Low Frequency Electric Field Distortion in the Vicinity of the WEBS Catamaran

1/Lt Carl E. Baum
Air Force Weapons Laboratory

Abstract

The WEBS catamaran, even though made of insulating dielectric materials, can significantly distort electromagnetic fields in its vicinity. Specifically the distortion of the low frequency vertical electric field is approximately calculated. Under certain conditions this distortion can introduce significant errors into low frequency electric field measurements on the catamaran.

I. Introduction

In measurements of the close-in electromagnetic pulse (EMP) from a nuclear explosion, not only must we be careful in designing sensors for the electromagnetic fields because of the adverse environment, but also we must be careful that the presence of other objects does not significantly distort the fields our sensors are to measure. Such other objects include the mounting platform for the sensors. Since we are dealing with a medium (air) with non-linear and time-varying conductivity, we may not be able to unfold data which have been significantly distorted.

In the WEBS (Weapons Effects Buoy System) the principal sensor platform is a catamaran with about 2 1/2 meters between the two hulls, each about 7 1/2 meters long, together with some structural supports.[1] Neglecting the internal equipment (which may also contribute to the field distortion) a cross section of one of the catamaran hulls would be something like that illustrated in figure 1A. Basically the hull is an insulating shell (fiberglass). As such it does not affect the fields in the same manner as a metal shell. Nevertheless, in a conducting environment this insulating shell can significantly distort the electromagnetic fields in its near vicinity.

To illustrate this field distortion let us use an approximate equivalent cross section for the catamaran hull as in figure 1B. Here, for simplicity, we use a cylindrical shell where a/d is about 60. Specifically, we shall consider the distortion of the low frequency electric fields. By low frequency we require that the skin depth (or wavelength as appropriate) in the conducting air is sufficiently large such that in the absence of the catamaran the electric field would be a uniform vertical (y direction) electric field, E_o, over dimensions large compared to the catamaran. Assuming a time stationary, linear conductivity for the air (and other electrical parameters as appropriate) we can calculate the electric field distribution as a function of frequency around this idealized catamaran hull. Of course, the air conductivity is non-linear and time varying but our calculations should at least qualitatively indicate the extent of the low-frequency electric field distortion. There are probably other field distortions also present in the vicinity of the catamaran, but these may be more difficult to estimate and are not considered in this note.

II. Field Distortion

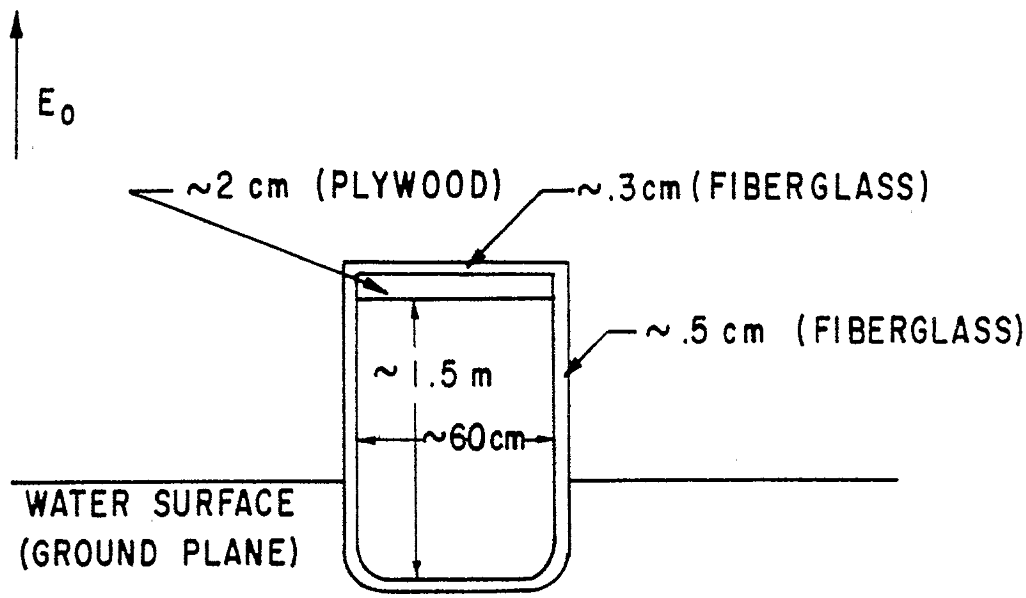
Consider then the problem illustrated in figure 1B where the length of the insulating shell is much larger than the radius so that we have only a two dimensional problem. In the case of the catamaran we have a conducting medium (sea water) which terminates the electric field. In the case we are calculating this can be approximately considered as the plane of symmetry, y = 0.

The general solution of Laplace's equation in cylindrical coordinates is of the form (with no z dependence)

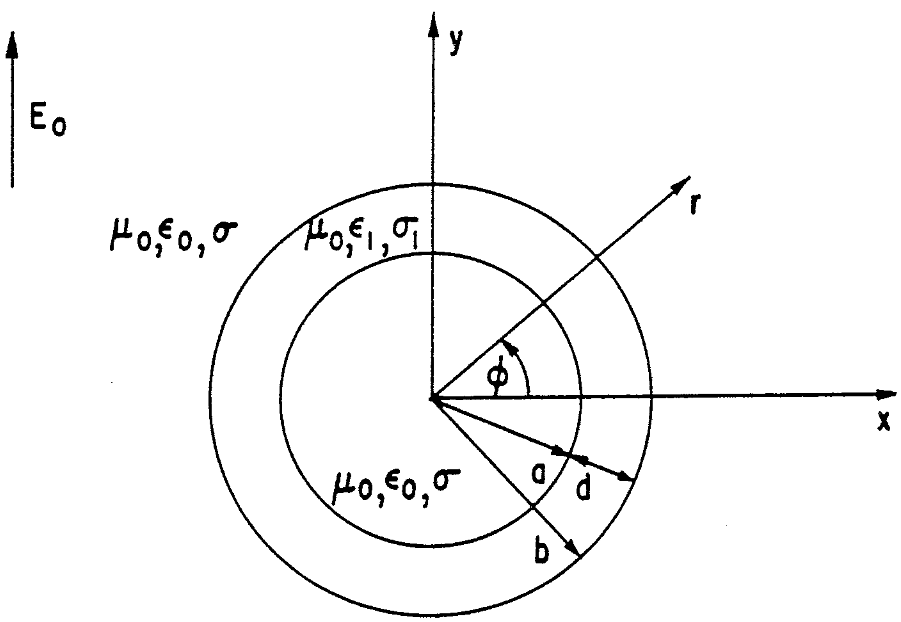
$$V = (a_o + b_o \ln(r)) + \sum_{n=1}^{\infty} [a_n r^n \begin{Bmatrix} \sin(n\phi) \\ \cos(n\phi) \end{Bmatrix} + b_n r^{-n} \begin{Bmatrix} \sin(n\phi) \\ \cos(n\phi) \end{Bmatrix}] \qquad (1)$$

where V is the electric potential.

1. See EMP Measurement Note II for a description of the WEBS catamaran.



A. APPROXIMATE PHYSICAL CROSS SECTION



B. APPROXIMATE EQUIVALENT CROSS SECTION

FIGURE 1. CROSS SECTION OF THE CATAMARAN HULL

We have the electric field

$$\vec{E} = -\nabla V \tag{2}$$

and thus

$$E_r = -\frac{\partial V}{\partial r} \tag{3}$$

and

$$E_\phi = -\frac{1}{r}\frac{\partial V}{\partial \phi} \tag{4}$$

Since for r >>b we must have

$$E_r \approx E_o \sin(\phi) \tag{5}$$

and

$$E_\phi \approx E_o \cos(\phi) \tag{6}$$

only the n=1 terms in equation (1) have any significance to match the distant uniform field. Thus, we have

$$V = (a_1 r + b_1 \frac{1}{r})\sin(\phi) \tag{7}$$

which is sufficient for the potential in the three regions.

Let us then define the potentials in the three regions as

$$V_{in} = -a_{in}\, r \sin(\phi) \quad \text{for } r \le a \tag{8}$$

$$V_s = -(a_s\, r + b_s \frac{1}{r}) \sin(\phi) \quad \text{for } a \le r \le b \tag{9}$$

$$V_{out} = -(E_o\, r + b_{out} \frac{1}{r}) \sin(\phi) \quad \text{for } b \le r \tag{10}$$

This gives us the electric field components

$$E_{r_{in}} = a_{in} \sin(\phi) \tag{11}$$

$$E_{r_s} = (a_s - b_s \frac{1}{r^2}) \sin(\phi) \tag{12}$$

$$E_{r_{out}} = (E_o - b_{out} \frac{1}{r^2}) \sin(\phi) \tag{13}$$

and

$$E_{\phi_{in}} = a_{in} \cos(\phi) \tag{14}$$

$$E_{\phi_s} = (a_s + b_s \frac{1}{r^2}) \cos(\phi) \tag{15}$$

$$E_{\phi_{out}} = (E_o + b_{out}\frac{1}{r^2})\cos(\phi) \tag{16}$$

For convenience define

$$\eta_o = \sigma + j\omega\varepsilon_o \tag{17}$$

$$\eta_1 = \sigma_1 + j\omega\varepsilon_1 \tag{18}$$

and

$$\eta_r = \frac{\eta_1}{\eta_o} \tag{19}$$

Then at $r = a$ and $r = b$ we have the boundary conditions that E_ϕ and ηE_r are continuous. At $r = a$ we have

$$a_s + \frac{b_s}{a^2} = a_{in} \tag{20}$$

$$\eta_r(a_s - \frac{b_s}{a^2}) = a_{in} \tag{21}$$

or

$$a_s\ (1-\eta_r) + \frac{b_s}{a^2}(1+\eta_r) = 0 \tag{22}$$

At $r = b$ we have

$$a_s + \frac{b_s}{b^2} = E_o + \frac{b_{out}}{b^2} \tag{23}$$

$$\eta_r(a_s - \frac{b_s}{b^2}) = E_o - \frac{b_{out}}{b^2} \tag{24}$$

Thus,

$$\frac{b_{out}}{b^2} = \frac{1}{2}[\ a_s\ (1-\eta_r) + \frac{b_s}{b^2}(1+\eta_r)] \tag{25}$$

$$E_o = \frac{1}{2}[\ a_s\ (1+\eta_r) + \frac{b_s}{b^2}(1-\eta_r)] \tag{26}$$

Combining equations (22) and (26) we have

$$2E_o(1-\eta_r) = b_s[\frac{(1-\eta_r)^2}{b^2} - \frac{(1+\eta_r)^2}{a^2}] \tag{27}$$

$$2E_o\ \frac{(1+\eta_r)}{a^2} = a_s[\ \frac{(1+\eta_r)^2}{a^2} - \frac{(1-\eta_r)^2}{b^2}] \tag{28}$$

Thus, combining equations (25), (27), and (28) and defining a new parameter, b', we have

$$b' = \frac{1}{E_o}\frac{b_{out}}{b^2} = \frac{(1+n_r)(1-n_r)}{(1+n_r)^2-(\frac{a}{b})^2(1-n_r)^2} + \frac{(1-n_r)(1+n_r)}{(1-n_r)^2-(\frac{b}{a})^2(1+n_r)^2} \quad (29)$$

or

$$b' = \left[1 - \left(\frac{a}{b}\right)^2\right] \frac{1-n_r^2}{(1+n_r)^2-(\frac{a}{b})^2(1-n_r)^2} \quad (30)$$

For d<<b this can be reduced to

$$b' \approx \frac{1 - n_r^2}{1+n_r^2 + \frac{2b}{d} n_r} \quad (31)$$

The potential outside the shell is then (from equations (10) and (29))

$$V_{out} = - E_o r(1+\frac{b^2}{r^2} b') \sin(\phi) \quad (32)$$

The magnitude of the scattered field, E_{scat}, (the total field minus the uniform field) is given by (from equations (5), (6), (13), and (16))

$$\frac{E_{scat}}{E_o} = (\frac{b}{r})^2 \; |b'| \quad (33)$$

Suppose now that $|n_r|$ <<1 which is the case when only the conductivities are important (compared to $\omega\varepsilon$ as in equations (17) and (18))and the conductivity (produced by the radiation) is much greater in the air than in the fiberglass hull. Then we have

$$b' \approx [1 + 2\frac{b}{d} n_r]^{-1} \quad (34)$$

and if $\left|2\frac{b}{d} n_r\right|$<<1 (as could be the case)

$$b' \approx 1 \quad (35)$$

In this last case we have essentially 100% distortion of the field right next to the cylindrical shell. The potential and field distributions for this last case in which negligible current density (conduction and displacement) penetrates the shell is plotted in normalized form (E_o = 1, b = 1) in figure 2 (for positive x and y). This is just the conformal transformation

$$w = z - \frac{1}{z} \quad (36)$$

where

$$w = u+jv \quad (37)$$

$$z = x+jy \quad (38)$$

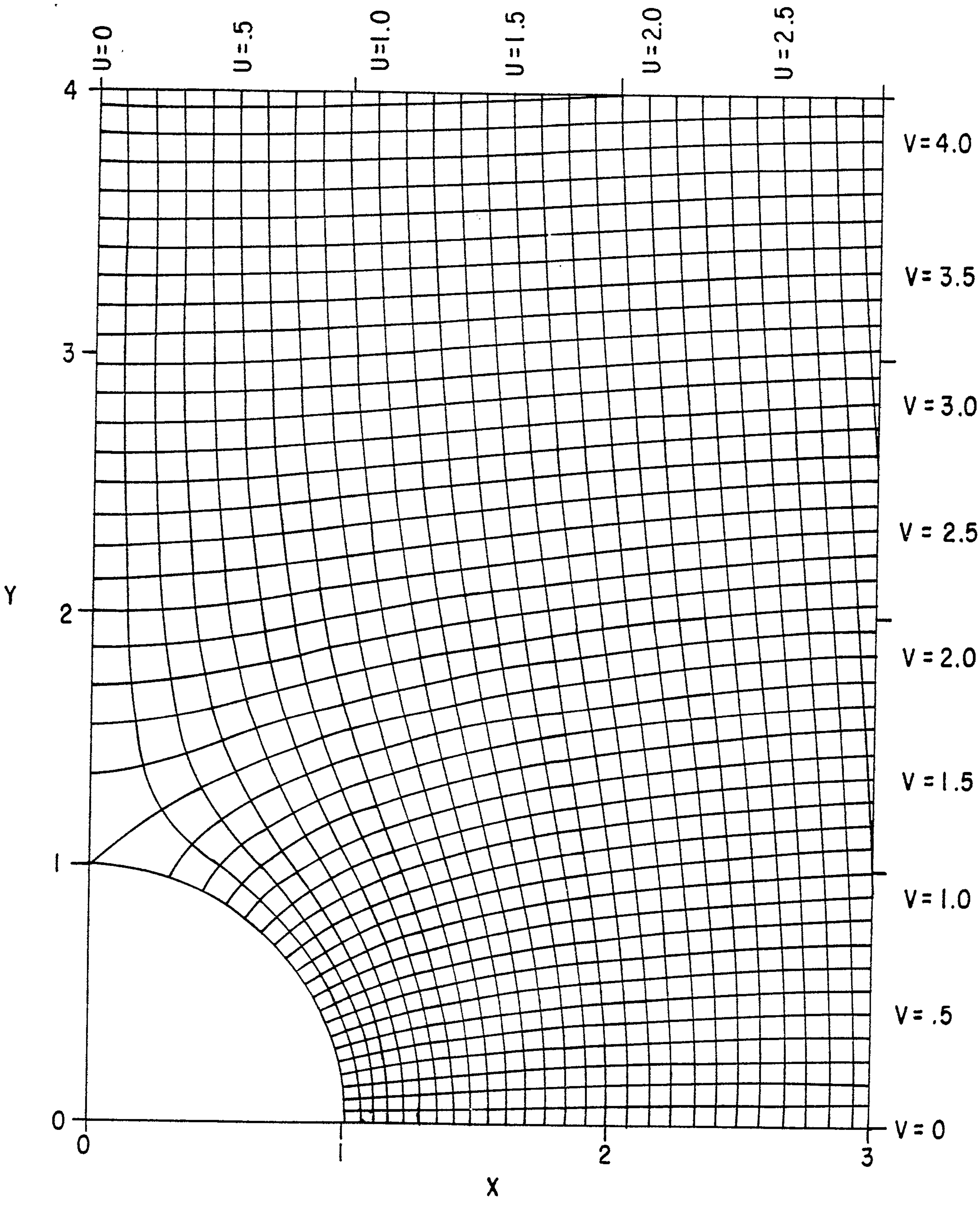


FIGURE 2. FIELD DISTRIBUTION AROUND INSULATING CYLINDER IN CONDUCTING MEDIUM

and where lines of constant v are equipotentials and lines of constant u are electric field lines. From this last figure we can get an idea of the extent of the low frequency electric field distortion.

III. Summary

When using the WEBS catamaran for a platform for electromagnetic field measurements we must be careful of field distortions in the vicinity of the catamaran. In particular, as we discussed, the low frequency vertical electric field can be significantly distorted. In the actual case the field distortion is somewhat more complicated because of our idealization of the catamaran hull as a cylindrical shell and particularly because of our ignoring the nonlinear and time varying characteristics of the air conductivity. Also, there will probably be other field distortions when the skin depth (or wavelength as appropriate) approaches the characteristic dimensions of the catamaran. Likewise, the other equipment in or on the catamaran can distort the field. However, these latter distortions are difficult to estimate.

In general, we should be cautious whenever we protrude objects above the ground or water surface in the vicinity of our electromagnetic field measurements. In some cases a conducting object will significantly distort the fields. (Imagine a conducting catamaran.) However, in other cases (such as in a conducting medium) an insulating object can also significantly distort the fields. As much as possible it would seem a good practice to have no unnecessary objects above the ground or water surface. Any objects which must protrude above the surface need to be designed to avoid such problems if at all possible.

We would like to thank Mr. Robert Mercer for his calculational assistance in the preparation of this note.