Physics Notes

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The Electron and the Ilectron

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Abstract

The electron was the first elementary particle to be discovered. We know its observed mass, its charge, its angular momentum and we assume that it has infinite lifetime. For more than a century no consensus has been reached over the equation of motion for a radiating electron and the standard model would have us believe that the electron is a point particle. We briefly consider the history of the equation of motion of the electron and point to the reason for the failure of what was considered to be the correct result of classical physics. Furthermore, a given mass has an event horizon, and any mass approaching a point would lie within and so be a black hole, which would rapidly disappear via Hawking radiation. Because a point particle with mass cannot spin, it has been declared that the electron angular momentum is "a purely quantum mechanical effect". The prediction of *zitterbewgung (trembling motion in German)* by Schrödinger offered a partial solution, in that it suggested that the angular momentum was due to circular motion around a point, but still left the problem of a massive particle (the electron) travelling at c, the speed of light. We have the rules for accelerating electrons to exist in stationary states, but no explanation as to why they do not radiate. It was known from the beginning that the electron must have mechanical mass. Unable to separate this mass it was assumed to be 'pushed into a non-observable realm'. Despite these problems and lack of explanations, Quantum Electro Dynamics (QED) continues to produce the most accurate predictions ever produced by theory.

We also consider a complex combination of charge and mass which leads to the consideration of possible 5th force and a new particle, which we call the ilectron. The ilectron has derived properties that make it a contender as a WIMP.

1. Introduction

The electron and in particular its motion has a long history. The Abraham-Lorentz equation of motion is

$$m(\dot{v} - \tau\ddot{v}) = F \qquad (1)$$

Jackson [1] writes ' ..can be criticised on the grounds that is second order in time.....' and '...manifests itself immediately in the so-called runaway solutions.' The derivation is much the same in Heitler [2], Jimenez and Campos [3], Erber [4] and Dirac [5]. The problem is caused by linearizing a non-linear equation [6].

The electron is a negatively charged massive particle with the attributes listed in Table 1.

Attribute	Fundamental "Particle" – the electron
rest mass m_0	$9.10938188 \times 10^{-31}$ kg
charge (- q)	$1.602176462 \times 10^{-19}$ C
classical radius r_e	$2.81795518 \times 10^{-15}$ m
quantum mechanical radius	0
angular momentum $\Omega = \hbar / 2$	5.27285×10^{-35} kg m^2/s
magnetic moment	$1.001159652\ \mu_B$ μ_B is the Bohr Magneton, $= 9.27400899 \times 10^{-24}$ J/T

Table 1. Attributes of the electron

All these quantities except for r_e have been determined by measurement. The radius is determined by equating the integral of the electric field energy to the kinetic energy ($m_0 c^2$) as follows.

$$m_0 c^2 = \frac{1}{4\pi\varepsilon_0} \int_{r_e}^{\infty} \frac{q^2}{r^2}\, dr = \frac{q^2}{4\pi\varepsilon_0 r_e} \qquad (1)$$

The result is

$$r_e = \frac{q^2}{4\pi\varepsilon_0 m_0 c^2} \qquad (2)$$

with ε_0 = permittivity of free space = $8.8541878 \times 10^{-12}$ F/m.

This is to be understood as meaning, if anything, that this is the order of magnitude of the size of the electron. Quantum physicists now regard the electron both as a point particle and a wave, but a point particle is in conflict with relativity. If the electron had zero radius, the integral in equation (1) would diverge and consequently a free electron would have infinite electrostatic energy.

Classically, the electron has to have a non zero radius, and this implies a structure. To form a model it is necessary to include not only field energy, but also a mechanical mass (pages 31-32 of [1]).

Measurements suggested (page 84 of [3]), and solutions to Dirac's equation (page 116 of [7]) confirmed that the electron has angular momentum. Let us consider a spinning spherical mass *m,* radius *r* spinning about its centre, as shown in Figure 1. Such a solid body has a moment of inertia I given by

$$I = \frac{2}{5} m r^2 \tag{3}$$

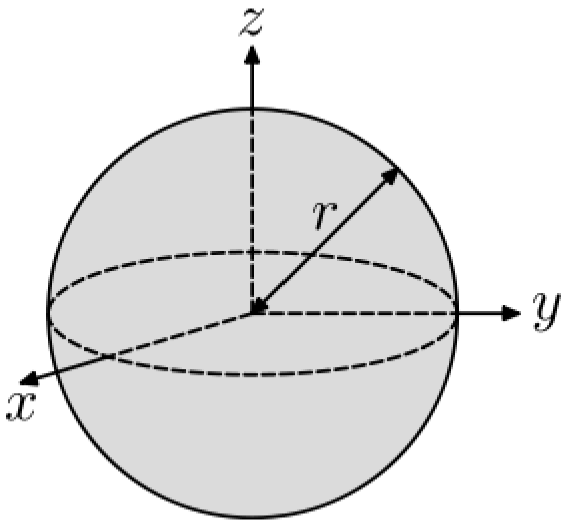

Figure 1. A solid body of mass *m* and radius *r*

its kinetic energy is

$$E = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{2}{5} m_0 r^2\right)\omega^2 = \frac{1}{5} m_0 r^2 \omega^2 \tag{4}$$

where ω is the angular velocity. The angular momentum is

$$\Omega = I\omega = \frac{2}{5} m_0 r^2 \omega \tag{5}$$

If we reduce the radius to zero, the angular momentum goes to zero and a point particle is not acceptable classically. This is why the electron spin is described as *"a purely quantum mechanical effect".*

An obvious interpretation of the solution to Dirac's equation for a stationary electron [2] is that it is moving in a circle of radius $\lambda\!\!\!^{-}/2$ where $\lambda\!\!\!^{-}$ is the reduced Compton wavelength. Using a classically derived equation of motion that introduces the classical stationary state [6] produces a radius some 5% greater, and a velocity some 5% lower, that is consistent with relativity. It is this rotational motion that was termed "*zitterbewgung*" by Schrödinger, the quantum mechanical result being that the velocity was c.

Before the development of quantum mechanics the structure of atoms had been determined by experiment and the results were in conflict with classical physics. The application of ideas contributed by Planck, de Broglie, Einstein, Schrödinger and Heisenberg, culminating in Dirac's equation, laid down the principles of quantum mechanics. Below we consider the specific failure that spurred the development of quantum mechanics followed by a brief look at a classical model of zitterbewgung.

This is followed by a discussion of results generated by a complex transformation of the electron leading to speculation about a new particle, its properties and implications.

2. Classical Approach to the Hydrogen Atom

Consider the Hydrogen atom illustrated in Figure 2.

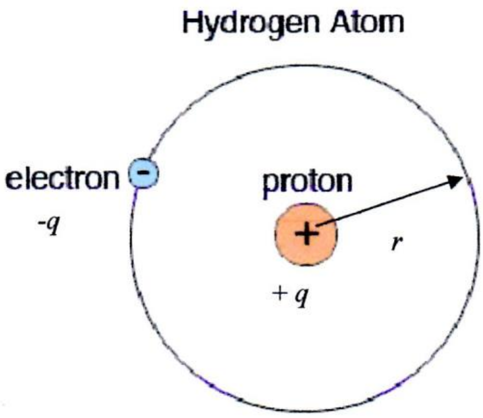

Figure 2. The Hydrogen atom with a single orbiting electron

The electron experiences a centripetal force and a centripetal acceleration. For any object to stay in a circular motion, there is a centripetal (center- seeking) force, However, as per Newton's laws, there is a reaction force that is centrifugal. This pseudo-force that is centrifugal is balanced by the Coulcomb force so that the electron can stay in its circular orbit.

$$\frac{m_o v^2}{r} = \frac{Z\,q^2}{4\pi\,\varepsilon_o r^2} \tag{6}$$

where v is the speed of the electron, r the orbit radius, and Z = atomic number (= 1 for Hydrogen) and ε_o = permittivity of free space = 8.8541878 × 10^{-12} F/m. Equation (6) can also be written so as to relate the speed and orbit radius as,

$$v = \frac{q}{\sqrt{4\pi\varepsilon_o m_o r}} \qquad \text{or} \qquad r = \frac{q^2}{4\pi\,\varepsilon_o\, m_o\, v^2} \tag{7}$$

The result of the orbit radius is consistent with equation (2) from energy considerations. However, classically, an accelerating electron radiates energy (E) at a rate given by the Larmor formula [4]

$$\frac{dE}{dt} = m_0 \tau\, \dot{v}^2 \ \ (J\,/\,s) \tag{8}$$

$$\tau = \frac{q^2}{6\pi\varepsilon_0 m_0 c^3} = 6.27 \times 10^{-24} \text{ (s)} \tag{9}$$

The velocity is $\sim \alpha c$ where α is the fine structure constant given by

$$\alpha = \frac{q^2}{4\pi\varepsilon_0 \hbar c} = 7.29735257\, x\, 10^{-3} \cong \frac{1}{137} \tag{10}$$

and the radius is known as the Bohr radius and is given by

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_0 q^2} = 0.529177 \times 10^{-10} \text{ (m)} \tag{11}$$

Writing E_k for the kinetic energy, the lifetime of the orbiting electron is given by

$$\frac{E_k}{\frac{dE}{dt}} = \frac{\frac{1}{2} m_0 v^2}{m_0 \tau \frac{v^4}{r^2}} = \frac{a_0^2}{2\tau\alpha^2 c^2} = 4.67 \times 10^{-11} \text{ (s)} \tag{12}$$

which is hardly long enough to build a universe!

This problem was overcome by wave mechanics by specifying rules for the motion of electrons in atoms. Dirac's development of his relativistic equation incorporated these rules, but surprisingly produced the result that electrons not acted on by any force, move about at the velocity of light.

3. Classical Explanation of Zitterbewgung

An equation of motion developed in (equation 7.2 of [6]) for a radiating electron, modified by the Stationary State Hypothesis and applied to an electron with no applied forces is

$$\frac{d}{dt}\frac{\mathbf{v}}{\beta} + \frac{i\,\tau_i}{\beta^4}\left\{\dot{\mathbf{v}}^2 + \frac{(\mathbf{v}.\dot{\mathbf{v}})^2}{c^2\beta^2}\right\}\frac{\mathbf{v}}{v^2} = 0 \tag{13}$$

where $\beta = \sqrt{1-(v^2/c^2)}$ and the radiation constant τ_i is

$$\tau_i = \frac{q^2}{6\pi\varepsilon_0 m_i c^3} \tag{14}$$

This equation has the expected solution of zero acceleration, but it also has a solution

$$v = v_0 \exp\left\{-i\,\beta^3 \frac{t}{\tau_i}\right\} = v_0 \exp\{-i\omega t\} \tag{15}$$

$$r = \frac{v_0 \tau_i}{\beta^3} \tag{16}$$

and the observed rest mass becomes

$$m_0 = \frac{m_i}{\beta} \sim 3.2635 m_i \qquad (17)$$

where m_i is the intrinsic mass, which includes both mechanical mass and electromagnetic mass. Equating the angular momentum to the known value of the electron spin

$$m_i \tau_i \frac{v_0^2}{\beta^4} = \frac{\hbar}{2} \qquad (18)$$

Solving this for v_0

$$\frac{v_0}{c} = \eta_e = \frac{\sqrt{1+\frac{3}{\alpha}} - 1}{\sqrt{\frac{3}{\alpha}}} \qquad (19)$$

and the angular velocity is found to be

$$\omega = \frac{v_0^2}{c^2}\frac{4\alpha}{3\tau_0} = \eta_e^2 \frac{4\alpha}{3\tau_0} \qquad (20)$$

The quantum mechanical result for the frequency of the electron "*Zitterbewgung*" is

$$\omega = \frac{4\alpha}{3\tau_0} \qquad (21)$$

a consequence of Dirac's failure to include the mechanical mass of the electron .

The rotation radius for the electron is given by

$$r_s = \frac{v_0 \tau_0}{\left[1 - \frac{v_0^2}{c^2}\right]^2} = \frac{3}{4}\frac{c^2 \tau_0}{\alpha v_0} = \frac{3}{4}\frac{c\tau_0}{\alpha \eta_e} \qquad (22)$$

Introducing the Compton Wavelength via the relation

$$\lambda\!\!\!^{-} = \frac{3c\tau_0}{2\alpha} \qquad (23)$$

the radius is given by

$$2r_s = \left(\frac{c}{v_0}\right)\lambda\!\!\!^{-} = \frac{\lambda\!\!\!^{-}}{\eta_e} = 1.05053537\lambda\!\!\!^{-} \qquad (24)$$

This result is consistent with the uncertainty principle , the uncertainty of the momentum being $\sqrt{2}mr\omega$ and the uncertainty in position $\sqrt{2}r$ giving

$$\Delta p \Delta x = \left(\sqrt{2}\, mr\omega\right) \text{x} \left(\sqrt{2}\, r\right) = 2mr^2\omega = h/(2\pi) = \hbar \qquad (25)$$

Furthermore this model [6] allows the calculation of the fine structure constant to the same accuracy as the QED calculation [7]. With this classical relativistic model of the electron equation of motion together with a model of the hydrogen atom, it was possible to calculate FSC by first determining an approximation for a constant using the then available value of FSC and no other constant. The constant was approximately 34031.256. For the theory the constant had to be prime, and 34031 is prime. Further development gave 34031-0.00016. Then inverting problem and calculating FSC assuming 34031, gives a value within about 1σ of the Gabrielse result and is only $1.3\text{x}10^{-11}$ from the latest mean CODATA result.

4. The Complex Electron

It is often advantageous to form a complex variable from two real variables, as in complex analysis, by dividing by suitably defined dimensional constants so as the transformed variables have the same dimension. We can do this for the charge and mass of elementary particles, as follows. Considering the electrostatic and the gravitational force between two electrons, we have

$$f_q = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{K q_1 q_2}{r^2} \tag{26}$$

$$f_m = -\frac{m_1 m_2}{4\pi g r^2} = \frac{-G m_1 m_2}{r^2} \tag{27}$$

where, in the usual notation

$$\frac{1}{4\pi\varepsilon_o} = K = 9 \ x \ 10^9 \ (N m^2 / C^2) \tag{28}$$

$$\frac{1}{4\pi g} = G = 6.670 \ x \ 10^{-11} \ (N m^2 / Kg^2) \tag{29}$$

In the gravitational force above, the convention is a negative force for attraction and a positive force for repulsion.

We now define two constants

$$d_1 = \sqrt{\frac{\varepsilon_0}{g}} = 0.861x10^{-10} \ (C / Kg) \quad and \quad d_2 = \sqrt{\frac{g}{\varepsilon_0}} = 1.16 \ x \ 10^{10} \ (Kg / C) \tag{30}$$

We observe that a dimensionless constant for the electron is given by

$$d_3 = \frac{q}{m_o} d_2 = \frac{q}{m_o}\sqrt{\frac{g}{\varepsilon_o}} = \frac{1.6x10^{-19} \ C}{9.1x10^{-31} Kg} \ d_2 = 1.75x10^{11} d_2 = 2.03x10^{21} = \sqrt{\frac{F_{em}}{F_G}} \tag{31}$$

This approach could be extended to all the elementary particles, in particular the proton and the neutron. It is noted that every fundamental particle that has non-zero mass will have an associated constant "d_3". This constant would vanish for a neutral particle $(q = 0)$ such as a neutron.

We can now express mass and charge with the same dimension by forming a complex Mass (M) or a complex charge (Q). There are various ways to form these combinations, as follows.

$$M = m + i\,q\,d_2 \;=\; m + i\,q\sqrt{\frac{g}{\varepsilon_o}} \quad (32a) \quad and \quad Q = q + i\,m\,d_1 \;=\; q + i\,m\sqrt{\frac{\varepsilon_o}{g}} \qquad (32b)$$

$$M \;=\; \frac{m}{\sqrt{g}} \;+\; i\;\frac{q}{\sqrt{\varepsilon_o}} \quad (32c) \qquad and \qquad Q \;=\; \frac{q}{\sqrt{\varepsilon_o}} \;+\; i\,\frac{m}{\sqrt{g}} \qquad (32d)$$

For example, if we use the combination of equation (32 c) and consider complex or combined forces, between two complex masses

$$M_1 \;=\; \frac{m_1}{\sqrt{g}} \;+\; i\;\frac{q_1}{\sqrt{\varepsilon_o}} \qquad and \qquad M_2 \;=\; \frac{m_2}{\sqrt{g}} \;+\; i\,\frac{q_2}{\sqrt{\varepsilon_o}} \qquad (33)$$

we have the complex force given by

$$F_M = \frac{-M_1 M_2}{4\pi\, r^2} \qquad (34)$$

Expanding this expression using equations (32 c),

$$F_M = \left\{ -\frac{m_1 m_2}{4\pi\; g\, r^2} + \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \right\} - i\,\left\{ \frac{m_1 q_2 + m_2 q_1}{4\pi\sqrt{g\varepsilon_0}\, r^2} \right\} = f_m \;+ i\; f_{im} \qquad (35)$$

Note that the real part gives the conventional gravitational and Coulomb forces.

If we now choose M_2 to be a neutral particle by setting $q_2 = 0$

$$F_M^{*} = f_m - i\,\left\{ \frac{m_2\; q_1}{4\pi\sqrt{g\,\varepsilon_0}\, r^2} \right\} \qquad (36)$$

Setting

$$q_1 = m_{im}\sqrt{\frac{\varepsilon_0}{g}} \qquad (37)$$

we obtain

$$F_M^{*} \;=\; f_m + i\; f_{im} \qquad \text{where} \qquad f_{im} = -\frac{m_2 m_{im}}{4\pi\; g\; r^2} \qquad (38)$$

The strength of a force is given by Matt Strassler [5]

$$S = \frac{r^2 f}{\hbar c} \qquad (39)$$

Comparing the strength of this imaginary force to the electric force, and assuming the neutral particle is a neutron

$$S_{im/e} = \frac{m_n}{q}\sqrt{\frac{\varepsilon_0}{g}} \sim 9.00x10^{-19} \qquad (40)$$

The strengths of the four standard forces together with this new force are given below in Table 2.

Strong	Weak	Electro Magnetic	Imaginary Electro- Mass	Gravity
0.11	0.020.	0.007 (=α)	6.6×10^{-21}	2.6×10^{-35}

Table 2. Strengths of the five forces

These strengths are calculated in the regime where they are effective, the strong force within the nucleus, and the weak force within a nucleon while the electromagnetic force operates outside the electron. A force is considered weak if $F\,r^2 << (\hbar c)$ and is considered strong if $F\,r^2 \approx (\hbar c)$. In particle physics the electro-mass force would be undetectable, but if the neutral 'particle' is a neutron star, this force would be the dominant force acting on electrons. A typical neutron star contains ~25×10^{56} neutrons.

5. The Ilectron

Recognising that imaginary mass can be interpreted as charge, and imaginary charge as mass, we can consider the ***imaginary electron,*** where we interchange the mass and charge, but this transformation is not merely an algebraic transform. We must transform the intrinsic mass of the electron to obtain the new charge, the charge transforming as before.

$$i[m_i, q] = [im_i, iq] = \left[\sqrt{\frac{g}{\varepsilon_0}}q, \sqrt{\frac{\varepsilon_0}{g}}m_i\right] = [m_{ii}, q_i] \tag{41}$$

Evaluating the components

$$m_{ii} = q_e\sqrt{\frac{g}{\varepsilon_0}} = 1.86\times10^{-9}\ \text{kg} \quad \text{and} \quad q_i = m_i\sqrt{\frac{\varepsilon_o}{g}} = \frac{m_0}{3.2635}\sqrt{\frac{\varepsilon_0}{g}} = 2.17 \text{ x } 10^{-41}\ \text{C} \tag{42}$$

In equation (42), m_{ii} is the intrinsic mass equivalent of the electron's charge and q_i is the charge equivalent of electron's intrinsic mass. We are calling this new particle with its mass and charge given by equation (42) as the "**ilectron**" or an "***imaginary electron***".

6. Ilectron Spin and Magnetic Moment

If we assume the spin of $\hbar/2$ is transferred to the new mass we can determine the magnetic moment. The fine structure constant is given by

$$\alpha = \frac{q^2}{4\pi\varepsilon_0\hbar c} \tag{43}$$

Replacing q by q_i

$$\alpha_i = \frac{q_i^2}{4\pi\varepsilon_0\hbar c} = \left(\frac{q_i}{q}\right)^2\alpha = \frac{7.85^2\times10^{-82}}{1.602^2\times10^{-38}} = 2.401\times10^{-43} \tag{44}$$

From equation (19) replacing α by α_i, the rotational velocity is

$$\frac{v_0}{c} = \eta_e = \frac{\sqrt{1+\frac{3}{\alpha_i}}-1}{\sqrt{\frac{3}{\alpha_i}}} = \sqrt{1+\frac{\alpha_i}{3}} - \sqrt{\frac{\alpha_i}{3}} = 1-2.829\times10^{-22} \tag{45}$$

From equation (16)

$$r = \frac{v_0 \tau_{ii}}{\left[1 - \frac{v_0^2}{c^2}\right]^{3/2}} \tag{46}$$

where

$$\tau_{ii} = \frac{q_i^{\ 2}}{6\pi\varepsilon_0 m_i c^3} = \frac{2}{3}\frac{\alpha_i \hbar}{m_{ii} c^2} = \frac{2}{3}\left(\frac{q_i}{q}\right)^2 \frac{\alpha\hbar}{c} = \frac{2}{3}\left(\frac{q_i}{q}\right)^2 \frac{\lambda\!\!\!^{-}}{c} \tag{47}$$

and $\lambda\!\!\!^{-}$ is the reduced Compton radius. The radius of rotation is then

$$r = \frac{2}{3}\left(\frac{q_i}{q}\right)^2 \frac{\lambda\!\!\!^{-}}{c} \frac{v_0}{\left[1 - \frac{v_0^2}{c^2}\right]^{3/2}} = \frac{2}{3}\frac{2.401\times10^{-43}}{7.297\times10^{-3}}\frac{3.861\times10^{-13}}{13.46\times10^{-33}} = 6.3\times10^{-18} m \tag{48}$$

The magnitude of the magnetic moment is then given by

$$\mu_m = q_i v_0 r = 1.36\times10^{-22} qc \times \frac{6.3\times10^{-18}}{9.27\times10^{-24}} \mu_B = 1.48\times10^{-27} \mu_B \tag{49}$$

The above ideas have been criticised as 'mere speculation', but where would science be if some of us did not occasionally break away from what is known.

7. *Electrons and Black Holes*

Briefly, it is worthwhile to look at some analogous relationship between the electron and a black hole. The classical electron radius is given by the Lorentz formula [6] (also equations (2) and (7) above),

$$r_0 = \frac{q_e^2}{4\pi\varepsilon_0 m_0 c^2} = 2.817\ x\ 10^{-15} m \tag{50}$$

Replacing $q_e / \sqrt{\varepsilon_0}$ by $m_0 / \sqrt{g}$ in the above, the radius of the electron is given by

$$r_0 = \frac{m_0}{4\pi g c^2} \tag{51}$$

A black hole is a region of space and time with a very strong gravitational pull so that no particle or EM radiation can escape from it. The boundary from which no escape is possible is called the “event horizon”. The above formula for the radius of an electron is similar to the formula for the radius of a Schwarzschild black hole

$$r_S = \frac{m}{2\pi\ g c^2} = \frac{2Gm}{c^2} \tag{52}$$

Examples of Schwarzschild radii of some common planets are:

sun (3 km) , earth (8.7 mm), Moon (0.11 mm) and Jupiter (2.2m). If the Schwarzschild radius exceeds the physical radius, the object is a black hole. Hence these planets are not black holes.

Alternatively we could estimate the Schwarzschild radius for an electron as

$$r_{es} = \frac{m_e}{4\pi\, g\, c^2} = 6.76 \times 10^{-58} m \tag{53}$$

whereas the classical radius of the electron is 2.82 x 10^{-15} m. With the classical radius of an electron being much larger than its Schwarzschild radius, it is not a black hole.

8. Summarizing Remarks

By forming a complex electron we have raised the possibility of a new force many orders weaker than the electromagnetic force, yet still fourteen orders greater than gravity. Continuing with this idea we constructed a hypothetical imaginary electron which turns out to have a substantial mass of ~1.9×10^{-9} kg and effectively no charge or magnetic moment.

The ilectron would appear to be a good contender for a WIMP (a weakly interacting massive particle), a so far undetected hypothetical particle in one explanation of dark matter. If the ilectron exists, presumably it has an anti-particle and an ilectron meeting an anti-ilectron would result in a pair of high energy gamma photons. Assuming a similar imbalance in the relative numbers as is found for other particle pairs, such interactions would be rare. The detection of these gamma rays would in all probability require the design of new detectors.

The ilectron has no impact on positron. The anti-particle to the ilectron is the ipositron.

The mass of the ilectron is 1.86 x10^{-9} kg which converts to ~10^{21} MeV = 10^{15} TeV
The maximum energy of the LHC in CERN is 13TeV. The LHC would have to be upgraded by a factor of around 8x10^{13} to produce an ilectron!

At this time the only possibility of confirmation is if the ilectron meets its anti-particle, the ipositron and the resulting gammas pass through our detectors. Gammas above 100 TeV are classified as ultra-high energy, and so far none have been detected.

If they are the elusive WIMP they would have to provide a mass of ~90% of the Milky Way. The mass of the sun is ~2x10^{30} kg and so the number of ilectrons required is

$$N = [\, 2 \times 10^{30} \text{ kg} / 1.86 \times 10^{-9}] \sim 10^{39} \tag{54}$$

With the volume of the Milky Way being ~10^{48} m^3 the required density is

$$\rho_N = 10^{-8}\ m^{-3} \tag{55}$$

Detection of gammas of this energy would not only support their existence but would also enable estimates of the density of ilectrons. Scattering of these gammas with electrons may also contribute to an explanation of gamma ray bursts.

From classical theory, the electron has a radius which is larger than its Schwarzschild radius, and hence, is not a black hole. This observation of the similarity of the radius of the electron [equation (51)] and the radius of a black hole [equation (52)] raises an interesting question – if the charges of particles are quantized, are the masses of black holes quantized? That is a question for Astro-Physicists and may have been already answered.

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