Technical Memorandum *November 1969*

PULSE DATA PROCESSING

By: H. G. HEUBACH

Prepared for:

AIR FORCE WEAPONS LABORATORY
KIRTLAND AIR FORCE BASE
NEW MEXICO 87117

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ABSTRACT

This memorandum describes the processing of two sets of pulse data to obtain a transfer function. Limitations in the data and the accuracy and response of the data-processing system and their effects on the transfer function are noted.

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ILLUSTRATIONS

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I INTRODUCTION

It is usually the purpose of an EMP interaction study to define phenomenological relationships between a free-field (reference) pulse and current or voltage pulses induced by the free-field pulse in a test system of unique configuration. To this end, it is necessary to adequately define both the free-field pulse, or the approximation used in its stead, over that region in space where interaction with the system occurs and the induced pulses at those test points of interest within the system. For a reference pulse which is reasonably uniform over the region of interaction or which varies over that region in the same manner as would be expected of any free-field pulse of interest and which is highly reproducible from one test run to the next, a single set of measurements made at one point in the region will suffice to define a reference pulse for all measurements made at the system test points. This set of measurements must be adequate for defining the time-amplitude history of the reference pulse for its duration of interest and at the same time be adequate for defining the spectral content of the pulse over its bandwidth of interest. The same may be said of the measurements of the induced pulses. It should also be said that both the durations and bandwidths of interest for the reference pulse and an induced pulse should coincide.

For linear systems of interaction, one way of relating an induced pulse to the reference pulse is to calculate a transfer function, defined as the ratio of the Fourier Transform of the induced pulse to the Fourier Transform of the reference pulse. Such a transfer function may also be calculated for nonlinear systems of interaction but is of limited usefulness.

To investigate the possibilities of obtaining, from a collection of experimental data, some reasonably defined pulse waveforms and of obtaining from these waveforms some meaningful transfer functions, two fairly representative sets of measurements obtained at a certain test facility will be necessary. One of these sets purports to define the horizontal magnetic-field component of the reference pulse and the other to define a voltage pulse induced at one test point within the test system. These sets will be used as vehicles for presenting the data-processing sequence used to obtain a transfer function from field pulse data and for pointing up the requirements that the data-processing system places on the field data and vice versa.

II DATA REDUCTION

The data selected for reduction are in the form of polaroid photographs of oscilloscope traces of voltage pulses and are shown in Figure 1 (reference pulse) and Figure 2 (system test pulse). Data reduction consists of essentially two phases: digitization and data processing.

A. Digitization

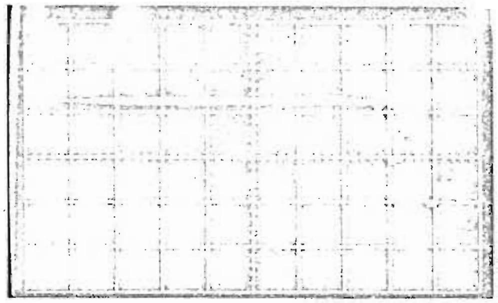
Digitization itself consists of four steps: slide-making, slide-projection and hand-tracing of the waveform, preparation for digitizing, and digitizing.

Slide-making consists of photographically producing negatives of the polaroids with about a four-to-one reduction and mounting these negatives in slide frames.

Slide-projection and hand-tracing consists of projecting the slides onto worksheets taped to a wall at a distance of about i

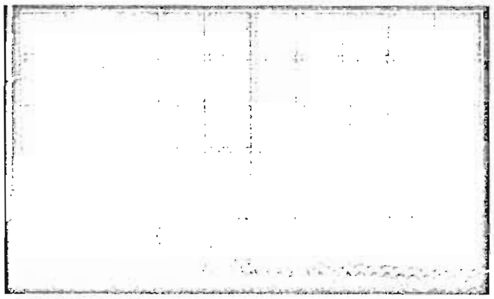
Slide-projection and hand-tracing consists of projecting the slides onto worksheets taped to a wall at a distance of about fifteen feet from the projector and at an enlargement of about thirty-to-one (or about seven-to-one over the original polaroids) and carefully tracing, with due regard for the peculiarities of oscilloscopes, the projected waveform onto the worksheets, along with any other pertinent information, such as oscilloscope gridlines, zero-amplitude lines, time calibration markers, etc. For the data presented here the only other pertinent information was the gridlines.

Preparation for digitizing consists of marking on the worksheets the correct placement and orientation of the zero-amplitude baseline, picking



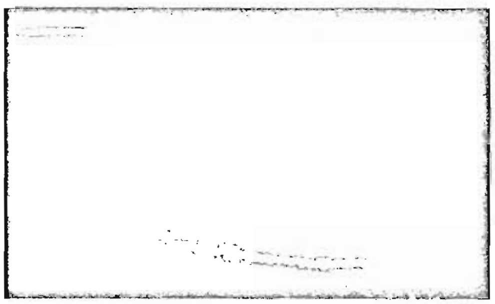
RECORD NO. 1

1 V/div. VERTICAL SENSITIVITY
10 ns/div. HORIZONTAL SENSITIVITY
1 μs RC TIME CONSTANT



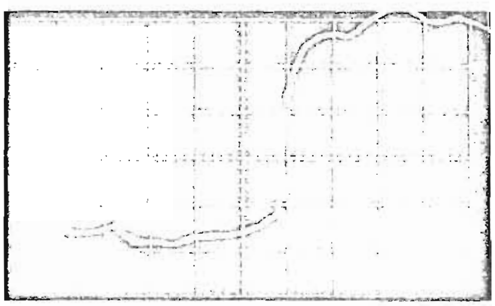
RECORD NO. 2

0.5 V/div. VERTICAL SENSITIVITY
20 ns/div. HORIZONTAL SENSITIVITY
1 μs RC TIME CONSTANT



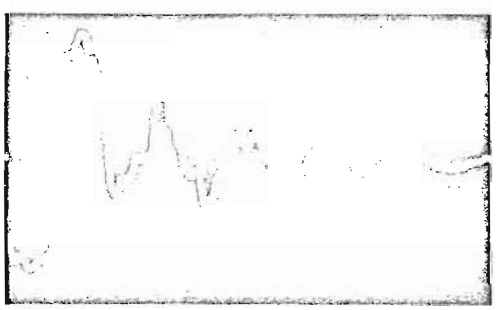
RECORD NO. 3

0.1 V/div. VERTICAL SENSITIVITY
50 ns/div. HORIZONTAL SENSITIVITY
10 μs RC TIME CONSTANT



RECORD NO. 4

2 V/div. VERTICAL SENSITIVITY
200 ns/div. HORIZONTAL SENSITIVITY
1 μs RC TIME CONSTANT



RECORD NO. 5

1 V/div. VERTICAL SENSITIVITY
1 μs/div. HORIZONTAL SENSITIVITY
1 μs RC TIME CONSTANT

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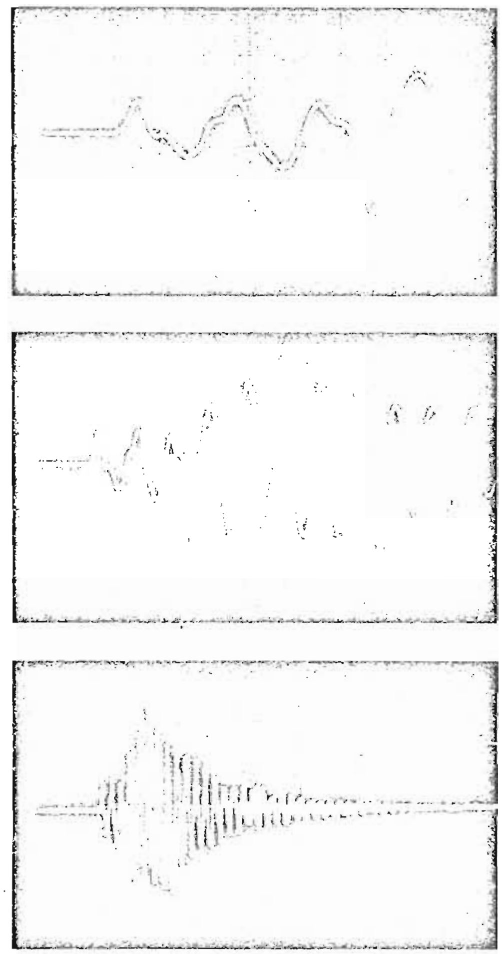
FIGURE 1 RAW DATA FOR REFERENCE PULSE



RECORD NO. 1

20 mV/div. VERTICAL SENSITIVITY
200 ns/div. HORIZONTAL SENSITIVITY

RECORD NO. 2

20 mV/div. VERTICAL SENSITIVITY
500 ns/div. HORIZONTAL SENSITIVITY

RECORD NO. 3

20 mV/div. VERTICAL SENSITIVITY
2 μs/div. HORIZONTAL SENSITIVITY

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FIGURE 2 RAW DATA FOR SYSTEM TEST PULSE

the time onset (zero time) of the waveform, noting down the digitizing sensitivities to be used (250, 500, or 1000 counts per inch), and specifying the sampling rate (so many counts per inch). For the data at hand the zero-amplitude baselines were assumed to be parallel to the horizontal gridlines, since there were no other compelling indications to the contrary. The sampling rates were chosen by applying a rule of thumb:* the sampling rate shall be at least ten times the highest frequency visible in the waveform to be sampled. Although it is not always necessary, in this case each waveform was sampled at one uniform rate. For the waveforms shown in Figure 1 (reference pulse) the sampling rates chosen corresponded to real-time rates of 2000 MHz, 2000 MHz, 1000 MHz, 1000 MHz, and 200 MHz for Records No. 1 through No. 5, respectively. For those shown in Figure 2 (system test pulse) the rates chosen were 200 MHz, 100 MHz, and 50 MHz.

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* The motivation behind this rule of thumb is as follows: a sampling rate at least twice the highest frequency visible in the time waveform is necessary if one wishes to avoid aliasing (folding of frequency components about integer multiples of one-half the sample rate) these visible, and hence presumably appreciable, high-frequency components into the lower frequency components; a sampling rate on the order of four times (appreciably greater than twice, at any rate) the highest frequency visible is necessary if one wishes to define the spectral content of the waveform for these visible frequencies (aliasing of frequency components just above these frequencies into these frequencies is thus avoided); a sampling rate on the order of ten times (or, a little over twice four times) is necessary if one wishes to define the high-frequency roll-off of the spectrum above the visible high frequencies (again, to avoid significant aliasing into regions of possible interest). In addition, a high sampling rate facilitates subsequent data processing: the digital waveform may be plotted by connecting the sampled values with straight lines to obtain an adequate representation of the original continuous waveform; the computer routine used to numerically evaluate the Fourier Integral requires that the representation resulting from connecting the sampled values with straight lines be a good approximation to the original continuous waveform; a larger number of samples yields a greater reliable range of spectral amplitudes (see Appendix); etc.

Digitizing consists of manually following the curves on the worksheet with an automated X-Y digitizer (Benson-Lehrer LARR V) which is set to sample at the rate specified on the worksheet. The digital output of the LARR V is recorded on magnetic tape.

The effect of sampling a continuous waveform at a uniform rate for a finite period of time is shown in Figure 3. Convolution of the curve, $h(\nu)$, shown in Figure 3 with the Fourier Transform, $g(\nu)$, of the continuous time waveform produces the Fourier Transform, $\tilde{g}(\nu)$, of the sample series:

$$\tilde{g}(\nu) = h(\nu) \otimes g(\nu) = \int_{-\infty}^{\infty} h(\xi)g(\nu - \xi)d\xi \quad .$$

The function $h(\nu)$ represents the combined effect of two separate phenomena: the effect, $h_1(\nu)$, of truncating the continuous time waveform and the effect, $h_2(\nu)$, of sampling the truncated continuous time waveform. The repetitive spikes occurring every $\nu_n = n\nu_s$, where ν_s is the sampling rate, shown in Figure 3(b) would have been true delta functions had the time waveform been sampled for all time, in which case the convolution would yield

$$\tilde{g}(\nu) = \sum_{n=-\infty}^{\infty} g(\nu - n\nu_s) = g(\nu) + \sum_{n=1}^{\infty} [g^*(n\nu_s - \nu) + g(n\nu_s + \nu)] \quad ,$$

which is the aliasing phenomenon.

Truncating the continuous time waveform at times 0 and T is equivalent to multiplying the continuous time waveform by a square pulse of amplitude unity and width T centered at time T/2. The Fourier Transform of this square pulse is

$$h_1(\nu) = \frac{T}{2\pi} \cdot \frac{\sin \pi\nu T}{\pi\nu T} \cdot e^{i\pi\nu T}$$

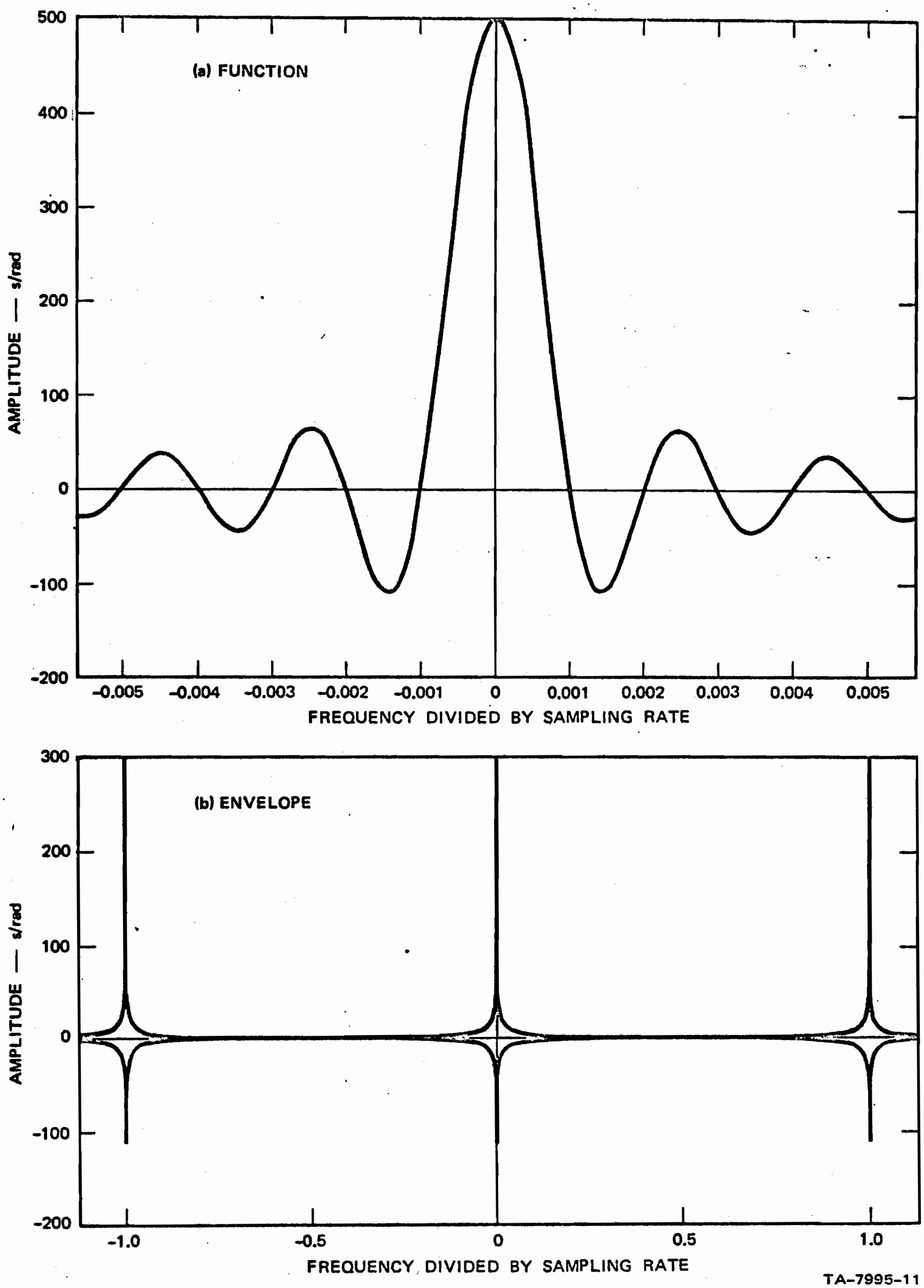

FIGURE 3 TYPICAL EFFECT OF SAMPLING A TIME-DOMAIN WAVEFORM AT A UNIFORM RATE FOR A FINITE DURATION

and the Fourier Transform of the truncated continuous time waveform is

$$\hat{g}(\nu) = h_1(\nu) \otimes g(\nu) \quad .$$

Sampling the truncated continuous time waveform at intervals $\tau = 1/\nu_s$ is equivalent to multiplying the truncated continuous time waveform by the function $\sum_{n=-\infty}^{\infty} \delta(t - n\tau)$. The Fourier Transform of this series of delta functions is

$$h_2(\nu) = \sum_{n=-\infty}^{\infty} e^{i2\pi\nu n\tau} = \sum_{n=-\infty}^{\infty} \delta(\nu - n\nu_s)$$

and the Fourier Transform of the sampled truncated continuous time waveform is

$$\tilde{g}(\nu) = h_2(\nu) \otimes \hat{g}(\nu) = h_2(\nu) \otimes h_1(\nu) \otimes g(\nu) \quad ,$$

or

$$\tilde{g}(\nu) = h(\nu) \otimes g(\nu) \quad ,$$

where

$$h(\nu) = h_2(\nu) \otimes h_1(\nu) \quad .$$

The function $h(\nu)$ shown in Figure 3 is for a typical case totaling 500 samples.

B. Data Processing

In this instance data processing consists of calibration, filtering, removal of differences in recording system characteristics, composite making, calculation of Fourier Integrals, removal of recording system characteristics, and calculation of a transfer function.

Calibration consists of transformation from the raw units of digitization to real-time units. Since no calibration information was available, the scope sweeps and vertical deflections were assumed to be linear, and the data were merely scaled to conform to the nominal deflection sensitivities shown in Figures 1 and 2. Lack of calibration is most regrettable, particularly in calculating a transfer function where frequency matchup is of crucial importance.

Filtering consists of "smoothing" the digitized waveform to remove, as much as possible, noise introduced in the process of tracing and digitizing and to minimize the effects of quantizing introduced in digitizing.* A typical smoothing filter is shown in Figures 4 and 5, which filter is applied in the time domain (Figure 4) as a convolution (weighted average). The smoothing filters applied to the data had cutoffs (~ 0 dB down, shown as a dashed line in Figure 5) at 300 MHz, 300 MHz, 150 MHz, 100 MHz, and 25 MHz for the waveforms shown in Figure 1 and 18 MHz, 15 MHz, and 8 MHz for those shown in Figure 2.

Removal of differences in recording system characteristics in the various waveforms used in obtaining a composite waveform is necessary if the differences are appreciable in the time interval or bandwidth

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* The noise introduced in the process of tracing and digitizing is primarily in the form of jitter caused by an unsteady hand following the curve and to a much lesser extent in the form of quantizing noise. (It is the purpose of digitization to "quantize": digital values are recorded as integer multiples of a least-significant unit--one least-significant unit equals 1/500th of an inch, for example.) The smoothing filter removes (diminishes by at least 40 dB) the high-frequency components of the noise (HF noise, sharp corners, etc.) but, obviously, does not affect the low-frequency components of the noise. The frequency band of this noise (both jitter and quantizing noise) is greatly dependent on the form and complexity of the curve being traced by hand; but it has been our experience that this frequency band falls mostly above the bandwidth of interest of the waveform being traced, so it is generally possible to separate unwanted signal from wanted signal by the application of a low-pass filter.

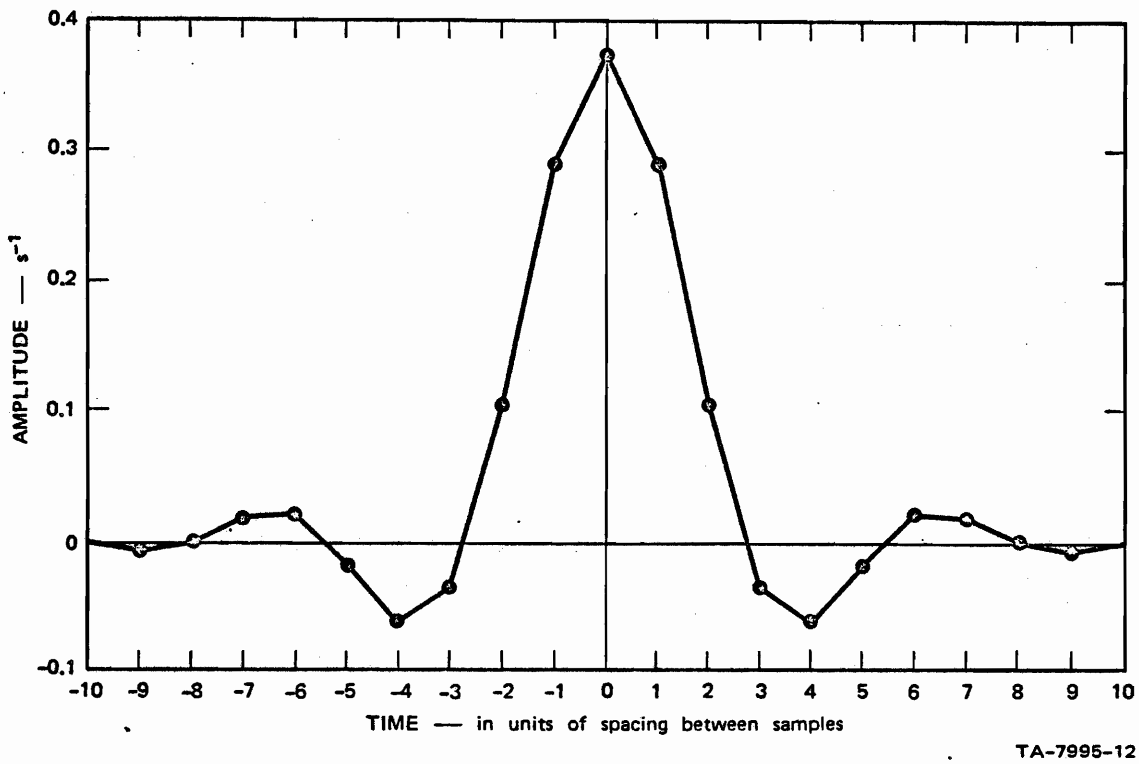

FIGURE 4 IMPULSE RESPONSE OF SMOOTHING FILTER

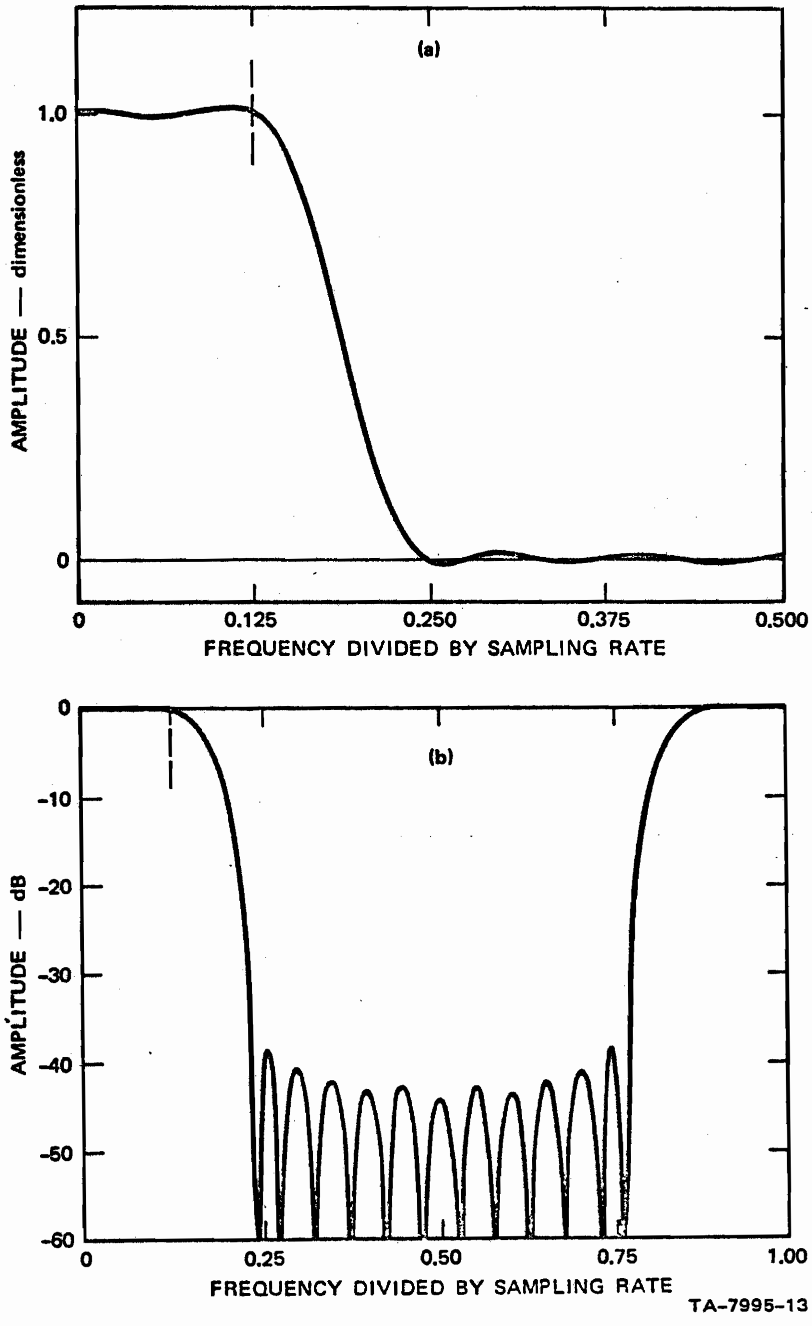

FIGURE 5 SMOOTHING FILTER RESPONSE

of interest. This step was omitted. The omission is particularly regrettable in that one of the five waveforms (Figure 1, Record No. 3) used in obtaining a composite representation of the reference pulse was recorded with an integrating circuit with a different RC time constant (10 μs as opposed to 1 μs) substituted in the recording system; and it is all the more regrettable in that, as a first approximation to correction (lacking more detailed information), this maverick waveform could easily have been corrected to take out the difference introduced by a different simple, theoretical RC integrator (one capacitor, one resistor). Figure 6 shows the response of the simple RC-integrator $[1/(1 + j\omega RC)]$ divided by the response of an ideal integrator $(1/j\omega)$. Note that there is a considerable difference in responses of the two RC-integrators for frequencies less than 1 MHz.

Constructing the composite consists of piecing together the various individual waveforms to form as complete a representation as possible of the original recorded waveform. The individual waveforms are selected to obtain the best possible resolution in time, amplitude, and bandwidth and are matched up against one another to obtain agreement of such common features as time and amplitude of peaks, time of zero-crossings, etc.; such matching frequently entails translation along the time axis of several waveforms and occasionally entails rescaling a waveform (the better the calibration, the less frequent the rescaling). The points of transition between individual waveforms are chosen to avoid introducing discontinuities in amplitude, phasing, or resolution of salient features (intuitive judgment is occasionally necessary--amplitude values are changed slightly to achieve a smooth match at the transition). All the records shown in Figure 1 were used in obtaining a composite representation of the reference pulse (Figures 7 and 8); these records, from fastest to slowest sweep rate, account for the following intervals in

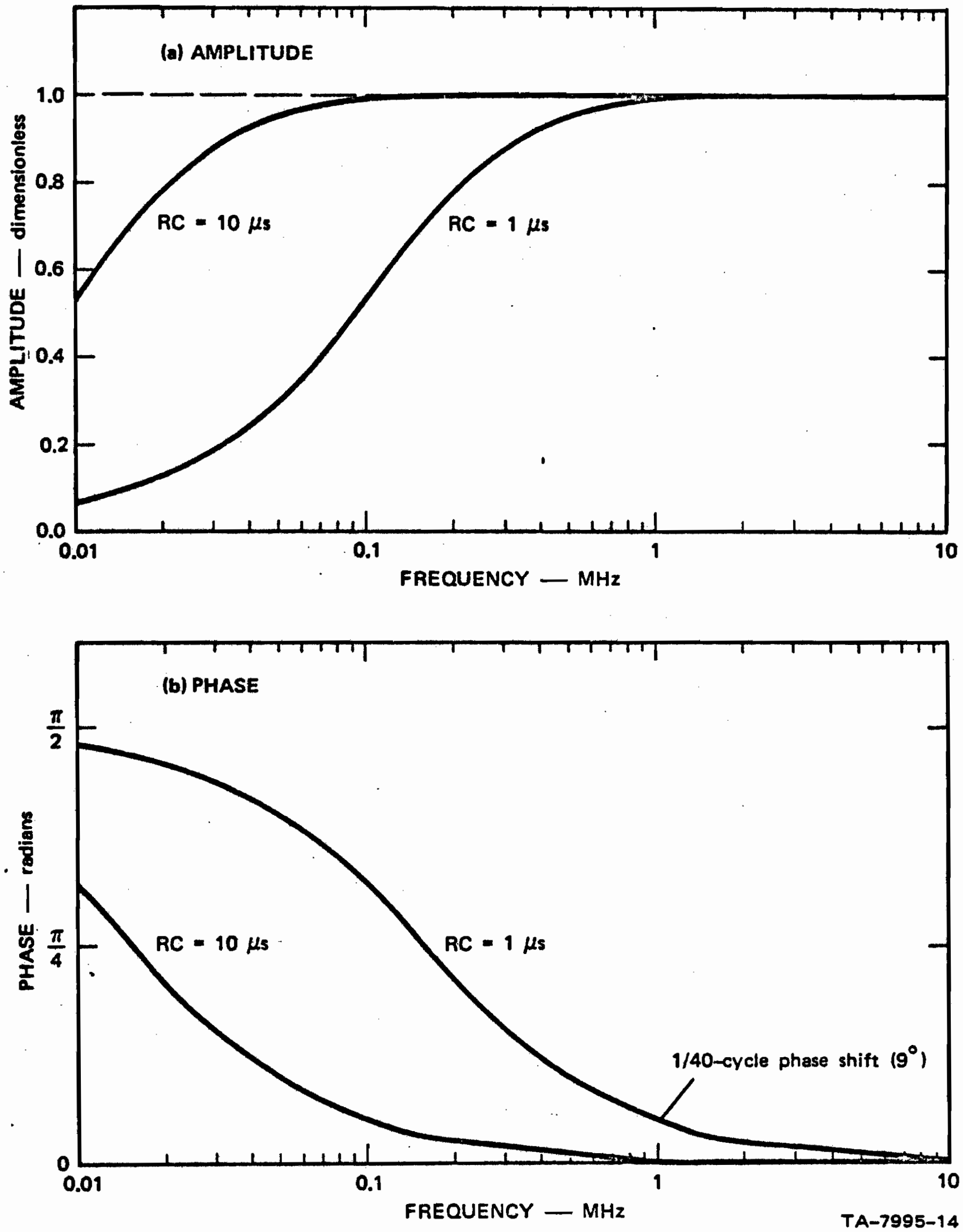

FIGURE 6 RESPONSE OF A SIMPLE RC INTEGRATOR RELATIVE TO AN IDEAL INTEGRATOR

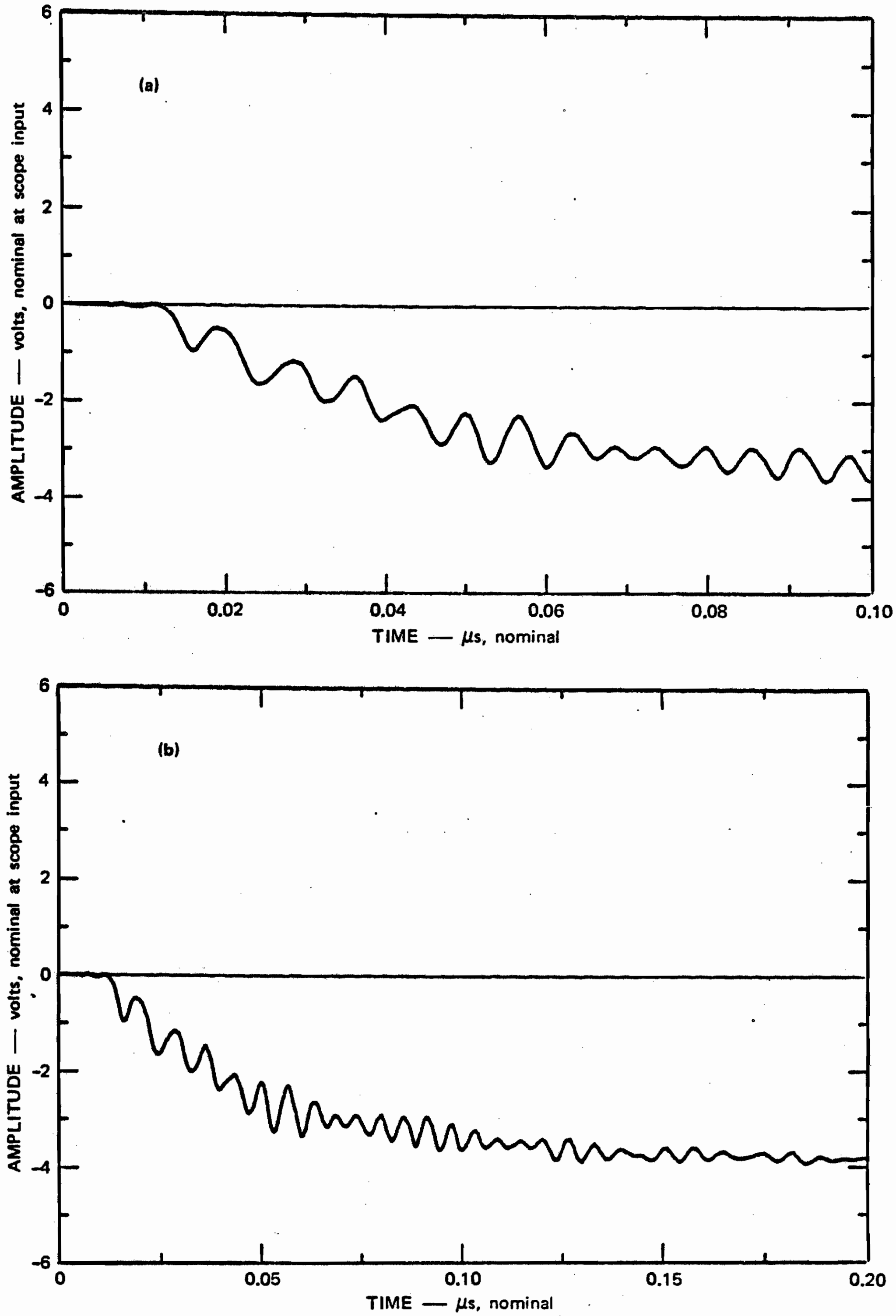

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FIGURE 7 REFERENCE H PULSE (RC-INTEGRATED B-DOT)—EARLY TIME

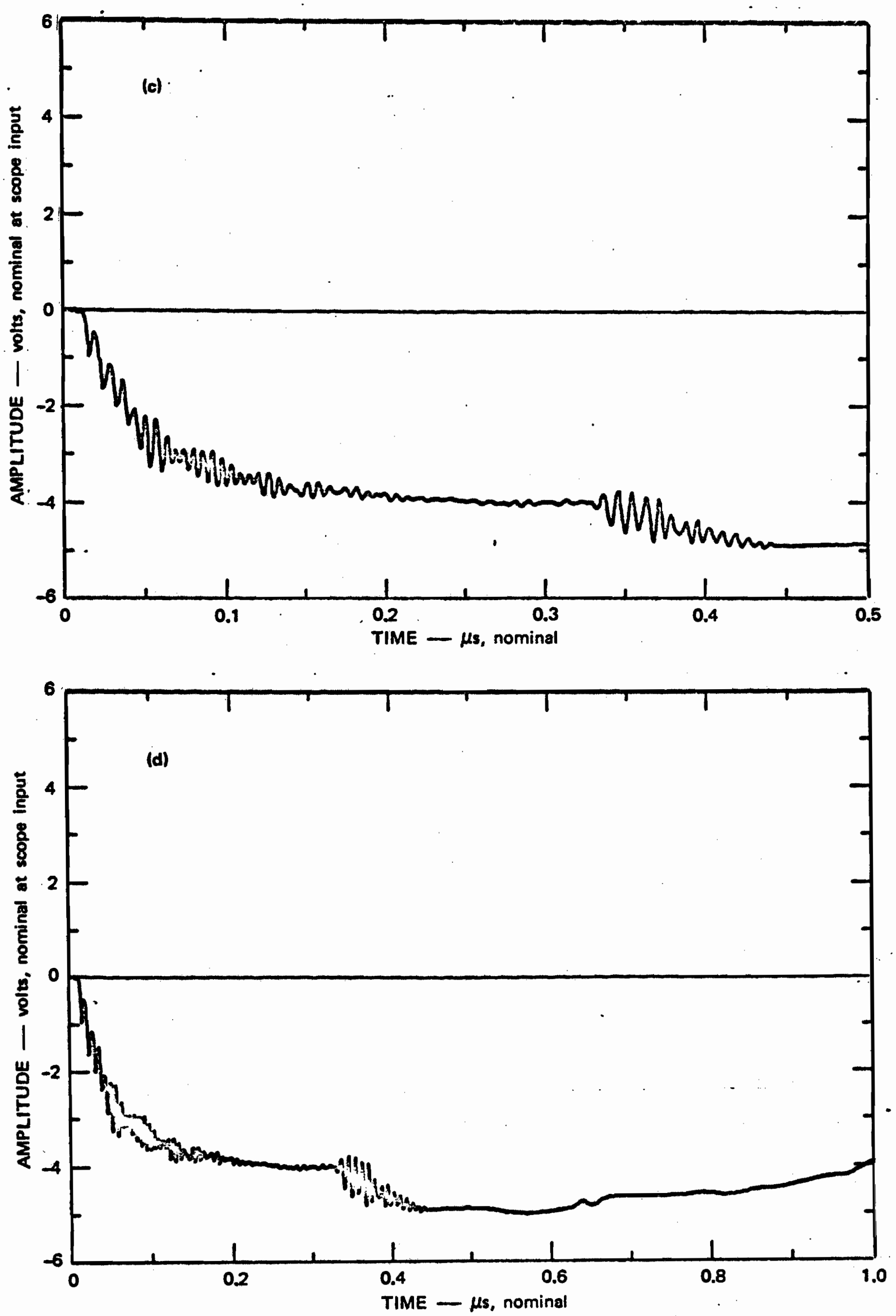

FIGURE 7 REFERENCE H PULSE (RC-INTEGRATED B-DOT)—EARLY TIME Concluded

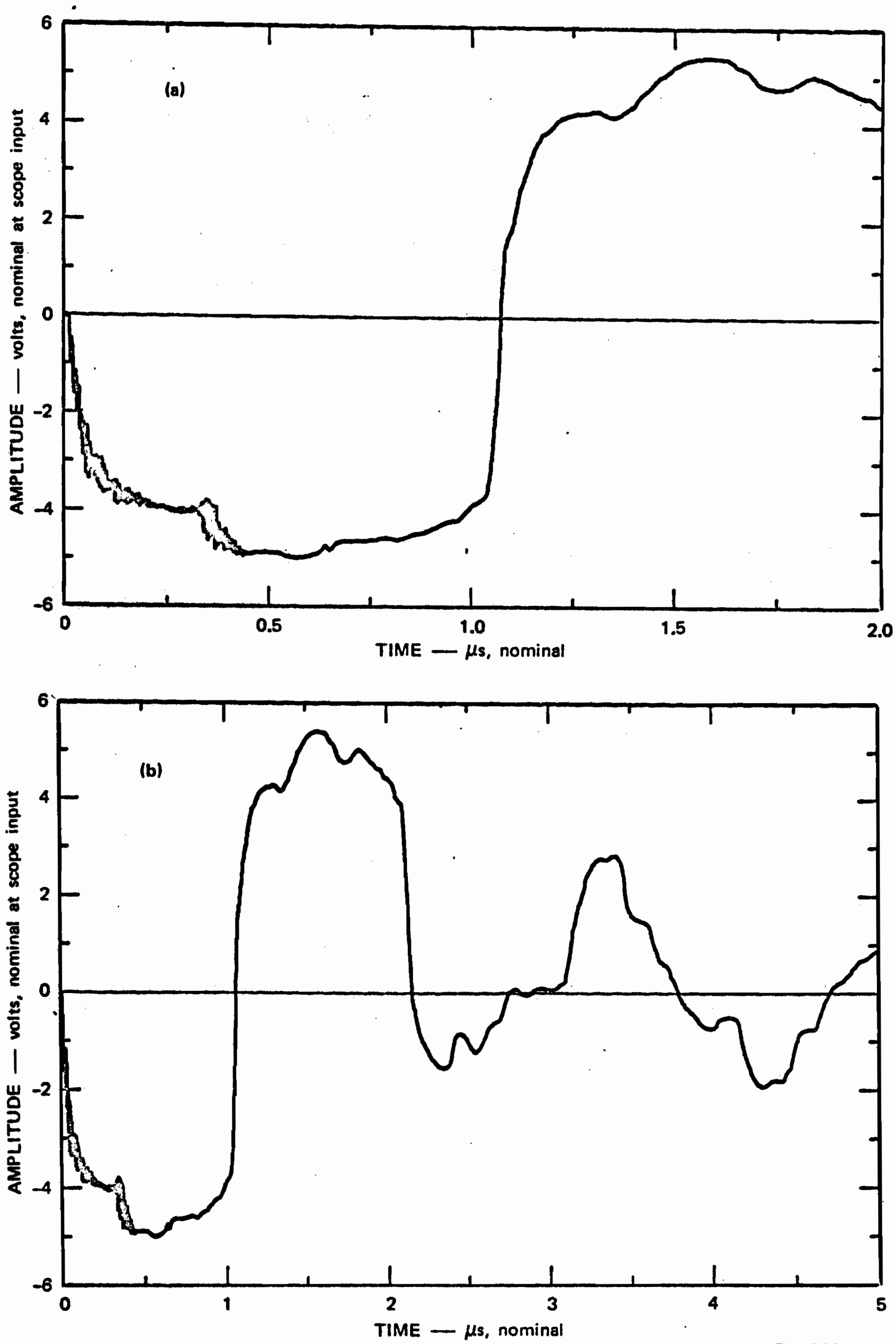

FIGURE 8 REFERENCE H PULSE (RC-INTEGRATED B-DOT)—LATE TIME

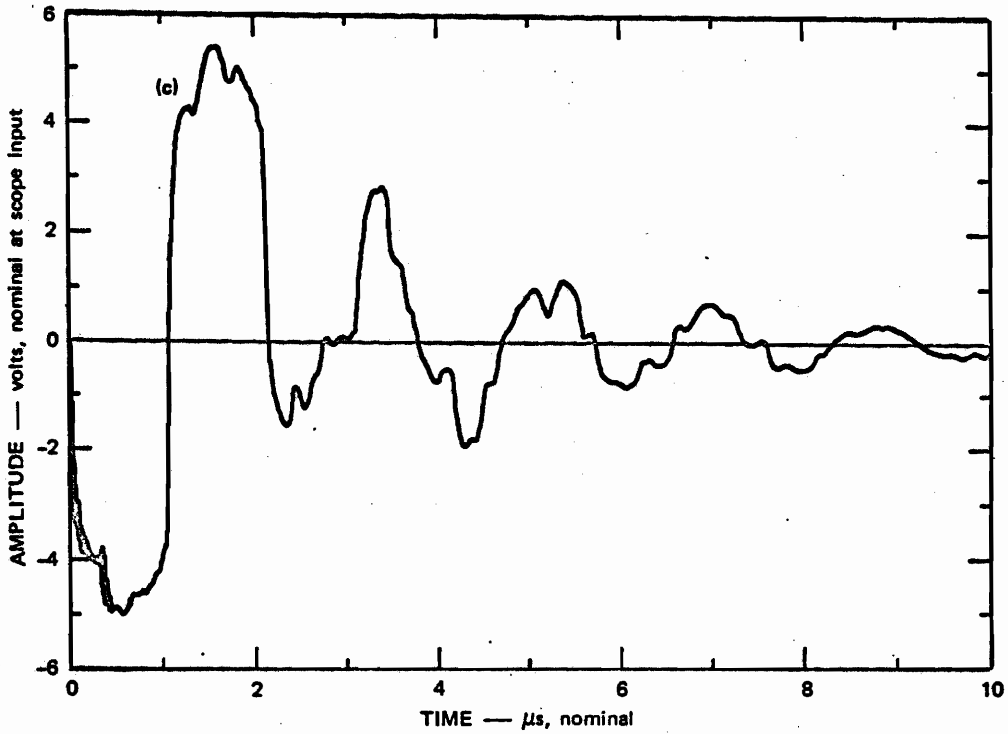

FIGURE 8 REFERENCE H PULSE (RC-INTEGRATED B-DOT)—LATE TIME Concluded

the definition of the composite waveform: 0-38 ns, 38-116 ns, 116-144 ns, 0.444-1.925 μs, 1.925-10.250 μs. In order to match up the five individual waveforms, it was necessary to rescale them: in the same order as above, the amplitude values were multiplied by 0.70, 1.73, 7.85, 1.00, 2.00.* All the records in Figure 2 were used in obtaining a composite representation of the system test pulse (Figure 9); these records, from fastest to slowest sweep rate, account for the following intervals in the definition of the composite waveform: 0-1.45 μs, 1.45-3.92 μs, 3.92-17.88 μs. It was not necessary to rescale any of the three individual waveforms.

Calculation of the Fourier Integral consists of assuming that the time functions are well approximated by a series of triangular impulses and of summing, with appropriate phase shift, the Fourier Transforms of the triangular impulses. It is assumed that the height of the triangular impulses is the amplitude of a sample point on the digital composite waveform, that the base is the time interval between the sample points adjacent to that sample point, and that it is centered in time at that sample point. (These assumptions are equivalent to assuming that the time function is well approximated by straight lines drawn between adjacent sample points of the digital composite waveform.) The response (transfer function) of this component of the data-processing system

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* The lack of agreement among the five individual waveforms as to the amplitude of the reference pulse casts some doubt on the validity of the composite representation of the reference pulse. Apparently, either the nominal values given for the vertical sensitivities of the oscilloscope traces are in error or the five traces do not portray the same waveform. In either case, the amplitude of the composite pulse is essentially arbitrary; in the latter case, the composite is definitely invalid.

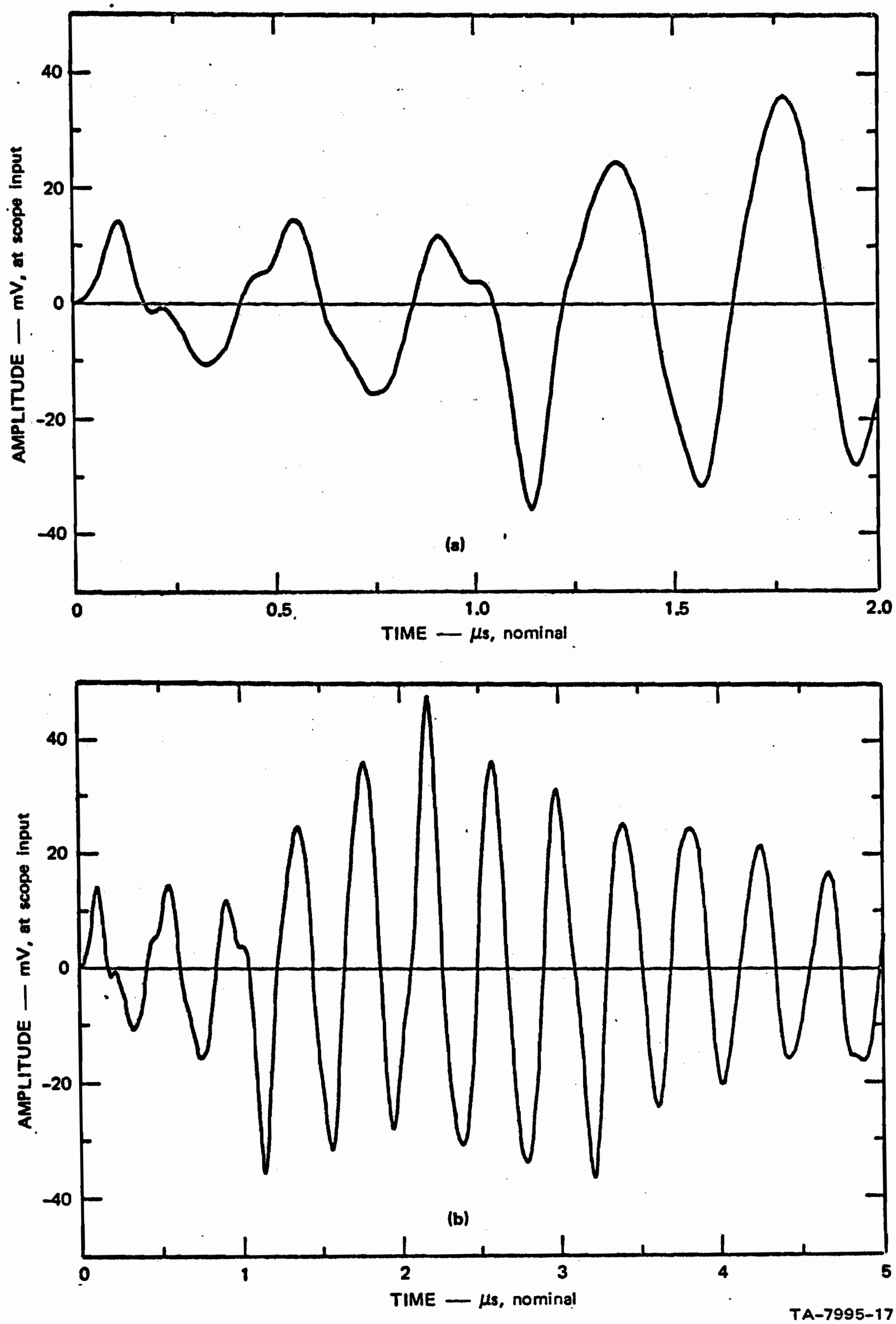
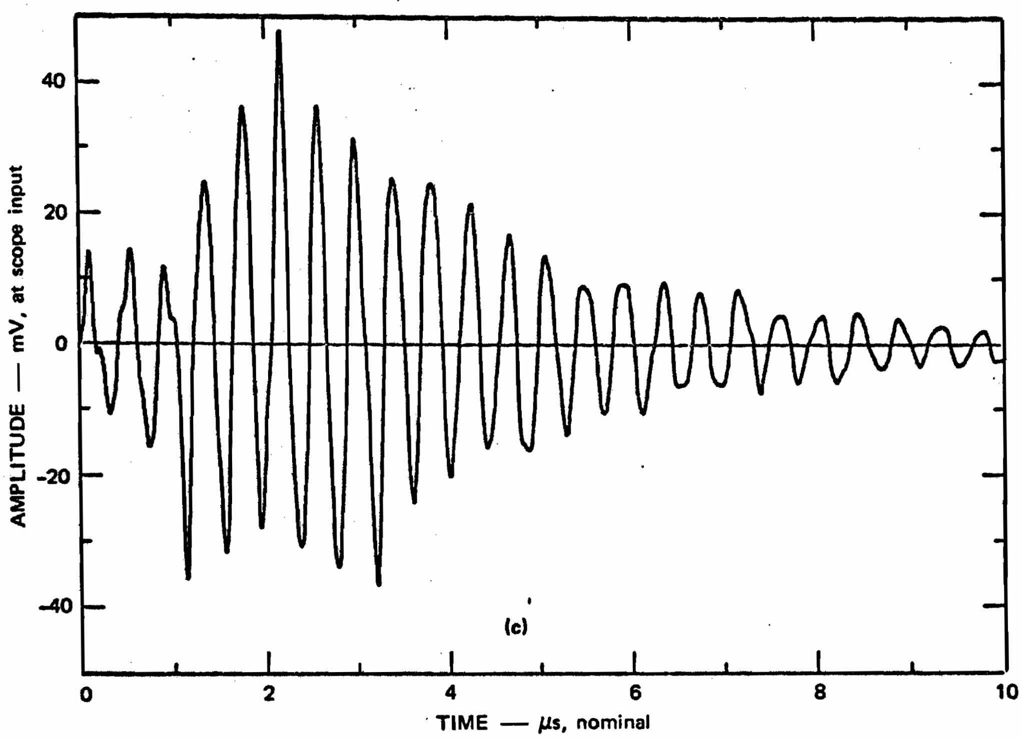


FIGURE 9 SYSTEM TEST PULSE



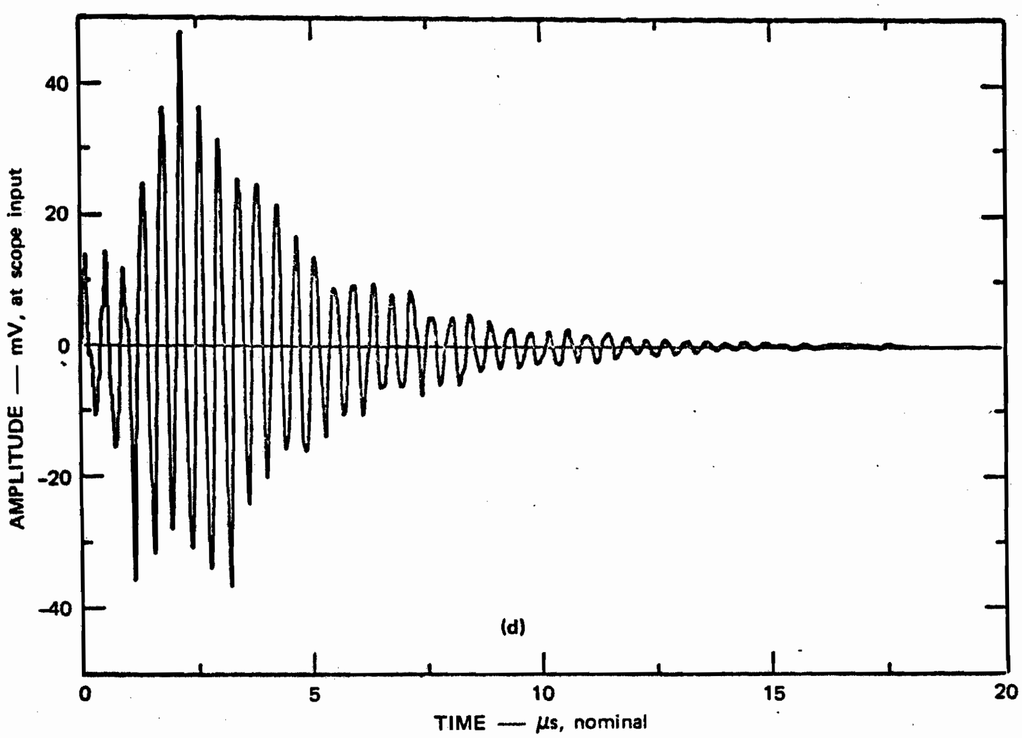


FIGURE 9 SYSTEM TEST PULSE Concluded

(Fourier Integral approximation) is shown in Figure 10.* The Fourier Transforms for the reference pulse and for the system test pulse obtained from this calculation are shown in Figures 12-15 and in Figures 16-19.

Removal of recording system characteristics consists of determining from the system calibration information the response (transfer function)--amplitude and phase as a function of frequency over the entire frequency band of interest--of each component of the recording system and of dividing the Fourier Transform of the recorded time function by the collective response of the recording system components. These recording system components presumably include, for the reference pulse, a free-field pulse sensor, a delay line, an RC integrator, a current or voltage probe, the probe termination, and an oscilloscope; for the system test pulse, they include a current or voltage probe, the probe termination, and an oscilloscope. Inasmuch as the necessary calibration information was not available, this step was omitted.

Calculation of a transfer function (Figures 20-22) consists of dividing the Fourier Transform of the system test pulse by the Fourier Transform of the reference pulse.†

A summary of the various sampling rates, sampling intervals, smoothing filters, etc. is presented in Table I.

---

* Figure 11 shows the combined effect on the Fourier Transform of the application in the time domain of the smoothing filters and the response of the approximation to the Fourier Integral.

† Since the two sets of the Fourier Transform values (reference pulse, system test pulse) were not calculated with the same frequency spacings, it was necessary to interpolate between adjacent amplitude values and adjacent phase values for the reference pulse Fourier Transform. (The computer program which performed the division automatically interpolates if necessary.) The effect of this interpolation on the resulting transfer function is extremely complicated and is not presented here. (It would appear that the effect on the early-time behavior--0-2 μs--of the impulse response is negligible.) Suffice it to say that this interpolation is not desirable nor should it be necessary.

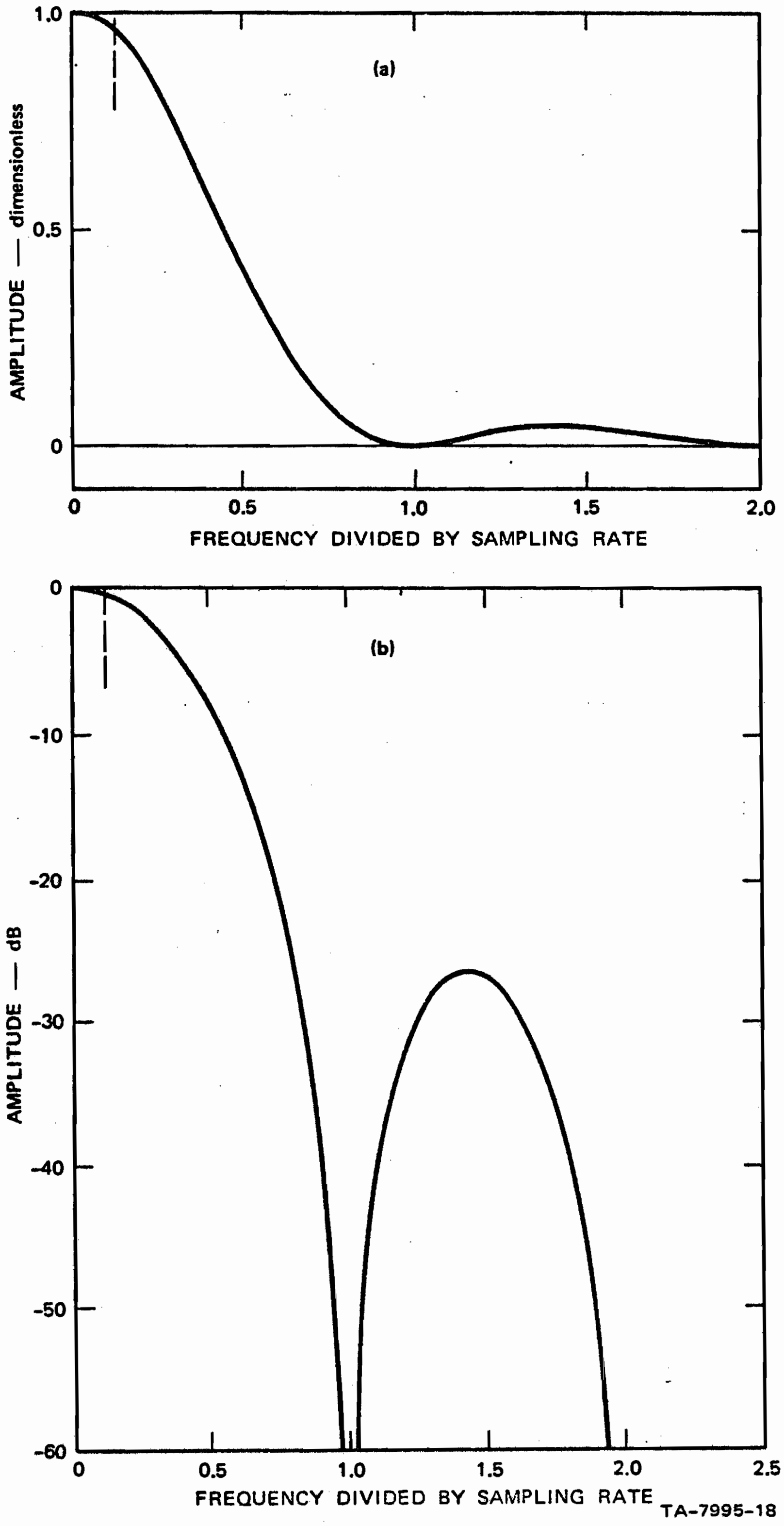


FIGURE 10 RESPONSE OF THE TRIANGULAR-IMPULSE SUMMATION APPROXIMATION TO THE FOURIER INTEGRAL

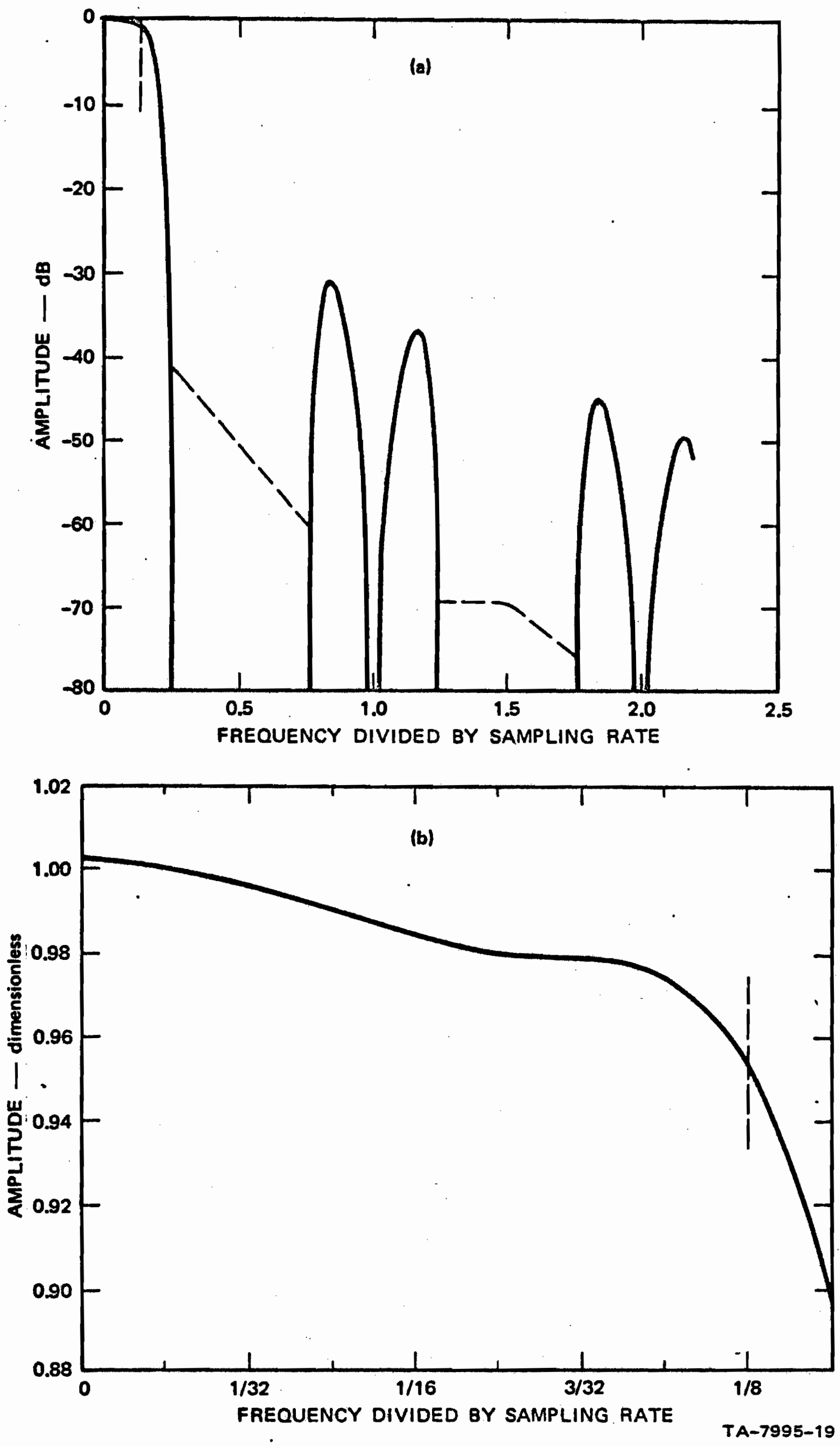


FIGURE 11 COMBINED EFFECT OF A TYPICAL SMOOTHING FILTER AND THE TRIANGULAR-IMPULSE SUMMATION APPROXIMATION TO THE FOURIER INTEGRAL

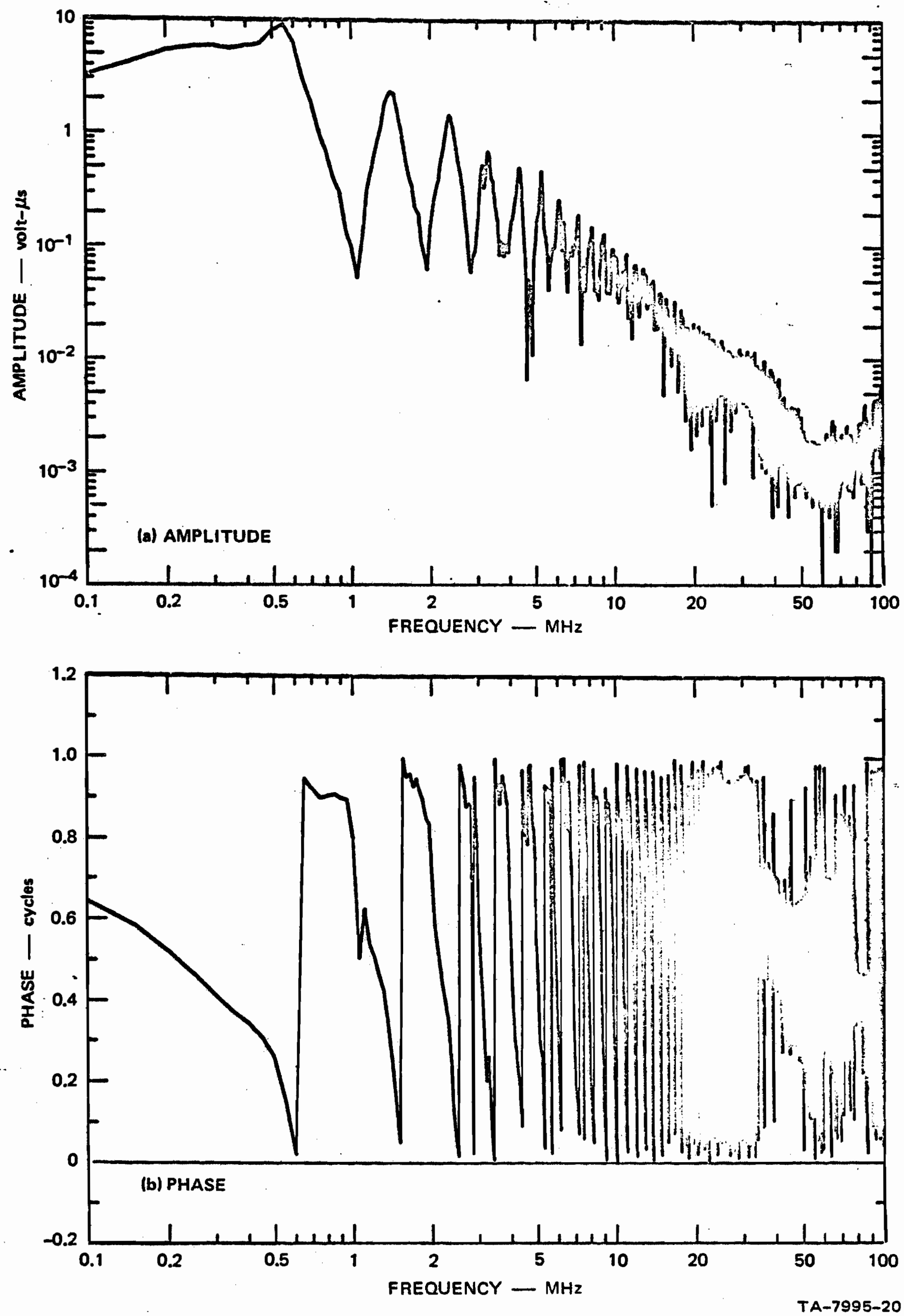


FIGURE 12 FOURIER AMPLITUDE AND PHASE OF THE REFERENCE H PULSE (RC-INTEGRATED B-DOT)—LOGARITHMIC FREQUENCY

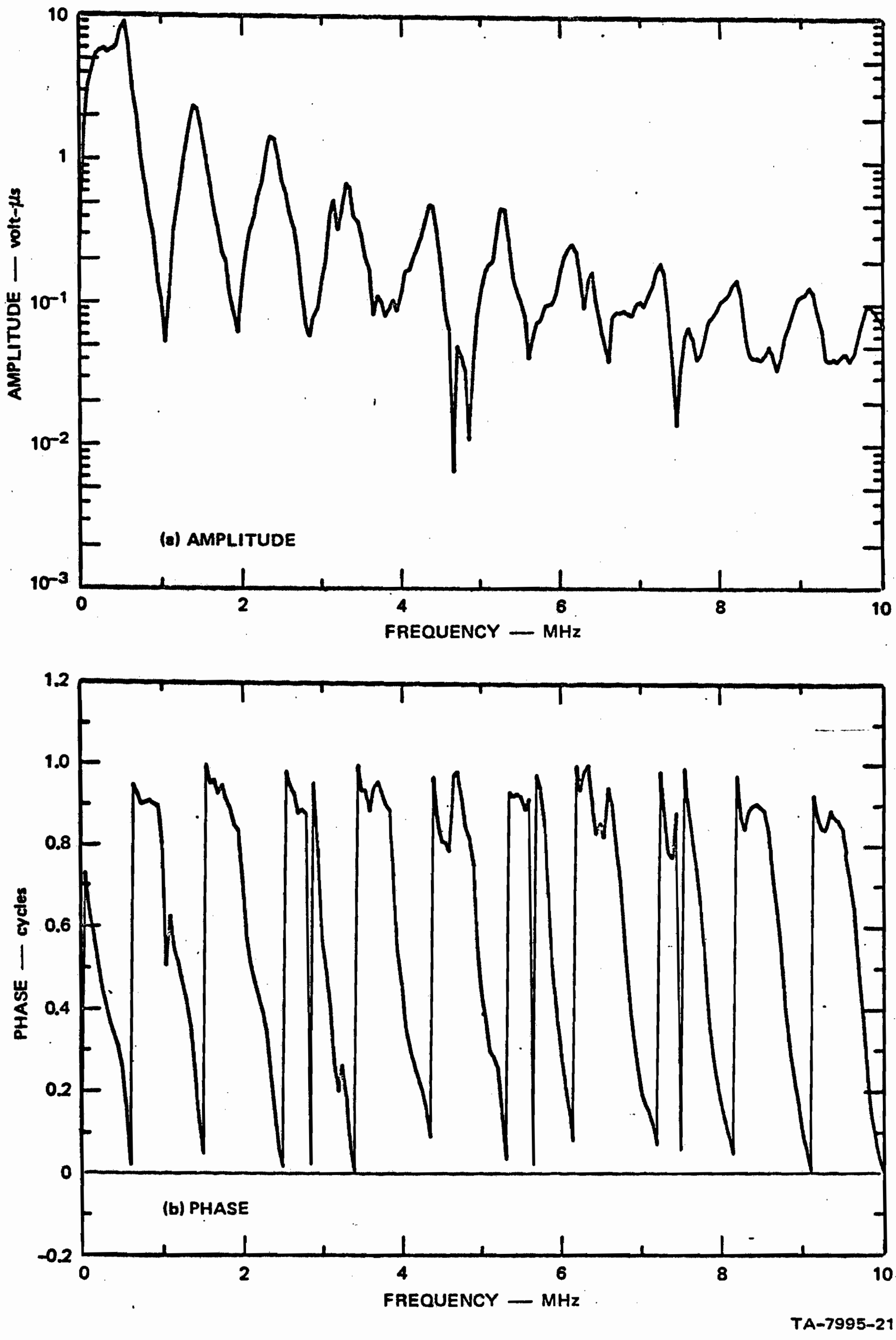


FIGURE 13 FOURIER AMPLITUDE AND PHASE OF THE REFERENCE H PULSE (RD-INTEGRATED B-DOT)—LINEAR FREQUENCY

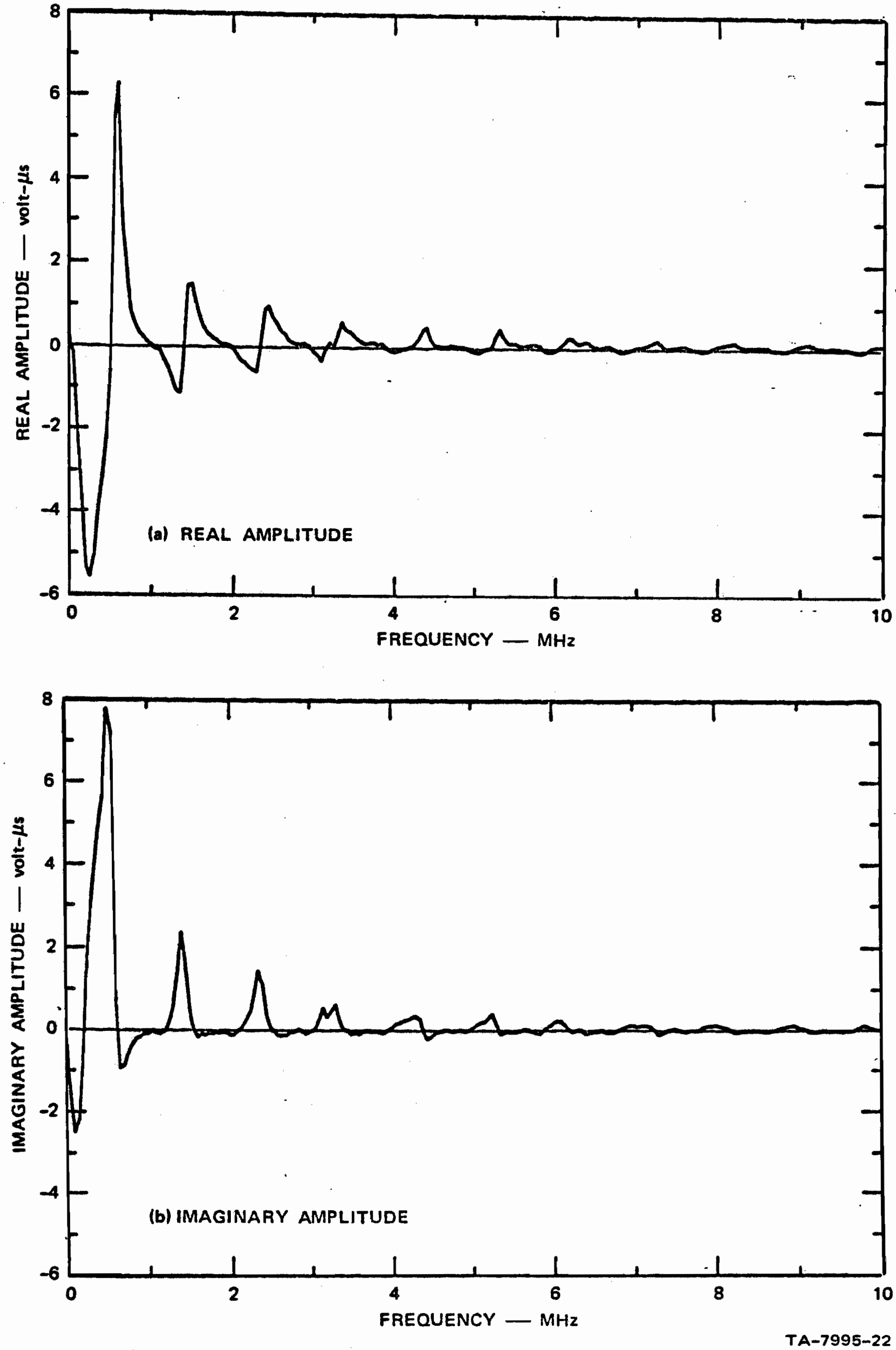


FIGURE 14 REAL AND IMAGINARY PARTS OF THE FOURIER AMPLITUDE OF THE REFERENCE H PULSE (RC-INTEGRATED B-DOT)

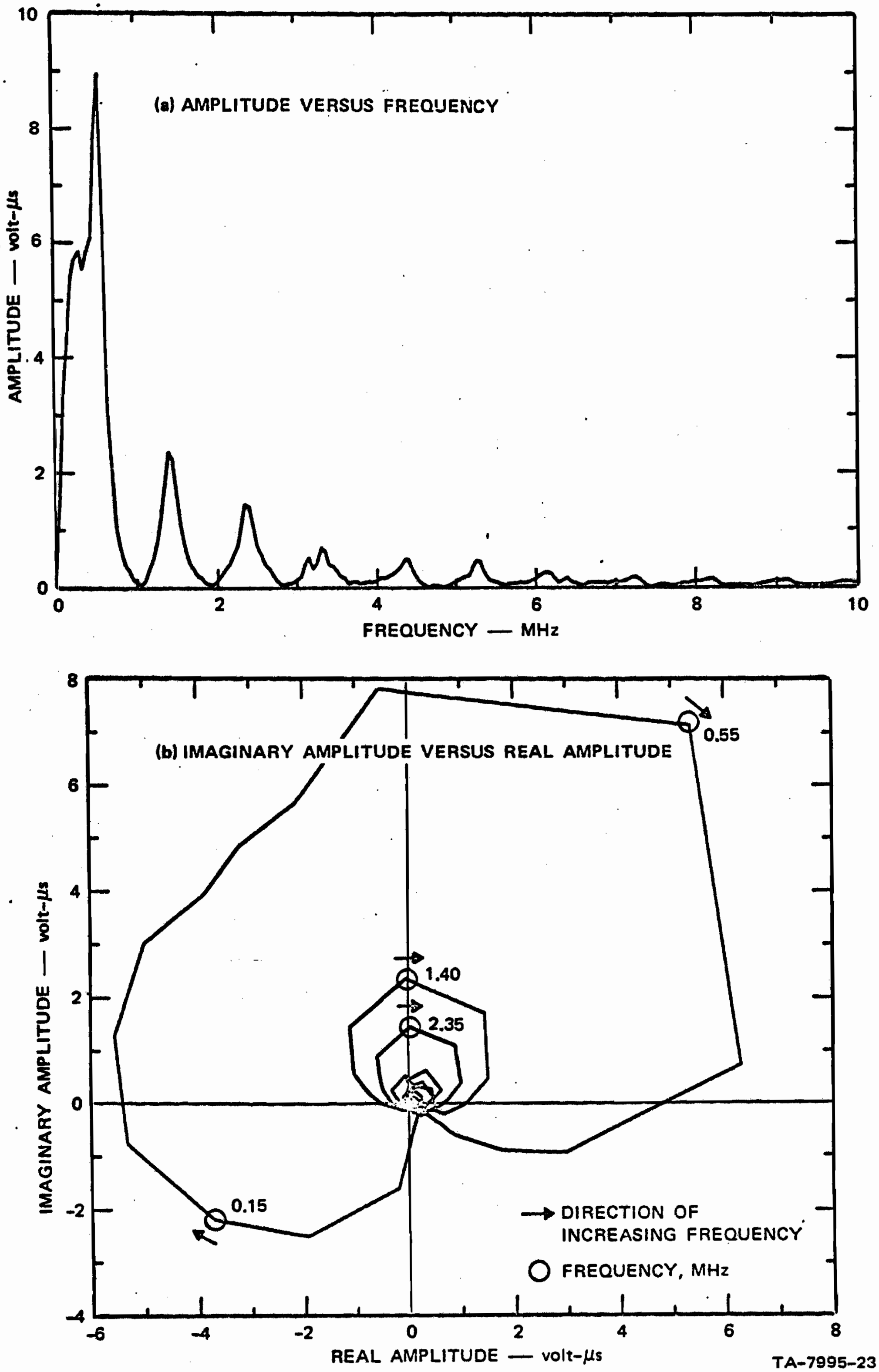


FIGURE 15 FOURIER AMPLITUDE OF THE REFERENCE H PULSE (RC-INTEGRATED B-DOT)

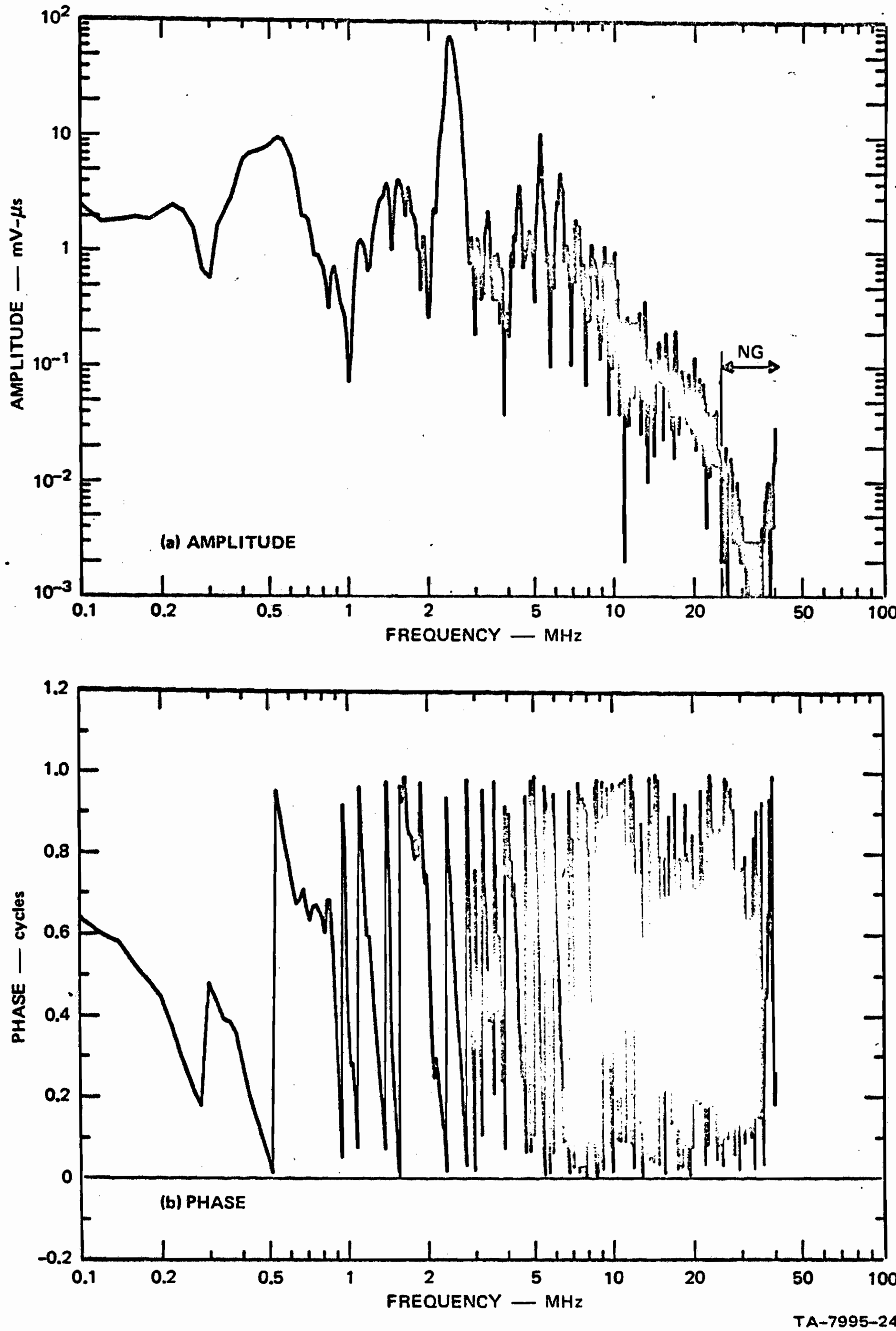


FIGURE 16 FOURIER AMPLITUDE AND PHASE OF THE SYSTEM TEST PULSE—LOGARITHMIC FREQUENCY

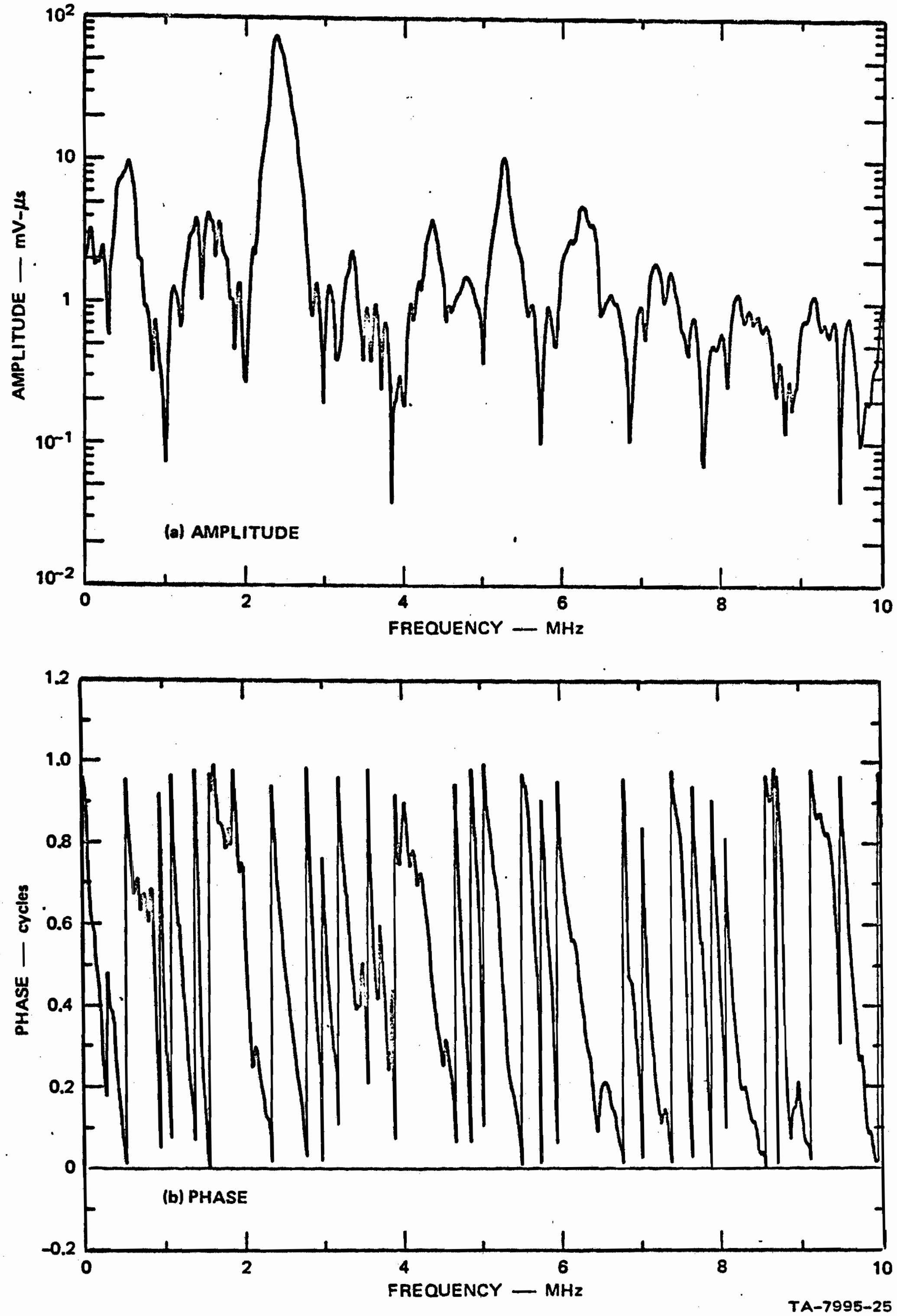


FIGURE 17 FOURIER AMPLITUDE AND PHASE OF THE SYSTEM TEST PULSE— LINEAR FREQUENCY

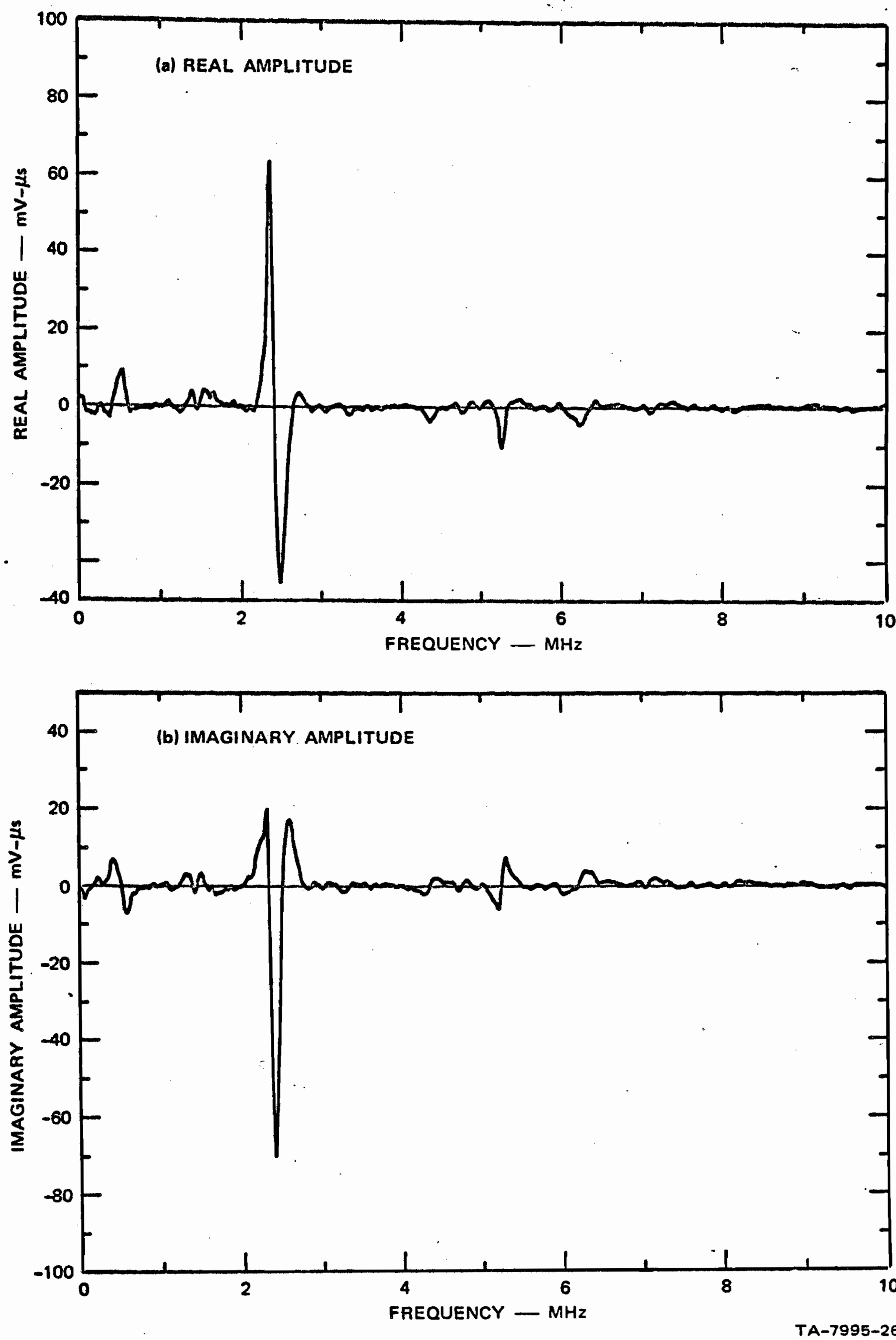


FIGURE 18 REAL AND IMAGINARY PARTS OF THE FOURIER AMPLITUDE OF THE SYSTEM TEST PULSE

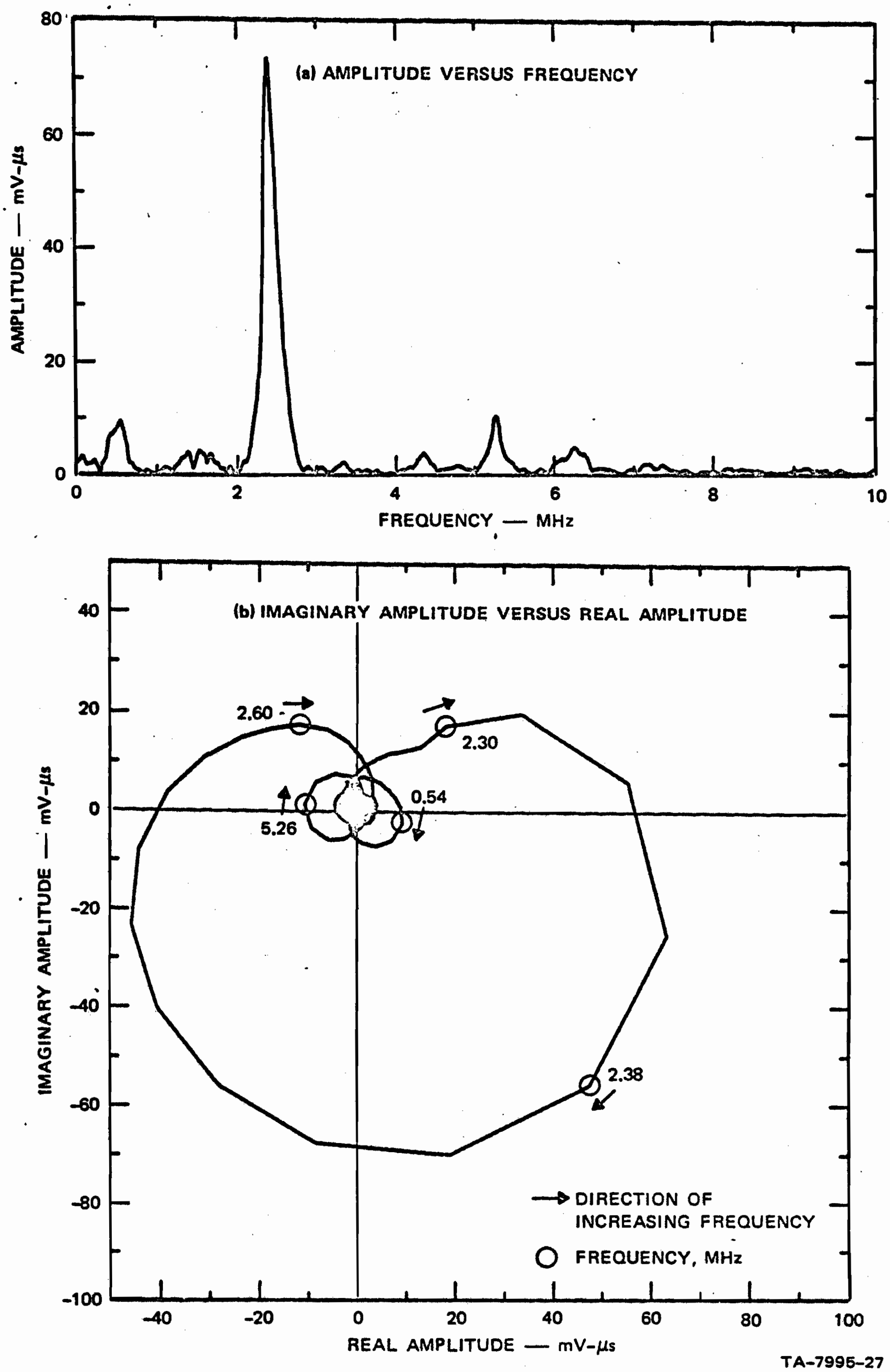


FIGURE 19 FOURIER AMPLITUDE OF THE SYSTEM TEST PULSE

III RESULTS

The results presented herein are in fact the products of the data-reduction system, showing the time history of the reference pulse, the time history of the systems test pulse, the Fourier Transform of the reference pulse, the Fourier Transform of the system test pulse, and the transfer function.

The validity of the digital composites of the pulse-time histories is judged by comparing the digital composites to the original oscilloscope traces with one criterion in mind: the digital composite must appear in every way identical to the original traces. Before proceeding with this application of this criterion, one qualification must be clearly stated: establishing a relationship of the oscilloscope traces, and hence of the digital composite waveform, to the actual time waveform recorded was possible in this case only by assuming that the oscilloscope amplification and deflection sensitivities were linear (or by some other equally, or more arbitrary assumption).

Comparison of the digital composite representation of the reference pulse (Figures 7 and 8) to the polaroids of the oscilloscope records of the reference pulse (Figure 1) shows that the composite is a good representation of the reference pulse, with three qualifications:

1. Since the absolute amplitudes of the various oscilloscope records do not agree with one another, the absolute amplitude of the composite does not agree with all oscilloscope records--an observation which does cast some doubt on the validity of the composite.

2. Owing to this uncertainty in amplitude, it was not possible to use the amplitude of the different records as an aid in piecing them together to form the composite, so that, due to the faintness in definition of the onset of the waveform in Records No. 2 and No. 3, it is possible that there may be one too many, or one too few, cycles of about 150 MHz riding on the onset of the composite waveform.

3. On the interval 150 to 350 ns (this segment was taken from Record No. 3, the maverick*) the amplitude of the composite is seen to be increasing slightly in magnitude, while the amplitude on this interval appears to be decreasing slightly on oscilloscope Records No. 4 and No. 5.

Comparison of the digital composite representation of the system test pulse (Figure 9) to the polaroids of the oscilloscope records of the system test pulse (Figure 2) shows that the composite is a good representation of the system test pulse, with no qualifications.†

The validity of the Fourier Transforms of the composite pulses is ascertained by comparison with the time histories of the composite pulses. Inasmuch as the computer program that performs the Fourier integration has been thoroughly checked out, the question of validity is really the question of determining the valid range of frequency and the valid range of amplitude of the Fourier Transform (bandwidth over which the time

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* Refer to the paragraph on removal of differences in recording system characteristics during data reduction, Section II-B.

† The high-frequency noise (50 MHz and greater) seen riding on the composite waveform (particularly noticeable at the onset as jitter at about 200 MHz) is not present in the composite. This noise has been introduced by a malfunctioning plotter (CRT).

history affords sufficient resolution in time and amplitude) and the question of determining to what extent the resolution in frequency, amplitude, and phase is sufficient.

Comparison of the Fourier Transform of the reference pulse (Figures 12-15) to the time history of the reference pulse (Figures 7 and 8) shows that the Fourier Transform is reliable over the frequency range 100 kHz to greater than 100 MHz and over an amplitude range of about 80 dB:* the range of the oscillations (varying between about 10-20 dB) about the trend of the amplitude is approximately the same throughout the entire frequency range (Figure 12(a)); the repetitive amplitude minima at about 950-kHz intervals (Figures 12(a) and 13(a)) correspond to the square-wave nature of the pulse (Figure 8(c)), which has a period a little greater than 2 μs; the amplitude maximum (spike at 540 kHz) would seem to correspond with the slightly greater than 500-kHz dominant frequency seen following the first cycle or so of the square wave; the amplitude spike at about 3.15 MHz and the following amplitude disturbances

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* Strictly speaking, a claim of 80-dB range in Fourier amplitude cannot be justified. However, intuition, past experience, and a statistical approach indicate that the claim is not unreasonable. From inspection of the two Fourier Transforms (Figures 12(a) and 16(a)) one sees that the transforms seem to preserve their character in regions where the amplitudes are 60 to 80 dB down from peak amplitude and do not seem to be appreciably noisier in these regions. The computer program that performs the Fourier Integral calculation was checked out with a trial waveform, the time-amplitude values of which were calculated from a known Fourier Transform, plotted with a thick pen, and digitized from the plot. The calculated Fourier Transform was found to be reliable over a 60-dB range for this one trial waveform. Comparisons made during past experiments of pulse-derived and CW-derived transfer functions have shown that agreement over ranges in excess of 60 dB is not uncommon. One statistical approach (presented in the Appendix as a continuation of this footnote) demonstrates that the range of reliable amplitudes for the reference pulse Fourier Transform is probably greater than approximately 60 dB and for the system test pulse is probably greater than approximately 50 dB.

fall in the band containing the dominant frequency distortions seen on the "square" wave; the upswing in the trend of the Fourier amplitude after about 60 MHz is believable, since it is necessary to account for the high-frequency ringing seen riding on the onset of the pulse (Figure 7); etc. Comparison of the real-amplitude-versus-frequency and imaginary-amplitude-versus-frequency plots (Figure 14), as well as the phase plot (Figure 13(b)) and polar plot (Figure 15), seems to indicate that the repetitive minima referred to above are true "zeroes" in the Fourier Transform, but these indications are inconclusive. The polar plot (Figure 15(b)) does indicate that the Fourier Transform has not been calculated at frequencies spaced sufficiently close together.

Comparison of the Fourier Transform of the system test pulse (Figures 16-19) to the time history of the system test pulse (Figure 9) shows that the transform is reliable over the frequency range 200 kHz to about 30 MHz and over an amplitude range of about 80 dB:* the large resonance at about 2.4 MHz (Figure 19(a)) corresponds to the dominant ringing of the system test pulse; the repetitive amplitude minima at about 970-kHz intervals is seen as a strong modulation of the amplitude envelope of the pulse, which modulations appear quite noticeably as a boosting about every 1 μs; the peak in Fourier amplitude at about 540 kHz can be seen in the time history of the pulse as a component riding under the resonant ringing; the very sharp minimum in amplitude at about 10.86 MHz corresponds to a distortion at about this frequency seen in the onset of the pulse; etc. It is difficult to tell whether or not the repetitive minima referred to above are true "zeroes." The polar plot (Figure 19(b)) indicates that the Fourier Transform has probably been calculated with sufficiently close frequency spacing. The Fourier Transform is not reliable beyond about 30 MHz.

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* See footnote on preceding page.

Table I

SUNDRY PARAMETERS AND CONSIDERATIONS PERTAINING TO THE PROCESSING OF THE PULSE RADAR

	Parameter	Waveform								Impulse Response		
		Reference Pulse					System Test Pulse					
		Record No. 1	Record No. 2	Record No. 3	Record No. 4	Record No. 5	Record No. 1	Record No. 2	Record No. 3			
time domain, f(t)	Number of samples, n_t	77	156	328	1481	1665	291	247	698	Not calculated		
		(3707 total)					(1236 total)					
	Time of first sample on interval, t_1	0 (μs)	0.038	0.116	0.444	1.925	0	1.45	3.92			
	Time of last sample on interval, t_2	0.038 (μs)	0.116	0.444	1.925	10.250	1.45	3.92	17.88			
	Time increment between samples, τ	0.5 (ns)	0.5	1.0	1.0	5.0	5	10	20			
	Sampling rate, $\nu_s = 1/\tau$	2000 (MHz)	2000	1000	1000	200	200	100	50			
	Corner (0 dB) of smoothing filter, ν_o	300 (MHz)	300	150	100	25	18	15	8			
	Half-width of filter cutoff (-6 dB), $\Delta\nu_{1/2}$	100	100	50	75	10	9	7.5	4			
$f(t) \rightarrow g(\nu)$	Highest meaningful frequency component, $\nu_{1/2} = \nu_o + \Delta\nu_{1/2}$	400 (MHz)	400	200	175	35	27	22.5	12			
	Highest possible frequency component, $\nu_n = 1/2\ \nu_s$	1000 (MHz)	1000	500	500	100	100	50	25			
	Coarsest sampling increment (frequency), $\delta\nu_{max} = 1/2\ t_2$	→	4310 (kHz)	1126	260	49	345	128	28			
frequency domain, $g(\nu)$	Sampling rate, T_s				4 (μs)	20	5	10	50	5	10	50
	Frequency increment between samples, $\delta\nu = 1/T_s$				250 (kHz)	50	200	100	20	200	100	20
	Frequency of first sample on interval, ν_1				40 (MHz)	0	30	15	0	30	15	0
	Frequency of last sample on interval, ν_2				100 (MHz)	40	40	30	15	40	30	15
	Number of samples, n_ν				240	801	50	150	751	50	150	751
					(1041 total)		(951 total)			(951 total)		
$g(\nu) \rightarrow f(t)$	Latest possible time component, $T_n = 1/2\ T_s$				2 (μs)	10	2.5	5	25	2.5	5	25
	Coarsest sampling increment (time), $\tau_{max} = 1/2\ \nu_2$				5 (ns)	12.5	12-1/2	16-2/3	33-1/3	12-1/2	16-2/3	33-1/3

Comparison of the Fourier Transform of the system test pulse with the Fourier Transform of the reference pulse shows that the two pulses are related, in that they have repetitive minima at about the same spacing in frequency (although apparently not identical spacing), somewhat equivalent "trends" in amplitude, and some significant features (such as a resonance at about 540 kHz) in common. They differ in that the Fourier Transform of the system test pulse is, for lack of a better term, more complicated; they also differ in other significant features (such as a second harmonic of the 2.4-MHz resonance appearing in the system test pulse Fourier Transform where the reference Fourier Transform shows a sharp, double-spiked minimum).

The validity of the transfer function (Figures 20-22) depends not only on the validity of the two Fourier Transforms used in its calculation, but to a very great extent on how well the two transforms line up against one another. Reliable features of the transfer function are its general trend (attenuation of the response below the 2.4-MHz resonance, boosting of the response above the 2.4-MHz response, and a roll-off in response starting at about 20 MHz), and by and large its detailed character (amplitude maxima and minima consistent in spacing with those for the system test pulse Fourier Transform and the amplitude range of this detail (10-30 dB) equivalent with the range (10-20 dB) for the system test pulse Fourier Transform). Somewhat disconcerting features are the double-spiked maximum at about 4.8 MHz, which is noticeable on the reference Fourier Transform as a minimum, and some evidence that the repetitive minima (zeroes ?) of the two transforms are not lining up. Just how disconcerting these features should be is difficult to assess: the double-spiked maximum is somewhat believable (the maximum, anyway, if not the double-spike) in that there is evidence in the system test

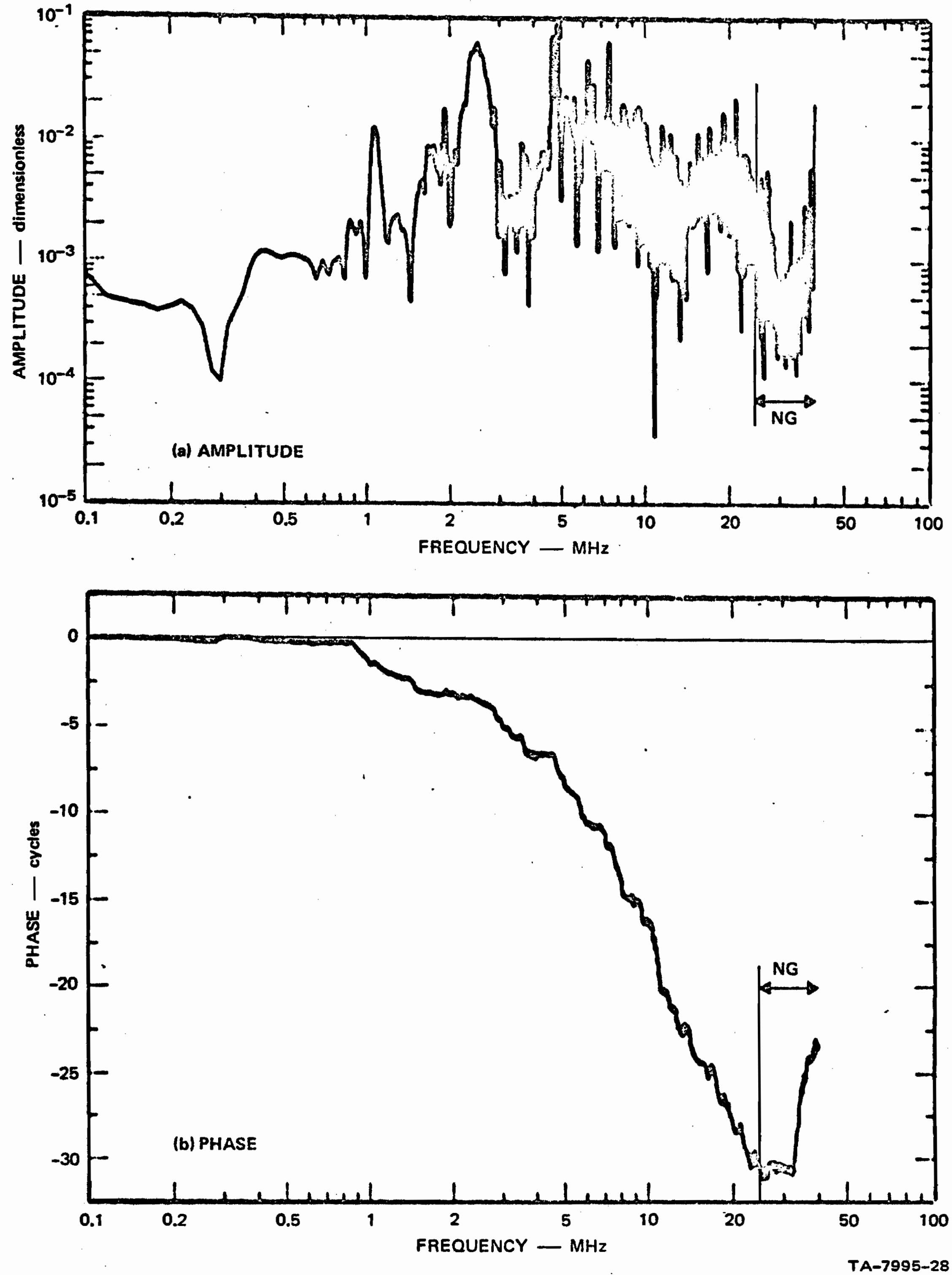

FIGURE 20 FOURIER AMPLITUDE AND PHASE OF THE TRANSFER FUNCTION—LOGARITHMIC FREQUENCY

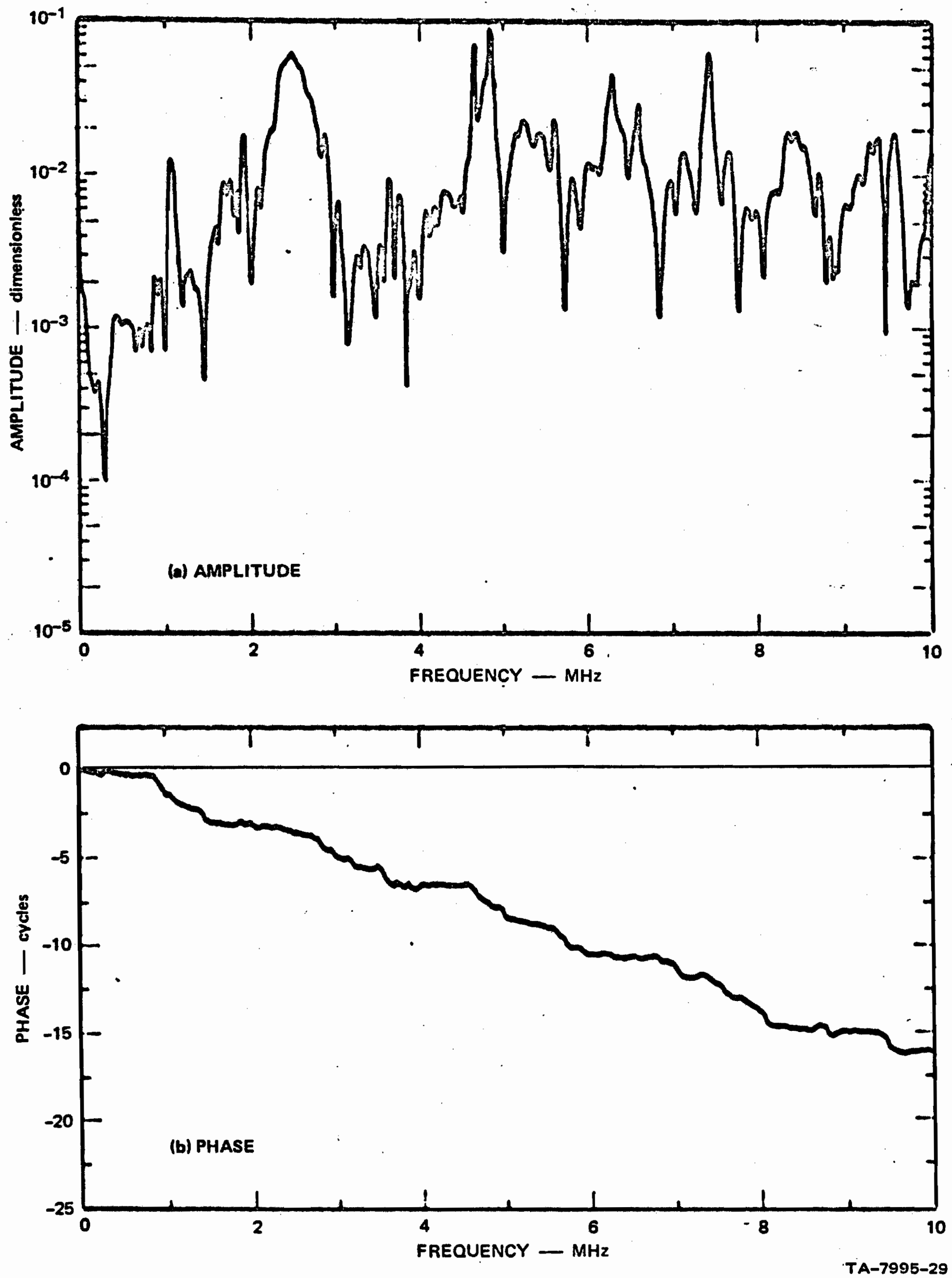

FIGURE 21 FOURIER AMPLITUDE AND PHASE OF THE TRANSFER FUNCTION—LINEAR FREQUENCY

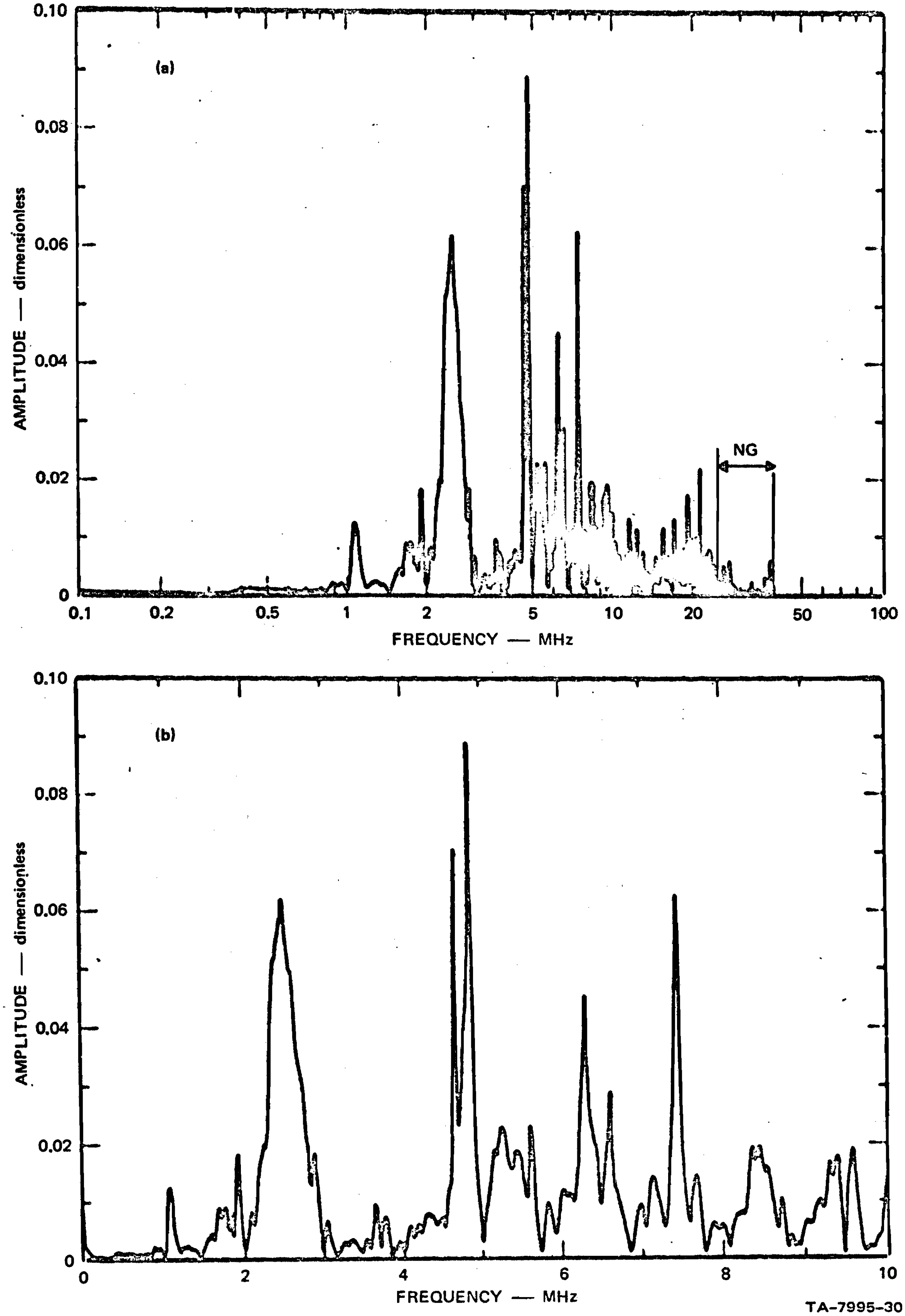

FIGURE 22 FOURIER AMPLITUDE OF THE TRANSFER FUNCTION

pulse transform that there is a resonance at about 4.8 MHz; the possible mismatch of zeroes is confused by the fact that the effect on the transfer function of the typical double-spike character of such a mismatch is apparently in character with the systems test pulse transform.

IV CONCLUSIONS

Since the character of the transfer function is consistent with the character of the system test pulse Fourier Transform, the transfer function is probably fairly reliable over the frequency range 200 kHz to 25 MHz. However, there are two unfortunate aspects to the calculation of this transfer function, both aspects directly attributable to the square-wave nature of the reference pulse, e.g., the repetitive minima in the reference Fourier Transform. It is safe to say that the repetitive minima in the reference pulse transform and in the system test pulse transform were probably not lined up exactly in calculating the transfer function. The seriousness of this misalignment is difficult to judge. The effect on this alignment of inadequate calibration information for the pulse time histories and of inadequate sampling of the reference pulse transform is a matter of speculation; but one can assert without speculating that adequate calibration and sampling would improve at least the analysis of the alignment problem by removing some uncertainties.

The second unfortunate aspect is the appearance of a probable resonance in the response of the test system at a point (approximately 4.8 MHz) where the reference pulse transform has a marked minimum. Consequently, this resonance is not well defined. In addition to the two unfortunate aspects, it should be noted that the transfer function (Figures 20-22) has not been corrected for the responses of the recording systems.

It is anticipated that the processing of data similar to that presented here would yield similar results. The transfer function so obtained would be by and large reliable but would suffer two defects: (1) possible mismatch of minima in the Fourier Transforms of the test and reference pulses, exacerbated by inadequate calibration of the pulse time-domain waveforms (but presumably ameliorated by calculating the transforms at the appropriate frequencies the next time around), and (2) diminished resolution of features of the transfer function occurring near those minima. In order to obtain a meaningful transfer function, it will also be necessary to remove the effects of the recording systems.

Since it may be surmised that a great deal of attention must be paid to accurately recording and processing the data in order to obtain a transfer function, it may be worthwhile to inquire into the usefulness of the transfer function concept as applied to the purposes of the experiment. The sole purpose of the transfer function is to explicitly relate the output of a system to the input to that system and, in relating the output to the input, to make it easier to predict the response of the system to a different input. The calculation of a transfer function (or some equivalent relationship) is necessary if the response of the system is to be calculated for a variety of different theoretical inputs or if experimental results obtained using different source functions (inputs) are to be compared directly. The calculation of a transfer function is a waste of time if different source functions, theoretical or experimental, are not to be considered or if the actual source function has a spectrum sufficiently flat across the bandwidth of interest to be used as a good approximation to an impulse. Additionally, the transfer function usefulness is necessarily limited to systems of linear response characteristics, although the calculation of transfer functions may be useful in pinpointing areas of nonlinear response.

In an EMP interaction study, where the source function is a simulated EMP, calculation of transfer functions is useful only if (1) the response of the system, or critical sybsystems thereof, exposed to the simulated

EMP is linear and (2) the assumed threat EMP is significantly different in form from the simulated EMP, or if the transfer functions are used to define the linear parts of the system so that a simulated pulse, such as from a cable-driver, may be injected directly into the system to investigate the nonlinear subsystems.

The burden of addressing oneself to the question of whether or not any transfer function obtained by considering one component of the electromagnetic field is meaningful to the purpose of the experiment has not been assumed by this writer. His feeling--and it is only that--is that the transfer function is also meaningful to the extent that the reference pulse can be described as in the first paragraph of the Introduction.

Appendix

DYNAMIC RANGE IN THE FREQUENCY DOMAIN

Given a continuous function of time, f(t), defined over the interval [0,T], suppose we have a measured version of f(t), which is contaminated by an error function $\varepsilon(t)$ such that

$$\tilde{f}(t) = f(t) + \varepsilon(t) \quad .$$

The average power in the desired signal is proportional to

$$P_s = \frac{1}{T}\int_0^T f^2(t)dt \quad ,$$

and the power in the error signal is proportional to

$$P_\varepsilon = \frac{1}{T}\int_0^T \varepsilon^2(t)dt \quad .$$

Suppose we can define a maximum f(t), f_{max}. Then the maximum P_s is

$$P_{smax} = \frac{1}{T}\int_0^T f^2_{max}\, dt$$

$$= f^2_{max} \quad .$$

The error functions will be assumed zero-mean, random, and stationary (such that the expected value of the squared error function will be independent of time). Then the expected value of P_ϵ will be

$$E\{P_\epsilon\} = \frac{1}{T}\int_0^T E\{\epsilon^2(t)\}dt$$

$$= \frac{1}{T}\int_0^T \sigma_\epsilon^2 dt$$

$$= \sigma_\epsilon^2 \quad .$$

A maximum signal-to-noise ratio (SNR) may be defined as

$$\Lambda = \frac{P_{smax}}{E\{P_s\}} = \frac{f_{max}^2}{\sigma_\epsilon^2} \quad .$$

The sources of the errors have been discussed in the body of the report, and it may be realistically assumed that the standard deviation of the error is proportional to f_{max},

$$\sigma_\epsilon = \alpha f_{max} \quad ,$$

and the maximum SNR becomes:

$$\Lambda = \frac{1}{\alpha^2} \quad .$$

Our primary interest lies in the frequency domain representation of the signal $\tilde{f}(t)$. We arrive at such a representation by applying the finite-time, discrete Fourier Transform to the sample values of $\tilde{f}(t)$.

$$\tilde{g}(j\Delta) = \tau \sum_{k=0}^{N-1} \tilde{f}(k\tau)e^{-i2\pi kj\Delta\tau}$$

$$= \tau \sum_{k=0}^{N-1} f(k\tau)e^{-i2\pi kj\Delta\tau} + \tau \sum_{k=0}^{N-1} \epsilon(k\tau)e^{-i2\pi kj\Delta\tau} , \quad \text{(A-1)}$$

$$j = 0, 1, 2, \ldots, N - 1 \quad .$$

The quantities τ and Δ are the intervals between samples in the time domain and in the frequency domain, respectively. Uniform sampling at a sufficient rate is assumed and

$$T = (N - 1)\tau \quad .$$

The quantity $\tilde{g}(j\Delta)$ represents an averaged version of $\tilde{g}(\nu)$--averaged over the interval $[j\Delta - \Delta/2, j\Delta + \Delta/2]$. This results from the fact that the time waveform is available only for a finite interval $[0, T]$. The frequency band, Δ, is given by

$$\Delta = \frac{1}{T} = \frac{1}{(N - 1)\tau} \quad . \quad \text{(A-2)}$$

Suppose that the signal energy is contained in only one of these bands and to illustrate let $f(k\tau)$ be equal to f_{max}. By using Eq. (A-2), the first term in Eq. (A-1) becomes

$$\tau \sum_{k=0}^{N-1} f_{max} e^{-i\left(\frac{2\pi kj}{N - 1}\right)} = f_{max} N\tau \quad .$$

The signal power in the narrow band Δ Hz in bandwidth is proportional to

$$f_{max}^2 = P_{smax} \quad .$$

We also know that the power in the error signal is proportional to σ_ϵ^2. If we further assume that the errors associated with different sample values are mutually independent, it is reasonable to conclude that the power spectral density of the error signal is essentially uniform with respect to frequency in the band $(N - 1)\Delta$ Hz wide. The error signal power in a band Δ Hz wide is proportional to $\sigma_\epsilon^2/(N - 1)$.

With the signal concentrated in a narrow band, we have in the frequency domain an improved SNR given by

$$\Lambda_\nu = \frac{f_{max}^2 (N - 1)}{\sigma_\epsilon^2} = \Lambda(N - 1) \quad .$$

It is unrealistic to assume that all of the signal energy is concentrated in a single narrow frequency band, so we will define a reduction factor for the SNR in terms of the ratio of the value of g at peak resonance to the $f_{max} N\tau$. The modified SNR becomes

$$\Lambda'_\nu = \Lambda(N - 1)\left(\frac{g \text{ pk. res.}}{f_{max} N\tau}\right)^2$$

$$= (N - 1)\left(\frac{g \text{ pk. res.}}{\alpha f_{max} \tau}\right)^2 \quad .$$

The dynamic range that may be anticipated in the frequency domain analysis is approximately the SNR and expressed in dB is

$$10 \log_{10} \left[(N - 1) \left(\frac{g \text{ pk. res.}}{\alpha f_{max} T} \right)^2 \right] .$$

We now apply this estimate to the pulse data. Referring to the oscilloscope traces of the waveforms (Figures 1 and 2), we see that $\alpha = 0.01$ **(1 percent of peak amplitude) is a reasonably conservative estimate.** For the reference pulse (refer also to Figures 8(b) and 15(a) and to Table I), we have

$$\alpha = 0.01$$

$$f_{max} \approx 5.5 \text{ V}$$

$$T \approx 10 \ \mu s$$

$$g \text{ peak resonance} \approx 9 \text{ V-}\mu s$$

$$N = 3707 .$$

Thus, the reliable range in spectral amplitude for the reference pulse is 60 dB from peak resonance.

For the system test pulse (refer also to Figures 9(d) and 19(a) and to Table(I), we have

$$\alpha = 0.01$$

$$f_{max} = 48 \text{ mV}$$

$$T \approx 20 \mu s$$

$$g \text{ peak resonance} \approx 74 \text{ mV-}\mu s$$

$$N = 1236 .$$

The reliable range in spectral amplitude for the system test pulse is 49 dB from peak resonance.