Mathematics Notes

Note 30

Comparisons of Three Inverse Fourier Transform Routines Used In Minuteman In-Place EMP Assessment

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Abstract

The signals which were measured during the Minuteman tests were erratic and covered several decades of the frequency domain. Because of these qualities only a few Fourier inverse transform routines could be applied. Three of these routines, two which are discrete methods and the third an analytic fit, are investigated and compared. One source of error in the transforms was concluded to arrive from the amount of energy in the signals, which are causal, that appeared in the negative time domain after the inverse transform.

Introduction:

One of the more difficult probelms encountered in MINUTEMAN (MM) assessment has been the Fourier inverse transforming of CW data from the on-going SAMSO EMP test program. Three methods for doing this inverse transform have been used in the MM In-Place assessment. These are:

1. Multi-Peak Fit: This is a routine to fit the frequency domain data with one or more peaks of the form:

$$g(f_o) = \frac{A \frac{\Delta f}{f_o}}{1 - \left(\frac{f}{f_o}\right)^2 + j2\delta\left(\frac{f}{f_o}\right)} \qquad (1)$$

where A = peak amplitude

Δf = peak width at $\frac{A}{\sqrt{2}}$

f_o = frequency at peak amplitude

$\delta = \Delta f/2f_o$

The Fourier transform of g(f) can be analytically obtained and is:

$$f^{-1}\,[g(f)] = 2\pi\Delta fAe^{-j\omega t}\sin\,\omega t$$

2. MIT: This is a discrete inverse Fourier transform routine written originally by Dikewood for use on unequally spaced frequency domain data points. The code has been modified by AFWL to use both real and complex parts of frequency domain data. There is no published documentation on the MIT routine.

3. TIMEOT: This is an inverse Fourier transform routine written by Boeing to handle unequally spaced frequency domain data points. (Ref 1.) This note reports on the use of these three inverse Fourier transform routines and the application/use of each in future EMP systems assessment efforts.

Results

The frequency domain data from the In-Place SVS program is defined on unequally spaced frequency data points. Since there is no known theory on predicting the errors when transforming unequally spaced data into the time domain, the method chosen to investigate these transform techniques was to apply them to several (much more than presented here) data sets and compare the results.

All three methods for doing the inverse Fourier transform were applied to the same extrapolated voltage signal at a particular circuit. This signal was predicted from voltage measurements made during the array drive portion of the SVS test conducted at Site Q-6. For the TIMEOT subroutine, some minor changes of the data had to be made prior to its input. These changes were:

1. Redefining the phase at the lowest frequency point to be zero.

2. Defining the highest frequency point as having zero amplitude and zero phase.

Figures 1 through 6 summarize the results of these three inverse Fourier transform methods. Figure one is a plot of one voltage signal measured during the MM test program. This signal was digitized and then input into the TIMEOT and MIT routines. The predicted time domain signals from these codes is plotted in figures 2 and 3. As part of the effort to determine how well the MIT and TIMEOT codes worked FFT (an AFWL subroutine to do Fourier Transforms) was applied to the TIMEOT and MIT predicted signals. Figures 4 and 5 show overlays of the initial signal (Figure 1) and the predicted signals from FFT. The last of these plots in figure 6 shows an overlay of the original signal and the curve which the Multi-Peak Fit (MPF) has fitted it with.

Table 1 presents four important results (from an assessment point-of-view) of applying the three Inverse Fourier transforms to three voltage signals (identified as RN# 45132, RN# 45133, and RN# 45134). The results are:

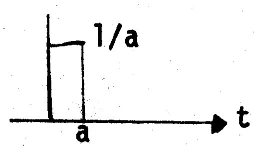
1. VHAP - The maximum value of the extrapolated voltage signal in the time domain.

*The vertical axis on plots 1,4,5, and 6 represents the magnitude of the inverse transform of an impulse function, times the transfer function of the circuit:

$$H(\omega) \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

where $H(\omega)$ is the transfer function of the circuit

$\delta(t) = \lim_{a \to 0}$



2. % ERROR - An energy check which tells how much energy is lost or gained in the inverse transform routine:

$$\frac{E_{freq} - E_{time}}{E_{freq}}$$

where E_{freq} is the energy in the frequency domain and E_{time} is the energy in the time domain.

3. TIME - Computer time to do the inverse transform on the CDC 6600.

4. RN - The run number which identifies the transfer function being used.

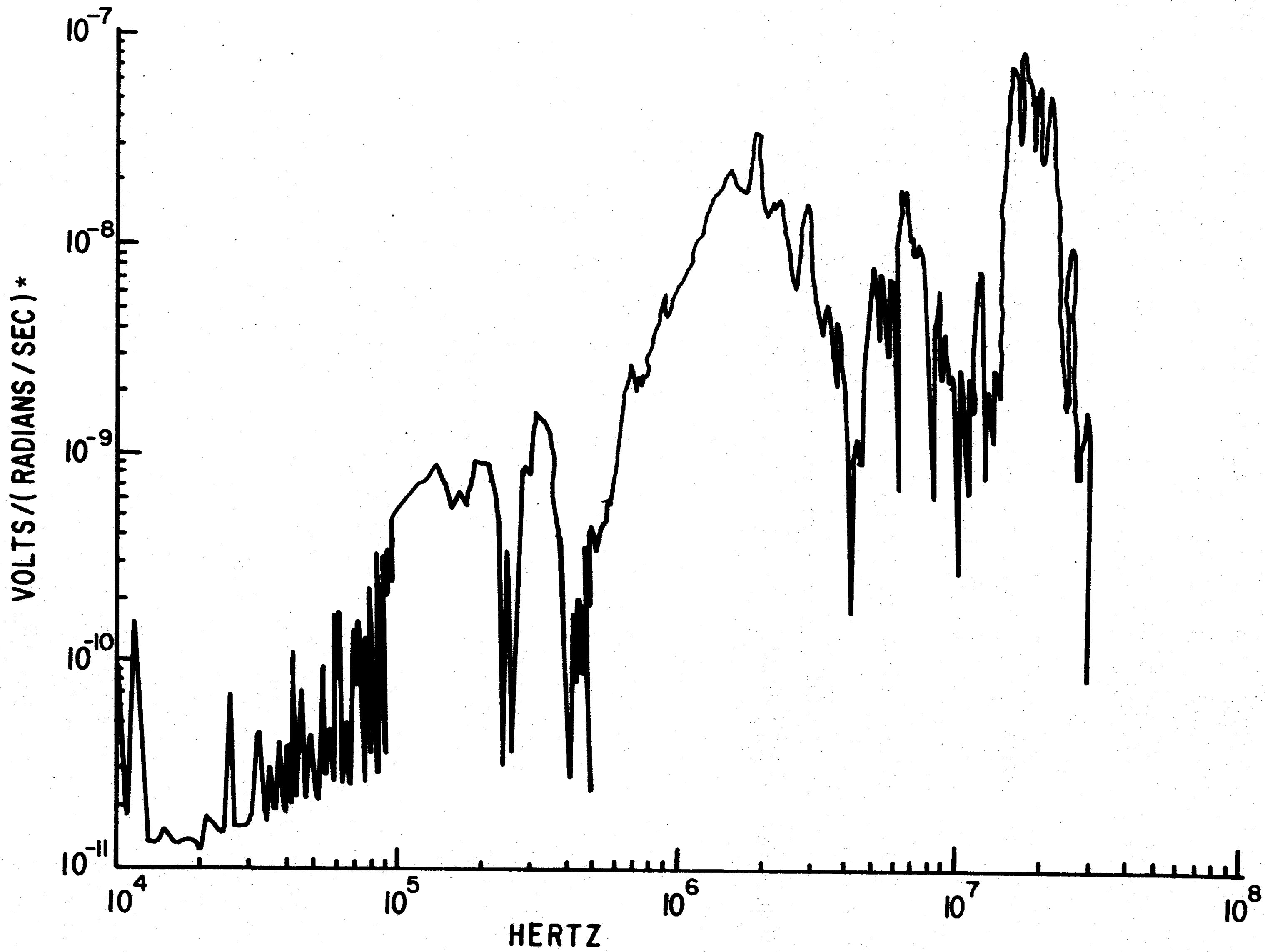


FIGURE 1. VOLTAGE MEASURED AT ONE OF THE CIRCUITS DURING THE MM CW TESTS.

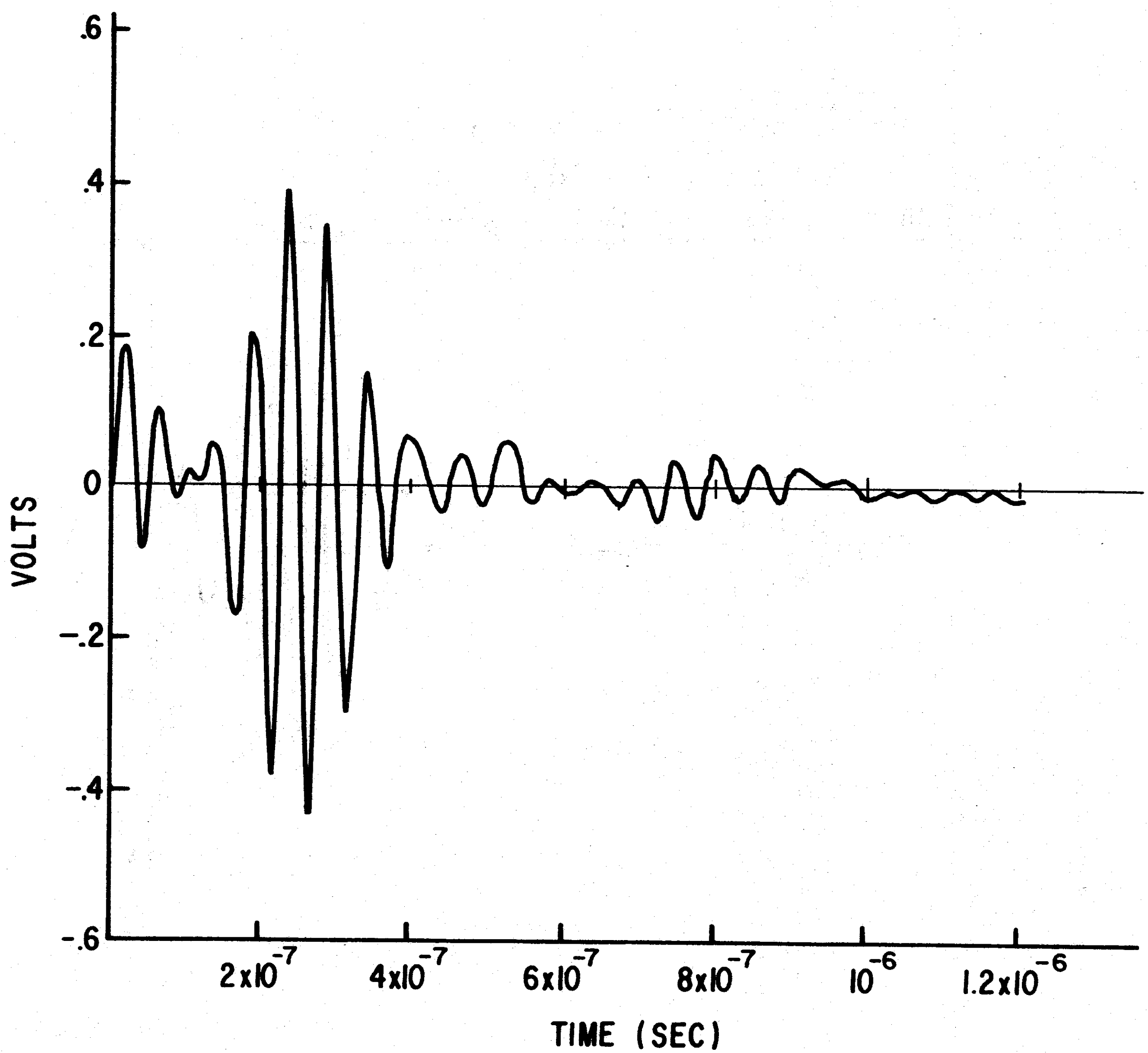


FIGURE 2. TIME DOMAIN FORM OF SIGNAL FROM TIMEOT ROUTINE

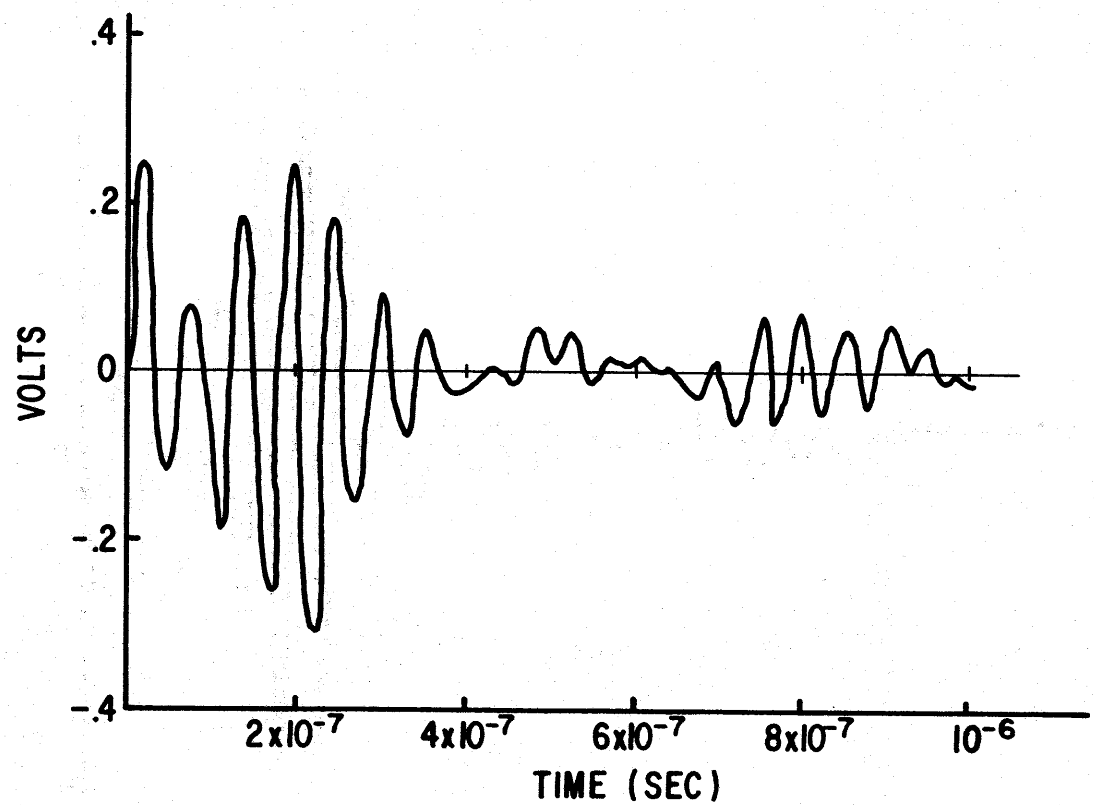


FIGURE 3. TIME DOMAIN FORM OF SIGNAL FROM MIT ROUTINE

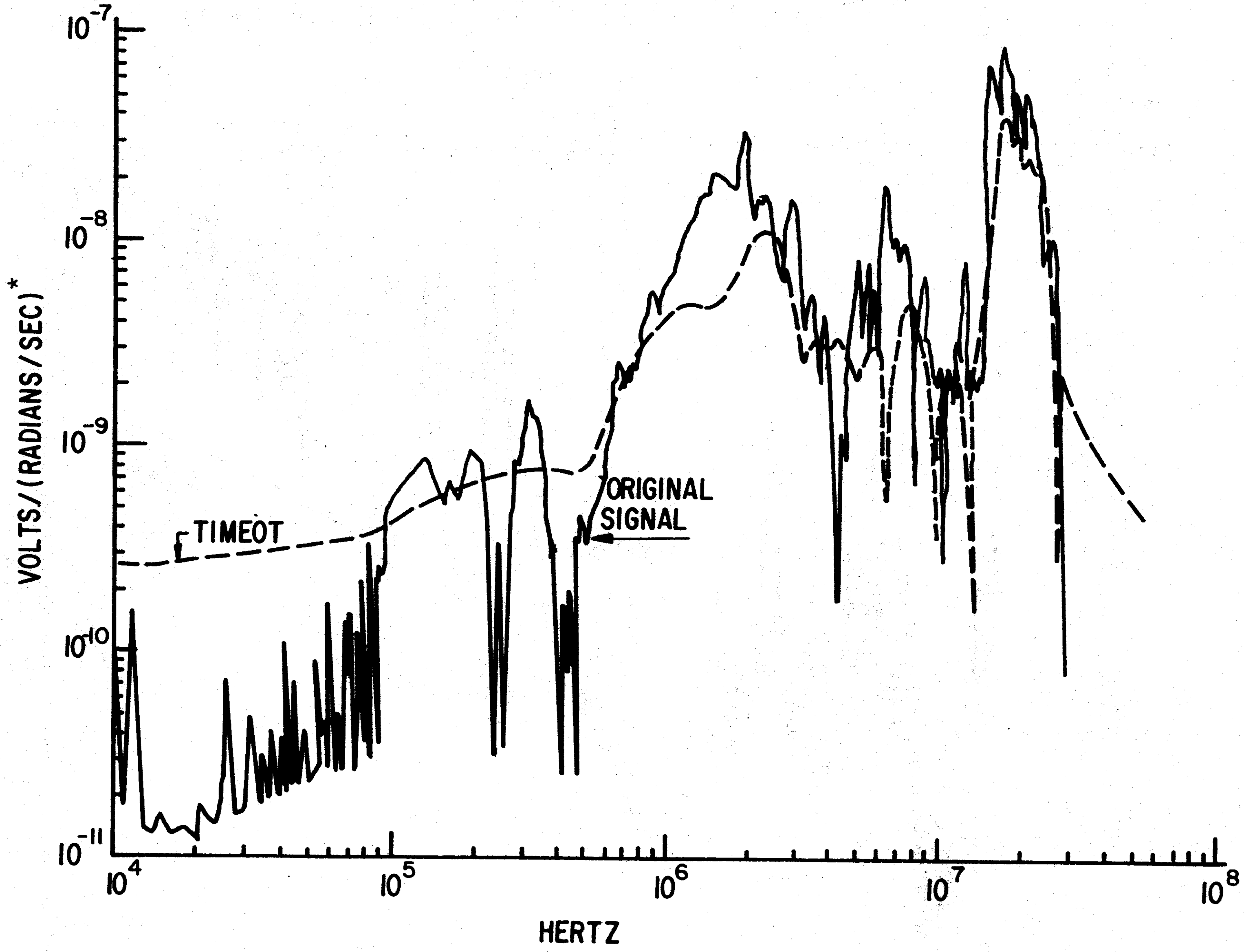


FIGURE 4. OVERLAY OF THE ORIGINAL SIGNAL AND FFT APPLIED TO TIM[redacted] PULSE

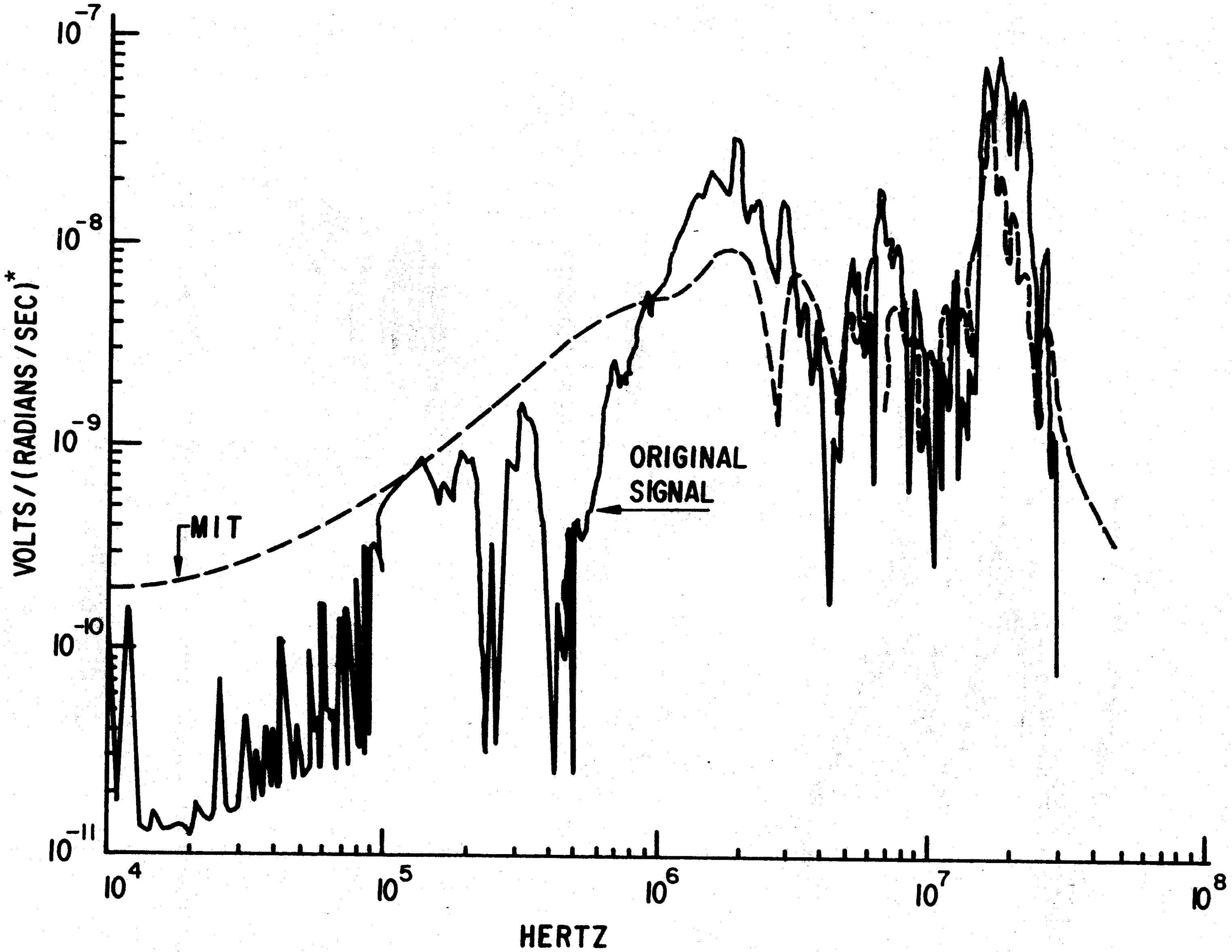


FIGURE 5. OVERLAY OF ORIGINAL SIGNAL AND RFT APPLIED TO MIT RESULTS

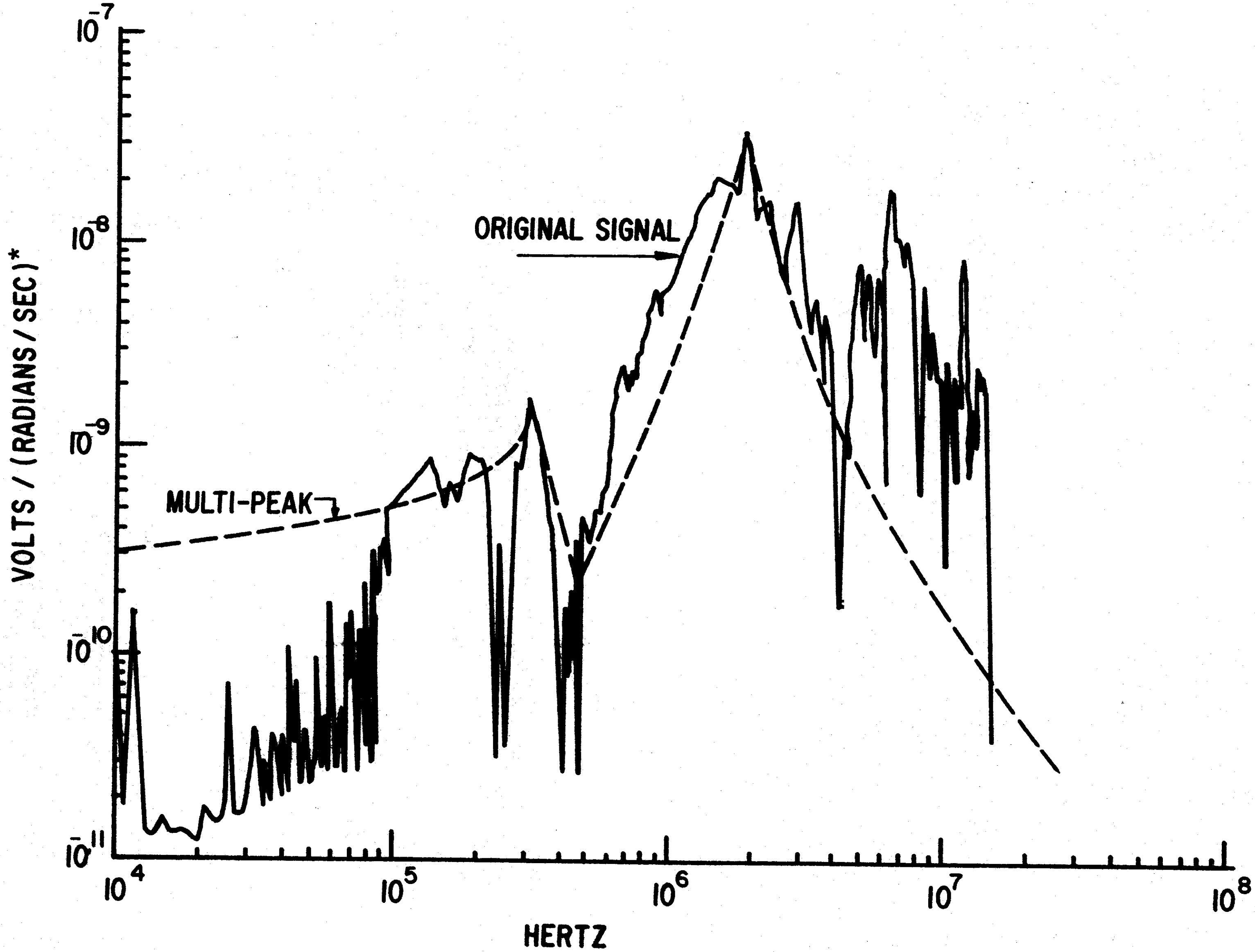


FIGURE 6. ORIGINAL SIGNAL AND MULTI-PEAK FIT OVERLAY

Inverse Transform Method	Run Number (RN)	(Time Domain)	CP Time On CDC 6600 (Seconds)	Parseval's Energy Check (% Error)
Multi-Peak Fit	45132	.62	3.5	-40.3
	45133	.85	3.5	-17.4
	45134	.63	3.5	31.9
MIT	45132	.31	300	82.8
	45133	.34	300	84.4
	45134	.15	300	88.5
TIMEOT	45132	.42	160	82.6
	45133	.31	160	84.1
	45134	.16	160	88.1

Table 1. Comparison of Inverse Transform Methods

Conclusions/Discussion

There are four important observations to be made from Table 1:

1. The Multi-Peak Fit method yields the largest predicted voltages at the circuit.

2. The computation time for the Multi-Peak Fit method is less than 1/50 of the time for the inverse Fourier subroutines, MIT or TIMEOT.

3. TIMEOT takes about 1/2 the computer time to run as MIT.

4. The Parseval's energy check is poor on the TIMEOT and MIT routines.

Before discussing the first two of these observations it is necessary to understand the details of the MPF. MPF was designed to be applied to the Fourier transform of the extrapolated signals of circuits in the MINUTEMAN system. It was known that the time domain response of these circuits had three qualities. These qualities were:

1. The response is real and finite;

2. the response is causal; and

3. previous tests had shown that the circuits responded with a damped sinusoid.

When these qualities are combined into a mathematical expansion, we get:

$$F(t) = \begin{cases} 0 & -\infty < t < 0 \\ Be^{-kt} \sin \omega t & 0 \leq t < \infty \end{cases} \qquad (2)$$

and the amplitude of the Fourier transform of this is

$$F(f) = \frac{A_o \frac{\Delta F}{F_o}}{1 - \left(\frac{F}{F_o}\right)^2 + j2\delta\left(\frac{F}{F_o}\right)} \qquad (3)$$

which appears as a peak in the frequency domain with:

f_o = the frequency where the peak of the signal occurs
= $2\pi\omega$

A_o = amplitude of peak
= $B/2\pi\Delta f$

Δf = peak width at $A_o/\sqrt{2}$
δ = $\Delta f/2f_o$

As can be seen in Figure 6, the extrapolated signal will have peaks at more than one frequency. In these cases MPF routine fits each of the peaks with its own F(f) and creates a final fit

$$G(f) = \sum_{I=1}^{N} \frac{A_I \quad \Delta F_I/F_I}{1 - \left(\frac{F}{F_I}\right)^2 + j2\delta\left(\frac{F}{F_I}\right)} \qquad (4)$$

where N is the number of peaks in the extrapolated signal. The number of peaks is chosen so that the total energy in the original signal and the energy in G(f) is nearly the same. Because we have no phase data that is considered to be valid, we assume that the time domain peaks for each of the F(f) fits arrives simultaneously and we estimate the peak of our total inverse transform of the extrapolated signal as:

$$V_p(t) = \text{peak voltage in time domain}$$
$$\leq \sum_{I=1}^{N} B_I(t)$$
$$\leq \sum_{I=1}^{N} 2\pi \Delta F A_I(f)$$

MPF then acts like a band-pass filter for N frequencies where the resulting signals are all initially in phase with each other. Thus, the predicted time domain peak $V_p(t)$ is larger than the peaks predicted by the MIT and TIMEOT routines. Finally, because the MPF does an analytic inverse transform, it will take much less computer time than the discrete Inverse Fourier transform subroutines for large amounts of data.

The fourth point of the poor energy checks indicates that either the MIT and TIMEOT routines are not working properly or that the signal being input into these subroutines is missing some information. As part of the investigation into this problem, another run was made with the TIMEOT and MIT routines. This time the subroutine described in reference 1 was used to calculate the amount of energy which is passed into negative time. The result of this run is shown in Table 2. It is most interesting that when one adds the energy in negative time to the energy in the predicted signal (for $0 \leq t < \infty$), the errors in the energy checks are all less that 5%. This leads one to suspect that the data is the cause of the error. It may be that one can reduce the energy check error by modifying the Fourier transform of the signal to make it look more causal to the subroutines.

In view of the results of this investigation into the three Inverse Fourier transforms, the following suggestions are made for any future assessment work:

1. Use the MPF to scan the large number of transfer functions measured during the tests. It appears that the MPF is a worse case approximation to the Fourier Transform of the signal and the MPF is much faster than the MIT and TIMEOT subroutines, so it could be used to scan all the data and eliminate all but a small set of interesting responses. These might be all signals with a peak amplitude which are within $\pm$20db of the circuit threshold.

2. Modify the signals to make them appear more causal. This would probably require some research into the

Fourier Inverse Transform Subroutine	Run Number	E^- Energy In Time Domain $-\infty < t > 0$	E^+ Energy In Time Domain $0 \leq t < \infty$	E^t Total Energy In Time Domain $E^- + E^+$	Energy Check $\frac{E(FREQ) - E^t}{E(FREQ)} \times 100$
MIT	45132	3.33×10^{-8}	7.09×10^{-9}	4.04×10^{-8}	1.7 %
	45133	3.45×10^{-8}	6.60×10^{-9}	4.11×10^{-8}	2.4 %
TIMEOT	45132	3.33×10^{-8}	7.16×10^{-9}	4.05×10^{-8}	1.7 %
	45133	3.45×10^{-8}	6.70×10^{-9}	4.12×10^{-8}	2.4 %

Table 2. Comparison of Energy in Time Domain Signal From MIT and TIMEOT

characteristics of the circuit response for very low (< 10^4Hz) and very high (> 5 X 10^7Hz) frequencies.

3. Use the TIMEOT routine to assess the interesting cases. Even with the poor energy checks shown in Table 1, figures 4, 5 and 6 show that the TIMEOT and MIT routines are better representations of our original signal than the MPF. When the research in suggestion 2 is completed, then these fits in figure 4 and 5 should be more accurate. Since TIMEOT was the quicker running routine it should be used for a final run on the data. The result should be a quicker assessment with more accurate predictions of circuit response.

References

1. Anderson, Stuart, Fourier Integrals of Unequally Spaced Data, D2-2690-2, The Boeing Company, January 1973, Unclassified.