LIGHTNING PHENOMENOLOGY NOTES

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EFFECTS OF CHANNEL MODEL CHARACTERISTICS ON THE PROPAGATION OF LIGHTNING RADIATED ELECTROMAGNETIC FIELDS

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ABSTRACT

A model was developed in a previous paper to calculate electromagnetic fields radiated by a complex lightning return stroke model in a realistic earth-ionospheric environment. The previous model is exercised in this paper to show the effects of velocity of propagation of the current pulse, ground conductivity, and channel geometry on the electromagnetic field spectrum. Parameters affecting the channel geometry include the number of line segments used to construct the channel and the degree of departure from a single straight line model (tortuosity). A signal function, which to first order eliminates the effects of the currentpulse waveform was also introduced in the previous work. This signal function is used in this paper to determine a best fit to a signal function derived from measured data. A best fit, determined by trial and error, was found for the velocity of current pulse propagation, v, of $v = c/2.8$, ground conductivity in Florida of $\sigma_g = 4 \times 10^{-4}$ mhos/m and a three segment model departing from a vertical column by about 12% of the total length of the column.

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Introduction

The lightning return stroke is often modeled as a point source if propagation effects due to the earth-ionosphere waveguide are considered (Ref. 1) For a perfectly conducting earth with no ionosphere, return stroke models consisting of a straight vertical column (Refs. 2, 3), or a set of arbitrarily oriented straight line segments (Ref. 4, 5) have been used. In Reference 6, hereinafter referred to as Paper I, a method of computing electromagnetic fields was presented that combines the complex return stroke model of LeVine and Meneghini with a complex treatment of electromagnetic field propagation in the earth ionosphere waveguide. Effects on the radiated electromagnetic field of lightning due to earth conductivity, degree of channel tortuosity and velocity of propagation of the current pulse can be calculated with that model. A tortuous channel is one that deviates from a straight line path for the return stroke channel. This paper is a compendium of parameter studies that show the effect of finite ground conductivity, channel tortuosity, and velocity of propagation of the return stroke current pulse on the propagation of the radiated fields. The number of segments required to represent the return stroke geometry as well as the effect of maintaining components of the velocity parameter constant are calculated in this paper. Finally, these three variables are used to adjust a signal function $A(\omega)$, introduced in Paper I, for a best fit to a signal function deduced from measured data. The resulting predicted waveform is calculated by multiplying the signal function by the spectrum of the nearby measured waveform and inverse transforming the result for comparison with the distant measured waveform. The result is presented in the last section of this paper.

Motivation for a Detailed Source Description

Modeling the complete electromagnetic behavior of a lightning return stroke is a very complex process because of the many physical processes involved. In completing such a model one includes as much of relevant physics as possible. Models are limited by state of the art in model development and by our perception of the lightning event. The model presented in Reference 6 extends the state of the art in lightning model development. In this note, we develop some techniques of deriving figures of merit from the data which describe the lightning event. The sensitivity of the lightning spectrum to variations in some of the figures of merit is also described.

While this model (this paper and Ref. 6) is much more complex than previous models and describes physical processes not previously combined into a single model, this model is only a beginning at quantitatively describing the lightning return stroke.

The remainder of this paper is a compendium of various theoretically derived spectra, relating different parameters of the model. It should be noted, however, that the lightning spectral details shown here are the result of a few simple physical processes. By observing the details of the field expressions for a line source as shown in Reference 6 and for perfectly conducting ground in Reference 4, the emitted electromagnetic fields appear to be radiated from the ends of the line segment. For example, lightning spectra appear as a hump with a series of minima beginning at a few kilohertz and becoming more closely spaced at higher frequencies. The first minimum then is representative of a frequency for which the corresponding time constant ($\tau = 1/f$) is the time for the leading edge of the return stroke current pulse to travel from the bottom of the channel (actually about 100 m from the ground) to the top of the channel. This time is the quotient of the length of the channel and the average velocity of propagation of the return stroke current pulse. Additional details may be derived using higher order minima as is noted in later sections.

All of the fields in this paper are found using the methods of Reference 6. In no case is 1/r scaling used.

The azimuth of the multi-segment models is chosen such that there is equal x- and y-directed excursion. A real return stroke channel has many randomly oriented segments. Without considering complex local geometries these segments are randomly distributed in azimuth. Therefore, a simple model should use the azimuth for equal x- and y-directed excursion of the components of the channel model. This choice has been used throughout this paper.

The Ideal Current Element Model

The point current element is the simplest model one can use to determine lightning generated electromagnetic fields. In this model, the lightning channel is assumed to be located at a single point. A current element is a current times a length (Ref. 7). Physically, this model ignores the propagation of the current along the channel and the effects of varying height above the ground. Therefore the minima noted earlier do not appear. Below, we compare a current element with a three segment tortuous channel. A diagram of the geometry used for the tortuous channel is shown as an insert in Figure 1.

The treatment of the lightning return stroke current as an ideal current element over an imperfectly conducting ground can lead to errors in prediction of the electromagnetic fields. As an example of the possible errors the spectrum of the electric field for an ideal current element which has magnitude 1.5 km times the triple exponential current of Reference 4 is compared to that for a tortuous channel with the same current in Figure 1, but in the complex case, the current waveform propagates along the channel.

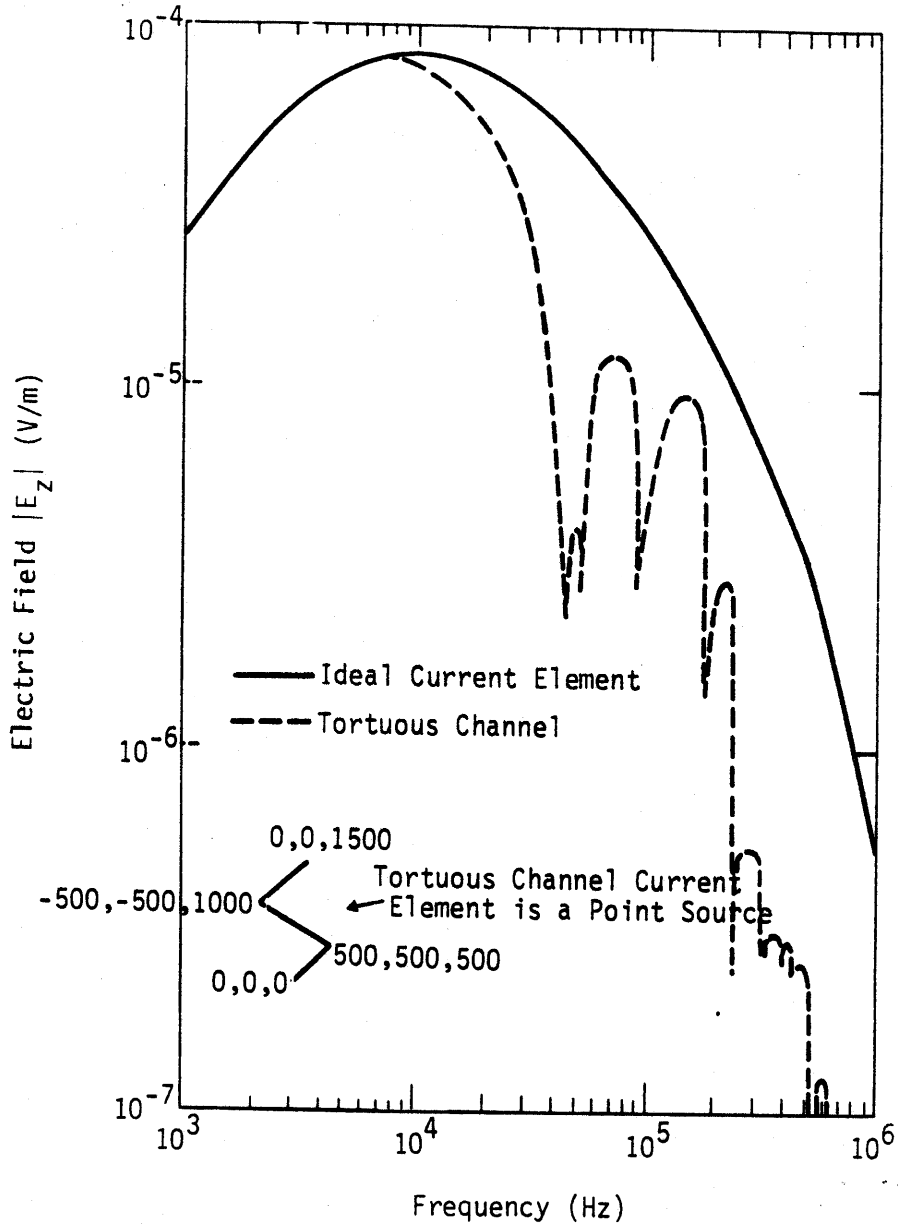

Figure 1. Spectra for ideal current element with triple exponential current compared with spectrum for three segment tortuous channel.

This triple exponential current is described by:

$$I(t) = [I_0(e^{-\alpha t} - e^{-\beta t}) + I_1 e^{-\gamma t}]\ U(t) \qquad (1)$$

where

$I_0 = 30$ kA

$I_1 = 2.5$ kA

$\alpha = 2 \times 10^4\ s^{-1}$

$\beta = 2 \times 10^5\ s^{-1}$

$\gamma = 1 \times 10^3\ s^{-1}$

Recent data analysis (Ref. 8) indicate that a faster rise time may be appropriate, but the methodology of this note remains the same.

This current waveform is used throughout this paper. The current element is located 750 m above the interface. For the tortuous channel the velocity parameter is $|\eta| = 1$, (where $\eta = c/v$ or $\eta = 1/\beta$, and v is the velocity of propagation of the return stroke current) but $\eta_z = 2.15$ indicating the degree of departure from the vertical column. The ideal current element has none of the nulls which are present in the finite length spectra. The nulls are due to radiation from various parts of the line segment interfering with radiation from other parts. Since the radiation can be pictured as coming from the end points (Refs. 4, 9) the nulls can be thought of as coming from interference of fields from the segment ends. The nulls are also compressed due to the decrease in the vertical component of the current pulse propagation velocity. The high frequency "tweets", caused by the horizontal extent of the tortuous channel model, are presented just below 1 MHz. Both sources are 100 km from the observer over ground which has conductivity $\sigma = 8 \times 10^{-3}$ and permittivity $\varepsilon = 12\ \varepsilon_0$.

The Velocity of Propagation of the Current Pulse

Another figure of merit important for determining electromagnetic fields from a lightning return stroke is the velocity of propagation of the current pulse. In antenna theory, we are accustomed to the current waveforms traversing the antenna at the same velocity as the fields, that of light. In the case of lightning the current propagates in a plasma and moves more slowly than that of light. A typical velocity might be about 1/3 the speed of light, or $\eta = 3$. In this section we will investigate the effects of varying the velocity of propagation of the current pulse on the emitted spectrum.

The nulls in the spectra occur at lower frequency for larger velocity parameters as shown in Figure 2. This phenomenon is a compression of the spectral details to lower frequencies. The model used for this calculation is the perfectly conducting ground model of Reference 4. This model was used, instead of the Paper I model, to keep the number of variables complicating the spectrum to a minimum. The perfectly conducting earth was used only for Figure 2. The return stroke channel is a 1.5 km vertical column. The observer is 100 km from the source along the x-axis. The spectral compression is quite severe and is apparent. This sensitivity will be used later to determine an estimate for that velocity from data.

In an actual return stroke, however, the velocity of propagation of the current pulse is not constant (Refs. 10-13). An undistorted waveform and constant velocity parameter has been assumed in this paper and Paper I. The sensitivity of the spectrum to variations of propagation velocity as a function of height is shown in Figure 3. The return stsroke model is again the 1.5 km vertical channel but in this case is divided into 5 vertical segments of 300 m in length. The velocity on each segment is chosen to be that at the center of the column for a linear velocity profile which as velocity parameter $\eta = 4$ at the bottom and $\eta = 10$ at the top. This

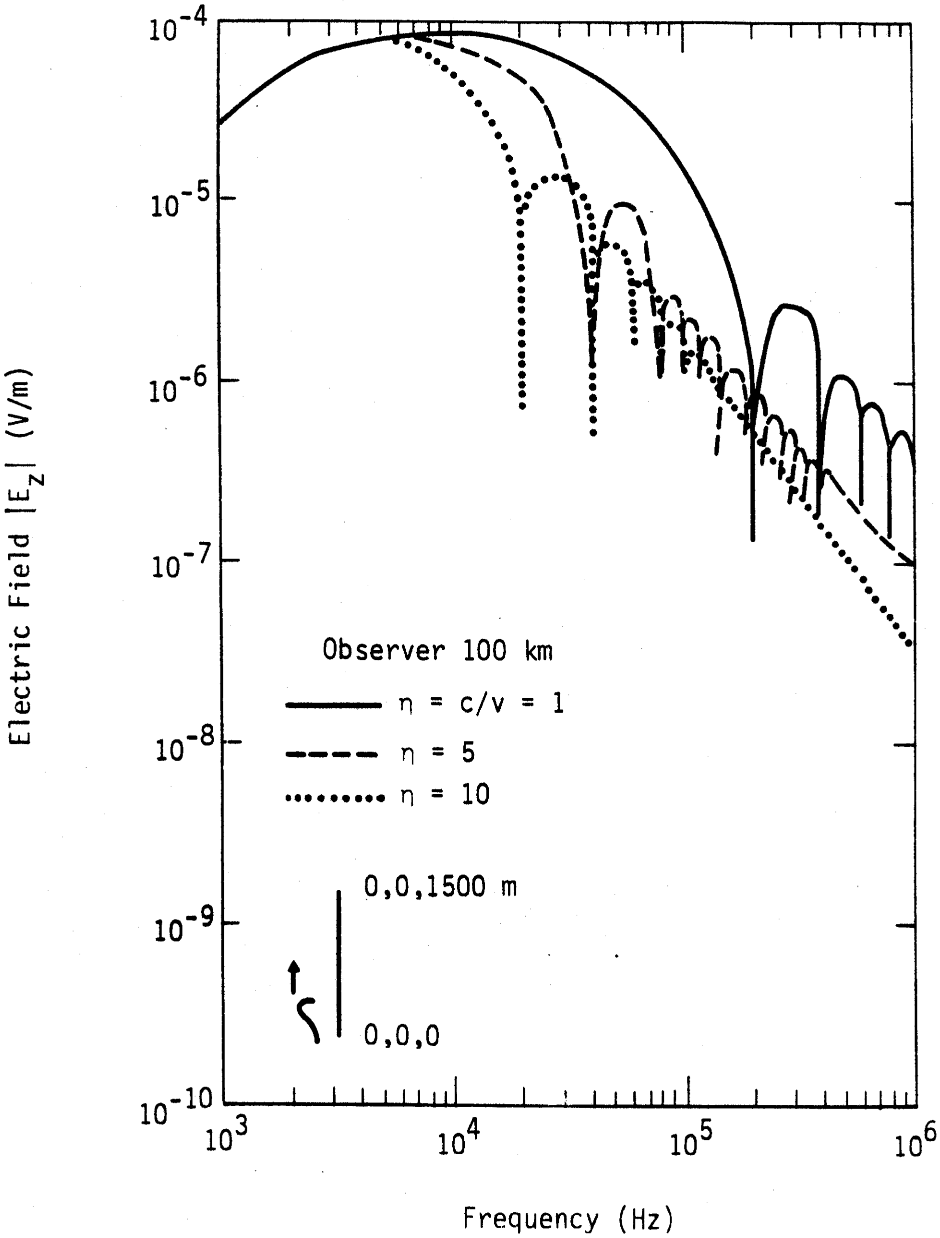

Figure 2. Electric field spectra for 1.5 km vertical column for various velocities of propagation of the current pulse.

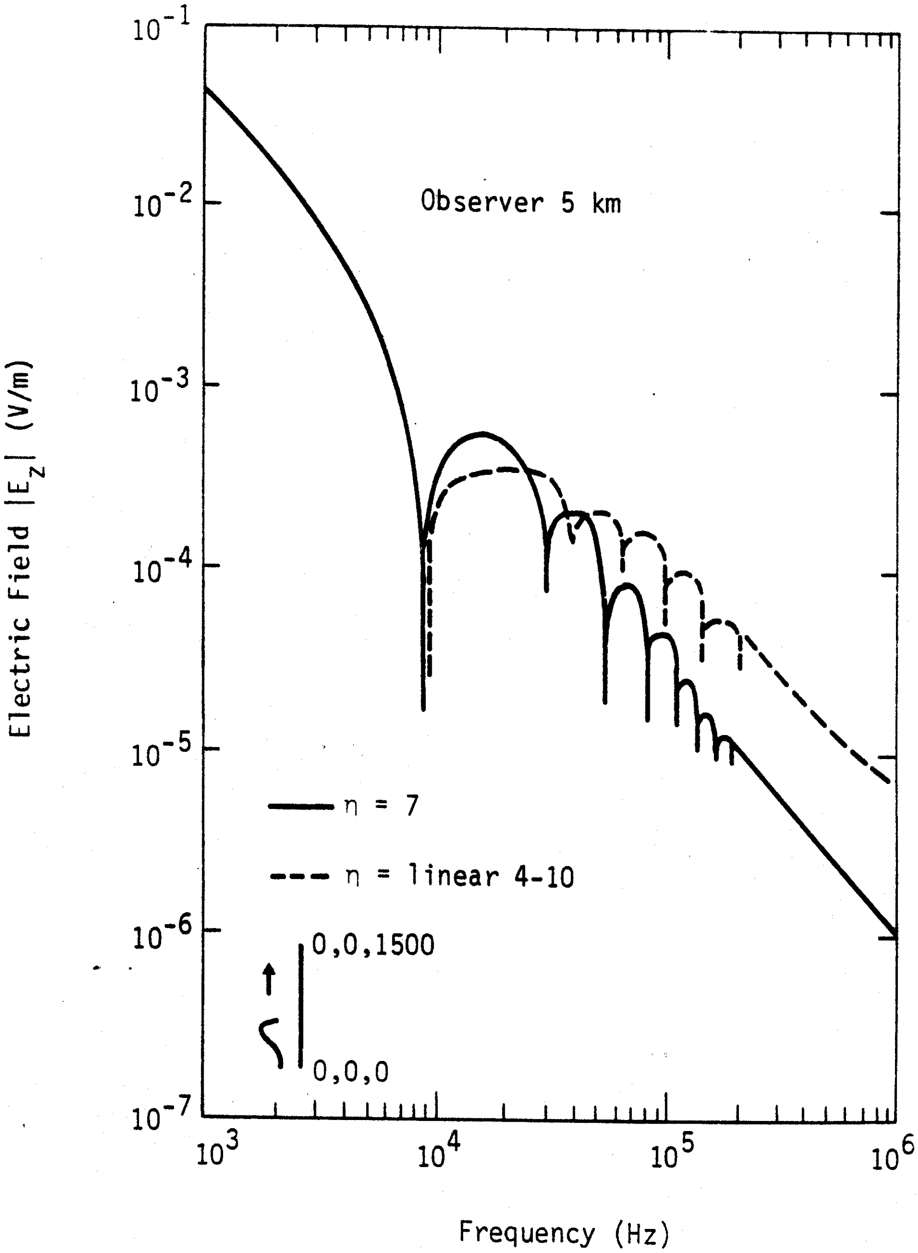

Figure 3. Electric field spectra for a 1.5 km vertical column showing the effect on the spectrum of a z-dependent velocity parameter.

spectrum is compared to a single segment output which has velocity parameter $\eta = 7$. These values were chosen to represent a range of the measured values presented in Reference 12. Since the error bars on the velocity measurements are fairly large (40% to 60% on the Boyle and Orville measurements) the linear variation in height is an adequate representation of the data, even though Reference 10 conclude an exponential fit is appropriate. There are clearly differences at high frequencies, demonstrating difficulty in choosing a single number to represent the velocity of propagation of the return stroke current pulse.

The vertical component of the velocity is also an important parameter since a vertical elemental current near a conducting interface is much more efficient at transmitting electromagnetic waves than a horizontal one for heights of the element above the interface much less than a wavelength. It might be expected then that for certain frequencies a tortuous channel spectrum might be matched by a vertical channel for which the proper propagation velocity has been chosen. The results of this experiment are shown in Figure 4. Choosing that velocity so that the transit time from bottom to top of the channel results in a matching of spectra up to about 100 kHz. Above that frequency the higher reflection efficiency of the horizontally polarized waves becomes evident. The vertical model with the geometrically chosen velocity parameter matches the propagation characteristics of most of the transmitted energy but not the energy transmitted by the horizontal excursion.

Boyle and Orville (Ref. 12) suggest using the Uman, et al., (Ref. 14) conclusion (derived in Ref. 15) that the electric field waveform follows the channel current waveform until the current reaches the top of the channel to determine the velocity parameter. The conclusion assumes that only the radiated (as opposed to inductive or quasistatic) vertical component of the electric field is important. Note that the assumed relation between channel current waveform is valid only for straight vertical channel segments. That is, when a bend in the channel is reached by the leading edge of the waveform the relation is no longer valid. Under these conditions

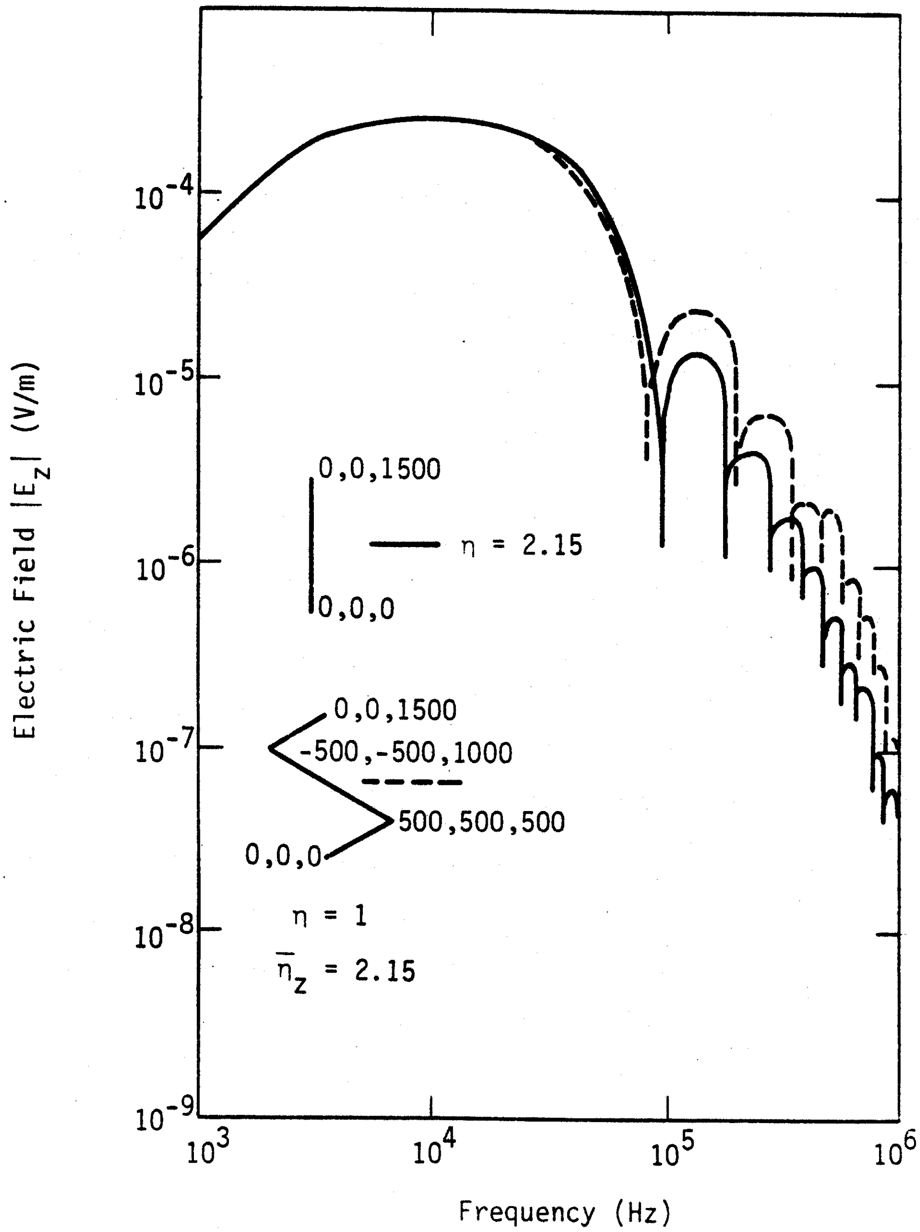

Figure 4. Electric field spectrum for a 1.5 km vertical column with velocity parameter corrected so that the transit time of the leading edge of the pulse is the same as that for the tortuous channel whose spectrum also appears.

the vertical component of the velocity parameter is the value that would likely be measured. At very early times (frequencies above 1 MHz) the horizontal component radiation becomes important and complicates the current from the field derivation of Reference 15. The ground wave attenuation is also most severe where this simple inverse current derivation is most accurate.

Effect of Channel Tortuosity on the Fields

Generally, in antenna theory, the far field pattern of the antenna is only weakly dependent on the detailed geometry of the antenna element for details small compared to a wavelength. Therefore, it may be expected that the number of segments required for a return stroke model might be small if certain quantities remain the same for different source geometries and currents. A few figures of merit are determined here. The idea is to pick figures of merit that if held constant, will maintain the same emitted spectrum for the return stroke, while varying the figures of merit will result in predictable changes in the spectrum. The ratio of the vertical excursion to horizontal excursion of the channel path must be maintained so that the two source polarizations, which propagate differently, maintain the same ratio. The peak current and total length of the channel cannot readily be determined from electromagnetic field values without also obtaining photographic data since the electromagnetic signals respond to the lightning stroke current element. The total vertical excursion was arbitrarily set at a 1.5 km for the example curves. As noted earlier both the velocity of propagation and the vertical component of the velocity of propagation need to be maintained so the positions of the nulls will remain the same. The velocities control when the source current causes radiation from the source, not where, and are thus important at all distances. Fortunately, these last two parameters remain constant, to first order, if the horizontal and vertical excursion is constant. The results of 2 return stroke models of 3 and 5 segments each maintaining the quantities described above are shown in Figure 5 for observer distance 5 km. For 5 km, 5

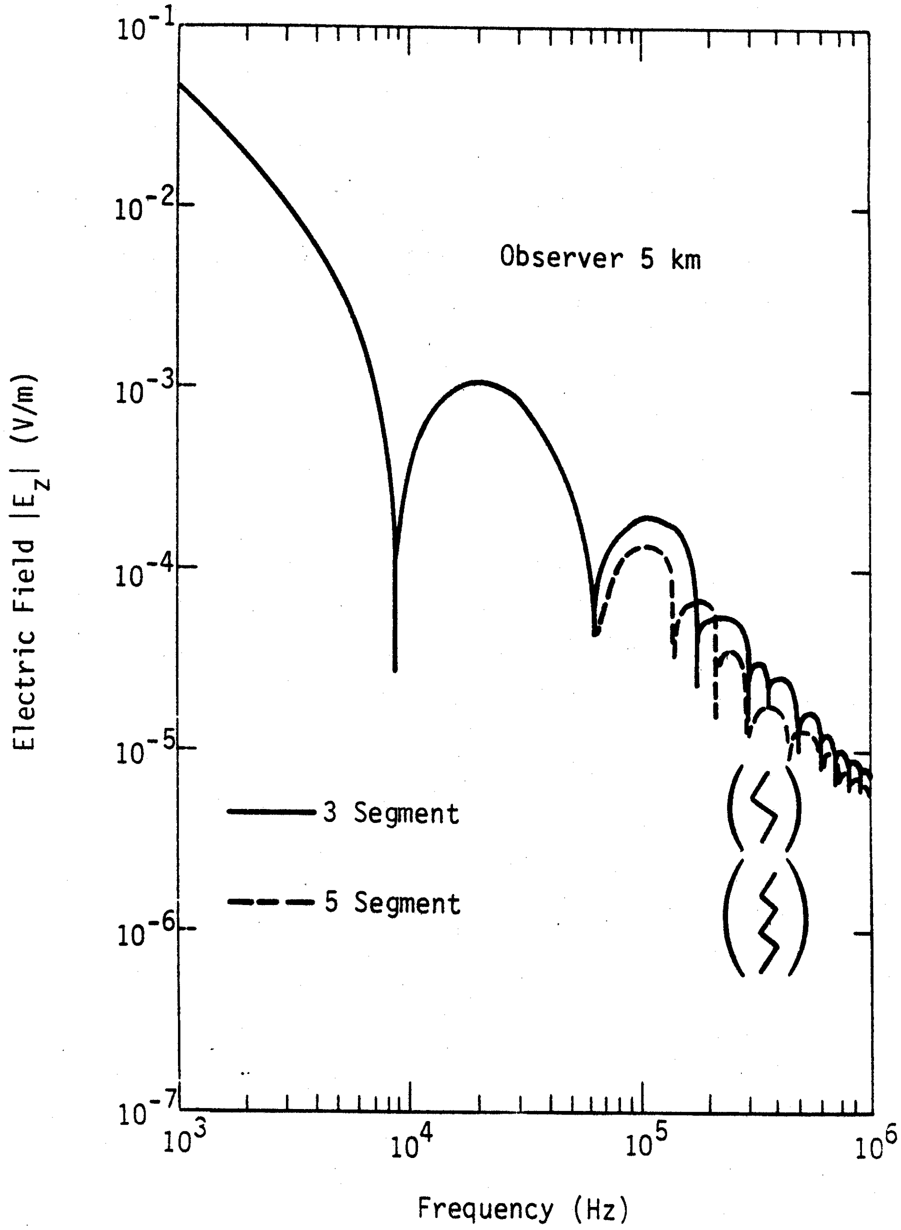

Figure 5. Electric field spectrum for 3 segment model compared to that for a 5 segment model. Horizontal and vertical extent of the two models is the same. The velocity parameter and its components are also conserved.

segments would be desirable but 3 is adequate since the two curves agree so well. As distance increases the 3 segment model becomes more representative. For limited frequencies and observation distances, the models which contain hundreds of segments are not necessary.

Another tortuous case for a cloud-to-cloud discharge is shown in Figure 6. In this case both a tortuous channel along with the equivalent, purely vertical and purely horizontal model spectra are shown. Neither is adequate alone to provide the correct spectrum.

An obvious anomaly in this set of curves is that the vertical component above is a stronger source than the three element case of which it is a part. The three element case even approaches the horizontal single element model at low frequencies. The reason for this behavior is that the vertical excursion of the three-element case is symmetric about the $z = 4$ km elevation. For wavelengths much longer than the vertical excursion the radiation by the vertical components of the three element case cancel. This result is further evidence that one should not replace a complex path with a single vertical current element without due consideration to the phase of the output of each element.

Conductivity

Usually, one does not know the average conductivity along a propagation path without actually propagating signals along the entire path. The local conductivity may be measured at many points but variations in ground water, the existence of lakes and shorelines, and various man-made and natural geometric variations complicate the path description. The sensitivity of a lightning return stroke transmission to a change of ground conductivity of an order of magnitude is shown in Figure 7. The general effect of decreasing the conductivity is to cause an increase in signal attenuation. Unfortunately, there is also a spectral compression, as described earlier in the description of Figure 2.

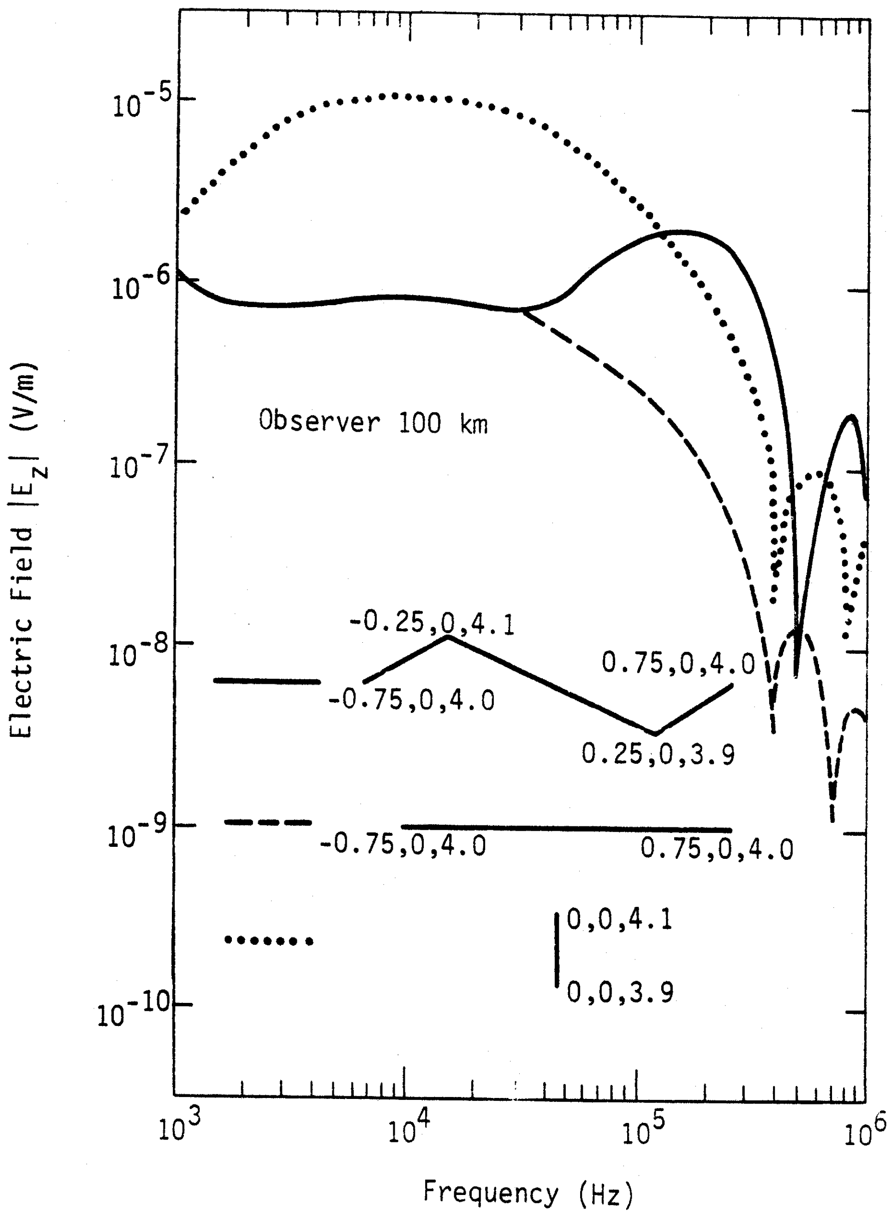

Figure 6. Electric field spectra for three models of a cloud-to-cloud stroke. Tortuous (3 segment) model is shown. Attempts to represent it as a single horizontal and single vertical segment are also shown.

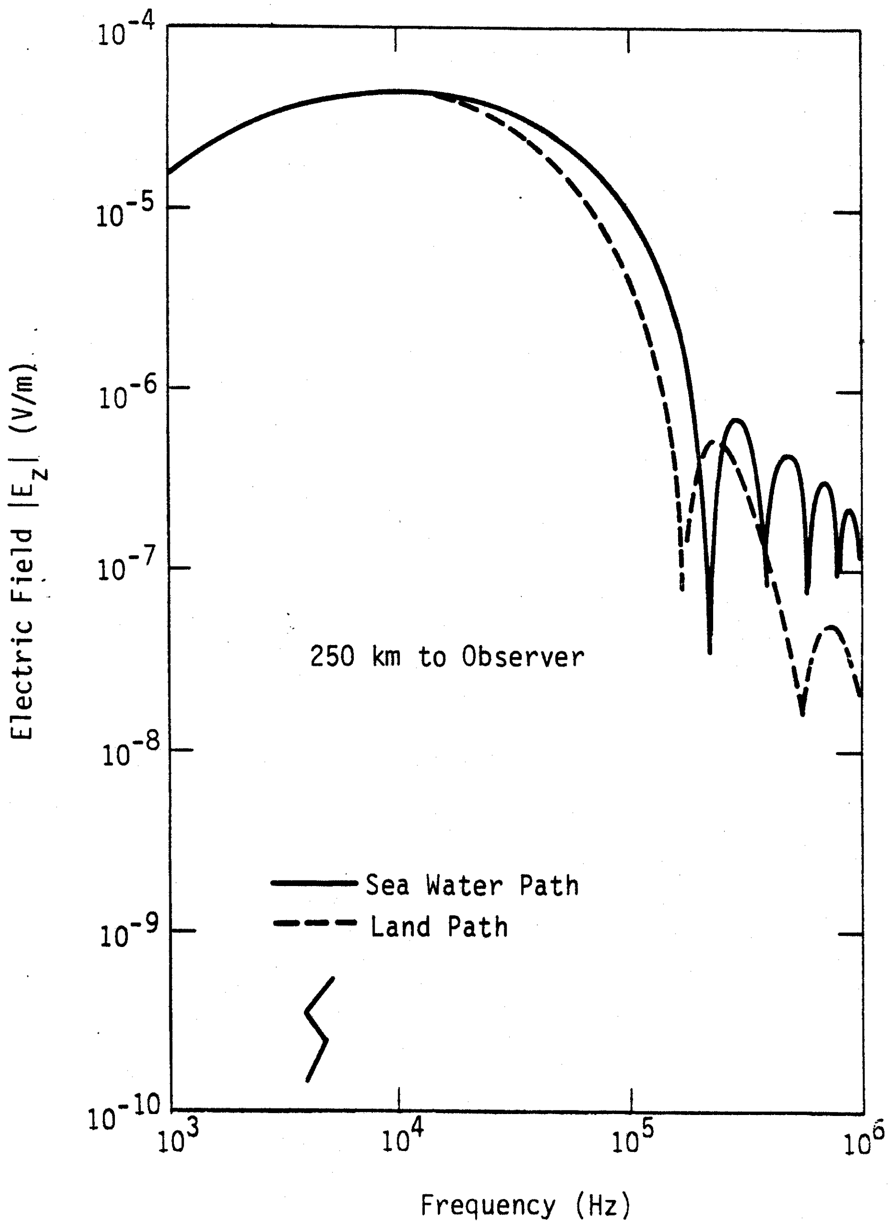

Figure 7. Effect on electric field spectrum of a large change in the ground constituency of the propagation path. Field also contains contribution from reflection from the ionosphere.

The Signal Function

The signal function, $A(\omega)$, is defined as

$$A(\omega) = \frac{|E_z(r_f,\omega)|}{|E_z(r_n,\omega)|} \frac{|\vec{r}_f|}{|\vec{r}_n|} \tag{2}$$

where $\vec{r}_n$ and $\vec{r}_f$ are near and far observation points, respectively. The signal function separates the 1/r geometric attenuation and explicit current effects from the field values. This function has the advantage of decreasing the sensitivity of the transmitted waveforms to some input parameters but not others. If it is assumed, as it is in the theoretical model in Paper I, that the waveform propagates up the channel without distortion then the division of the fields forming the signal function the current pulse divides out entirely. In addition, the scale factor describing the magnitude of the current element is also eliminated from the signal function.

Investigation of the sensitivity of the signal function to various input parameters resulted in the fitting of a signal function to a set of data. The data used are two simultaneous two-station waveforms from a lightning stroke propagating from an over ocean discharge point to Kennedy Space Flight Center as the first observation point to the University of Florida as the second observation point. The data was taken from Y. T. Lin's thesis (Ref. 16) and was presented in the last part of Paper I. The numerical transform is inaccurate at low frequencies because of the truncation of the waveform for $t > 150$ μs. The short propagation path is assumed to be over water. The conductivity of the second path alone is varied by changing the horizontal excursion. Finally, the velocity of propagation of the current pulse is varied along the channel model. No attempt is made to treat the apparent velocity changes caused by changes in the channel geometry. The fitting method illustrated here is present to show a fitting method rather than absolutely determining the specific parameter values.

Obtaining precise values of the parameters by the method outlined below from the data will have to await better data access. These waveforms were digitized from small figures in Reference 16. Digitization error as well as poor geometry definition of the channel limit the applicability of the results.

The first example calculation is shown in Figure 8. The return stroke model is a single vertical column with velocity parameter $\eta = 1$. The signal function derived from the data is shown as the dashed line. A signal function derived from field predictions from the model described in Paper I is denoted by the solid curve. The large central peak in the theoretical signal function is much larger than anything in the corresponding signal function derived from the data. It is also not co-located with any of the smaller peaks in the signal function derived from the data. In Figure 9 the result of adding 282 meters of horizontal excursion to the geometry used for the theoretical calculation is shown. The horizontal excursion is modeled using a three segment symmetric channel. The height of the peak is much smaller, but the location of the peak and the overall height of the theoretical curve is still incorrect. In Figure 10 the velocity parameter has been changed to $\eta = 2.8$, and the ground conductivity of the long path changed to $\sigma = 4 \times 10^{-4}$. The increased velocity parameter shifts the peak to a lower frequency as it is in the signal function derived from the data. The minima in the predicted signal function above about 200 kHz do not exist in the curve derived from the data, however. This omission may either be due to non-ideal terrain where the data was taken or due to limits in the data reduction performed for this thesis. Predicted and measured waveforms using the parameters of Figure 10 are shown in Figure 11. The rise time is approximately correct for the predicted waveform but there are numerous oscillations in the measured waveform that are not predicted. This deviation is to be expected because of the high frequency differences in Figure 11.

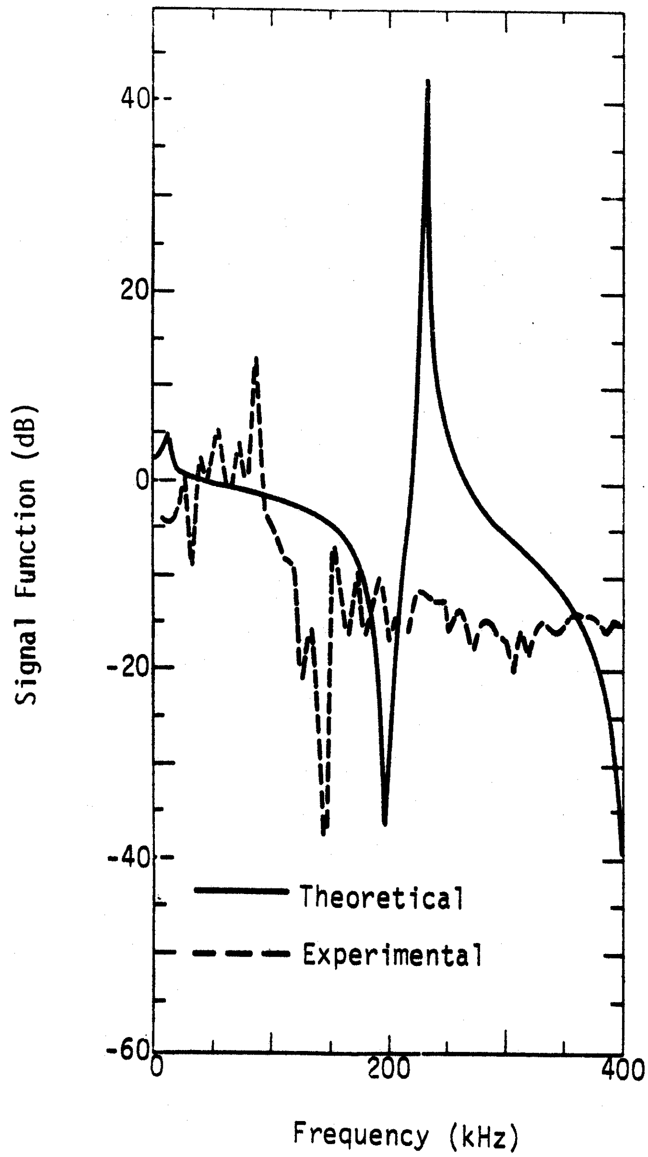

Figure 8. The signal function calculated from transient waveforms of Reference 16 are compared to the signal function predicted from the model in Paper I for velocity parameter $\eta = 1$, ground conductivity $\sigma = 8 \times 10^{-3}$ mho/m and horizontal excursion 0 m.

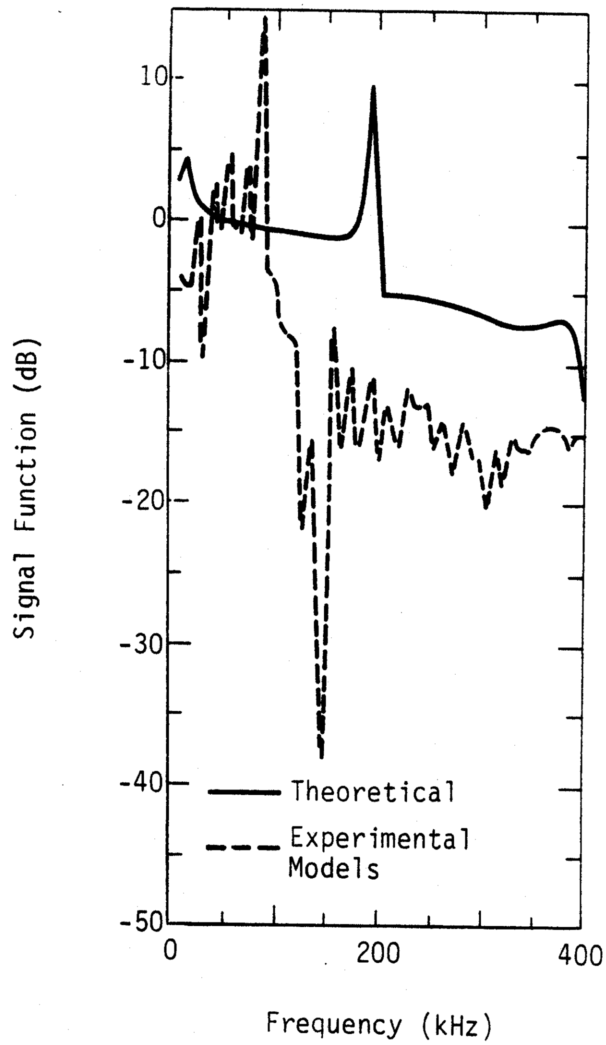

Figure 9. The signal function calculated from transient waveforms of Reference 16 are compared to the signal function predicted from the model in Paper I for velocity parameter $\eta = 1$, ground conductivity $\sigma = 8 \times 10^{-3}$ mho/m and horizontal excursion 282 m.

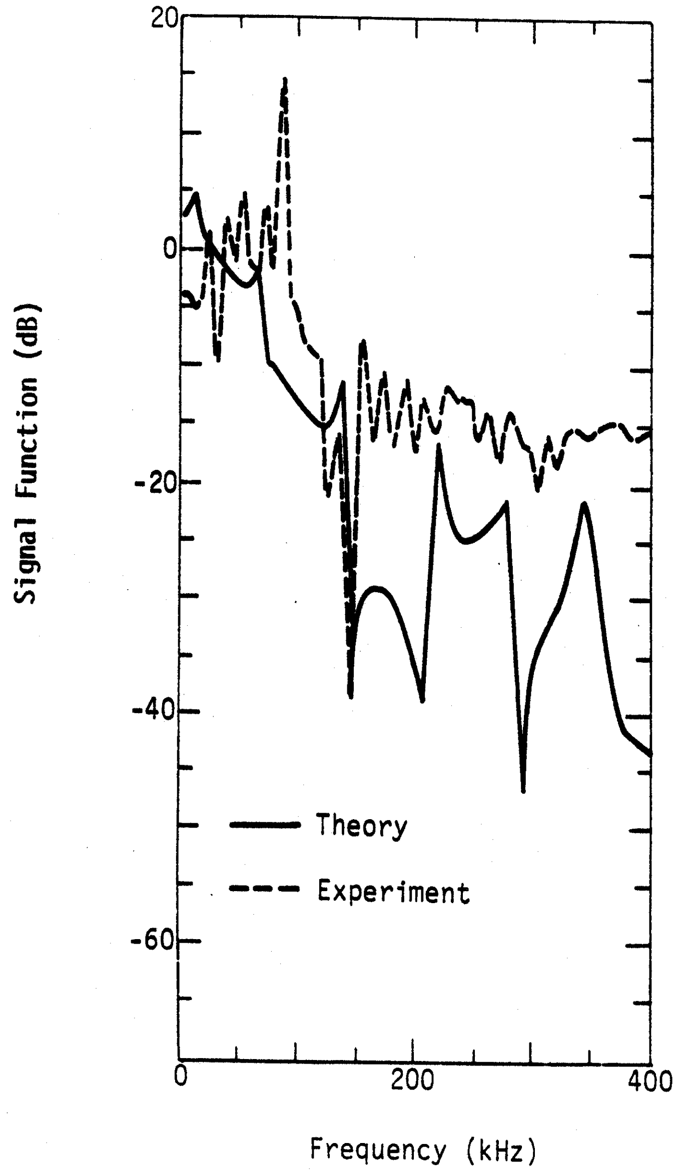

Figure 10. The signal function calculated from transient waveforms of Reference 16 are compared to the signal function predicted from the model in Paper I for velocity parameter $\eta = 2.82$, ground conductivity $\sigma = 4 \times 10^{-4}$ mho/m and horizontal excursion 282 m.

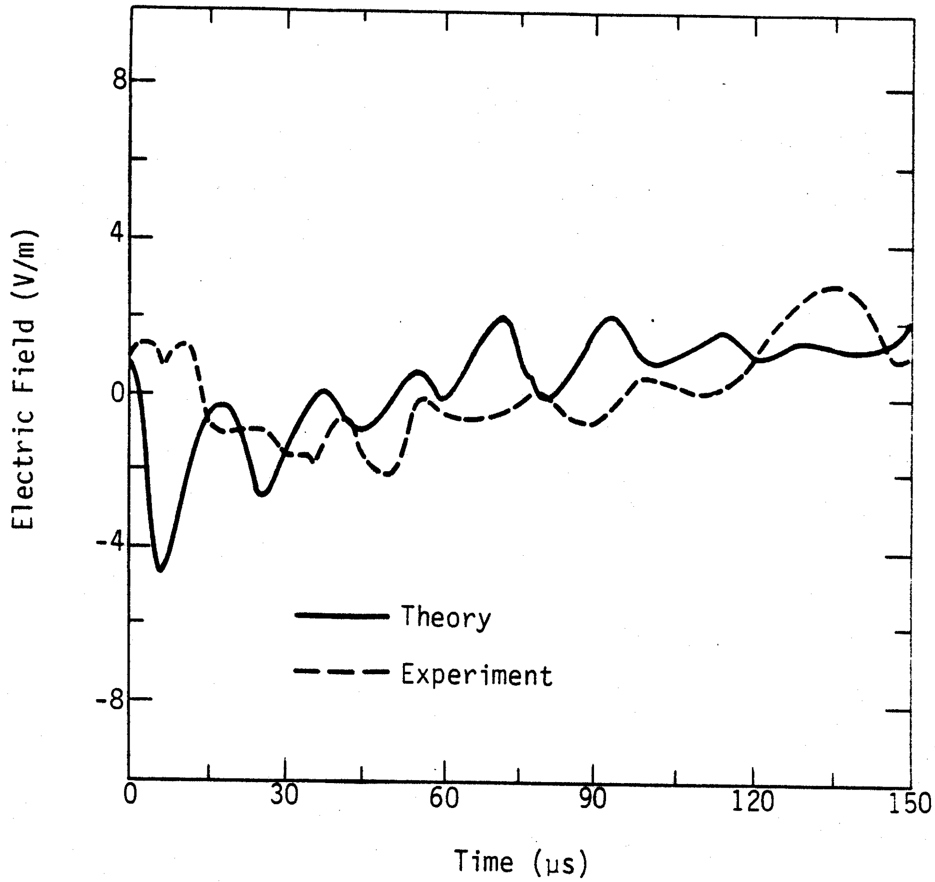

Figure 11. The 200 km transient waveform from Reference 16 compared to a predicted waveform by multiplying the 5.2 km transform from Reference 16 by the signal function calculated with this model and shown in Figure 10 and performing the inverse transform.

Conclusions

Each of the four early sections provided specific examples of the sensitivity of the spectrum radiated by a lightning return stroke to one of the parameters describing the details of the model: (1) Finite size of the channel, (2) Velocity of propagation of the current pulse, (3) Channel tortuosity, and (4) Ground conductivity. Variation of each of the parameters produced a pronounced effect on the predicted spectrum. Therefore, at least, those four parameters should be used in a model of the lightning return stroke. A model fitting that description along with a signal function which minimizes the effect of the lightning current on the resulting fields are used to determine a best fit to a signal function from data.

A best fit for the signal function was obtained for horizontal excursion of 282 m, vertical height 1500 m, a velocity parameter $\eta = 2.8$, and long path ground conductivity $\sigma_g = 4 \times 10^{-4}$. This velocity agrees with the limits set by data analysis in Reference 17 for leader propagation. The calculations here are entirely for return stroke. No attempt was made to calculate the effect of the shoreline, velocity parameter non-uniformity, or waveform distortion along the channel (Ref. 13). These uncertainties remain to be considered. Photographic data which quantifies the entire three dimensional geometry of the source would allow more information to be gathered about the velocity or velocity profile.

Because of the data sensitivity to these various parameters a model of the electromagnetic radiation of a lightning return stroke requires at least degrees of freedom describing some tortuosity, a velocity parameter (preferably height variable), and a finitely conducting ground. The interaction with the ionosphere is needed for times longer than about 500 μsec. Models of the lightning return strokes which are ideal current elements or models over perfectly conducting ground just do not contain sufficient physics to adequately describe the observed phenomena.

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