System Design and Assessment Notes

Note 13

An Example of How a Tool of Decision Analysis Might be Applied to Air Force System Vulnerability Assessment

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Abstract

This paper reports the result of a brief effort to construct a simple linear model for the allocation of Air Force resources to the assessment of weapons system nuclear effect vulnerability. The model was used to investigate four questions, as examples: 1. What are the consequences for optimal allocation of Air Force SV assessment resources of building and using a particular effect simulator? 2. What is the effect of optimism on optimal allocation of resources? 3. How might the size of the total resource pool affect optimal allocation of resources? 4. How might the choice of standards for valuing returns from a test affect optimal allocation of resources?

Summary and Conclusions

This is an informal report on a two-week, part time effort by the author and one programmer. The purpose of the effort was to produce data which would, hopefully, provide some indication whether applying decision analysis to allocation of SV resources could be a profitable investment for the Air Force.

Since only one decision analysis technique (viz., linear programming) was examined, and that only cursorily, no pretense is made that the subject of applying decision analysis to Air Force SV is exhausted. However, further pursuit of the topic, should this be deemed wise, must be left to professional operations researchers who have time for it. It is recommended that such further pursuit be continued "in-house".

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Introduction

The "Air Force system" discussed in this note is a set of bolas. A bola is a weapon consisting of, for example, three cords each a few feet long tied together at one end, with a rock tied to each at its other end. The bola is thrown at an animal to entangle it. The set of bolas is intended here to serve as a model for any weapons system of interest. This particular weapon will perform poorly if stress on the cords, prior to or in use, causes them to stretch or break either in flight or after contact with the target. Bola vulnerability to cord stress is intended to serve as a model for any threat to a weapons system, for example EMP (electromagnetic pulse) or other nuclear effects.

The calculations reported in this note were originally performed for a specific Air Force weapons system, nuclear effect threat, and set of vulnerability testing programs. Some of the assumptions of the model were then found to be questionable for that particular system, so the report was rewritten in terms of the bola system to virtually preclude any possibility that the reader might draw some conclusion which could be erroneous in the special case of a particular real system.

Part A. Construction of the Linear Model.

To construct the linear model we accept at the outset that certain axioms realistically represent the Bola cord stress vulnerability assessment situation. The axioms we used for this sample model are:

1. From 1972 to 1978, inclusive, some amount of money will be made available with which to assess Bola cord stress vulnerability. (Let C_{28} represent this amount.)

2. The sum of all expenditures during the assessment may not exceed C_{28}.

3. The amount of time available for the assessment is limited. (Let C_{31} represent this limit.)

4. The assessment is done by testing. Tests are performed by, for example, using simulators[1] (or, for some kinds of tests, sets of simulators). The amount of time required for a single test of fixed kind, multiplied by the number of tests of that kind performed, divided by the number of simulators (or simulator sets) of that kind built (which are used to test different bolas or bola throwers simultaneously), may not exceed C_{31}.

5. The number of bolas or bola throwers available for any one kind of test is bounded by the arsenal size. (Let C_{32} represent this bound.)

6. There are six kinds of tests which one could perform, viz.,

    a. CLM (cord length measurement) tests,

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1. In this note the terms "simulator" and "instrument" will be used interchangeably.

b. KRC (knot and rock count) tests,
c. EPA (exhaustive performance analysis) tests,
d. FTT (field throw test),
e. TA (trajectory alteration in flight) tests, and
f. CSE (analysis of Cord Stretch Effect on accuracy) tests.

7. For each kind of test, except CSE, there is an R&D (research and development) cost of the required instrument (set), a production cost for each instrument (set) actually built, and a cost to actually perform the test on one bola or bola thrower with the instrument. For CSE there is a cost to withdraw from the arsenal, test, and return to the arsenal a bola bombardier.

8. The objective of a test is an increase in the arsenal reliability in which one may have a reasonable confidence. We seek to maximize the sum of these returns over all kinds of tests. (Let Z represent this sum. This objective function will receive further attention below.)

(Note: Units of money will be cents. Units of time will be minutes.)

A more realistic, for example a more complete, set of axioms could of course be devised. For instance, cord modeling is wholly ignored. We wanted, however, to define a problem we could solve in a short time, without interfering significantly with our other responsibilities. We will be satisfied that we have done something worthwhile if the reader, detecting places where realism could be improved, proceeds to solve the more realistic problem himself. Our simplified model is intended only to furnish a guide as to how the reader might go about this.

The next step is to reduce the above axioms to mathematical expressions, linear ones if possible.

Consider first axiom 7. This axiom tells us that for CLM testing (see axiom 6a) we will have to spend $(c_1+c_6x_1+c_{11}x_6)$ altogether, where

c_1 = R&D cost of CLM instrument set,

c_6 = production cost per CLM instrument set,

x_1 = number of CLM instrument sets built,

c_{11} = cost to test one bola in CLM, and

x_6 = number of bolas tested in CLM.

(Please bear with the scattered choices of subscript values. To a large degree the reason for this will appear below.) There will be a similar three part term for the total cost of KRC, EPA, FTT, and TA testing, and a one part term for CSE testing. By axiom 2, therefore,

$$(c_1+c_6x_1+c_{11}x_6) + (c_2+c_7x_2+c_{12}x_7) + (c_3+c_8x_3+c_{13}x_8) +$$

$$+ (c_4+c_9x_4+c_{14}x_9) + (c_5+c_{10}x_5+c_{15}x_{10}) + c_{21}x_{11} \leq c_{28} ,$$

where the x's and the c's are defined in the dictionary formed by Tables I and II. We refer to this kind of mathematical expression as a constraint. By axiom 5, another constraint is

$$x_6 \leq c_{32} .$$

By axiom 4, another constraint is

$$c_{16}x_6 \leq c_{31}x_1 ,$$

Table I: Definitions of Variables.

x_1	$\triangleq$	Number of CLM (cord length measurement) instruments (sets) built.
x_2	"	" " KRC (knot and rock count) " " "
x_3	"	" " EPA (exhaustive performance analysis) instruments (sets) built.
x_4	"	" " FTT (field throw test) " " "
x_5	"	" " TA (trajectory alteration) " " "
x_6	"	" " bolas tested in CLM.
x_7	"	" " " " " KRC.
x_8	"	" " " " " EPA.
x_9	"	" " " " " FTT.
x_{10}	"	" " bolas tested under in-flight conditions.
x_{11}	"	" " bola bombardiers tested by CSE (analysis of Cort Stretch Effect on accuracy).

Table II (first of three pages).

Table II: Definitions of Constants.

<u>(This Table is in three pages, of which this is the first.)</u>

c_1 ≜ R&D (research and development) cost of CLM simulator (set).

c_2 " " " " " " " KRC " "

c_3 " " " " " " " EPA " "

c_4 " " " " " " " FTT " "

c_5 " " " " " " " TA " "

c_6 " Production cost to build one CLM simulator (set).

c_7 " " " " " " KRC " "

c_8 " " " " " " EPA " "

c_9 " " " " " " FTT " "

c_{10} " " " " " " TA " "

c_{11} " Cost to test one bola in CLM.

c_{12} " " " " " " " KRC.

Table II: Definitions of Constants.

(This Table is in three pages, of which this is the second.)

$c_{13} \triangleq$ Cost to test one bola in EPA.

c_{14} " " " " " " " FTT.

c_{15} " " " " " " under in-flight conditions.

c_{16} " " " " " " in CLM.

c_{17} " " " " " " " KRC.

c_{18} " " " " " " " EPA.

c_{19} " " " " " " " FTT.

c_{20} " " " " " " under in-flight conditions.

c_{21} " Cost " " " " bombardier by CSE.

c_{22} " Amount learned per bola tested in CLM.

c_{23} " " " " " " " KRC.

c_{24} " " " " " " " EPA.

Table II: Definitions of Constants.

(This Table is in three pages, of which this is the third.)

$c_{25} \triangleq$ Amount learned per bola tested in FTT.

c_{26} " " " " " " under in-flight conditions.

c_{27} " " " " " bombardier tested by CSE.

c_{28} " Total resources (budget) available for bola cord stress vulnerability testing from 1972 to 1978, inclusive.

c_{29} " Lower bound on interval of validity of linear approximation of amount learned per test for "large" number of tests (CLM, KRC, EPA, CSE).

c_{30} " Upper bound on number of bolas tested in FTT.

c_{31} " Total time available for tests.

c_{32} " Upper bound on number of CLM, KRC, EPA, and CSE tests, i.e., number of bolas in the arsenal.

c_{33} " Lower bound on number of bolas tested in FTT.

c_{34} " " " " interval of validity of linear approximation of amount learned per test for "small" number of tests (TA).

c_{35} " Upper bound on interval of validity of linear approximation of amount learned per test for "small" number of tests (TA).

where c_{16} is defined in the dictionary.

Using such mathematical expressions as the foregoing we can represent all of the first seven axioms. Representing the last axiom requires a little more work.

First we describe reliability as a function of the number of tests conducted. Equations for doing this are derived in other papers[2]. These equations provide reliability (R) as an implicit function not only of the number tested (L), but also of the number which failed the test (M, or L-M if you prefer to count successes instead of failures) and the confidence (C) which one is justified in having in a reliability in view of the test results. For this study we decided to fix C = 90% and try to increase R. (It would of course be easy enough to have fixed R and worked at increasing C, if that seemed more desirable. In fact, given the confidence equations at hand, that would have been easier. Also, one could study the effect on optimal resource allocation of varying the confidence one demands.) To investigate question 2 (see the Abstract), we calculated optimal allocation for a failure rate of 0% (i.e., M = 0) and then repeated the calculations for a failure rate of 10% (i.e., M = L/10). The two graphs in Figures I and II describe reliability as a function of the number of tests conducted for these two degrees of optimism. (Of course, the effect of even greater pessimism could be investigated, say by using a graph for which the failure rate is 30%. We didn't have time to do this. Also, to save time and because arsenal size is large, we set up graphs

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2. *Confidence and Reliability in a Finite Population,* 18 February 1971 (AFWL EMP System Design and Assessment Notes, Note 2), and *Confidence and Reliability in an Infinite Population,* 7 October 1971 (AFWL EMP System Design and Assessment Notes, Note 3), both by Chris Ashley.

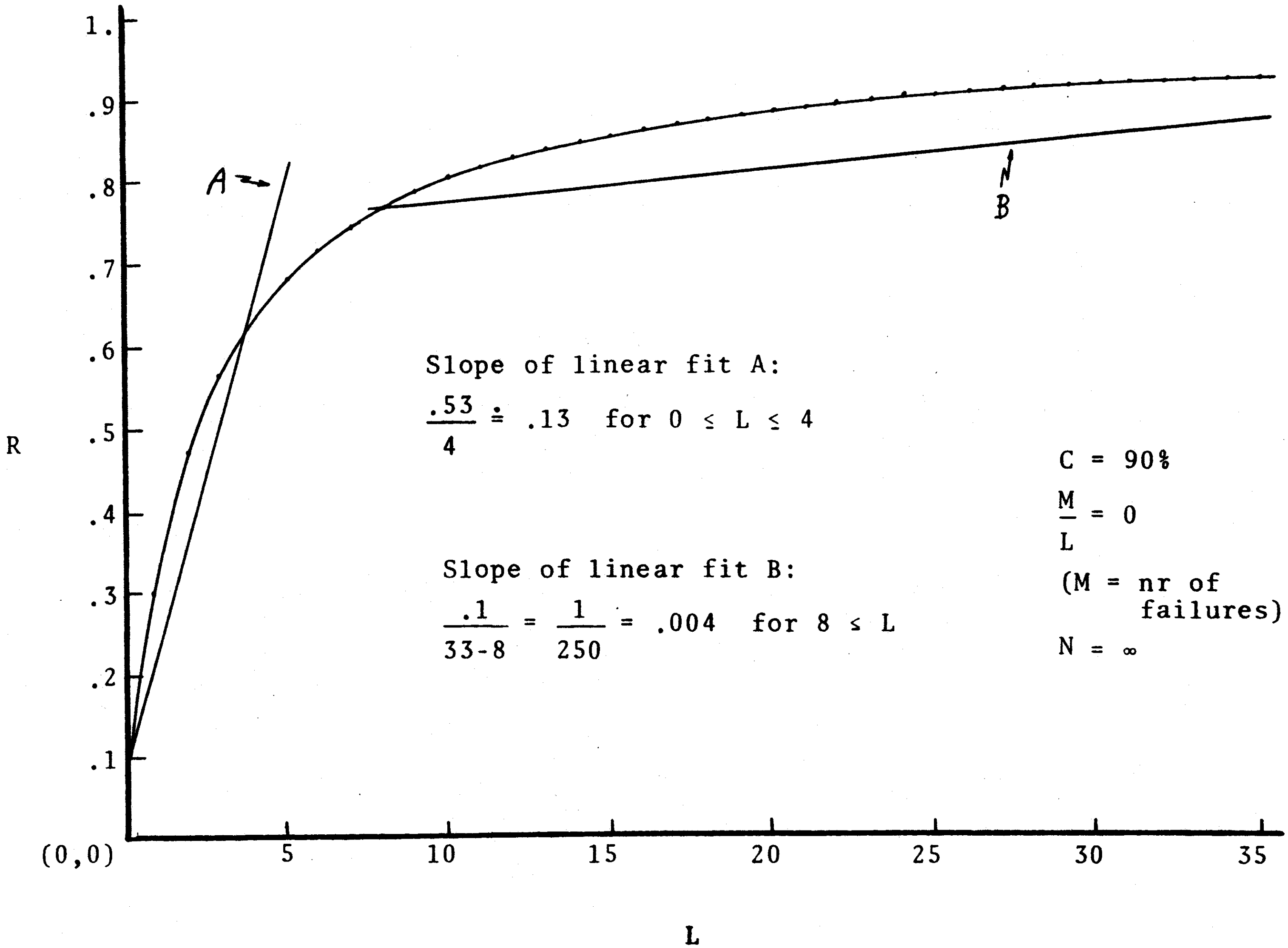
1.
.9
.8
.7
.6
.5
.4
.3
.2
.1
(0,0)
5
10
15
20
25
30
35
R
L
A
B
Slope of linear fit A:
.53/4 ≐ .13 for 0 ≤ L ≤ 4
Slope of linear fit B:
.1/(33-8) = 1/250 = .004 for 8 ≤ L
C = 90%
M/L = 0
(M = nr of failures)
N = ∞

Figure I.

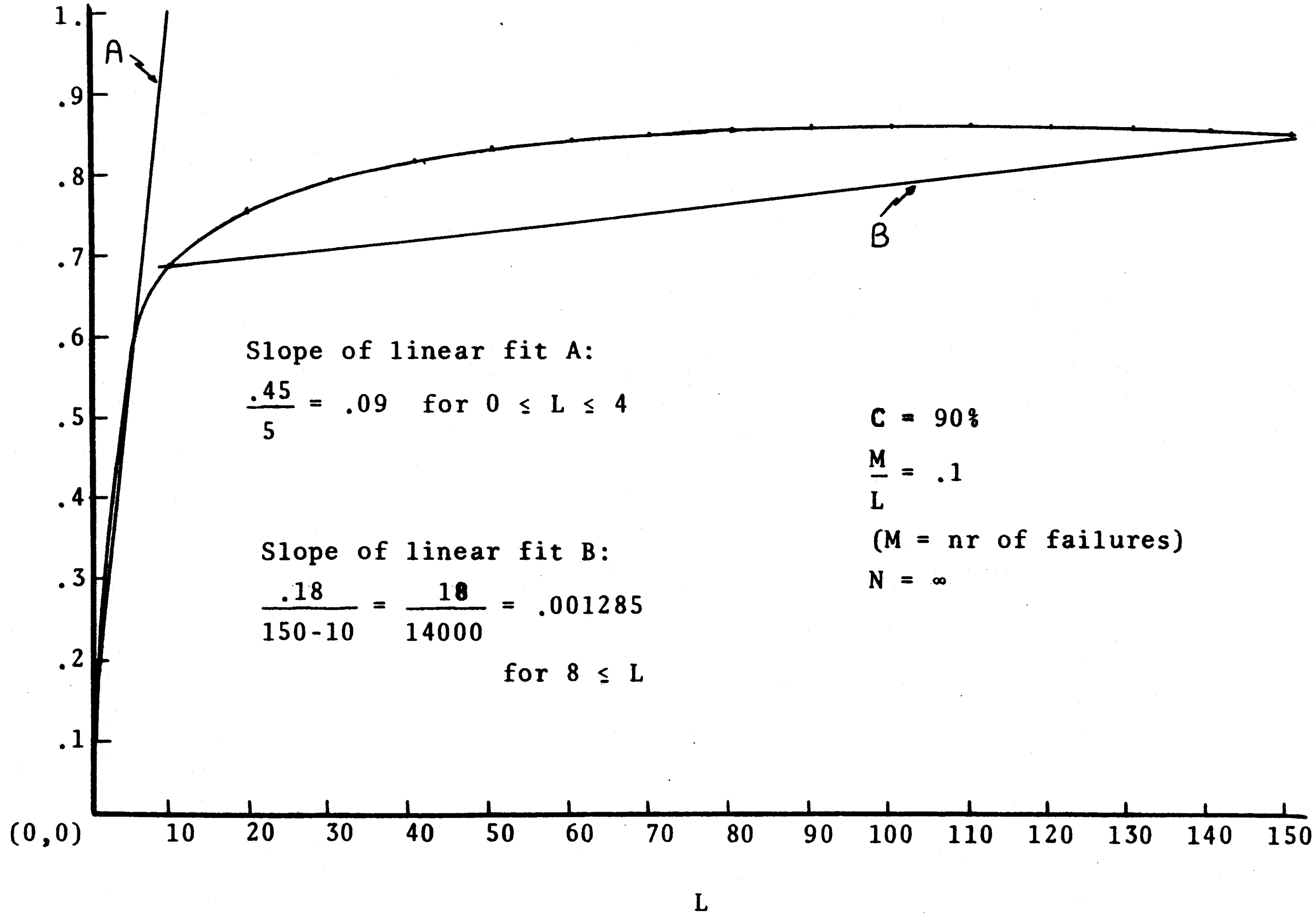
1.
.9
.8
.7
.6
.5
.4
.3
.2
.1
R
A
B
Slope of linear fit A:
.45/5 = .09 for 0 ≤ L ≤ 4
Slope of linear fit B:
.18/150-10 = 18/14000 = .001285
for 8 ≤ L
C = 90%
M/L = .1
(M = nr of failures)
N = ∞
(0,0)
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
L

Figure II.

using only the infinite population equations. We believe the realism sacrificed by this approximation is not significant.)

Second, we overlay linear fits on the relatively straight portions of these reliability graphs. Using these fits, instead of the curves themselves, we can write linear approximations of the reliability function. In doing this we incur responsibility for restricting use of the fits to regions where they are reasonably valid. Such restrictions provide new constraints. In general it would be possible that a constraint of this kind could make the model which we are building unrealistic. For example, what if a good linear fit could only be made for domain values greater than one million, so that the consequence for our model was that we had to put in a constraint that the number of bolas tested in the in-flight mode was at least one million? Not only would such a constraint be unrealistic, it would contradict our axiom 5, leaving us with no feasible region at all.

As it turns out, for this model no such dilemmas arise: a linear fit is available for each interval of numbers tested for which we need one. For example, in real life the number of in-flight tests is only a few. So, to stay in the region of realism, we need to fit the curves for numbers of tests from zero to, say, 4. In this interval the curves are steeply rising, yet have not reached their bend-over elbow. Examination of Figure I, the curve for a 0% failure rate, will show that the slope of the curve in this interval is, to a first order approximation, constant at about .53/4 = .1325, for $0 \leq L \leq 4$. So we can use .13 as the pay-off per in-flight test (assuming no failures) as long as the number of in-flight tests is not permitted to exceed 4.

Similarly, in real life the numbers of different kinds of in-hand tests being considered are in general 18 or more. But fits are available which have a region of validity from about 8 or 10 on up. It is true that for the "pessimistic" case (Figure II, 10% failure rate) a better fit could be had by constraining the number of tests to exceed 20. But we felt having the same constraints on both the optimistic and pessimistic runs, and having a better fit at lower values (since we suspected that's where the answers were going to be), was worth having a slightly poorer fit at high values of number tested.

So we have generated an objective function satisfying axiom 8, viz.,

$$c_{22}x_6+c_{23}x_7+c_{24}x_8+c_{25}x_9+c_{26}x_{10}+c_{27}x_{11} = Z ,$$

where these c's are defined in Table II. Now, since x_{10} is the number of in-flight tests conducted, the foregoing discussion might lead one to expect that c_{26} would be assigned the value .13 in this equation provided the additional constraint were imposed that

$$c_{34} \leq x_{10} \leq c_{35} ,$$

where c_{34} and c_{35} are assigned values like 0 and 4. Such additional constraints as this are indeed imposed, for the reasons laid out above, but further consideration of the problem results in not assigning c_{26} the value .13. These considerations are as follows.

To begin, we discovered that in order for many of the coefficients to have values of about 1 or greater, it was convenient to change systems of units. Therefore all slopes of linear fits were multiplied by one million. That is, instead of letting c_{26} = .13 units, we moved in the direction of letting C_{26} = 130,000 microunits. Then, in the interest of realism, we decided that it was not inconvenient to take account of the fact that, after all, one should learn much more from an EPA on a bola than from a CLM on that same bola. For after all, if the CLM is performed on L bolas and significant variation from the standard is detected in L/2 of them, then what one has learned is not that he may have 50% confidence in 50% arsenal reliability, but rather that he may have 50% confidence that not more than 50% of the arsenal is significantly different from the standard. If the "standard" bola happens to be excessively "stretchy", then one has achieved 50% confidence that at least P% of the bolas will survive cord stretch in hand, where P is notably less than 50. In short, learning that bola susceptibilities to stress aren't varying much is weaker than learning that bolas are stress-proof.

There are several ways of weighting the amounts learned from different kinds of tests. We chose two, to compare them. No doubt more sophisticated schemes for assigning values to c_{22} through c_{27} (see Table II) could be devised, but we had to choose two we could implement quickly. So we chose simple ones. The first simply makes the amount learned from a test to be directly proportional to the amount of time spent performing the test. In implementing this scheme we normalized time so the weighting given EPA is unity. Since (according to one source of information) an EPA on a bola takes on the order of 24 minutes and an in-flight test of a bola takes on the order of 18 minutes, therefore the amount learned from an in-flight test is weighted by the value .75. Consequently c_{26} = .75 * 130,000 = 97,500.

In the second scheme for deciding the amount learned in favor of arsenal reliability, per test, we kept all the previous conventions and added one new one. We decided that bola cord stress vulnerability in hand could reasonably be argued to be different than that in flight by a factor equal to the ratio of the probabilities of sustaining a stress in hand as opposed to in flight. (Note that we are assuming, implicitly, that all stresses are alike, whether sustained in hand or in flight. A study could easily have been included of the effect on optimal resource allocation of different assumptions about kinds of stresses, e.g., local differences in likely peak time domain amplitude). We took the ratio of these probabilities of incidence to be simply the ratio of times which the bola spends in the two states. Allowing for the fact that bolas could be held in hand for periods up to minutes, because of delays to determine what target to throw at or to set up fusillades of throws at one minute intervals, whereas the in-flight phase lasts only on the order of 3 seconds, we decided to use for a ratio of times the number 25. That is, we decided to investigate the effects on optimal resource allocation of the assumption that the typical bola is 25 times as likely to sustain a stress on a cord in hand as it is in flight because the typical bola spends 25 times as much time (after hostilities begin) in hand as it does in flight. To do this we simply repeated all the runs but with c_{26} and c_{27} set to values 1/25 as great as in the earlier set of runs.

Note in passing that in this simple study we made no effort to take into account the effects of multiple stresses (and the probability thereof) nor of synergism between cord stress and other forms of bola degradation.

One criticism which can be leveled against this cursory study, against which we have no defense, is that axiom 8 is set up in terms of the <u>sum</u> represented in the objective equation above, whereas it is by no means obvious that the returns from different kinds of tests are simply additive. Maybe they are additive, but we haven't shown that they are. Our response is that we hope the reader will try to run the problem with the more defensible objective function which he has in mind. In any case, Z, the "total amount learned in favor of arsenal reliability acceptable at the 90% confidence level", is rather difficult to interpret physically even if it is simple and cheap to maximize. It is not even easy to say what its units are. Despite this the values of Z resulting from optimal allocation, divided by one million and rounded to the nearest tenth, are included in the table of results (to be discussed later) for the reader who might want this information as an aid in improving the choice of objective function.

The foregoing discussion results in mathematical expression of the eight axioms as shown in Table III.

Table IV is the same as Table III, except rewritten into matrix notation. We used the CDC 6600 linear programming code OPTIMA to find the optimal solutions under the different conditions indicated by the four questions we undertook to look at, and rewriting the constraints as in Table IV was a convenience in preparing the data for input to OPTIMA. OPTIMA then solved the problems using the product form of the revised simplex method.

One final comment must be made about the construction of our model. Many of the quantities of interest must, in real life, have integer values only. Therefore the problem which we tackled is properly an integer programming problem, not just a linear programming problem. To solve an integer programming problem with only linear programming techniques can produce answers in the optimal solution set which are in

Table III: Constraints.

1. $(c_1+c_6x_1+c_{11}x_6) + (c_2+c_7x_2+c_{12}x_7) + (c_3+c_8x_3+c_{13}x_8) +$

   $+ (c_4+c_9x_4+c_{14}x_9) + (c_5+c_{10}x_5+c_{15}x_{10}) + c_{21}x_{11} \leq c_{28}$

2. $c_{29} \leq x_6 \leq c_{32}$
3. $c_{29} \leq x_7 \leq c_{32}$
4. $c_{29} \leq x_8 \leq c_{32}$
5. $c_{33} \leq x_9 \leq c_{30}$
6. $c_{34} \leq x_{10} \leq c_{35}$
7. $c_{29} \leq x_{11} \leq c_{32}$
8. $c_{16}x_6 \leq c_{31}x_1$
9. $c_{17}x_7 \leq c_{31}x_2$
10. $c_{18}x_8 \leq c_{31}x_3$
11. $c_{19}x_9 \leq c_{31}x_4$
12. $c_{20}x_{10} \leq c_{31}x_5$
13. $x_1 \geq 1$
14. $x_2 \geq 1$
15. $x_3 \geq 1$
16. $x_4 \geq 1$ (for odd numbered runs)

    $x_4 = 0$ (for even numbered runs)
17. $x_5 \geq 1$
18. $c_{22}x_6 + c_{23}x_7 + c_{24}x_8 + c_{25}x_9 + c_{26}x_{10} + c_{27}x_{11} = Z$.

Maximize Z.

Table III.

Table IV: Constraints.

(Where matrix shows no entry, read 0.)

$$
\begin{bmatrix}
c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{21} \\
 & & & & & 1 & & & & & \\
 & & & & & 1 & & & & & \\
 & & & & & & 1 & & & & \\
 & & & & & & 1 & & & & \\
 & & & & & & & 1 & & & \\
 & & & & & & & 1 & & & \\
 & & & & & & & & 1 & & \\
 & & & & & & & & 1 & & \\
 & & & & & & & & & 1 & \\
 & & & & & & & & & 1 & \\
 & & & & & & & & & & 1 \\
 & & & & & & & & & & 1 \\
-c_{31} & & & & & c_{16} & & & & & \\
 & -c_{31} & & & & & c_{17} & & & & \\
 & & -c_{31} & & & & & c_{18} & & & \\
 & & & -c_{31} & & & & & c_{19} & & \\
 & & & & -c_{31} & & & & & c_{20} & \\
1 & & & & & & & & & & \\
 & 1 & & & & & & & & & \\
 & & 1 & & & & & & & & \\
 & & & 1 & & & & & & & \\
 & & & & 1 & & & & & & \\
 & & & & & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & c_{27}
\end{bmatrix}
\cdot
\begin{bmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11}
\end{bmatrix}
=
\begin{bmatrix}
\le c_{28} - c_1 - c_2 - c_3 - c_4 - c_5 \\
\ge c_{29} \\
\le c_{32} \\
\ge c_{29} \\
\le c_{32} \\
\ge c_{29} \\
\le c_{32} \\
\ge c_{33} \\
\le c_{30} \\
\ge c_{34} \\
\le c_{35} \\
\ge c_{29} \\
\le c_{32} \\
\le 0 \\
\le 0 \\
\le 0 \\
\le 0 \\
\le 0 \\
\ge 1 \\
\ge 1 \\
\ge 1 \\
\ge 1^{*} \\
\ge 1 \\
= Z
\end{bmatrix}
$$

Maximize Z.

* for odd numbered runs;
= 0 for even numbered runs.

Table IV.

error by as much as a factor of 2. Not having the time to set up cutting planes, or to find some other way around this difficulty, we elected to simply solve thc problem by linear programming and hope the answers came out integers, or at least close enough to integers so the error induced by rounding would not be significant compared to the error induced by errors in estimates of the costs to build instruments, etc. This full-steam-ahead-and-damn-the-torpedos attitude has, in this effort, one other fact to excuse it. Both the presently most widespread techniques for solving integer programming problems (vix., the Method of Integer Forms developed by Gomory and the more recent alternative approach of Land and Doig) begin by obtaining an optimal continuous solution by the simplex method. Consequently, the reader who is concerned about this defect in our work can use the data in this report as a necessary first step in obtaining a more rigorous solution. For the majority of readers who are puzzled at how one should respond to an optimal solution which requires construction of 27.93 CLM instrument sets, the answer is to construct 28 and round down somewhere else to free the funds to complete that last one: such fine adjustment of the optimal solution set is not going to be noticeable beside the effects of budget uncertainty.

Part B. Some Results Using this Linear Model.

As mentioned in the Abstract of this paper, the model developed in Part A was used to investigate the effects on optimal resource allocation of four decisions, viz.,

1. whether or not to develop and use the FTT stress simulator;
2. whether or not to be exceedingly optimistic about the outcome of system tests, as to whether they will or will not indicate stress vulnerability;
3. whether a large or a small resource pool is allocated to bola cord stress vulnerability testing; and
4. whether to use one or another of two standards for valuing returns from any specific kind of bola cord stress test.

These questions were chosen because something could be done on them in an extremely short time. With a little more time these questions could be investigated more thoroughly, variations on these questions could be addressed, and other questions could be answered. Some variations on these questions which could have been addressed in no more than an additional week or two are the effects on optimal allocation of resources of:

a. having different confidence requirements than the 90% which was used throughout this effort;
b. making fixed reliability a requirement, instead of confidence, and making the objective to increase confidence instead of to increase reliability;

c. different sizes of resource pools besides the two, "large" and "small", which we looked at; and

d. other standards of valuing returns from different kinds of tests than the two we looked at.

Within the scope of the task which we undertook, then, we made 16 computer runs. Half these runs allowed the FTT stress simulator to be developed and used, and half allowed neither. Half the runs assumed testing would return uniformly successful reports of stress immunity, i.e., a 0% failure rate from all tests; and half assumed that 10% of all kinds of tests would indicate vulnerability. Half the runs assumed a seven year Air Force resource pool for all bola cord stress testing (including government administering agencies and testing contractors) of 400,000¢, and half the runs assumed only 175,000¢. Finally, half the runs weighted the amount learned from a test by a factor proportional to the time spent testing, and half weighted the amount learned by an additional factor proportional to the probability that the bola will be in the testing state (in-hand or in-flight) at the time it sustains the stress. (That last factor is discussed in more detail in Part A.)

The 16 computer runs which we made to exercise the linear model are summarized in Table V. Actually we made 22 runs altogether, needing a few to get a feel for the task. The "wasted" runs are not discussed in detail in this note.)

Of all the runs we made, the most expensive required less than 6 seconds of CDC 6600 CP (central processor) time. So the entire study used up less than 3 minutes of CP time. We estimate the cost of the study was, very roughly, as follows:

Table V: Definitions of Runs.

Run Number	FTT stress simulator	Anticipated failure fraction	Resource limit (cents)	Exposure time taken into account in valuing?
1	In	0	400,000	No
2	Out	0	400,000	No
3	In	0	175,000	No
4	Out	0	175,000	No
5	In	10%	400,000	No
6	Out	10%	400,000	No
7	In	10%	175,000	No
8	Out	10%	175,000	No
9	In	0	400,000	Yes
10	Out	0	400,000	Yes
11	In	0	175,000	Yes
12	Out	0	175,000	Yes
13	In	10%	400,000	Yes
14	Out	10%	400,000	Yes
15	In	10%	175,000	Yes
16	Out	10%	175,000	Yes

Table V.

task costs	$340.	salary to construct linear model
	250.	salary to set up runs for computer (including finding and learning how to use OPTIMA)
	30.	computer costs
documentation costs	360.	salary to write this report of study
	100.	salary for typing
	100.	reproduction
	120.	postage and handling
	+ 60.	overhead (offices, furnishings, utilities)
	$1360.	Total

One conclusion which might be drawn from this study, therefore, is that the cost of decision analysis is insignificant compared to the cost of one of any kind of Air Force weapons system nuclear effect vulnerability test. The expenses of optimizing resource allocation would be even less noticeable in the context of all active weapons systems and all (nuclear effects) threats.

The values which we assigned to the constants (defined in Table II) which appear in the constraints of our linear model (presented in Tables III and IV) are given in Table VI. This table is quite large, being 35 constants down by 16 runs across, and the layout required to fit it on standard sized sheets of typing paper will require a few moments of study to understand. Basically it is laid out in four sets of three pages each.

Most of the constants were assigned values which were unchanged for all runs. For example, the cost of building one trajectory alteration in flight simulator (set), after R&D is complete, is not

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the first.)

Run Number:	1	2	3	4
c_1	50	50	50	50
c_2	4,000	4,000	4,000	4,000
c_3	500	500	500	500
c_4	2,000	0	2,000	0
c_5	2,000	2,000	2,000	2,000
c_6	50	50	50	50
c_7	2,100	2,100	2,100	2,100
c_8	0	0	0	0
c_9	8,000	0	8,000	0
c_{10}	5,000	5,000	5,000	5,000
c_{11}	1,200	1,200	1,200	1,200
c_{12}	231	231	231	231

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the second.)

Run number:	1	2	3	4
c_{13}	13,900	13,900	13,900	13,900
c_{14}	269	0	269	0
c_{15}	10,000	10,000	10,000	10,000
c_{16}	3	3	3	3
c_{17}	1.4	1.4	1.4	1.4
c_{18}	24	24	24	24
c_{19}	.1	0	.1	0
c_{20}	18	18	18	18
c_{21}	300	300	300	300
c_{22}	500	500	500	500
c_{23}	233.5	233.5	233.5	233.5
c_{24}	4,000	4,000	4,000	4,000

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the third.)

Run number:	1	2	3	4
c_{25}	16.68	0	16.68	0
c_{26}	90,000	90,000	90,000	90,000
c_{27}	4	4	4	4
c_{28}	400,000	400,000	175,000	175,000
c_{29}	8	8	8	8
c_{30}	1,000	0	1,000	0
c_{31}	84	84	84	84
c_{32}	1,000	1,000	1,000	1,000
c_{33}	8	0	8	0
c_{34}	0	0	0	0
c_{35}	4	4	4	4

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the fourth.)

Run number:	5	6	7	8
c_1	50	50	50	50
c_2	4,000	4,000	4,000	4,000
c_3	500	500	500	500
c_4	2,000	0	2,000	0
c_5	2,000	2,000	2,000	2,000
c_6	50	50	50	50
c_7	2,100	2,100	2,100	2,100
c_8	0	0	0	0
c_9	8,000	0	8,000	0
c_{10}	5,000	5,000	5,000	5,000
c_{11}	1,200	1,200	1,200	1,200
c_{12}	231	231	231	231

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the fifth.)

Run number:	5	6	7	8
c_{13}	13,900	13,900	13,900	13,900
c_{14}	269	0	269	0
c_{15}	10,000	10,000	10,000	10,000
c_{16}	3	3	3	3
c_{17}	1.4	1.4	1.4	1.4
c_{18}	24	24	24	24
c_{19}	.1	.1	.1	.1
c_{20}	18	18	18	18
c_{21}	300	300	300	300
c_{22}	161	161	161	161
c_{23}	75	75	75	75
c_{24}	1,285	1,285	1,285	1,285

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the sixth.)

Run number:	5	6	7	8
c_{25}	5.36	0	5.36	0
c_{26}	67,500	67,500	67,500	67,500
c_{27}	1.285	1.285	1.285	1.285
c_{28}	400,000	400,000	175,000	175,000
c_{29}	8	8	8	8
c_{30}	1,000	0	1,000	0
c_{31}	84	84	84	84
c_{32}	1,000	1,000	1,000	1,000
c_{33}	8	0	8	0
c_{34}	0	0	0	0
c_{35}	4	4	4	4

Table VI (sixth of twelve pages).

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the seventh.)

Run number:	9	10	11	12
c_1	50	50	50	50
c_2	4,000	4,000	4,000	4,000
c_3	500	500	500	500
c_4	2,000	0	2,000	0
c_5	2,000	2,000	2,000	2,000
c_6	50	50	50	50
c_7	2,100	2,100	2,100	2,100
c_8	0	0	0	0
c_9	8,000	0	8,000	0
c_{10}	5,000	5,000	5,000	5,000
c_{11}	1,200	1,200	1,200	1,200
c_{12}	231	231	231	231

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the eighth.)

Run number:	9	10	11	12
c_{13}	13,900	13,900	13,900	13,900
c_{14}	269	0	269	0
c_{15}	10,000	10,000	10,000	10,000
c_{16}	3	3	3	3
c_{17}	1.4	1.4	1.4	1.4
c_{18}	24	24	24	24
c_{19}	.1	0	.1	0
c_{20}	18	18	18	18
c_{21}	300	300	300	300
c_{22}	500	500	500	500
c_{23}	233.5	233.5	233.5	233.5
c_{24}	4,000	4,000	4,000	4,000

Table VI (eighth of twelve pages).

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the ninth.)

Run number:	9	10	11	12
c_{25}	16.68	0	16.68	0
c_{26}	3,600	3,600	3,600	3,600
c_{27}	.16	.16	.16	.16
c_{28}	400,000	400,000	175,000	175,000
c_{29}	8	8	8	8
c_{30}	1,000	0	1,000	0
c_{31}	84	84	84	84
c_{32}	1,000	1,000	1,000	1,000
c_{33}	8	0	8	0
c_{34}	0	0	0	0
c_{35}	4	4	4	4

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the tenth.)

Run number:	13	14	15	16
c_1	50	50	50	50
c_2	4,000	4,000	4,000	4,000
c_3	500	500	500	500
c_4	2,000	0	2,000	0
c_5	2,000	2,000	2,000	2,000
c_6	50	50	50	50
c_7	2,100	2,100	2,100	2,100
c_8	0	0	0	0
c_9	8,000	0	8,000	0
c_{10}	5,000	5,000	5,000	5,000
c_{11}	1,200	1,200	1,200	1,200
c_{12}	231	231	231	231

Table VI (tenth of twelve pages).

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the eleventh.)

Run number:	13	14	15	16
c_{13}	13,900	13,900	13,900	13,900
c_{14}	269	0	269	0
c_{15}	10,000	10,000	10,000	10,000
c_{16}	3	3	3	3
c_{17}	1.4	1.4	1.4	1.4
c_{18}	24	24	24	24
c_{19}	.1	.1	.1	.1
c_{20}	18	18	18	18
c_{21}	300	300	300	300
c_{22}	161	161	161	161
c_{23}	75	75	75	75
c_{24}	1,285	1,285	1,285	1,285

Table VI: Values of Constants.

(This Table is in twelve pages, of which this is the twelfth.)

Run number:	13	14	15	16
c_{25}	5.36	0	5.36	0
c_{26}	2,700	2,700	2,700	2,700
c_{27}	.0514	.0514	.0514	.0514
c_{28}	400,000	400,000	175,000	175,000
c_{29}	8	8	8	8
c_{30}	1,000	0	1,000	0
c_{31}	84	84	84	84
c_{32}	1,000	1,000	1,000	1,000
c_{33}	8	0	8	0
c_{34}	0	0	0	0
c_{35}	4	4	4	4

affected by whether the FTT stress simulator is pursued or not. Referring to Tables V and II, therefore, we would not expect to see any change in the value of c_{10} between even numbered and odd numbered runs. In fact, to answer the first of the four questions it is necessary only to set $c_4, c_9, c_{14}, c_{25}, c_{30}$, and c_{33} to zero for the even numbered runs, leaving all other values of constants unchanged from the preceding run. This is what was done.

Because of this, out of 35 constants 20 were assigned values which were left unchanged through all 16 runs. Consequently, the accuracy with which the value of any one of these 20 constants was chosen affects the accuracy of the conclusions of every one of the 16 runs. Frequently in decision analysis it is discovered that the most expensive (time consuming) task is determining the values of the data which go into the problem; given that data, calculating the optimal values for the variables is quite straightforward. One of the caveats which must be included in this report, therefore, is that the brief time spent on the effort did not permit a determination of indisputable values for all the constants. Several of the values used for these example runs, in fact, are no more than guesses made by relatively informed people in a minute or so without reference to any written costing data.

The conclusion one should draw from the foregoing is that, before the calculated optimal values presented by this report are given a great deal of weight, the reader should carefully scrutinize the values used for input data given in Table VI. In cases where he believes different values would be more accurate, he should revise the input table and rerun the problem to obtain a set of results compatible with his beliefs about costs and times. For example, assumptions about times required to run different kinds of tests were required to implement the valuing schemes described earlier. Specifically, we assumed that 3 minutes were required to CLM a bola, and 1.4 minutes were required to KRC a bola (cf. Table VI, c_{16} and c_{17}). The valuing algorithm therefore concluded that one learns about twice as much from CLM, per bola, as from KRC. The reader with more accurate data may dispute this, and any results calculated from it. If so, he should run the problem again using his more accurate data. Another example is the value we chose for CSE tests.

With all these reservations we set up and made our runs using the data in Table VI. The results are presented in Table VII, which is to be interpreted using Table I.

The first thing to notice in Table VII is that x_{10} is forced to 0 in the entire second half of the runs, viz., runs 9 through 16, inclusive. This result sheds much light on the fourth question. In short, if it be assumed that bolas are 25 times as likely to be subjected to cord stress in hand as they are in flight, because they will in battle probably spend 25 times as much time in hand as they will in flight, and if it be assumed that therefore what is learned from a trajectory alteration test has only 1/25 the value it would otherwise have, then one can conclude (assuming also our model is accurate) trajectory alteration tests are so expensive none should be done.

Three comments must be made about this conclusion. First, if we had chosen a time factor different from 25 the results might have come out differently. In fact, the first half of the runs (runs 1 through 8, inclusive) essentially used a factor of 1, and those runs in general tried to require as

Table VII: Results of Runs.

(This Table is in four pages, of which this is the first.)

Run Number:	1	2	3	4
x_1	1.0	1.0	1.0	1.0
x_2	13.35	14.11	1.0	1.0
x_3	2.29	2.29	2.29	2.29
x_4	1.0	0.	1.0	0.
x_5	1.0	1.0	1.0	1.0
x_6	8.0	8.0	8.0	8.0
x_7	800.93	846.62	8.0	8.0
x_8	8.0	8.0	8.0	8.0
x_9	8.0	0.	8.0	0.
x_{10}	4.0	4.0	2.41	3.63
x_{11}	8.0	8.0	8.0	8.0
Z	58.3	59.4	25.5	36.4
FTT Stress Simulator	In	Out	In	Out
Optimism	High	High	High	High
Resource limit	400,000	400,000	175,000	175,000
Time valued	No	No	No	No

Table VII (first of four pages).

Table VII: Results of Runs

(This Table is in four pages, of which this is the second.)

Run Number:	5	6	7	8
x_1	1.0	1.0	1.0	1.0
x_2	13.35	14.11	1.0	1.0
x_3	2.29	2.29	2.29	2.29
x_4	1.0	0.	1.0	0.
x_5	1.0	1.0	1.0	1.0
x_6	8.0	8.0	8.0	8.0
x_7	800.93	846.62	8.0	8.0
x_8	8.0	8.0	8.0	8.0
x_9	8.0	0.	8.0	0.
x_{10}	4.0	4.0	2.41	3.63
x_{11}	8.0	8.0	8.0	8.0
Z	34.2	34.5	17.5	25.7
FTT Stress Simulator	In	Out	In	Out
Optimism	Moderate	Moderate	Moderate	Moderate
Resource limit	400,000	400,000	175,000	175,000
Time valued	No	No	No	No

Table VII (second of four pages).

Table VII: Results of Runs

(This Table is in four pages, of which this is the third.)

Run Number:	9	10	11	12
x_1	1.0	1.0	1.0	1.0
x_2	15.86	16.62	1.76	2.52
x_3	2.29	2.29	2.29	2.29
x_4	1.0	0.	1.0	0.
x_5	1.0	1.0	1.0	1.0
x_6	8.0	8.0	8.0	8.0
x_7	951.31	996.99	105.44	151.13
x_8	8.0	8.0	8.0	8.0
x_9	8.0	0.	8.0	0.
x_{10}	0.	0.	0.	0.
x_{11}	8.0	8.0	8.0	8.0
Z	25.8	26.9	6.1	7.1
FTT Stress Simulator	In	Out	In	Out
Optimism	High	High	High	High
Resource limit	400,000	400,000	175,000	175,000
Time valued	Yes	Yes	Yes	Yes

Table VII (third of four pages).

Table VII: Results of Runs

(This Table is in four pages, of which this is the fourth.)

Run Number:	13	14	15	16
x_1	1.0	1.0	1.0	1.0
x_2	15.86	16.62	1.76	2.52
x_3	2.29	2.29	2.29	2.29
x_4	1.0	0.	1.0	0.
x_5	1.0	1.0	1.0	1.0
x_6	8.0	8.0	8.0	8.0
x_7	951.31	996.99	105.44	151.13
x_8	8.0	8.0	8.0	8.0
x_9	8.0	0.	8.0	0.
x_{10}	0.	0.	0.	0.
x_{11}	8.0	8.0	8.0	8.0
Z	8.3	8.6	1.95	2.3
FTT Stress Simulator	In	Out	In	Out
Optimism	Moderate	Moderate	Moderate	Moderate
Resource limit	400,000	400,000	175,000	175,000
Time valued	Yes	Yes	Yes	Yes

many trajectory alteration tests as possible. So somewhere between the time (or, if you will, probability) factor of 1 and the time factor of 25 there is a value where the optimum number of trajectory alteration tests is forced to neither the maximum possible nor the minimum possible. A few more computer runs would suffice to find this equilibrium value. In any case it is clear that our number 25 should be examined carefully before the conclusion is accepted.

Second, other value weighting schemes might keep the factor 25 but, because of other considerations introduced, might not conclude trajectory alteration tests are economically unfeasible. In particular, if consideration is given to the thought that a bola would seem more vulnerable in flight, after it has left the protection of its bombardier and when stress removal increases CEP much more because the bola is in motion, than it is in hand, then a value weighting scheme might be preferred which attributed to trajectory alteration tests greater value because of the more useful information they would be expected to yield for stress immunization programs.

Third, the constraints allowed the number of in-flight tests to be forced to 0 but required nonetheless that x_5, the number of Trajectory Alteration simulators built, be at least 1. That is, the model didn't "know" that it could, by eliminating trajectory alteration tests, save not only testing costs but also additional stress simulator construction and R&D costs (cf. Table III). (Had the model "known" this, it would have eliminated trajectory alteration in-flight tests even more readily. This third comment therefore suggests caution "in the opposite direction" from that suggested by the preceding two comments.) There are two corollaries to this comment. The first is that, in light of the knowledge we have now available that the optimal value of x_{10} is 0, runs 9 through 16, inclusive, should be repeated with constraint 17 replaced with constraint 17', viz., $x_5 = 0$, which would result in distribution among the other variables of the trajectory alteration simulator production and R&D resources, to increase their optimal values. The values assigned to these variables are not really optimal until this has been done. The second corollary is that the next thing to do in sophisticating the model, if not to install integer programming, may well be to find and install some device for informing the model that $x_i < .5 \Rightarrow x_{i-5} = 0$ for $6 \leq i \leq 10$.

The second thing to notice about Table VII is that the even numbered runs invariably resulted in higher values of Z than did the immediately preceding run, regardless of which of the two valuing schemes was used, regardless of the size of the resource pool used (400,000 cents or 175,000 cents), and regardless of the level of optimism about the anticipated failure rate. This result suggests that the FTT stress simulator might indeed be a poor investment.

The obvious comment is that it might be wise to check this conclusion about field throw testing by building into the test weighting a factor proportional to the peak time domain stress used during the test (as a guard against the possibility of non-linearity) and/or a factor proportinal to the fraction of the bola stressed. Such additional factors could be incorporated and the runs repeated in no more than a few additional days, and if despite this the same result persisted one could be a good deal surer of the conclusion.

The third thing to notice about Table VII is that there is absolutely no difference at all in optimal allocation of resources regardless of whether a 0% or a 10% failure rate is anticipated. The allocations entered on the first page of the table are identical to those entered on the second, and those on the third page are identical to those on the fourth.

Two comments arise here. First, if it is really thought, as suggested for other reasons above, that the bola might be more vulnerable in flight than in hand, then an additional run might be advisable in which 10% failure is assumed for trajectory alteration (and cord stretch effect) tests and 0% is assumed for in-hand tests. (After all, 6 seconds of CP only costs 25¢, so what's to lose?) Secondly, one could investigate the effects of downright pessimism, e.g., assuming, say, a 40% failure rate.

The final (fourth) thing to notice about Table VII is the interesting synergism which varying the resource pool has with introducing time (probability) into the valuing of different kinds of tests. When time (probability) valuing is absent, decreasing the resource pool results straightforwardly in cutting the number of KRC tests to the minimum allowed by the model and then decreasing trajectory alteration tests as necessary to balance the budget. EPA tests are not affected, and all other kinds of tests are already minimal. On the other hand, when time (probability) is introduced into valuing, the reaction to shrinking resources is quite different. EPA tests are still not affected, and CLM, FTT, and CSE tests are still maintained at minimal numbers regardless. But KRC is no longer forced to take the first brunt of economizing entirely alone. Instead, trajectory alteration and KRC are reduced together, neither one to the minimum permissible number of tests. This suggests that the weighting factor value of 25 which we used is in a range of values to which the effects of varying the resource pool are rather sensitive. Close examination of the factor 25 was suggested for other reasons above; it is now seen that such examination should be performed bearing in mind also this interplay with the effects on optimal resource allocation of the size of the resource pool.