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ON AN OPTIMUM RECEIVER BANDWIDTH CRITERION FOR RESOLVING PULSED SIGNALS PROPAGATED THROUGH A DISPERSIVE CHANNEL

R. D. Jones
Satellite Systems Division
Sandia Laboratories
Albuquerque, New Mexico
87115

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ABSTRACT

An optimum receiver bandwidth criterion is described for resolving pulsed sinusoidal signals with a Gaussian envelope propagated through a dispersive channel. Only quadratic-type phase distortion is considered, and the receiver is assumed to have a Gaussian impulse response. The optimum bandwidth is that of the matched filter required for optimum linear processing of the signal at the receiver input.

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Using a simple model, Wait[1] has derived an expression for the optimum receiver bandwidth which minimizes the stretching out of the detected envelope of a Gaussian-modulated sinusoidal signal propagated through a dispersive channel. In his paper Wait assumed that the receiver had a Gaussian impulse response and that the dispersive channel introduced a quadratic phase-type distortion. According to this derivation, the optimum receiver bandwidth γ_o is given by

$$\gamma_o = \left[\left(1/W^2\right) - \left(1/\alpha^2\right)\right]^{-1/2} \quad \text{rad/sec} , \tag{1}$$

where W is the "channel bandwidth" and α is the source bandwidth. However, it should be noted that Eq. (1) yields a real value for γ only for $W < \alpha$; i.e., γ_o is the optimum receiver bandwidth for minimizing distortion only when the dispersive channel bandwidth is less than the bandwidth of the source signal.

In this paper a more general criterion for an optimum receiver bandwidth is presented. The same simple dispersive channel model described by Wait is used, but perturbation of the channel by white Gaussian noise is assumed. In order to preserve the simplicity of analysis and to facilitate comparison with Wait's result, we also adopt a Gaussian-modulated sinusoidal pulse for a source signal and assume that the receiver has a Gaussian impulse response. The approach used here is analagous to characterizing the signal source and dispersive channel by a generator and a source impedance. The optimum receiver, therefore, should be represented by a load impedance which is the complex conjugate of the source impedance, thereby effecting a maximum power transfer from source to load.

[1] James R. Wait, "On the optimum receiver Bandwidth for propagated pulsed signals," Proc. IEEE, Vol. 57, October 1969, pp. 1784-1785.

As a starting point it is appropriate to derive Wait's expression for the optimum bandwidth by using an alternative approach. Following Wait, a field component of the source signal as a function of time is given by

$$f_o(t) = F_o e^{-\alpha^2 t^2} e^{j\omega_o t}, \quad -\infty < t < \infty . \tag{2}$$

The corresponding frequency spectrum $F_o(\omega)$ is

$$F_o(\omega) = F_o\left(\sqrt{\pi}/\alpha\right) \exp\left[-\left(\omega - \omega_o\right)^2/4\alpha^2\right], \tag{3}$$

which peaks strongly near $\omega = \omega_o$ for $(\omega_o/\alpha) \gg 1$; i.e., the source signal can be considered monochromatic.

In the vicinity of ω_o, the transfer function of the dispersive propagation path can be approximated by

$$P(\omega) \approx |P(\omega_o)| \exp\left[-j\Phi(\omega)\right] , \tag{4}$$

where

$$\Phi(\omega) \approx \omega_o\tau + (\omega - \omega_o)\tau_g + (\omega - \omega_o)^2/4W^2 . \tag{5}$$

By definition, τ is the phase delay of the channel, τ_g is the group delay, and W is the dispersive channel bandwidth. The field component at the input of the receiver can be obtained from

$$f_i(t) \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} F_o(\omega)P(\omega)e^{j\omega t}\, d\omega . \tag{6}$$

In his derivation, Wait includes the transfer function $R(\omega)$ of the receiver channel in the integrand of Eq. (6) and obtains an expression for the detected signal. Instead of doing this. let us perform the operation indicated in (6), but suppress the carrier term for convenience and obtain the envelope of a field component at the input of the receiver as

$$f_1(t - \tau_g) = C \exp\left[-\alpha^2(t - \tau_g)^2\left(1 - \frac{\alpha^2}{\alpha^2 - jW^2}\right)\right] , \tag{7}$$

where C is a proportionality constant. The amplitude of the envelope is given by

$$|f_1(t - \tau_g)| = C \exp\left[-\alpha^2(t - \tau_g)^2\left(1 - \frac{\alpha^4}{W^4 + \alpha^4}\right)\right] . \tag{8}$$

For $W \gg \alpha$, the dispersion is negligible, and in comparing (8) with (2) it can be observed that the envelope of the received signal is simply that of the source signal delayed by the group delay τ_g.

For convenience, we take C equal to unity and write (8) in the form

$$f_1(t - \tau_g) = e^{-\delta^2(t - \tau_g)^2} , \tag{9}$$

where

$$\delta^2 = \alpha^2\left(1 - \frac{\alpha^2}{\alpha^2 - jW^2}\right) . \tag{10}$$

Let γ be the receiver bandwidth. Then, because the receiver has a Gaussian impulse response, it can be written in the form

$$f_r(t - \tau_g) = e^{-\gamma^2(t - \tau_g)^2} . \tag{11}$$

According to the addition formula for standard deviations, the envelope of the received signal after detection is

$$f(t - \tau_g) = e^{-\delta^2(t - \tau_g)^2} * e^{-\gamma^2(t - \tau_g)^2} = \frac{1}{\delta\gamma}\sqrt{\pi\beta}\, e^{-\beta^2(t - \tau_g)^2} , \tag{12}$$

where

$$\frac{1}{\beta^2} = \frac{1}{\delta^2} + \frac{1}{\gamma^2} = \frac{1}{\alpha^2} + \frac{1}{\gamma^2} + \frac{1}{W^2} . \tag{13}$$

The magnitude of the last exponential in (12) is exp $\left[-(t - \tau_g)^2/t_d^2\right]$,

where

$$t_d^2 = \left(\frac{1}{\alpha^2} + \frac{1}{\gamma^2}\right) + \left(\frac{1}{\alpha^2} + \frac{1}{\gamma^2}\right)^{-1} \frac{1}{W^4} , \tag{14}$$

which is Eq. (9) in Wait's paper. As Wait observes, t_d is a minimum for

$$\frac{1}{\alpha^2} + \frac{1}{\gamma_o^2} = \frac{1}{W^2} , \tag{15}$$

and solving for γ_o, Wait's criterion for the optimum bandwidth is obtained as

$$\gamma_o = \left[(1/W^2) - (1/\alpha^2)\right]^{-1/2} . \tag{1}$$

At this point it is reasonable to propose a different optimum receiver bandwidth criterion which holds when the bandwidth of the dispersive channel is greater than the bandwidth of the source signal, i.e., for $W > \alpha$. This criterion is that the receiver should simply act as a matched filter so as to provide optimum linear processing of the received signal. This form of signal processing transforms the raw signal data, available at the receiver input and assumed to be corrupted by white Gaussian noise, into a form suitable for obtaining maximum resolution between a sequence of similar signals. Following North, VanVleck and Middleton,[2] in the frequency domain the matched-filter transfer function, $H(\omega)$, is the complex conjugate function of the spectrum of the signal that is to be processed in an optimum fashion. That is, the transfer function is given by

$$H(\omega) = kF_i^*(\omega) \exp\left[-j\omega T_d\right] , \tag{16}$$

where $F_i(\omega)$ is the spectrum of the input signal, $f_i(t)$, and T_d is an arbitrary delay constant required to make the filter physically realizable. For present purposes, the delay term can be ignored and the optimum filter is

$$H(\omega) = F_i^*(\omega) , \tag{17}$$

or, in the time domain, the matched-filter impulse response is given by

$$h(t) = f_i(-t) . \tag{18}$$

Therefore, it follows from (9) that the optimum receiver bandwidth for a Gaussian-modulated sinusoidal pulse is

$$\gamma_o' = \mathcal{R}e\,\delta , \tag{19}$$

or from (10),

$$\gamma_o' = \frac{\alpha W^2}{\sqrt{\alpha^4 + W^4}} = \frac{\alpha}{\sqrt{1 + (\alpha/W)^4}} . \tag{20}$$

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[2]See, for example, Charles E. Cook and Marvin Bernfeld, Radar Signals, Academic Press, New York, 1967, Chapter 2.

From Eq. (20), as the dispersive channel bandwidth increases without limit (i.e., the dispersion approaches zero), γ_o' approaches α. This, of course, means in the case of dispersionless transmission that the optimum bandwidth for the receiver is equal to the bandwidth of the source signal.

A consideration of the optimum receiver bandwidth for a non-Gaussian quasi-monochromatic signal is beyond the scope of this paper. However, the following approach might provide an estimate of the optimum bandwidth, again where it is assumed that the receiver has a Gaussian impulse response.

For a measure of bandwidth of an arbitrary quasi-monochromatic signal, it seems appropriate to use the second moment about a suitably chosen point of $|F(\omega)|^2$, where $F(\omega)$ is the spectrum of the source signal. That is, one takes

$$\alpha = \left[\int_{-\infty}^{\infty} \omega^2 |F(\omega)|^2 d\omega\right]^{1/2} \tag{21}$$

as the value of α is (20). In this case the optimum receiver bandwidth is suggested as

$$\gamma_o'' = \frac{W^2\left[\int_{-\infty}^{\infty} \omega^2 |F(\omega)|^2 d\omega\right]^{1/2}}{\left\{W^4 + \left[\int_{-\infty}^{\infty} \omega^2 |F(\omega)|^2 d\omega\right]^2\right\}^{1/2}} . \tag{22}$$

By the reciprocal spreading principle, quasi-monochromatic signals, which are band-limited by definition, must be of long duration. It seems reasonable to assume that their spectra would not be grossly dissimilar from a Gaussian pulse and that therefore Eq. (22) could be used to obtain an estimate of the required bandwidth.

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