Lightning Phenomenology Notes

Note 19

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Return-Stroke Initiation

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Abstract

The far fields from a lightning return stroke are a function of the spatio-temporal distribution of the return-stroke currents. This paper introduces a conical-transmission-line model of the return stroke immediately following leader closure. Based on this model, return-stroke speed is computed as well as the far fields. The impact of this model, as well as other factors, on the far fields is estimated.

I. Introduction

The structure of the currents in a lightning return stroke is rather complex and relatively little is known. While some evidence indicates a propagation speed of c/3 is appropriate at late times, there are limits in the time resolution which leave this uncertain. This paper gives another approximate model showing that the speed may be near c at early times, and that geometric considerations may introduce another factor of 2 in the relation of far fields to current.

A conical transmission-line model is introduced, appropriate to early times in the return stroke. This leads to a current increasing proportional to time, and extending over a length also proportional to time. Performing the requisite integral far fields are related to the currents, showing the dependence of the far fields on the spatial form of the current.

Combining this model with other factors, various estimates of the ratio of far fields to return-stroke currents are possible. An order-of-magnitude variation in this ratio is shown to be plausible. This makes estimation of return-stroke currents from far fields similarly uncertain.

II. Conditions Immediately Prior to Return Stroke

A previous paper [4] discusses the propagation of a leader tip through air. While this view is somewhat idealized, it can still provide some insight into some of the physical processes involved. In this model the leader tip is approximated as a circular cone, in which there is a conducting core carrying most of the current along the axis. The charge is assumed to primarily reside on the conical surface, forming the surface of some (expanding) corona. This cone propagates along its axis at some speed bounded by the speed of light.

Consider now that this leader tip approaches a conducting plane as indicated in fig. 2.1. This plane could also be an approximate symmetry plane between downward and upward propagating leaders. The lower leader in the figure might then be an upward propagating leader or an image of the downward propagating leader. On the $z = 0$ plane the electric field is parallel to the z axis. With respect to this symmetry plane the fields are antisymmetric [8].

Of course, one may question the suitability of a conical model of a leader approaching closure to form a return stroke. However, it has some attractive features. As the leader propagates through the air, the corona near the tip has had only a small time to propagate outward, while farther behind the tip the corona has had more time to propagate outward. This argues in the direction of a small corona near the tip with a larger corona farther back. For present purposes let us approximate this as a cone. This is also the static limit for air breakdown around a wire connected normal to a conducting ground plane [3].

If there is a constant electric field normal to the surface of the corona (say the "breakdown" field E_b of a few MV/m), then the surface charge density (assuming most of the charge near the corona surface) is a constant $\varepsilon_0 E_b$. Let the corona radius (for $z > 0$) be

$$\Psi_0(z) = z \tan(\alpha_0) \tag{2.1}$$

so the charge per unit length is

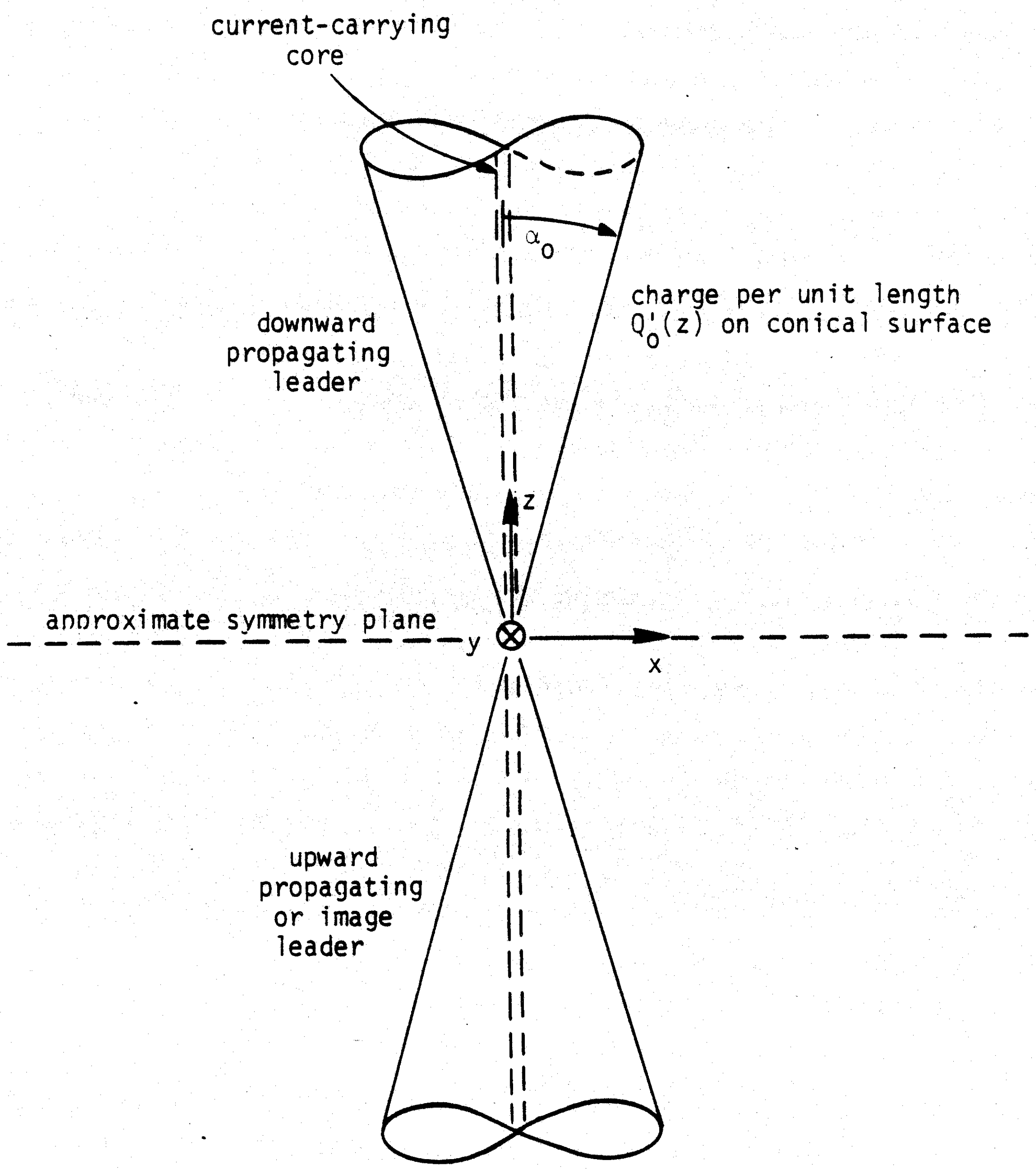

Fig. 2.1. Leader(s) Immediately Prior to Return Stroke

$$Q_0'(z) = -2\pi\varepsilon_0 E_b \frac{1}{\cos(\alpha_0)} \Psi_0(z)$$

$$= -2\pi\varepsilon_0 E_b \frac{\sin(\alpha_0)}{\cos^2(\alpha_0)} z$$

$$= Kz$$

$$K = -2\pi\varepsilon_0 E_b \frac{\sin(\alpha_0)}{\cos^2(\alpha_0)} \simeq -2\pi\varepsilon_0 E_b \alpha_0 \quad \text{for small } \alpha_0 \tag{2.2}$$

Note the minus sign, consistent with negative charge in the cloud propagating to earth.

As discussed in [4] the leader itself radiates an electromagnetic signal, but that is not the subject here. For present purposes we approximate the leader cone as stationary compared to the wave which propagates up the cone after closure at $z = 0$. As discussed later, this return-stroke wave propagates up the cone at near the speed of light.

III. Corona Model of Return Stroke

As discussed in [1,7] one can form a transmission-line model of the lightning arc. The telegrapher equations are

$$
\begin{aligned}
\frac{\partial V}{\partial z} &= \frac{\partial}{\partial z}\left(\frac{Q'}{C'}\right) = -L' \frac{\partial I}{\partial t} \\
\frac{\partial I}{\partial z} &= -\frac{\partial Q'}{\partial t} = -\frac{\partial}{\partial t}(C'V) \quad \text{(continuity)}
\end{aligned}
\tag{3.1}
$$

Here the voltage is defined with respect to some equivalent (approximate) reference conductor. This has been taken as some circular cylinder (centered on the z axis) of radius Ψ_∞. In the region near arc closure at $\vec{r} = \vec{0}$ a conical transmission line is more appropriate for the wave expanding from $\vec{r} = 0$. Note that

$$
\begin{aligned}
\vec{r} &= x\vec{1}_x + y\vec{1}_y + z\vec{1}_z \quad \text{(cartesian coordinates)} \\
&= \Psi\vec{1}_\Psi + z\vec{1}_z \quad \text{(cylindrical coordinates)} \\
&= r\vec{1}_r \quad \text{(spherical coordinates)} \\
\Psi^2 &= x^2 + y^2 \\
r^2 &= \Psi^2 + z^2 \\
x &= \Psi \cos(\phi) \\
y &= \Psi \sin(\phi) \\
\Psi &= r \sin(\theta) \\
z &= r \cos(\theta)
\end{aligned}
\tag{3.2}
$$

Now for small θ we have $r \approx z$, so r and z are roughly interchangeable in this case for the variables in the transmission-line model.

Now in the corona model we have in spherical coordinates for the inductance per unit length

$$L' = \mu_0 f_L$$

$$f_L = \frac{1}{2\pi} \ell n[\cot(\frac{\beta}{2})] \simeq \frac{1}{2\pi} \ell n(\frac{2}{\beta}) \text{ for } \beta \to 0 \tag{3.3}$$

β = effective half-angle in conical approximation of current carrying core

The capacitance term is different in that it depends on the corona. Before the return stroke it is governed by the cone half-angle α_o as

$$C'_o = \frac{\varepsilon_o}{f_{C_o}}$$

$$f_{C_o} = \frac{1}{2\pi} \ell n[\cot(\frac{\alpha_o}{2})] \simeq \frac{1}{2\pi} \ell n(\frac{2}{\alpha_o}) \quad \text{for } \alpha_o \to 0 \tag{3.4}$$

$\alpha_o \equiv$ effective half angle in conical approximation of charge carrying corona before return stroke

After the return stroke passes the capacitive term is different. In general the corona is depleted to some degree with an effective half angle α_1 as

$$C'_1 = \frac{\varepsilon_o}{f_{C_1}}$$

$$f_{C_1} = \frac{1}{2\pi} \ell n[\cot(\frac{\alpha_1}{2})] \simeq \frac{1}{2\pi} \ell n[\frac{2}{\alpha_1}] \quad \text{for } \alpha_1 \to 0 \tag{3.5}$$

$\alpha_1 \equiv$ effective half angle in conical approximation of charge carrying corona after return stroke

This term (α_1) is discussed in a later section.

Note that for present purposes the $z = 0$ symmetry plane is taken as reference conductor in the transmission-line model. So our present calculations are in terms of the corona of one leader ($z > 0$) and are applicable to the other leader by symmetry.

Now in the usual corona model [1,7] we assume a corona "radius" or angle α in this case related by some breakdown electric field E_b as in (2.1) and (2.2). As α changes (from say α_o) the charge per unit length and the capacitance both change. As discussed before, this gives a nonlinear set of transmission-line equations. For a transmission line with a constant capacitance per unit length the wave velocity is

$$v_o = [L'C']^{-1/2} = c\,\frac{\ell n[\cot(\frac{\beta}{2})]}{\ell n[\cot(\frac{\alpha}{2})]}$$

$$= c \quad \text{for } \beta = \alpha \tag{3.6}$$

For one with varying capacitance per unit length (as a function of Q') we have

$$v(Q') = \left\{\frac{1}{L'}\,\frac{d}{dQ'}\left[\frac{Q'}{C'(Q')}\right]\right\}^{1/2} \tag{3.7}$$

For a general α, (2.2) gives

$$Q' = -2\pi\varepsilon_o E_b\,\frac{\sin(\alpha)}{\cos^2(\alpha)} \simeq -2\pi\varepsilon_o E_b\alpha \quad \text{for } \alpha \to 0 \tag{3.8}$$

The capacitance per unit length is

$$C' = \frac{\varepsilon_o}{f_c}$$

$$f_c = \frac{1}{2\pi}\,\ell n[\cot(\frac{\alpha}{2})] \simeq \frac{1}{2\pi}\,\ell n[\frac{2}{\alpha}] \quad \text{for } \alpha \to 0 \tag{3.9}$$

$$f_c \simeq \frac{1}{2\pi}\,\ell n[\frac{-4\pi\varepsilon_o E_b}{Q'}] \quad \text{for } Q' \to 0$$

Then we have [1]

$$[\frac{v(Q')}{c}]^2 = \frac{1}{f_L}\,\frac{d}{d|Q'|}\,[|Q'|f_c(Q')]$$

$$= 1 - \frac{\Delta + \frac{1}{2\pi}}{f_L} \tag{3.10}$$

$$\Delta \simeq \frac{1}{2\pi}\,\ell n(\frac{\alpha}{\beta})$$

$$= 0 \quad \text{for } \alpha = \beta$$

In this nonlinear case then even for $\alpha = \beta$ the velocity is less than c but tends to c as f_L becomes large (along with f_c), i.e.,

$$v(Q') \to c \text{ as } f_L,\ f_C \to \infty \quad \text{for constant } f_L - f_C$$
$$\text{or } \alpha, \beta \to 0 \quad \text{for constant } \frac{\alpha}{\beta} \tag{3.11}$$

So it would seem that for thin coaxial leaders the wave velocity can be approximated as c.

IV. Initial Speed of Return-Stroke Electromagnetic Shock Wave

As discussed in the previous section the wave velocity is near c provided the cone angle α of the leader is small, and $|\alpha/\beta|$ is bounded both above and away from zero. Another way to view this matter is to consider an electromagnetic shock wave as in [7]. In this situation let α_0 represent the corona in front of the shock wave and α_1 represent the corona behind.

Considering charge conservation across a shock front moving at speed v we have

$$\begin{aligned} I_0 &= 0 \\ I_1 &= -[Q'_0 - Q'_1]\, v \end{aligned} \tag{4.1}$$

Considering energy conservation there we have the energy per unit length ahead of the shock

$$W'_0 = \frac{1}{2} C'_0 V_0^2 = \frac{1}{2} Q'_0 V_0 = \frac{1}{2} \frac{Q'^2_0}{C'_0} \tag{4.2}$$

Behind the shock

$$W'_1 = \frac{1}{2} \frac{Q'^2_1}{C'_1} + \frac{1}{2} L'_1 I_1^2 \tag{4.3}$$

At the shock interface there is a power (in the +z direction)

$$P_1 = V_1 I_1 = \frac{Q'_1}{C'_1} I_1 \tag{4.4}$$

Conserving energy (no significant additional losses) we have

$$v[W'_0 - W'_1] - P_1 = 0 \tag{4.5}$$

Combining (4.1) and (4.5) together with some assumptions concerning C'_0 and C'_1 (via Q'_0 and Q'_1) gives the electromagnetic shock in [7] which involves a circular-cylindrically shaped corona (at least in front of the shock).

In this paper we make different assumptions concerning the corona, specifically that it is shaped as a circular cone with a fairly small half-cone angle. From (3.4) and (3.5) if α_0 and α_1 represent the corona before and after an EM shock front, respectively, then note that

$$f_{C_o} - f_{C_1} = \frac{1}{2\pi} \ell n \frac{\cot(\frac{\alpha_o}{2})}{\cot(\frac{\alpha_1}{2})}$$

$$= \frac{1}{2\pi} \ell n \frac{\frac{2}{\alpha_o} + O(\alpha_o)}{\frac{2}{\alpha_1} + O(\alpha_1)} \quad \text{as } \alpha_o, \alpha_1 \to 0$$

$$= \frac{1}{2\pi} \ell n(\frac{\alpha_1}{\alpha_o}) \; [1 + O(\alpha_o^2) + O(\alpha_1^2)] \quad \text{as } \alpha_o, \alpha_1 \to 0 \qquad (4.6)$$

For α_1/α_o bounded both above and away from zero, then the above difference is bounded above and below. Normalizing to f_{C_o}

$$\frac{f_{C_o} - f_{C_1}}{f_{C_o}} = \frac{\ell n(\frac{\alpha_1}{\alpha_o})[1 + O(\alpha_o^2) + O(\alpha_1^2)}{\ell n(\frac{2}{\alpha_o})[1 + O(\alpha_o^2)]}$$

$$\to 0 \text{ as } \alpha_o, \alpha_1 \to 0 \text{ for constant } \frac{\alpha_1}{\alpha_o} \qquad (4.7)$$

So the fractional change in the per-unit-length capacitance across the shock front is small.

If there is a discontinuity across a shock front we can consider a wave propagating back from the shock front with

$$\frac{V_1 - V_o}{I_1} = -Z_1 \qquad (4.8)$$

$$Z_1 = \sqrt{\frac{L_1'}{C_1'}} \equiv \text{transmission-line impedance behind shock front}$$

Rewriting (4.1) as

$$I_1 = -[C_0' V_0 - C_1' V_1]\, v$$

$$= \varepsilon_0 \left[\frac{V_1}{f_{C_1}} - \frac{V_0}{f_{C_0}} \right] v$$

$$= C_1' \left[V_1 - \frac{f_{C_1}}{f_{C_0}} V_0 \right] v$$

$$= C_1' \left\{ V_1 - V_0 + [1 - \frac{f_{C_1}}{f_{C_0}}]\, V_0 \right\} v$$

$$= C_1' \left\{ V_1 - V_0 + \frac{\ell n(\frac{\alpha_1}{\alpha_0})\, [1 + O(\alpha_0^2) + O(\alpha_1^2)]}{\ell n(\frac{2}{\alpha_0})\, [1 + O(\alpha_0^2)]} V_0 \right\} v \quad \text{as } \alpha_0, \alpha_1 \to 0$$

$$\to C_1' \{V_1 - V_0\}\, v \quad \text{as } \alpha_0, \alpha_1 \to 0 \quad \text{for constant } \frac{\alpha_1}{\alpha_0} \tag{4.9}$$

Combining these gives

$$v = \frac{1}{C_1' Z_1} = [L_1' C_1']^{-1/2} \quad \text{as } \alpha_0, \alpha_1 \to 0 \text{ for constant } \frac{\alpha_1}{\alpha_0} \tag{4.10}$$

This is just the propagation speed behind the shock front.

Now since

$$C_1' = \frac{\varepsilon_0}{f_{C_1}} = \frac{\varepsilon_0}{f_{C_0}} \frac{f_{C_0}}{f_{C_1}}$$

$$\to C_0' \text{ as } \alpha_0, \alpha_1 \to 0 \text{ for constant } \frac{\alpha_0}{\alpha_1} \tag{4.11}$$

then the propagation speed in front of the shock front is the same as that behind. Interpreting this differently the shock front is not a shock front at all, but rather the wavefront is the same as that on a conventional transmission line.

Going a step further, note that the geometric factor for capacitance purposes approaches that for inductances for narrow half-cone angles, i.e.,

$$f_{C_0} = \frac{1}{2\pi} \ln[\cot(\frac{\alpha_0}{2})$$

$$= f_L \frac{\ln[\cot(\frac{\alpha_0}{2})]}{\ln[\cot(\frac{\beta}{2})]}$$

$$= f_L \quad \text{for } \beta = \alpha_0$$

$$(4.12)$$

$$\frac{f_{C_0} - f_L}{f_L} = \frac{\ln\left[\frac{\cot(\frac{\alpha_0}{2})}{\cot(\frac{\beta}{2})}\right]}{\ln[\cot(\frac{\beta}{2})]}$$

$$\doteq \frac{\ln(\frac{\beta}{2})[1 + O(\alpha_0^2) + O(\beta^2)]}{\ln(\frac{2}{\beta})[1 + O(\alpha_0^2)]}$$

$$\to 0 \quad \text{as } \alpha_0, \beta \to 0 \quad \text{for constant } \frac{\alpha_0}{\beta}$$

In this case since

$$L'_1 \to L'_0 \quad \text{as } \alpha_0, \beta \to 0 \text{ for constant } \frac{\alpha_0}{\beta} \qquad (4.13)$$

then

$$v \to [L'_0 C'_0]^{-1/2} \quad \text{as } \alpha_0, \beta \to 0 \text{ for constant } \frac{\alpha_0}{\beta}$$

$$= [\mu_0 \varepsilon_0]^{-1/2}$$

$$= c \qquad (4.14)$$

The shock-wave solution then gives a speed of c for a thin conical leader upon which the return stroke propagates. Then the nonlinear shock-wave speed tends to the usual transmission-line speed for a thin leader, and this speed tends to c, the speed of light.

V. Solution of Telegrapher Equations for Conical Initial Corona

With the approximation of a constant capacitance per unit length, combining with a wave velocity of c, let us find an appropriate solution of the telegrapher equations. Note that as a wave propagates out from the origin in fig. 2.1 it encounters an increasing $|Q_0'(z)|$ which is to be collapsed. This implies an increasing $|I|$, at least just behind the wavefront. Note that behind the wavefront we assume that the resistance per unit length is negligible. This is partly due to the return-stroke wavefront having passed some position of interest on the current-carrying core. In front of the wavefront only the leader has passed, which we expect carries somewhat less current.

So let us look for solutions of (3.1) with

$$
\begin{aligned}
Z_1 &= \left[\frac{L_1'}{C_1'}\right]^{1/2} \simeq f_{g_1} Z_0 = \text{transmission-line impedance} \\
Z_0 &= \left[\frac{\mu_0}{\varepsilon_0}\right]^{1/2} = \text{wave impedance of free space} \\
f_{g_1} &\simeq f_{L_1} \simeq f_{C_1} \simeq \frac{1}{2\pi}\,\ell n\left[\cot\left(\frac{\alpha_1}{2}\right)\right] \simeq \frac{1}{2\pi}\,\ell n\left(\frac{2}{\alpha_1}\right) \\
\alpha_1 &\equiv \text{effective half angle of return stroke} \\
v &\simeq c = [\mu_0\varepsilon_0]^{-1/2} \equiv \text{speed of light}
\end{aligned}
\tag{5.1}
$$

At the wavefront a jump discontinuity can be handled by the charge conservation requirement in (4.1).

So let us try a current linearly increasing with t which we take as

$$
I_1 = -\zeta c^2 K t \quad \text{for } z < ct \tag{5.2}
$$

with K as in (2.2) and ζ to be computed. Applying (3.1) behind the wavefront note that

$$
\frac{\partial I_1}{\partial z} = 0 = -\frac{\partial Q_1'}{\partial t} \quad \text{for } z < ct \tag{5.3}
$$

so Q_1' is not a function of t. Furthermore

$$
\frac{\partial Q_1'}{\partial z} = -\frac{1}{c^2}\frac{\partial I_1}{\partial t} = \zeta K \quad \text{for } z < ct \tag{5.4}
$$

This has a solution

$$Q_1' = \zeta K z \quad \text{for } z < ct \tag{5.5}$$

where the charge per unit length has been made zero at the origin to keep the symmetry with respect to the z = 0 plane.

Next apply charge conservation at the wavefront (z = ct) from (4.1) giving

$$\begin{aligned} I_1\Big|_{z=ct} &= -\zeta c^2 K t = -c[Q_o' - Q_1']\Big|_{z=ct} \\ &= -c^2[1 - \zeta]\, Kt \\ \zeta &= \frac{1}{2} \quad \text{(a constant)} \end{aligned} \tag{5.6}$$

So the solution in (5.2) and (5.5) can also match the conditions at the wave-front. This solution is quite simple in form and is depicted in fig. 5.1.

Now α_1 is the half-cone angle of the return stroke. This can be estimated from (5.5) and (5.6) with (2.2) as

$$\alpha_1 \simeq \frac{1}{2}\alpha_o \quad \text{for small } \alpha_o \tag{5.7}$$

Estimating the time derivative of the current

$$\frac{dI_1}{dt} = -\zeta c^2 K \simeq \pi\varepsilon_o E_b c^2 \alpha_o \simeq 2\pi\varepsilon_o E_b c^2 \alpha_1 \tag{5.8}$$

and taking an estimate of the breakdown field

$$E_b \simeq 2\,\frac{\text{MV}}{\text{m}} \tag{5.9}$$

and an observed current derivative [6]

$$\frac{dI_1}{dt} \simeq 10^{11}\,\frac{\text{A}}{\text{s}} \tag{5.10}$$

gives

$$\begin{aligned} \alpha_1 &\simeq .01 \text{ radian} \simeq 0.6^0 \\ \alpha_o &\simeq .02 \text{ radian} \simeq 1.2^0 \end{aligned} \tag{5.11}$$

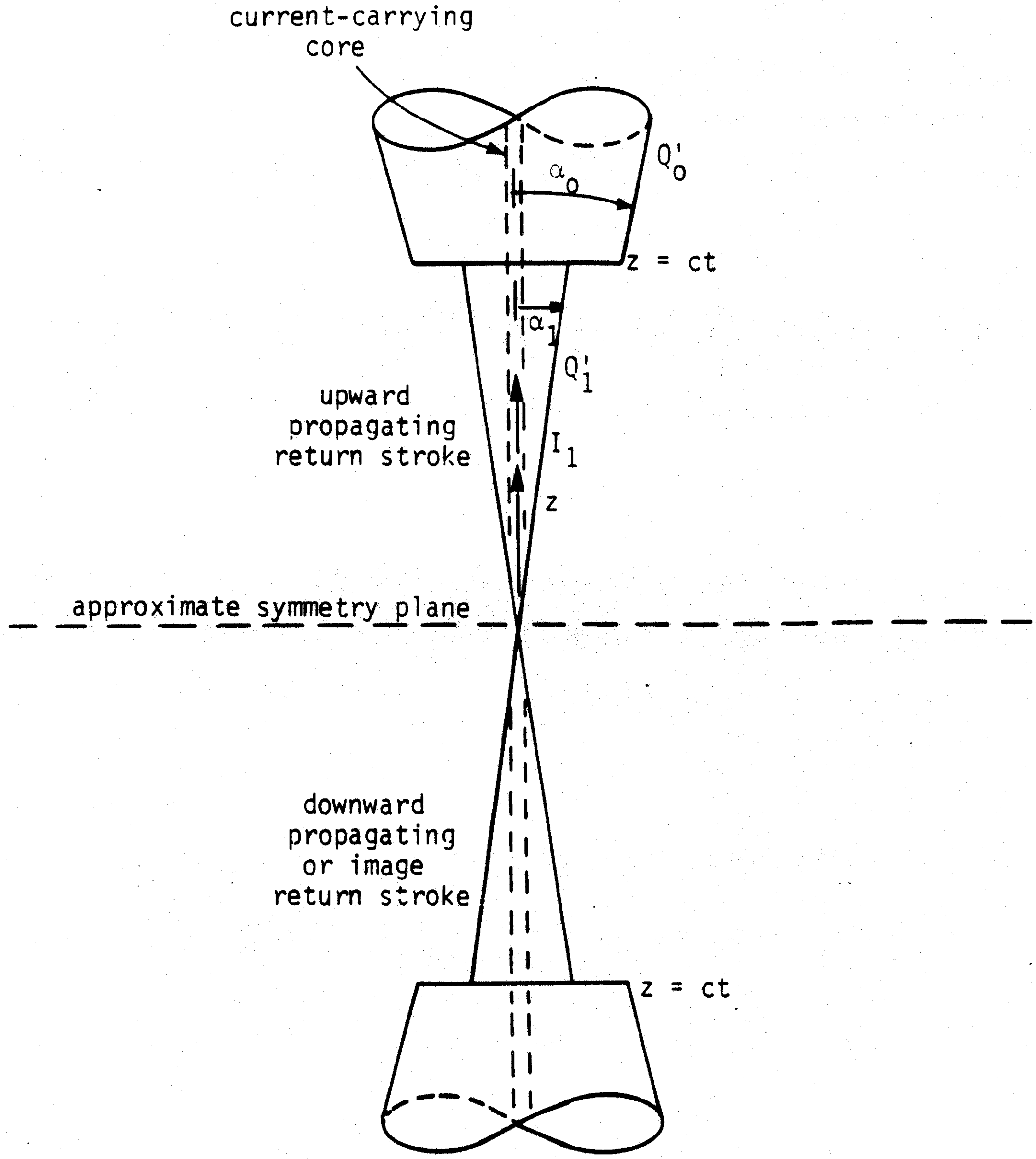

Fig. 5.1. Return Stroke Following Leader Closure

VI. Radiated Fields

As discussed in [2,5] the far fields from lightning can be represented as

$$\vec{E}_f = -\frac{\mu_o}{4\pi r}\,\overleftrightarrow{1}_t \cdot \vec{T}\,(t - \frac{r}{c})$$

$$\vec{H}_f = -\frac{1}{4\pi r_c}\,\vec{1}_1 \times \vec{T}(t - \frac{r}{c}) \qquad (6.1)$$

$$\vec{1}_1 = \vec{1}_r \equiv \text{direction of propagation to an observer at } \vec{r}$$

$$\overleftrightarrow{1}_t \equiv \overleftrightarrow{1} - \vec{1}_r\vec{1}_r \equiv \text{transverse dyad}$$

$$\overleftrightarrow{1} \equiv \vec{1}_x\vec{1}_x + \vec{1}_y\vec{1}_y + \vec{1}_z\vec{1}_z \equiv \text{identity dyad}$$

The far-field radiation is governed by an effective source vector

$$\vec{T}(t) = \frac{\partial}{\partial t}\int_V \vec{J}(\vec{r}\,',\ t + \frac{\vec{1}_r\cdot\vec{r}\,'}{c})\ dV' \qquad (6.2)$$

$$\vec{J}(\vec{r}\,',t) \equiv \text{current density in lightning}$$

Specializing this to a thin current path we have

$$\vec{T}(t) = \vec{1}_z \frac{\partial}{\partial t}\int I(z',\ t + \frac{z'\cos(\theta)}{c})\ dz' \qquad (6.3)$$

Considering only the current in the return stroke region for $0 < z < ct$ and in the direction $\theta = \pi/2$ (i.e., broadside) we have

$$\vec{T}(t) = \vec{1}_z T_1 \qquad (6.4)$$

$$T_1 = \frac{\partial}{\partial t}\int_0^{ct} I_1(z',t)\ dz'$$

Note that this does not consider the current in the leader. Furthermore, T_1 does not include any image or downward propagating return stroke; this is a separate factor.

Now from (5.2) we have

$$T_1 = -\zeta c^2 K \frac{\partial}{\partial t} \{t \int_0^{ct} dz'\} = -\zeta c^3 K \frac{\partial t^2}{\partial t}$$

$$= -2\zeta c^3 Kt = 2cI \qquad (6.5)$$

Here the velocity of the return stroke is c. Now note the factor of 2; this is due to the conical geometry which produces both a current proportional to t and the size of the region of integration proportional to t.

VII. Comparison of Radiated Fields for Various Kinds of Return-Stroke Currents

For normalization define an effective source (in scalar form) in the form

$$T = \eta c I \tag{7.1}$$

where η can be written as a product of factors to account for various phenomena as

$$\eta = \prod_{n=1}^{N} \eta_n \tag{7.2}$$

Let η_1 represent a geometry factor as

$$\eta_1 = \begin{cases} 1 \text{ for cylindrical geometry} \\ 2 \text{ for conical geometry (present case)} \end{cases} \tag{7.3}$$

where the case of cylindrical geometry corresponds to the traditional approximation of a constant return-stroke current propagating up the leader [6]. Which model is most appropriate depends on what times are being considered. While the return stroke starts out with conical geometry, by the time it has traveled tens of meters (or perhaps a little more) it encounters a more fully developed corona and the cylindrical geometry becomes more appropriate [7].

Another factor is the speed of propagation (normalized to c) of the return stroke. Designating this factor as η_2 we have

$$\eta_2 = \begin{cases} 1/3 \text{ for later times in return stroke (cylindrical model [7])} \\ 1 \text{ for early times in return stroke (conical model)} \end{cases} \tag{7.4}$$

It is the speed of c/3 that has been typically used [9], but this may only apply for later times.

Now a commonly used factor η_3 represents the doubling of the fields associated with the image of the return stroke below a ground plane (such as the earth's surface). This assumes that the electromagnetic field measurements are made at this surface. Thus we take

$$\eta_3 = 2 \tag{7.5}$$

The EM source (the return stroke) may or may not be initiated at the ground surface. While as in a "subsequent" return stroke the source may be at the ground surface (so that the "downward" propagating return stroke is actually an image) this is not necessarily the case [6]. For an initial return stroke, in particular, the downward propagating leader can be met by an upward propagating leader from the ground. The resulting return stroke has both upward and downward propagating parts (which are both imaged in the ground). Thus we have

$$\eta_4 = \begin{cases} 2 \text{ for "initial return strokes"} \\ 1 \text{ for "subsequent return strokes"} \end{cases} \tag{7.6}$$

Another factor [6] is the possibility of multiple channels simultaneously radiating. There is some evidence for this [2,5]. However, this may be an infrequent event. In any case let

$$\eta_5 \equiv \text{number of "simultaneously" radiating channels - an effective number, not necessarily an integer} \tag{7.7}$$

Considering only the above factors, let us look at extreme cases. Then

$$\eta = \begin{cases} 8\eta_5 & \text{(maximum)} \\ 2/3 & \text{(minimum)} \end{cases} \tag{7.8}$$

The ratio of maximum to minimum is $12\eta_5$ with η_5 some positive integer (but not very large). This is over an order of magnitude variation. The detailed form of any relation of current (or its time derivative) to far fields is quite variable which, without detailed knowledge of the spatial form of the return-stroke current, makes such a relationship difficult.

Other factors may also be discovered. For example, we have not considered the effect of the direction to the observer not being orthogonal to the direction of the current.

VIII. Summary

This paper has explored yet another model of the lightning return stroke, one applicable shortly following the initiation of the return stroke. This model increases the ratio of far field to current by a factor of 2. Combining this with other factors gives an order of magnitude variation of this ratio.

Before one can use far fields to infer lightning return-stroke currents (including time derivatives), it is necessary to establish the spatio-temporal form of these currents. Various experiments and mathematical models are needed to resolve this difficulty.

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