EMP Measurement Notes

Note I

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Radial Position Accuracy for EMP Measurements on a Surface Nuclear Test

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Abstract

Radial position accuracy criteria are developed for EMP and related measurements on a surface nuclear test. These criteria are considered both for the placement of measurement stations relative to ground zero and for the knowledge of these relative positions after the stations are in place. The significant quantities which influence these criteria are the radius of measurement, the smallest mean free path of the significant nuclear radiation, and the radius of the ionized region or the radius out to which the nuclear radiation is significant.

I. Introduction

In measuring the EMP from a surface nuclear explosion we are confronted with several problems concerning the accuracy of our measurement. One of these pertains to the position of measurement devices relative to ground zero and can be stated in a double question: how accurately should our sensors be located with respect to their preplanned locations; and after they are in position how accurately should this position be known? Thus, we should establish two appropriate sets of criteria.

For measurements over land we can quite accurately survey our measurement locations, both before and after installation. However, in some circumstances, e.g., over water measurements as with the WEBS (Weapons Effects Buoy System), accurate placement of the measurement stations may be a distinct problem. Even after the systems are in place their location cannot be determined as accurately as land based measurements. The purpose of this note is to develop some simple expressions from which the position accuracy can be determined for a given measurement accuracy. Note that we shall be talking only of accuracy related to position and not to the many other sources of error.

We should also note at this point that while we are discussing position accuracy from the viewpoint of the measurement location, the same criteria will hold for our prepositioning and after-the-fact knowledge of the position of ground zero. We shall for now ignore variations in such things as height above the surface (ground or water), since it is the relative horizontal position of ground zero and the measurement location at detonation time which we are considering. Altitude variations of either the detonation point or the measurement location are not considered in this note, but are indeed important.

II. Model for Accuracy Relationships

To relate position inaccuracy to a deviation in the quantities to be measured, we need to know the sensitivity of these quantities to position changes. For any given position we can then relate our positional accuracy to the measurement most sensitive to position.

In measurements of the EMP we are usually concerned with the electric and magnetic field (and various other electromagnetic parameters) and the nuclear radiation, which constitutes the source terms (through the Compton current and the conductivity) for the fields. For convenience we can separate the measurement locations into two groups: those where the nuclear radiation is important and those where it is not. This is done by the use of the approximate concept of the saturation radius or ionized sphere radius, r_i. There are several ways of defining this parameter, including:

1. The radius inside which the conduction currents "dominate" over the displacement currents.

2. The radius at which the peak conductivity equals the highest significant radian frequency component times the permittivity.

3. The radius at which the dominant radian frequency times the permittivity equals some "typical" conductivity.

However, we can see that these are all approximate concepts based on the general nature of the physics of the problem. Since, as we shall see later, the position requirements are more stringent inside r_i, we shall be safe in our accuracy criteria if we overestimate r_i. Perhaps a better method is to choose r_i based on the variation of fields with distance as determined by the best calculations available. We expect to see slow variation of field strength with position for radii smaller than r_i, at least for the electric field, based on present models for calculation of the fields.

Since our models for EMP assume azimuthal symmetry around ground zero we will ignore variations with azimuth. In a real application we shall have to assure ourselves that variations with azimuthal position are small compared with variations with radial distance. Actually the requirements on azimuthal position are probably less stringent than those on radial position. In addition, the various quantities being measured will depend on height of measurement (from the surface) but this is somewhat more involved and is not considered here.

III. Accuracy Expressions

Let us consider two regions for determining our accuracy from variations in radial position, r. For $r < r_i$ the determining quantity will be the nuclear radiation because of its comparatively rapid variation with position, in comparison to the typical variation of the electric and magnetic fields with radius. For $r > r_i$ the determining quantity will be the electromagnetic fields, at least for those positions at which the nuclear radiation is insignificant.

A. Inside the ionized region: $r < r_i$

For simplicity we characterize the variation of the radiation intensity with radius, $f_1(r)$, as

$$f_1(r) = \frac{e^{-\frac{r}{r_\gamma}}}{r^2} \qquad (1)$$

The γ-ray mean free path, r_γ, is taken as about 200 meters for 2.5 MeV γ rays in STP air and is a total scattering mean free path. In general, equation (1) is meant to characterize whichever significant component of the nuclear radiation (for EMP purposes) has the largest relative change with position. Since the total scattering mean free path for 14.1 MeV neutrons is also about 200 meters in STP air we need make no distinction from the γ rays for this analysis. However, fission neutrons and X rays have shorter mean free paths. Typical fission neutron mean free paths (total scattering) are approximately 100 meters in STP air while X-ray mean free paths are strongly energy dependent and significantly shorter than γ-ray mean free paths. Thus, if we are trying to measure these last two quantities or if we are concerned with a region in which the X rays significantly affect the Compton current or ionization (compared with the γ-rays) we should use the appropriate shorter mean free paths in equation (1). For generality, r_γ can be used to represent the shortest mean free path of interest to each particular application.

Let us calculate from equation (1) a relative error, η_1, by varying our measurement radius, r, by an amount, Δr. Thus, let

$$1 + \eta_1 = \frac{f_1(r+\Delta r)}{f_1(r)}$$

$$= \left(\frac{r}{r+\Delta r}\right)^2 e^{-\frac{\Delta r}{r_\gamma}}$$

$$= \left(1+\frac{\Delta r}{r}\right)^{-2} e^{-\frac{\Delta r}{r_\gamma}} \qquad (2)$$

For $|\Delta r|$ small compared to both r and r_γ equation (2) reduces to

$$1+\eta_1 \approx \left(1-2\,\frac{\Delta r}{r}\right)\left(1-\frac{\Delta r}{r_\gamma}\right) \qquad (3)$$

or

$$1+\eta_1 \approx 1 - 2\,\frac{\Delta r}{r} - \frac{\Delta r}{r_\gamma} \qquad (4)$$

and

$$\eta_1 \approx -2\,\frac{\Delta r}{r} - \frac{\Delta r}{r_\gamma} \qquad (5)$$

This last equation can be inverted to solve for Δr as

$$\Delta r \approx -\eta_1 \left[\frac{2}{r} + \frac{1}{r_\gamma}\right]^{-1} \qquad (6)$$

Thus, for small $|\eta_1|$, our desired relative error level, we can readily calculate the required $|\Delta r|$.

We can note from equation (6) two general regions of interest. For $r < 2r_\gamma$ the $1/r^2$ part of $f_1(r)$ is the more important, while for $r > 2r_\gamma$ the $e^{-\frac{r}{r_\gamma}}$ part of $f_1(r)$ is more important. As we vary r we can also see for small $|\eta_1|$ the maximum value of $|\Delta r|$ is

$$|\Delta r|_{max} \approx |\eta_1|\, r_\gamma \qquad (7)$$

This is appropriate for use where $r \gg 2\, r_\gamma$. On the other hand for $r \ll 2r_\gamma$ and small $|\eta_1|$

$$\Delta r \approx -\eta_1 \frac{r}{2} \qquad (8)$$

B. Outside the ionized region: $r > r_i$

Since we are now concerned with the variation with radius of the electric and magnetic fields, we characterize this variation by the first order term in the expansion for a radiating dipole:

$$f_2(r) = \frac{1}{r} \tag{9}$$

Note that we are assuming no attenuation with distance due to the finite conductivity of the ground or water surface. If we wished to include this we could use something like

$$f_2'(r) = \frac{e^{-\frac{r}{r_\sigma}}}{r} \tag{10}$$

where r_σ is the shortest attenuation distance for frequencies of interest in the pulse at the given r. However, we shall ignore this last effect in our subsequent calculations.

From equation (9) we can calculate a relative error, η_2, for a given Δr as

$$1+\eta_2 = \frac{f_2(r+\Delta r)}{f_2(r)}$$

$$= \frac{r}{r+\Delta r}$$

$$= \left(1+\frac{\Delta r}{r}\right)^{-1} \tag{11}$$

For $|\Delta r|$ small compared to r we have

$$1+\eta_2 \approx 1 - \frac{\Delta r}{r} \tag{12}$$

or

$$\eta_2 \approx -\frac{\Delta r}{r} \tag{13}$$

or

$$\Delta r \approx -\eta_2 r \tag{14}$$

Thus, for a given $|\eta_2|$ our allowable $|\Delta r|$ constantly increases with r. However, we should note that this only applies for $r << r_\sigma$ but since the high frequencies are preferentially attenuated the principal frequencies become lower with distance and thus r_σ increases with r. We would need to look at the actual measurement situation to obtain an estimate of the appropriate attenuation distance. However, if we use a more stringent requirement (such as that developed for inside the ionization region) we should generally be safe because the fields should fall off no faster than the nuclear radiation which is the source for the fields. This presumes that the electrical properties of the surface (ground or water) are reasonably independent of position.

IV. Accuracy Requirements

Now that we have related our position accuracy to our measurement accuracy we can discuss our accuracy requirements. As mentioned in the first section we can reasonably divide such requirements into two sets of criteria: one for placement of the measurement stations and another for knowing the position of these stations after they are in place. Let us consider these separately.

As reasonable way to consider the requirement for placement accuracy is to relate it to the criteria on which any particular measurement location is chosen in the first place. Generally, this is decided on the basis of adequate coverage, over the range of interest, of the physical quantity to be measured. In this case, we are concerned with (among other things) electromagnetic fields and nuclear radiation. Thus, if we look at the quantity to be measured which varies the most rapidly with distance we can allow for a considerable amount of variation of this quantity as long as adequate coverage is maintained. A factor of two or three (opinions will vary as to the exact number) variation might not be unreasonable. There is some latitude in an "ideal" measurement array anyway.

Returning to the previous section then we can establish some approximate Δr's for placement of measurement stations. Inside the ionized region (see equation (2)) we can consider two cases. For $r > 2r_\gamma$ our criterion becomes that $|\Delta r|$ be less than about r_γ, about 200 meters for γ rays but possibly less in cases where other radiation is of interest. For $r < 2r_\gamma$ our criterion becomes that $|\Delta r|$ be less than about $r/2$. Outside the ionized region (see equation (11)), ignoring attenuation, our criterion becomes that $|\Delta r|$ be less than about $2r/3$. However, if attenuation is of concern then we can use the same criteria as inside the ionized region to be safe. In any case we should use the more stringent requirements out to a distance at which the contribution of the radiation is negligible. These approximate criteria are based on about a factor of three variation in the measured quantities and should be regarded as minimum requirements. If the $|\Delta r|$'s for placement of the measurement stations can be reasonably reduced without excessive difficulty they should be.

The requirement for knowledge of the actual position of the measurement station at detonation time can be determined by relating the error in the position of the measurement station to the other errors introduced into the measurements. However, the basic accuracies of the electromagnetic field and nuclear radiation measurements are not always known very well. Perhaps the best we can do is find the $|\Delta r|$'s which give a variation in the measured quantities of about ten percent, the engineering rule of thumb. Letting η_1 and η_2 be $\pm$ 0.1 we can use equations (7), (8), and (14) to determine $|\Delta r|$ in the various regions of interest where, as before, we may wish to use the criteria for inside the ionized region to larger radii for safety. For example, for $r > 2r_\gamma$ we have a $|\Delta r|$ of 0.1 r_γ or about 20 meters. Again these criteria should be regarded as minimum requirements. Errors do add up. Thus, to the extent practical the error attributable to an inaccurate position should be made small compared to the other errors. Our goal in knowledge of the radial position of our measurement locations (relative to ground zero) should then be typically an accuracy of a few meters.

V. Summary

In summary, then, for EMP and related measurements on a surface nuclear test we have two general problems concerning the horizontal position accuracy of our measurement stations. First we must physically place our measurement devices in their desired locations. For positions at which the nuclear radiation is significant our radial placement error should be at least within the smallest significant mean free path (about 200 meters for γ rays), decreasing for radii less than about 2 mean free paths. Second, after we have our measurement stations in place, we need to know the actual radii (from ground zero at detonation time) of the stations to within about a tenth of a mean free path (if the radiation is significant), also decreasing for radii less than about 2 mean free paths. For radii outside the ionized region (where the nuclear radiation is unimportant) our position accuracy requirements significantly decrease but careful analysis is necessary to determine just how much they decrease.

However, these above requirements are minimal. To the extent reasonable and practical these $|\Delta r|$'s should be decreased. These requirements pertain to the radial direction, not the azimuthal direction which generally has less stringent accuracy requirements. However, if we apply about the same criteria for azimuthal distance variations as for radial distance variations we should be safe. We should emphasize that it is not the absolute horizontal coordinates of our measurement location which are important so much as their horizontal coordinates relative to ground zero.