Measurement Notes

Note 31

2 October 1986

Winding Topology for Transformers

Carl E. Baum
Air Force Weapons Laboratory

Abstract

In order to improve the high-frequency performance of pulse transformers, one of the problems to be addressed is the reduction of the leakage inductance associated with leakage flux between primary and secondary windings. This note introduces the concept of making such windings out of coaxial (or higher order multiaxial) cables, with the outer shields of primary and secondary windings bonded together, so as to effectively remove this leakage inductance.

Foreword

I would like to thank Joe Martinez of Dikewood Division of Kaman Sciences for the numerical computations in Figure 3.2.

I. Introduction

The subject is transformers, pulse or broadband transformers to be specific. The concept of an ideal transformer is common in electrical circuit theory. However, practical transformers have limitations. At the low-frequency end, the transformer core (a highly permeable ($\mu >> \mu_0$) medium) provides only a finite inductance to the primary and secondary windings, so that the impedance looking into the primary winding tends to zero at zero frequency, irrespective of the load impedance (even open circuit) on the secondary winding.

Another problem with transformer design concerns high frequency performance. There is in general not complete linkage of magnetic flux between primary and secondary windings. The difference, or leakage flux, limits the high-frequency transfer function. In order to effectively remove the associated leakage inductance one can bring the primary and secondary windings physically together to exclude magnetic flux between them. However, it is desirable to preserve an electrical isolation between certain portions of these windings. This is accomplished by constructing the windings from coaxial cables (or cables with higher order multiaxial shields) so that the outermost shields of the primary and secondary windings are in continuous electrical contact (thereby excluding leakage magnetic flux), while maintaining isolation between appropriate portions of internal conductors.

Various types of these transformer windings are considered. These allow voltage step-up and step-down, pulse inversion, and single-ended to differential signal conversion (and conversely).

II. Mutual Inductance Between Turns on a Ferromagnetic Core

One of the concerns in transformer design is the problem of the efficiency of coupling magnetic flux from one winding on a transformer core to another such winding. As in Figure 2.1 consider a loop (winding turn) number 1 with current I_1 flowing in it. Passing through this loop is a core of highly permeable material ($\mu >> \mu_o$). This core is assumed to be itself a closed loop, such as a toroid, but its cross section can be circular, square, or rectangular (typical shapes). In the figure only a small section of the core is shown.

As indicated in Figure 2.1 let the nth turn have current I_n and voltage V_n. Let there be a voltage source driving turn 1, and all other turns be open circuited (zero current). Now associated with I_1 there is a magnetic flux Φ_1, most of which is ideally in the magnetic core. Associated with this there is

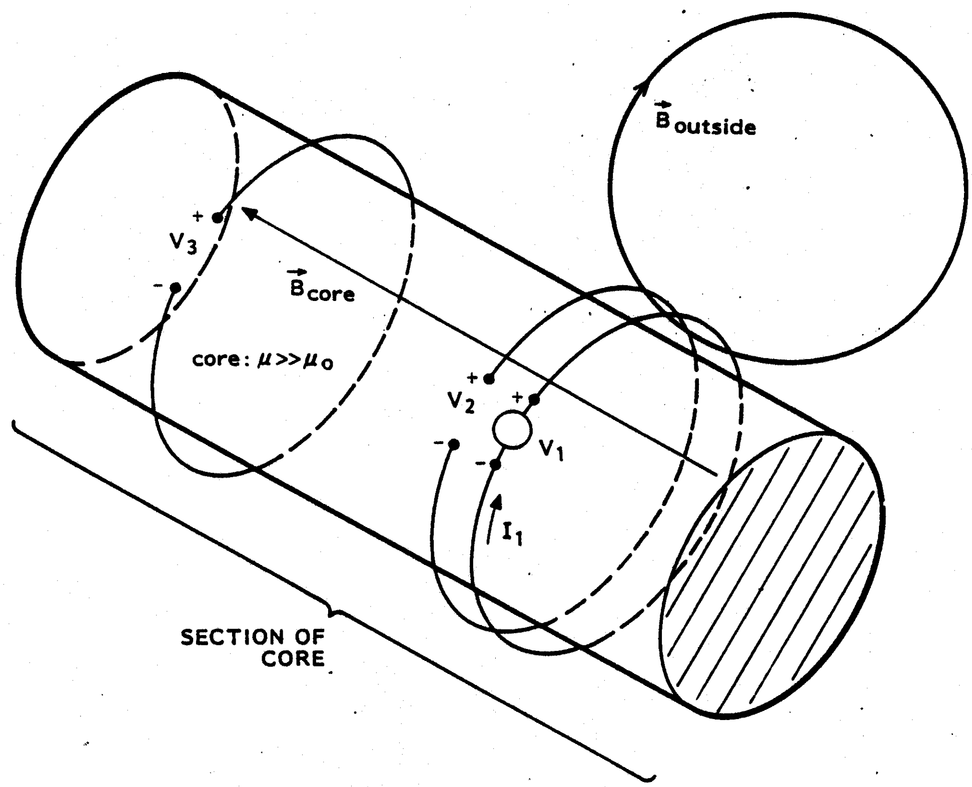


Figure 2.1. Windings on a Permeable Core

$$V_1 = \frac{d\Phi_1}{dt} = L_1 \frac{dI_1}{dt}$$

$$\Phi_1 = L_1 I_1$$

$$L_1 \equiv \text{self inductance of turn 1} \tag{2.1}$$

The value of L_1 is dependent on the permeability μ, cross section area, and length (or better length of magnetic flux paths in the core for a "closed loop" core) of the core. While the core conductivity and permittivity are also significant, they are assumed to be small enough to be neglected in our present considerations.

Now consider some turn designated by n=3 that is not very close to the first turn. Not all of Φ_1 links the third turn. As indicated in Figure 2.1 some magnetic flux lines illustrated as $\vec{B}_{outside}$, while linking turn 1 do not link turn 3, while the flux in the core (at least most of it) illustrated as $\vec{B}_{core}$ links all the turns. The flux Φ_3 linking the third turn has

$$0 < \Phi_3 < \Phi_1 \tag{2.2}$$

with

$$V_3 = \frac{d\Phi_3}{dt}$$

$$\Phi_3 = M_{3,1} I_1$$

$$V_3 = M_{3,1} \frac{dI_1}{dt} \tag{2.3}$$

$$M_{3,1} = M_{1,3} \equiv \text{mutual inductance between turns 1 and 3}$$

It is this mutual inductance that makes a transformer, but note

$$0 < M_{3,1} < L_1 \tag{2.4}$$

The difference between the self and mutual inductance is sometimes referred to as a leakage inductance ("leakage" of flux lines associated with $\vec{B}_{outside}$), for which

$$0 < \ell_{3,1} \equiv L_1 - M_{3,1} \qquad (2.5)$$

It will be one of our design considerations to minimize this leakage inductance.

Let us put this in the form of an equivalent circuit as in Figure 2.2. Let there be a load R on turn 3 so that

$$\frac{\tilde{V}_3}{\tilde{I}_3} = -R \qquad (2.6)$$

$\sim$ ≡ Laplace transform (2 sided over time) to complex frequency s.

The mutual inductance provides a voltage source in the secondary which gives

$$\tilde{V}_3 = \frac{sM_{3,1}\, R}{R + sL_3}\, \tilde{I}_1 \qquad (2.7)$$

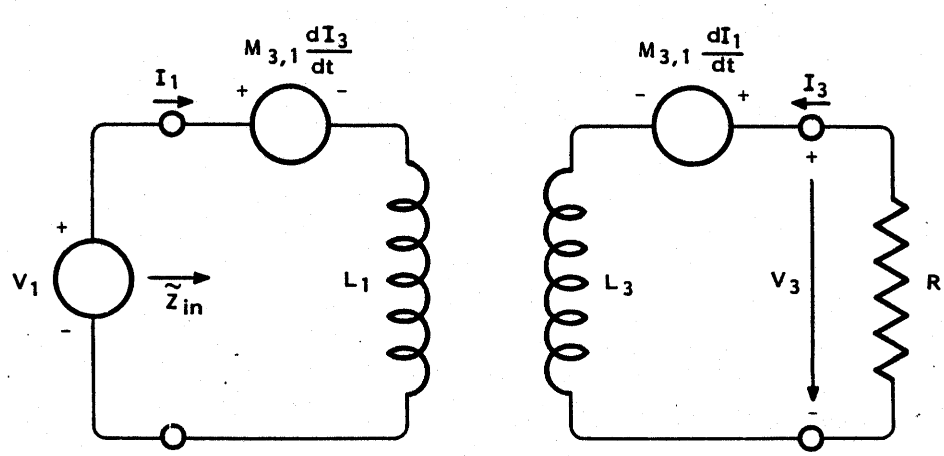


Figure 2.2. Equivalent Circuit for Two Turns on Permeable Core

In the primary I_3 provides a voltage source giving

$$\tilde{V}_1 = sM_{3,1}\,\tilde{I}_3 + sL_1\,\tilde{I}_1 = \left\{- s^2 \frac{M_{3,1}^2}{R + sL_3} + sL_1\right\}\tilde{I}_1 \tag{2.8}$$

This gives a voltage transfer function

$$\tilde{T} \equiv \frac{\tilde{V}_3}{\tilde{V}_1} = \left\{\frac{L_1}{M_{3,1}} + s\left[L_1\,L_3 - M_{3,1}^2\right]\frac{1}{M_{3,1}\,R}\right\}^{-1} \tag{2.9}$$

and an input impedance at turn 1

$$\tilde{Z}_{in} \equiv \frac{\tilde{V}_1}{\tilde{I}_1} = \frac{sL_1\,R + s^2\left[L_1\,L_3 - M_{3,1}^2\right]}{R + sL_3} \tag{2.10}$$

Looking at the voltage transfer function observe that if

$$L_1\,L_3 = M_{3,1}^2 \tag{2.11}$$

then

$$\tilde{T} = \frac{M_{3,1}}{L_1} \tag{2.12}$$

which is frequency independent (ideally). Furthermore, the input impedance simplifies to

$$\tilde{Z}_{in} = R\left\{\frac{L_3}{L_1} + \frac{R}{sL_1}\right\}^{-1} \tag{2.13}$$

If we further assume that turns 1 and 3 are effectively the same (except for translation (including rotation) along a uniform core) then we have

$$L_1 = L_3 \equiv L \tag{2.14}$$

and

$$\tilde{T} = 1$$

$$\tilde{Z}_{1,1} = R \left\{ 1 + \frac{R}{sL} \right\}^{-1} \qquad (2.15)$$

This approaches an ideal transformer in that the leakage inductance is

$$\ell_{3,1} = 0 \qquad (2.16)$$

However, there is still the matter of the self inductance of the turns. Ideally $L=\infty$, but in practice one makes L large. At the high frequency end (2.16) seems to remove all problems, but this neglects propagation times on the windings since the foregoing analysis is quasi static.

Referring to Figure 2.1 once more, note that the leakage inductance is made zero as in (2.16) by having all the flux from turn 1 link turn 3. Considering $\vec{B}_{outside}$ in the illustration, this is accomplished by moving turn 3 to the position indicated by turn 2, adjacent to turn 1. In the limit turn 2 takes the "same" position as turn 1 so that no flux appears between them. Actually the two turns are of finite size and can be bonded side by side to eliminate intervening flux. This will be realized later by using cables with exterior shields bonded together.

III. Wires Near a Highly Permeable Half Space

In order to estimate the leakage flux consider the problem illustrated in Figure 3.1. Turn 1 (as in the previous section) is now wire 1 carrying current I_1 in the z direction centered along the line

$$(x,y) = (x_1, 0) \tag{3.1}$$

Then x_1 is the spacing of the wire from a highly permeable medium ($\mu >> \mu_o$ for $x<0$). The other half space ($x>0$) is assumed to have permeability μ_o (as in free space).

In this idealized problem there is an infinite magnetic flux surrounding wire 1 (or equivalently an infinite inductance) because of the current I_1 returning at ∞. However, if wire 3 is introduced at

$$(x,y) = (x_3, y_3) \tag{3.2}$$

then the flux per unit length between wires 1 and 3 is bounded, as is the leakage inductance per unit length $\ell'_{3,1}$.

Define a normalized complex coordinate

$$\zeta \equiv \frac{x}{x_1} + j\,\frac{y}{x_1} \tag{3.3}$$

and a complex potential function

$$w \equiv u + jv \tag{3.4}$$

If we let $\mu \to \infty$ in the left half space we can consider the x=0 plane as a magnetic conductor, i.e.

$$H_y \Big|_{x=0} = 0 \tag{3.5}$$

Then the magnetic field is perpendicular to the x=0 plane. In our convention v is chosen as the magnetic potential function, so the magnetic field is perpendicular to contours of constant v, and hence x=0 corresponds to contours of constant v.

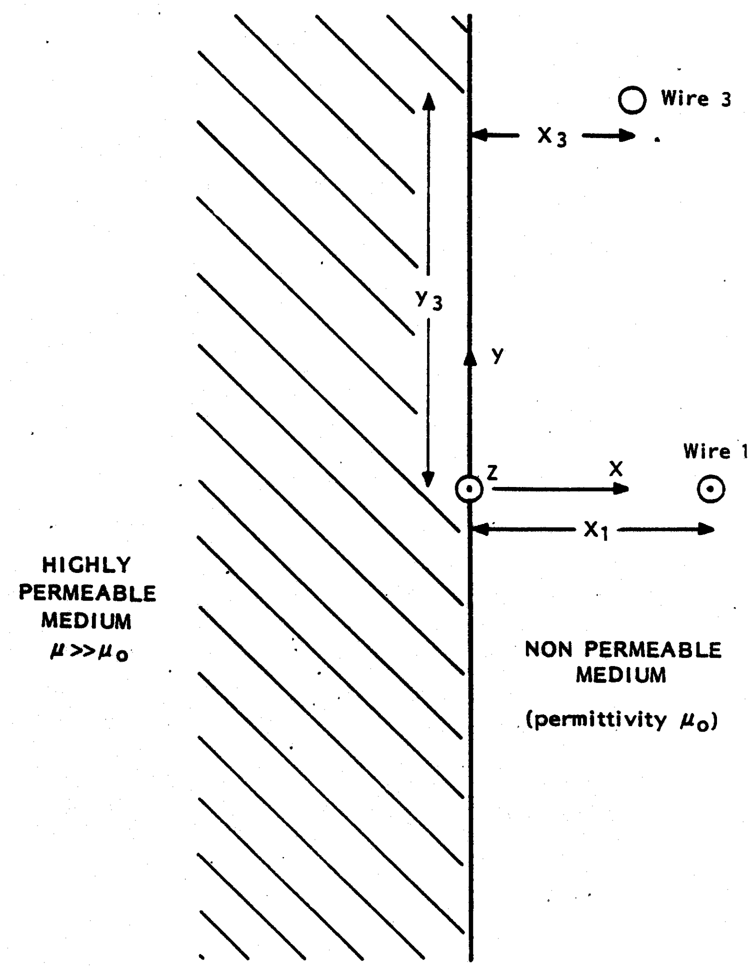


Figure 3.1. Wires Near a Highly Permeable Medium

The appropriate conformal transformation has been discussed in [3]. For our current problem we have the special case

$$\zeta = \left[e^{2w} + 1\right]^{1/2}$$

$$w = \frac{1}{2} \ln(\zeta^2 - 1) \qquad (3.6)$$

Now w is singular ($u = -\infty$) at

$$\zeta = \begin{cases} +1, & \text{wire 1 center} \\ -1, & \text{image of wire 1 center} \end{cases} \qquad (3.7)$$

The surface of the "magnetic conductor" (x=0) is described by

$$\mathrm{Re}[\zeta] = 0$$

$$v = \pm \frac{\pi}{2} \quad \text{for} \quad u \geq 0 \qquad (3.8)$$

This complex potential is illustrated in Figure 3.2.

The leakage inductance per unit length between wires 1 and 3 can be described by

$$\ell'_{3,1} = \mu_o f_g$$

$$f_g = \frac{\Delta u}{\Delta v} \qquad (3.9)$$

Now the change of v around wire 1 is

$$\Delta v = \pi \qquad (3.10)$$

write

$$\Delta u = u_3 - u_1$$

$$u_3 \equiv \text{potential of wire 3 (at center)}$$

$$u_1 \equiv \text{potential at "radius" of wire 1} \qquad (3.11)$$

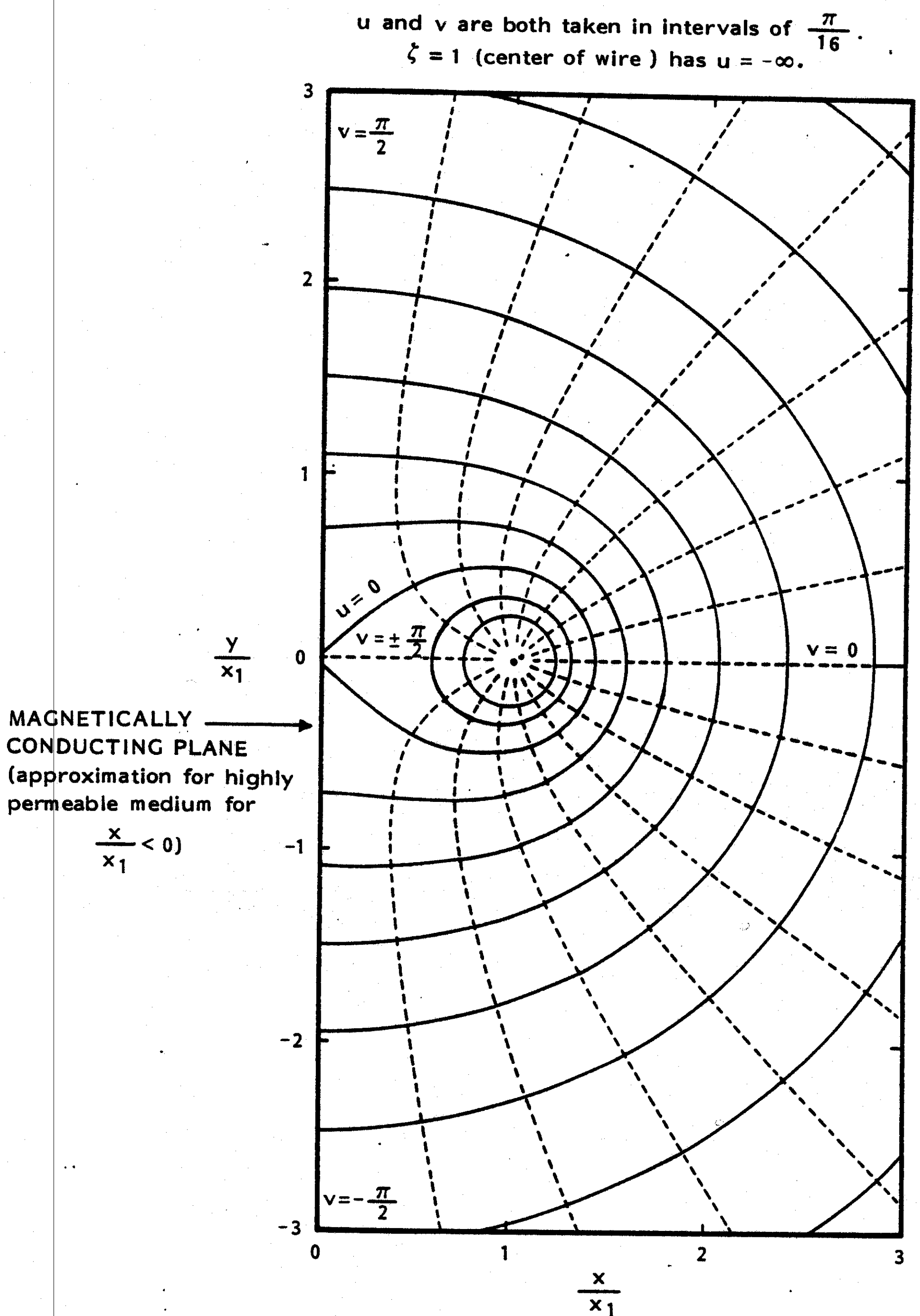


Figure 3.2. Complex Potential for Wire Near a Magnetically Conducting Plane

Considering wire 3 we have

$$u_3 = \frac{1}{2} \ln\left(\left|\zeta_3^2 - 1\right|\right)$$

$$\zeta_3 = \frac{x_3}{x_1} + j\,\frac{y_3}{x_1}$$

$$u_3 = \frac{1}{4} \ln\left(\left[\left(\frac{x_3}{x_1}\right)^2 - \left(\frac{y_3}{x_1}\right)^2 - 1\right]^2 + 4\left(\frac{x_3}{x_1}\right)^2\left(\frac{y_3}{x_1}\right)^2\right) \tag{3.12}$$

Considering wire 1, let it have radius d. Then we have for positions near wire 1

$$\zeta_1 = 1 + \nu$$

$$\nu = \frac{d}{x_1}\, e^{j\phi_1}$$

$$-\pi < \phi_1 \leq \pi \tag{3.13}$$

So the complex potential is

$$w_1 = \frac{1}{2} \ln(2\nu + \nu^2) = \frac{1}{2}\left[\ln(2\nu) + \ln\left(1 + \frac{\nu}{2}\right)\right]$$

$$= \frac{1}{2} \ln(2\nu) + O(\nu)$$

$$u_1 \simeq \frac{1}{2} \ln\left(|2\nu|\right) = \frac{1}{2} \ln\left(\frac{2d}{x_1}\right) = -\frac{1}{2} \ln\left(\frac{x_1}{2d}\right) \tag{3.14}$$

The geometric factor is then

$$f_g \simeq \frac{1}{2}\left\{\frac{1}{2} \ln\left(\left[\left(\frac{x_3}{x_1}\right)^2 - \left(\frac{y_3}{x_1}\right)^2 - 1\right]^2 + 4\left(\frac{x_3}{x_1}\right)^2\left(\frac{y_3}{x_1}\right)^2\right) + \ln\left(\frac{x_1}{2d}\right)\right\} \tag{3.15}$$

This has interesting special cases. Let $x_3 = x_1$ so that wires 1 and 3 have the same spacing from the highly permeable medium, giving

$$f_g \simeq \frac{1}{2}\left\{\frac{1}{2}\,\ell n\left(\left(\frac{y_3}{x_1}\right)^4 + 4\left(\frac{y_3}{x_1}\right)^2\right) - \ell n\left(\frac{2d}{x_1}\right)\right\} \tag{3.16}$$

Considering the magnetic flux which does not enter the core (equivalent to placing wire 3 at the origin) has

$$\zeta_3 = 0$$

$$u_3 = 0$$

$$f_g \simeq \frac{1}{2}\,\ell n\left(\frac{x_1}{2d}\right) \tag{3.17}$$

In any event this geometric factor (and hence $\ell'_{3,1}$) is set to zero by making

$$u_3 = u_2\ ,\ \Delta u = 0$$

so that wire 3 is moved to wire 1.

IV. Twin Windings

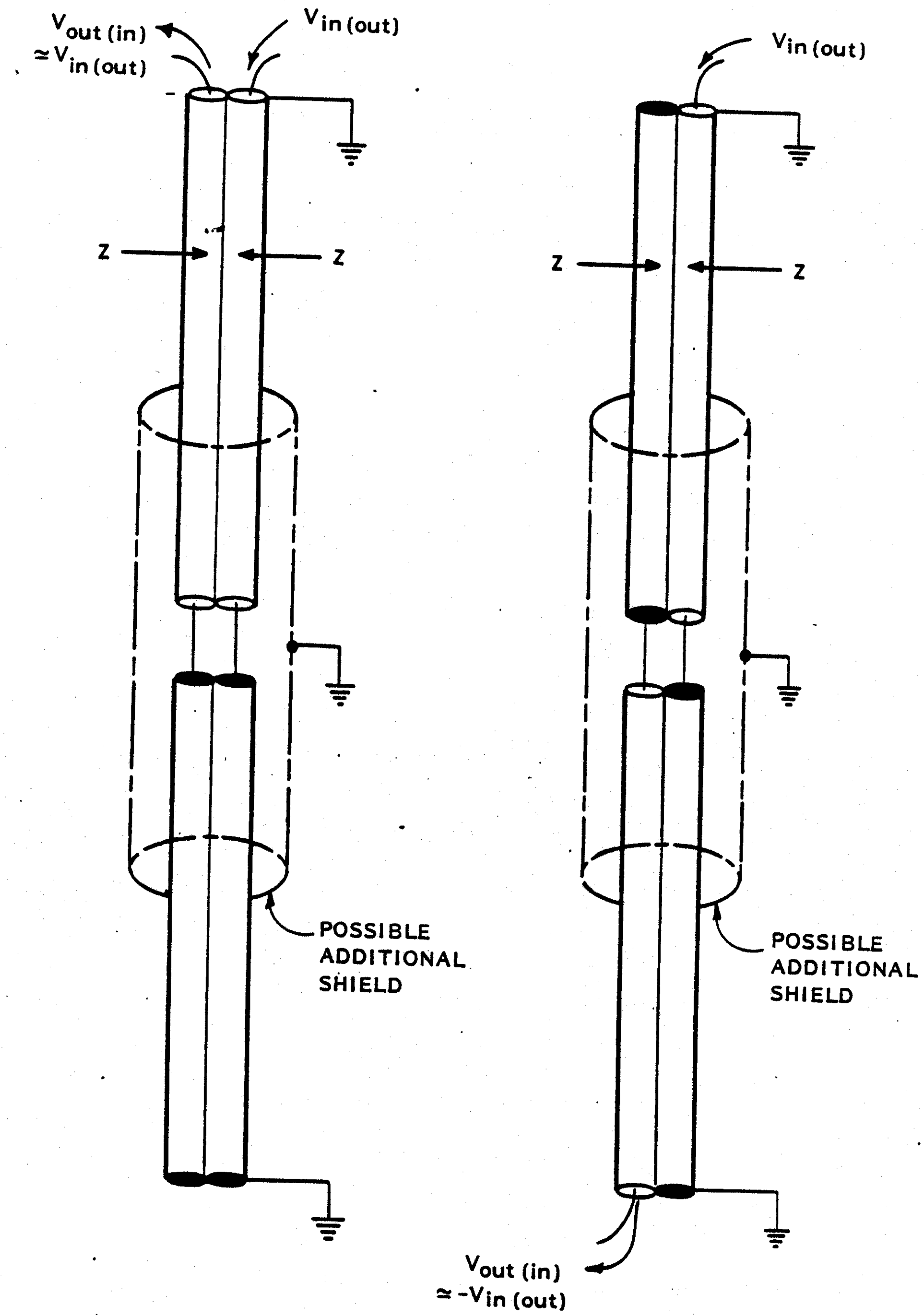
In order to realize the concept of making two transformer windings (primary and secondary) into one winding as far as the magnetic flux is concerned, let us generalize some earlier considerations concerning the windings in loops. Two previous papers [1,2] have shown some of the possibilities concerning the generalization of single conductors to coaxial cables, multiaxial cables, and other configurations concerning the use of Moebius gaps in loop structures. Here we generalize such configurations to transformer windings.

Now the basic idea is to have the actual windings (in a DC sense) as center conductors of a coaxial or higher-order structure. The shields (outermost) are bonded together to make the resulting winding act as a single winding on the transformer core.

Consider then that there is some large inductance to raise the impedance on the outside of the outermost cable shields. At the gaps in the cables where signals are matched between various transmission lines, then let us neglect for present purposes any waves on the outside. Signals propagate from one transmission line to another with appropriate care for impedance matching.

As an elementary example let us match (twin) one coaxial winding with another as illustrated in Figure 4.1 as a 1 to 1 transformer or 1 to -1 transformer (inverter). Note that, since the coaxial shields are bonded together as one conductor, no magnetic flux can pass between the two conductors (assuming perfectly conducting shields). A small exception to this concerns the magnetic flux linking the center conductors of the coaxes at the small gap region where the signals are transmitted from one coax to another. Note at this junction (and at the ends of the windings) certain of the coaxes have no internal signal. This is indicated by a shorting conductor connecting the center conductor to the shield (or connecting two shields in later examples).

Note the ground symbols at each end of the shields of the twin coaxes. Since the twin coaxes form a winding on some transformer core these points can be connected together, perhaps via some common conductor (such as an overall shield or some other conductor. Note also the possible addition of a shield around the twin coax to distribute the signal input position (to the center



a. 1 to 1, single-ended to single-ended (or with two such, differential to differential).

b. 1 to -1 (inverter), single-ended to single-ended (or with two such, differential to differential).

Figure 4.1. Twin Coaxial Simple Windings

conductors) to two positions on the winding. Including the possible addition of another ground on this shield, this additional shield effectively breaks the length of the winding in two, as far as external waves are concerned. (A shorter winding has resonances at a higher frequency.) This still leaves the question of what should be the impedance between the additional shield and the twin-coaxial shields. If waves induced inside both ends of this additional shield are to be terminated in the twin coaxes (two in parallel or Z/2), then this impedance for the inside of the additional shield would be Z/4. This is analogous to techniques used in loop design for magnetic-field sensors [1,2], but the combination of the two windings as one introduces a significant difference since the signals from this additional transmission line propagate into both primary and secondary coaxial windings. Using the multiaxial concept in [1] yet additional shields can be added to further increase the number of distributed signal inputs to the windings to 4, 8, etc., and thereby correspondingly break up the length of the winding (and raise the resonant frequency of the winding segments).

The example in Figure 4.1 is given in a single-ended form. However, by using two such twin-coaxial windings (on a common core) with each winding driven in opposite polarity (with winding sense to make magnetic fluxes add in the core) then this type of winding can take differential form. This allows a differential to differential transformer in either 1 to 1, or 1 to -1 (inverter) form.

Yet another variation utilizing the windings of Figure 4.1 is to combine those in both A and B with the input signals (primary windings) connected in parallel, but the output signals (secondary windings) being in series (differential output). This gives a transformer ratio of 1 to 2 (single ended to differential). In this case the two primary-winding transmission lines have a net parallel impedance Z/2, while the secondary-winding transmission lines have a net series impedance (differential) 2Z. Figure 4.2 shows how such a transformer might be wound. This has two twin-coaxial windings, each of 5 turns. Note that the direction of magnetic flux density in the core is maintained by the sense of both windings. At the ends of the windings the shields of the two windings are connected together and to ground (say a local external shield containing the transformer). Note that no additional conductors should pass through the transformer core because such

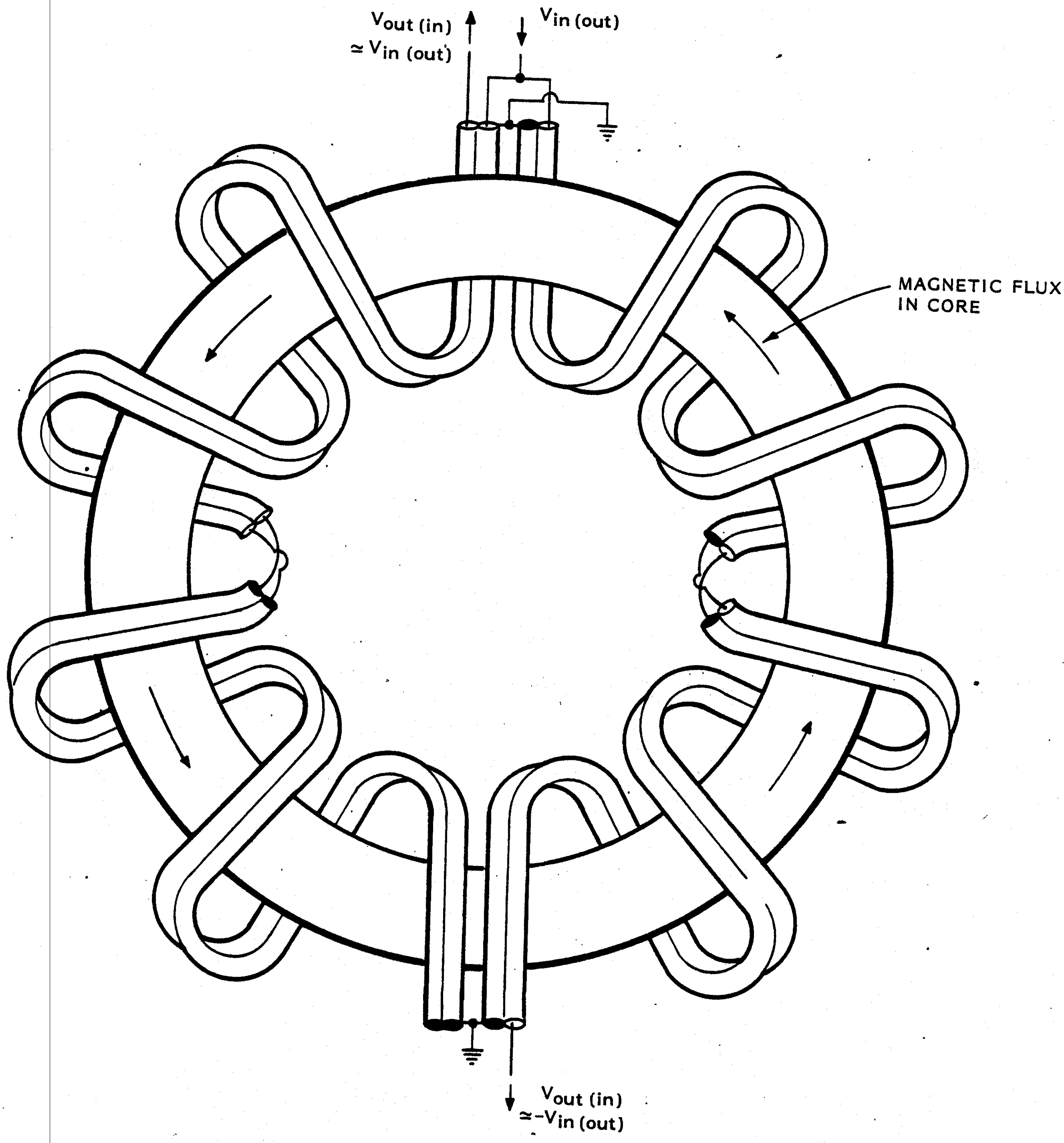


Figure 4.2. Example of Single-Ended to Differential Twin Coaxial Transformer (Balun) with 1 to 2 Turns Ratio

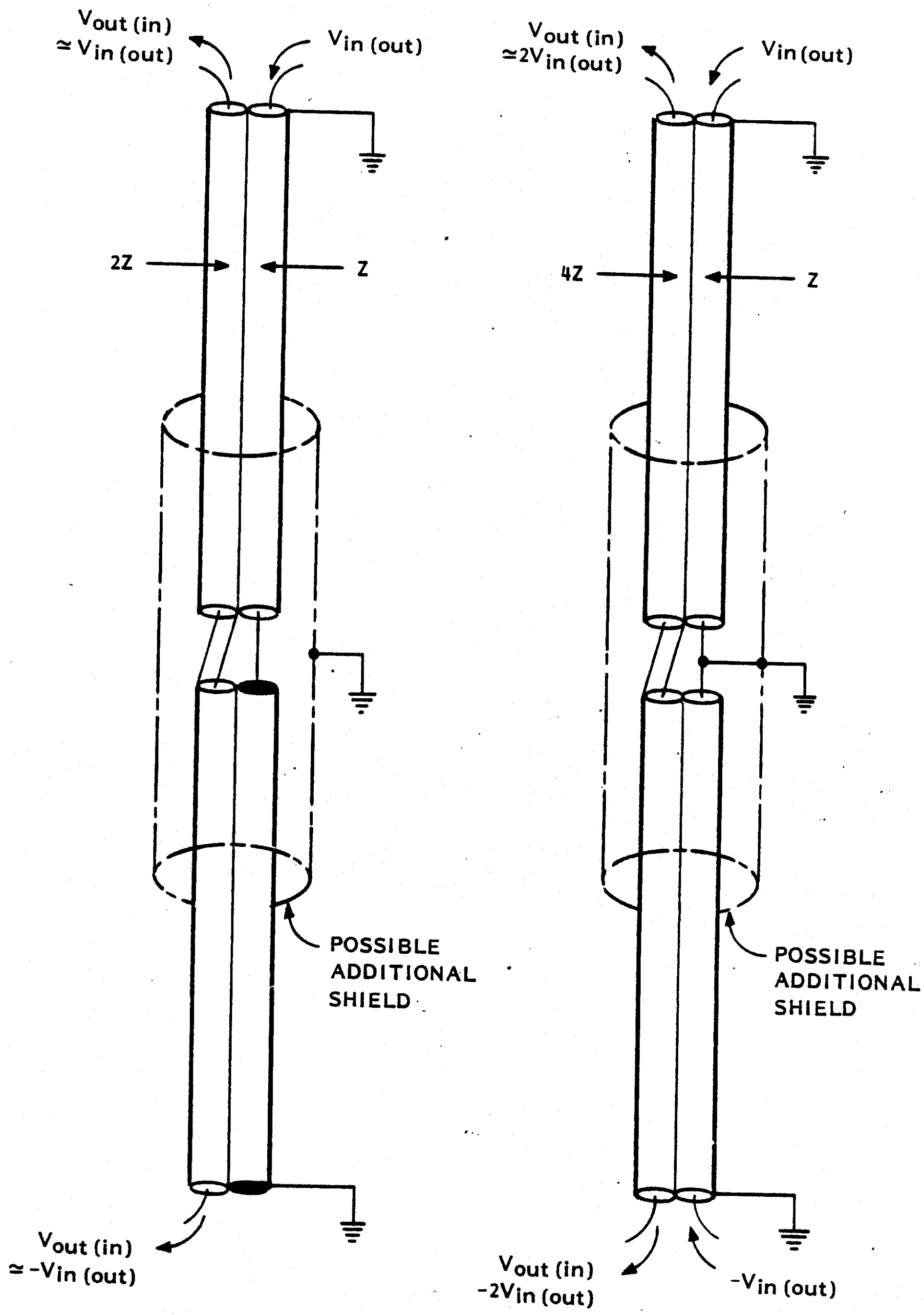
can create an additional "winding" (such as a shorted turn) on the transformer. While the discussion has centered on a single-ended to differential transformer (balun), it also functions as a differential to single-ended transformer with a 2 to 1 turns ratio.

Let us briefly consider a few alternate winding topologies to show that this general class of transformer windings admits in principle an infinite number of winding topologies. However, the cable impedances required will limit the number of practical cases.

Figure 4.3 shows two related examples involving the use of a Moebius gap [1] which effectively doubles the number of turns on the secondary winding. In Figure 4.3a we have single ended to differential while Figure 4.3b shows differential to differential. As in Figure 4.1 one can have an additional shield around the twin-coaxial structure to distribute the signal input around the winding. One can also use Moebius gaps in both primary and secondary to make a 1 to 1 differential to differential transformer.

Higher order winding topologies are also possible. Using the multiax concept [1], Figure 4.4 shows the case of a triaxial secondary bonded to a coaxial primary. Preserving the impedance matching from coaxial primary (with two inputs) to the transmission line formed of the secondary coaxial and triaxial shields, the latter waves in turn are impedance matched into the secondary coax (with a doubled impedance). Note that the two inputs to the primary are fed in parallel giving an impedance of Z/2. There are three ground points on the winding connected together. In this case the winding is naturally divided into two parts as far as external waves are concerned. Beyond this one can add additional shields around this winding centered on the external gaps as in Figure 4.1.

By reversing which coax is used for the signal output on the secondary (or by reversing both input coaxes on the primary) the configuration of 4.4 becomes an inverter with 1 to -2 turns ratio (analogous to the change from Figure 4.1a to Figure 4.1b). By taking this configuration and combining it with the previous (in Figure 4.4) one can make a single-ended to differential transformer with 1 to 4 turns ratio. Note in this case there are four parallel inputs to the primary. This configuration is a generalization of that in Figure 4.2.



a. 1 to 2, single ended to differential.

b. 1 to 2, differential to differential.

Figure 4.3. Two Coaxial Windings with Moebius Gap in the Secondary Winding

Yet another variation on the theme of Figure 4.4 has the gap in the secondary coax changed to a Moebius gap, giving a differential output from opposite ends of the winding. The impedance of the secondary coax is raised to 4Z. The resulting transformer is then single ended to differential with 1 to 4 turns ratio.

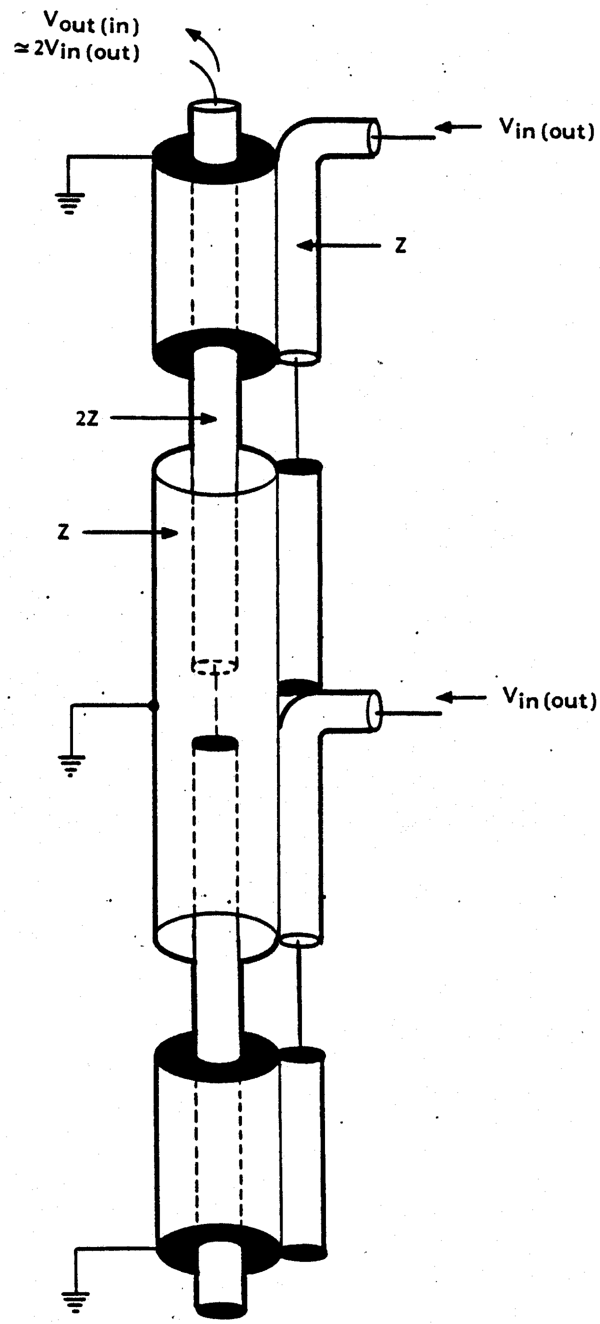


Figure 4.4. Coaxial/Triaxial Single-Ended to Single-Ended Transformer with 1 to 2 Turns Ratio

V. Summary

This note has introduced some new concepts in transformer windings. By using coaxial (and multiaxial) cables as the windings, with outermost shields of primary and secondary windings bonded together, the problem of transformer leakage inductance can be largely eliminated.

In these special winding configurations the primary and secondary windings are effectively reduced to a single winding as far as the external properties (such as interaction with the transformer core) are concerned. This single effective winding still has positive length on which waves can propagate. Such waves involve not only the winding lengths (pitch, etc.), but also the detailed high-frequency electromagnetic properties of the transformer core. So the question of leakage inductance is shifted to problems (at generally higher frequencies) involving core properties, winding geometries, and loading (signal introduction positions) along the windings. This is a subject for further research.

References

1. C.E. Baum, A Technique for the Distribution of Signal Inputs to Loops, Sensor and Simulation Note 23, July 1966.

2. C.E. Baum, The Multiple Moebius Strip Loop, Sensor and Simulation Note 25, August 1966.

3. C.E. Baum, Design of a Pulse-Radiating Dipole Antenna as Related to High-Frequency and Low-Frequency Limits, Sensor and Simulation Note 69, January 1969.