System Design and Assessment Notes
Note 15

An Organized, Six Step Approach to System EMP Vulnerability Assessment

Sgt. Mitchell A. Skinner
Capt. W. D. Wilson
Air Force Weapons Laboratory
29 June 1973

Abstract

To ensure an organized approach to the EMP vulnerability assessment of a deployed system, a method of six steps has been developed that yields a quantitative assessment of the deployed force survivability.

The six steps used in this assessment approach are:

1. Identification of system critical equipment,

2. Identification of mission critical circuits in each critical equipment,

3. Construction of an Assessment Matrix to identify available data,

4. Extrapolation of sub-threat level test data to threat-level,

5. Calculation of individual, system probability of survival curves, and

6. Determination of force survivability from individual system survivabilities.

The details of these six steps with an application to a strategic missile system are explained in this report.

DISCUSSION

STEP 1: Identify System Critical Equipment

The identification of system critical equipment requires that one have a detailed system description available. Further one must have an understanding of each of the subsystems that constitute the system to be assessed.

The process can be described in three sequential steps. These are as follows:

A. Determine if the subsystem is mission critical.

In short one must determine if the subsystem is required for the system to perform its mission. For instance a ground communications facility will be equipped with overhead fluorescent lighting for use by maintenance personnel. This would not be considered mission critical since the facility can process messages without this subsystem.

On the other hand a digital message processing computer used to relay command and control information would be considered to be a mission critical subsystem.

Note that this interpretation of a mission critical subsystem describes a purely series system, i.e., failure of any subsystem results in system failure.

B. Determine if the subsystem is potentially susceptible to EMP effects.

For each subsystem determined to be mission critical, it is necessary to determine if the subsystem is potentially susceptible to EMP induced currents.

For example, the skin of an aircraft would be considered to be mission critical but would not be potentially susceptible to EMP.

A receiver-transmitter determined to be mission critical would be considered to be potentially susceptible to EMP induced currents since it would usually contain sensitive semiconductor components.

C. Subsystems that are both Mission Critical and Potentially Susceptible are Critical Equipment.

Note that the subsystem must be mission critical and potentially susceptible. This process often will significantly reduce the number of subsystems to be analyzed.

STEP 2: Identify Mission Critical Circuits

It is necessary to determine for each critical subsystem which circuits and/or devices can cause a mission failure if upset or damaged by EMP induced currents.

For example, a multi-purpose weapons system computer on-board an aircraft has been determined to be a critical subsystem. Input logic circuits which could alert the computer memory or shut-down the computer operation would be considered to be critical circuits.

Another example might be the rocket motor in an aircraft attack missile. For this critical subsystem the electroexplosive devices that ignite the rocket motor would be considered to be critical circuits. If the electroexplosive device is fired prematurely, a catastrophic failure would occur.

STEP 3: Construct an "Assessment Matrix."

The purpose of an "Assessment Matrix" is to identify available data on each critical circuit for -

a. Each method of simulation,

b. Each system configuration, and

c. Each point of entry.

For any particular application, the Assessment Matrix may take a different form. The basic idea is to identify what data is available on system response to EMP. Often the matrix shows where large amounts of data are lacking. An example of an "Assessment Matrix" for an airborne weapons system computer is shown in Figure 1.

Note that the example matrix in Figure 1 indicates where data is available and also where no data is available. The location numbers identify measurements made at particular critical circuits.

STEP 4: Extrapolate Data to Full-Field

Often the data on any particular critical circuit is taken for a simulation drive level that is not full-field or criteria level. It is then necessary to determine a relation/scale factor between the simulator environment and the nuclear environment. This scale factor is then used to extrapolate the test data to full-field level.

EXAMPLE:

WEAPON SYSTEMS COMPUTER

LOCATION NUMBER ↓ / METHOD OF DRIVE →	ARRAY ILLUMINATION	DIRECT DRIVE	RES ILLUMINATION
112	Photo No. 11723	Photo No. 11725	No Data
117	Photo No. 10721	No Data	Photo No. 10729
942	No Data	Photo No. 10112	Photo No. 10091
1071	Photo No. 9820	No Data	Photo No. 10081

ASSESSMENT MATRIX EXAMPLE

FIGURE 1

Fourier transform theory has been found useful for computing a linear extrapolation to full-field level. For example let

$F(\omega)$ = Fourier transform of critical circuit signal in simulator,

$E_1(\omega)$ = Fourier transform of simulator field,

$E_2(\omega)$ = Fourier transform of nuclear environment fields, and

$F_F(\omega)$ = Fourier transform of critical circuit signal linearly extrapolated to the nuclear environment field.

Then, $$F_F(\omega) = F(\omega) \times \frac{E_2(\omega)}{E_1(\omega)} \qquad \text{(Equ. 1)}$$

The extrapolated signal at the critical circuit as a function of time is just the inverse Fourier transform of $F_F(\omega)$, or $f_F(t)$.

Note that if the simulated environment differs from the actual environment by some constant factor only, then it is only a matter of multiplying the measured signal by a constant factor, i.e., Fourier transform techniques are not required.

STEP 5: Calculate Individual System Survivability

The calculation of individual system survivability is a two part process. Part 1 requires that one compare the extrapolated critical circuit data to the critical circuit upset/damage thresholds.

Part 2 requires that one use measurement, simulation, and threshold uncertainties to determine subsystem probabilities of survival.

To make these concepts more clear to the reader, the following example is presented.

Example: Individual System Survivability

Let V_F = critical circuit signal peak value (magnitude only) extrapolated to full-field,

V_T = critical circuit threshold value (magnitude only),

$P_{V_T}(x)$ = the cumulative probability distribution function of V_T, or just the probability that V_T is less than or equal to x, i.e.,

$P_{V_T}(x) = \text{prob}\{V_T \leq x\}$, and

$p_{V_F}(x)$ = the probability density function of V_F, or just the probability that V_F is in the interval (x, x + dx) divided by dx, i.e.,

$$p_{V_F}(x) = \frac{\text{prob}\{x < V_F < x + dx\}}{dx}$$

First it can be stated that a failure occurs when the critical circuit signal exceeds or equals the threshold value or -

$$V_F \geq V_T \ .$$

An equivalent statement is that a failure occurs when the critical circuit signal is in the interval x to x plus dx <u>and</u> the threshold value is less than x.

The probability of this event is just

$$dP_F = p_{V_F}(x) P_{V_T}(x) dx \ .$$

The total probability of failure is just the integral of these infinitesimal probabilities of failure over all possible values and therefore the integral is performed over the interval $(0, +\infty)$. The result is

$$P_F = \int_0^{\infty} p_{V_F}(x) P_{V_T}(x) dx \ . \qquad \text{(Equ. 2)}$$

Graphically this is just the area under the curve of the product of $P_{V_T}(x)$ and $p_{V_F}(x)$.

The cumulative distribution function of V_T is determined by a statistical study of all variables affecting the threshold data. These variables include

- Hardware Variables.

- Measurement Variables.

- Waveform Variables.
- Multiple Drive Effects.
- Point of Entry Effects.
- Drive Time Variables (non-stationarity).

The density function of V_F is determined by a statistical study of all variables affecting the test data and extrapolation. These variables include

- Environment Variables.
- System Variables.
- Data Reduction Variables.
- Data Retrieval Variables.
- Extrapolation Variables.

Once P_F is calculated for each critical circuit, then the system probability of failure is calculated to be within the bounds given below.

UPPER BOUND (Mutually Exclusive Failures)

$$P_F = \min\{P_{F_1} + P_{F_2} + \cdots + P_{F_N}, 1.0\}$$

LOWER BOUND (Totally Dependent Failures)

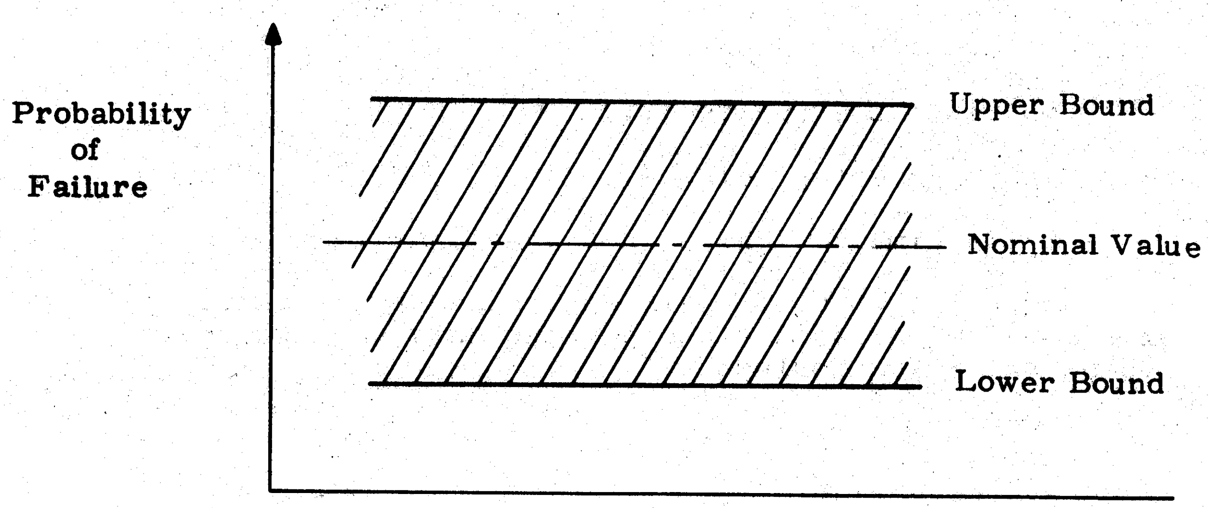
$$P_F = \max\{P_{F_i}\}, \quad (i = 1, 2, \cdots, N)$$

NOMINAL CASE (Independent Failures)

$$P_F = 1 - (1 - P_{F_1})(1 - P_{F_2})\cdots(1 - P_{F_N})$$

Note that these formulas were previously derived in System Design and Assessment Note Number 11, "Determination of System Probability of Failure from Subsystem Probabilities of Failure," M. Skinner, AFWL.

The results of the calculations can be plotted to show the region of uncertainty in the system probability of failure. This is shown in figure 2.



BOUNDS ON PROBABILITY OF FAILURE

Figure 2

<u>STEP 6</u>: Determine Force Survivability

The essence of the problem is to use statistical techniques to determine from observations on a few items what the composition of all the items will be. Typically one may have calculated the probability of failure for ten aircraft from a force of two hundred and fifty aircraft.

A method for this problem has been presented in Note 2 of the System Design and Assessment Notes, by Chris Ashley, AFWL, entitled "Confidence and Reliability in a Finite Population." This note starts from the assumptions that

a. Systems are assessed to be either survivable or not survivable to the effects of a particular EMP,

b. Systems were selected randomly from the force for testing, and

c. All combinations of so many systems either survivable or not survivable are equally likely.

From these assumptions the results of tests on the systems can be used to make quantitative statements about the force survivability.

The expression derived in Note 2 for confidence in the force reliability is given in equation (3) below:

$$C(R) = \frac{\sum_{I=M}^{N(1-R)} \left[\binom{I}{M} \binom{N-I}{L-M} \right]}{\sum_{M=M}^{N-L+M} \left[\binom{J}{M} \binom{N-J}{L-M} \right]} \qquad \text{(Equ. 3)}$$

C(R) is the confidence to be associated with a force reliability of R, i.e., a fraction R or more of the force will survive with confidence C(R).

L equals the number of systems tested. M equals the number of systems that fail the test. N is the number of elements in the force.

A typical example would be if 5 systems are tested and 1 fails the test, then L = 5, M = 1. If it is desired to evaluate C(R) for a reliability of 0.65 or 65%, equation (3) gives C(R) = 0.68. That is, one can have a 68 percent confidence that 65 percent or more of the force will pass the test. It should be noted that one can have 100 percent confidence that at least 4 of the systems will pass the test. These are the 4 that have in fact already passed the test. For any higher reliability the confidence will be less than 100 percent and will decrease as the reliability is increased. In fact, the confidence is zero that the reliability is 100 percent if one system failed the test.

APPLICATION: Strategic Missile System

An application of these six steps to a strategic missile system is presented to demonstrate to the reader how the process is applied. The results are fictitious although the methodology is representative of a real case.

STEP 1: Identify Critical Subsystems

For this application it will be assumed that the system of interest is an intercontinental ballistic missile (ICBM) during its first stage of flight. Further it will be assumed that a detailed analysis and the test program have shown that there are two critical subsystems.

The first critical subsystem is an on-board computer that guides and controls the ICBM during flight. The second critical subsystem is a stage separation device that initiates stage separation at the termination of the stage 1 flight.

Each subsystem is both mission critical and potentially susceptible to EMP effects.

STEP 2: Identify Mission Critical Circuits

For the on-board computer subsystem it will be assumed that two wires into the subsystem control critical memory and shutdown functions. These wires will be designated wire A and wire B, respectively.

For the stage separation subsystem it will be assumed that a single wire is connected to an electro-explosive device (EED). This EED is electrically activated to initiate the stage separation. This wire will be designated as wire C.

STEP 3: Construct the Assessment Matrix

For this example two methods of drive have been used to obtain data on the ICBM survivability. These will be denoted as Simulator #1 and Simulator #2. The Assessment Matrix for this case is given in Figure 3.

STEP 4: Data Extrapolated to Full-Field

Although this step should be a straightforward application of linear systems theory, i.e., Fourier transform techniques, often the quality or sparseness of the data makes the calculation difficult. The author will not elaborate on this point in this note but will only warn the reader.

It will be assumed that this process can be performed within some error bounds and that the results are as listed below:

Wire A = +20 dBV

Wire B = +30 dBV

Wire C = +10 dBV

where $\text{dBV} = 20 \log_{10} \frac{\text{volts}}{1 \text{ volt}}$.

Thus 0 dBV = 1 volt peak.

STEP 5: Calculate Individual System Survivability

For this application it will be assumed that a review and analysis of all variables affecting both threshold and response

LOCATION NUMBER ↓ \ METHOD OF DRIVE →	SIMULATOR #1	SIMULATOR #2
WIRE A (ON-BOARD COMPUTER) (MEMORY INPUT)	WAVEFORM A - 1 - 10	WAVEFORM A - 2 - 17
WIRE B (ON-BOARD COMPUTER) (SHUT-DOWN INPUT)	NO DATA	WAVEFORM B - 2 - 21
WIRE C (STAGE SEPARATION) (EED ACTIVATE INPUT)	WAVEFORM C - 1 - 34	NO DATA

<u>ASSESSMENT MATRIX FOR APPLICATION</u>

FIGURE 3

data have been performed. Further it will be assumed that the results from this variables analysis show that both the responses and thresholds are log-normally distributed with means and standard deviations as listed below:

WIRE A:

Full-Field Response;	Mean Value	= +20 dBV.
	Standard Deviation	= +7 dBV.
Threshold;	Mean Value	= +30 dBV.
	Standard Deviation	= +7 dBV.

WIRE B:

Full-Field Response;	Mean Value	= +30 dBV.
	Standard Deviation	= +9 dBV.
Threshold;	Mean Value	= +60 dBV.
	Standard Deviation	= +9 dBV.

WIRE C:

Full-Field Response;	Mean Value	= +10 dBV.
	Standard Deviation	= +3 dBV.
Threshold;	Mean Value	= +25 dBV.
	Standard Deviation	= +6 dBV.

The calculation of probability of failure for Wire A will be presented in detail. Equation 2 of this note takes the following form for this application:

$$P_F = \int_{-\infty}^{+\infty} p_{\tilde{V}_F}(x) P_{\tilde{V}_T}(x)\,dx$$

where $\tilde{V}_F$ = the log-normally distributed critical circuit peak value,

$\tilde{V}_T$ = the log-normally distributed threshold value,

$$p_{\tilde{V}_F}(x) = \frac{1}{\sqrt{2\pi}\sigma_{V_F}} \exp\left\{-\frac{1}{2}\frac{(x - \bar{V}_F)^2}{\sigma^2_{V_F}}\right\}$$

$$P_{\tilde{V}_T}(x) = \int_{-\infty}^{x} p_{\tilde{V}_T}(y)\,dy$$

$$p_{\tilde{V}_T}(y) = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{V}_T}} \exp\left\{-\frac{1}{2}\frac{(y - \bar{V}_T)^2}{\sigma^2_{V_T}}\right\}$$

$\bar{V}_T$ = mean value of $\tilde{V}_T$, and

$\bar{V}_F$ = mean value of $\tilde{V}_F$.

For this example,

$\sigma_{V_F} = +7$

$\sigma_{V_T} = +7$

$\bar{V}_F = +20$ and

$\bar{V}_T = +30$.

Note also that the limits of the integral now span over the interval $(-\infty, +\infty)$ since the calculation is done for the logarithms of the magnitudes.

Fortunately this integral is of the form used in determining error rates for digital communication systems.[1]

The result of equation 4 is given as $P_F = 0.15$ for wire A. The results for wire B and C are

P_F , wire B = 0.014

P_F , wire C = 0.060

[1] Van Trees, H. L., "Detection, Estimation, and Modulation Theory," Part I.

These can then be used to calculate the Upper bound, the Lower bound, and the Nominal case for the system probability of failure. These results are listed below:

UPPER BOUND, $P_F = 0.15 + 0.014 + 0.060 = 0.22$

LOWER BOUND, $P_F = \max\{0.15, 0.014, 0.060\} = 0.15$

NOMINAL VALUE, $P_F = 1 - (0.85)(0.986)(0.94) = 0.21$

<u>STEP 6</u>: Determine Force Survivability

The present techniques available require that one determine whether or not the system is survivable to the effects of EMP. It will be defined for this example that the system is survivable if the probability of failure is less than 0.01.

Suppose that 3 ICBM's have been tested and assessed. Further suppose that one has been assessed as survivable and two have been assessed as not survivable to EMP effects, and that the total number of such ICBM's is 2000. Then it can be shown using equation #3 that one can have a confidence of 90% that the reliability of the force is 31.6%.

It has been the purpose of this application to show to the reader how this method can be applied. It has not, however, been the purpose of this note to discuss all the details inherent in performing the six steps.