

*Research Report*

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DIGITAL COMPUTER SOLUTION OF LAPLACE'S  
EQUATION INCLUDING DIELECTRIC SURFACES

Jack E. Boers, 5245

**SANDIA LABORATORIES**



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ABSTRACT

A digital computer program has been written for the solution of Laplace's Equation and the plotting of the equipotentials. The program simulates both axially symmetric and rectangular configurations, including dielectric interfaces, on a large matrix of up to 201 x 201 points. It is written in the CDC 3600-6600 Fortran IV programming language and employs the Stromberg-Carlson 4020 computer recorder for plotting. Solutions with errors of less than 1 percent are easily obtained in less than 10 minutes on the CDC 3600 and less than 2 minutes on the CDC 6600. A detailed description of the input data is presented.

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# DIGITAL COMPUTER SIMULATION OF LAPLACE'S EQUATION INCLUDING DIELECTRIC SURFACES

## Introduction

This report presents a brief description and detailed operating instructions for a digital computer program which will solve Laplace's Equation,

$$\nabla^2 V = 0, \quad (1)$$

including dielectric interfaces, and plot the equipotentials. Either axially symmetric or rectangular, two-dimensional configurations may be solved. The program is written in the Fortran IV programming language for either the CDC 3600 or CDC 6600 digital computers, and the Stromberg-Carlson 4020 computer recorder. The 3600 computer requires 1 to 10 minutes of execution time and the 6600 less than 2 minutes. Errors of less than 1 percent are generally easily attainable.

## The Simulation Method

The method used is basic relaxation of a rectangular array of points, laid out in squares. The simulation is carried out on a large matrix, typically 201 x 101 points (or 201 x 201 on the 6600 computer), all or any portion of which may be employed for a particular problem. Potentials on all electrodes and boundary points must be specified in the input data. All electrode (or dielectric) surfaces must lie on matrix points; i.e., no partial matrix increments may be employed. Laplace's Equation, expressed in difference form, is solved iteratively at all points within the electrode configuration.

A flow chart for the program is shown in Fig. 1. Initializing the matrix consists of putting linear voltage distributions between electrodes. The "coarse" matrix consists of all the odd-numbered points within the simulation region. Relaxation on the coarse matrix is much faster in terms of both the number of passes required through the matrix

and the time required per pass. The loop between the coarse and fine matrices is necessary, since the dielectric interface calculations are performed only on the fine matrix relaxation. The plots produced by the program are the most immediately useful output, while the printed output permits more accurate determination of voltages and fields. The program listing and input and output data are described in the appendices.

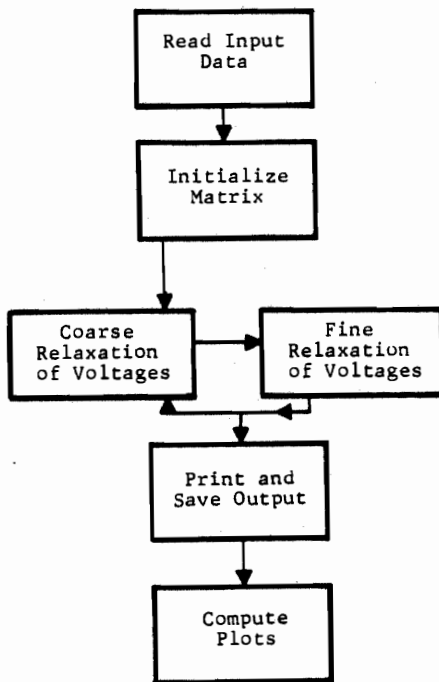


Fig. 1 Flow chart for program.

The difference equations are derived from Laplace's Equation expressed in axisymmetric coordinates:

$$\frac{\partial^2 V}{\partial z^2} + \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} = 0. \quad (2)$$

Rectangular coordinates are obtained by letting  $r$  become large. The difference equations for a uniform dielectric region are well known:

$$V_{i,j} = \frac{V_{i+1,j} + V_{i,j+1} + V_{i-1,j} + V_{i,j-1}}{4} + \frac{(V_{i,j+1} - V_{i,j-1})\Delta r}{8r_j} \quad (3)$$

for the general point, and

$$V_{i,j} = \frac{1}{6} (4 V_{i,j+1} + V_{i-1,j} + V_{i+1,j}), \quad (4)$$

for points on the axis, where an array of squares ( $\Delta z = \Delta r$ ) is employed and  $i, j, r_j$ , and  $r$  are described in Fig. 2.

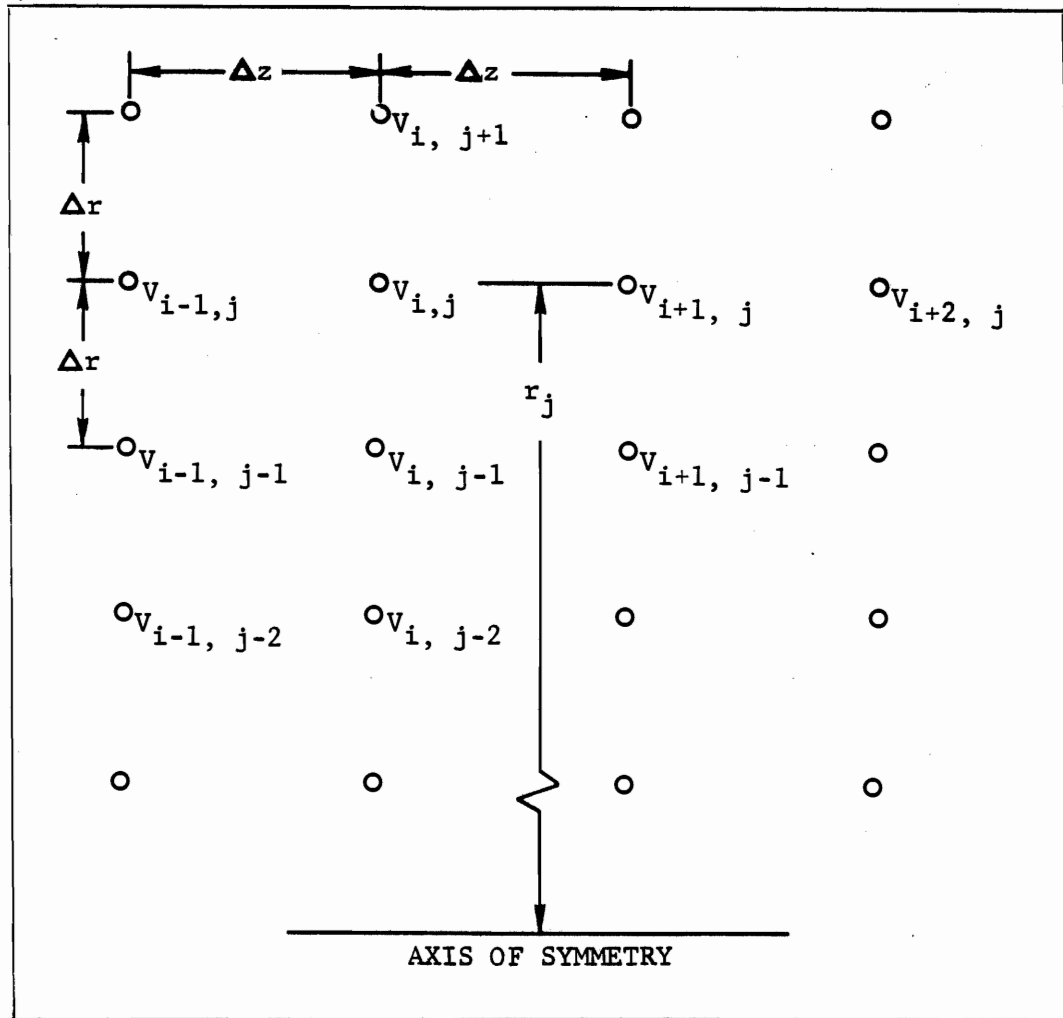


Fig. 2 Definitions of voltage matrix quantities.

Dielectric interfaces which are parallel, perpendicular, and at  $\pm 45$  degrees to the axis are handled in the following manner. Equation 2 could probably be solved exactly for a dielectric interface passing through a matrix point, but the expression would be unwieldy at best.

Solutions for dielectric interfaces parallel, perpendicular, and  $\pm 45$  degrees to the axis of symmetry are obtained fairly easily (see below) and are employed in the program.

For the case of a dielectric surface parallel to the axis of symmetry, Eq. 2 becomes in difference form

$$\begin{aligned} \epsilon_{12} \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta z)^2} + \frac{\epsilon_2(V_{i,j+1} - V_{i,j}) - \epsilon_1(V_{i,j} - V_{i,j-1})}{(\Delta r)^2} \\ + \frac{1}{2} \left( \frac{\epsilon_2(V_{i,j+1} - V_{i,j})}{\Delta r(r_j + \frac{\Delta r}{2})} + \frac{\epsilon_1(V_{i,j} - V_{i,j-1})}{\Delta r(r_j - \frac{\Delta r}{2})} \right) = 0, \end{aligned} \quad (5)$$

where  $\epsilon_2$  is the dielectric constant above (away from the axis) the point  $i,j$ ,  $\epsilon_1$  is the dielectric constant below (nearer the axis) the point  $i,j$ , and  $\epsilon_{12}$  is undefined. This can be solved for  $V_{i,j}$ , assuming a square matrix, e.g.,  $\Delta z = \Delta r$ ,

$$V_{i,j} = \frac{\epsilon_1 V_{i,j-1} + \epsilon_2 V_{i,j+1} + \epsilon_{12}(V_{i+1,j} + V_{i-1,j}) + \frac{\Delta r}{2r_j} \left( \frac{\epsilon_2 V_{i,j+1}}{1 + \frac{\Delta r}{2r_j}} - \frac{\epsilon_1 V_{i,j-1}}{1 - \frac{\Delta r}{2r_j}} \right)}{2\epsilon_{12} + \epsilon_1 + \epsilon_2 + \frac{\Delta r}{2r_j} \left( \frac{\epsilon_2}{1 + \frac{\Delta r}{2r_j}} - \frac{\epsilon_1}{1 - \frac{\Delta r}{2r_j}} \right)}. \quad (6)$$

For a dielectric surface at 45 degrees to the axis, Eq. 4 becomes

$$\begin{aligned} \frac{\epsilon_2(V_{i-1,j} - V_{i,j}) - \epsilon_1(V_{i,j} - V_{i+1,j})}{\Delta z^2} \\ + \frac{\epsilon_2(V_{i,j+1} - V_{i,j}) - \epsilon_1(V_{i,j} - V_{i,j-1})}{(\Delta r)^2} \\ + \frac{1}{2} \left( \frac{\epsilon_2(V_{i,j+1} - V_{i,j})}{\Delta r(r_j + \frac{\Delta r}{2})} + \frac{\epsilon_1(V_{i,j} - V_{i,j-1})}{\Delta r(r_j - \frac{\Delta r}{2})} \right) = 0. \end{aligned} \quad (7)$$

Again solving for  $V_{i,j}$ , one obtains

$$V_{i,j} = \frac{\epsilon_2(V_{i,j+1} + V_{i-1,j}) + \epsilon_1(V_{i+1,j} + V_{i,j-1}) + \frac{\Delta r}{2r_j} \left( \frac{\epsilon_2 V_{i,j+1}}{1 + \frac{\Delta r}{2r_j}} - \frac{\epsilon_1 V_{i,j-1}}{1 - \frac{\Delta r}{2r_j}} \right)}{2(\epsilon_1 + \epsilon_2) + \frac{\Delta r}{2r_j} \left( \frac{\epsilon_2}{1 + \frac{\Delta r}{2r_j}} - \frac{\epsilon_1}{1 - \frac{\Delta r}{2r_j}} \right)} \quad (8)$$

Similarly, for a surface at -45 degrees,

$$V_{i,j} = \frac{\epsilon_2(V_{i+1,j} + V_{i,j+1}) + \epsilon_1(V_{i-1,j} + V_{i,j-1}) + \frac{\Delta r}{2r_j} \left( \frac{\epsilon_2 V_{i,j+1}}{1 + \frac{\Delta r}{2r_j}} - \frac{\epsilon_1 V_{i,j-1}}{1 - \frac{\Delta r}{2r_j}} \right)}{2(\epsilon_1 + \epsilon_2) + \frac{\Delta r}{2r_j} \left( \frac{\epsilon_2}{1 + \frac{\Delta r}{2r_j}} - \frac{\epsilon_1}{1 - \frac{\Delta r}{2r_j}} \right)} \quad (9)$$

For surfaces perpendicular to the axis of symmetry, the difference equation becomes

$$\epsilon_{21} \left( \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta r)^2} + \frac{1}{2r_j} \frac{V_{i,j+1} - V_{i,j-1}}{\Delta r} \right) + \frac{\epsilon_2(V_{i+1,j} - V_{i,j}) - \epsilon_1(V_{i,j} - V_{i-1,j})}{(\Delta z)^2} = 0, \quad (10)$$

where  $\epsilon_1$  is the dielectric on the left and  $\epsilon_2$  is to the right. Solving this equation for  $V_{i,j}$ , one obtains

$$V_{i,j} = \frac{\epsilon_{21}(V_{i,j+1} + V_{i,j-1}) + \frac{\epsilon_{21}\Delta r}{2r_j}(V_{i,j+1} - V_{i,j-1}) + \epsilon_2 V_{i+1,j} + \epsilon_1 V_{i-1,j}}{2\epsilon_{21} + \epsilon_1 + \epsilon_2}, \quad (11)$$

In comparing Eqs. 6, 8, and 9 it can be shown that if  $\epsilon_{12} = 1/2(\epsilon_1 + \epsilon_2)$ , the denominators are the same. Also, if  $(\Delta r/2r_j)^2 \ll 1$ , then  $\epsilon_{21} = 1/2(\epsilon_1 + \epsilon_2)$  makes the denominator of Eq. 11 approximately equivalent to the others. These results have been verified on the computer. It can also be shown that if  $\epsilon_1 = \epsilon_2$  and  $(\Delta r/2r_j)^2 \ll 1$ , these equations reduce to Eq. 3.



Equations 6, 8, 9, and 11 are employed in the solution of all points along dielectric interfaces. Since only surfaces which lie exactly on matrix points are employed in the simulation, no further cases will be solved.

### Accuracy of the Program

The difference equations required for the discrete simulation on the digital computer are derived from Taylor's series expansion about zero.

$$f(x) = f(o) + xf'(o) + \frac{x^2}{2!} f''(o) + \dots + \frac{x^n}{n!} f^{(n)}(\alpha x)$$

$$0 < \alpha < 1. \quad (12)$$

Assuming a matrix increment  $h$ , Eq. 12 can be solved for the first and second derivatives as follows:

$$f'(o) = \frac{f_1 - f_{-1}}{2h} - \frac{h^2}{6} f'''(o) + O(h^4) \quad (13)$$

and

$$f''(o) = \frac{f_1 - 2f_o + f_{-1}}{h^2} - \frac{h^2}{12} f^{(iv)}(o) + O(h^4), \quad (14)$$

where  $f_1 = f(h)$ ,  $f_o = f(o)$ , and  $f_{-1} = f(-h)$ . If only the first term on the right-hand side of either equation is employed, the error is seen to be the order of  $h^2$ .

Results from the computer have indicated that the error in one-dimensional cases is indeed the order of  $h^2$ . In two dimensions (as we have here), the error seems to be proportional to  $h$ , generally one-half to one-third of  $h$ . This gives an error of less than 1 percent for most interesting cases. Results for cases where the exact solutions are known have generally been within 1/2 percent when a 50 x 50 matrix is employed.

## The Computer Program

The iterative procedure consists of applying the difference equations at appropriate points within the area to be analyzed. The procedure for relaxing the potentials on the V matrix is quite straightforward. The first subscript (usually I) begins at the left edge of the matrix and increases axially in the +Z direction. The second subscript (usually J) begins at the axis and increases radially. The relaxation operations are carried out starting at the left end of the matrix and passing up each column in sequence across the matrix.

This method of computing the potentials requires a knowledge of the locations of electrodes relative to the radial columns. This is accomplished by supplying as input data integer matrices which specify J coordinates of electrodes and dielectric surfaces. Computations are carried out from the first matrix [JA(I)] to the second matrix [JB(I)], then from the third [JC(I)] to the fourth [JD(I)], and similarly from JE(I) to JF(I) and JG(I) to JI(I), a total of eight matrices. The axis may be included in the computations by specifying appropriate values of JA(I) as zero. Any finite or infinite offset from the axis may also be specified. Dielectric surfaces parallel and at  $\pm 45$  degrees are specified by setting the upper limit of one of these matrix pairs equal to the lower limit of the next pair; e.g.,  $JB(I) = JC(I)$ , or  $JD(I) = JE(I)$ , or  $JF(I) = JG(I)$ . Surfaces perpendicular to the axis are specified separately in the input data.

A complete plot of the configuration analyzed with up to 24 equipotential surfaces can be plotted. In addition, up to 10 additional plots of small areas of the matrix may be produced on a magnified scale so that details may be examined more closely.

A second program has been written to permit the analyzing of a small area of a plot in greater detail. This program takes the data from a previous run and blows up the scale of a section of the matrix; e.g., a small area is taken out of the initial result and is subdivided to provide a larger number of points. This blown-up section may then be rerun to obtain greater accuracy.

### Input Data

Input data to the program consists of five groups of data. These control the flow, set dielectric constants, define the configuration, set voltages, and determine the plotting in the program.

Flow is controlled by integers on the first card. These integers are:

- |      |  |
|------|--|
| MM   | The number of passes through the matrix in the relaxation process, generally 20 to 50.   |
| NVEM | The number of points which may still be in error at convergence, generally 2.  |
| NZ   | The axial length of the matrix to be employed in the analysis, 201 or less for a 201 x 101 matrix.   |
| NR   | The radial extent of the matrix to be employed, 101 or less for a 201 x 101 matrix.  |
| NRO  | The radial offset of the matrix in matrix squares, zero if the axis corresponds to the bottom edge of the matrix, 9999 for rectangular configurations.   |
| MU   | Controls the saving of data at the end of the computations and restoring of data from earlier runs on magnetic tape:<br><ol style="list-style-type: none"><li>(1) Neither saves nor restores data.</li><li>(2) Saves data.</li><li>(3) Restores and saves data.</li><li>(4) Restores data but does not save.</li></ol> |

JJA,JJB,\*\*\*  
JJG,IV,  
LOG                    Controls potential setting (see below).

IRN                    The number of the data set on tape to be restored.

The second card contains the error criterion and the relative dielectric constants. EPSV is the maximum fractional change that may occur at a given matrix point from one iteration to the next. If the change is less than EPSV, a point is considered converged and no longer in error. A typical value for EPSV is 0.0005 (0.05 percent).

EP1, EP2, EP3, and EP4 are the relative dielectric constants in the regions between the matrices JA-JB, JC-JD, JE-JF, and JG-JI, respectively. If the entire area has the same dielectric constant, all of these can be set equal to 1.0.

Radial dielectric interfaces are specified on the next two cards. IDI specifies the number of such interfaces up to a maximum of four. The relative dielectric constants on each side of the four surfaces are specified by EPV1, EPV2, ..., EPV5. EPV1 is to the left and EPV2 is to the right of the first surface, EPV2 is to the left, and EPV3 is to the right of the second surface, etc. The locations of the interfaces are specified on the next card on the ID matrix. The first term of this matrix gives the column number (I) of the first interface; the second and third terms give respectively the lower and upper "J" coordinates. This sequence is repeated for the remaining three interfaces.

The electrode configuration is specified by the integer matrices described above. Some restrictions and requirements will be listed here. As described above, dielectric surfaces are specified by making the upper limit matrix values, e.g., JB(I), the same as the next lower limit matrix, e.g., JC(I). The dielectric interface must be a straight line, parallel to or at  $\pm 45$  degrees to the axis of symmetry-bottom edge of the matrix. As a direct result of this, any electrode parallel to or at  $\pm 45$  degrees to the axis must be at least one matrix square thick,

e.g.,  $JC(I) \geq JB(I) + 1$ , or it will be treated as a dielectric interface. If the axis is to be included, as the bottom edge of the plot,  $JA(I)$  should be zero in the region where the potential is to be computed on the axis.

The normal procedure in relaxing the potentials is to compute the matrix points starting at  $JA(I)$ , proceeding up the column to  $JB(I)$ , then from  $JC(I)$  to  $JD(I)$ , from  $JE(I)$  to  $JF(I)$  and  $JG(I)$  to  $JI(I)$ . The computation from  $JA(I)$  to  $JB(I)$  always occurs, but if  $JC(I)$  is zero, the computation stops there; if  $JC(I) \geq 0$  and  $JE(I) = 0$ , the computation stops at  $JD(I)$ ; and similarly, if  $JE(I) > 0$  and  $JG(I) = 0$ , the computation stops at  $JF(I)$ . The computations stop at the point where  $JC(I)$  or  $JE(I)$  or  $JG(I) = 0$ .

The potentials on the electrodes and boundary points are set by the next section of the data. Nonzero potentials must be set along the lower and upper surfaces of electrodes within the configuration. For each potential along each section of the electrode, the potential can be set by specifying the integer matrix to be used to store the potentials, the lower and upper  $I$  limits, and the potential to be stored. The number of potentials to be set are specified by  $JJA$ ,  $JJB$ ,  $JJC$ , etc., on the first card (see above). The lower and upper " $I$ " limits and the potentials to be set are specified on individual cards, in sequence, such that the potentials are set along  $JA$ ,  $JB$ ,  $JC$ , etc., in that order. There may be surfaces on which no potentials need be set, e.g., those at zero potential and dielectric interfaces; in this case some of the  $JJA$ ,  $JJB$ , etc., may be zero. The potentials within electrodes and on vertical (radial) surfaces will be set automatically, provided the potentials on the top and bottom surfaces are set correctly.

One exception to the above potential setting routines occurs in columns containing dielectric interfaces. In these columns potentials on or within electrodes must be set by the routines described below.

It may be that all the integer matrices are not needed to describe the configuration. If this is the case, the  $JJC$ ,  $JJE$ , or  $JJG$  should be set equal to  $-1$ ; e.g., if only  $JA$  and  $JB$  matrices are needed,  $JJC = -1$  will both eliminate the need for reading in the remaining matrices and eliminate the use of these matrices in any further computations.

Linearly and logarithmically varying voltages may be set along vertical (radial) lines and linearly varying voltages may be set along horizontal (axial) lines. The number of vertical and horizontal linear potentials are specified by IV and the number of logarithmically varying potentials by LOG on the first data card. The number of horizontal linear distributions is given by the first two digits of IV and the vertical distributions by the last two digits of IV; e.g., IV = 102 would indicate 1 horizontal and 2 vertical potential lines to be set.

The first of these data cards shows the vertical linear distributions. On each card are given I, the column number; JL, the lower J limit; JH, the upper J limit; VSL, the voltage at (I,JL); and VSH, the voltage at (I,JH). The second group is identical in form to the first but sets logarithmic potentials. The last group sets horizontal lines and has IL, the left "I" coordinate; IH, the right "I" coordinate; J, the row number; VSL, the voltage at (IL,J); and VSH, the voltage at (IH,J). Note that the potentials of all boundary points (with the possible exception of the axis) must be set by the input data.

The next group of cards gives the information needed for plotting. If no plot is desired, these three cards should be blank. If a plot is desired, up to 24 equipotentials may be specified. Any equipotential may be plotted except for zero. If the zero equipotential is desired, a value such as 0.0001 can be used. The values must be set consecutively; e.g., if ten equipotentials are desired, the first ten numbers must be set with the remaining 14 zero.

If it is desired to blow up any square areas of the plot, it is necessary to specify these areas on up to 10 cards giving the left and right and bottom and top limits on consecutive cards.

The data set is terminated with one blank card. The program may be terminated by 2 additional blank cards, or any number of additional data sets may be added. The last data set must be terminated with a total of 3 blank cards.

### Sample Plot

Figures 3, 4, and 5 show a configuration with three dielectrics and two electrodes. The dielectrics range from  $\epsilon_r = 1.0$  at the bottom to  $\epsilon_r = 81$  at the top. Figure 3 shows the complete configuration. Figures 4 and 5 are enlargements of the area about the  $\epsilon_r = 2.5$  dielectric.

Input data for this run are shown in Table 1 and are described in detail in Appendix A. Total execution time was less than 2 minutes on the CDC 6600 computer.

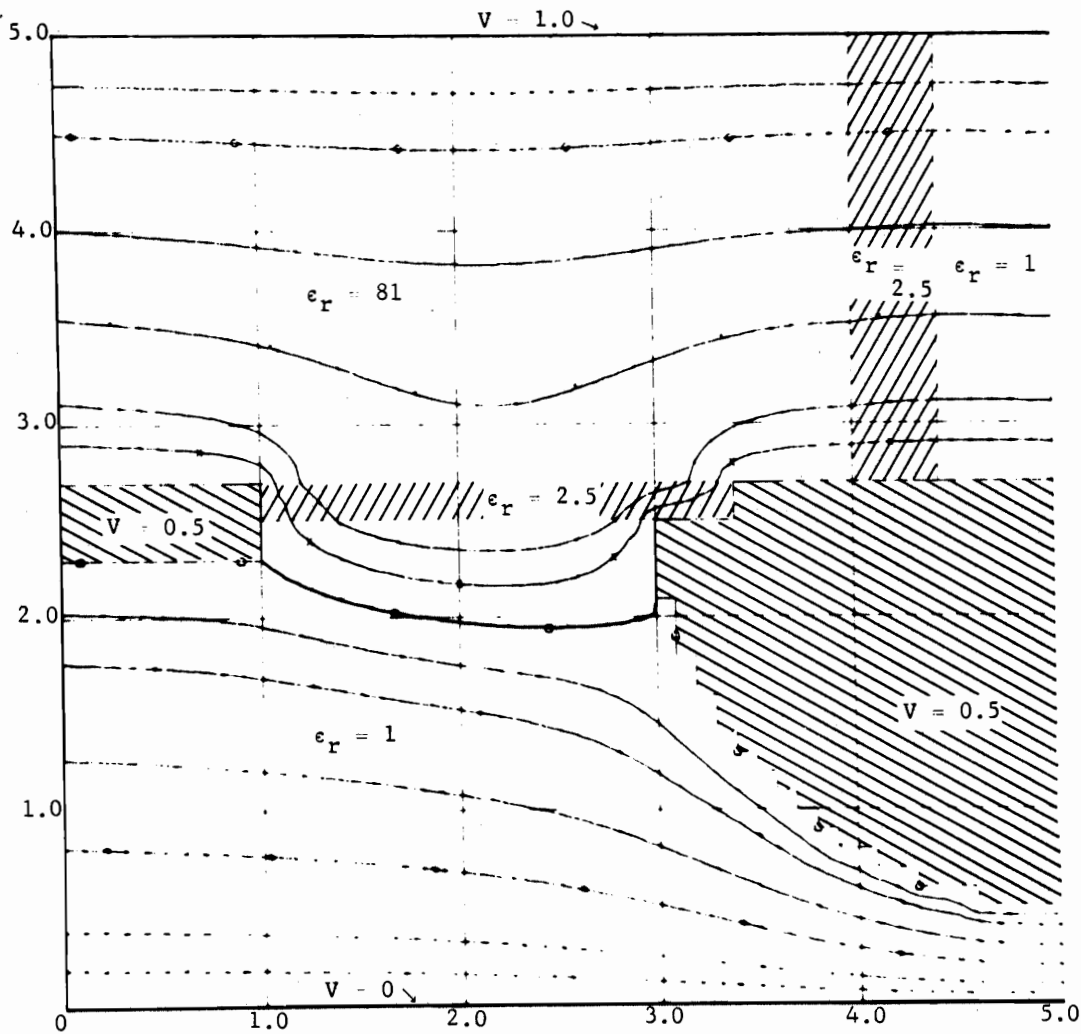


Fig. 3 Plot of multiple dielectric configuration.

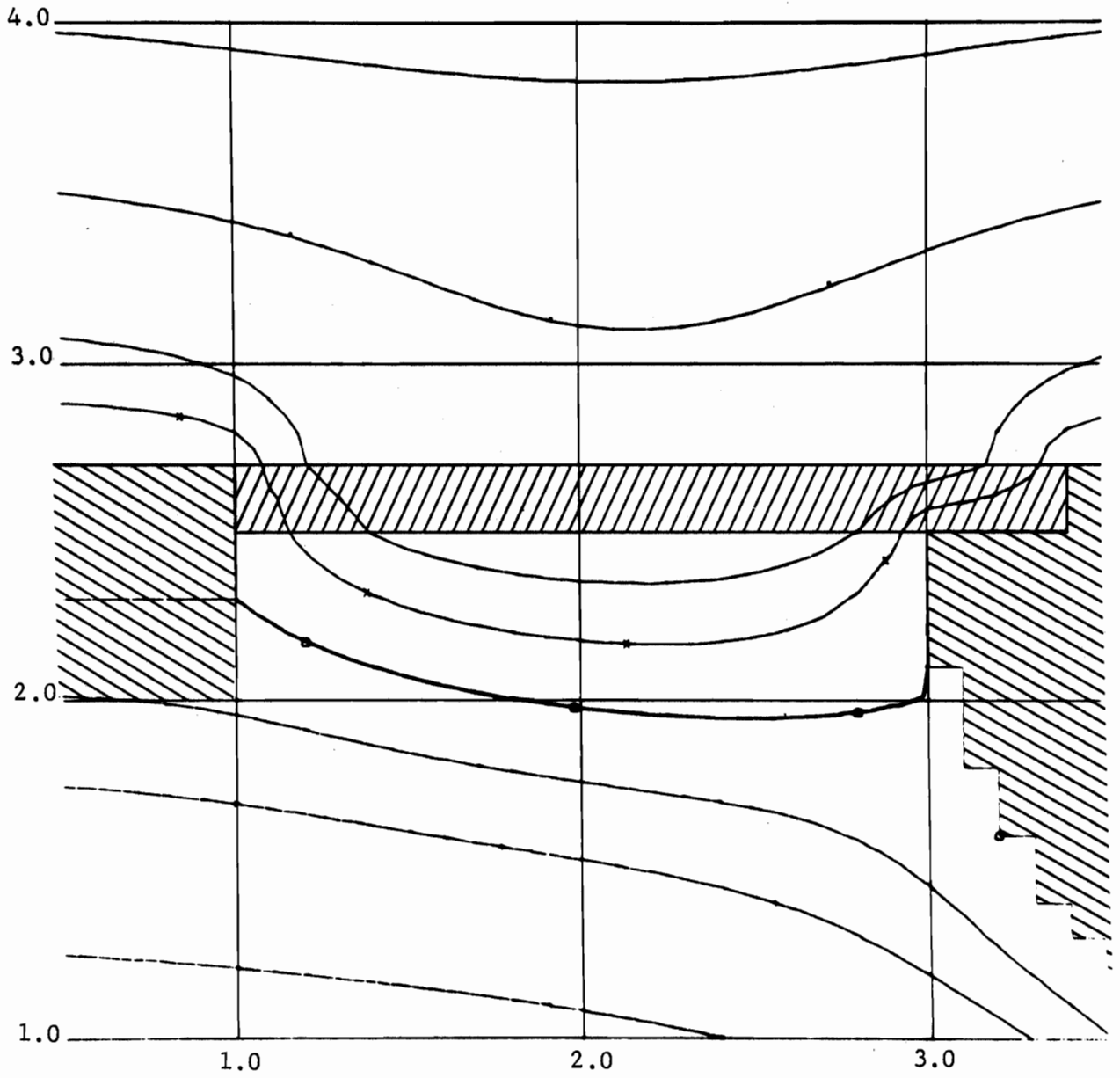


Fig. 4 Enlargement of dielectric interface area.





TABLE 1. Sample Data Set

01	60	2	51	51	51	1	0	1	2	2	1	1	-1	0	4	4	0	0
02	0.0005			1.0		2.5		81.0		81.0								
03	2		81.0			2.5		1.0										
04	41	28	51	46	28	51												
05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
06	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
07	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
08	24	24	24	24	24	24	24	24	24	24	24	26	26	26	26	26	26	26
09	26	26	26	26	26	26	26	26	26	26	22	19	17	15	14	13	12	11
10	9	8	8	7	7	7	6	6	6	6	6							
11	28	28	28	28	28	28	28	28	28	28	28	26	26	26	26	26	26	26
12	26	26	26	26	26	26	26	26	26	26	26	26	26	26	28	28	28	28
13	0	28	28	28	28	0	28	28	28	28	28							
14	51	51	51	51	51	51	51	51	51	51	51	28	28	28	28	28	28	28
15	28	28	28	28	28	28	28	28	28	28	28	28	28	28	51	51	51	51
16	0	51	51	51	51	0	51	51	51	51	51							
17												28	28	28	28	28	28	28
18	28	28	28	28	28	28	28	28	28	28	28	28	28	28				
19												0						
20	0											51	51	51	51	51	51	51
21	51	51	51	51	51	51	51	51	51	51	51	51	51	51				
22												0						
23	1	51				0.5												
24	1	11				0.5												
25	31	51				0.5												
26	1	11				1.0												
27	31	51				1.0												
28	12	34				0.5												
29	12	34				1.0												
30	41	/	41			0.5		0.5										
31	46	6	41			0.5		0.5										
32	41	42	51			1.0		1.0										
33	46	42	51			1.0		1.0										
34	1	1	24			0.0		0.5										
35	51	1	6			0.0		0.5										
36	1	28	51			0.5		1.0										
37	51	28	51			0.5		1.0										
38	0.0001			0.05		0.1		0.2		0.3		0.4		0.45		0.5		
39	0.5001			0.55		0.6		0.7		0.8		0.9		0.95		0.9999		
40																		
41	6	36	11	41														
42	6	16	21	31														
43																		
44																		
45																		

APPENDIX A

Sample Data Set

APPENDIX A  
Sample Data Set

The data set shown in Table 1 is explained in detail in this appendix. The original configuration is shown in Fig. A-1, where two cylindrical electrodes at a normalized potential of 0.5 are connected by a dielectric with a relative dielectric constant of 2.5. These cylinders are enclosed in two other cylinders held at potentials of 0 and 1.0, with  $\epsilon_r = 1.0$  and  $\epsilon_r = 81.0$ , respectively, between them and the intermediate electrodes. The axis of symmetry is at -50 on the scale shown.

The data on the first card are:

MM = 60      The maximum number of passes through the voltage matrix during each of the three relaxation cycles.

NVEM = 2      The maximum number of errors to be left in the potential relaxation at convergence.

NZ = 51      The number of axial matrix points employed.

NR = 51      The number of radial matrix points employed.

NRO = 50      The number of matrix increments between the lower edge of the drawing and the axis of symmetry.

MU = 1      No data are to be saved.

JJA = 0      No potentials are to be set along JA.

JJB = 1      One potential is to be set along JB.

JJC = 2      Two potentials are set along JC.

JJD = 2      Two potentials are set along JD.

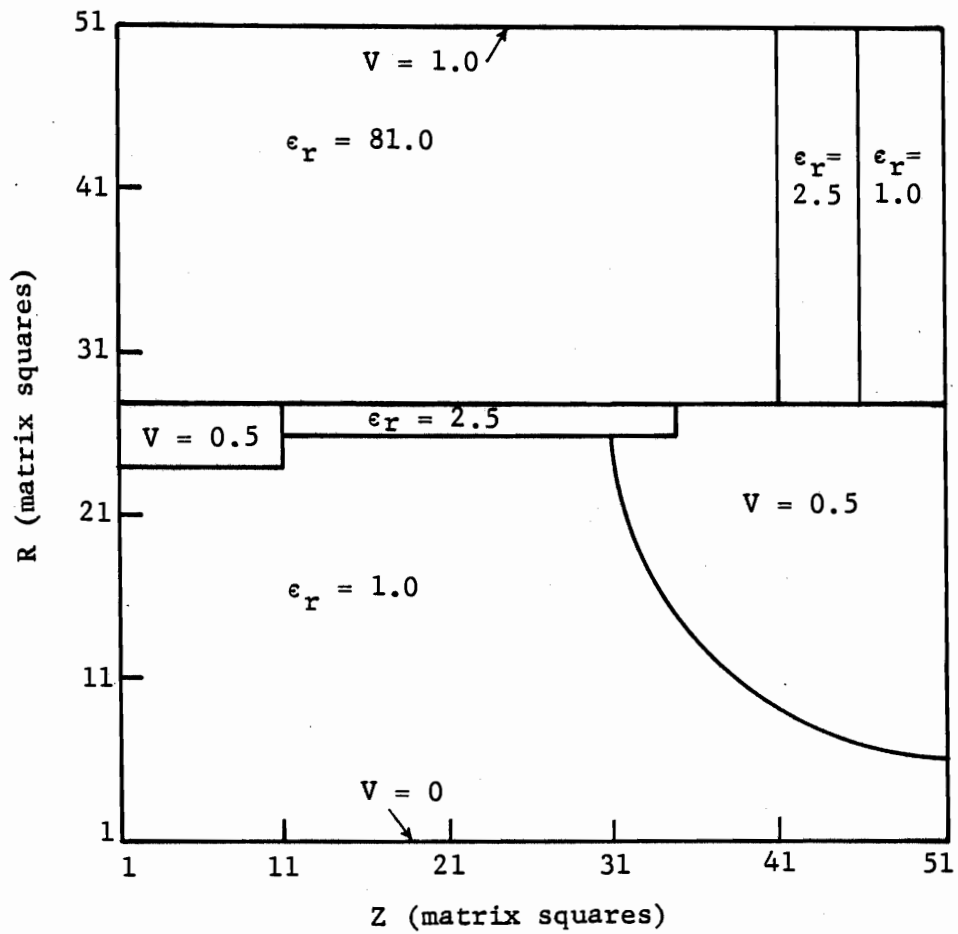


Fig. A-1 Configuration for sample data set.

JJE = 1    One potential is set along JE.

JJF = 1    One potential is set along JF.

JJG = -1    No potentials are set along JG, and the JG and JI matrices are not employed and will not be read as input data.

JJI = 0    Ignored, since JJG is negative.

IV = 004    Four linearly varying potentials are set vertically but none are set horizontally.

LOG = 4    Four logarithmically varying potential distributions are set.

IRN = 0    Ignored, since data are not restored.

The second card gives the error limit as 0.05 percent and specifies the relative dielectric constants as EP1 = 1.0, EP2 = 2.5, and EP3 as 81.0. EP4 has been set equal to 81.0 (equal to EP3) but is not employed in the relaxation process.

The next two cards specify the two vertical dielectric interfaces. IDI = 2 indicates there are two surfaces; the relative dielectric constants from the left to the right are 81.0, 2.5, and 1.0; the last two relative dielectric constants are not employed and are left blank. The second card specifies the column numbers 41 and 46 and the lower and upper limits, 28 to 51 in both cases.

The next 3 cards (numbers 5, 6, and 7) make up the JA matrix which in this case is simply the bottom edge of the matrix.

The JB matrix is specified by the cards 8 through 10. This matrix follows the lower edge of the left intermediate electrode from I = 1 to 11 and then the lower surface of the  $\epsilon_r = 2.5$  dielectric over

to the right electrode. It then follows the lower edge of the right intermediate electrode from  $I = 31$  to  $51$ . On this curved portion the matrix points which lie nearest the surface are employed.

The matrix JC is specified by cards 11, 12, and 13. These values follow the top of the left electrode through  $I = 11$ , and then follow the bottom of the dielectric from  $I = 12$  to  $30$ , corresponding exactly to JB in this region. The top of the right electrode is specified from  $I = 31$  to  $51$  on this matrix except at  $I = 41$  and  $46$ , the location of the vertical dielectric interfaces, at which points  $JC(I)$  is zero.

The matrix JD specified the outer (top) electrode for  $I \leq 11$  and  $\geq 35$ ; from  $I = 12$  to  $34$  the upper surface of the  $\epsilon_r = 2.5$  dielectric is specified. Again  $JD(I)$  is zero at  $I = 41$  and  $46$ , the radial dielectric surfaces.

The top surface of the dielectric is specified by the values of JE between  $I = 12$  and  $34$ . The remaining portions of JE are zero since they are not needed.

The remaining portion of the outer cylinder is given by the matrix JF from  $I = 12$  to  $34$ . Again, the remainder of this matrix is zero since it is not needed.

The next seven cards set potentials along surfaces defined by the matrices above. It is not necessary to set the potential along the bottom edge of the matrix since the potential matrix is initially all zero.

Card number 23 sets the potentials along the bottom edges of the intermediate electrodes and sets an initial potential along the bottom edge of the  $\epsilon_r = 2.5$  dielectric. Potentials along dielectric surfaces do not have to be set but a good initial guess will speed convergence.

Cards 24 and 25 set the potentials along the top edge of the intermediate electrodes as specified by the matrix JC. Cards 26 and 27 set potentials along the two portions of the outer electrode specified by the matrix JD.

The initial potential of the upper edge of the  $\epsilon_r = 2.5$  dielectric, as specified by JE, is set by card number 28. The potential of the section of the outer electrode given by the JF matrix is set by card number 29.

All points within electrodes will be set to the potentials on their surfaces, and all points between electrodes will be set initially by making linear distributions between them. Since all points on the boundaries of the matrix must be specified, with the possible exception of axis values if the bottom edge of the matrix is the axis, the potentials along the right and left edges must also be set.

The exception to the above potential setting routines occurs in the columns containing radial dielectric interfaces, I = 41 and 46 in this case. Potentials within the intermediate electrode and at the top electrode must be set. In this case it is also convenient to set the initial potentials on the dielectric surfaces. The four cards specified by IV are employed here as cards 30 through 33.

Since Fig. A-1 represents a portion of an axially symmetric system, it would be preferable to set the potentials along the radial edges logarithmically. This is accomplished by the 34<sup>th</sup> through 37<sup>th</sup> cards. These specify potentials in column 1 from 1 to 24 and 28 to 51, and in column 51 from 1 to 6 and 28 to 51, respectively. The number of these cards was specified above on card number 1 by LOG.

The next three cards give the values of the equipotentials to be plotted. Note that the zero potential cannot be plotted. If no plot is desired, three blank cards should be used here.

Since a plot is desired, up to ten areas may be plotted on a larger scale. In this case two areas are to be expanded. These are shown in Figs. 4 and 5 in the main body of the report. A blank card (number 43) terminates the data set.

At this point a second data set may be employed or two blank cards (numbers 44 and 45) may be inserted to terminate the program.



APPENDIX B

Data Sheets for Plotting Program

LEARS DATA

MM	4	NVEM	NZ	12	NR	16	NRO	MU	24	JJA	JJB	32	JJC	JJD	JJE	44	JJF	JJG	JJI	56	IV	LOG	IRN	72			
																										1	18I4
EPSV	9		€1	18		€2	27		€3	36		€4	45													2	8E9.2
IDI	4		€V1	13		€V1	22		€V3	31		€V4	40		€V5	49										3	I4,5E92
I(1)	JL(1)	JU(1)	12	I(2)	JL(2)	JU(2)	24	I(3)	JL(3)	JU(3)	36	I(4)	JL(4)	JU(4)	48											4	12I4
		9		18		27		36		45		54		60													
																										2013	JA
																										2013	JB

	9	18	27	30	36	45	54	60	
									2013 JC
									2013 JD
									2013 JE









APPENDIX C

Listing of the Program and Subroutines

## APPENDIX C

### Listing of the Program and Subroutines

The program consists of a main program called LEARS, which performs the relaxation of the potentials, and several subroutines which do the plotting, the saving of data, and the restoring of data.

Plotting is carried out by the subroutines EQPLOT, CDPLOT, SCAN, TRACE, CALC, and PILINE. EQPLOT is in overall charge of the plotting while SCAN, TRACE, and CALC do the equipotential computations. CDPLOT sorts the data for the various sections of the plotted output, e.g., the enlarged sections. PILINE transmits the equipotential points to the plotting subroutines.

The North American Aviation Subroutines for the Stromberg-Carlson 4020 as modified for the CDC 6600 computer at Sandia Laboratories, Albuquerque, are employed. Tape number 10 is employed as the plot output tape.

RDTAPE and WTTAPE are employed in the restoring and saving of data, respectively. The tape employed must initially have the binary number 999 written on it before these routines are used. These routines use tape number IST.

Tapes 51 through 60 are system scratch tapes and need not be user tapes.



PROGRAM LEARS (INPUT,OUTPUT,TAPE10,TAPE4,TAPE51,TAPE52,TAPE53,  
1 TAPE54,TAPE55,TAPE56,TAPE57,TAPE58,TAPE59,TAPE60)

LAPLACES EQUATION AXISYMMETRIC AND RECTANGULAR SIMULATION

INCLUDING DIFFERENT DIELECTRICS

BY J. E. BOERS ORG 5245

DIMENSION V(201,201), JA(201), JB(201), JC(201), JD(201), JE(201),  
1 JF(201),RI(201), JG(201), JI(201)  
DIMENSION RIE1(201), RIE2(201), RI1(201), RI2P(201), RI2M(201),  
1 RI3(201), RIE3(201), RI3M(201), RI4(201)  
DIMENSION VCON(24), REC(1200), Z(2000), R(2000), IPT(3,3), INX(8),  
1 INY(8)  
DIMENSION SV(100), XYM(40), IRIT(10), IZ(2000), IR(2000)

DIMENSION ID(12)

COMMON NZ,NR,IX,IY,IDX,IDY,ISS,IT,NP,N,PY,REC,CV,R,Z,IPT,INX,INY,  
1 V,XL,YL,S,U,NT,MT,IZX,DX,DY,XMAX,SCALE,YMAX,VCON,JSYM,LSYM,JT  
2 ,SV,XYM,IRIT,NPLOT,IZ,IR,MLNTH,IRB,IRT,IZL,IZR

IZX = 1  
ISTP = 399  
IST = 4  
NVR = 40401

5 READ 1000,MM,NVEM,NZ,NR,NR0,MU,JJA,JJB,JJC,JJD,JJE,JJF,JJG,JJI,IV,  
1 LOG,IRN,ISN,  
1 EPSV,EP1,EP2,EP3,EP4

IF (NZ) 2,2,3  
2 CALL EXTFLM(0)  
CALL EXIT  
3 CONTINUE

READ 1030,IDI,EPV1,EPV2,EPV3,EPV4,EPV5,(ID(I),I=1,12)

READ 1005,(JA(I),I=1,NZ)  
READ 1005,(JB(I),I=1,NZ)  
IF (JJC) 612,615,615  
612 DO 613 I=1,NZ  
JC(I) = 0  
613 JJ(I) = 0  
GO TO 630  
615 READ 1005,(JC(I),I=1,NZ)  
READ 1005,(JD(I),I=1,NZ)  
IF (JJF) 617,620,620  
617 DO 618 I=1,NZ  
JE(I) = 0  
618 JF(I) = 0  
GO TO 630  
621 READ 1005,(JE(I),I=1,NZ)  
READ 1005,(JF(I),I=1,NZ)  
IF (JJG) 622,625,625  
622 DO 623 I=1,NZ  
JG(I) = 0

```

623 JI(I) = 0
    GO TO 630
625 READ 1005,(JG(I),I=1,NZ)
    READ 1005,(JI(I),I=1,NZ)
630 CONTINUE
C
    DO 7 I=1,NVR
C
    7 V(I) = 0.0
C
    IF (MU=2) 9,9,8
C
    8 CALL RDTAPE (IST,ISTP,IRN,V,NVR)
    9 CONTINUE
C
    IF (JJA) 12,12,11
11 DO 10 K=1,JJA
    READ 1010,IL,IH,VS
    DO 10 I=IL,IH
    J = JA(I)
10 V(I,J) = VS
C
12 IF (JJB) 17,17,13
13 DO 15 K=1,JJB
    READ 1010,IL,IH,VS
    DO 15 I=IL,IH
    J = JB(I)
15 V(I,J) = VS
C
17 IF (JJC) 34,22,18
18 DO 20 K=1,JJC
    READ 1010,IL,IH,VS
    DO 20 I=IL,IH
    J = JC(I)
20 V(I,J) = VS
C
22 IF (JJD) 34,27,23
23 DO 25 K=1,JJD
    READ 1010,IL,IH,VS
    DO 25 I=IL,IH
    J = JD(I)
25 V(I,J) = VS
C
27 IF (JJE) 34,31,28
28 DO 30 K=1,JJE
    READ 1010,IL,IH,VS
    DO 30 I=IL,IH
    J = JE(I)
30 V(I,J) = VS
C
31 IF (JJF) 34,33,32
32 DO 35 K=1,JJF
    READ 1010,IL,IH,VS
    DO 35 I=IL,IH
    J = JF(I)
35 V(I,J) = VS
C
33 IF (JJG) 34,24,21
21 DO 36 K=1,JJG

```

```
    READ 1010,IL,IH,VS
    DO 36 I=IL,IH
      J = JG(I)
36  V(I,J) = VS
```

C

```
24  IF (JJI) 34,34,19
19  DO 37 K=1,JJI
    READ 1010,IL,IH,VS
    DO 37 I=IL,IH
      J = JI(I)
37  V(I,J) = VS
34  CONTINUE
```

C

C

C

#### DIELECTRIC SURFACE CONSTANTS

```
DO 40 J=2,NR
RI(J) = 0.0
IF (NR0-9999) 38,39,39
38  RI(J) = 0.125/FLOAT(NR0+J-1)
39  RIE1(J) = 2.0*(EP1+EP2)+RI(J)*(EP2/(0.25+RI(J))-EP1/(0.25-RI(J)))
    RIE2(J) = 2.0*(EP2+EP3)+RI(J)*(EP3/(0.25+RI(J))-EP2/(0.25-RI(J)))
    RIE3(J) = 2.0*(EP3+EP4)+RI(J)*(EP4/(0.25+RI(J))-EP3/(0.25-RI(J)))
    RI1(J) = -RI(J)*EP1/(0.25-RI(J))+EP1
    RI2P(J) = EP2+RI(J)*EP2/(0.25+RI(J))
    RI2M(J) = EP2-RI(J)*EP2/(0.25-RI(J))
    RI3M(J) = EP3-RI(J)*EP3/(0.25-RI(J))
    RI4(J) = EP4+RI(J)*EP4/(0.25+RI(J))
40  RI3(J) = RI(J)*EP3/(0.25+RI(J))+EP3
    E12 = (EP1+EP2)*0.5
    E23 = (EP2+EP3)*0.5
    E34 = (EP3+EP4)/2.0
    EPV12 = 0.5*(EPV1+EPV2)
    EPV23 = 0.5*(EPV2+EPV3)
    EPV34 = 0.5*(EPV3+EPV4)
    EPV45 = 0.5*(EPV4+EPV5)
    NZ1 = NZ-1
    NZ2 = NZ-2
    MN = MM/2
```

C

C

C

#### LINEAR VOLTAGES

```
LINI = IV/100
LINJ = IV-100*LINI
IF (LINJ) 47,47,43
43  DO 45 K=1,LINJ
    READ 1015,I,JL,JH,VSL,VSH
    V(I,JL) = VSL
    V(I,JH) = VSH
    JL = JL+1
    JH = JH-1
    DV = (VSH-VSL)/FLOAT(JH-JL+2)
    DO 45 J=JL,JH
45  V(I,J) = V(I,J-1)+DV
```

C

C

C

#### LOG VOLTAGES

```
47  IF (LOG) 51,51,48
48  DO 50 K=1,LOG
```

```

READ 1015 ,I,JL,JH,VSL,VSH
V(I,JL) = VSL
V(I,JH) = VSH
JL = JL+1
JH = JH-1
RO = JL-2+NR0
RT = JH+NR0
DV = (VSH-VSL)/ALOG(RT/RO)
DO 50 J=JL,JH
50 V(I,J) = VSL+DV*ALOG(FLOAT(J-1+NR0)/RO)

```

C  
C  
C

#### HORIZONTAL LINEAR

```

51 IF (LINI) 54,54,53
53 DO 52 K=1,LINI
READ 1015,IL,IH,J,VSL,VSH
V(IL,J) = VSL
V(IH,J) = VSH
IL = IL+1
IH = IH-1
DV = (VSH-VSL)/FLOAT(IH-IL+2)
DO 52 I=IL,IH
52 V(I,J) = V(I-1,J)+DV
54 CONTINUE

```

C  
C  
C

#### INITIAL POTENTIALS

```

IF (MU-2) 55,55,795
55 DO 120 I=2,NZ1
JL = MAX0(2,JA(I)+1)
JH = JB(I)-1
VD = (V(I,JH+1)-V(I,JL-1))/FLOAT(JH-JL+2)
DO 72 J=JL,JH
72 V(I,J) = V(I,J-1)+VD
IF (JJC) 120,75,75
75 IF (JC(I)) 120,120,80
80 JL = JC(I)+1
85 JH = JD(I)-1
95 DV = (V(I,JH+1)-V(I,JL-1))/FLOAT(JH-JL+2)
DO 100 J=JL,JH
100 V(I,J) = V(I,J-1)+DV
IF (JJE) 120,105,105
105 IF (JE(I)) 120,120,110
110 JL = JE(I)+1
JH = JF(I)-1
DV = (V(I,JH+1)-V(I,JL-1))/FLOAT(JH-JL+2)
DO 115 J=JL,JH
115 V(I,J) = V(I,J-1)+DV
IF (JJG) 120,116,116
116 IF (JG(I)) 120,120,117
117 JL = JG(I)+1
JH = JI(I)-1
DV = (V(I,JH+1)-V(I,JL-1))/FLOAT(JH-JL+2)
DO 118 J=JL,JH
118 V(I,J) = V(I,J-1)+DV
120 CONTINUE

```

C  
C

#### ELECTRODE POTENTIALS

C

```
DO 780 I=1,NZ
IF (IDI) 985,985,965
965 IF (I-ID(1)) 970,780,970
970 IF (I-ID(4)) 975,780,975
975 IF (I-ID(7)) 980,780,980
980 IF (I-ID(10)) 985,780,985
985 CONTINUE
IF (JA(I)=1) 695,695,680
680 JL = 1
JH = JA(I)-1
VS = V(I,JH+1)
DO 690 J=JL,JH
IF (V(I,J)) 690,685,690
685 V(I,J) = VS
690 CONTINUE
695 CONTINUE
IF (JC(I)) 700,700,705
700 JH = NR
GOTO 710
705 JH = JC(I)-1
710 JL = JB(I)+1
VS = V(I,JL-1)
DO 715 J=JL,JH
IF (V(I,J)) 715,714,715
714 V(I,J) = VS
715 CONTINUE
IF (JC(I)) 780,780,720
720 IF (JE(I)) 725,725,730
725 JH = NR
GO TO 735
730 JH = JE(I)-1
735 JL = JD(I)+1
VS = V(I,JL-1)
DO 740 J=JL,JH
IF (V(I,J)) 740,739,740
739 V(I,J) = VS
740 CONTINUE
IF (JE(I)) 780,780,745
745 IF (JG(I)) 750,750,755
750 JH = NR
GO TO 760
755 JH = JG(I)-1
760 JL = JF(I)+1
VS = V(I,JL-1)
DO 765 J=JL,JH
IF (V(I,J)) 765,764,765
764 V(I,J) = VS
765 CONTINUE
IF (JG(I)) 780,780,770
770 JH = NR
JL = JI(I)+1
VS = V(I,JL-1)
DO 775 J=JL,JH
IF (V(I,J)) 775,774,775
774 V(I,J) = VS
775 CONTINUE
780 CONTINUE
```



```

205 CONTINUE
C
  IF (JJG) 209,207,207
207 DO 206 I=3,NZ2,2
  IF (JG(I)) 206,206,191
191 JL = 3+(JG(I)/2)*2
  JH = JI(I)-2
  IF (JL-JH) 206,193,193
193 DO 201 J=JL,JH,2
  VNU = (V(I+2,J)+V(I-2,J)+V(I,J+2)+V(I,J-2))*0.25+RI(J)*(V(I,J+2)-
  1 V(I,J-2))
  IF (ABS(V(I,J)/VNU-1.0)=EPSV) 201,201,196
196 NE = NE+1
201 V(I,J) = VNU
206 CONTINUE
C
209 K = K+1
  IF (NE=NVEM) 215,215,210
210 IF (K=MM) 125,215,215
215 PRINT 1020,NE
  DO 325 I=3,NZ2,2
  JL = 2+(JA(I)/2)*2
  JU = JB(I)-1
  DO 300 J=JL,JU,2
300 V(I,J) = (V(I,J+1)+V(I,J-1))*0.5
  IF (JC(I)) 325,325,305
305 JL = 2+(JC(I)/2)*2
  JU = JD(I)-1
  DO 310 J=JL,JU,2
310 V(I,J) = (V(I,J+1)+V(I,J-1))*0.5
  IF (JE(I)) 325,325,315
315 JL = 2+(JE(I)/2)*2
  JU = JF(I)-1
  DO 320 J=JL,JU,2
320 V(I,J) = (V(I,J+1)+V(I,J-1))*0.5
C
  IF (JG(I)) 325,325,316
316 JL = 2+(JG(I)/2)*2
  JU = JF(I)-1
  DO 321 J=JL,JU,2
321 V(I,J) = (V(I,J+1)+V(I,J-1))*0.5
C
325 CONTINUE
  DO 355 I=2,NZ2,2
  JL = 1+JA(I)
  JU = JB(I)-1
  DO 330 J=JL,JU
330 V(I,J) = (V(I+1,J)+V(I-1,J))*0.5
  IF (JC(I)) 355,355,335
335 JL = JC(I)+1
  JU = JD(I)-1
  DO 340 J=JL,JU
340 V(I,J) = (V(I+1,J)+V(I-1,J))*0.5
  IF (JE(I)) 355,355,345
345 JL = JE(I)+1
  JU = JF(I)-1
  DO 350 J=JL,JU
350 V(I,J) = (V(I+1,J)+V(I-1,J))*0.5

```

```

      IF (JG(I)) 355,355,346
346 JL = JG(I)+1
      JU = JI(I)-1
      DO 351 J =JL,JU
351 V(I,J) = (V(I+1,J)+V(I-1,J))*0.5
355 CONTINUE
C
      K = 0
370 NE = 0
C
      DO 460 I=2,NZ1
      JL = JA(I)+1
      JU = JB(I)-1
      DO 420 J=JL,JU
      IF (J-1) 400,400,405
400 VNU = (V(I+1,J)+V(I-1,J)+4.0*V(I,J+1))/6.0
      GO TO 410
405 VNU = (V(I+1,J)+V(I-1,J)+V(I,J+1)+V(I,J-1))*0.25+RI(J)*(V(I,J+1)-
1 V(I,J-1))
410 IF (ABS(V(I,J)/VNU-1.0)-EPSV) 420,420,415
415 NE = NE+1
420 V(I,J) = VNU
C
      IF (JJC) 460,425,425
425 IF (JC(I)) 460,460,430
430 JL = JC(I)+1
      JU = JD(I)-1
      DO 440 J=JL,JU
      VNU = (V(I+1,J)+V(I-1,J)+V(I,J+1)+V(I,J-1))*0.25+RI(J)*(V(I,J+1)-
1 V(I,J-1))
      IF (ABS(V(I,J)/VNU-1.0)-EPSV) 440,440,435
435 NE = NE+1
440 V(I,J) = VNU
C
      IF (JJE) 460,443,443
443 IF (JE(I)) 460,460,445
445 JL = JE(I)+1
      JU = JF(I)-1
      DO 455 J=JL,JU
      VNU = (V(I+1,J)+V(I-1,J)+V(I,J+1)+V(I,J-1))*0.25+RI(J)*(V(I,J+1)-
1 V(I,J-1))
      IF (ABS(V(I,J)/VNU-1.0)-EPSV) 455,455,450
450 NE = NE+1
455 V(I,J) = VNU
C
      IF (JJG) 460,442,442
442 IF (JG(I)) 460,460,446
446 JL = JG(I)+1
      JU = JI(I)-1
      DO 456 J=JL,JU
      VNU = (V(I+1,J)+V(I-1,J)+V(I,J+1)+V(I,J-1))*0.25+RI(J)*(V(I,J+1)-
1 V(I,J-1))
      IF (ABS(V(I,J)/VNU-1.0)-EPSV) 456,456,451
451 NE = NE+1
456 V(I,J) = VNU
C
460 CONTINUE
C

```



```

DO 586 I=2,NZ1
IF (JJC) 589,495,495
495 IF (JB(I)-JC(I)) 535,500,535
500 J = JB(I)
IF (IABS(J-JB(I-1))-1) 520,510,505
505 IF (IABS(J-JB(I+1))-1) 520,515,535
510 IF (J-JB(I-1)) 530,520,525
515 IF (J-JB(I+1)) 525,520,530
520 V(I,J) = (RI1(J)*V(I,J-1)+RI2P(J)*V(I,J+1)+E12*(V(I+1,J)+V(I-1,J))
1)/RIE1(J)
GO TO 535
525 V(I,J) = (RI2P(J)*V(I,J+1)+RI1(J)*V(I,J-1)+EP1*V(I+1,J)+EP2*V(I-1,
1 J))/RIE1(J)
GO TO 535
530 V(I,J) = (RI2P(J)*V(I,J+1)+RI1(J)*V(I,J-1)+EP1*V(I-1,J)+EP2*V(I+1,
1 J))/RIE1(J)
535 CONTINUE

```

C

```

IF (JJE) 586,540,540
540 IF (JE(I)) 586,586,545
545 IF (JD(I)-JE(I)) 585,550,585
550 J = JD(I)
IF (IABS(J-JD(I-1))-1) 570,560,555
555 IF (IABS(J-JD(I+1))-1) 570,565,585
560 IF (J-JD(I-1)) 580,570,575
565 IF (J-JD(I+1)) 575,570,580
570 V(I,J) = (RI3(J)*V(I,J+1)+RI2M(J)*V(I,J-1)+E23*(V(I+1,J)+V(I-1,J))
1)/RIE2(J)
GO TO 585
575 V(I,J) = (RI3(J)*V(I,J+1)+RI2M(J)*V(I,J-1)+EP2*V(I+1,J)+EP3*V(I-1,
1 J))/RIE2(J)
GO TO 585
580 V(I,J) = (RI3(J)*V(I,J+1)+RI2M(J)*V(I,J-1)+EP2*V(I-1,J)+EP3*V(I+1,
1 J))/RIE2(J)
585 CONTINUE

```

C

```

IF (JJG) 586,541,541
541 IF (JG(I)) 586,586,546
546 IF (JF(I)-JE(I)) 586,551,586
551 J = JF(I)
IF (IABS(J-JF(I-1))-1) 571,561,556
556 IF (IABS(J-JF(I+1))-1) 571,566,586
561 IF (J-JF(I-1)) 581,571,576
566 IF (J-JF(I+1)) 576,571,581
571 V(I,J) = (RI4(J)*V(I,J+1)+RI3M(J)*V(I,J-1)+E34*(V(I+1,J)+V(I-1,J))
1)/RIE3(J)
GO TO 586
576 V(I,J) = (RI4(J)*V(I,J+1)+RI3M(J)*V(I,J-1)+EP3*V(I+1,J)+EP4*V(I-1,
1 J))/RIE3(J)
GO TO 586
581 V(I,J) = (RI4(J)*V(I,J+1)+RI3M(J)*V(I,J-1)+EP3*V(I-1,J)+EP4*V(I+1,
1 J))/RIE3(J)

```

C

586 CONTINUE

C

589 K = K+1

C

C

RADIAL DIELECTRICS

```

C      IF (IDI)960,960,915
C
915 M = 1
    I = ID(M)
    JL = ID(M+1)+1
    JH = ID(M+2)-1
    DO 920 J=JL,JH
920 V(I,J) = (EPV12*((1.0+RI(J))*V(I,J+1)+(1.0-RI(J))*V(I,J-1))+EPV2*
1 V(I+1,J)+EPV1*V(I-1,J))/(4.0*EPV12)
    IF (IDI-1) 960,960,925
925 M = 4
    I = ID(M)
    JL = ID(M+1)+1
    JH = ID(M+2)-1
    DO 930 J=JL,JH
930 V(I,J) = (EPV23*((1.0+RI(J))*V(I,J+1)+(1.0-RI(J))*V(I,J-1))+EPV3*
1 V(I+1,J)+EPV2*V(I-1,J))/(4.0*EPV23)
    IF (IDI-2) 960,960,935
935 M = 7
    I = ID(M)
    JL = ID(M+1)+1
    JH = ID(M+2)-1
    DO 940 J=JL,JH
940 V(I,J) = (EPV34*((1.0+RI(J))*V(I,J+1)+(1.0-RI(J))*V(I,J-1))+EPV4*
1 V(I+1,J)+EPV3*V(I-1,J))/(4.0*EPV34)
    IF (IDI-3) 960,960,945
945 M = 10
    I = ID(M)
    JL = ID(M+1)+1
    JH = ID(M+2)-1
    DO 950 J=JL,JH
950 V(I,J) = (EPV45*((1.0+RI(J))*V(I,J+1)+(1.0-RI(J))*V(I,J-1))+EPV5*
1 V(I+1,J)+EPV4*V(I-1,J))/(4.0*EPV45)
C
960 CONTINUE
C
    IF (K-MN) 590,595,595
590 IF (NE-NVEM) 595,595,370
C
595 PRINT 1020,NE
    IF (L-3) 122,600,600
600 CONTINUE
C
    IF (MU-4) 900,910,910
900 IF (MU-2) 910,905,905
C
905 CALL WTTAPE (IST,ISTP,IRN,V,NVR)
910 CONTINUE
C
    DO 615 J=1,NR
605 PRINT 1050,J,(V(I,J),I=1,NZ)
C
    READ 1025,VCON
    PRINT 1025,VCON
    IF (VCON(1)) 610,5,610
610 CALL EQPLOT
C

```

6 0 TO 5

C  
C  
C

FORMATS

1000 FORMAT (18I4/8E9.2)  
1005 FORMAT (20I3)  
1010 FORMAT (2I4,E9.2)  
1015 FORMAT (3I4,2E9.2)  
1020 FORMAT (/I10)  
1025 FORMAT (8E9.2)  
1030 FORMAT (14,5E9.2/12I4)  
1050 FORMAT (15/(10E12.5))  
END

```

SUBROUTINE EQPLOT
COMMON NZ, NR, IX, IY, IDX, IDY, ISS, IT, NP, N, PY, REC, CV, R, Z, IPT, INX, INY,
1 V, XL, YL, S, U, NT, MT, IZX, DX, DY, XMAX, SCALE, YMAX, VCON, JSYM, LSYM, JT
2 , SV, XYM, IRIT, NPLOT, IZ, IR, MLNTH, IRB, IRT, IZL, IZR
DIMENSION V(201,201), REC(1200), Z(2000), R(2000), IPT(3,3), INX(8),
1 INY(8), VCON(24)
DIMENSION SV(100), XYM(40), IRIT(10), IZ(2000), IR(2000)
DIMENSION ISYMT(12)
C
IF (IZX) 5,6,5
5 ISYMT(1) = 42
  ISYMT(2) = 44
  ISYMT(3) = 16
  ISYMT(4) = 63
  ISYMT(5) = 55
  ISYMT(6) = 58
  ISYMT(7) = 38
  ISYMT(8) = 42
  ISYMT(9) = 44
  ISYMT(10) = 16
  ISYMT(11) = 63
  ISYMT(12) = 55
  CALL ENTFLM(10)
C
6 CONTINUE
C
RNR = 0.1*FLOAT(NR-1)
K = 0
C
CALL SMXYV(0,0)
CALL GRIDIV (1,0.0,RNR,0.0,RNR,1.0,1.0,-0,-0,1,1,2,2)
C
IRB = NYV(0.0)
IRT = NYV(RNR)
IZL = NXV (0.0)
IZR = NXV (RNR)
IF (NZ-NR) 25,25,10
10 XYM(1) = RNR
  XYM(2) = RNR+RNR
  XYM(3) = 0.0
  XYM(4) = RNR
  K = K+1
  IF (NZ-NR=NR) 25,25,20
20 XYM(5) = RNR+RNR
  XYM(6) = 3.0*RNR
  XYM(7) = 0.0
  XYM(8) = RNR
  K = K+1
25 READ 1000,IL,IW,JB,JT
1000 FORMAT (4I4)
  IF (IL) 35,35,30
30 XYM(4*K+1) = FLOAT (IL-1)*0.1
  XYM(4*K+2) = FLOAT (IW-1)*0.1
  XYM(4*K+3) = FLOAT (JB-1)*0.1
  XYM(4*K+4) = FLOAT (JT-1)*0.1
  K = K+1
  GO TO 25

```

```

35 CONTINUE
C
  NPLOT = K
  DO 36 I=1,NPLOT
    ITP = 50*I
    REWIND ITP
  36 IRIT(I) = 0
C
    II = 1
    DO 115 L=1,24
      IF (VCON(L)) 120,120,100
100  CV = VCON(L)
      GO TO (105,110),II
105  JSYM = 0
      LSYM = 0
      II = 2
      GO TO 115
110  JSYM = 25
      LL = L/2
      LSYM = ISYMT(LL)
      II = 1
115  CALL SCAN
120  CONTINUE
C
  K = 0
  IF (NPLOT) 60,60,40
40  DO 55 I=1,NPLOT
    CALL HOLDIV(1)
    CALL GRIDIV (1,XYM(4*K+1),XYM(4*K+2),XYM(4*K+3), XYM(4*K+4),1.0,1.
1    ,-0,-0,1,1,2,2)
    K = K+1
    IMAX = IRIT(I)
    ITP = I*50
    REWIND ITP
    DO 50 KK=1,IMAX
      READ TAPE ITP,KI,JSYM,LSYM,(IZ(J),IR(J),J=1,KI)
50  CALL PILINE (IZ,IR,KI,JSYM,LSYM)
55  CONTINUE
C
60  RETURN
    END

```

```

SUBROUTINE COPLOT
COMMON NZ,NR,IX,IY,IDX,IDY,ISS,IT,NP,N,PY,REC,CV,R,Z,IPT,INX,INY,
1 V,AL,YL,S,U,NT,MT,IZX,DX,DY,XMAX,SCALE,YMAX,VCON,JSYM,LSYM,JT
2 ,SV,XYM,IRIT,NPLOT,IZ,IR,MLNTH,IRB,IRT,IZL,IZR
C
DIMENSION V(201,201), REC(1200), Z(2000), R(2000),IPT(3,3),INX(8),
1 INY(8), VCON(24)
DIMENSION SV(100), XYM(40), IRIT(10), IZ(2000), IR(2000)
K = 0
DO 20 I=1,N
K = K+1
IZ(K) = MAX(Z(I))
IF (IZ(K)) 10,10,15
10 K = K-1
GO TO 20
15 IR(K) = NYV(R(I))
20 CONTINUE
C
IF (K) 27,27,23
C
23 CALL PILINE (IZ,IR,K,JSYM,LSYM)
C
27 IF (NPLOT) 55,55,28
28 DO 50 KK=1,NPLOT
CALL XSCALV(XYM(4*KK-3),XYM(4*KK-2),IZL,1023-IZR)
CALL YSCALV(XYM(4*KK-1),XYM(4*KK),IRB,1023-IRT)
K = 0
DO 40 I=1,N
K = K+1
IZ(K) = MAX(Z(I))
IF (IZ(K)) 30,30,35
30 K = K-1
GO TO 40
35 IR(K) = NYV(R(I))
IF (IR(K)) 37,37,40
37 K = K-1
40 CONTINUE
IF (K) 50,50,45
45 IRIT(KK) = IRIT(KK)+1
ITP = KK+50
WRITE TAPE ITP,K,JSYM,LSYM,(IZ(I),IR(I),I=1,K)
50 CONTINUE
RNR = 0.1*FLOAT(NR-1)
CALL XSCALV (0.0,RNR,IZL,1023-IZR)
CALL YSCALV (0.0,RNR,IRB,1023-IRT)
55 RETURN
END

```

SUBROUTINE SCAN

C

```
COMMON NZ, NR, IX, IY, IDX, IDY, ISS, IT, NP, N, PY, REC, CV, R, Z, IPT, INX, INY,  
1 V, XL, YL, S, U, NT, MT, IZX, DX, DY, XMAX, SCALE, YMAX, VCON, JSYM, LSYM, JT  
2 ,SV, XYM, IRIT, NPLOT, IZ, IR, MLNTH, IRB, IRT, IZL, IZR
```

C

```
DIMENSION V(201,201), REC(1200), Z(2000), R(2000), IPT(3,3), INX(8),  
1 INY(8), VCON(24)  
DIMENSION SV(100), XYM(40), IRIT(10), IZ(2000), IR(2000)
```

C

```
MLNTH = 201  
NREC = 1200  
NP = 0  
NT = NR  
MT = NZ  
IF (IZX) 10,10,5  
5 IPT(1,1) = 8  
IPT(1,2) = 1  
IPT(1,3) = 2  
IPT(2,1) = 7  
IPT(2,3) = 3  
IPT(3,1) = 6  
IPT(3,2) = 5  
IPT(3,3) = 4  
INX(1) = -1  
INX(2) = -1  
INX(3) = 0  
INX(4) = 1  
INX(5) = 1  
INX(6) = 1  
INX(7) = 0  
INX(8) = -1  
INY(1) = 0  
INY(2) = 1  
INY(3) = 1  
INY(4) = 1  
INY(5) = 0  
INY(6) = -1  
INY(7) = -1  
INY(8) = -1  
IZX = 0  
10 CONTINUE  
DO 15 J=1, NREC  
15 REC(J) = 0.0  
ISS = 0
```

C

```
20 MT1 = MT-1  
DO 40 I=1, MT1  
IF (V(I,1)-CV) 25,40,40  
25 IF (V(I+1,1)-CV) 40,30,30  
30 IF (V(I+1,1)-V(I,1)-1.0) 35,40,35  
35 IX = I+1  
IY = 1  
IDX = -1  
IDY = 0
```

C

```
CALL TRACE
```

```

C
40 CONTINUE
C
  NT1 = NT-1
  DO 60 I=1,NT1
    IF (V(MT,I)-CV) 45,60,60
45  IF (V(MT,I+1)-CV) 60,50,50
50  IF (V(MT,I+1)-V(MT,I)-1.0) 55,60,55
55  IX = MT
    IY = I+1
    IDX = 0
    IDY = -1
    CALL TRACE
60  CONTINUE
C
65  DO 85 I=1,MT1
    MT2 = MT-I+1
    IF (V(MT2,NT)-CV) 70,85,85
70  IF (V(MT2-1,NT)-CV) 85,75,75
75  IF (V(MT2-1,NT)-V(MT2,NT)-1.0) 80,85,80
80  IX = MT2-1
    IY = NT
    IDX = 1
    IDY = 0
    CALL TRACE
C
85  CONTINUE
C
  DO 105 I=1,NT1
    NT2 = NT+1-I
    IF (V(1,NT2)-CV) 90,105,105
90  IF (V(1,NT2-1)-CV) 105,95,95
95  IF (V(1,NT2-1)-V(1,NT2)-1.0) 100,105,100
100 IX = 1
    IY = NT2-1
    IDX = 0
    IDY = 1
C
    CALL TRACE
C
105 CONTINUE
    ISS = 1
    NT1 = NT-1
    MT1 = MT-1
    DO 140 J=2,NT1
    DO 140 I=1,MT1
    IF (V(I,J)-CV) 110,140,140
110 IF (V(I+1,J)-CV) 140,115,115
115 IF (V(I+1,J)-V(I,J)-1.0) 120,140,120
120 COM = MLNTH*(I+1)+J
    IF (NP) 125,135,125
125 DO 130 ID = 1,NP
    IF (REC(ID)-COM) 130,140,130
130 CONTINUE
135 IX = I+1
    IY = J
    IDX = -1
    IDY = 0

```



C CALL TRACE  
140 CONTINUE  
RETURN  
END

SUBROUTINE TRACE

```

C
COMMON NZ,NR,IX,IY,IDX,IDY,ISS,IT,NP,N,PY,REC,CV,R,Z,IPT,INX,INY,
1 V,XL,YL,S,U,NT,MT,IZX,DX,DY,XMAX,SCALE,YMAX,VCON,JSYM,LSYM,JT
C
2 SV,XYM,IRIT,NPLOT,IZ,IR,MLNTH,IRB,IRT,IZL,IZR
C
DIMENSION V(201,201), REC(1200), Z(2000), R(2000),IPT(3,3),INX(3),
1 INY(8), VCON(24)
DIMENSION SV(100), XYM(40), IRIT(10), IZ(2000), IR(2000)
C
PY = 0.0
5 JT = 0
N = 0
IX0 = IX
IY0 = IY
ISX = IDX+2
ISY = IDY+2
IS = IPT(ISX,ISY)
JTB = 0
ISO = IS
IF (ISO=8) 15,15,10
10 ISO = ISO-8
15 IT = 0
C
20 CALL CALC
C
N = N
C
IF (IT+JT=1) 30,30,25
25 XS = Z(N-1)
YS = R(N-1)
Z(N-1) = Z(N)
R(N-1) = R(N)
Z(N) = XS
R(N) = YS
30 IS = IS+1
JT = IT
35 IF (IS=9) 45,40,40
40 IS = IS-8
45 IDX = INX(IS)
IDY = INY(IS)
IX2 = IX+IDX
IY2 = IY+IDY
JTB = JTB+1
IF (JTB=2000) 60,60,50
C
50 PRINT 1000,CV,Z(N),R(N)
C
1000 FORMAT(1H0,23HA CONTOUR LINE AT LEVEL,F6.3,21H WAS TERMINATED AT Z
1=E12.5,3H R=,F7.3,47HBECAUSE IT CONTAINED MORE THAN 2000 PLOT POIN
2TS)
C
DO 55 K=1,N
R(K) = (R(K)-1.0)*0.1
55 Z(K) = (Z(K)-1.0)*0.1
C
CALL CDPLOT

```

```

C      RETURN
60 IF (ISS) 85,85,65
65 IF (IX-IX0) 105,70,105
70 IF (IY-IY0) 105,75,105
75 IF (IS-IS0) 105,80,105
C
80 CALL CALC
C
      GO TO 160
85 IF (IX2) 90,150,90
90 IF (IX2=MT) 95,95,150
95 IF (IY2) 100,150,100
100 IF (IY2=NT) 105,105,150
105 IF (CV=V(IX2,IY2)) 110,110,20
110 IF (IDX**2+IDY**2=1) 115,135,115
115 DCP = (V(IX,IY)+V(IX2,IY)+V(IX,IY2) +V(IX2,IY2))*0.25
      IF (DCP=CV) 20,120,120
120 IF (INX(IS=1)) 125,130,125
125 IX = IX+IDX
      IDX = -IDX
      PY = 2.0
C
      CALL CALC
C
      IX = IX+IDX
      GO TO 135
130 IY = IY+IDY
      IDY = -IDY
      PY = 2.0
C
      CALL CALC
C
      IY = IY+IDY
135 IF (V(IX-1,IY)=CV) 140,145,145
140 NP = NP+1
      REC(NP) = MLNTH*IX+IY
145 IS = IS+5
      IX = IX2
      IY = IY2
      GO TO 35
150 XT = MT
      IF (V(IX-1,IY)=CV) 155,160,160
155 NP = NP+1
      REC(NP) = MLNTH*IX+IY
160 CONTINUE
      DO 165 K=1,N
      Z(K) = (Z(K)-1.0)*0.1
165 R(K) = (R(K)-1.0)*0.1
C
      CALL CDPLOT
C
      RETURN
      END

```

SUBROUTINE CALC

```

C      COMMON NZ, NR, IX, IY, IDX, IDY, ISS, IT, NP, N, PY, REC, CV, R, Z, IPT, INX, INY,
1     V, XL, YL, S, U, NT, MT, IZX, DX, DY, XMAX, SCALE, YMAX, VCON, JSYM, LSYM, JT
C     2 , SV, XYM, IRIT, NPLOT, IZ, IR, MLNTH, IRB, IRT, IZL, IZR
C
C     DIMENSION V(201,201), REC(1200), Z(2000), R(2000), IPT(3,3), INX(8),
1     INY(8), VCON(24)
C     DIMENSION SV(100), XYM(40), IRIT(10), IZ(2000), IR(2000)
C
      IT = 0
      N = N+1
      IF (IDX**2+IDY**2-1) 30,5,30
5     IF (IDX) 20,10,20
10    Z(N) = IX
      W = IY
      IY2 = IY+IDY
      DY = IDY
15   R(N) = ((V(IX,IY)-CV)/(V(IX,IY)-V(IX,IY2)))*DY+W
      RETURN
20   R(N) = IY
      W = IX
      DX = IDX
      IX2 = IX+IDX
25   Z(N) = ((V(IX,IY)-CV)/(V(IX,IY)-V(IX2,IY)))*DX+W
      RETURN
30   IX2 = IX+IDX
      IY2 = IY+IDY
      W = IX
      U = IY
      DX = IDX
      DY = IDY
      DCP = (V(IX,IY)+V(IX2,IY)+V(IX,IY2)+V(IX2,IY2))*0.25
      IF (PY=2,0) 35,40,35
35   IF (DCP=CV) 40,40,55
40   AL = V(IX,IY)-DCP
45   S = 0.5*(AL+DCP-CV)/AL
50   Z(N) = S*DX+W
      R(N) = S*DY+U
      PY = 0.0
      RETURN
55   IT = 1
      AL = V(IX2,IY2)-DCP
60   S = 0.5*(AL+DCP-CV)/AL
65   Z(N) = -S*DX+W+DX
      R(N) = -S*DY+U+DY
      RETURN
      END

```

```
SUBROUTINE PILINE (IZ,IR,N,JSYM,LSYM)
DIMENSION IZ(1), IR(1)
K = 0
DO 20 I=2,N
K = K+1
IF (K=JSYM) 20,10,20
10 CALL PLOTV (IZ(I),IR(I),LSYM)
K = 0
20 CALL LINEV (IZ(I-1),IR(I-1),IZ(I),IR(I))
RETURN
END
```

```
SUBROUTINE ROTAPE (IST,ISTP,ISN,V,NVR)
  DIMENSION V(1)
  NVR = NVR
  REWIND IST
  5 READ TAPE IST,K
  IF (K-ISTP) 7,20,7
  7 IF (K-ISN) 10,15,10
  10 READ TAPE IST
  GO TO 5
  15 READ TAPE IST, (V(I),I=1,NVR)
  PRINT 1005,ISN
  REWIND IST
  RETURN
  20 PRINT 1000,ISN
  REWIND IST
  CALL EXIT
1000 FORMAT (I6,22H IS NOT A SAVED NUMBER)
1005 FORMAT (I6,9H RESTORED)
END
```

```
SUBROUTINE WTTAPE (IST,ISTP,ISN,V,NVR)
  DIMENSION V(1)
  NVR = NVR
  REWIND IST
  5 READ TAPE IST,K
    IF (K-ISTP) 10,25,10
  10 IF (K-ISN) 20,15,20
  15 ISN = ISN+1
  20 READ TAPE IST
    GO TO 5
  25 BACKSPACE IST
    WRITE TAPE IST,ISN
    WRITE TAPE IST, (V(I), I=1,NVR)
    PRINT 1000,ISN
1000 FORMAT (23H DATA SAVED UNDER ISN =,I4)
    WRITE TAPE IST,ISTP
    REWIND IST
    RETURN
  END
```

APPENDIX D

Data Output



## APPENDIX D

### Data Output

The output from the program falls into four categories: plots, matrix printout, convergence information, and data saving information. The most immediately useful information can be obtained from the plots, typical examples of which were shown in Figs. 3, 4, and 5.

The matrix is printed out by rows. Each row is printed in sequence, starting at the bottom edge of the matrix. The number of each row is also printed to facilitate finding any particular section of the matrix.

Convergence information is printed as the first six integers before the matrix printout. A typical printout sequence might be

```
1894
 683
 530
  33
 106
  36
```

where the first, third, and fifth numbers are the "errors" left on the matrix after the coarse matrix relaxation, and the second, fourth, and sixth numbers are the errors after the fine matrix relaxation. The trend of both sequences of numbers should be to become smaller and, ideally, the last number would be zero, indicating no "errors" left on the matrix. Experience has shown, however, that whenever order-of-magnitude reductions in the size of these numbers have been obtained the program is rapidly reaching convergence, and there is very little to be gained by increasing (MM) the number of passes through the matrix. If these numbers are not indicating convergence, MM should be increased.

Data saving and restoring information is accomplished by the sub-routines RDTAPE and WTTAPE which will respectively read and write information onto and from a magnetic tape. If data are to be restored, the

saved number from the previous run must be specified as the restore number (IRN). If data are restored, the comment (IRN) RESTORED is printed in the output. The comment (IRN) IS NOT A SAVED NUMBER is printed and the program is terminated if the record number requested cannot be found on the tape.

If data are saved, the comment DATA SAVED UNDER IRN = \_ \_ \_ \_ is printed. It is not necessary to specify IRN, since the subroutine (WTTAPE) will automatically choose an unused number. Before the tape is used by this subroutine for the first time, the number 999 (ISTP) must be written as the first record by a separate program.

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Defense Atomic Support Agency  
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E. H. Beckner, 5240  
J. B. Gerardo, 5243  
T. H. Martin, 5245 (5)  
J. E. Boers, 5245 (10)  
D. L. Johnson, 5245  
K. R. Prestwich, 5245  
L. Bradley, 5245  
F. W. Neilson, 7360  
M. Cowan, 7340  
T. B. Cook, 8000  
G. A. Fowler, 9000  
B. F. Murphey, 9100  
J. D. Plimpton, 9112  
L. C. Baldwin, 3412  
B. R. Allen, 3422  
B. F. Hefley, 8232 (5)  
C. H. Sproul, 3428-2 (25)  
M. S. Goldstein, 3412 (2)