Physics Notes
Note 16

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A Note on the Stationary State Model of the Hydrogen Atom

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An inconsistency in the derivation of the dynamics of the hydrogen atom [1] led to the omission of an improved approximation in the implicit formula for the fine structure constant. This note removes the inconsistency.

1 INTRODUCTION

1.1 In [1] the equation for the angular velocity of the electron in the ground state was found to be

$$\rho^3 + \rho^2 - \frac{\tau^2 k}{r_0^3} = 0 \tag{1.1}$$

and the approximate solution

$$\rho \sim \pm\sqrt{\frac{\tau^2 k}{r_0^3}}\left[1 - \frac{1}{2}\sqrt{\frac{\tau^2 k}{r_0^3}}\right] \tag{1.2}$$

obtained, where $\rho = \omega\tau$ and $k = e^2/m$. The orbital radius was then assumed to be a_0, the Bohr radius. The correct procedure is to obtain the radius on the assumption that the angular momentum is $\hbar$.

2. ORBITAL RADIUS CORRECTION

2.1 The angular momentum is given by

$$\Omega = m r_0^2 \omega \tag{2.1}$$

Imposing the known orbital spin

$$m r_0^2 \sqrt{\frac{k}{r_0^3}}\left[1 - \frac{1}{2}\sqrt{\frac{\tau^2 k}{r_0^3}}\right] = \hbar \tag{2.2}$$

This reduces to

$$\sqrt{\frac{a_0}{r_0}} = \left(1 - \frac{1}{3}\alpha^3\left[\frac{a_0}{r_0}\right]^{3/2}\right) \tag{2.3}$$

Setting $r_0 = a_0 + \delta$ and making use of the binomial theorem

$$r_0 = a_0\left[1 + \frac{2}{3}\alpha^3\left(1 - \alpha^3\right)\right] \tag{2.4}$$

or to $0(\alpha^3)$

$$r_0 = a_0\left[1 + \frac{2}{3}\alpha^3\right] \tag{2.5}$$

3. CORRECTION TO THE ANGULAR VELOCITY

3.1 It follows that the angular velocity is

$$\omega = \frac{\alpha c}{a_0}\left(1 - \frac{4}{3}\alpha^3\right) \tag{3.1}$$

4. CORRECTION TO α FORMULA

4.1 This correction makes a small change to the formula for α (see equation 12.18 in [1]), specifically

$$p = 2\frac{s^{3/2}}{3\alpha}\left[\sqrt{1 + \frac{3}{\alpha}} - 1\right]^2 \times \left[1 - \frac{\alpha^2}{2}\left(1 - \gamma^2\right)\left(1 + \frac{3\alpha^2\gamma^2}{2}\right)\right]^3 \frac{1}{\left(1 - \frac{4\alpha^3}{3}\right)} = 34031.01845 s^{3/2} \tag{4.1}$$

5 REFERENCE

Ref [1] An Investigation into the Motion of a Classical Charged Particle, I.L. Gallon, Physics Note No 15, University of New Mexico, Albuquerque