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Interaction Type and Reciprocal Probability Model for Spin-Half Measurements*

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Abstract

We use *three new constructs* to obtain a *new* probability model for the spin-half measurements, the simplest EPR problem, under the usual positive probability theory and without instantaneous interaction at-a-distance. The constructs are: 1) the ***reciprocal PDFs*** (probability density functions); 2) the ***spin types***; 3) the ***measuring equipment's observation co-generating variable responses.***

From the average of repeated measurements on a $\hat{z}$-polarized spin state $"\hat{z}"$, we *explicitly deduced* that there are the two spin types, named A and B. For $"\hat{z}"$, the spin orientation **r.v.** (random variable) $\hat{R}_S$ of the two types have the conditional PDFs $f_{\hat{R}_S}(\hat{r}|"\hat{z}",A) = 4\,\hat{r}\cdot\hat{z}\,u(\hat{r}\cdot\hat{z})$ and $f_{\hat{R}_S}(\hat{r}|"\hat{z}",B) = 2\,u(\hat{r}\cdot\hat{z})$, respectively. For an $\hat{a}$-directed measuring equipment, its response r.v. $\hat{R}_M$ has both types conditional PDFs $f_{\hat{R}_M}(\hat{r}|\hat{a},A) = 1$ and $f_{\hat{R}_M}(\hat{r}|\hat{a},B) = 2\,|\hat{r}\cdot\hat{a}|$. When an equipment encounters a particle, their probabilistic measurement interaction is modeled as $\hat{R}_I = \hat{R}_M \otimes \hat{R}_S$, the reciprocal product r.v. of $\hat{R}_M$ and $\hat{R}_S$ where the former responses in kind to type of the latter. The model predicts all one particle spin measurement probabilities. Then, assigning the joint PDF $f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0") = \delta(\hat{r}_1 + \hat{r}_2)$ to the *source* r.v. $(\hat{R}_{S1}, \hat{R}_{S2})$ of a singlet pair "0", the model immediately predicts the pair's **QM** (quantum mechanics) measurement probabilities by a *conditional* probability expansion under *independent* measuring PDFs and *independent* counting probabilities at separate measurement sites. The EPR "paradox" is shown to be another counter-intuitive example in conditional probability.

Moreover, we showed the *necessity* of the type attribute by a self-contradict that follows from, and exhibits the *mutual incompatibility* of, *four* premises. The well-known first ***three*** are: QM measurements, positive probability, and independence of space-like separated measurement equipments. The *heretofore unrecognized but tacitly used **fourth** is: disallowing interaction types*. The self-contradict, implied by the four premises, is a mathematically false relation in the second moment, the Bell inequality. Whence all related difficulties. The new model deletes and replaces the fourth, explicitly and quantitatively. Further, we suggest an experiment in He spectroscopy to test and demonstrate a factor-of-two observable effect of the spin types. Finally, we summarize, including a list of implications.

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1 Background

The simplest example of the well-known EPR wave-packet reduction and QM (quantum mechanics) incompleteness problem [1] is the spin-half particles' singlet-pair measurements [2]. Consider a *population* of many such singlet-pairs, each prepared in the spin zero state $"0"$. Randomly sample a singlet-pair from the population. Measure the spin of one member of the sampled pair at location x_1 on $\hat{a}$ and referred to as #1, e.g. by passing it through a Stern-Gerlach magnet aligned along an arbitrarily fixed unit vector $\hat{a}$. The measurement result S_1 is $+\hbar/2$ **xor** (exclusively either or) $-\hbar/2$. Repeating this sampling and measurement procedure *many* times, the accumulative results tend to be 50% $+\hbar/2$ and 50% $-\hbar/2$. The same holds for S_2, measurement results to the other members of the sampled pairs measured *separately* at x_2 on $\hat{b}$ and referred to as #2. But if the measurement results are *paired* by keeping track of the outcomes of *both* S_1 and S_2 for *each pair*, then the many repetitions tend to the joint probability [1] [3]

$$Pr\{S_1=+,\ S_2=+|"0";\hat{a},\hat{b}\}=\frac{1-\hat{a}\cdot\hat{b}}{4} \tag{1}$$

Since a measured $S=-$ on $\hat{a}$ is same as $S=+$ on $-\hat{a}$, the other three outcomes' probabilities are obvious. They are referred to as etc. when needed and imply the covariance

$$cov(S_1,\ S_2|"0",\ \hat{a},\ \hat{b})=-\hat{a}\cdot\hat{b} \tag{1-1}$$

The (1) is derived readily in QM and verified by experiments [4].

Now, (1) exhibits that the paired measurements' results are *not* statistically independent. This of course is anticipated because two members of a singlet-pair come from a common source at which they are *prepared* to be correlated, namely 0 total spin angular momentum state $"0"$. The *puzzle* is

[1] The *usual* probability theory, with its frequency interpretation and *explicit* notation, is used. E.g., Pr{X=x|y} is the conditional probability that the random variable X realizes as x given that the condition y occurred; Pr{X=x} = Σ Pr{X=x|y_i }*Pr{Y=y_i } if {y_i} are disjoint and exhaustive; cov(X, Y) = E{XY}-E{X}E{Y} where E{X} is expected value of X. Also, spin values are in units of $\hbar/2$ and $S=+$ stands for $S=+\hbar/2$, etc.

that the seemingly simple (1) has heretofore *defied all previous probability modeling efforts under only two obvious and general constraints*: 1) the usual positive probability theory; 2) the finite speed of light and its implied independence of space-like separated measuring processes.

These two constraints are cherished because 1) should tally *any* physical frequency of occurrences [5] and 2) is *the* foundation of relativity [6]. But all such modeling attempts on (1), many by brilliant researchers, to conceptually reconcile QM and relativity, failed. Finally Feynman showed that *all these previous models* must invoke *negative* probabilities [7]. In fact, insisting positive **PDFs** (probability density functions) to these models for (1) leads to self-contradictions, e.g. the Bell inequality [8] in their second moments. The above are well-known [9].

2 New Model Synopsis

We introduce and quantify *three new concepts* to probabilistically model (1) under the constraints 1) and 2), following a long strenuous effort. They are the ***reciprocal PDFs***, the ***spin types***, and the ***measuring equipment's active role in co-generating*** a measured outcome. The intuitive physical picture follows.

There are two types of spin-half particles, A and B, type being a *new attribute* aside from the polarization. But an $\hat{a}$-directed spin measuring equipment carries *both* types of measuring responses. The particle and the equipment have nothing to do with each other before their measurement encounter. *During* the encounter, the measuring equipment *responds* to the measured particle *according* to the particle's *type*. Their interaction is probabilistically modeled by an interaction **r.v.** (random variable) $\hat{R}_I$ whose PDF is the *reciprocal product* of the particle's PDF and the equipment's PDF, the former depending only on the particle's polarization and the latter only on the equipment's direction. These PDFs of the same type are naturally reciprocal to each other. Then the $\hat{R}_I$'s random sampling realization $\hat{r}$ determines the spin measurement *outcome*. Measuring equipments space-like separated are physically and statistically independent so that their joint measuring PDF is the usual product of their individual marginal PDFs.

To present the model, we first show the relevant mathematics. Then we model and solve the one particle spin in a polarized state. We next apply the solution to all one particle states. Then we apply it to predict the two particles singlet-pair measurement statistics. An explicit exhibit of the EPR "paradox" as a conditional probability illusion follows. Finally, we show the necessity of the model and summarize.

3 Reciprocal Probability and Preliminaries

3.1 Reciprocal PDFs

A PDF is a real-valued non-negative scalar function which integrates over its defined domain to a normalized 1. Consider r.v.s X and Y with respective PDFs $f_X(\xi)$ and $f_Y(\xi)$ and **CDFs** (cumulative distribution functions) $F_X(\xi),\ F_X(\leq x0)=0$ and $F_Y(\xi),\ F_Y(\leq y0)=0$. The two r.v.s are defined to be *reciprocal* to each other and form a *reciprocal product* r.v. denoted by $Z \equiv X \otimes Y \equiv Y \otimes X$ *if* the product of their PDFs $f_Z(\xi) \equiv f_X(\xi)\ f_Y(\xi)$ remains a PDF, which is the PDF of Z.[2] Its CDF $F_Z(\xi)$ has $F_Z(\leq z0)=0,\ z0=max(x0,\ y0)$. Here, the r.v.s X and Y, and Z, can each be multidimensional of the same dimension with its own joint PDF. Notice that $(Z_1,\ Z_2)=(X_1,\ X_2)\otimes(Y_1, Y_2)$ does *not* imply $Z_i = X_i \otimes Y_i$ unless we have independent $\{X_i\}$ and independent $\{Y_i\}$.

3.2 Some Simple Properties

One, immediately following definition the reciprocal PDFs can not be discrete. They must be continuous. *Two*, many distribution families, but not the normal distribution, have natural reciprocals. E.g., the exponential PDF Ep{λ}, $e^{-x/\lambda}/\lambda,\ x \in [0,\infty)$, has the reciprocal Ep{1- λ} for λ in (0,1). Their reciprocal product's PDF is Ep{λ(1−λ)}. The beta PDF Be{m,n}, $x^{m-1}(1-x)^{n-1}\ \Gamma(m+n)/(\Gamma(m)\,\Gamma(n))$ $x \in [0,\ 1]$, has a continuous set of reciprocals Be{m', n'} and forms the reciprocal products Be{m+m'-1, n+n'-1}. For integer orders, the Be{m', n'} becomes discrete, such as the Be{5,4}, Be{20,15}, Be{76,56}, Be{285,209}, etc. to Be{3,1}; and the Be{1,2}, Be{2,6}, Be{6,21}, Be{21,77} etc. to Be{2,2}.

Three, the reciprocal product r.v. is a Fourier complement of the independent r.v. sum. I.e., the **CF** (characteristics function) of the sum $Z1 = X1 + Y1$ is the product of the CFs of $X1$ and $Y1$. And the PDF of $Z1$ is the convolution of the latter's PDFs. Complementarily, the CF of $Z = X \otimes Y$ is the convolution of the CFs of X and Y. And of course the PDF of Z is the product of the latters' PDFs. This Fourier complementariness may imply interesting physics, relating to the uncertainty principle which is just the mathematical inequality $\sigma_t^{\,2}\,\sigma_\omega^{\,2} \geq 1/4$ for PDFs $|f(t)|^2$ and $|F(\omega)|^2$ of normalized Fourier transform pairs $f(t)$ and $F(\omega)$.

Four, the reciprocal product r.v. $Z = X \otimes Y$ can be viewed mathematically as a *deterministic* mapping function

$$Z = g(X) \qquad (2\text{-}1)$$

---

[2] This is *not* the joint PDF $f_{X,Y}(x,y) = f_X(x)\ f_Y(y)$ of two independent r.v.s X and Y with respective PDFs $f_X(x)$ and $f_Y(y)$.

that maps a r.v. X under *an influence* of both the reciprocal PDFs of X and Y. The mapping function $g()$ solves

$$f_X(g(\eta))\, f_Y(g(\eta))\, \frac{d}{d\eta} g(\eta) = f_X(\eta) \;\Leftrightarrow\; F_{X\otimes Y}(g) = F_X(\eta) \qquad (2\text{-}2)$$

which has a unique solution for scalar r.v.s but has infinite solutions in general. The probability distribution of realizations of the r.v. $X \otimes Y$ is identical to that of the r.v. $g(X)$.

3.3 Some Integrals

Define $d\hat{r} \equiv d\Omega_{\hat{r}} / 4\pi$ as the normalized infinitesimal solid angle in the 3-dimensional unit-vector $\hat{r}$ orientation space. We present some integrals for later use:

$$\int d\hat{r}\; u(\hat{r}\cdot\hat{z}) = \int d\hat{r}\; |\hat{r}\cdot\hat{z}| = 1/2 \qquad (3)$$

$$\int d\hat{r}\; u(\hat{r}\cdot\hat{z})\; \hat{r}\cdot\hat{a} = \hat{a}\cdot\hat{z}/4 \qquad (4\text{-}1)$$

$$\int d\hat{r}\; u(\hat{r}\cdot\hat{z}) |\hat{r}\cdot\hat{a}| = 1/4 \qquad (4\text{-}2)$$

$$\int d\hat{r}\; u(\hat{r}\cdot\hat{z})\; u(\hat{r}\cdot\hat{a})\; \hat{r}\cdot\hat{a} = (1+\hat{a}\cdot\hat{z})/8 \qquad (5\text{-}1)$$

$$\int d\hat{r}\; u(\hat{r}\cdot\hat{z})\; \hat{r}\cdot\hat{z}\;\; sgn(\hat{r}\cdot\hat{a}) = \int d\hat{r}\; u(\hat{r}\cdot\hat{z})\; |\hat{r}\cdot\hat{a}|\;\; sgn(\hat{r}\cdot\hat{a}) = \hat{a}\cdot\hat{z}/4 \qquad (5\text{-}2)$$

$$\int d\hat{r}\; u(\hat{r}\cdot\hat{a})\; u(\hat{r}\cdot\hat{b}) = (1-\theta_{\hat{a}\hat{b}}/\pi)/2 \qquad (6)$$

Here, $u(\xi)$ is the step function, $sgn(\xi)$ the sign function, and $\theta_{\hat{a}\hat{b}}$ ($\leq \pi$) the angle between $\hat{a}$ and $\hat{b}$. The (3), (4-1), and (6) are obvious. The (4-2) is easily seen by graphically rotating $\hat{a}$ away from $\hat{z}$ in the integrand of (4-1). The (5-1) immediately follows (4) and implies (5-2).

4 One Particle Model

4.1 Formulation --- Polarized State

Consider the sampling and measurement experiments of Sec. 1, but to a population of $"\hat{z}"$ polarized particles at only one site. The average of the many repetitions' $\hat{a}$-measurement results S, each + xor -, is $\hat{a}\cdot\hat{z}$. This *phenomenology alone*, with definition of statistical average, implies

$$Pr\{S=+|"\hat{z}",\hat{a}\} = (1+\hat{a}\cdot\hat{z})/2 \qquad (7)$$

Without loss of generality, we model the measurement interaction by a unit-vector r.v. $\hat{R}_I$ and conditionally expand on it the left side of (7)

$$Pr\{S=+|"\hat{z}",\hat{a}\} = \int d\hat{r}\;\; Pr\{S=+|"\hat{z}",\hat{a},\hat{R}_I=\hat{r}\}\; f_{\hat{R}_I}(\hat{r}|"\hat{z}",\hat{a}) \qquad (8)$$

Next, conforming with the + and - observations, we make the simplest intuitive *assumption* that *given* $\hat{R}_I$'s realization $\hat{r}$ the measurement outcome is deterministically + xor - *solely* according to the sign of the *realized* $\hat{a}\cdot\hat{r}$, i.e.

$$Pr\{S=+|"\hat{z}",\hat{a},\hat{R}_I=\hat{r}\}=u(\hat{a}\cdot\hat{r}) \quad (9)$$

$$Pr\{S=-|"\hat{z}",\hat{a},\hat{R}_I=\hat{r}\}=u(-\hat{a}\cdot\hat{r}) \quad (10)$$

Now, we *postulate* that the interaction r.v. $\hat{R}_I$ be the *reciprocal product* of the measuring equipment's response r.v. $\hat{R}_M$ and the measured particle's orientation r.v. $\hat{R}_S$, i.e. $\hat{R}_I=\hat{R}_M\otimes\hat{R}_S$ with

$$f_{\hat{R}_I}(\hat{r}|"\hat{z}",\hat{a})=f_{\hat{R}_M}(\hat{r}|\hat{a})\ f_{\hat{R}_S}(\hat{r}|"\hat{z}") \quad (11)$$

Substituting (8) to (11) into (7) gives the *integral equation*

$$\int d\hat{r}\ u(\hat{r}\cdot\hat{a})\ f_{\hat{R}_M}(\hat{r}|\hat{a})\ f_{\hat{R}_S}(\hat{r}|"\hat{z}")=(1+\hat{a}\cdot\hat{z})/2 \quad (12)$$

4.2 Solution

The integral equation (12), assuming the obvious physical-space orientation isotropy which dictates that $f_{\hat{R}_M}(\hat{r}|\hat{a})$ be a function of $\hat{r}\cdot\hat{a}$ and $f_{\hat{R}_S}(\hat{r}|"\hat{z}")$ be a function of $\hat{r}\cdot\hat{z}$, has *two* set of solutions. They are *unique*, within a physical smoothness excluding "infinitely oscillating" mathematical functions. Denoted as *type A and type B*, they are

$$f_{\hat{R}_M}(\hat{r}|\hat{a},A)=1 \quad (13\text{-}1)$$

$$f_{\hat{R}_S}(\hat{r}|"\hat{z}",A)=4\,\hat{r}\cdot\hat{z}\ u(\hat{r}\cdot\hat{z}) \quad (13\text{-}2)$$

and

$$f_{\hat{R}_M}(\hat{r}|\hat{a},B)=2\,|\hat{r}\cdot\hat{a}| \quad (14\text{-}1)$$

$$f_{\hat{R}_S}(\hat{r}|"\hat{z}",B)=2\ u(\hat{r}\cdot\hat{z}) \quad (14\text{-}2)$$

From (3) to (4), clearly all these are PDFs and the $\hat{R}_M$ and $\hat{R}_S$ of the same type are reciprocal.

When substituted back, of course either type renders (12) an identity and reproduces the measurement probability (7) with the expected value $\hat{a}\cdot\hat{z}$ to S. But the expected value can also be viewed from the interaction r.v. $\hat{R}_I$. E.g., for type A we have

$$E\{S|"\hat{z}",\hat{a},A\}=\int d\hat{r}\ f_{\hat{R}_I}(\hat{r}|"\hat{z}",\hat{a},A)\,E\{S|"\hat{z}",\hat{a},A,\hat{r}\}$$

$$=\int d\hat{r}\ f_{\hat{R}_M}(\hat{r}|\hat{a},A)\,f_{\hat{R}_S}(\hat{r}|"\hat{z}",A)\ sgn(\hat{r}\cdot\hat{a}) \quad (15\text{-}1)$$

$$=\hat{a}\cdot\hat{z} \quad (15\text{-}2)$$

Here, (15-1) uses (9) to (11); (15-2) uses (13), (5-2).

The physical interpretation is that there are two spin types, A and B, of spin-half particles. Such a particle carries, and is characterized by, a r.v. $\hat{R}_S|"p",T)$ whose PDF depends *only* on its polarization state "p" and spin type T, given by (13-2) and (14-2) if $"\hat{z}"$ polarized. However, a measuring equipment carries both types of response r.v.s $\{\hat{R}_M|\hat{a},A),\hat{R}_M|\hat{a},B)\}$, each of which has a PDF depending *only* on its type and the equipment direction setting $\hat{a}$, *given by (13-1) and (14-1)*. Only during the particle- equipment measurement encounter, the measuring equipment responds to and interacts with the measured particle according to the latter's type, *locally*. The interaction is random and modeled by a r.v. $\hat{R}_I = \hat{R}_M \otimes \hat{R}_S$, the reciprocal product of $\hat{R}_S$ and $\hat{R}_M$ which of the same type are naturally reciprocal to each other. Then the interaction randomly samples a *realization* $\hat{r}$ of $\hat{R}_I$ to determine the measured spin reading and its outgoing polarization state: $S = +$ and $"+\hat{a}"$ polarized if $\hat{r}\cdot\hat{a} > 0$; $S = -$ and $"-\hat{a}"$ if $\hat{r}\cdot\hat{a} < 0$.[3]

Notice an asymmetry in the measuring and the measured. The former's state and r.v.s are not altered by the measurement, but the latter's are (except the trivial case of measuring $"\hat{z}"$ on $\hat{z}$). Notice also a symmetry. A measuring equipment does not just sort a pre-realized deterministic attribute of the measured object, as treated in classical physics. Nor it is limited to sample from a pre-described r.v. of the measured object and sort, as practiced in QM. It *co-founds* the interaction r.v. *and* does the sampling realization and sorting. All on-site, during, and *is* the measurement. From a pre-prejudiced, unphysical, view that disallowed this observation co-founding role to measuring tool, one would proclaim that the measuring equipment "distorts" the measured attribute before recording and reporting it.

To sum up, the spin-half probability model is

$$Pr\{S=+|"p",\hat{a}\} = \sum_j P(t_j)\int d\hat{r}\; Pr\{S=+|"p",\hat{a},t_j,\hat{r}\}\; f_{\hat{R}_I}(\hat{r}|"p",\hat{a},t_j) \qquad (16\text{-}1)$$

$$Pr\{S=+|"p",\hat{a},t_j,\hat{r}\} = u(\hat{a}\cdot\hat{r}) \qquad (16\text{-}2)$$

$$f_{\hat{R}_I}(\hat{r}|"p",\hat{a},t_j) = f_{\hat{R}_M}(\hat{r}|\hat{a},t_j)\; f_{\hat{R}_S}(\hat{r}|"p",t_j) \qquad (16\text{-}3)$$

Here, "p" = spin polarization state; $\hat{a}$ = measuring equipment's direction setting; t_j = j_th spin type = A, B; $P(t_j)$ = probability that the particle spin type T, a new attribute additional to "p" and not dependent on $\hat{a}$, is t_j; and $\hat{R}_M$ is reciprocal to $\hat{R}_S$ and has PDFs (13-1), (14-1). The explicit solution of (16) for a general multi-states quantum system is not found yet.

4.3 <u>Application to Unpolarized State</u>

First, a unpolarized state $"U"$ has no preferred direction. The simplest model is to *assign*, to both types, a uniform PDF to its $\hat{R}_S$, i.e.

$$f_{\hat{R}_S}(\hat{r}|"U",A) = f_{\hat{R}_S}(\hat{r}|"U",B) = 1 \qquad (17\text{-}1)$$

Then, applying the above model (16), with (13-1), (14-1), (17) and using (3), (4-2), predicts

$$Pr\{S=+|"U",A,\hat{a}\} = \int d\hat{r}\; u(\hat{r}\cdot\hat{a})\; 1\; 1 = 1/2 \qquad (17\text{-}2)$$

---

[3] The $\hat{a}\cdot\hat{r} = 0$ case is of zero measure. It does not matter whether, or how, it is assigned + or - as long as done self-consistently.

$$Pr\{S=+|"U",B,\hat{a}\}=\int d\hat{r}\; u(\hat{r}\cdot\hat{a})\; 2|\hat{r}\cdot\hat{a}|\;\; 1=1/2 \qquad (17\text{-}3)$$

This is the observed, and QM calculated, result.

Second, using (4-2), we can assign the PDFs

$$f_{\hat{R}_S}(\hat{r}|"U",A)=2|\hat{r}\cdot\hat{z}''| \qquad (18\text{-}1)$$

$$f_{\hat{R}_S}(\hat{r}|"U",B)=1 \qquad (18\text{-}2)$$

with any $\hat{z}''$, simultaneously. And we still get the measurement probability 1/2, same as (17).

Third, the state $"U"$ can not be measured without being changed. Thus, the PDF of its $\hat{R}_S$ can and needs only to reflect property of the population; and we can model $"U"$ as a type-unrestricted population of polarized particles of which *each* is polarized along its $\hat{z}'$ direction, with a corresponding PDF (13-2) xor (14-2) to *its* $\hat{R}_S$, while the whole population's set $\{\hat{z}'\}$ is *uniformly* distributed in all 4π solid angles. I.e.,

$$f_{\hat{R}_S}(\hat{r}|"U",t)=\int d\hat{z}'\; f_{\hat{R}_S}(\hat{r}|"U",t,\hat{z}')\; f_{\hat{Z}'}(\hat{z}'|"U",t)=\int d\hat{z}'\; f_{\hat{R}_S}(\hat{r}|"\hat{z}'",t)\;\; 1=1 \qquad (19)$$

where type t = A, B. This model to $\hat{R}_S$ predicts the measurement statistics (17-2, -3) just as well.

Thus, a unpolarized spin state $"U"$ has its spin r.v. $\hat{R}_S$ not uniquely modeled. E.g., it can be interpreted at least three ways as: 1) a type unrestricted population consisting of *individually unpolarized* particles each with a $\hat{R}_S$ given by (17-1); 2) similar to 1) but with its $\hat{R}_S$ specified by (18); 3) a population of *individually polarized* particles with their polarizations distributed uniformly in all directions. This non-uniqueness to underlying modeling, in view of measurement result being indirectly and probabilistically co-generated by the measuring equipment, is no surprise.

5 Application to Singlet-Pair Measurements

5.1 Joint Probability

Reconsider the paired measurements in Sec 1. The above model requires *two members of a singlet-pair to be different types*. Keeping them type symmetric, we assume the simplest and often used anti-correlated joint PDF for the *source* pair's spin orientation r.v.s $\hat{R}_{S1}$ and $\hat{R}_{S2}$

$$f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0";A,B)=\delta(\hat{r}_1+\hat{r}_2)=f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0";B,A) \qquad (20)$$

Suppose the #1 is type A and #2 type B. Apply model (16). The reciprocal-product interaction r.v. $(\hat{R}_{I1},\hat{R}_{I2})=(\hat{R}_{M1},\hat{R}_{M2})\otimes(\hat{R}_{S1},\hat{R}_{S2})$ has the joint PDF

$$f_{\hat{R}_{I1},R_{I2}}(\hat{r}_1,\hat{r}_2|"0";\hat{a},\hat{b};A,B)=f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0";A,B)\; f_{\hat{R}_{M1},\hat{R}_{M2}}(\hat{r}_1,\hat{r}_2|\hat{a},\hat{b};A,B) \qquad (21)$$

Now the separate *measuring equipments* at x_1 and x_2 are not related and, each *locally* seeing *its own* direction-setting and encountered spin type, are *physically and statistically* independent. I.e., the joint PDF of their r.v.s $\hat{R}_{M1}$ and $\hat{R}_{M2}$ is

$$f_{\hat{R}_{M1},\hat{R}_{M2}}(\hat{r}_1,\hat{r}_2|\,\hat{a},\hat{b};A,B)=f_{\hat{R}_{M1}}(\hat{r}_1|\,\hat{a},A)\;f_{\hat{R}_{M2}}(\hat{r}_2|\hat{b},B) \tag{22}$$

where the factors are given by (13-1) and (14-1). Similarly, given realizations $\hat{r}_1$ and $\hat{r}_2$, the conditional measurements S_1 and S_2 are local and independent, i.e.

$$\begin{aligned}Pr\{S_1=+,\;S_2=+|"0";\hat{a},\hat{b};A,B;\hat{r}_1,\hat{r}_2\} &= Pr\{S_1=+|"0",\hat{a},A,\hat{r}_1\}\;Pr\{S_2=+|"0",\hat{b},B,\hat{r}_2)\\ &= Pr\{S_1=+|\hat{a},\hat{r}_1\}\;Pr\{S_2=+|\hat{b},\hat{r}_2\} \qquad (23)\end{aligned}$$

where the last factors are specified by (16-2).

With (20) to (23), the model (16) predicts

$$\begin{aligned}Pr\{S_1=+,\;S_2=+|"0";\hat{a},\hat{b};A,B\} &= \int d\hat{r}_1\int d\hat{r}_2\;Pr\{S_1=+|\hat{r}_1,\hat{a})\;Pr\{S_1=+|\hat{r}_2,\hat{b})\\ &\qquad f_{\hat{R}_{M1},\hat{R}_{M2}}(\hat{r}_1,\hat{r}_2|\hat{a},\hat{b};A,B)\;f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0";A,B) \qquad (24\text{-}1)\\ &= \int d\hat{r}_1\,u(\hat{r}_1\cdot\hat{a})\int d\hat{r}_2\,u(\hat{r}_2\cdot\hat{b})\,1\;2|\hat{r}_2\cdot\hat{b}\,|\,\delta(\hat{r}_1+\hat{r}_2) \qquad (24\text{-}2)\\ &= (1-\hat{a}\cdot\hat{b})/4 \qquad (24\text{-}3)\end{aligned}$$

where (24-3) has used (5-1). Now let p_{AB} be the probability that #1 be type A and #2 be type B, and p_{BA} be vice versa; thus $p_{AB}+p_{BA}=1$. But exchanging $A\leftrightarrow B$ in (24) renders $1\;2|\hat{r}_2\cdot\hat{b}|\rightarrow 2|\hat{r}_1\cdot\hat{a}|\;1$ in (24-2) and the same (24-3). Thus, *regardless* of the singlet-pairs' spin type arrival probabilities at x_1 and x_2, the new probability model under constraints 1) and 2) predicts

$$\begin{aligned}Pr\{S_1=+,S_2=+|"0";\hat{a},\hat{b}\} &= p_{AB}\,Pr\{S_1=+,S_2=+|"0";\hat{a},\hat{b};A,B\}+(A\leftrightarrow B)\\ &= (1-\hat{a}\cdot\hat{b})/4 \qquad (25)\end{aligned}$$

This is the puzzling (1).

5.2 <u>Marginal Probabilities</u>

Suppose in the *paired* measurements we are interested only in one site's measured results, say S_1 at x_1 on $\hat{a}$ and do not care about the outcomes of #2 at x_2 on $\hat{b}$. It is straightforward to calculate the marginal and conditional probabilities and PDFs. E.g., (25) and etc. immediately imply for S_1

$$Pr\{S_1=+|"0";\hat{a},\hat{b}\} = Pr\{S_1=+,S_2=+|"0";\hat{a},\hat{b}\}+Pr\{S_1=+,S_2=-|"0";\hat{a},\hat{b}\}=1/2 \tag{26}$$

This is the observed probability. The *same* holds for S_2 on $\hat{b}$.

But (26) can be obtained and interpreted from the *marginal* interaction r.v. $\hat{R}_{I1}$. Integrating out $\hat{R}_{I2}$ from (21) gives

$$f_{\hat{R}_{I1}}(\hat{r}_1|"0";\hat{a},\hat{b};A,B)=\int d\hat{r}_2\;f_{\hat{R}_{I1},\hat{R}_{I2}}(\hat{r}_1,\hat{r}_2|"0";\hat{a},\hat{b};A,B)=2|\hat{r}_1\cdot\hat{b}| \tag{27-1}$$

and similarly

$$f_{\hat{R}_{1l}}(\hat{r}_1|"0";\, \hat{a},\hat{b};B,A) = 2\,|\hat{r}_1 \cdot \hat{a}| \tag{27-2}$$

Therefore

$$f_{\hat{R}_{1l}}(\hat{r}_1|"0";\, \hat{a},\hat{b}) = p_{AB}\; 2\,|\hat{r}_1 \cdot \hat{b}| + p_{BA}\; 2\,|\hat{r}_1 \cdot \hat{a}| \tag{28}$$

It *appears* unsettling that the measuring direction $\hat{b}$ at x_2, whatever its measurement outcome, enters the marginal PDF of the interaction r.v. $\hat{R}_{1l}$ at x_1. And one is even tempted to literally interpret particle #1 as having an spin orientation r.v. $\hat{R}_{S1}$ whose PDF be $2|\hat{r}_1 \cdot \hat{b}|$ when it is type A, with a probability p_{AB}, and be 1 when type B with a p_{BA}. This interpretation is just (18), one description of the unpolarized state $"U"$. The appearance is just an artifice caused by our insisting, in the first place, on *obtaining and organizing the necessary common information* to combine the separate measurements into a paired experiment and then *choosing* to view part of the whole information in a particular way. Regardless, from (28) and using (4-2) the model predicts the observed probability

$$Pr\{S_1 = +|"0";\hat{a},\hat{b}\} = \int d\hat{r}_1\;\; u(\hat{r}_1{\cdot}\hat{a})\;\, f_{\hat{R}_{1l}}(\hat{r}_1|"0";\hat{a},\hat{b}) = 1/2 \tag{29}$$

on any $\hat{a}$, same as (26) and as (18) did.

A different, and intuitively more appealing, interpretation is to view the #1 all *alone*, by itself and without even mentioning its companion #2. From the singlet *source* r.v.s' joint PDF (20), integrating out $\hat{R}_{S2}$ gives the marginal PDF for $\hat{R}_{S1}$

$$f_{\hat{R}_{S1}}(\hat{r}_1|"0",A) = f_{\hat{R}_{S1}}(\hat{r}_1|"0",B) = 1 \tag{30}$$

This contains no information about #2. It is just another modeling representation of $"U"$ described at (17). Thus, when viewed *individually* the model *predicts* either members of the singlet-pair be a unpolarized spin $"U"$ with a measurement probability (17-2, 3), or (29), or (26). As remarked earlier in Sec 4.3, the PDF modeling of unpolarized spin's orientation r.v. $\hat{R}_S$ is not unique.

5.3 Conditional Probabilities

The joint probability space of measurement r.v.s (S_1, S_2) consists of the four outcomes (+,+), (+,-), (-,+), (-,-) as sample space and their respective probabilities given by (25) etc. Their conditional probability of say S_1 given S_2=+, using only (+,+) and (-,+) as its sample space and the (25) and (26), is

$$Pr\{S_1 = +|"0";\hat{a},\hat{b};S_2 = +\} = \frac{Pr\{S_1 = +,S_2 = +|"0";\hat{a},\hat{b}\}}{Pr\{S_2 = +|"0";\hat{a},\hat{b}\}} = \frac{1-\hat{a}\cdot\hat{b}}{2} \tag{31}$$

This is of course the observed, and QM calculated, conditional probability to the S_2=+ subset of paired measurement repetitions. But it can be viewed and obtained from the joint probability space of the interaction r.v.s $(\hat{R}_{1l},\hat{R}_{12})$, as follows.

From (9) and (10), the condition of given a measured S_2=+ on $\hat{b}$ is equivalent to $\hat{R}_{12} \cdot \hat{b} > 0$ in the $(\hat{R}_{11}, \hat{R}_{12})$ probability space. Thus, the corresponding conditional PDF of $\hat{R}_{11}$ is

$$f_{\hat{R}_{11}}(\hat{r}_1|"0";\hat{a},\hat{b};A,B;S_2=+) = \frac{\int_{\hat{r}_2\cdot\hat{b}>0} d\hat{r}_2\, f_{\hat{R}_{11},\hat{R}_{12}}(\hat{r}_1,\hat{r}_2|"0";\hat{a},\hat{b};A,B)}{\int_{\hat{r}_2\cdot\hat{b}>0} d\hat{r}_2\, f_{\hat{R}_{12}}(\hat{r}_2|"0";\hat{a},\hat{b};A,B)} = 4\ \hat{r}_1\cdot(-\hat{b})\ u(\hat{r}_1\cdot(-\hat{b})) \quad (32\text{-}1)$$

when #1 is type A and #2 is type B. Similarly

$$f_{\hat{R}_{11}}(\hat{r}_1|"0";\hat{a},\hat{b};B,A;S_2=+) = \frac{\int_{\hat{r}_2\cdot\hat{b}>0} d\hat{r}_2\, f_{\hat{R}_{11},\hat{R}_{12}}(\hat{r}_1,\hat{r}_2|"0";\hat{a},\hat{b};B,A)}{\int_{\hat{r}_2\cdot\hat{b}>0} d\hat{r}_2\, f_{\hat{R}_{12}}(\hat{r}_2|"0";\hat{a},\hat{b};B,A)} = 2\ |\hat{r}_1\cdot\hat{a}|\ \ 2u(\hat{r}_1\cdot(-\hat{b})) \quad (32\text{-}2)$$

when #1 is type B and #2 is type A. Therefore

$$f_{\hat{R}_{11}}(\hat{r}_1|"0";\hat{a},\hat{b};S_2=+) = p_{AB}\ \ 4\ \hat{r}_1\cdot(-\hat{b})\ u(\hat{r}_1\cdot(-\hat{b})) + p_{BA}\ \ 2|\hat{r}_1\cdot\hat{a}|\ 2u(\hat{r}_1\cdot(-\hat{b})) \quad (33)$$

Substituting into the model (16), any one of these conditional PDFs predicts the observed conditional probability (31) as they should.

From (32) and (33), the reciprocal probability model $\hat{R}_{11} = \hat{R}_{M1} \otimes \hat{R}_{S1}$ explicitly states that for a singlet-pair, given one member be whatever type and measured + on $\hat{b}$, the other member must be the other type and *appears* $"-\hat{b}"$ polarized. This appearance concludes just as in QM, but only as the effective result of a conditional constraining in the $(\hat{R}_{11}, \hat{R}_{12})$ joint probability space. I.e., the measurement result + on #2 *does not* makes any physical change that alters #1 from a unpolarized $"U"$ to a polarized $"-\hat{b}"$. Rather, it *reflects* the necessary common information, inherent in the *joint probability space* and originating from the correlated singlet-pair source, in partitioning and restricting to an observation-constrained conditional sample subspace such that a realization condition of #2 restricts possibilities to #1. Although it is well-known that statistical correlation implies a common agent, which correlates, but not a physical causality, this may still seem strange due to our lack of experience in combining and sorting probabilistic events.[4] Such counter-intuitive and apparent "paradoxes" are abundant in conditional probabilities as a result of sampling from a conditioned sub- sample space, even when the *underlying joint probability space is disarmingly obvious* [10].

However, the full content of the common information inherent in (24) and (25) implies more than just these marginal and conditional PDFs and probabilities. It further dictates that the measuring

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[4] As a simple example in *classical* physics, consider the experiments of Sec. 1 but on coin-tossing to a population of coin pairs, each with two coins oppositely biased say one with p_H=0.9 and p_T=0.1 and the other vice versa. Then, independent tosses at x_1 and x_2 give Pr{H_1,H_2}=0.09 and Pr{H_1,T_2}=0.41, etc. Thus the unconditional Pr{H_1}=0.5 but the conditional Pr{H_1|T_2}=0.82, etc. I.e., for a *pair*, knowing #2's tossed result changes the odds of #1.

equipments do not merely sample and sort an attribute of the measured object, they *co-found* the interaction attribute, necessarily. This is shown next.

6 Necessity of Model and Remarks

6.1 Unphysical Cases

If we disallow or fail to recognize the type attribute, then symmetry dictates that measuring equipments must have the same uniform PDF to their $\hat{R}_M$ so as to *not* co-founding the interaction r.v.. I.e., forcing $f_{\hat{R}_M}(\hat{r}|\hat{a},T) \equiv 1$ in (24-1), then instead of (24-2, 3) and (25), we get

$$Pr\{S_1 = +,\ S_2 = +|"0";\hat{a},\hat{b}\} = \int d\hat{r}_1 \int d\hat{r}_2\ Pr\{S_1 = +|\hat{r}_1,\hat{a}\}\ Pr\{S_2 = +|\hat{r}_2,\hat{b}\}\ f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0") \quad (34)$$

$$= \int d\hat{r}_1\, u(\hat{r}_1 \cdot \hat{a}) \int d\hat{r}_2\, u(\hat{r}_2 \cdot \hat{b})\ 1\ \ 1\ \ \delta(\hat{r}_1 + \hat{r}_2) = \theta_{\hat{a}\hat{b}} / (2\pi) \quad (35)$$

The hypothetical (35), intuitive but unphysical, was often used in previous effort to exhibit the puzzle of (1) [11].

Notice that *even* in this hypothetical case we have

$$f_{\hat{R}_{S1}}(\hat{r}_1|"0";\hat{a},\hat{b};S_2 = +) = 2u(\hat{r}_1 \cdot (-\hat{b})) \quad (36)$$

Thus the condition of #2's measurement S_2=+ on $\hat{b}$ *appears* to alter the PDF of #1 from marginal isotropic to conditional $-\hat{b}$-hemispheric. Again, it is just a matter of informing of and constraining to partitioned sample subspace (Footnote 4, p10). However, this $\hat{R}_M$-disallowed model, intuitively appealing and mathematically self-consistent, just does *not* describe actual physics.

Another unphysical case is, using (20) to (22) and (27), the conditional PDF

$$f_{\hat{R}_{11}|\hat{R}_{12}}(\hat{r}_1|"0";\hat{a},\hat{b};\hat{R}_{12} = \hat{r}_2) = \delta(\hat{r}_1 + \hat{r}_2) \quad (37)$$

I.e., given the condition $\hat{R}_{12} = \hat{r}_2$ implies $\hat{R}_{11} = -\hat{r}_2$ and, of course, (37), (30), (20) are correctly related. But this condition is *not* the physically realized observation S_2=+ described in Sec. 5.3. Further, the conditional PDF of $\hat{R}_{S1}$ given $\hat{R}_{S2} = \hat{r}_2$, a condition also *not* physically realized, is same as the right side of (37).

6.2 Necessity of Type and $\hat{R}_M$

More strongly, the measuring equipment's $\hat{R}_M$*-disallowed* probability model (34), which physically invoked only the independence of separated measuring processes, is *incompatible* with the observed probability (1) *no matter what* $Pr\{S = +|\hat{r},\hat{a}\}$ and $f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0")$ are *used unless they* *go negative*. This was shown by Feynman [7]. We just exhibit this incompatibility by a familiar *self-contradiction*, next.

First, we *force* (34) equal to (1)

$$\frac{1-\hat{a}\cdot\hat{b}}{4} = Pr\{S_1=+,\ S_2=+|"0";\hat{a},\hat{b}\} = \int d\hat{r}_1\int d\hat{r}_2\ Pr\{S_1=+|\hat{r}_1,\hat{a})\ Pr\{S_2=+|\hat{r}_2,\hat{b})\ f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0") \tag{38}$$

Thus, from (38) and etc. the covariance of $\{S_1,S_2|"0",\ \hat{a},\ \hat{b}\}$ is

$$-\hat{a}\cdot\hat{b} = \int d\hat{r}_1\int d\hat{r}_2\, E\{S_1|"0",\hat{a},\hat{r}_1\}\ E\{S_2|"0",\hat{b},\hat{r}_2\}\ f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0") \tag{39}$$

where the obvious $E\{S_1|"0",\ \hat{a}\}=0$ etc. were used. Now by definition the conditional S_1 under a given $\hat{r}_1$ is deterministically +1 xor -1, thus $|E\{S_1|"0",\hat{a},\hat{r}_1\}|=1$, etc.

Then, *insisting non-negative* PDFs, evaluating the (39) at $\hat{a}=\hat{b}$ implies $E\{S_1|"0",\hat{a},\hat{r}_1\}$ $=-E\{S_2|"0",\hat{a},\hat{r}_2\}$ *inside* its integral. Therefore, the difference between covariances of $\{S_1,S_2|"0",\ \hat{a},\ \hat{b}\}$ and $\{S_1,S_2|"0",\ \hat{a},\ \hat{c}\}$ becomes

$$\begin{aligned}-\hat{a}\cdot\hat{b}+\hat{a}\cdot\hat{c} &= \int d\hat{r}_1\int d\hat{r}_2\, E\{S_1|"0",\hat{a},\hat{r}_1\}\,(E\{S_2|"0",\hat{b},\hat{r}_2\}-E\{S_2|"0",\hat{c},\hat{r}_2\})\, f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0")\\ &= \int d\hat{r}_1\int d\hat{r}_2\, E\{S_1|"0",\hat{a},\hat{r}_1\}E\{S_2|"0",\hat{b},\hat{r}_2\}\,(1+E\{S_1|"0",\hat{b},\hat{r}_1\}\ E\{S_2|"0",\hat{c},\hat{r}_2\})\, f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0")\end{aligned} \tag{40}$$

Taking absolute value and *insisting again a non-negative* PDF, the (40) results in

$$|\hat{a}\cdot\hat{b}-\hat{a}\cdot\hat{c}| \le \int d\hat{r}_1\int d\hat{r}_2\ (1+E\{S_1|"0",\hat{b},\hat{r}_1\}\ E\{S_2|"0",\hat{c},\hat{r}_2\})\, f_{\hat{R}_{S1},\hat{R}_{S2}}(\hat{r}_1,\hat{r}_2|"0") = 1-\hat{b}\cdot\hat{c} \tag{41}$$

for *any* unit vectors $\hat{a},\ \hat{b},\hat{c}$. The (41) is of course the Bell inequality [8]. It is a mathematically false statement[5] and is the self-contradiction.

6.3 A Suggested Experiment

For the model to have physical consequences besides merely solving a logical difficulty, there must be other experimentally observable implications. A conceptually feasible experiment is suggested next to demonstrate such a consequence.

The new model dictates that any two electrons in a singlet state must be of different types. Consider the He atoms. At room temperature their two electrons are virtually all in the $1s^2$ ground state as singlet pairs. Suppose the He atoms are ionized into He^+ ions, with one electron remained behind still mostly in the ground state $1s^1$. Then the ions are accelerated to form a low-density ion

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[5] Can be seen easily over a wide range of angles among $\hat{a},\ \hat{b},\ \hat{c}$. E.g., when they are coplanar: if π/8 apart then 1<=cos(π/8) thus √2<=1; if θ<<1 apart then 3<=1. Also, lumping the $(\hat{r}_1,\hat{r}_2)$ into a $\hat{\underline{r}}$ makes no difference in deducing (41).

beam and passed through a neutral gas, letting the He^+ re-capture an electron, to make a neutral He beam. Now the re-captured electron in the neutral He should quickly radiate a photon of ~20 eV to go to the ground state and join the other electron as a singlet pair $1s^2$. Using usual QM assuming identical electrons the rate of this radiation, ρ_{QM}, can be accurately calculated as a function of temperature, ion and neutral atom densities, beam velocity, and position along the beam path. And the observed rate ρ_M can be measured.

But according to the new model, the re-captured electron's type could be either same as or different from the electron remained, with a 50%-50% probability each. And only in the later cases of different types can they reconfigure into the $1s^2$ singlet ground state. In the former cases the two same type electrons can *not* radiate a photon and settle to this ground state. One of them has to collide and exchange with an electron of an appropriate neighbor atom before the two can radiate and go into the singlet ground state. For a low density beam this collision exchange occurs in a much longer time scale. Thus the interaction types of the new model predict a radiation rate $\rho_N = \rho_{QM}/2$, a factor of two lower than the QM prediction. The observed radiation rate ρ_M should be able to clearly distinguish them.

7 Summary

The probabilistic rules and their implied wave packet collapse when measurement occurs have been perplexing ever since the beginning of QM. In this paper we *explicitly* resolved the spin-half measurement problem by a *new* probability model, under the usual positive probability theory and the independence of space-like separated measuring equipments and processes. Aside from a new notion of reciprocal r.v.s, the model is mathematically unremarkable.

But physically it shows that in an observation the measuring equipment and the measured object, facilitated by the new type attribute, co-found the interaction r.v. $\hat{R}_I = \hat{R}_M \otimes \hat{R}_S$ *during* the measurement encounter and then sample $\hat{R}_I$ to determine the observed outcome. These r.v.s should underlie the respective QM wave functions. Moreover, in order to preserve the three accepted premises of QM result, independent measurement processes at separated sites, and positive probability, it is *necessary* to have such a interaction type attribute and an observation co-founding role of the measuring equipment. This necessity, physically part of the whole phenomena, logically stands apart from the sufficiency of the explicitly solved reciprocal probability model.

A profound implication of the model appears to be that the measuring equipments and the observed objects each carries *not* a pre-realized value of a r.v. but the r.v. itself, i.e. the PDF rule. Not only that the joint PDF of a common *source's* r.v.s can be carried means QM is *incomplete* and a common agent must exist to link them. *And* post-QM physics must address attributes of the measuring and the measured simultaneously. Also that what observed is necessarily co-generated by, and seen through the eye of, the observer suggests that the underlying objective physical truth may be *scientifically inaccessible and therefore irrelevant.*

To conclude, we recapitulate the implications. 1) If physical observations are necessarily *co-generated* by both the observing equipment and the observed object *during* the measurement interaction, post-QM physics must address *both* attributes at the *outset*. 2) Since any non-independent joint probability is a consequence of a *common third* agent, the correlated joint PDF of the *source* particles exposes that QM is *incomplete*, as Einstein maintained. 3) That the reciprocal PDFs are continuous may indicate a "true" *continuum* underling all discrete observations. 4) The measurement uncertainties and their probability modeling may indicate that *our* understanding of nature be limited to *some dice it appears to play*. 5) An experiment, conceptually simple and 'small science', is suggested to demonstrate an observable consequence of the spin types. 6) Research to isolate, measure effects of, and generalize the interaction type may open new areas of physics.

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* This paper is partly based on the author's: unpublished Logicon RDA report "Type and Probability Model for Spin-Half Measurements", first draft (Feb. 1988), re-edited and distributed (June 1988), expanded (March 1989); presentation at 1989 APS Annual Meeting in San Francisco, APS Bulletin 34, no. 1, GI6, 83; equations and drawings photographed in "Feynman's Last Blackboards", Physics Today, 88 (Feb. 1989) and description letter, ibid., 15 (July 1989).

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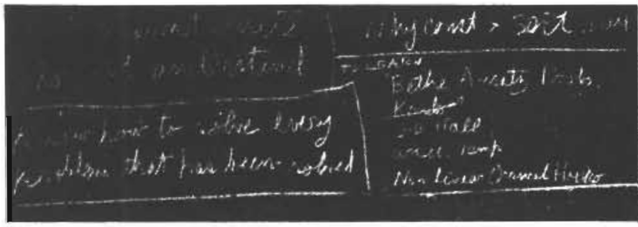
Attachment A

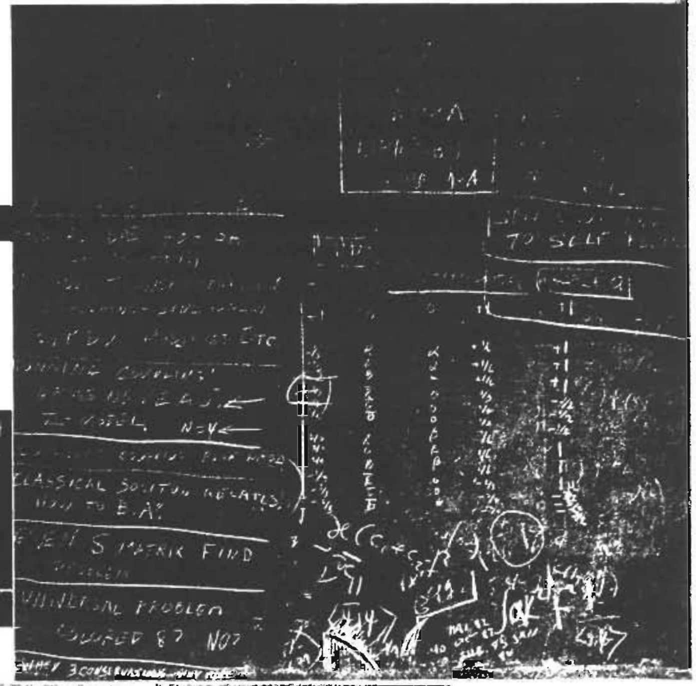
"Feynman's Office: The Last Blackboards"

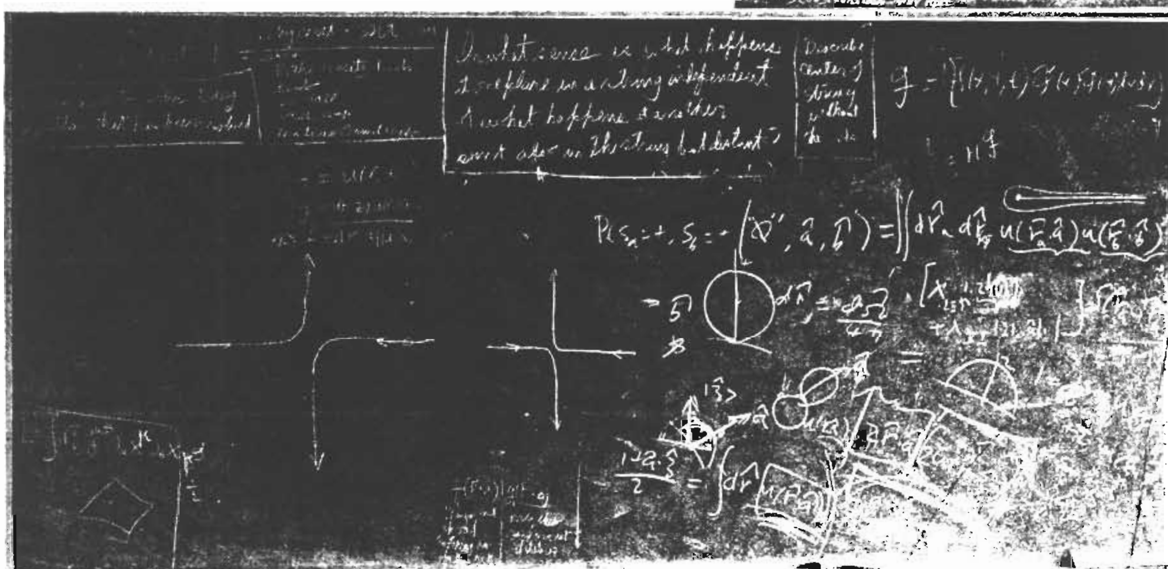
Physics Today, February 1989 p.88, reprint with permission from The California Institute of Technology, May 1994.

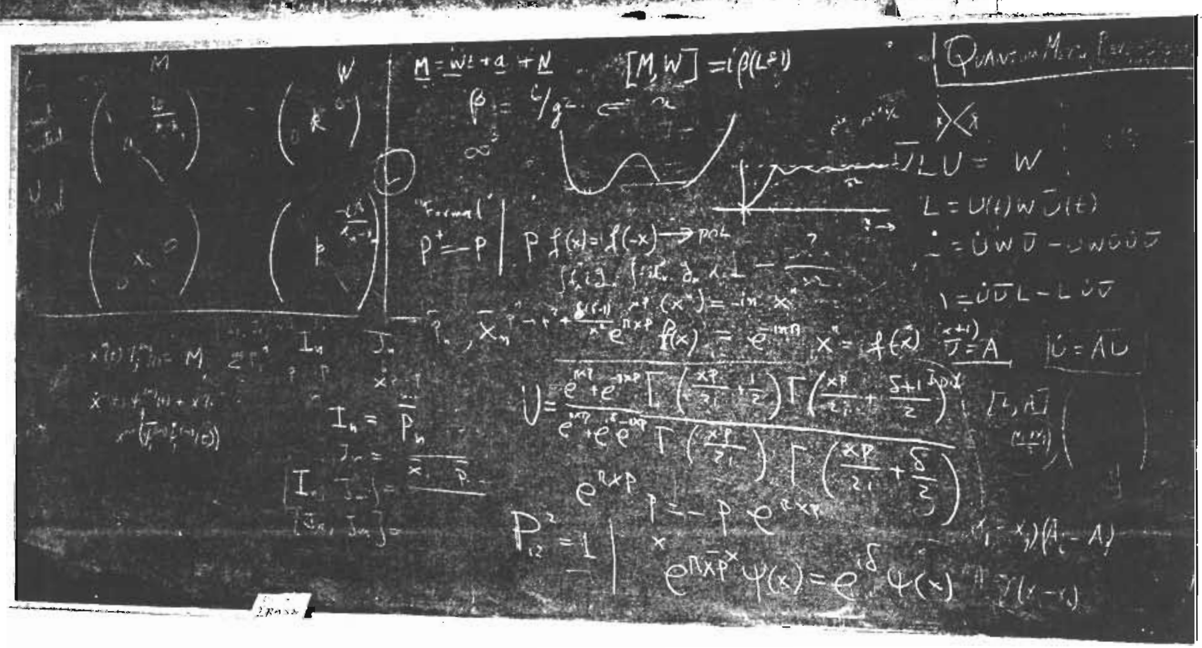
FEYNMAN'S OFFICE: THE LAST BLACKBOARDS

ALL PHOTOS ROBERT PAZ/ARCHIVES, CALIFORNIA INSTITUTE OF TECHNOLOGY









Attachment B

"Clearing the Slate About Feynman's 'Last Blackboards' "

Physics Today, July 1989 p.15, reprint with permission from Physics Today, May 10 1994.

Clearing the Slate About Feynman's 'Last Blackboards'

I am submitting this letter to identify the equations and figures on one of Richard Feynman's "last blackboards," shown in the middle of page 88 of the February issue. Except for the blocked-off diagram at the lower left corner and the boxed statements at the top, the blackboard's contents come from the last discussion I held with Feynman, on the afternoon of 14 January 1988. He had kindly allowed me to present to him the new probability model on spin types I have obtained to solve the Bohm spin-$\frac{1}{2}$ case for the Einstein–Podolsky–Rosen "paradox" of quantum mechanics.

As Feynman sat in his big chair at the right end of the blackboard, I erased it, except for those portions mentioned above, and started writing from the center, toward the right. This part is exhibited virtually intact in the PHYSICS TODAY photo. Within a couple of minutes, as soon as I presented the one-particle integral equation solutions that lead to two new types of spin-$\frac{1}{2}$ particles and to the measuring tool's type-dependent measurement interaction, Feynman became intensely interested. With his

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eyes beaming, he walked to the blackboard and we continued working on the left part, which, after several intermediate erasures, was full at the end of the discussion. The photo shows that most of what we wrote on that half of the blackboard was later erased and replaced by two scattering-like drawings, which may or may not represent Feynman's later thinking on the spin problem. But his writing on the two-particle-type probability densities and the corresponding measuring tool densities, as well as his equation for g on the top right, is mostly still there. I had not named the two types and used subscripts I and II in the exhibited formulas. Feynman said to just call them A and B. His use of this notation can be seen above the left "scattering" drawing.

It is inappropriate for me to explain the new model of spin types here (it has been further disseminated and is to be published), except to mention that Feynman's two lectures in 1980 and 1982 at Caltech on using negative probabilities to understand the EPR paradox inspired me to work on this well-known fundamental problem. He had allowed me quite a few privileged discussions over the years, beginning in 1966 with my taking his advanced quantum mechanics course at Caltech, and including several occasions since 1980 on which I sought to specifically understand the EPR problem. But this last occasion was the only such opportunity I had in his last two years.

Knowing that he had been struggling with cancer, I found it a great inspiration to see his usual healthy clear and quick mind and high spirits during the discussion. He probed the key notion of independent reciprocal probability densities profoundly. He was very pleased with and encouraging of the new idea. He asked me to write it up and see him again as soon as possible. And he promised to think more about it. I immediately shared this inspiration with several colleagues. But I did not know, and only learned retrospectively from his secretary, Helen Tuck, that he was quite ill even then.

In about two weeks I wrote the draft up and brought it to his office. He was ill and not in. I left it with Mrs. Tuck. He never saw it.

Feynman's last blackboards speak of his generosity to others and his unceasing quest for scientific truth. What the great teacher taught, we will carry on.

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