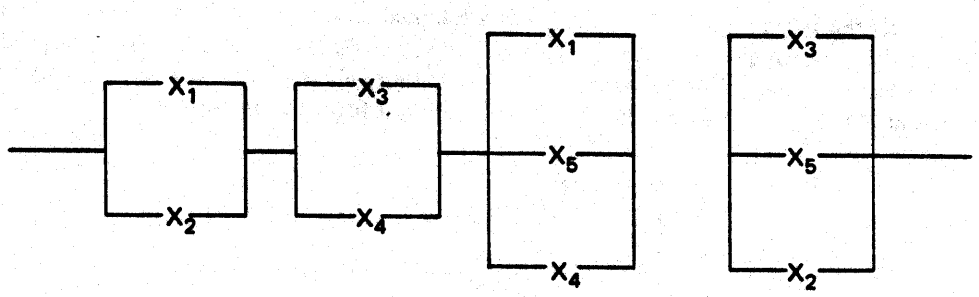
The notation $\vec{x} > \vec{y}$ means $x_i \geq y_i$ for all $i = 1, \ldots, n$; and $x_j > y_j$ for some j (at least one). The set B of all minimal cuts is called the minimal cut set. Physically the elements of $\vec{y}$ which are zero (failed components) correspond to the smallest set of components which by failing cause the structure to fail. A representation of the structure could be made by imagining the failed components of a minimal cut to act in parallel and the set of all such parallel structures to be acting in series. For each of these parallel structures, associate a parallel structure function β. If there are s minimal cuts then there are s structure functions β_j

$$\beta_j = 1 - \prod_{i \in B_j} (1 - y_i) \tag{140}$$

where the product is over the failed components of the j^{th} minimal cut. The representation of this structure constructed from the minimal cut set is denoted by Φ_L

$$\Phi_L = \prod_{j=1}^{s} \beta_j. \tag{141}$$

As an example, once again consider the bridge structure. The representation of the bridge is diagrammed below.



The parallel structure functions are

$$\beta_1 = 1 - (1 - x_1)(1 - x_2) \qquad \beta_3 = 1 - (1 - x_1)(1 - x_5)(1 - x_4)$$
$$\beta_2 = 1 - (1 - x_3)(1 - x_4) \qquad \beta_4 = 1 - (1 - x_3)(1 - x_5)(1 - x_2) \tag{142}$$

and the structure representation Φ_L is

$$\Phi_L = \beta_1\beta_2\beta_3\beta_4. \tag{143}$$

5.2 Bounds on the Reliability of a Structure

We naturally now wish to consider the relationship between the reliability of the representations we have defined and the reliability of the structure itself. Our interest in these representations stems naturally from the simple forms for their reliabilities. For Φ_u

$$R_{\Phi_u} = 1 - \prod_{j=1}^{r} [1 - P(\alpha_j = 1)] \tag{144}$$

$$P(\alpha_j = 1) = \prod_{i \in A_j} p_i. \tag{145}$$

The product in equation (145) is over the working components of the j^{th} minimal path. Similarly,

$$R_{\Phi_L} = \prod_{k=1}^{s} P(\beta_k = 1) \tag{146}$$

and

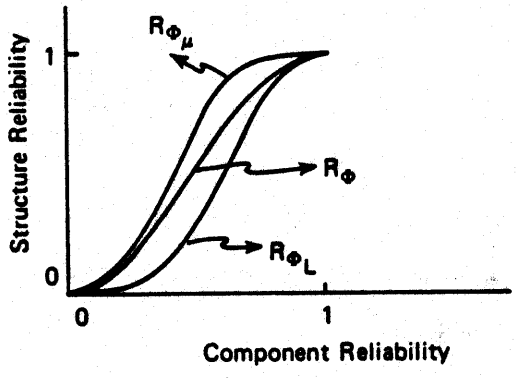
$$P(\beta_k = 1) = 1 - \prod_{i \in B_k} (1 - p_i). \tag{147}$$

The product in equation (147) is over the failed components of the k^{th} minimal cut.

The reliabilities R_{Φ_L}, R_Φ, and R_{Φ_u} satisfy the following inequality (for the proof, see ref. 7):

$$R_{\Phi_L} \leq R_\Phi \leq R_{\Phi_u}. \tag{148}$$

The equality on the left holds if the structure has a minimal cut set whose elements are mutually disjoint, for example, the minimal cuts have no elements in common. Similarly the equality on the right holds if the structure has a minimal path set whose elements are mutually disjoint. For a typical structure satisfying equation (110) the various reliabilities have the general form shown below.



In general, as the component reliability approaches 1, R_{Φ_L} becomes a very good approximation to R_Φ; as the components become unreliable, R_{Φ_u} is a good approximation to R_Φ. Since most structures have reliable components, most of the attention in the literature is directed toward the construction of algorithms which generate the set of minimal cuts. References 9 through 11 discuss some useful techniques. The interested reader should consult them for further details.

5.3 Equipment Reliability in Communication Networks in an EMP Environment

Up to now, we have only considered structure reliability in terms of the failure probabilities of its components. In communication networks the network is also considered to be failed between two terminals if the terminals cannot be connected because the network is busy; hence, the calculation must be modified to include the conditional probabilities that the various elements of the system (nodes, lines, trunks, etc.) are busy. A quantity called the network unserviceable probability is thus calculated. When one has a perfectly reliable network where the paths can never fail, the network is unserviceable only when all paths are busy. The unserviceable probability is referred to as the blocking probability. Discussions of some useful calculations are given in references 12 and 13.

Usage considerations on a network impose a considerable complication on any calculation, and most analysts resort to a simulation of the system in order to obtain useful results. Many simulation studies of the AUTOVON network (whose nodes are going to be stressed by various EMP simulators) have been carried out (ref. 14 through 17). These reports describe in detail the various methods of calculation and the performance of the network.

In spite of our earlier attempts to build models which might have some validity in the calculation of the reliability of a communication network in an EMP environment, the most promising approach appears to be a simulation of the network and its nodes. The calculations will include the simultaneous consideration of EMP upset and damage of nodes and links, network usage, and the inherent failure probabilities of its elements. The relevant details of the proposed calculation can be found in references 18, 19, and 20.

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